

Suppose a linear program has i th matrix constraint

$$\begin{aligned}\mathbf{a}_i \mathbf{x} &\leq b_i \text{ or} \\ \mathbf{a}_i \mathbf{x} &\geq b_i \text{ or} \\ \mathbf{a}_i \mathbf{x} &= b_i\end{aligned}$$

and j th variable constraint

$$\begin{aligned}x_j &\leq 0 \text{ or} \\ x_j &\geq 0 \text{ or} \\ x_j &\text{ free (unrestricted).}\end{aligned}$$

Let \mathbf{A} be the matrix with i th row \mathbf{a}_i , and let \mathbf{A}_j be the j th column of \mathbf{A} . Then

If the LP is a **minimization** problem, then

$$\begin{array}{llll} \text{the matrix constraint} & \mathbf{a}_i \mathbf{x} \geq b_i & \text{dualizes to} & \text{the variable constraint} & y_i \geq 0 \\ & \mathbf{a}_i \mathbf{x} \leq b_i & & & y_i \leq 0 \\ & \mathbf{a}_i \mathbf{x} = b_i & & & y_i \text{ free} \end{array}$$

$$\begin{array}{llll} \text{the variable constraint} & x_j \geq 0 & \text{dualizes to} & \text{the matrix constraint} & \mathbf{y}^T \mathbf{A}_j \leq c_j \\ & x_j \leq 0 & & & \mathbf{y}^T \mathbf{A}_j \geq c_j \\ & x_j \text{ free} & & & \mathbf{y}^T \mathbf{A}_j = c_j \end{array}$$

If the LP is a **maximization** problem, then

$$\begin{array}{llll} \text{the matrix constraint} & \mathbf{a}_i \mathbf{x} \geq b_i & \text{dualizes to} & \text{the variable constraint} & y_i \leq 0 \\ & \mathbf{a}_i \mathbf{x} \leq b_i & & & y_i \geq 0 \\ & \mathbf{a}_i \mathbf{x} = b_i & & & y_i \text{ free} \end{array}$$

$$\begin{array}{llll} \text{the variable constraint} & x_j \geq 0 & \text{dualizes to} & \text{the matrix constraint} & \mathbf{y}^T \mathbf{A}_j \geq c_j \\ & x_j \leq 0 & & & \mathbf{y}^T \mathbf{A}_j \leq c_j \\ & x_j \text{ free} & & & \mathbf{y}^T \mathbf{A}_j = c_j \end{array}$$