Technische Universität Dresden • Fakultät Informatik

Data Preperation for PMC-Visualization

Bachelorarbeit zur Erlangung des ersten Hochschulgrades

Bachelor of Science (B.Sc.)

vorgelegt von

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Contents

Abstract

Lorem ipsum

1 Introduction

2 Preliminaries

2.1 Mathematical Fundamentals

e.g.strongly connected components, equivalence relation, more?

we denote $[a]_R$ for the equivalence class with the representative a under the equivalence relation R

how much should be included? ...probably no set theory

2.2 Transition Systems

Motivation of transition systems

The following definition is directly taken form Principles of Modelchecking, Baier p. 20

Definition 2.1. A *transition system* TS is a tuple $(S, Act, \longrightarrow, I, AP, L)$ where

- S is a set of states,
- Act is a set of actions,
- $\longrightarrow \subseteq S \times Act \times S$ is transition relation,
- $I \subseteq S$ is a set of initial states,
- AP is a set of atomic propositions, and
- $L : S \rightarrow \mathcal{P}(AP)$

A transition system is called *finite* if S, AP and L are finite.

explanation of components

2.3 Markov Chain

NOTES BEGIN

- Markov Chain (MC)
- transition systems to markov chains: nondeterministic choices replaced by probablistic
- successor chosen according to probability distribution
- distribution only dependent on current state s (not path)
- system evolution not dependent on history but only current state $\rightarrow$ *memory-less property*

NOTES END

Definition 2.2. A *(discrete-time) Markov chain* is a tuple $\mathcal{M} = (S, \mathbf{P}, l_{init}, AP, L)$ where

- S is a countable, nonempty set of states,
- $\mathbf{P} : S \times S \to [0,1]$ is the *transition probability function*, such that for all states s:

$$\sum_{s' \in S} \mathbf{P}(s, S') = 1.$$

- $l_{init} : S \to [0,1]$ is the *initial distribution*, such that $\sum_{s \in S} l_{init}(s) = 1$, and
- AP is a set of atomic propositions and,
- $L : S \to \mathcal{P}(AP)$ a labeling function.

$\mathcal{M}$ is called *finite* if S and AP are finite. For finite $\mathcal{M}$, the *size* of $\mathcal{M}$, denoted $size(\mathcal{M})$, is the number of states plus the number of pairs $(s, s') \in S \times S$ with $\mathbf{P}(s, s') > 0$.

NOTES BEGIN

- Probability Function **P** specifies for each state s the probability **P** (s,s') of moving from s to s' in one step.
- constraint on **P** ensures that **P** is distribution
- $l_{init}(s)$ specifies system evolution starts in s
- states s with $l_{init}(s) > 0$ are considered *initial states*
- states s' with $\mathbf{P}(s, s') > 0$ are view as possible successors of s
- has no actions
  "As compositional approaches for Markov models are outside the scope of this monograph, actions are irrelevant in this chapter and are therefore omitted."

NOTES END

2.4 Markov Decision Process

NOTES BEGIN

- Markov decision process (MDP)
- idea: Adding nondeterminism to markov chains. MDPs permit both probabilistic and nondeterministic choices
- probabilistic choices: possible outcomes for of randomized actions -¿ requires statistical experiments to obtain adequate distributions that model average behavior of the environment

- information not available or guarantee about system properties is required -¿ nondeterminism
- Another example: randomized distributed algorithms. Non-determinism: interleaving behavior: nondeterministic choice which process, probabilistic: have rather restricted set of actions that have a random nature
- used for abstraction in markov chains: states grouped by AP and have a wide range of transition probabilities -¿ essentially nondeterminism -¿ transition probabilities are replaced by nondeterminism

NOTES END

Definition 2.3. A *Markov decision process* is a tuple $\mathcal{M} = (S, Act, \mathbf{P}, l_{init}, AP, L)$ where

- S is a countable set of states,
- Act is a set of actions,
- $\mathbf{P} : S \times Act \times S \to [0, 1]$ is the transition probability function such that for all states $s \in S$ and actions $\alpha \in Act$:

$$\sum_{s' \in S} \mathbf{P}(s, \alpha, s') \in \{0, 1\},$$

- $l_{init} : S \to [0, 1]$ is the initial distribution such that $\sum_{s \in S} l_{init}(s) = 1$,
- AP is a set of atomic propositions and
- $L : S \to \mathcal{P}(AP)$ a labeling function.

An action α is *enabled* in state s if and only if $\sum_{s' \in S} \mathbf{P}(s, \alpha, s') = 1$. Let $Act(s)$ denote the set of enabled actions in s. For any state $s \in S$, it is required that $Act(s) \neq \emptyset$. Each state s' for which $\mathbf{P}(s, \alpha, s') > 0$ is called an *α-successor* of s.

An MDP is called *finite* if S, Act and AP are finite.

NOTES BEGIN

- $\mathbf{P}(s, \alpha, t)$ can be arbitrary real numbers in $[0, 1]$ (sum up to 1 or 0 for fixed s and α), for algorithmic purposes rational
- unique initial distribution l_{init}. Could be generalized to set of $l_{init}$with nondeterministic choice at the beginning. For sake of simplicity: one single distribution
- operational behavior:
  - starting state s_0 yielded by $l_{init}$with $l_{init}(s_0) > 0$

  - nondeterministic choice of enabled action (i.e. Probability sums up to one)
  - probabilistic choice of state (action fixed by nondeterministic selection)

- $MC = MDP \iff \forall s \in S : |Act(s)| = 1$
- $\implies$ MCs are a proper subset of MDPs

NOTES END

3 View

Views are the central objective of this thesis. The purpose of a view is to obtain a simplification of a given transition system (TS). It is an independent TS derived from a given TS and represents a (simplified) view on the given one - hence the name. Thereby the original TS is retained.

3.1 Grouping Function

The conceptional idea of a view is to group states by some criteria and structure the rest of the system accordingly. To formalize the grouping we define a dedicated function.

Definition 3.1. Let TS $= (S, Act, \longrightarrow, I, AP, L)$ be a transition system an M be an arbitrary set. We call any function $F : S \to M$ a *grouping function*. switched to M instead of $\mathbb{N}$

Two states are grouped (should be Definition?) to a new state if and only if the grouping function maps them to the same value. The definition offers an easy way of defining groups of states and labels them with a natural number. It is also very close to the actual implementation later on. The exact mapping depends on the desired grouping. In order to define a new set of states for the view, we define an equivalence relation R based on a given grouping function F.

Definition 3.2. Let F be a grouping function. We define the equivalence relation $R := \{(s_1, s_2) \in S \times S \mid F(s_1) = F(s_2)\}$

R is an equivalence relation because the equality relation is one. The property directly conveys to R. We observe that two states s_1, s_2 are grouped to a new state if and only if $(s_1, s_2) \in R$. This is the case if and only if $s_1, s_2 \in [s_1]_R = [s_2]_R$ where $[s_i]_R$ for $i \in \{1, 2\}$ denotes the equivalence class of R.

3.2 Formal Definition

The definition of a view is dependent on a given transition system and a grouping function F. We derive the equivalence relation R as in Definition 3.2 and use its equivalence classes $[s]_R$ ($s \in S$) as states for the view. The rest of the transition system is structured accordingly.

Definition 3.3. Let TS $= (S, Act, \longrightarrow, I, AP, L)$ be a transition system and F a grouping function. A *view* TS_F is a transition system $(S', Act', \longrightarrow', I', L')$ that is derived from TS with the grouping function F where

- $S' = \{[s]_R \mid s \in S\}$
- $Act' = Act$
- $\longrightarrow' = \{([s_1]_R, \alpha, [s_2]_R) \mid \exists s_1 \in [s_1]_R \exists s_2 \in [s_2]_R : ([s_1]_R, \alpha, [s_2]_R) \in \longrightarrow\}$
- $I' = \{[s']_R \in S' \mid \exists s \in [s']_R : s \in I\}$

- $L' : S' \to \mathcal{P}(AP), [s]_R \mapsto \bigcup_{s \in [s]_R} \{L(s)\}$

and R is the equivalence relation according to Definition 3.2.

Note that the definition is in a most general form in the sense that if in a view a property accounts to one part of some entity the whole entity receives the property i.e.

- $(s_1, \alpha, s_2) \in \longrightarrow \;\Rightarrow\; ([s_1]_R, \alpha, [s_2]_R) \in \longrightarrow'$
- $s \in I \;\Rightarrow\; [s]_R \in I'$
- $\forall s \in S : L(s) \in L'([s]_R)$

3.3 Composition of Views

In essence views are simplification generated from a transitionsystem. It seems rather obvious that the composition of views is a very practical feature. Therefore in this chapter we will introduce, formalize and discuss different notions of compositions. All variants will ensure that the effect caused by each partaking views also takes effect in the composed view. Moreover it is to be generated in a way that the effect of each individual view can be reverted from the composed one. There may be restrictions in the order of removal.

3.3.1 Parallel Composition

One of the most uncomplicated ideas is to group states that match in the function value of all given grouping function s. This idea is parallel in the sense that a set of grouping function is combined to a new grouping function in one single step.

Definition 3.4. Let $F_1, F_2, \dots, F_n$ be grouping function s. A *parallel composed grouping function* is a grouping function $F_{F_1||F_2||\dots||F_n} : S \to M$, $s \mapsto (F_1(s), F_2(s), \dots, F_n(s))$.

A view of such a grouping function is as usual created in in accordance with definition 3.3. Furthermore it is obvious that the effect of a view is considered in the new grouping function and hence the view. Since we define a new grouping function, the given ones and its views are retained.
The operator $||$ used in the definition is derived from the operator used in electric circuits, when the respective elements are connected in parallel.
If we want to speak about this grouping function in a general way, where it is only of importance that we refer to this type of composition and the given grouping function s are of no importance, we will denote a *parallel composition grouping function* with $F_{||}$.

4 View Examples

In this chapter we will introduce and discuss some view examples created by the author. Their purpose is to understand the idea and concept of a view and get to know some views that might be useful in real world applications i.e. in the Project of Dr. Max Korn.

4.1 Transition Systems

First we will take a look at views that can be applied to transition systems, MCs and MDPs and hence only use the properties of transition systems. That is they neither utilize any probabilistic properties of an MC nor any properties that arise from the combination of nondeterminism and the probability distribution.

4.1.1 Atomic Propositions

The *Atomic Propositions View* groups all states to a new state that have the same set of atomic propositions.

Definition 4.1. For a given TS let S be the set its states, L its labeling function and M an arbitrary set. The grouping function for the Atomic Propositions View is defined with $F_{AP} : S \to M, s \mapsto L(s)$.

We define its grouping function with $F_{AP} : S \to M, s \mapsto L(s)$ i.e. for all $s \in S : F_{AP}(s) = L(s)$. So it is $F_{AP}(s_1) = F_{AP}(s_2) \iff L(s_1) = L(s_2)$. According to definition 3.2 for $\tilde{s} \in S$ it is $[\tilde{s}]_R = \{s \in S \mid L(s) = L(\tilde{s})\}$.

By this we obtain the view $TS_{F_{AP}}$ for a given transition system TS where: $S' = \bigcup_{s \in S}\{[s]_R\} = \bigcup_{a \in AP}\{\{s \in S \mid L(s) = a\}\}$. All other components are constructed as in definition 3.3.

tikz example

example from the database of max

4.1.2 Initial States

The *Initial State View* groups all initial states into one single state. All other states are left untouched. We define its grouping function with $F_I : S \to M$ with

$$s \mapsto \begin{cases} \emptyset, & \text{if } s \in I \\ \{s\}, & \text{otherwise} \end{cases}$$

For $s_1, s_2 \in S$ it is $F_I(s_1) = F_I(s_2)$ if and only if $s_1, s_2 \in I$ or $s_1 = s_2$. According to definition 3.2 it is $[s]_R = \{s \in S \mid F(s) = \emptyset\}$ for $s \in I$ and $[s]_R = \{s \in S \mid F(s) = \{s\}\} = \{s\}$ for $s \notin I$.

By this we obtain the view TS_{F_I} for a given transition system TS where: $S' = \bigcup_{s \in S}\{[s]_R\} = \{s \in S \mid s \in I\} \cup \bigcup_{s \in S \setminus I}\{\{s\}\}$.

All other components are constructed as in definition 3.3.

4.1.3 Outgoing Actions

The *OutAction View* groups states that share some property regarding their outgoing actions. Several variants are feasible. The most obvious one is to group states that *have* a given outgoing action. Its grouping function looks as follow: $F_{\exists\alpha} : S \to M$ with

$$s \mapsto \begin{cases} \alpha, & \text{if } \exists s' \in S : (s, \alpha, s') \in \longrightarrow \\ s, & \text{otherwise} \end{cases}$$

For $s_1, s_2 \in S$ it is $F_{\exists\alpha}(s_1) = F_{\exists\alpha}(s_2)$ if and only if there exist $s_a, s_b \in S$ with $(s_1, \alpha, s_a), (s_2, \alpha, s_b) \in \longrightarrow$ (i.e. they have the same outgoing action α) or $s_1 = s_2$. In accordance with definition 3.2 it is $[s]_R = \{s \in S \mid F_{\exists\alpha}(s) = \alpha\}$ if there exists $s' \in S$ so that $(s, \alpha, s') \in \longrightarrow$ and it is $[s]_R = \{s \in S \mid F_{\exists\alpha}(s) = s\} = \{s\}$ otherwise.

Thereby we obtain the view $TS_{F_{\exists\alpha}}$ for a given transition system TS where $S' = \bigcup_{s\in S}\{[s]_R\} =: S_1 \cup S_2$ where $S_1 := \{s \in S \mid \exists s' \in S : (s, \alpha, s') \in \longrightarrow\} = \{s \in S \mid s$ has outgoing action $\alpha\}$ and $S_2 := \bigcup_{s\in S\setminus S_1}\{s\}$.

Since actions are a very important part of transition systems as well as of its more powerful siblings MDPs and MCs it seems useful to further enhance this view and look at variants of it. Instead of only grouping states that only *have* outgoing actions we could also quantify the amount of times that action should be outgoing.

For example we could require that a given action has to be outgoing a minimum amount of times. For this we define the grouping function $F_{n\leq\alpha} : S \to M$ with

$$s \mapsto \begin{cases} \alpha, & \text{if } \exists s_1, \dots, s_n \in S : (s, \alpha, s_1), \dots, (s, \alpha, s_n) \in \longrightarrow \\ s, & \text{otherwise} \end{cases}$$

where $n \in \mathbb{N}$ and $|\{s_1, \dots, s_n\}| = n$.

For $s_1, s_2 \in S$ it is $F_{n\leq\alpha}(s_1) = F_{n\leq\alpha}(s_2)$ if and only if there exist distinct $s_{a_1}, \dots, s_{a_n} \in S$ and distinct $s_{b_1}, \dots, s_{b_n} \in S$ so that $(s_1, \alpha, s_{a_1}), \dots, (s_1, \alpha, s_{a_n}) \in \longrightarrow$ and $(s_2, \alpha, s_{b_1}), \dots, (s_2, \alpha, s_{b_n}) \in \longrightarrow$ or $s_1 = s_2$. According to 3.2 it is $[s]_R = \{s \in S \mid F_{n\leq\alpha}(s) = \alpha\}$ and $[s]_R = \{s \in S \mid F_{n\leq\alpha}(s) = s\} = \{s\}$ otherwise.

By this we obtain the view $TS_{F_{n\leq\alpha}}$ for a given transition system TS where $S' = \bigcup_{s\in S}\{[s]_R\} =: S_1 \cup S_2$ where $S_1 := \{s \in S \mid \exists s_1, \dots, s_n \in S : (s, \alpha, s_1), \dots, (s, \alpha, s_n) \in \longrightarrow, |\{s_1, \dots, s_n\}| = n\} = \{s \in S \mid$ the action α is outgoing at least n times $\}$ and $S_2 := \bigcup_{s\in S\setminus S_1}\{s\}$.

4.2 Markov Chain

4.3 Markov Decision Process

4.4 Comparison of the Examples

5 Outlook

References