CS-E4800 Artificial Intelligence Algorithms for Planning with Partial Observability

Jussi Rintanen

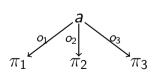
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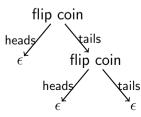
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Approaches to planning with partial observability (non-probabilistic):

- State-space search in belief space (instead of state space)
 - Algorithms for AND-OR tree search
 - Used with heuristic lower bound functions
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- Others exist

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- Set $I \subseteq S$ of initial states
- Set $G \subseteq S$ of goal states

Example

- DayOff = {Saturday, Sunday}
- $\bullet \ \ \mathsf{WorkingDay} = \{\mathsf{Monday}, \mathsf{Tuesday}, \mathsf{Wednesday}, \mathsf{Thursday}, \mathsf{Friday}\}$

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- ullet "No observations" corresponds to $e_{ op} \in E$ such that $S_{e_{ op}} = S$
- We do not assume the disjointness of observations: $S_e \cap S_{e'} = \emptyset$ for $e \in E$ and $e' \in E$ such that $e \neq e'$

Image Operations

Successors of states w.r.t. an action:

Image

```
img_a(B) = \{s'|s \in B, sR_as'\}
```

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$$\mathsf{img}_{\mathsf{a}}(B) = \{s' | s \in B, sR_{\mathsf{a}}s'\}$$

Predecessors of states w.r.t. an action:

 $\mathsf{preimg}_{a}(B) = \{s | s' \in B, sR_{a}s'\}$

Strong Pre-Image

 $\mathsf{spreimg}_{\textit{a}}(\textit{B}) = \{\textit{s} \in \textit{S} | \emptyset \subset \mathsf{img}_{\textit{a}}(\{\textit{s}\}) \subseteq \textit{B}\}$

Successor Belief States

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Given current belief state $B \subseteq S$ and action a, the successor belief states B' of B are all non-empty $\operatorname{succ}_a^o(B) = \operatorname{img}_a(B) \cap S_o$ for some $o \in E_a$

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If $|E_a| = 1$, that is, action a has only one observation, then there is only one successor belief state.

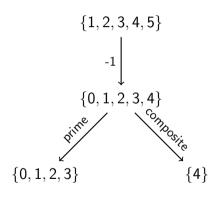
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- Search for plans by algorithms for AND-OR trees:
 - OR-nodes: choice of action
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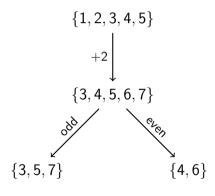
- Image $img_a(B)$ for successors of states in B
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- Search for plans by algorithms for AND-OR trees:
 - OR-nodes: choice of action
 - AND-nodes: choice of successor (observation)
 - belief state for root node I
 - belief state for leaf nodes $\subseteq G$
 - Algorithms:
 - Standard Depth First-Search (DFS) generalized to AND-OR trees
 - Informed algorithms: AO* (Nilsson 1980) and its various improvements



Actions:

- Effect: -1, Observations: prime, composite
- Effect: +2, Observations: even, odd
- Effect: mod 2, Observations: -

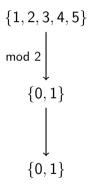
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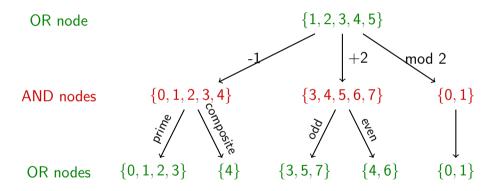
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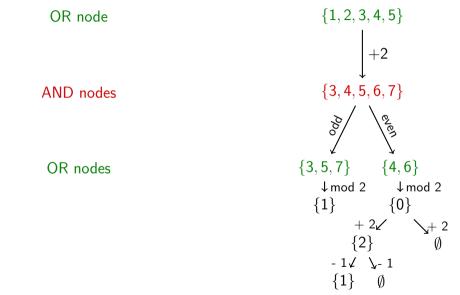


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- 1: **procedure** findplan(B,path,G) 2: **if** $B \subseteq G$ **then return** ϵ :
- 3: **if** $B_0 \subseteq B$ for some $B_0 \in \text{path then return NONE}$:
- 4: for each $a \in A$ do 5:
 - if a not applicable in some $s \in B$ then continue:
 - success := true:
- 6:
 - subplans := \emptyset ;
- 7: 8:
 - subplan := findplan(img₂(B) \cap S₀,path \cup {B},G);
- for each $o \in E_3$
- - **if** subplan = NONE **then** success := false; break;
- 11: $subplans := subplans \cup \{(o, subplan)\};$
 - **if** success **then return** (a;subplans);
- 12:

return NONE:

9:

10:

Backward Search: Full Observability

Algorithm much simpler when problem fully observable

Algorithm (Full Observability)

- P := G
- $P_{old} := P$
- $P := P \cup \bigcup_{a \in A} \operatorname{spreimg}_a(P)$
- If $I \subseteq P$, then return TRUE
- If $P = P_{old}$ then return FALSE
- **o** Go to 2

Solution found.

No more states reached.

Backward Search: Idea

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- Starting point: the set of goal states G (goals reachable by ϵ)

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- Generate more and bigger belief states from which goals can be reached (with bigger/longer plans)
- Starting point: the set of goal states G (goals reachable by ϵ)
- Given belief states \mathcal{B} from which goals reachable by $\leq i$ steps: Find state sets \mathcal{B} and actions a such that
 - for every $o \in E$, succ_a $(B) \subseteq B_0$ for some $B_0 \in \mathcal{B}$, and
 - $B \not\subseteq B_0$ for all $B_0 \in \mathcal{B}$

Goals reachable from B by $\leq i + 1$ steps

Backward Search: Backup Operation

backup(P): Find a plan that can reach goals from a new set of states

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- Try every action $a \in A$

 - ② Try every $\{(\pi_1, B_1), \dots, (\pi_m, B_m)\} \subseteq P$
 - ① Let $B = \mathsf{spreimg}_a((S_{e_1} \cap B_1) \cup \cdots \cup (S_{e_m} \cap B_m))$
 - ② If $B \nsubseteq B_i^*$ for every $i \in \{1, \ldots, n\}$, then return $(a; \{(e_1, \pi_1), \ldots, (e_m, \pi_m)\}, B)$

Backward Search: Backup Operation

backup(P): Find a plan that can reach goals from a new set of states

- Let $P = \{(\pi_1^*, B_1^*), \dots, (\pi_n^*, B_n^*)\}$
- Try every action $a \in A$
 - ① Let $E_a = \{e_1, \dots, e_m\}$ (observations possible after action a)
 - ② Try every $\{(\pi_1, B_1), \dots, (\pi_m, B_m)\} \subseteq P$
 - ① Let $B = \operatorname{spreimg}_a((S_{e_1} \cap B_1) \cup \cdots \cup (S_{e_m} \cap B_m))$
 - ② If $B \nsubseteq B_i^*$ for every $i \in \{1, \ldots, n\}$, then return $(a; \{(e_1, \pi_1), \ldots, (e_m, \pi_m)\}, B)$
- ullet Return \emptyset (No new plan could be found)

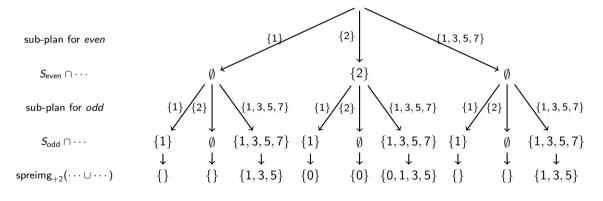
For state space $\{0,1,2,3,4,5,6,7\}$, from $P = \{(\epsilon,\{1\})\}$, backup produces

- {2} by action -1,
- \emptyset by action +2, and
- $\{1, 3, 5, 7\}$ by action mod 2.

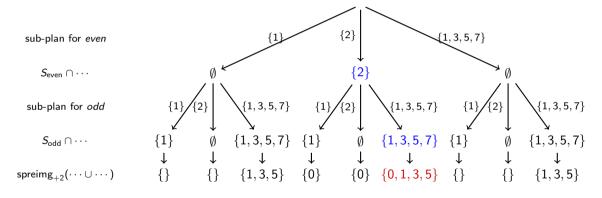
This results in

 $P = \{(\epsilon, \{1\}), ((+1, \{(\textit{prime}, \epsilon)\}), \{2\}), ((\textit{mod } 2, \{(\textit{odd}, \epsilon)\}), \{1, 3, 5, 7\})\}$

Given $P = \{(\epsilon, \{1\}), ((-1, \{(prime, \epsilon)\}), \{2\}), ((mod 2, \{(odd, \epsilon)\}), \{1, 3, 5, 7\})\}$ and action +2, backup goes through the following search tree.



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Only $\operatorname{spreimg}_{+2}((S_{\operatorname{even}} \cap \{2\}) \cup (S_{\operatorname{odd}} \cap \{1,3,5,7\})) = \{0,1,3,5\} \text{ not already covered by } P$ It corresponds to plan $(+2,\{(\operatorname{even},(-1,\{(\operatorname{prime},\epsilon)\})),(\operatorname{odd},(\operatorname{mod}2,\{(\operatorname{odd},\epsilon)\}))\})$

Backward Search: Algorithm

Algorithm

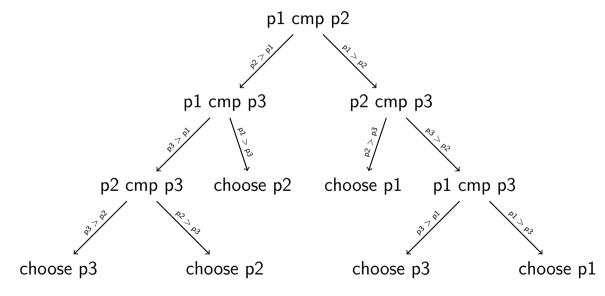
- $P := \{(\epsilon, G)\}$
- $(\pi, B_{\text{new}}) := \text{backup}(P)$
- **1** If $B_{\text{new}} = \emptyset$, stop
 No new set could be found. No solution exists.

- Go to 2

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 - Compare weights of packages i and j, with result w(i) > w(j) or w(j) > w(i)
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- Plan: Weigh packages to identify the heavy one, and choose it



Example: Problem Representation

- State space S consists of all (w_1, w_2, w_3, p) such that
 - $w_i \in \{1, 2, 3\}$ for $i \in \{1, 2, 3\}$
 - $w_1 \neq w_2$, $w_1 \neq w_3$, $w_2 \neq w_3$
 - $p \in \{1, 2, 3\}$

(weights of packages)

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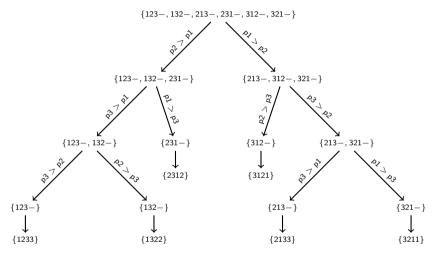
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- $W_1 \neq W_2$, $W_1 \neq W_3$, $W_2 \neq W_3$
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- Goal states are $(w_1, w_2, w_3, p) \in S$ such that p = i and $w_i = 3$: $G = \{(1, 2, 3, 3), (1, 3, 2, 2), (2, 1, 3, 3), (2, 3, 1, 2), (3, 1, 2, 1), (3, 2, 1, 1)\}$

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- Action: Choose i, turns state (w_1, w_2, w_3, p) to (w_1, w_2, w_3, i)
- Action: Compare *i-j* produces the observations
 - GT if $w_i > w_i$
 - LT if $w_i < w_i$

from (w_1, w_2, w_3, p) , and does not change the state



There are 18 initial states. Above, 123— is all sequences with prefix 123, that is {1231, 1232, 1233}.

Goal states:

```
P_0 = \{(\epsilon, \{(1, 2, 3, 3), (1, 3, 2, 2), (2, 1, 3, 3), (2, 3, 1, 2), (3, 1, 2, 1), (3, 2, 1, 1)\})\}
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```

Add states from which goals reached with *Choose 3*:

```
P_1 = P_0 \cup \{((\mathsf{Choose\ 3}; \{(e_\top, \epsilon)\}), \{(1, 2, 3, 1), (2, 1, 3, 1), (1, 2, 3, 2), (2, 1, 3, 2), (1, 2, 3, 3), (2, 1, 3, 3)\})\}
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Add states from which goals reached with *Choose 2*:

```
P_2 = P_1 \cup \{((\mathsf{Choose}\ 2; \{(e_\top, \epsilon)\}), \{(1, 3, 2, 1), (2, 3, 1, 1), (1, 3, 2, 2), (2, 3, 1, 2), (1, 3, 2, 3), (2, 3, 1, 3)\})\}
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```

Add states from which goals reached with *Choose 1*:

```
P_3 = P_2 \cup \{((\mathsf{Choose}\ 1; \{(e_\top, \epsilon)\}), \{(3, 1, 2, 1), (3, 2, 1, 1), (3, 1, 2, 2), (3, 2, 1, 2), (3, 1, 2, 3), (3, 2, 1, 3)\})\}
```

 $P_3 = \{(\epsilon, G), ((\mathsf{Choose}\ 1; \pi_1), B_1), ((\mathsf{Choose}\ 2; \pi_2), B_2), ((\mathsf{Choose}\ 3; \pi_3), B_3)\}$

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Add states from which belief states in P_3 reached with *Compare 2-3*:

$$P_4 = P_3 \cup \{((\mathsf{Compare 2-3}; \{(e_{\mathsf{GT}}, \pi_2), (e_{\mathsf{LT}}, \pi_3)\}), B_2 \cup B_3)\}$$

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Add states from which belief states in P_3 reached with *Compare 1-3*:

$$P_5 = P_4 \cup \{((\mathsf{Compare 1-3}; \{(e_{\mathsf{GT}}, \pi_1), (e_{\mathsf{LT}}, \pi_3)\}), B_1 \cup B_3)\}$$

$$P_3 = \{(\epsilon, G), ((Choose 1; \pi_1), B_1), ((Choose 2; \pi_2), B_2), ((Choose 3; \pi_3), B_3)\}$$

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This now includes the two bottom levels of the plan given earlier Additional steps will construct the rest of that plan...

Conclusion

- We have discussed algorithm for planning with partial observability
- Based on three basic operations:
 - Compute successor state set with respect to a given action
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- Two main approaches:
 - Forward search: Plan constructed top-down
 - Backward search: Plan constructed bottom-up
- Complexity exponentially higher than with full-observability
- Number of belief states exponentially higher than number of states