Introduction to Stochastic Programming

Javier Contreras ©, Luis Baringo ©, UCLM

Department of Electrical Engineering and Automation, Aalto University, Espoo, Finland

20-26 January 2020

Table of contents

Introduction

Why is stochastic programming needed?

Lack of perfect information

- Common in problems of different areas: engineering, economics, finances
- Decisions need to be made even with lack of perfect information
- Decision-making problems under uncertainty

Decision-making problems

- Adequately formulated as optimization problems
- If input data are well-defined: deterministic problem
- However, input data are generally uncertain

Random variables

- Stochastic programming is used to formulate and solve problems with uncertain parameters
- Each uncertain parameter is modeled as a random variable
- In stochastic programming, random variables are usually represented by a finite set of realizations or scenarios $\lambda(\omega)$, $\omega = 1, \ldots, N_\Omega$
- Each realization $\lambda(\omega)$ is associated with a probability $\pi(\omega)$ defined as:

$$\pi(\omega) = P(\omega | \lambda = \lambda(\omega)), \text{where} \sum_{\omega \in \Omega} \pi(\omega) = 1 \qquad (1)$$

Example 1

Price at noon on a future day of the Spanish electricity market

Cumulative distribution function

Assuming a finite set of scenarios, random variable λ can be characterized by its cumulative distribution function (cdf):

$$F_{\lambda}\left(\eta\right) = P\left(\omega | \lambda\left(\omega\right) \leq \eta\right) = \sum_{\omega \in \Omega | \lambda(\omega) \leq \eta} \pi\left(\omega\right), \forall \eta \in \Re, \tag{2}$$

where $\Re$ is the set of real numbers

Statistical moments

Mean ($\bar{\lambda}$):

$$\bar{\lambda} = \mathrm{E}\left\{\lambda\right\} = \sum_{\omega \in \Omega} \pi\left(\omega\right) \lambda\left(\omega\right) \tag{3}$$

Variance (σ_{λ}^2) and standard deviation (σ_{λ}):

$$\sigma_{\lambda}^2 = \mathrm{Var}\left\{\lambda\right\} = \sum_{\omega \in \Omega} \pi\left(\omega\right) \left(\lambda\left(\omega\right) - \bar{\lambda}\right)^2 \tag{4}$$

Example 2

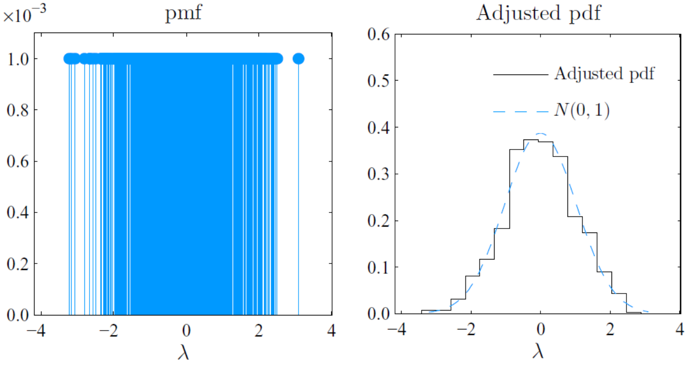
Mean, variance, and standard deviation of the pool price at noon considered in Example 1

Probability mass function (pmf)

- It is the probability of a discrete random variable being equal to a given value
- The equivalent of the pmf for continuous variables is the probability density function (pdf): The integral of the pdf over an interval gives the probability of the random variable falling within that interval
- From a graphical point of view, the pmf is not appropriate for representing random variables with many realizations. It is preferable to use the adjusted pdf:
    1. The base of each bar represents a range of values of the random variable
    2. The height of each bar is equal to the sum of the probabilities of all realizations falling within the interval spanned by the base of the bar divided by the width of the bar

Example 3

Random variable λ is represented by one thousand scenarios randomly generated from a normal distribution N(0,1) pmf and adjusted pdf using 15 intervals:



Stochastic processes

- A random variable whose value evolves over time is known as a stochastic process
- Examples:
  - Pool price over one week
  - Electricity demand over a month
- A stochastic process is constituted by a set of dependent random variables sequentially arranged in time
- For each time period, the corresponding random variable (e.g. the price at noon) depends on the other random variables (e.g., the prices in other hours of the day)
- Example: The price at noon is a random variable, while the collection of random variables corresponding to the hourly prices of the day constitutes a stochastic process.

Stochastic processes

- Stochastic processes spanning a given time horizon can be represented by scenarios:
- Stochastic process $\boldsymbol{\lambda}$ can be presented by vectors $\boldsymbol{\lambda}(\omega)$, $\omega = 1, \ldots, N_\Omega$, where ω is the scenario index and N_Ω is the number of scenarios
- $\boldsymbol{\lambda}$ contains the set of dependent random variables constituting the stochastic process
- We denote $\boldsymbol{\lambda}_\Omega = \{\boldsymbol{\lambda}(1), \ldots, \boldsymbol{\lambda}(N_\Omega)\}$

Stochastic processes

- Each realization $\boldsymbol{\lambda}(\omega)$ is associated with a probability $\pi(\omega)$

$$\pi(\omega) = P(\omega | \boldsymbol{\lambda} = \boldsymbol{\lambda}(\omega)), \text{where} \sum_{\omega \in \Omega} \pi(\omega) = 1 \tag{5}$$

- Example: If $\boldsymbol{\lambda}$ represents the 24 hourly electricity prices of tomorrow, $\boldsymbol{\lambda}(\omega)$ is a 24×1 vector representing one possible realization of this prices

Example 4

Stochastic process: hourly pool prices at two consecutive hours

Scenarios

- A convenient manner to characterize stochastic processes is through scenarios
- A scenario is a single realization of a stochastic process
- It is critical to generate a sufficient number of scenarios to cover the most plausible realizations of the considered stochastic process
- A very large number of scenarios may lead to intractable problems
- Scenario reduction procedures are generally required

Example 5

Scenarios describing the stochastic process defined in Example 4

Stochastic programming problems

- In decision making under uncertainty, the decision maker has to make optimal decisions throughout a decision horizon with incomplete information
- Over the considered decision horizon, a number of stages is defined
- Each stage represents a point in time where decisions are made or where uncertainty partially or totally vanishes
- The amount of information available is usually different from stage to stage
- According to the number of stages: Two-stage and multi-stage stochastic programming problems

Two-stage problems

- Let us consider a decision-making problem where decisions are made at two stages
- There exists a stochastic process $\boldsymbol{\lambda}$ represented by a set of scenarios $\boldsymbol{\lambda}_{\Omega}$
- Two different decision variable vectors, $\boldsymbol{x}$ and $\boldsymbol{y}$, are involved in this problem
- Decision-making process:
  1. Decision $\boldsymbol{x}$ is made
  2. The stochastic process $\boldsymbol{\lambda}$ is realized as $\boldsymbol{\lambda}(\omega)$
  3. Decision $\boldsymbol{y}(\boldsymbol{x}, \omega)$ is made

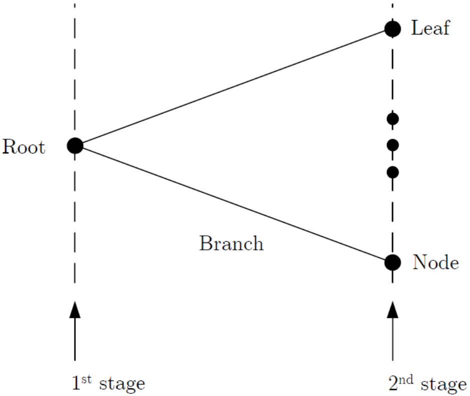
Type of decisions

In the previous decision-making process, there are two kinds of decisions:

1. *First-stage* or *here-and-now* decisions: Decisions made before the realization of the stochastic process. Variables representing here-and-now decisions do not depend on each realization of the stochastic process
2. *Second-stage* or *wait-and-see* decisions: Decisions made after knowing the actual realization of the stochastic process. Decisions do depend on each realization vector of the stochastic process

Scenario tree

Scenario tree for a two-stage problem:



Formulation

- For all decisions (first- and second-stage decisions) to be optimal, they need to be derived simultaneously by solving a single optimization problem
- General expression of a two-stage stochastic linear programming problem:

$$\begin{aligned} \text{Minimize}_{\boldsymbol{x}} \quad & z = \boldsymbol{c}^{\mathrm{T}} \boldsymbol{x} + \mathrm{E}\left\{Q\left(\omega\right)\right\} && (6)\\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} && (7)\\ & \boldsymbol{x} \in X && (8) \end{aligned}$$

where

$$\begin{aligned} Q\left(\omega\right) = \Big\{ \; \text{Minimize}_{\boldsymbol{y}(\omega)} \quad & \boldsymbol{q}\left(\omega\right)^{\mathrm{T}} \boldsymbol{y}\left(\omega\right) && (9)\\ \text{subject to} \quad & \boldsymbol{T}\left(\omega\right)\boldsymbol{x} + \boldsymbol{W}\left(\omega\right)\boldsymbol{y}\left(\omega\right) = \boldsymbol{h}\left(\omega\right) && (10)\\ & \boldsymbol{y}\left(\omega\right) \in Y && (11)\\ \Big\}, \forall \omega \in \Omega \end{aligned}$$

Formulation

Under rather general assumptions, the previous two-stage stochastic programming problem can be equivalently expressed as follows:

$$\begin{aligned} \text{Minimize}_{\boldsymbol{x},\boldsymbol{y}(\omega)} \quad & z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \sum_{\omega \in \Omega} \pi(\omega)\, \boldsymbol{q}^{\mathrm{T}} \boldsymbol{y}(\omega) && (12) \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} && (13) \\ & \boldsymbol{T}(\omega)\, \boldsymbol{x} + \boldsymbol{W}(\omega)\, \boldsymbol{y}(\omega) = \boldsymbol{h}(\omega), \forall \omega \in \Omega && (14) \\ & \boldsymbol{x} \in X, \boldsymbol{y}(\omega) \in Y, \forall \omega \in \Omega && (15) \end{aligned}$$

Formulation

- Stochastic programming problems can be mathematically formulated using either a *node-variable* formulation or a *scenario-variable* formulation
- Node-variable formulation:
  - It relies on variables associated with decision points
  - Compact formulation
  - Well-suited for a direct solution approach
- Scenario-variable formulation:
  - It relies on variables associated with scenarios
  - It requires a larger number of variables and constraints
  - Exploitable structure that is well suited for decomposition

Example 6

An electricity consumer is facing both uncertain electricity demand and price for next week. For simplicity, we consider that both price and demand are uncertain but constant throughout the week. Scenario data pertaining to demand and price are provided in the table below. Additionally, this consumer has the possibility of buying up to 90 MW at $45/MWh throughout next week by signing a bilateral contract before next week, i.e., before knowing the actual demand and pool price it has to face.

Scenario	Probability	Demand [MW]	Price [$/MWh]
1	0.2	110	50
2	0.6	100	46
3	0.2	80	44

Example 7

Solve Example 6 considering a scenario-variable formulation

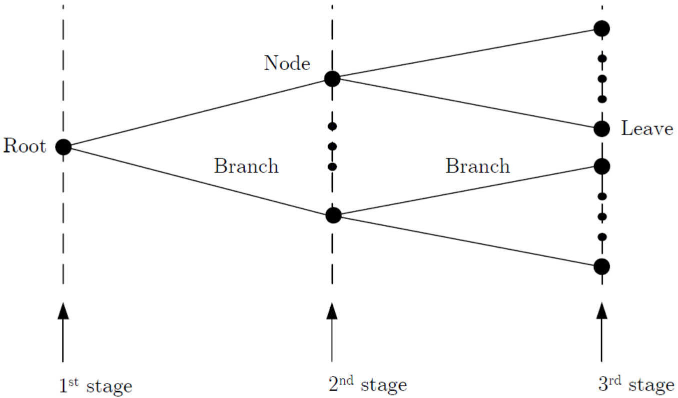
Multi-stage problems

- In some cases, decision-making problems comprise more than two stages
- Multi-stage stochastic programming problems are needed
- Decision-making process:
    1. Decisions $\boldsymbol{x}^1$ are made
    2. The stochastic process $\boldsymbol{\lambda}^1$ is realized as $\boldsymbol{\lambda}^1\left(\omega^1\right)$
    3. Decisions $\boldsymbol{x}^2\left(\boldsymbol{x}^1, \omega^1\right)$ are made
    4. The stochastic process $\boldsymbol{\lambda}^2$ is realized as $\boldsymbol{\lambda}^2\left(\omega^2\right)$
    5. Decisions $\boldsymbol{x}^3\left(\boldsymbol{x}^1, \omega^1, \boldsymbol{x}^2, \omega^2\right)$ are made

  Steps 4-5 are then repeated until the last stage

Scenario tree

Scenario tree for a three-stage problem:



Non-anticipativity constraints

- It is important to establish the non-anticipativity constraints of decisions: If the realizations of the stochastic processes are identical up to stage k, the values of the decision variables must be then identical up to stage k
    1. Decisions $\boldsymbol{x}^1$ are independent of each future realization of the set of stochastic processes $\{\boldsymbol{\lambda}^1, \ldots, \boldsymbol{\lambda}^{r-1}\}$
    2. Decisions $\boldsymbol{x}^2$ depend on each realization of the stochastic process in the first stage, $\boldsymbol{\lambda}^1$, but they are unique for all possible values of the stochastic process that are realized in the future $\{\boldsymbol{\lambda}^2, \ldots, \boldsymbol{\lambda}^{r-1}\}$
    3. Decisions $\boldsymbol{x}^2$ are wait-and-see decisions with respect to $\boldsymbol{\lambda}^1$ and here-and-now decisions with respect to $\{\boldsymbol{\lambda}^2, \ldots, \boldsymbol{\lambda}^{r-1}\}$

Formulation

The general expression of a multi-stage stochastic linear programming problem with r stages is as follows:

$$\text{Minimize}_{\boldsymbol{x}^1} \quad z = \boldsymbol{c}^{1,\mathrm{T}} \boldsymbol{x}^1 + \mathrm{E}_{\omega_1} \left\{ Q^1 \left(\boldsymbol{x}^1, \omega^1 \right) \right\} \tag{16}$$

$$\text{subject to} \quad \boldsymbol{A}^1 \boldsymbol{x}^1 = \boldsymbol{b}^1 \tag{17}$$

$$\boldsymbol{x}^1 \in X^1 \tag{18}$$

where

$$\begin{aligned}
Q^1 \left(\boldsymbol{x}^1, \omega^1 \right) = \{ & \\
\text{Minimize}_{\boldsymbol{x}^2(\omega^1)} \quad & \boldsymbol{c}^{2,\mathrm{T}} \left(\omega^1 \right) \boldsymbol{x}^2 \left(\omega^1 \right) + \mathrm{E}_{\omega_2} \left\{ Q^2 \left(\boldsymbol{x}^1, \omega^1, \boldsymbol{x}^2 \left(\omega^1 \right), \omega^2 \right) \right\} \\
\text{subject to} \quad & \boldsymbol{T}^{1,1} \left(\omega^1 \right) \boldsymbol{x}^1 + \boldsymbol{T}^{1,2} \left(\omega^1 \right) \boldsymbol{x}^2 \left(\omega^1 \right) = \boldsymbol{h}^1 \left(\omega^1 \right) \\
& \boldsymbol{x}^2 \left(\omega^1 \right) \in X^2 \\
& \} , \forall \omega^1 \in \Omega^1
\end{aligned} \tag{19}$$

Formulation

where

$$Q^2\left(\boldsymbol{x}^1,\omega^1,\boldsymbol{x}^2\left(\omega^1\right),\omega^2\right)=\{$$

$$\begin{aligned}
\text{Minimize}_{\boldsymbol{x}^3(\omega^1,\omega^2)}\ & \boldsymbol{c}^{3,\mathrm{T}}\left(\omega^1,\omega^2\right)\boldsymbol{x}^3\left(\omega^1,\omega^2\right)\\
& +\mathrm{E}_{\omega_3}\left\{Q^3\left(\boldsymbol{x}^1,\omega^1,\boldsymbol{x}^2\left(\omega^1\right),\omega^2,\boldsymbol{x}^3\left(\omega^1,\omega^2\right),\omega^3\right)\right\}\\
\text{subject to}\ & \boldsymbol{T}^{2,1}\left(\omega^1,\omega^2\right)\boldsymbol{x}^1+\boldsymbol{T}^{2,2}\left(\omega^1,\omega^2\right)\boldsymbol{x}^2\left(\omega^1\right)\\
& +\boldsymbol{T}^{2,3}\left(\omega^1,\omega^2\right)\boldsymbol{x}^3\left(\omega^1,\omega^2\right)=\boldsymbol{h}^2\left(\omega^1,\omega^2\right)\\
& \boldsymbol{x}^3\left(\omega^1,\omega^2\right)\in X^3\\
& \},\forall\omega^1\in\Omega^1,\forall\omega^2\in\Omega^2
\end{aligned}\tag{20}$$

$$\vdots$$

Formulation

$$\vdots$$

where

$$Q^{r-1}\left(\boldsymbol{x}^1, \omega^1, \ldots, \omega^{r-1}\right) = \{$$

$$\begin{aligned}
\text{Minimize}_{\boldsymbol{x}^r(\omega^1,\ldots,\omega^{r-1})} \quad & \boldsymbol{c}^{r,\mathrm{T}}\left(\omega^1, \ldots, \omega^{r-1}\right) \boldsymbol{x}^r\left(\omega^1, \ldots, \omega^{r-1}\right) \\
\text{subject to} \quad & \boldsymbol{T}^{r-1,1}\left(\omega^1, \ldots, \omega^{r-1}\right) \boldsymbol{x}^1 \\
& \quad + \boldsymbol{T}^{r-1,2}\left(\omega^1, \ldots, \omega^{r-1}\right) \boldsymbol{x}^2\left(\omega^1\right) + \ldots \\
& \quad + \boldsymbol{T}^{r-1,r}\left(\omega^1, \ldots, \omega^{r}\right) \boldsymbol{x}^r\left(\omega^1, \ldots, \omega^{r-1}\right) \\
& \quad = \boldsymbol{h}^{r-1}\left(\omega^1, \ldots, \omega^{r-1}\right) \\
& \boldsymbol{x}^r\left(\omega^1, \ldots, \omega^{r-1}\right) \in X^r \\
& \}, \forall \omega^1 \in \Omega^1, \ldots, \forall \omega^{r-1} \in \Omega^{r-1}
\end{aligned} \tag{21}$$

Formulation

Under rather general assumptions, the previous multi-stage stochastic programming problem can be equivalently expressed as follows:

$$
\begin{aligned}
\text{Minimize}_{\boldsymbol{x},\boldsymbol{y}(\omega)} \ \ & z = \boldsymbol{c}^{1,\mathrm{T}}\boldsymbol{x}^1 + \sum_{\omega^1 \in \Omega^1} \pi\left(\omega^1\right) \left\{ \boldsymbol{c}^{2,\mathrm{T}}\left(\omega^1\right) \boldsymbol{x}^2\left(\omega^1\right) \right. \\
& + \sum_{\omega^2 \in \Omega^2} \pi\left(\omega^2\right) \left[\boldsymbol{c}^{3,\mathrm{T}}\left(\omega^1,\omega^2\right) \boldsymbol{x}^3\left(\omega^1,\omega^2\right) + \dots \right. \\
& + \sum_{\omega^{r-1} \in \Omega^{r-1}} \pi\left(\omega^{r-1}\right) \times \\
& \left. \left. \left(\boldsymbol{c}^{r,\mathrm{T}}\left(\omega^1,\dots,\omega^{r-1}\right) \boldsymbol{x}^r\left(\omega^1,\dots,\omega^{r-1}\right)\right)\right]\right\} && (22) \\
\text{subject to} \ \ & \boldsymbol{A}^1\boldsymbol{x}^1 = \boldsymbol{b}^1 && (23) \\
& \boldsymbol{x}^1 \in X^1 && (24)
\end{aligned}
$$

Formulation

$$\text{s.t.} \quad \boldsymbol{T}^{1,1}\left(\omega^1\right)\boldsymbol{x}^1 + \boldsymbol{T}^{1,2}\left(\omega^1\right)\boldsymbol{x}^2\left(\omega^1\right) = \boldsymbol{h}^1\left(\omega^1\right), \forall \omega^1 \in \Omega^1 \tag{25}$$

$$\boldsymbol{x}^2\left(\omega^1\right) \in X^2, \forall \omega^1 \in \Omega^1 \tag{26}$$

$$\begin{aligned} &\boldsymbol{T}^{2,1}\left(\omega^1,\omega^2\right)\boldsymbol{x}^1 + \boldsymbol{T}^{2,2}\left(\omega^1,\omega^2\right)\boldsymbol{x}^2\left(\omega^1\right) \\ &\quad + \boldsymbol{T}^{2,3}\left(\omega^1,\omega^2\right)\boldsymbol{x}^3\left(\omega^1,\omega^2\right) \\ &\quad = \boldsymbol{h}^2\left(\omega^1,\omega^2\right), \forall \omega^1 \in \Omega^1, \forall \omega^2 \in \Omega^2 \end{aligned} \tag{27}$$

$$\boldsymbol{x}^3\left(\omega^1,\omega^2\right) \in X^3, \forall \omega^1 \in \Omega^1, \forall \omega^2 \in \Omega^2 \tag{28}$$

$$\dots$$

$$\begin{aligned} &\boldsymbol{T}^{r-1,1}\left(\omega^1,\dots,\omega^{r-1}\right)\boldsymbol{x}^1 + \boldsymbol{T}^{r-1,2}\left(\omega^1,\dots,\omega^{r-1}\right)\boldsymbol{x}^2\left(\omega^1\right) + \dots \\ &\quad + \boldsymbol{T}^{r-1,r}\left(\omega^1,\dots,\omega^{r}\right)\boldsymbol{x}^r\left(\omega^1,\dots,\omega^{r-1}\right) \\ &\quad = \boldsymbol{h}^{r-1}\left(\omega^1,\dots,\omega^{r-1}\right), \forall \omega^1 \in \Omega^1,\dots,\forall \omega^{r-1} \in \Omega^{r-1} \end{aligned} \tag{29}$$

$$\boldsymbol{x}^r\left(\omega^1,\dots,\omega^{r-1}\right) \in X^r, \forall \omega^1 \in \Omega^1,\dots,\forall \omega^{r-1} \in \Omega^{r-1} \tag{30}$$

Example 8

Consider an electric energy producer with a production capacity of 120 MW.
This producer needs to determine its selling strategy during a future market horizon of 6 days, divided into two 3-day periods.
The bilateral contracts available to this producer are the following:

1. A selling contract of up to 50 MW at \$25/MWh at the beginning of the 6-day period and spanning the entire 6-day period.
2. A selling contract of up to 40 MW at \$26/MWh just after the first 3-day period and covering the last 3-day period.

Contract	Duration	Hours	Price	Power cap
A	Entire 6-day period	144	25 \$/MWh	50 MW
B	2nd 3-day period	72	26 \$/MWh	40 MW

Example 8

Complementarily to bilateral contracting, the producer can sell energy in the pool at constant power during the first and second 3-day periods:

1. Pool prices for the first 3-day period are \$27/MWh with probability 0.4 and \$23/MWh with probability 0.6
2. If the price during the first 3-day period is \$27/MWh, prices during the second 3-day period are \$28/MWh with probability 0.4 and \$26/MWh with probability 0.6
3. Alternatively, if the price during the first 3-day period is \$23/MWh, prices during the second 3-day period are \$24/MWh with probability 0.4 and \$22/MWh with probability 0.6

Example 9

Solve Example 8 considering a node-variable formulation

Quality metrics

- We should justify why stochastic programming problems are solved instead of deterministic ones
- Deterministic problems are obtained out of stochastic ones by replacing the random variables by their expected values
- Deterministic problems are simpler and easier to solve than stochastic ones
- Quality metrics:
  1. Expected value of perfect information (EVPI)
  2. Value of stochastic solution (VSS)
  3. Out-of-sample techniques

Expected value of perfect information

- The EVPI represents the quantity that the decision maker is willing to pay for obtaining perfect information about the future
- We denote by $z^{\mathrm{S}*}$ the optimal value of the objective function of a two-stage stochastic programming problem
- We denote by $z^{\mathrm{P}*}$ the optimal value of the objective function of a two-stage stochastic programming problem in which non-anticipativity constraints are relaxed
- The EVPI for a maximization problem is computed as:

$$\mathrm{EVPI}_{\mathsf{max}} = z^{\mathrm{P}*} - z^{\mathrm{S}*} \tag{31}$$

- The EVPI for a minimization problem is computed as:

$$\mathrm{EVPI}_{\mathsf{min}} = z^{\mathrm{S}*} - z^{\mathrm{P}*} \tag{32}$$

Example 10

Compute the EVPI for Example 7

Example 11

Compute the EVPI for Example 8

Value of stochastic solution

- The VSS is a measure to quantify the advantage of using a stochastic programming approach over a deterministic one
- In the deterministic problem associated with a stochastic programming problem:
  1. The random variables are replaced by their expected values
  2. The solution to this deterministic problem provides values for the first-stage variables
  3. The original stochastic programming problem is solved with fixed values of the first-stage variables
- This modified problem decomposes by scenario and is generally easy to solve

Value of stochastic solution

- We denote by $z^{\mathrm{S}*}$ the optimal value of the objective function of the stochastic programming problem
- We denote by $z^{\mathrm{D}*}$ the optimal value of the objective function of the modified stochastic problem (with fixed first stage decisions)
- The VSS for a maximization problem is computed as:

$$\mathrm{VSS}_{\mathsf{max}} = z^{\mathrm{S}*} - z^{\mathrm{D}*} \tag{33}$$

- The VSS for a minimization problem is computed as:

$$\mathrm{VSS}_{\mathsf{min}} = z^{\mathrm{D}*} - z^{\mathrm{S}*} \tag{34}$$

Example 11

Compute the VSS for Example 6

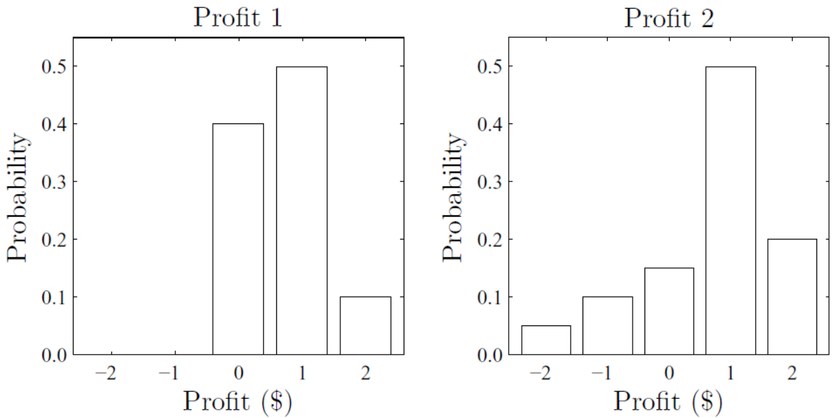
Example 12

Compute the VSS for Example 9

Out-of-sample assessment

- To evaluate the effectiveness of the decisions obtained by solving a stochastic programming problem in terms of actual real-world outcomes
- Additionally, to compare the effectiveness in terms of actual outcomes of alternative decisions, e.g., the decision obtained by solving a stochastic programming model and that obtained via a simplified deterministic one
- Procedure:
  1. Solve a stochastic programming model to obtain the decision to be made at time t with all the available information up to time $t-1$
  2. Implement the decision obtained by solving the stochastic programming problem of step 1 above, and observe the outcome obtained that corresponds to time t
  3. Steps 1 and 2 above are repeated for times $t = 1, \ldots, T$ to statistically characterize the resulting outcome

Risk


Profit 1
Probability
0.5
0.4
0.3
0.2
0.1
0.0
−2
−1
0
1
2
Profit ($)
Profit 2
Probability
0.5
0.4
0.3
0.2
0.1
0.0
−2
−1
0
1
2
Profit ($)

Risk

- Stochastic programming problems maximize (minimize) an objective function representing an expected profit (cost)
- These problems constitute *risk-neutral* models
- The decision maker only focuses on the expected value of the profit (cost), ignoring the rest of parameters characterizing the distribution of the profit (cost)
- If risk is accounted for, the decision maker also assesses the profit values in the worst scenarios, in addition to the expected value of the profit
- In this case, the decision maker becomes a *risk-averse* agent

Risk

- A variety of stochastic programming models can be set up depending on the relevance of the expected objective function value vs. its variability
- Different decisions result in different outcomes in terms of expected objective value and objective function variability
- Different alternatives:
  - Maximizing expected profit and disregarding profit variability
  - Maximizing profit and limiting the variance of the profit distribution it achieves
  - Others
- The risk-aversion level is an input parameter
- A decision maker should know in advance the level of risk he/she is willing to assume

Solving stochastic programming problems

- The size of a stochastic programming problem (in terms of number of variables and constraints) grows with the number of scenarios
- Stochastic programming problems become easily large-scale involving millions of variables and constraints
- It is important to select carefully the number of scenarios to properly represent the stochastic processes involved
- Scenario generation and reduction methods should be adequately selected

Solving stochastic programming problems

- Stochastic programming problems present often an exploitable structure
- The number of constraints involving variables across scenarios is comparatively small with respect to the number of variables and constraints pertaining to any particular scenario
- These linking constraints are mostly non-anticipativity conditions
- This structure is particularly exploitable if a scenario-variable formulation is used

Solving stochastic programming problems

- Stochastic programming problems generally include complicating constraints: Constraints that if relaxed render the resulting problem easy to solve as it decomposes by scenario
- Decomposition techniques are particularly suited to tackle stochastic programming problems
- Examples of decomposition techniques: Benders' decomposition, Lagrangian relaxation

References

1. A.J. Conejo, M. Carrión, J.M. Morales. Decision-making under uncertainty in electricity markets. Springer, New York, 2010
2. J.R. Birge and F. Louveaux. Introduction to stochastic programming. Springer-Verlag, New York, 1997