A Multi-Stage Stochastic Non-Linear Model for Reactive Power Planning Under Contingencies

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***Abstract*—This paper presents a model for long-term reactive power planning where a deterministic nonlinear model is expanded into a multi-stage stochastic model under load uncertainty and an N-k contingency analysis. Reactive load shedding is introduced in the objective function to measure the reactive power deficit after the planning process. The objective is to minimize the sum of investment costs (IC), expected operation costs (EOC) and reactive load shedding costs optimizing the sizes and locations of new reactive compensation equipment to ensure power system security in each stage along the planning horizon. An efficient scenario generation and reduction methodology is used for modeling uncertainty. Expected benefits are calculated to establish the performance of the expected value with perfect information (EVPI) and the value of the stochastic solution (VSS) methodologies. The efficacy of the proposed model is tested and justified by the simulation results using the Ward-Hale 6-bus and the IEEE 14-bus systems.**

***Index Terms*—Contingencies, expected benefits, multi-stage, reactive power planning, scenario tree, stochastic programming, uncertainty modeling.**

NOMENCLATURE

The following notation is used throughout the paper:

Acronyms

AMPL	A Modeling Language for Mathematical Programming.
EV	Expected value solution.
EVPI	Expected value with perfect information.
SS	Stochastic solution.
VSS	Value of the stochastic solution.
WSS	Wait-and-see solution.

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Sets

$\mathbf{B}$	Set of buses of the system.
$\mathbf{L}$	Set of transmission lines.
$\mathbf{PQ}$	Set of $\mathbf{pq}$ buses.
$\mathbf{PV}$	Set of $\mathbf{pv}$ buses.
$\mathbf{S}$	Set of scenarios.
$\mathbf{SH}$	Set of candidate buses to install new reactive power sources.
$\mathbf{SHE}$	Set for existing shunt reactive power sources.
$\mathbf{Slack}$	*Slack* bus.
$\mathbf{T}$	Set of periods.
$\mathbf{TAP}$	Set for under-load variable tap transformers.

Indexes

a	Index for $\mathbf{Slack}$ bus.
e	Index for existing shunt reactive power sources.
i	Index for buses of the system.
j	Index for $\{\mathbf{PV} \cup Slack\}$ buses.
k	Index for candidate buses for reactive sources installation.
l	Index for $\mathbf{PQ}$ buses.
m	Index for under-load variable tap transformers.
n	Index for transmission lines.
p	Index for $\mathbf{PV}$ buses.
s	Index for scenarios.

Constants

C_{FCk}, C_{FRk}	Investment costs of capacitive and inductive reactive sources in bus k in US$.
C_{VCk}, C_{VRk}	Operating costs of capacitive and inductive reactive sources in bus k in US$/MVAr.
$C_{\mathrm{LSC}i}, C_{\mathrm{LSR}i}$	Reactive capacitive and inductive load shedding costs in bus i in US$.
$P^G_{p,t,s}$	Active power of generator p in period t in scenario s.

$P^D_{l,t,s}, Q^D_{l,t,s}$	Active and reactive demand in bus l in period t in scenario s.
$\overline{QC}_k, \overline{QR}_k$	Capacity of capacitive and inductive new sources k.
$\overline{QC}_e, \overline{QR}_e$	Capacity of capacitive and inductive existing sources e.
r_t	Annual interest rate in period t.
$\lvert\bar{S}_n\rvert$	Complex power capacity of line n.
$\underline{\mathrm{Tap}}_m, \overline{\mathrm{Tap}}_m$	Minimum and maximum tap limits of transformer m.
$\underline{V}_i, \bar{V}_i$	Minimum and maximum voltage in bus i.
y_n	Admittance of transmission line n.
α_t	Transmission line state factor in period t.
$\pi_{t,s}$	Probability of occurrence in period t in scenario s.

Variables

$p_{i,t,s}, q_{i,t,s}$	Active and reactive power injections in bus i in period t in scenario s.
$p^G_{e,t,s}$	Active power in the $slack$ bus in period t in scenario s.
$q^G_{j,t,s}$	Reactive power of generator j in period t in scenario s.
$qc_{k,t,s}, qr_{k,t,s}$	Capacitive and inductive reactive power of the new sources to be installed in bus k in period t in scenario s.
$qc_{e,t,s}, qr_{e,t,s}$	Capacitive and inductive reactive power of the existing sources installed in bus e in period t in scenario s.
$s_{n,t,s}$	Complex power flow in transmission line n in period t in scenario s.
$\mathrm{tap}_{m,t,s}$	Tap of under-load variable transformer m in period t in scenario s.
$v_{i,t,s}, \delta_{i,t,s}$	Voltage magnitude and angle in bus i in period t in scenario s.
$wc_{k,t}, wr_{k,t}$	Binary decision variables: 1 if qc or qr are built in bus k in period t, 0 otherwise.
$yc_{i,t,s}, yr_{i,t,s}$	Capacitive and inductive reactive power shedding in bus k in period t in scenario s.

I. Introduction

POWER systems operators and planners have a growing interest in the influence of reactive power in planning and operation problems. This interest stems mainly from the stochastic nature of the problem, demand increase, the need to operate systems based on economic considerations, and the fact that in many electrical systems there is a trend towards operating near the operational limits. This has led to concerns about safety in the operation (control and voltage stability), since voltage instability is recognized as one of the biggest threats in electrical systems operation. It must be kept in mind that reactive power has a significant influence on the safe and economical operation of an electrical power system.

Unknown data abound in decision-making problems in the real world. This lack of perfect information is common in problems belonging to different fields such as engineering, economics, finance, etc. Decision-making problems in power system planning are no exception. In fact, uncertainty is present in most decision-making problems faced by electricity market agents, planners, and system operators. However, decisions need to be made even though there is a lack of perfect information. This is what motivates the use of *stochastic programming* models for decision making under uncertainties.

Reactive power planning is one of the most complex problems in power systems and is divided into two parts: reactive equipment investment planning and system operation planning. Investment planning determines the location, capacity and type of reactive compensation and occurs when reactive power resources placed on system are unable to meet system constraints. Based on investment planning, operation planning determines the settings of voltage control devices such as capacitive and reactive banks, static compensators, synchronous compensators, excitation systems of synchronous generators and under-load transformers with automatic tap changing control. These preventive controls ensure a safe, stable and economic system operation [1].

In reactive power planning, decisions can be made based on three different situations: 1) certainty, 2) uncertainty, and 3) risk conditions. There are many random events that should be considered in reactive planning. These events are demand, availability and capacity of existing and new generating units, and transmission network topology. These random events influence the model's random variables. Some of these random variables are: power flow through transmission lines, existing generating units production, tap position in under-load transformers with automatic tap changer control, and the number of reactive sources to be installed and their generation levels.

A. Literature Review and State of the Art

Optimization techniques have been developed in the past few years to address the reactive planning problem. Both classical and combinatorial optimization techniques have been widely applied to solve this problem.

In [2], the performance of combinatorial algorithms such as real coded genetic algorithm (RCGA), differential evolution (DE), hybrid differential evolution and particle swarm optimization (HDEPSO), particle swarm optimization (PSO), and improved particle swarm optimization (IPSO) is compared, where the problem is formulated as a nonlinear mixed integer multi-objective problem.

A multi-stage tabu search (TS) is used to solve a two-stage reactive power planning problem in [3]. The first stage obtains the optimal installation of capacitive reactive power resources and the second stage determines the optimal operation of reactive power resources.

In [4] the PSO technique was applied for solving the reactive planning problem, including line flows and improving the system voltage profile. The PSO algorithm uses binary and continuous variables for devices such as under-load transformers with automatic control of tap change and reactive power sources.

An evolutionary particle swarm optimization (EPSO) algorithm is used in [5] to solve the reactive planning problem, considering different contingencies and load levels. This application is done taking into account the methodology's stochastic nature without explicitly showing load stages and contingency scenarios.

A model that is based on probabilistic chance-constraint programming is used in [6]. The model is formulated considering both generation and demand as probabilities distributions. The solution method presented to solve the optimization problem is based on Monte Carlo simulation and genetic algorithms.

Several classical optimization algorithms such as linear programming [7]–[9], quadratic programming [10] nonlinear programming [11], [12], mixed integer nonlinear programming [13], [14], decomposition methods [15], [16], methods based on successive linear programming [17], [18], mathematical programming techniques such as gradient method [19], Newton method [20], have been also used to solve deterministic reactive power optimization problems.

B. Aims and Contributions

A common characteristic of reactive power planning classical models is that they point out where new power sources should be added, what type of reactive sources should be installed, and their capacities. In these models the nominal, maximum and minimum load levels are taken into account, only considering the end stage when solving the entire planning problem.

Based on the planning horizon we can distinguish two types of reactive power planning problems: 1) static reactive power planning, and 2) dynamic reactive power planning. The dynamic reactive power planning determines when the new reactive sources should be installed.

None of the papers previously mentioned have considered either the timing to install reactive power sources in a multi-stage deterministic or stochastic reactive power planning model or an explicit analysis of uncertainties through the use of algorithms for generation and reduction of scenarios that explicitly show the behavior of uncertainties. They have only investigated static reactive power planning, solving the reactive power planning problem using classical or combinatorial optimization based on a deterministic model. Although stochastic analysis has been implemented in some cases, this analysis was based on the stochastic nature of the combinatorial optimization techniques, i.e., the modeling technique of the stochastic parameters is not discussed. Changes or adjustments in certain parameters of these algorithms are performed by the users to modify the final solutions, resulting in a relatively low performance.

The consequences of ignoring uncertainty can lead to the installation of a smaller number of new reactive power sources in the power system as well as a low reliability, because certain stochastic parameters of low or medium probability are not considered in the analysis, but, nevertheless, may occur leading to power system to operate in critical conditions.

In this paper a multi-stage stochastic reactive power planning model is developed considering the reactive power planning time as an optimization variable, as well as load uncertainties that are taken into account using scenario trees along the time horizon. Scenario trees are reduced using an optimal scenario reduction technique in order to get a reasonable number of scenarios while maintaining key information related to uncertainties. This model allows N-k contingency analysis in each stage of the time horizon. These considerations are used to calculate a set of solutions in order to find the best planning that is adapted to future changes so that an optimum solution is obtained. Reactive load shedding is introduced in the objective function to measure the reactive power deficit after the planning process. The objective is to minimize the sum of investment costs (IC), expected operation costs (EOC), and reactive load shedding costs optimizing the sizes and locations of new reactive compensation equipment to ensure power system security in every stage of the planning horizon.

It should be noted that the proposed model is not constrained by uncertainty modeling. This means that the modeling of uncertainty can be done using either one or all of the stochastic factors for the entire system; the number of levels of demand can be greater, all without loss of generality. This model allows us to evaluate all cases simultaneously without having to run the program several times.

Given that the reactive power planning model is a non-convex, non-linear, large scale optimization problem, the solution obtained and presented in this paper is a local optimal.

The reactive sources planning problem addressed in this paper consists of a static mathematical model that considers the system operating under normal conditions and contingencies. The reactive sources considered in the model are sources of slow and fast action [21]. Slow-action sources are those in which their control devices have a response time which is not able to perform reactive compensation under contingencies. These sources include capacitor banks and fixed or switching reactors that respond to demand increases. This equipment can be operated in the face of predictable maintenance actions on transmission lines, equipment, etc., to cope with the increased demand for reactive power. The fast-action elements in the model, SVCs, STATCOMs and the limitation of available generation capacity of synchronous generators around on operation point, may be operated against unpredictable actions (such as the failure of a transmission line) in order to have a stable operation of the system.

C. Paper Organization

This paper is organized as follows: Section II presents the deterministic model for reactive power planning. Section III describes the multi-stage deterministic model for reactive power planning. Section IV presents the multi-stage stochastic mathematical model proposed for reactive power planning, and presents the Y_{bus} matrix formulation for contingency analysis. Section V describes the uncertainty model used in this paper for multi-stage scenario tree demand generation and reduction. Section VI describes two values used in stochastic problems to measure stochasticity, the *expected value of perfect information* (EVPI) and the *value of the stochastic solution* (VSS). In Section VII two case studies are presented to show the advantages of a multi-stage stochastic model instead of a deterministic model. In Section VII-A and B the numerical results obtained with the Ward-Hale 6-bus and IEEE 14-bus systems are analyzed in steady state and under contingencies. Expected benefits are calculated to measure the performance of the methodology used. Finally, Section VIII presents some final conclusions.

II. Deterministic Model

Deterministic optimization models assume that the future is perfectly known. The deterministic model presented in this section is cast as a nonlinear programming problem, and formulated as:

$$\min_{\substack{\mathbf{v},\boldsymbol{\delta},\mathbf{tap},\mathbf{qc},\mathbf{qr},\\ \mathbf{wc},\mathbf{wr},p^G,\mathbf{q}^G}} \left\{ \sum_{k\in\mathbf{SH}} C_{FC\,k}\cdot wc_k + \sum_{k\in\mathbf{SH}} C_{FR\,k}\cdot wr_k + \sum_{k\in\mathbf{SH}} C_{VC\,k}\cdot qc_k + \sum_{k\in\mathbf{SH}} C_{VR\,k}\cdot qr_k \right\} \tag{1}$$

s.t :

$$P_p^G + p_a^G - P_l^D - p_i(\mathbf{v},\delta,\mathbf{tap}) = 0 \tag{2}$$
$$q_j^G - Q_l^D - q_k(\mathbf{v},\delta,\mathbf{tap}) + q_{ck} - q_{rk} + q_{ce} - q_{re} = 0 \tag{3}$$
$$\underline{Q}_j^G \le q_j^G \le \bar{Q}_j^G \tag{4}$$
$$\underline{V}_i \le v_i \le \bar{V}_i \tag{5}$$
$$\underline{\mathrm{Tap}}_m \le \mathrm{tap}_m \le \overline{\mathrm{Tap}}_m \tag{6}$$
$$|s_n| \le \bar{S}_n \tag{7}$$
$$0 \le qc_e \le \overline{QC}_e \tag{8}$$
$$0 \le qr_e \le \overline{QR}_e \tag{9}$$
$$0 \le qc_k \le \overline{QC}_k \cdot wc_k \tag{10}$$
$$0 \le qr_k \le \overline{QR}_k \cdot wr_k \tag{11}$$
$$wc_k \in \{0,1\} \tag{12}$$
$$wr_k \in \{0,1\} \tag{13}$$
$$\forall : \{i\in\mathbf{B}, e\in\mathbf{SHE}, k\in\mathbf{SH}, p\in\mathbf{PV}, a\in\{\mathbf{Slack}\}, j\in\{\mathbf{PV}\cup\mathbf{Slack}\}, l\in\mathbf{PQ}, m\in\mathbf{TAP}, n\in\mathbf{L}\}.$$

The objective function in (1) aims to minimize the installation costs of new capacitive and inductive reactive power compensators and the operation costs of capacitive and inductive reactive power compensators subject to: nodal active and reactive balance equations in (2) and (3), upper and lower reactive power generation limits in (4), upper and lower voltage limits in (5), upper and lower tap limits in (6), power transfer capability in (7), upper and lower capacitive and inductive for existing reactive power sources in (8) and (9), upper and lower capacitive and inductive for new reactive power sources in (10) and (11), and binary decision variables for the installation of capacitive and inductive reactive power sources in (12) and (13).

III. Multi-Stage Deterministic Model

The multi-stage deterministic model is presented in this section considering the time to install reactive power compensators. In this model the reactive load shedding term is considered as explained below:

$$\min_{\substack{\mathbf{v},\delta,\mathbf{tap},\mathbf{qc},\mathbf{qr},\mathbf{wc},\\ \mathbf{wr},p^G,\mathbf{q}^G}} f = D_F(K_F + K_V) + K_{\mathrm{LS}} \tag{14}$$

s.t :

$$P_{p,t}^G + p_{a,t}^G - P_{l,t}^D - P_{i,t}(\mathbf{v},\delta,\mathbf{tap}) = 0 \tag{15}$$
$$q_{j,t}^G - Q_{l,t}^D - Q_{k,t}(\mathbf{v},\delta,\mathbf{tap}) + q_{ck,t} - q_{rk,t} + q_{ce,t} - q_{re,t} = 0 \tag{16}$$
$$\underline{Q}_j^G \le q_{j,t}^G \le \bar{Q}_j^G \tag{17}$$
$$\underline{V}_i \le v_{i,t} \le \bar{V}_i \tag{18}$$
$$\underline{\mathrm{Tap}}_m \le \mathrm{tap}_{m,t} \le \overline{\mathrm{Tap}}_m \tag{19}$$
$$|s_{n,t}| \le \bar{S}_n \tag{20}$$
$$0 \le qc_{e,t} \le \overline{QC}_e \tag{21}$$
$$0 \le qr_{e,t} \le \overline{QR}_e \tag{22}$$
$$0 \le qc_{k,t} \le \overline{QC}_k \cdot wc_{k,t} \tag{23}$$
$$0 \le qr_{k,t} \le \overline{QR}_k \cdot wr_{k,t} \tag{24}$$
$$wc_{k,t-1} \le wc_{k,t} \tag{25}$$
$$wr_{k,t-1} \le wr_{k,t} \tag{26}$$
$$wc_k \in \{0,1\} \tag{27}$$
$$wr_k \in \{0,1\} \tag{28}$$
$$yc_{i,t} \ge 0 \tag{29}$$
$$yr_{i,t} \ge 0 \tag{30}$$
$$\forall : \{i\in\mathbf{B}, e\in\mathbf{SHE}, k\in\mathbf{SH}, p\in\mathbf{PV}, a\in\{\mathbf{Slack}\}, j\in\{\mathbf{PV}\cup\mathbf{Slack}\}, l\in\mathbf{PQ}, m\in\mathbf{TAP}, n\in\mathbf{L}, t\in\mathbf{T}\}.$$

The objective function in (14) aims to minimize the investment costs of new capacitive and inductive reactive power compensators, the operation costs of capacitive and inductive reactive power compensators as well as the reactive load shedding costs. Reactive load shedding occurs in any bus, expressing the reactive power deficit of the planning process. In mathematical terms, the reactive load shedding term states the degree of infeasibility of the problem after the planning process.

The objective function in (14) is made up of the D_F, K_F, K_V, and K_{LS} factors. The D_F factor is mathematically defined as

$$D_F = \sum_{t\in\mathbf{T}} \frac{1}{(1+r)^t} \tag{31}$$

where D_F refers to the discount factor used to calculate the net present value of objective function in stage $t \in T$.

The K_F and K_V factors represent the investment and operation costs in each stage along the time horizon:

$$K_F = \sum_{k\in\mathbf{SH}} C_{FC\,k}(wc_{k,t} - wc_{k,t-1}) + \sum_{k\in\mathbf{SH}} C_{FR\,k}(wc_{k,t} - wc_{k,t-1}) \tag{32}$$

$$K_V = \sum_{s\in\mathbf{S}} \pi_s \left(\sum_{k\in\mathbf{SH}} C_{VCk} \cdot qc_{k,t} + \sum_{k\in\mathbf{SH}} C_{VRk} \cdot qr_{k,t} \right). \tag{33}$$

The K_{LS} factor represents the reactive load cost due to shedding in each stage along the time horizon:

$$K_{\mathrm{LS}} = \sum_{i\in\mathbf{B}} C_{\mathrm{LSC}i} \cdot yc_{i,t} + \sum_{i\in\mathbf{B}} C_{\mathrm{LSR}i} \cdot yr_{k,t}. \tag{34}$$

The objective function of the multi-stage deterministic model is subject to the same set of constraints of the single-stage deterministic model, but the constraints are evaluated in each stage now. Two new constraints are introduced in the multi-stage deterministic model, (25) and (26). They are the inter-period constraints, whose variables are associated with the investment costs in the objective function in (14), ensuring that these costs are charged only once in each stage.

IV. Multi-Stage Stochastic Mathematical Model for Reactive Power Planning

The mathematical model for the multi-stage stochastic solution of reactive power planning is as follows:

$$\min_{\substack{\mathbf{v},\delta,\mathbf{tap},\mathbf{qc},\mathbf{qr},\mathbf{wc},\\ \mathbf{wr},p^G,\mathbf{q^G}}} f = D_F(K_F + K_V) + K_{\mathrm{LS}} \tag{35}$$

s.t :

$$P^G_{p,t,s} + p^G_{e,t,s} - P^D_{l,t,s} - p_{i,t,s} = 0 \tag{36}$$

$$q^G_{j,t,s} - Q^D_{l,t,s} - q_{k,t,s} + q_{ck,t,s} - q_{rk,t,s} + q_{ce,t,s} - q_{re,t,s} = 0 \tag{37}$$

$$\underline{Q}^G_j \le q^G_{j,t,s} \le \bar{Q}^G_j \tag{38}$$

$$\underline{V}_i \le v_{i,t,s} \le \bar{V}_i \tag{39}$$

$$\underline{\mathrm{Tap}}_m \le \mathrm{tap}_{m,t,s} \le \overline{\mathrm{Tap}}_m \tag{40}$$

$$|s_{n,t,s}| \le \bar{S}_n \tag{41}$$

$$0 \le qc_{e,t,s} \le \overline{QC}_e \tag{42}$$

$$0 \le qr_{e,t,s} \le \overline{QR}_e \tag{43}$$

$$0 \le qc_{k,t,s} \le \overline{QC}_k \cdot wc_{k,t} \tag{44}$$

$$0 \le qr_{k,t,s} \le \overline{QR}_k \cdot wr_{k,t} \tag{45}$$

$$wc_{k,t-1} \le wc_{k,t} \tag{46}$$

$$wr_{k,t-1} \le wr_{k,t} \tag{47}$$

$$wc_k \in \{0,1\} \tag{48}$$

$$wr_k \in \{0,1\} \tag{49}$$

$$yc_{i,t,s} \ge 0 \tag{50}$$

$$yr_{i,t,s} \ge 0 \tag{51}$$

$$\forall : \{i \in \mathbf{B}, e \in \mathbf{SHE}, k \in \mathbf{SH}, p \in \mathbf{PV}, a \in \{\mathbf{Slack}\}, j \in \{\mathbf{PV} \cup \mathbf{Slack}\}, l \in \mathbf{PQ}, m \in \mathbf{TAP}, n \in \mathbf{L}, s \in \mathbf{S}, t \in \mathbf{T}.$$

The objective function in (35) is made up of the D_F, K_F, K_V, and K_{LS} factors. D_F and K_F are mathematically defined as in (31) and (32), and K_V and K_{LS} are now evaluated in each scenario:

$$K_V = \sum_{k\in\mathbf{SH}} C_{VCk} \cdot qc_{k,t,s} + \sum_{k\in\mathbf{SH}} C_{VRk} \cdot qr_{k,t,s} \tag{52}$$

$$K_{\mathrm{LS}} = \sum_{i\in\mathbf{B}} C_{\mathrm{LSC}i} \cdot yc_{i,t,s} + \sum_{i\in\mathbf{B}} C_{\mathrm{LSR}i} \cdot yr_{k,t,s} \tag{53}$$

where (52) represents the expected operation costs (EOC) and (53) the expected costs of reactive load shedding.

The model formulated in (35)–(51) is a multi-stage stochastic nonlinear model with a large number of variables and uncertainty parameters. In this model reactive load shedding is introduced in the objective function to measure the reactive power deficit of the planning process. As mentioned, the objective is to minimize the sum of investment costs (IC), expected operation costs (EOC) and reactive load shedding costs by optimizing sizes and locations of new reactive compensation equipment to ensure power system security in every stage.

Due to the fact that the stochastic parameters have a deep influence on the state and control variables of the system, it is important to keep in mind this relationship for the problem formulation. The power balance constraints in (36) and (37) and the complex power flow through the branches in (41) are related to the $\mathbf{Y_{bus}}$ matrix that contains information about the power system elements. The values of the matrix change when contingencies are considered. Therefore, taking into account these considerations, the changes in the $\mathbf{Y_{bus}}$ matrix when contingencies are considered are explained as follows.

The $\mathbf{Y_{bus}}$ matrix formulation in each stage is shown in (54) and (55). Equation (55) considers the contingency analysis due to line outages in any stage, where a new $\mathbf{Y_{bus}}$ contingency matrix is calculated for each contingency in any stage. The outage of a certain line is determined by α_t, if $\alpha_t = 0$ there is no line contingency ($\mathbf{Y^0_{bus,t}}$), otherwise, if $\alpha_t = 1$, a contingency occurs ($\mathbf{Y^C_{bus,t}}$):

$$\mathbf{Y_{bus,t}} = \mathbf{Y^0_{bus,t}} - \mathbf{Y^C_{bus,t}} \tag{54}$$

$$\mathbf{Y_{bus,t}} = \begin{bmatrix} \alpha_t \cdot y_{kk} & -\alpha_t \cdot y_{km} \\ -\alpha_t \cdot y_{mk} & \alpha_t \cdot y_{mm} \end{bmatrix} \tag{55}$$

where y_{kk}, y_{km}, y_{mk}, and y_{mm} are the branch admittances.

V. Uncertainty Modeling

Uncertainties related to power systems planning must be treated properly, especially when decisions are aimed toward a long-term planning where the level of uncertainty is higher. The key point is how to deal with uncertainty over time. An accurate modeling of uncertainty should combine all uncertain information using probability trees within operating scenarios and system operation states throughout the planning horizon.

In stochastic programming, one way to do it is to replace the random variables for their expectations and then solve the deterministic mathematical problem. Another way is to consider

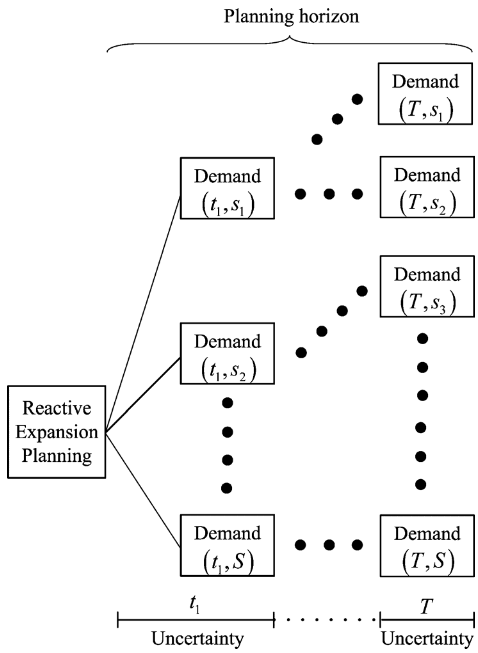

Fig. 1. Uncertainty modeling.

all scenarios with the corresponding probabilities and then select the best plan among all scenarios.

The number of outcomes needed to properly represent an uncertain parameter is usually very large. Traditionally, scenario-reduction techniques are used to reduce the cardinality of the set of outcomes while retaining the statistical properties of the uncertain parameters. Once the scenario tree with the original data is obtained, its size must be reduced to make it computationally affordable. In general, the original tree is too large. It is therefore necessary to develop an optimal reduction scenario technique. Many approaches are available for scenario generation such as Markov chains [22], sequential importance sampling [23], among others [24]–[26].

It is assumed that the normal probability density functions (PDF) of the system loads are available to all buses along the planning horizon. A scenario tree is represented by a finite number of nodes. It starts at the root node and finally terminates at the leaf nodes. Each node in the scenario tree has an individual predecessor node, but presumably several successors. Each path from the root node to the leaf nodes is defined as a scenario.

In this paper the methodology proposed in [27] is used, based on an approximate model involving a smaller number of scenarios so that the optimal values of the original problem and the approximate one remain close to each other. Therefore, scenario reduction techniques are applied to reduce the number of scenarios by deleting the scenarios with small probabilities or bundling similar scenarios. The reduction algorithms determine a subset of the initial scenarios and assign new probabilities to the preserved scenarios.

Uncertainty is modeled by a multi-stage scenario tree demand over the planning horizon, as shown in Fig. 1. Demand levels are associated with a given load profile. Each level of demand is associated with the probability extracted from an annual load curve, usually considering its duration.

VI. Expected Value of Perfect Information and Value of the Stochastic Solution

There are two key values in stochastic problems that allow us to quantify the model's stochasticity. These values are the EVPI and the VSS.

EVPI represents the loss of profit due to the presence of uncertainty, in other words, it measures the maximum amount (of money) that a decision maker would be willing to pay for complete and accurate information about the future [26]. EVPI measures the value of knowing the future with certainty [28], [29].

EVPI is the difference between the stochastic solution (SS) and the "wait-and-see" solution (WSS). The SS refers to the "here-and-now" solution of the stochastic problem. WSS is the expected value of the optimal solution, i.e., the weighted sum of the overall objective function for each scenario, knowing that each scenario will occur with certainty. EVPI is denoted as

$$\mathrm{EVPI} = \mathrm{SS} - \mathrm{WSS}. \tag{56}$$

VSS is the cost of ignoring uncertainty when a decision is made [26], it calculates the benefit of knowing the distributions of the stochastic variables [28]. In other words, VSS is used to calculate the importance of uncertainty in the optimization problem. VSS indicates the loss of profit due to the presence of uncertainty in the problem.

VSS is defined as the difference between the estimated result of using the expected value solution (EV) and the value of the objective function of the stochastic problem taking into account all possible scenarios. VSS is denoted as:

$$\mathrm{VSS} = \mathrm{EV} - \mathrm{SS} \tag{57}$$

where EV is the expected result when replacing the random variables with their expected values in the optimization problem, and SS is the "here-and-now" solution corresponding to the objective function value of the stochastic problem considering all possible scenarios [29].

In a multi-stage stochastic problem the expected benefits, EVPI and VSS, can also be calculated in each stage of the planning horizon. However, in this paper the analysis is performed considering the overall optimal solution of the entire planning horizon.

VII. Case Studies

To validate the proposed model, numerical results are presented for the Ward-Hale 6-bus system [30] and the IEEE 14-bus system [31]. The multi-stage stochastic mixed integer nonlinear program is solved by using the optimization solver KNITRO 8 [32] in AMPL v2012 [33], in a Dell PowerEdge R910x64 PC server, 128 GB of RAM and 1.87 GHz under the Windows Server 2008 operating system.

In these examples, the following considerations have been made:

- Two states of the power system are considered: 1) steady-state operation and 2) operation under contingencies.
- A 7% annual interest rate is utilized to calculate the present value of investment in the planning horizon, using the values listed in [34] and [35] as a reference.

TABLE I
WARD-HALE SYSTEM DEMAND DATA

Bus	Demand PDF in period t_0	
	$P^{\diamond}$	$Q^{\dagger}$
3	$P \sim N(55.00, 10.20)$	$Q \sim N(13.00, 15.40)$
5	$P \sim N(30.00, 13.30)$	$Q \sim N(18.00, 18.20)$
6	$P \sim N(20.00, 17.20)$	$Q \sim N(5.00, 16.20)$

$\diamond \rightarrow$ in MW; $\dagger \rightarrow$ in MVAr

TABLE II
INVESTMENT AND OPERATING COSTS AND MAXIMUM CAPACITY OF NEW REACTIVE SOURCES DATA

$\mathrm{Bus}_{\mathrm{SH}}$	$C^{\star}_{\mathrm{FC}}$, $C^{\star}_{\mathrm{FR}}$	$C^{\ddagger}_{\mathrm{VC}}$, $C^{\ddagger}_{\mathrm{VR}}$	$\overline{\mathrm{QC}}^{\dagger}$, $\overline{\mathrm{QR}}^{\dagger}$
1	500.00	5.00	50.00
2	480.00	2.00	80.00
3	300.00	4.00	60.00
4	500.00	4.50	70.00
5	440.00	2.50	90.00
6	370.00	3.10	75.00

$\star \rightarrow$ in \$; $\ddagger \rightarrow$ in \$/MVAr; $\dagger \rightarrow$ in MVAr

TABLE III
CONTINGENCY RANKING

Contingency	T_1	T_2	T_3
$N - K_1, K_1 = 1$	$\{L1-6\}$	$\{\emptyset\}$	$\{\emptyset\}$
$N - K_2, K_2 = 1$	$\{\emptyset\}$	$\{L1-6\}$	$\{\emptyset\}$
$N - K_3, K_3 = 1$	$\{\emptyset\}$	$\{\emptyset\}$	$\{L1-6\}$

- All buses are candidate buses of the set SH and the demand in stage T_0 is the average demand.
- In scenario generation, the load at bus i in period t must be greater than the load at bus i in period $t-1$, namely, $D_{i,t,s} \geq D_{i,t-1,s}$.
- The values of $C_{\mathrm{LSC}i}$ and $C_{\mathrm{LSR}i}$ are equal to \$10 000.
- The upper and lower voltage limits in all buses are [0.95–1.05] p.u.
- The upper and lower tap limits of the under-load tap changing transformers are [0.9–1.1] p.u.
- EVPI and VSS are computed considering steady-state operation.
- The annual expected load growth rates are assumed to be 6% for all loads.

A. Ward-Hale 6-Bus System

This system consists of 6 buses, 5 transmission lines, 2 generators, 2 under-load tap changing transformers and 2 reactive capacitive power sources.

Table I shows the PDF of the loads in period T_0.

Table II shows the investment and operation costs and maximum capacity of the new reactive capacitive and inductive compensators that can be installed.

Table III shows the set of contingencies for each stage $N - K_t$. For instance, for the case $N - K_1, K_1 = 1$ means that in stage T_1 a contingency (N-1) occurs, which is the outage of line L1-6. But, in the next stages, the line is reconnected. This consideration allows us to carry out the optimization of the sizing and allocation of reactive power sources ensuring a secure and economic operation for any demand scenario in any stage.

TABLE IV
RESULTS OF THE MULTI-STAGE DETERMINISTIC REACTIVE POWER PLANNING MODEL

Per	Bus	qc Cap.†	qc Disp.†	IC*	OC*
Steady-state operation					
1	–	–	–	–	–
2	–	–	–	–	–
3	5	60	11.28	224.89	36.84
$N - K_1, K_1 = 1$					
1	6	75	33.73	345.79	97.73
2	–	–	–	–	–
3	6	–	6.23	–	15.75
$N - K_2, K_2 = 1$					
1	–	–	–	–	–
2	6	75	48.25	323.17	130.64
3	6	–	6.23	–	15.75
$N - K_3, K_3 = 1$					
1	–	–	–	–	–
2	–	–	–	–	–
3	3 6	60 75	12.24 49.13	546.92	164.27

$\dagger \rightarrow$ in MVAr; $* \rightarrow$ in \$

Considering demand uncertainty, the probabilities of the original tree in each stage are: $T_0 = \{1.00\}$, $T_1 = \{0.250, 0.500, 0.250\}$, $T_2 = \{0.062, 0.125, 0.062, 0.125, 0.250, 0.125, 0.062, 0.1250.062\}$, $T_3 = \{0.016, 0.031, 0.016, 0.031, 0.062, 0.031, 0.016, 0.031, 0.016, 0.031, 0.062, 0.031, 0.062, 0.125, 0.062, 0.031, 0.062, 0.031, 0.016, 0.031, 0.016, 0.031, 0.062, 0.031, 0.016, 0.031, 0.016\}$.

After optimal reduction of the original tree, the probabilities to be preserved are: $T_0 = \{1.00\}$, $T_1 = \{0.141, 0.625, 0.234\}$, $T_2 = \{0.141, 0.094, 0.328, 0.203, 0.187, 0.047\}$, $T_3 = \{0.141, 0.094, 0.187, 0.141, 0.141, 0.062, 0.141, 0.047, 0.047\}$.

Table IV shows the results of the optimal reactive power planning considering steady state and also operating under contingency using the multi-stage deterministic model in (14)–(30).

The results obtained assuming the demand to be the average demand in each period are shown in Table IV. No investment is required in the initial planning stage, T_0. The results show that in steady-state operation there is no need to install reactive power compensators in the system in stages T_1 and T_2. This is equivalent to saying that the reactive power dispatch of the existing reactive sources is sufficient to ensure the normal operation of the system. A capacitive reactive source whose maximum capacity is 60 MVAr must be installed in bus 5 in stage T_3, with an installation cost (IC) of \$224.89 and an operation cost (OC) of \$36.84 to dispatch 11.28 MVAr. The total cost is \$261.73.

For the other cases operating under contingency, Table IV shows that most investments occur in the fourth case, where an investment in two capacitive reactive sources, one in bus 3 and the other in bus 6, is required in stage T_3. The total cost is \$711.19.

In all the cases analyzed with the multi-stage deterministic reactive power planning model, $yc_{i,t} = yr_{i,t} = 0$, i.e., load

TABLE V
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—STEADY-STATE OPERATION

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	6	75	28.69	345.79	11.72
2	1	6	–	13.45	0.00	75.02
	2	6	–	43.44		
	3	6	–	66.24		
3	1	3	60	0.24	244.89	84.51
		6	–	5.79		
	2	3	–	9.61		
		6	–	18.76		
	3	3	–	16.69		
		6	–	28.16		
	4	3	–	23.62		
		6	–	39.73		
	5	3	–	20.50		
		6	–	33.12		
	6	6	–	0.35		
	7	3	–	3.21		
		6	–	9.96		

† → in MVAr; ∗ → in $

TABLE VI
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—$\mathrm{N}-\mathrm{K}_1, \mathrm{K}_1=1$

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	3	60	30.96	626.16	108.67
		6	75	64.60		
	2	6	–	33.73		
	3	6	–	7.20		
2	1	3	–	2.47	0.00	62.10
		6	–	8.92		
	2	3	–	11.61		
		6	–	21.43		
	3	3	–	18.47		
		6	–	30.46		
3	1	3	–	0.24	0.00	84.52
		6	–	5.79		
	2	3	–	9.61		
		6	–	18.76		
	3	3	–	16.69		
		6	–	28.16		
	4	3	–	23.62		
		6	–	39.73		
	5	3	–	20.50		
		6	–	33.12		
	6	6	–	0.35		
	7	3	–	3.21		
		6	–	9.97		

† → in MVAr; ∗ → in $

TABLE VII
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—$\mathrm{N}-\mathrm{K}_2, \mathrm{K}_2=1$

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	6	75	28.69	345.79	11.72
2	1	3	60	17.97	698.75	270.94
		6	–	54.03		
	2	3	–	45.92		
		6	–	73.88		
	3	3	–	50.36		
		4	70	29.28		
		6	–	75.00		
	4	6	–	35.32		
	5	3	–	4.52		
		6	–	42.23		
	6	6	–	13.87		
3	1	3	–	0.24	0.00	84.51
		6	–	5.79		
	2	3	–	9.71		
		6	–	18.76		
	3	3	–	16.69		
		6	–	28.16		
	4	3	–	23.62		
		6	–	39.73		
	5	3	–	20.50		
		6	–	33.12		
	6	3	–	0.35		
	7	3	–	3.21		
		6	–	9.86		

† → in MVAr; ∗ → in $

TABLE VIII
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—$\mathrm{N}-\mathrm{K}_3, \mathrm{K}_3=1$

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	6	75	28.69	345.79	11.72
2	1	6	–	13.45	0.00	75.02
	2	6	–	43.44		
	3	6	–	66.24		
3	1	3	60	12.24	653.04	327.09
		6	–	49.13		
	2	3	–	38.51		
		6	–	70.36		
	3	3	–	50.27		
		4	70	20.13		
		6	–	75.00		
	4	3		51.12		
		4	–	62.39		
		6	–	75.00		
	5	3	–	50.45		
		4	–	40.23		
		6	–	75.00		
	6	3	–	5.36		
		6	–	43.02		
	7	3	–	19.93		
		6	–	55.70		
	8	6	–	28.83		
	9	6	–	13.49		

† → in MVAr; ∗ → in $

shedding is not required and the problem is feasible for all cases with the considerations established initially.

Tables V –VIII show the results of the optimal reactive power planning considering steady-state and under-contingency operation using the multi-stage stochastic model in (35)–(51). The expected operating cost (EOC) of the reactive power dispatch is computed in each stage in the reduced probability tree, where the investment cost (IC) is also computed in each stage.

Considering demand uncertainty in the three stages, no investment is required at the initial planning stage, T_0. As it can be seen, not all the investments are required in the same stage. For the steady-state operation case, the overall cost is \$761.93, requiring two reactive power sources, whereas for the three under-contingency operation cases the overall costs are: for $N-K_1, K_1=1$, \$881.45, and two reactive power sources are required, for $N-K_2, K_2=1$, \$1411.71, requiring two reactive power sources, and for $N-K_3, K_3=1$, \$1412.06, requiring two reactive power sources. The details of the results are shown in Tables V –VIII. The feasibility indicator is $yc_{i,t}=yr_{i,t}=0$.

TABLE IX
IEEE 14-BUS SYSTEM DEMAND DATA

Bus	Demand PDF in period t_0	
	$P^{\diamond}$	$Q^{\dagger}$
2	$P \sim N(21.70, 12.50)$	$Q \sim N(12.70, 14.00)$
3	$P \sim N(94.20, 16.60)$	$Q \sim N(19.00, 8.20)$
4	$P \sim N(47.80, 18.50)$	$Q \sim N(3.90, 11.50)$
5	$P \sim N(7.60, 10.20)$	$Q \sim N(1.60, 18.20)$
6	$P \sim N(11.20, 17.20)$	$Q \sim N(7.50, 15.45)$
9	$P \sim N(29.50, 15.40)$	$Q \sim N(16.60, 12.20)$
10	$P \sim N(9.00, 16.60)$	$Q \sim N(5.80, 6.80)$
11	$P \sim N(3.50, 17.80)$	$Q \sim N(1.80, 9.80)$
12	$P \sim N(6.10, 7.20)$	$Q \sim N(1.60, 15.15)$
13	$P \sim N(13.50, 11.20)$	$Q \sim N(5.80, 7.40)$
14	$P \sim N(14.90, 7.40)$	$Q \sim N(5.00, 10.14)$

$\diamond \rightarrow$ in MW; $\dagger \rightarrow$ in MVAr

To obtain the expected value of perfect information we need to compute WSS first, which is the mean value of all the deterministic solutions for each one of the possible levels of demand. With this consideration, WSS is \$891.84.

From (56), EVPI is

$$\begin{aligned} \text{EVPI} &= \$1107.71 - \$891.84 \\ &= \$215.87. \end{aligned}$$

If it were possible to know the future demand, the cost would be \$891.84 instead \$1107.71, saving \$215.87. Since it is impossible to know the future demand in advance, the best thing to do is to use SS as the best result. These results show that the stochastic model solution, taking into account randomness in the stochastic variables, is a good approximation since it is not too far from the result obtained by WSS.

From (57), VSS is

$$\begin{aligned} \text{VSS} &= \$1107.71 - \$761.92 \\ &= \$345.79. \end{aligned}$$

This means that, considering demand uncertainty, the investment and operation cost of the new reactive power sources is \$1107.71 instead of \$761.92, increasing the cost in \$344.79 due to the presence of uncertainty in the problem.

B. IEEE 14-Bus System

This test system consists of 14 buses, 17 transmission lines, 5 generators, 3 under-load tap changing transformers and 1 fixed reactive capacitive power source.

With the aim of putting under stress the reactive power generating capacity of existing generators in the base period, reactive power in each generation bus is reduced by a 50% factor.

Table IX shows the PDFs of the loads in period T_0.

Table X shows the investment and operation costs and the maximum capacity data.

In this case, a N-2 contingency is considered in each stage, consisting of the outage of line L2-5 and the load tap changer T4-9, ($\text{cr} = \{\text{L2}-5, \text{T4}-9\}$), as shown in Table XI.

Considering demand uncertainty, the probabilities of the original tree in each stage are: $T_0 = \{1.00\}$, $T_1 = \{0.250, 0.500, 0.250\}$, $T_2 = \{0.250, 0.125, 0.087, 0.125, 0.062, 0.125, 0.062, 0.1250.037\}$, $T_3 = \{0.075, 0.021, 0.016, 0.024, 0.062, 0.031, 0.016, 0.031, 0.016, 0.031, 0.037, 0.031, 0.062, 0.110, 0.037, 0.045, 0.042, 0.045, 0.016, 0.052, 0.016, 0.052, 0.037, 0.031, 0.016, 0.031, 0.016\}$.

TABLE X
INVESTMENT AND OPERATING COSTS AND MAXIMUM CAPACITY OF NEW REACTIVE SOURCES

Bus_{SH}	$C^{\star}_{\text{FC}}$, $C^{\star}_{\text{FR}}$	$C^{\ddagger}_{\text{VC}}$, $C^{\ddagger}_{\text{VR}}$	$\overline{\text{QC}}^{\dagger}$, $\overline{\text{QR}}^{\star}$
1	500.00	5.00	70.00
2	400.00	2.00	50.00
3	480.00	3.00	45.00
4	370.00	3.50	85.00
5	500.00	2.70	65.00
6	380.00	3.40	80.00
7	460.00	4.50	75.00
8	450.00	3.80	55.00
9	650.00	4.20	95.00
10	400.00	5.10	85.00
11	560.00	3.70	80.00
12	520.00	4.10	90.00
13	480.00	5.10	55.00
14	510.00	4.25	65.00

$\star \rightarrow$ in \$; $\ddagger \rightarrow$ in \$/MVAr; $\dagger \rightarrow$ in MVAr

TABLE XI
CONTINGENCY RANKING

Contingency	T_1	T_2	T_3
$N-K_1, K_1 = 2$	cr	$\{\emptyset\}$	$\{\phi\}$
$N-K_2, K_2 = 2$	$\{\emptyset\}$	cr	$\{\emptyset\}$
$N-K_3, K_3 = 2$	$\{\emptyset\}$	$\{\emptyset\}$	cr

After optimal reduction of the original tree, the probabilities to be preserved are: $T_0 = \{1.00\}$, $T_1 = \{0.234, 0.672, 0.094\}$, $T_2 = \{0.047, 0.187, 0.203, 0.469, 0.094\}$, $T_3 = \{0.047, 0.047, 0.141, 0.062, 0.141, 0.141, 0.187, 0.141, 0.094\}$.

Tables XII–XVI show the details of the results for optimal reactive power planning considering steady-state and under-contingency operation for the multi-stage deterministic and stochastic models. In this case the feasibility indicator is $yc_{i,t} = yr_{i,t} = 0$.

To calculate the expected value of perfect information, WSS is computed for the mean value of all the deterministic solutions for each one of the possible levels of demand. WSS is \$998.63.

From (56), EVPI is

$$\begin{aligned} \text{EVPI} &= \$1266.22 - \$998.63 \\ &= \$267.59. \end{aligned}$$

The cost of knowing the future demand would be \$998.63 instead \$1266.22, saving \$267.59.

From (57), VSS is

$$\begin{aligned} \text{VSS} &= \$1266.22 - \$934.31 \\ &= \$341.91. \end{aligned}$$

Considering demand uncertainty, the investment and operation cost of the new reactive sources is \$1266.22 instead of \$934.31, losing \$331.91 due to the presence of uncertainty in the problem.

The results show that the control and state variables are set to their optimum values (and within their limits) for all scenarios in each period of the planning horizon.

TABLE XII
RESULTS OF THE MULTI-STAGE DETERMINISTIC REACTIVE POWER PLANNING MODEL—STEADY-STATE OPERATION

Steady-state operation					
Per	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	–	–	–	–	–
2	4	85	5.24	323.17	16.01
3	4	–	12.25	–	34.99
$\mathrm{N}-\mathrm{K}_1, \mathrm{K}_1 = 2$					
Per	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	2	50	10.90	373.83	20.38
2	2	–	5.48	–	9.57
3	2	–	12.84	–	20.95
$\mathrm{N}-\mathrm{K}_2, \mathrm{K}_2 = 2$					
Per	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	–	–	–	–	–
2	2	50	18.68	349.38	32.63
3	2	–	12.84	–	20.95
$\mathrm{N}-\mathrm{K}_3, \mathrm{K}_3 = 2$					
Per	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	–	–	–	–	–
2	2	50	5.48	349.38	9.57
3	2	–	28.94	–	47.25

† → in MVAr; ∗ → in $

TABLE XIII
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—STEADY-STATE OPERATION

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	4	85	62.67	345.79	47.97
2	1	4	–	83.97	331.91	62.94
		6	80	9.54		
	2	6	–	5.03		
	3	4	–	67.14		
		6	–	8.42		
3	1	4	–	85.00	0.00	145.71
		6	–	24.92		
	2	4	–	79.48		
		6	–	9.50		
	3	4	–	85.00		
		6	–	49.08		
	4	4	–	62.98		
		6	–	7.62		
	5	4	–	48.06		
		6	–	4.64		
	6	4	–	19.46		
		6	–	17.61		
	7	4	–	4.25		
		6	–	20.02		
	8	6	–	11.87		

† → in MVAr; ∗ → in $

TABLE XIV
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—$\mathrm{N}-\mathrm{K}_1, \mathrm{K}_1 = 2$

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	4	85	77.50	345.79	81.56
	2	4		10.12		
2	1	4	–	83.97	331.91	62.94
		6	80	9.54		
	2	6	–	5.03		
	3	4	–	67.14		
		6	–	8.42		
3	1	4	–	85.00	0.00	145.71
		6	–	24.92		
	2	4	–	79.48		
		6	–	9.50		
	3	4	–	85.00		
		6	–	49.08		
	4	4	–	62.98		
		6	–	7.61		
	5	4	–	48.06		
		6	–	4.64		
	6	4	–	19.46		
		6	–	17.61		
	7	4	–	4.25		
		6	–	20.02		
	8	6	–	11.87		

† → in MVAr; ∗ → in $

TABLE XV
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—$\mathrm{N}-\mathrm{K}_2, \mathrm{K}_2 = 2$

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	4	85	62.67	345.79	47.97
2	1	4	–	85.00	331.91	87.59
		6	80	25.19		
	2	6	–	16.77		
	3	4	–	77.23		
		6	–	13.05		
	4	6	–	4.85		
3	1	4	–	85.00	0.00	145.71
		6	–	24.92		
	2	4	–	79.48		
		6	–	9.50		
	3	4	–	85.00		
		6	–	49.08		
	4	4	–	62.98		
		6	–	7.62		
	5	4	–	48.06		
		6	–	4.64		
	6	4	–	19.46		
		6	–	17.61		
	7	4	–	4.25		
		6	–	20.02		
	8	6	–	11.87		

† → in MVAr; ∗ → in $

VIII. CONCLUSION

This paper presents a new multi-stage stochastic nonlinear model for long-term reactive power planning. The model is developed to deal with demand uncertainty in the planning horizon using an efficient scenario tree generation and a scenario tree reduction methodology. The developed model allows the user to analyze contingencies in any planning stage or demand scenario due to its dynamic nature. The mathematical structure of the model is able to decide which two reactive sources should be installed: slow ones (fixed or switched banks) for foreseeable demand variations and fast ones (SVC, STATCOM) used when contingencies appear. EVPI and VSS values are computed to show the impact of uncertainties. To ensure the security of the power system under contingencies, the N-k contingency criterion is also applied to the deterministic and stochastic mathematical models. Numerical results have shown the improvement obtained solving the reactive power planning problem using the multi-stage stochastic method as well as the applicability of the proposed mathematical model. Our future research studies for reactive power planning will include: discrete banks represented

TABLE XVI
RESULTS OF THE MULTI-STAGE STOCHASTIC REACTIVE POWER PLANNING MODEL—$\mathrm{N}-\mathrm{K}_3, \mathrm{K}_3=2$

Per	Scen	Bus	qc Cap.†	qc Disp.†	IC*	EOC*
1	1	4	85	62.67	345.79	47.97
2	1	4	–	83.97	331.91	62.94
		6	80	9.54		
	2	6	–	5.03		
	3	4	–	67.14		
		6	–	8.42		
3	1	4	–	85.00	0.00	183.97
		6	–	43.05		
	2	4	–	85.00		
		6	–	19.81		
	3	4	–	85.00		
		6	–	69.33		
	4	4	–	72.13		
		6	–	13.06		
	5	4	–	53.87		
		6	–	13.03		
	6	4	–	28.00		
		6	–	23.46		
	7	4	–	16.24		
		6	–	21.32		
	8	4	–	5.03		
		6	–	19.23		

† → in MVAr; ∗ → in $

by reactive variables and the modeling of other uncertainties, such as the equivalent capability of reactive power sources and the location of new generators.

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