$P = \{ x \in \mathbb{R}^n : Ax = b, \; x \geq 0 \}$

d is <u>feas. direction</u> at $x \in P \overset{2}{\Longleftrightarrow}$ $d \neq 0 : x + \theta d \in P$, for some $\theta > 0$ $\overset{1}{\Longleftrightarrow}$ $\begin{cases} Ad = 0 \\ d_i \geq 0, \; \forall i \text{ for which } x_i = 0 \end{cases}$

1. d is feas. direction $\Rightarrow$ $Ad = 0$ ✓, $d_i \geq 0$ for $x_i = 0$

   - d is feas. direction : $\exists \theta > 0$ s.t. $x + \theta d \in P$

     $A(x + \theta d) = b$
     $Ax = b$
     $\Big\} \; \theta Ad = 0 \Rightarrow Ad = 0$

     $\hookrightarrow \begin{cases} A\bar{x} = b \\ \bar{x} \geq 0 \end{cases}$

   - $x_i = 0 \Rightarrow \cancel{x_i}^{\,0} + \theta \overset{>0}{d_i} \geq 0 \Rightarrow d_i \geq 0.$

2. $Ad = 0$, $d_i \geq 0$ for $x_i = 0$ $\Rightarrow$ d is feas. direction. ($x + \theta d \in P$ for some $\theta > 0$.)

- $Ad = 0 \Rightarrow A(x + \theta d) = b$, for all θ.

| $Ax = b$ ✓
| $x \geq 0$

- $x_i = 0$ then $d_i \geq 0 \Rightarrow x_i + \theta \underbrace{d_i}_{\geq 0} \geq 0$

$x_i > 0$, $x_i + \theta d_i \geq 0$ for some $\theta > 0$, as long as

$$0 < \theta < x_i / d_i$$