MS-E2121 - Linear optimization **DEADLINE:**
Homework 3 **28.02.2022 @23:55**

In this homework, you will be requested to complete the implementation of the revised simplex method, using the skeleton code provided here.

In addition you are requested to provide a report in pdf format answering the questions below.

Please submit your answers to this page before the given deadline. Late submissions will be penalised by 5 points per day after the deadline.

Problem 3.1: Implementing the revised simplex method

You are requested to complete the functions comprising the implementation of the revised simplex method. Each function represents a step of the method.

Each of these functions requires no more than a few lines of code. It is broken down in so many parts because it allows for testing the steps individually, making it easier to find mistakes (bugs) in the implementation. Make sure you test each of the functions by giving small examples for which you know what would be the expected behaviour.

Complete in the skeleton code the following functions:

1. `calculate_cr`
2. `select_entering_variable`
3. `calculate_reverse_basic_directions`
4. `calculate_step_length`
5. `select_leaving_variable`
6. `update_x!`
7. `update_B_ind!`
8. `update_invB!`

In addition, provide in your report an explanation of what each function accomplishes and how it does so. Use small numerical examples (the same you used to test your functions) to illustrate the expected behaviour.

Hint 1: The provided notebook includes cells for testing your function implementations using the LP problem from Exercise 4.4 (for which you have the model solutions here).

Hint 2: The function `select_leaving_variable` should return the row index corresponding to the leaving variable. For example, in the example provided, the initial basic variables are x_1, x_2 and x_4 and the function should return the index 3, corresponding to the leaving variable x_4.

Problem 3.2: Implementing the two-phase simplex method

Complete the function `two_phase` to implement the two-phase simplex method, using your implementation of the revised simplex method.

In addition, provide in your report an explanation of how a given problem is modified for the first phase and how the outputs of the first phase are used in the second phase.

Problem 3.3: Comparing variable selection rules

Solve the problems below using your implementation of the two-phase simplex method.

Provide in your report the optimal solutions and and compare the number of iterations required to find the optimal solution using Dantzig's or Bland's rule.

Hint 1: Recall that max. $f(x)$ is equivalent to min. $-f(x)$ (pay attention to reporting the correct sign for the objective function value though.)

Hint 2: You can also use Problem 3.3(a) to test the function `revised_simplex`, since a trivial initial BFS can be obtained.

a)

$$\begin{aligned}
\text{max. } & 10x_1 + 12x_2 + 12x_3 \\
\text{s.t.: } & x_1 + 2x_2 + 2x_3 \leq 20 \\
& 2x_1 + x_2 + 2x_3 \leq 20 \\
& 2x_1 + 2x_2 + x_3 \leq 20 \\
& x_1, \ldots, x_3 \geq 0.
\end{aligned}$$

b)

$$\begin{aligned}
\text{max. } & x_1 + x_2 + x_3 + x_4 \\
\text{s.t.: } & x_1 + x_2 + x_3 + x_4 \leq 20 \\
& x_1 + 2x_2 + 3x_4 \leq 24 \\
& 2x_1 + x_2 + 2x_3 \geq 16 \\
& 2x_1 + 3x_2 + x_3 + x_4 = 20 \\
& x_1, \ldots, x_4 \geq 0.
\end{aligned}$$

c)

$$\begin{aligned}
\text{min. } & x_1 - 2x_2 + 3x_3 + x_4 \\
\text{s.t.: } & x_1 + 2x_2 + 3x_3 + x_4 \leq 15 \\
& 2x_1 + x_2 + 5x_3 \geq 10 \\
& 2x_1 + 2x_2 + x_3 + x_4 \geq 10 \\
& x_1, \ldots, x_4 \geq 0.
\end{aligned}$$