In [1]:
```
# ASTR 513: Problem Set #1
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#from astropy.cosmology import w0waCDM
#cosmo = w0waCDM(H0=70, Om0=0.3, Ode0=0.7)
import matplotlib.pyplot as plt
from astropy.constants import c, G, kpc, R_sun, pc
import pandas as pd
import numpy as np
from sympy import *
import seaborn as sns
from scipy import integrate
sns.set(style='ticks', palette='Set2')
sns.set_style({'font.family': [u'serif']},{'text.size': '.3'})
#plt.rc('font',size=14,family='serif',style='normal',
       #variant='normal',stretch='normal',weight='normal')
%matplotlib inline
```
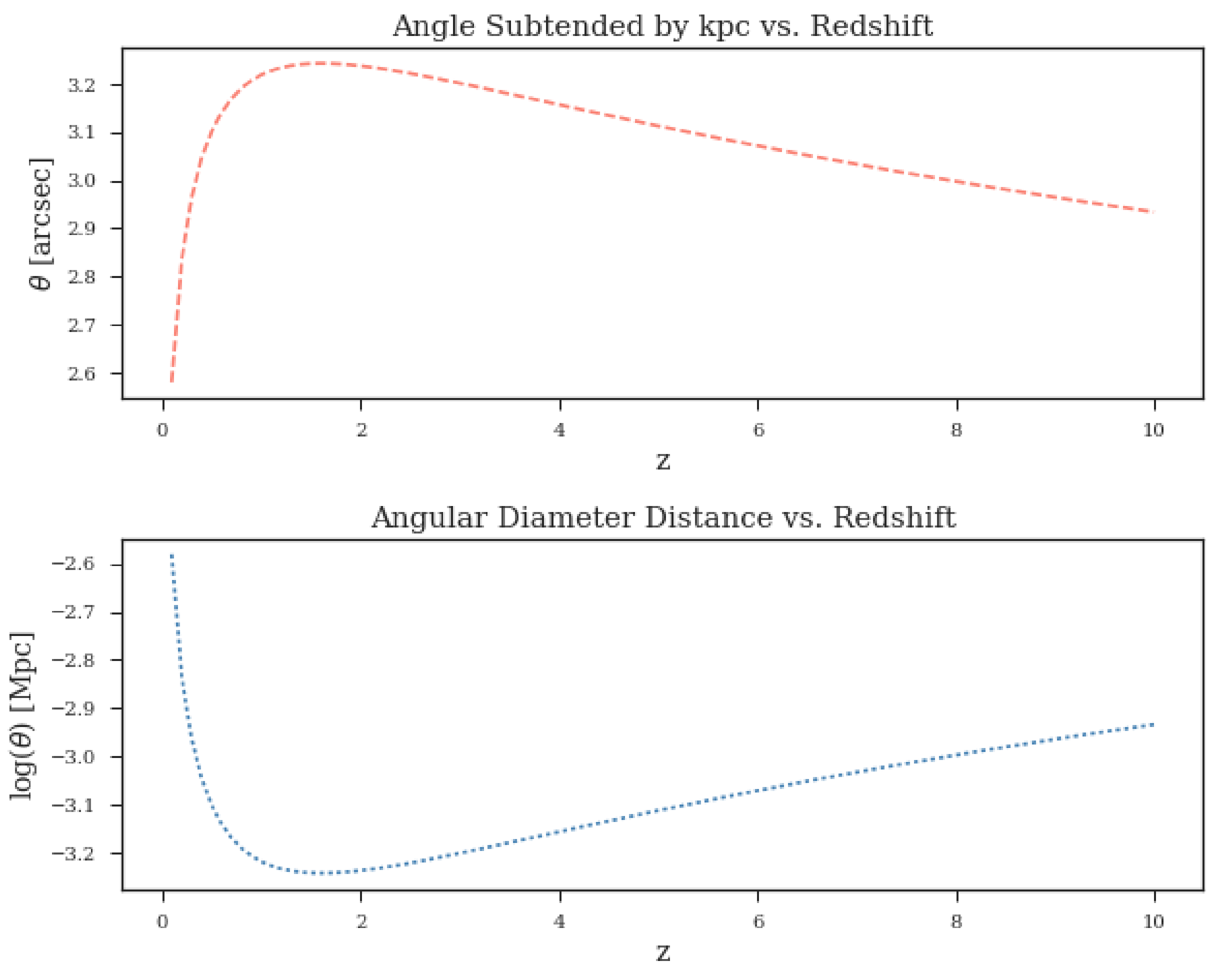

# *Problem 1: Angular Diameter Distances*

In [2]:
```
# Code up the cosmology calculations
omega_m, omega_lam, h = 0.3, 0.7, 0.7   # 1 - omega_m = omega_lam, flat
H_0 = h * 100 # MPC/h
z   = np.linspace(0.1,10, num=100)
c   = 3e5  #km/s

def angsize_per_kpc(lumin_dist,diameter,redshift):
    d_theta = (diameter*(1 + redshift)**2.)/lumin_dist
    return d_theta

def Hz(z): # Hubble function
    return 100*h*(omega_m*(1+z)**3+omega_lam)**.5

def Dconf(z): # conformal distance
    return c*integrate.romberg(dConfDistdz, 0, z)

def dConfDistdz(z):
    return 1./Hz(z)

def DA(z): # angular diameter distance
    return Dconf(z)/(1+z)

def Dhoriz(z):
    return integrate.quad(dConfDistdz, 1000, np.inf)
```

In [3]:
```
# Part A
# Galaxy-sized object with fixed-radius R = 1 kpc
# Plot angular size observed between z = 0.001 - 10
ang_diam = [DA(z[i]) for i in range(100)]
len(ang_diam)
d_theta  = 1./np.array(ang_diam)

fig = plt.figure(figsize=(10, 8))
ax1 = fig.add_subplot(211)
ax2 = fig.add_subplot(212)
plt.subplots_adjust(hspace = 0.4)

ax1.plot(z,np.log10(ang_diam),color='Salmon',linestyle='--')
ax1.set_title('Angle Subtended by kpc vs. Redshift',size=15)
ax1.set_ylabel(r'$\theta$ [arcsec]',size=14)
ax1.set_xlabel('z',size=14)

ax2.plot(z,np.log10(d_theta),':',color='SteelBlue')
ax2.set_title('Angular Diameter Distance vs. Redshift',size=15)
ax2.set_ylabel(r'log($\theta$) [Mpc]',size=14)
ax2.set_xlabel('z',size=14)
plt.show()
```




*Where does is the angular sized minimized? What is the size?*

```
The angular size of the 1 kpc galaxy is minimized at z~1.6 and is 2.
5".
```

In [4]:
```
# Part B
# Calculate comoving distance assuming velocity c
# "Comoving horizon"
cmov_horiz = Dhoriz(z)
#print(cmov_horiz[0]*c)
```

The comoving horizon is ~494.6 ± 0.003 Mpc.

In [9]:
```
# Part C
# What is the angle of subtended by the comoving horizon at z = 1000?
#cmov_d = Dconf(1000)
ang_cmovhoriz = c*cmov_horiz[0] / cmov_d
#print(ang_cmovhoriz*60)
```

The angle subtended by the comiving horizon at z = 1000 is 2.2 degrees.

# *Problem 2: The Expansion of the Universe*

## Part A:

At late times when Dark Energy dominates, the Friedmann equation from:

$$H^2 = H_0^2\ [\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0} + (1 - \Omega_{m,0} - \Omega_{r,0} - \Omega_{\Lambda,0})(1+z)^2]$$

becomes:

$$H^2 = H_0^2\ \Omega_{\Lambda,0}.$$

Since we know,

$$H = \left(\frac{\dot{a}}{a}\right)$$

we can determine that

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\ \Omega_{\Lambda,0}$$
$$= \sqrt{H_0^2\ \Omega_{\Lambda,0}}$$
$$= H_0\sqrt{\Omega_{\Lambda,0}}$$

$$\dot{a} = H_0\sqrt{\Omega_{\Lambda,0}}\ a$$

$$\frac{da}{dt} = H_0\sqrt{\Omega_{\Lambda,0}}\ a$$

$$\int \frac{da}{a} = \int H_0\sqrt{\Omega_{\Lambda,0}}\ dt$$

$$ln(a) = H_0\sqrt{\Omega_{\Lambda,0}}\ t$$

$$\therefore\ \ a = \ e^{H_0\sqrt{\Omega_{\Lambda,0}}\ t}$$

and the scale factor a is expanding exponentially.

## Part B:

*Friedmann eq 1:*

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}$$

where $k = 0$ in a flat universe.

*Friedmann eq 2:*

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi}{3}(\rho + 3p)$$

*Hubble constant*

$$H(t) = \frac{\dot{a}}{a}$$

For $H(t)$ to decrease monotonically:

$$\dot{H(t)} = \frac{\dot{a}}{a}$$
$$= \frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2$$
$$= -\frac{4\pi}{3}(\rho + 3p) - \frac{8\pi G\rho}{3}$$

where $(\rho + p) \geq 0$ must be true.

# *Problem 3: Densities & Sizes*

## Part A:

In [6]:
```
# Calculate density density of Hydrogen and Helium at z=0
# Compare to present day sea-level density
from astropy import units
H_0    = 70 * units.km / units.s / units.Mpc #cm/s/Mpc
p_crit = 3*(H_0.cgs)**2 / (8*np.pi*G.cgs)
#print(p_crit)

H_p  = p_crit * 0.75
He_p = p_crit * 0.25

m_H  = 1.673 * 10**(-24) * units.g
m_He = 6.647 * 10**(-24) * units.g
m_N  = m_He * 28

N_H  = H_p / m_H
N_He = He_p / m_He
#print(N_H, N_He)

p_air =  1.225 * units.kg / units.m**3
#print(H_p)
#print(p_air.cgs)
#print(N_H/(p_air.cgs/(2*m_N)))

p_m0 = p_crit*omega_m
#print(p_m0,omega_m,H_p)
z_Hatsea = (p_m0/H_p)**(1./3.) - 1
#print(z_Hatsea)
```

Using the assumption that the current universe is at the critical density, we can calculate the critical density:

$$\rho_c = \frac{3H^2}{8\pi G}.$$

Using a $H_0$ = 70 km/s/Mpc,

$$\rho_c = 9.2 \times 10^{-30}\ g/cm^3$$

and knowing that 25% by mass is Helium, leaving 75% to be comprised of Hydrogen, we determine that the present number densities of Hydrogen and Helium are $4.13 \times 10^{-6}$ $cm^{-3}$ and $3.46 \times 10^{-7}$ $cm^{-3}$, respectively.

Compared to the current sea-level density of molecules in the Earth's atmosphere (assuming all Nitrogen for simplicity), $3.29 \times 10^{18}$ $cm^{-3}$, the number density of Hydrogen is $1.25 \times 10^{-22}$ % the earth's sea-level density of molecules.

To determine what redshift the number density of hydrogen would be equal to the number density of molecules in our atmosphere, we need to use

$$\rho_{m@z} = \rho_{m,0}\ (1+z)^3$$

and solve for z,

$$(1+z)^3 = \frac{\rho_{sea\ level}}{\rho_{m,0}}$$
$$(1+z) = \sqrt[3]{\frac{\rho_{sea\ level}}{\rho_{m,0}}}$$
$$z = \sqrt[3]{\frac{\rho_{sea\ level}}{\rho_{m,0}}} - 1$$

given $\rho_{H@z} = \rho_{sea\ level} = 1.23 \times 10^{-3}\ g/cm^3$ and $\rho_{H,0} = 6.9 \times 10^{-30}\ g/cm^3$.

We find that the number density of Hydrogen will equal the density of sea-level molecules at z = 5.6 $\times$ $10^8$.

### Part B:

In [7]:
```
# Calculate comoving radius of spheres of different masses
# For dwarf galaxy mass, MW galaxy mass, galaxy cluster mass
p_m0 = p_crit*omega_m

def comov_radius(rho_m0,mass):
    return (mass/(4/3*np.pi*rho_m0))**(1./3.)

# Dwarf mass galaxy
M_d   = 10**10. * units.Msun
R_d_cmov = comov_radius(p_m0,M_d.cgs)
print('Dwarf Gxy R_comov',R_d_cmov/pc.cgs/10**6.)

# Milky Way mass galaxy
M_mw   = 10**12. * units.Msun
R_mw_cmov = comov_radius(p_m0,M_mw.cgs)
print('MW Gxy R_comov',R_mw_cmov/pc.cgs/10**6.)

# Galaxy Cluster
M_gc   = 10**15. * units.Msun
R_gc_cmov = comov_radius(p_m0,M_gc.cgs)
print('Gxy Cluster R_comov',R_gc_cmov/pc.cgs/10**6.)
```

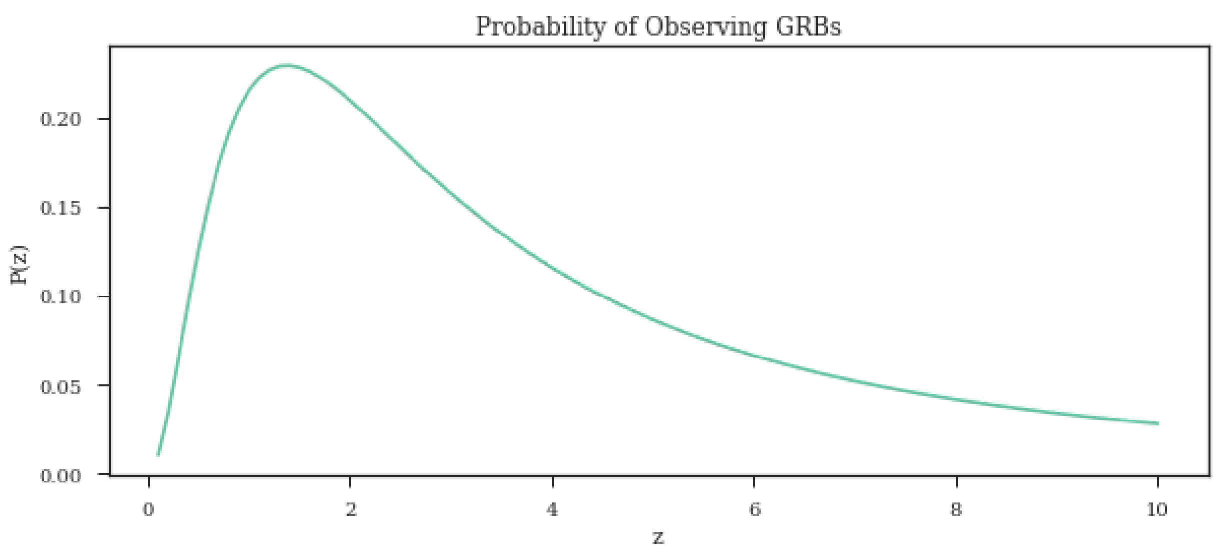
```
Dwarf Gxy R_comov 0.38826791886455997
MW Gxy R_comov 1.802180036651814
Gxy Cluster R_comov 18.021800366518132
```

The comoving radii of spheres that have halo masses of a dwarf galaxy ($10^{10}$ $M_\odot$), a Milky Way mass galaxy ($10^{12}$ $M_\odot$), and a galaxy cluster ($10^{15}$ $M_\odot$) are 0.39 Mpc, 1.8 Mpc, and 18 Mpc, respectively.

## *Problem 4: The Cosmic Volume*

In [8]:

```
# Assume Einstein deSitter cosmology (flat with W_m = 1)
# Rate of gamma ray bursts = star formation rate
# comoving star formation rate density [M-sun yr^-1 Mpc^-3] = constant (no z
# Calculate the probability of observing a GRB at given redshift.
# NOTE: RATES WILL REDSHIFT

def comov_volume(z):
    dp_GRB_dz = (4.*np.pi*c*Dconf(z)**2)/Hz(z)
    stuff = dp_GRB_dz
    return stuff

p_sfr = 1.

prob_GRB_dz = [(p_sfr*comov_volume(z[i])/(1+z[i])) for i in range(100)]
norm = sum(prob_GRB_dz)*(z[-1] - z[0])/100
#print(dp_GRB_dz)

fig = plt.figure(figsize=(10, 4))
ax1 = fig.add_subplot(111)

ax1.plot(z,(prob_GRB_dz/norm))
ax1.set_title('Probability of Observing GRBs')
ax1.set_ylabel('P(z)')
ax1.set_xlabel('z')
plt.show()
```




In [ ]: