

1. According to Lagrange multiplier,

$$l(\lambda) = f(x) - \lambda g(x)$$

$$f(a) = a^T B a$$

$$g(a) = a^T W a - 1 = 0 \quad (\text{evaluating } g(a) = 0 \text{ according to the law})$$

$$l(\lambda) = a^T B a - \lambda (a^T W a - 1)$$

$$\frac{dl}{da} = \frac{d}{da} (a^T B a) - \lambda \frac{d}{da} (a^T W a - 1)$$

$$= (1 \times B a) + a^T (B \times 1) - \lambda (1 \times W a + a^T W \times 1 - 0)$$

$$= B a + a^T B - \lambda (W a + a^T W)$$

$$= B a + B^T a - \lambda (W a + W^T a) \quad [B \& W \text{ are co-variance \& symmetrical}]$$

$$= a(B + B^T) - \lambda (W + W^T) a$$

To find the maxima, let it equal to 0.

$$a(B + B^T) - \lambda a(W + W^T) = 0$$

$[(W + W^T)]^{-1} (B + B^T) a = \lambda a$. Assuming W & B are symmetric, this is a standard eigenvalue problem.