CS2180 Artificial Intelligence Lab (Jan-May 2023)

Department of Computer Science and Engineering
Indian Institute of Technology Palakkad

Assignment 1: Simulations (Given: 17 Jan 2023, Due: 31 Jan 2023 2pm)

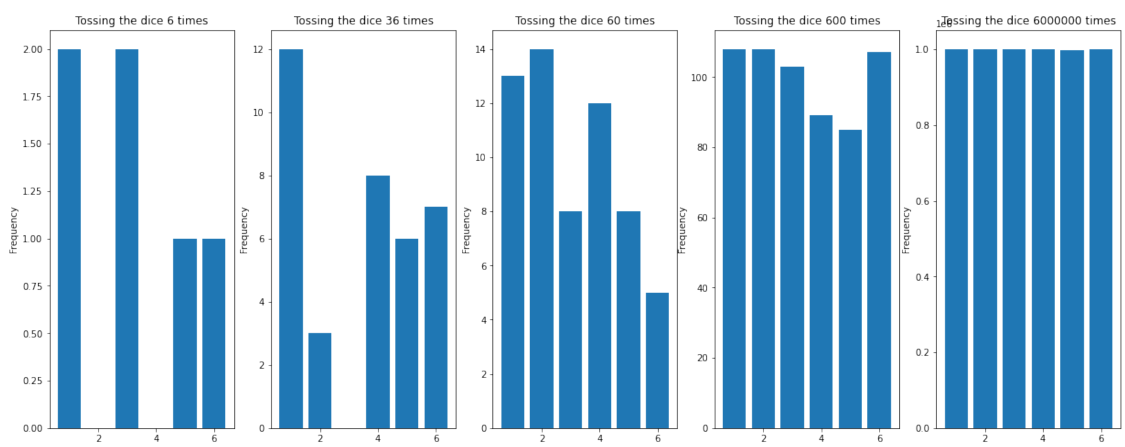
General instructions

- Solutions are to be typed in the `.ipynb` file provided and uploaded in the lab course page in Moodle before the due date.
- Your code should be well commented and should be compatible with python3.
- For this assignment, you are allowed to import the libraries `random` and `mathplotlib` of python3. No other libraries may be imported.
- For questions involving constructing plots, sample outputs may be provided. Your answers need not exactly match them. However if your plots are significantly deviating from what is given, it is possibily an indication that your code is doing something incorrect.

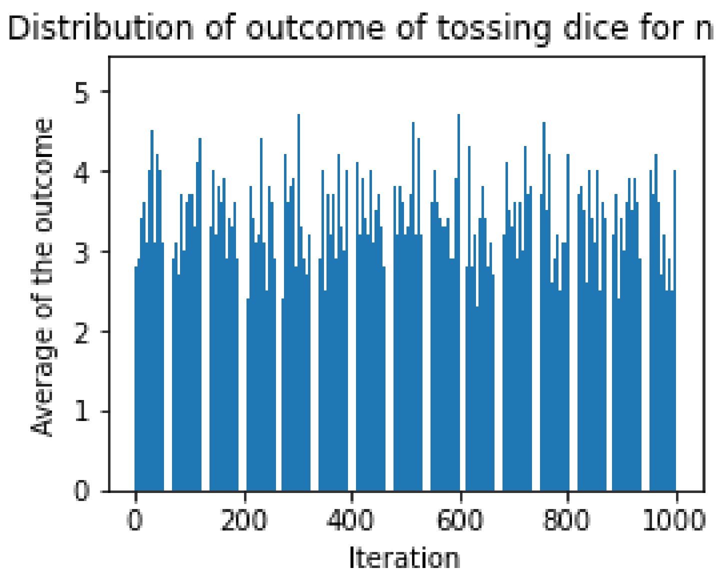
1 Law of Large Numbers

(a) Write a function `simulateDice(n)` that rolls a six-sided die n times and returns the frequencies of the results, i.e., the number of times 1 appears, the no. of times 2 appears, etc.

(b) For each $n \in \{6, 6^2, 60, 600, 6 \times 10^6\}$, run `simulateDice(n)` and plot a bar chart with outcomes of die rolls against the frequencies. A sample output looks as follows.



(c) Write a function `avgDice(n)` that calls `simulateDice(n)` and compute the average of the n outcomes. Run avgDice(n) 1000 times and plot a bar chart with iterations (i.e., 1 to 1000) against the average values. A sample output looks for $n = 10$ as follows.



(d) Repeat part (c) for each $n \in \{10^2, 10^3, 10^5\}$. How does the chart change with n?

2 Birthday Paradox

(a) Write a function `simulateBday(n)` that generates n random birthdays (i.e., n random integers between 1 and 366) and determine the number of pairs of same birthdays.

(b) Run `simulateBday(n)` 100 times and determine the number x of times that the n birthdays have an equal pair. Treat x/n as the probability of two same birthdays.

(c) Run `simulateBday(n)` for $n \in [100]$ where $[k]$ denotes the set $\{1, 2, \ldots, k\}$.

(d) What is the minimum n that guarantees two same birthdays? Identify a range of values of n that makes the probability of two same birthdays greater than 0.5.

3 Monty Hall Game

You are on a game show and given the choice of picking one of the three boxes (X, Y, Z). Two of the boxes are empty and one contains a gift. You pick a box, say X, and the host (who knows what is inside each of the boxes) opens another box, say Y, which is empty. You now have the option of retaining your choice (X) or switching it (to Z). Is it to your advantage to switch your choice in order to get the gift? This advantage is defined as the fraction of times switching the choice leads to a win when the game is played 1000 times.

4 Game of Dice

Simulate the following dice game.

1. Roll two six-sided dice.
   - If the sum is 7 or 11 on the first roll, you win.
   - If the sum is 2, 3 or 12 on the first roll, you lose.
   - If the sum is 4, 5, 6, 8, 9 or 10 on the first roll, that sum becomes your point.
2. Continue rolling the dice until the sum is 7 (game lost) or equal to your point (game won).

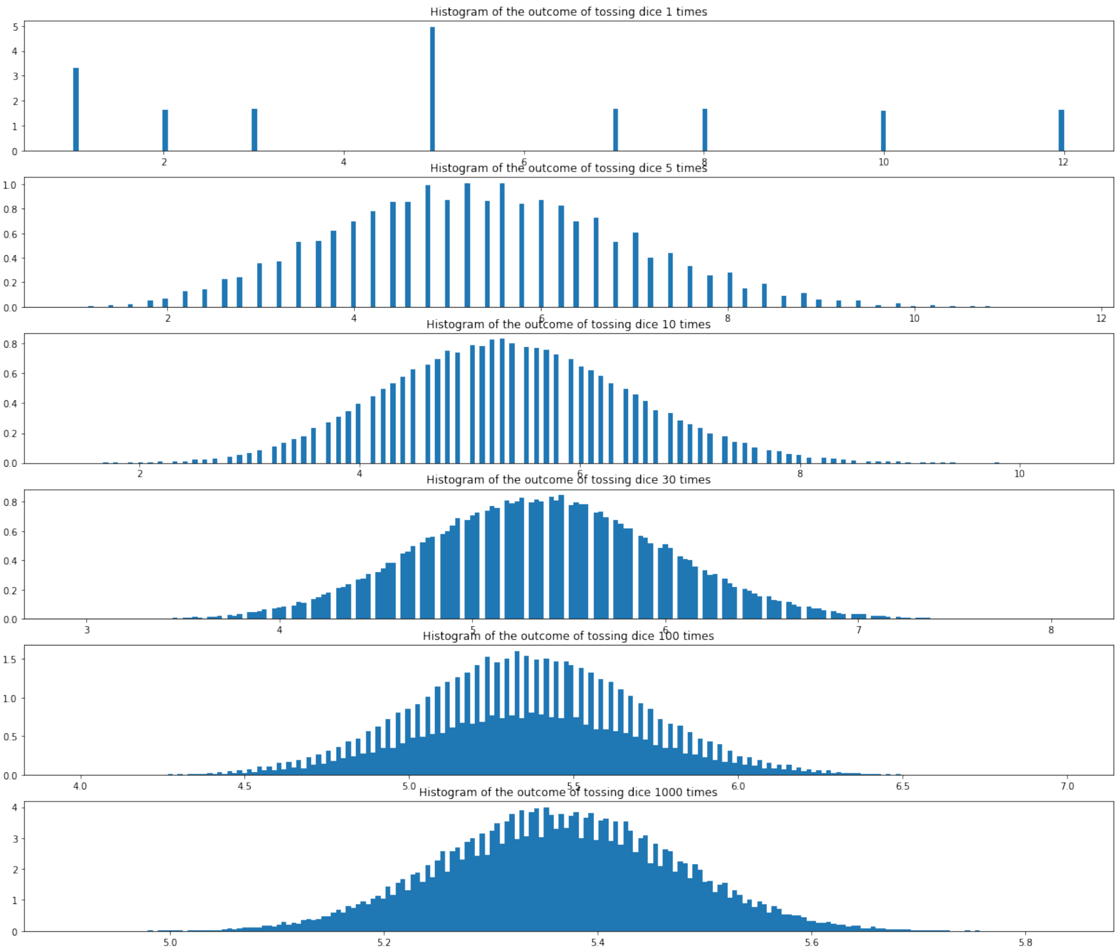
If n games are played and q of these games are won, then the chance of winning is q/n. Sample simulation of the game for $n = 10$ is as follows. What is the chance of winning at this game?

```
---
Player rolled 1 + 1 = 2
Point is 2
Player loses
---
Player rolled 5 + 3 = 8
Point is 8
Player rolled 6 + 3 = 9
Player rolled 6 + 6 = 12
Player rolled 6 + 5 = 11
Player rolled 5 + 2 = 7
Player loses
---
Player rolled 2 + 4 = 6
Point is 6
Player rolled 5 + 1 = 6
Player wins
---
Player rolled 6 + 3 = 9
Point is 9
Player rolled 3 + 3 = 6
Player rolled 1 + 2 = 3
Player rolled 3 + 4 = 7
Player loses
---
Player rolled 2 + 5 = 7
Point is 7
Player wins
---
Player rolled 6 + 4 = 10
Point is 10
Player rolled 3 + 3 = 6
Player rolled 6 + 1 = 7
Player loses
---
Player rolled 2 + 6 = 8
```

```
Point is 8
Player rolled 4 + 1 = 5
Player rolled 4 + 4 = 8
Player wins
---
Player rolled 4 + 5 = 9
Point is 9
Player rolled 3 + 1 = 4
Player rolled 1 + 5 = 6
Player rolled 1 + 6 = 7
Player loses
---
Player rolled 5 + 6 = 11
Point is 11
Player wins
---
Player rolled 3 + 4 = 7
Point is 7
Player wins
---
Total wins =  5
```

5 Central Limit Theorem

From a set $S = \{1, 1, 2, 3, 5, 5, 5, 7, 8, 10, 12\}$ of numbers, pick $n = 1$ numbers at random and compute the average. Repeat this 10^5 times and plot the frequency of the average values (i.e., how many times each average value occurs) as a histogram. Repeat this experiment with $n \in \{5, 10, 30, 100, 1000\}$.


Histogram of the outcome of tossing dice 1 times
Histogram of the outcome of tossing dice 5 times
Histogram of the outcome of tossing dice 10 times
Histogram of the outcome of tossing dice 30 times
Histogram of the outcome of tossing dice 100 times
Histogram of the outcome of tossing dice 1000 times