Indian Institute of Technology Palakkad
Department of Computer Science and Engineering

CS5616 Computational Complexity

January – May 2024

Mid Exam | Total Points – 42
Name: | 2 March 2023
Roll no: | 10.00 – 11.30 AM

1. (12 points) (a) (4 points) Let $\mathsf{REC} = \{M \mid L(M) \text{ is recursive}\}$ and

$$\mathsf{REN} = \{M \mid L(M) \text{ is recursive enumerable}\}.$$

   Find the smallest integers i and j such that $\mathsf{REC} \in \Pi_i$ and $\mathsf{REN} \in \Delta_j$.

   (b) (4 points) Is the language $K = \{M_x\#x \mid M_x \text{ halts on } x\}$ undecidable ? Justify.

   (c) (4 points) Recall that $\mathsf{AH} = \bigcup_{i \geq 0} \Sigma_i$. Argue that for any $i \geq 0$, $\mathsf{AH} \neq \Sigma_i$.

2. (10 points) For the following statements, either prove them true by a providing a proof or prove them false by a counter-example. Let A, B be languages.

   (a) (2 points) If $A \leq_T B$ and A is undecidable then B is undecidable.

   (b) (2 points) If $A \leq_T B$ and A is decidable then B is decidable.

   (c) (2 points) For some pair of languages A, B, we have $A \leq_T A \cup B$ and $B \leq_T A \cup B$.

   (d) (2 points) For any pair of languages A, B, we have $A \leq_T A \cup B$ and $B \leq_T A \cup B$.

   (e) (2 points) If A is Σ_1-complete and $B \subseteq A$, then B cannot be Σ_1-complete.

3. (10 points) Answer both the parts.

   (a) (5 points) Show that $\mathsf{MP}^{\mathsf{MP}}$ is Σ_2-complete.

   (b) (5 points) Define MPXOR as follows:

$$\{(M_1, M_2, x) \mid (x \in L(M_1) \text{ and } x \notin L(M_2)) \text{ or } (x \notin L(M_1) \text{ and } x \in L(M_2))\}$$

   Here M_1 and M_2 are valid Turing machine descriptions and $x \in \Sigma^*$. Show that $\mathsf{MPXOR} \in \Delta_2$.

4. (10 points) Consider a lexicographic total ordering of strings in Σ^*.

   (a) (5 points) Show that a recursively enumerable set is recursive if and only if there exists an enumeration machine that enumerates it in the increasing lexicographic order.

   (b) (5 points) Show that every infinite recursively enumerable set must contain an infinite recursive set.