INDIAN INSTITUTE OF TECHNOLOGY PALAKKAD
Department of Computer Science and Engineering
CS5616 Computational Complexity
January – May 2024

IIT PALAKKAD

Problem Set – 4 | Total Points – 50

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Given on 15 Mar
Due on 5 Apr

Instructions

- Use of resources other than class notes and references is forbidden.
- Collaboration is not allowed. Credit will be given for attempts and partial answers.

1. (10 points) [**Resource ?**]

   Consider the following definitions where M is a valid Turing machine encoding and $x \in \Sigma^*$. We use $\perp$ for value being undefined. Check if the follows are a resource using Blum's axioms. In case they don't, give a formal argument.
   (*Hint*: Decidability of the policing language !)

   (a) (5 points) (Head turns)

   $$h(M,x) = \begin{cases} \#\text{ turns that input head of } M \text{ makes on } x \text{ before halting} & M \text{ halts on } x \\ \perp & \text{otherwise} \end{cases}$$

   (b) (5 points) (Count) Let q_0 be a state in the finite control of M.

   $$c(M,x) = \begin{cases} \#\text{ of times } M \text{ visits a state } q_0 \text{ until it accepts or rejects} & M \text{ halts on } x \\ \perp & \text{otherwise} \end{cases}$$

   Solution:

   (a)

   (b)

2. (10 points) [**Oblivious Turing machines**]

   A Turing machine M is *oblivious* if for every input $x \in \Sigma^*$ and every $i \in \mathbb{N}$, the location of $M's$ head at the i^{th} step of execution is only a function of $|x|$ and i. In other words, the head movement does not depend on x. For a time constructible $t(n)$ show that if $L \in \mathsf{DTIME}(t(n))$, then there is a two tape oblivious Turing machine that decides L in $O(t(n)^2)$ time.

Solution:

3. (15 points) [**More on crossing sequences**]

   In class, we saw $o(\log \log n)$ space restricted Turing machines must be regular. Here, we show similar results for time restricted machines.

   (a) (5 points) Show that there exists a non-regular language that can be accepted by a one tape $O(n \log n)$ time Turing machine.

   (b) (10 points) Show that any language accepted by a one tape deterministic Turing machine in time $o(n \log n)$ must be regular.

Solution:

(a)

(b)

4. (15 points) [**Definition of space**]

   In this question, we explain why the definition of space complexity counts the cells that the machine scans and not just the ones that are written. Let $L = \{w\#w \mid w \in \{0,1\}^*\}$.

   (a) (5 points) Show that $L \in \mathsf{DSPACE}(\log n)$.

   (b) (10 points) Show that there exists a non-determinisitic Turing machine that writes only on cell to the right of left end marker symbol of its worktape and accepts $\overline{L}$. The machine may scan any number of worktape cells without writing on them.

Solution:

(a)

(b)