IIT PALAKKAD

Indian Institute of Technology Palakkad
Department of Computer Science and Engineering

CS5616 Computational Complexity

January – May 2024

Problem Set – 3	Total Points – 50
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Instructions

- Use of resources other than class notes and references is forbidden.
- Collaboration is not allowed. Credit will be given for attempts and partial answers.

1. (10 points) (**RE vs co-RE**) Show that the set

$$\{M \mid M \text{ halts on all inputs of length less than } 42\}$$

is recursively enumerable, but is its complement is not.

Solution:

$L = \{M \mid M \text{ halts on all inputs of less than } 42\}$

$L = \{M | \forall x \in \Sigma^* (|x| < 42 \implies M \text{ halts on x})\}$

$L = \{M | \forall x \in \Sigma^* (|x| < 42 \implies \exists t \in \mathbb{N}, t > 0 M \text{ halts in } t \text{ steps})\}$

$L = \{M | \forall x \in \Sigma^*, \exists t \in \mathbb{N}, t > 0 (|x| < 42 \implies M \text{ halts in } t \text{ steps})\}$

The decidable predicate R here is $\{(M, x, t) \mid M \text{ halts on } x \text{ in } t \text{ steps}\}$

By the characterisation of arithmetic hierarchy, we can conclude that $L \in \Sigma_1$, Hence L is recursively enumerable.

2. (10 points) (**Alternate definition for** Δ_i) Let A be any language. Define $\mathcal{D}^A$ be the class of all languages L such that L is decidable in A. Similarly, $\mathcal{SD}^A$ be the class of all L such that L is semi-decidable in A and co$\mathcal{SD}^A$ be the class of all languages whose complement is in $\mathcal{SD}^A$.

   (a) (5 points) Show that $\mathcal{D}^A = \mathcal{SD}^A \cap \text{co}\mathcal{SD}^A$.

   (b) (5 points) For any $i \geq 1$, by definition, $\Delta_i = \Sigma_i \cap \Pi_i$. Show that

$$\Delta_i = \{L \mid \text{ there exists } A \in \Sigma_{i-1} \text{ such that } L \text{ is decidable in A}\}.$$

Solution:

(a)

(b)

3. (10 points) (**Closure properties of** Σ_n, Π_n). Fix any $i \geq 1$. Show that Σ_i as well as Π_i are closed under intersection and union.

Solution:

4. (20 points) (**Rice's theorem**) Identify if the following are (0) properties of SD languages, (1) non-trivial properties and (2) non-monotone properties. If (1) / (2) is true, apply Rice's theorems suitably and give your conclusions. A direct use of diagonalisation or reductions does not fetch any credit.

   (a) (4 points) given a Turing machine M, $L(M)$ is not regular ?

   (b) (4 points) given a Turing machine M, does there exist a non-empty regular set L' such that $L' \subseteq L(M)$?

   (c) (4 points) given M, does M represent a DFA that accepts some string with equal number of 0s and 1s ?

   (d) (4 points) given a Turing machine M, is $L(M) \in \Pi_{42}$?

Solution:

(a)

(b)

(c)

(d)