



IIT PALAKKAD

INDIAN INSTITUTE OF TECHNOLOGY PALAKKAD

Department of Computer Science and Engineering

CS5616 Computational Complexity

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Problem Set – 2

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Total Points – 50

Given on 09 Feb

Due on 16 Feb

Instructions

- Use of resources other than class notes and references is forbidden.
- Collaboration is not allowed. Credit will be given for attempts and partial answers.

1. (10 points) [**Properties of \leq_m**] Show that \leq_m relation is reflexive and transitive over languages in Σ^* . Is \leq_m symmetric ? Argue.

Solution: Write your answer here.

2. (20 points) [**Reduction by containment !**] Let $L_1, L_2 \subseteq \Sigma^*$ where $L_1 \subseteq L_2$. Consider the statements **(1)** “ $L_2 \leq_m L_1$ ” **(2)** “ $L_1 \leq_m L_2$ ”
 - (a) (5 points) Give a pair of languages where **(1)** is true. Provide appropriate justification if no such pair exists.
 - (b) (5 points) Give a pair of languages where **(2)** is true. Provide appropriate justification if no such pair exists.
 - (c) (10 points) Prove or disprove the following statements (a), (b).
 - (a) “for any L_1, L_2 with $L_1 \subseteq L_2$, **(1)** is true.”
 - (b) “for any L_1, L_2 with $L_1 \subseteq L_2$, **(2)** is true.”
3. (10 points) [**Proof of Rice’s theorem 1**] In the proof of Rice’s theorem for showing undecidability done in class, we assumed that the non-trivial property \mathcal{P} is false for \emptyset . Explain how can this assumption be removed. [*Hint: Use \overline{HP} !*]
4. (10 points) [**Rice’s theorem ?**] Define the language $\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$.
 - (a) Describe the language $\overline{\text{TOTAL}}$ in a way similar to the language TOTAL .
 - (b) Show that the languages TOTAL as well as $\overline{\text{TOTAL}}$ are not semi-decidable.