



IIT PALAKKAD

INDIAN INSTITUTE OF TECHNOLOGY PALAKKAD

Department of Computer Science and Engineering

CS5616 Computational Complexity

January May 2024

Problem Set – 2

Name: Your name

Roll no: 11223344

Total Points – 50

Given on 09 Feb

Due on 16 Feb

### Instructions

- Use of resources other than class notes and references is forbidden.
- Collaboration is not allowed. Credit will be given for attempts and partial answers.

1. (10 points) [**Properties of  $\leq_m$** ] Show that  $\leq_m$  relation is reflexive and transitive over languages in  $\Sigma^*$ . Is  $\leq_m$  symmetric ? Argue.

**Solution:** Write your answer here.

2. (20 points) [**Reduction by containment !**] Let  $L_1, L_2 \subseteq \Sigma^*$  where  $L_1 \subseteq L_2$ . Consider the statements **(1)** “ $L_2 \leq_m L_1$ ” **(2)** “ $L_1 \leq_m L_2$ ”
  - (a) (5 points) Give a pair of languages where **(1)** is true. Provide appropriate justification if no such pair exists.
  - (b) (5 points) Give a pair of languages where **(2)** is true. Provide appropriate justification if no such pair exists.
  - (c) (10 points) Prove or disprove the following statements (a), (b).
    - (a) “for any  $L_1, L_2$  with  $L_1 \subseteq L_2$ , **(1)** is true.”
    - (b) “for any  $L_1, L_2$  with  $L_1 \subseteq L_2$ , **(2)** is true.”
3. (10 points) [**Proof of Rice’s theorem 1**] In the proof of Rice’s theorem for showing undecidability done in class, we assumed that the non-trivial property  $\mathcal{P}$  is false for  $\emptyset$ . Explain how can this assumption be removed. [*Hint: Use  $\overline{\text{HP}}$  !*]
4. (10 points) [**Rice’s theorem ?**] Define the language  $\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$ .
  - (a) Describe the language  $\overline{\text{TOTAL}}$  in a way similar to the language  $\text{TOTAL}$ .
  - (b) Show that the languages  $\text{TOTAL}$  as well as  $\overline{\text{TOTAL}}$  are not semi-decidable.