

EXTENDS *Integers*

VARIABLES *pc0, pc1, turn, intr0, intr1*

$TypeOK \triangleq$   
 $\wedge pc0 \in 0 \dots 7$   
 $\wedge pc1 \in 0 \dots 7$   
 $\wedge turn \in \{0, 1\}$   
 $\wedge intr0 \in \{0, 1\}$   
 $\wedge intr1 \in \{0, 1\}$

asynchronous system

for  $i = 0, 1$

the algorithm is

```

0 : while(True){
1 :   //non critical section
2 :   intr[i] = True
3 :   turn = 1 - i
4 :   while(turn  $\triangleq$  1 and intr[1 - i]  $\triangleq$  1)//wait
5 :   //critical section
6 :   intr[i] = 0
7 : }
```

$Init0 \triangleq$   
 $\wedge turn = 0$   
 $\wedge intr0 = 0$   
 $\wedge pc0 = 0$

$Init1 \triangleq$   
 $\wedge turn = 0$   
 $\wedge intr1 = 0$   
 $\wedge pc1 = 0$

$Init \triangleq Init0 \wedge Init1$

$L01 \triangleq$   
 $\wedge pc0 = 0$   
 $\wedge pc0' = 1$   
 $\wedge UNCHANGED \langle turn, intr0, intr1 \rangle$

$L12 \triangleq$   
 $\wedge pc0 = 1$   
 $\wedge pc0' = 2$   
 $\wedge UNCHANGED \langle turn, intr0, intr1 \rangle$

$$\begin{aligned}
L23 &\triangleq \\
&\wedge pc0 = 2 \\
&\wedge pc0' = 3 \\
&\wedge intr0' = 1 \\
&\wedge \text{UNCHANGED } \langle intr1, turn \rangle
\end{aligned}$$

$$\begin{aligned}
L34 &\triangleq \\
&\wedge pc0 = 3 \\
&\wedge pc0' = 4 \\
&\wedge turn' = 1 \\
&\wedge \text{UNCHANGED } \langle intr0, intr1 \rangle
\end{aligned}$$

$$\begin{aligned}
L44 &\triangleq \\
&\wedge pc0 = 4 \\
&\wedge pc0' = 4 \\
&\wedge turn = 1 \\
&\wedge intr1 = 1 \\
&\wedge \text{UNCHANGED } \langle intr0, intr1, turn \rangle
\end{aligned}$$

$$\begin{aligned}
L45 &\triangleq \\
&\wedge pc0 = 4 \\
&\wedge pc0' = 5 \\
&\wedge (turn = 0 \vee intr1 = 0) \\
&\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle
\end{aligned}$$

$$\begin{aligned}
L56 &\triangleq \\
&\wedge pc0 = 5 \\
&\wedge pc0' = 6 \\
&\wedge \text{UNCHANGED } \langle intr0, intr1, turn \rangle
\end{aligned}$$

$$\begin{aligned}
L67 &\triangleq \\
&\wedge pc0 = 6 \\
&\wedge pc0' = 7 \\
&\wedge intr0' = 0 \\
&\wedge \text{UNCHANGED } \langle intr1, turn \rangle
\end{aligned}$$

$$\begin{aligned}
L70 &\triangleq \\
&\wedge pc0 = 7 \\
&\wedge pc0' = 0 \\
&\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle
\end{aligned}$$

$$SLOGP \triangleq \text{UNCHANGED } \langle pc0, intr0, intr1, turn \rangle$$

for the second system

$$\begin{aligned}
M01 &\triangleq \\
&\wedge pc1 = 0 \\
&\wedge pc1' = 1
\end{aligned}$$

$\wedge \text{ UNCHANGED } \langle turn, intr0, intr1 \rangle$

$M12 \triangleq$

$\wedge pc1 = 1$

$\wedge pc1' = 2$

$\wedge \text{ UNCHANGED } \langle turn, intr0, intr1 \rangle$

$M23 \triangleq$

$\wedge pc1 = 2$

$\wedge pc1' = 3$

$\wedge intr1' = 1$

$\wedge \text{ UNCHANGED } \langle intr0, turn \rangle$

$M34 \triangleq$

$\wedge pc1 = 3$

$\wedge pc1' = 4$

$\wedge turn' = 0$

$\wedge \text{ UNCHANGED } \langle intr0, intr1 \rangle$

$M44 \triangleq$

$\wedge pc1 = 4$

$\wedge pc1' = 4$

$\wedge turn = 0$

$\wedge intr0 = 1$

$\wedge \text{ UNCHANGED } \langle intr0, intr1, turn \rangle$

$M45 \triangleq$

$\wedge pc1 = 4$

$\wedge pc1' = 5$

$\wedge (turn = 1 \vee intr0 = 0)$

$\wedge \text{ UNCHANGED } \langle turn, intr0, intr1 \rangle$

$M56 \triangleq$

$\wedge pc1 = 5$

$\wedge pc1' = 6$

$\wedge \text{ UNCHANGED } \langle intr0, intr1, turn \rangle$

$M67 \triangleq$

$\wedge pc1 = 6$

$\wedge pc1' = 7$

$\wedge intr1' = 0$

$\wedge \text{ UNCHANGED } \langle intr0, turn \rangle$

$M70 \triangleq$

$\wedge pc1 = 7$

$\wedge pc1' = 0$

$\wedge \text{ UNCHANGED } \langle turn, intr0, intr1 \rangle$

$$SLOGQ \triangleq \text{UNCHANGED } \langle pc1, intr0, intr1, turn \rangle$$

$$\begin{aligned} Next\_First &\triangleq \\ &\vee L01 \\ &\vee L12 \\ &\vee L23 \\ &\vee L34 \\ &\vee L45 \\ &\vee L56 \\ &\vee L67 \\ &\vee L70 \\ &\vee SLOGP \end{aligned}$$

$$\begin{aligned} Next\_Second &\triangleq \\ &\vee M01 \\ &\vee M12 \\ &\vee M23 \\ &\vee M34 \\ &\vee M45 \\ &\vee M56 \\ &\vee M67 \\ &\vee M70 \\ &\vee SLOGQ \end{aligned}$$

$$Next \triangleq (Next\_First \wedge \text{UNCHANGED } pc1) \vee (Next\_Second \wedge \text{UNCHANGED } pc0)$$

$$Mutual\_Exclusion \triangleq (pc0 \neq 5) \vee (pc1 \neq 5)$$

we need justice conditions, because here in this example, it can happen that  
the scheduler never schedules one process  
we want to avoid such runs ?????

justice conditions

$$\begin{aligned} J00 &\triangleq pc0 \neq 0 \\ J02 &\triangleq pc0 \neq 2 \\ J03 &\triangleq pc0 \neq 3 \end{aligned}$$

we cannot write  $pc0 \neq 4$ , because that's not a requirement, it should be proven

$$\begin{aligned} J04 &\triangleq \neg((pc0 = 4) \wedge ((turn = 0) \vee (intr1 = 0))) \\ J05 &\triangleq pc0 \neq 5 \\ J06 &\triangleq pc0 \neq 6 \\ J07 &\triangleq pc0 \neq 7 \end{aligned}$$

for the process 1

$$\begin{aligned} J10 &\triangleq pc1 \neq 0 \\ J12 &\triangleq pc1 \neq 2 \end{aligned}$$

$$J13 \triangleq pc1 \neq 3$$

$$J14 \triangleq \neg((pc1 = 4) \wedge ((turn = 0) \vee (intr0 = 0)))$$

$$J15 \triangleq pc1 \neq 5$$

$$J16 \triangleq pc1 \neq 6$$

$$J17 \triangleq pc1 \neq 7$$

the below justice conditions are to ensure that the scheduler is fair

$$J \triangleq$$

$$\wedge J00$$

$$\wedge J02$$

$$\wedge J03$$

$$\wedge J04$$

$$\wedge J05$$

$$\wedge J06$$

$$\wedge J07$$

$$\wedge J10$$

$$\wedge J12$$

$$\wedge J13$$

$$\wedge J14$$

$$\wedge J15$$

$$\wedge J16$$

$$\wedge J17$$

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