Model Checking - Exercise Sheet 2

1. - Give the parse tree of the formula $AFp \to AFq$ according to the syntax rules of construction of CTL formula. Recall that $\to$ has lower precedence than unary operators.
   - Give the parse tree of the formula $AFp \to AFq$ according to the syntax rules of construction of CTL* formula. Recall that $\to$ has lower precedence than unary operators.
   - Let K be a Kripke structure and s be a state of K. Check whether it is true that K, s satisifies the CTL formula $AFp \to AFq$ if and only if K, s satisifies the CTL* formula $AFp \to AFq$. Justify your answer using semantics of CTL and CTL* semantics of K, s satisfying a CTL (resp. CTL*) formula.

2. Consider the LTL formula $\phi = Fp \to Fq$ and the CTL formula $\psi = AFp \to AFq$. Describe a Kripke structure K with exactly two states s_0 and s_1 such that $K, s_0 \not\models \phi$, but $K, s_0 \models \psi$. Justify your answer using the semantics of LTL and CTL semantics of K, s satisfying a LTL (resp. CTL) formula.

3. Prove that the CTL formula $\psi = AFp \to AFq$ has no equivalent formula possible in LTL. (Hint: consider a model for ψ from which some transitions are removed so that it does not satisfy ψ.)

4. Convert the CTL formula $\phi = AFAGp$ into an equivalent CTL formula ψ in which only operators used are $\neg$, $\wedge$, EU, EG (along with $\top$). Show the steps in your conversion with appropriate justifications.

5. Draw the parse tree of the CTL formula ψ which you obtained as answer to the previous question. Consider the Kripke structure K given in Figure 1. Apply the bottom-up labelling procedure for states of K based on this parse tree that we used for the CTL model checking algorithm. While processing each node of the parse tree, write how the labellings of each state of K is getting updated.

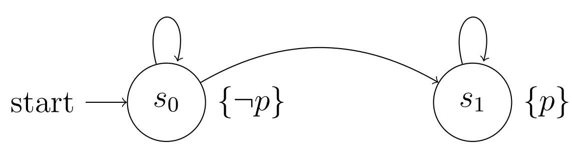

Figure 1: Model K

6. Consider the LTL formula $\phi = FGp$ and the CTL formula $\psi = AFAGp$. Give a Kripke structure that demonstrates that these are not equivalent. Also show that for any Kripke structure K and state s of K, if $K, s \models \psi$ then $K, s \models \phi$.

7. Consider the CTL formulae $\phi = AXAFp$ and $\psi = AFAXp$. Give a Kripke structure that demonstrates that these are not equivalent.

8. In the CTL model checking algorithm discussed, we had a fix point algorithm for identifying the set of states that satisfy $E[\phi U \psi]$ provided we have already identified the set of states that satisfy ϕ and ψ. In a similar way, obtain a fix-point algorithm for identifying the set of states that satisfy $A[\phi U \psi]$ (Use the recurrence for AU in terms of AU and AX and do not convert AU to other temporal operators.) Analyze the running time of your procedure.

9. Construct a Non-deterministic Büchi automaton over the alphabet $\{a, b, c\}$ whose language has only two strings *ababab...* and *bcbcbc.....* Justify your answer. Using this (an the concatenation rule with regular languages), construct a Non-deterministic Büchi automaton over the alphabet $\{a, b, c\}$ whose language has precisely all infinite strings which either has *ababab...* as a suffix or has *bcbcbc...* as a suffix. Justify your answer.

10. Using the definitions of safety property and liveness property, show that the only language over an alphabet Σ which is both a safety property and a liveness propery is the trivial language Σ^ω.

11. Consider the language L defined by the LTL formula $\phi = G(p \rightarrow XXq)$.

    - Give some minimal bad prefixes for L. Give some bad prefixes for L which are not minimal. Justify your answers using the definitions of bad prefixes and minimal bad prefixes.
    - Show that L is a regular safety property, by describing the set of bad prefixes of L as a regular expresson. Justify the correctness of your solution.
    - Construct an NFA whose language is the set of bad prefixes of L. Let this automata be M.
    - Suppose K is a Kripke structure for which you want to answer whether $K \models \phi$. We had reduced this problem to invariant checking on a specific product automata of K and the NFA M. Based on M, give the invariant formula that you would check on the product automaton to answer whether $K \models \phi$. (You need not construct the product automaton explicitly, because you do not know what K is explicitly).

12. Suppose L is a language over alphabet Σ which is a regular safety property. Suppose you have an NFA that accepts precisely the strings that are bad prefixes for L. Analyze how we may obtain an NFA that accepts precisely that strings that are minimal bad prefixes for L.