

EXTENDS *Integers*

VARIABLES *pc0, pc1, turn, intr0, intr1*

$TypeOK \triangleq$   
 $\wedge pc0 \in 0 \dots 7$   
 $\wedge pc1 \in 0 \dots 7$   
 $\wedge turn \in \{0, 1\}$   
 $\wedge intr0 \in \{0, 1\}$   
 $\wedge intr1 \in \{0, 1\}$

a) adding weak fairness constraints on steps where applicable and b) check  
*i)* whether all runs satisfy starvation freeness *i.e* a process requesting to enter *CS* at any instant will be allowed to enter *CS* at a future instant *ii)* whether all runs mandate strict sequencing between process 1 and process 2 in critical section  
show a trace where strict sequencing need not hold  
Ref pg 36 of handbook of model checking

asynchronous system

for  $i = 0, 1$

the algorithm is

```
0 : while( True){
1 :   //non critical section
2 :   intr[i] = True
3 :   turn = 1 - i
4 :   while( turn  $\triangleq$  1 and intr[1 - i]  $\triangleq$  1) //wait
5 :   //critical section
6 :   intr[i] = 0
7 : }
```

weak fairness :

$Init0 \triangleq$   
 $\wedge turn = 0$   
 $\wedge intr0 = 0$   
 $\wedge pc0 = 0$

$Init1 \triangleq$   
 $\wedge turn = 0$   
 $\wedge intr1 = 0$   
 $\wedge pc1 = 0$

$Init \triangleq Init0 \wedge Init1$

$L01 \triangleq$   
 $\wedge pc0 = 0$

$$\begin{aligned} &\wedge pc0' = 1 \\ &\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle \end{aligned}$$

$$\begin{aligned} L12 &\triangleq \\ &\wedge pc0 = 1 \\ &\wedge pc0' = 2 \\ &\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle \end{aligned}$$

$$\begin{aligned} L23 &\triangleq \\ &\wedge pc0 = 2 \\ &\wedge pc0' = 3 \\ &\wedge intr0' = 1 \\ &\wedge \text{UNCHANGED } \langle intr1, turn \rangle \end{aligned}$$

$$\begin{aligned} L34 &\triangleq \\ &\wedge pc0 = 3 \\ &\wedge pc0' = 4 \\ &\wedge turn' = 1 \\ &\wedge \text{UNCHANGED } \langle intr0, intr1 \rangle \end{aligned}$$

$$\begin{aligned} L44 &\triangleq \\ &\wedge pc0 = 4 \\ &\wedge pc0' = 4 \\ &\wedge turn = 1 \\ &\wedge intr1 = 1 \\ &\wedge \text{UNCHANGED } \langle intr0, intr1, turn \rangle \end{aligned}$$

$$\begin{aligned} L45 &\triangleq \\ &\wedge pc0 = 4 \\ &\wedge pc0' = 5 \\ &\wedge (turn = 0 \vee intr1 = 0) \\ &\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle \end{aligned}$$

$$\begin{aligned} L56 &\triangleq \\ &\wedge pc0 = 5 \\ &\wedge pc0' = 6 \\ &\wedge \text{UNCHANGED } \langle intr0, intr1, turn \rangle \end{aligned}$$

$$\begin{aligned} L67 &\triangleq \\ &\wedge pc0 = 6 \\ &\wedge pc0' = 7 \\ &\wedge intr0' = 0 \\ &\wedge \text{UNCHANGED } \langle intr1, turn \rangle \end{aligned}$$

$$\begin{aligned} L70 &\triangleq \\ &\wedge pc0 = 7 \\ &\wedge pc0' = 0 \end{aligned}$$

$\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle$

$SLOGP \triangleq \text{UNCHANGED } \langle pc0, intr0, intr1, turn \rangle$

for the second system

$M01 \triangleq$

$\wedge pc1 = 0$

$\wedge pc1' = 1$

$\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle$

$M12 \triangleq$

$\wedge pc1 = 1$

$\wedge pc1' = 2$

$\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle$

$M23 \triangleq$

$\wedge pc1 = 2$

$\wedge pc1' = 3$

$\wedge intr1' = 1$

$\wedge \text{UNCHANGED } \langle intr0, turn \rangle$

$M34 \triangleq$

$\wedge pc1 = 3$

$\wedge pc1' = 4$

$\wedge turn' = 0$

$\wedge \text{UNCHANGED } \langle intr0, intr1 \rangle$

$M44 \triangleq$

$\wedge pc1 = 4$

$\wedge pc1' = 4$

$\wedge turn = 0$

$\wedge intr0 = 1$

$\wedge \text{UNCHANGED } \langle intr0, intr1, turn \rangle$

$M45 \triangleq$

$\wedge pc1 = 4$

$\wedge pc1' = 5$

$\wedge (turn = 1 \vee intr0 = 0)$

$\wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle$

$M56 \triangleq$

$\wedge pc1 = 5$

$\wedge pc1' = 6$

$\wedge \text{UNCHANGED } \langle intr0, intr1, turn \rangle$

$M67 \triangleq$

$$\begin{aligned}
& \wedge pc1 = 6 \\
& \wedge pc1' = 7 \\
& \wedge intr1' = 0 \\
& \wedge \text{UNCHANGED } \langle intr0, turn \rangle
\end{aligned}$$

$$\begin{aligned}
M70 & \triangleq \\
& \wedge pc1 = 7 \\
& \wedge pc1' = 0 \\
& \wedge \text{UNCHANGED } \langle turn, intr0, intr1 \rangle
\end{aligned}$$

$$SLOGQ \triangleq \text{UNCHANGED } \langle pc1, intr0, intr1, turn \rangle$$

$$\begin{aligned}
Next\_First & \triangleq \\
& \vee L01 \\
& \vee L12 \\
& \vee L23 \\
& \vee L34 \\
& \vee L45 \\
& \vee L56 \\
& \vee L67 \\
& \vee L70 \\
& \vee SLOGP
\end{aligned}$$

$$\begin{aligned}
Next\_Second & \triangleq \\
& \vee M01 \\
& \vee M12 \\
& \vee M23 \\
& \vee M34 \\
& \vee M45 \\
& \vee M56 \\
& \vee M67 \\
& \vee M70 \\
& \vee SLOGQ
\end{aligned}$$

$$Next \triangleq (Next\_First \wedge \text{UNCHANGED } pc1) \vee (Next\_Second \wedge \text{UNCHANGED } pc0)$$

$$Mutual\_Exclusion \triangleq (pc0 \neq 5) \vee (pc1 \neq 5)$$

we need justice conditions, because here in this example, it can happen that  
the scheduler never schedules one process  
we want to avoid such runs ?????

justice conditions

$$\begin{aligned}
J00 & \triangleq pc0 \neq 0 \\
J02 & \triangleq pc0 \neq 2 \\
J03 & \triangleq pc0 \neq 3
\end{aligned}$$

we cannot write  $pc0 \neq 4$ , because that's not a requirement, it should be proven

$$J04 \triangleq \neg((pc0 = 4) \wedge ((turn = 0) \vee (intr1 = 0)))$$

$$J05 \triangleq pc0 \neq 5$$

$$J06 \triangleq pc0 \neq 6$$

$$J07 \triangleq pc0 \neq 7$$

for the process 1

$$J10 \triangleq pc1 \neq 0$$

$$J12 \triangleq pc1 \neq 2$$

$$J13 \triangleq pc1 \neq 3$$

$$J14 \triangleq \neg((pc1 = 4) \wedge ((turn = 0) \vee (intr0 = 0)))$$

$$J15 \triangleq pc1 \neq 5$$

$$J16 \triangleq pc1 \neq 6$$

$$J17 \triangleq pc1 \neq 7$$

the below justice conditions are to ensure that the scheduler is fair

$$J \triangleq$$

$$\wedge J00$$

$$\wedge J02$$

$$\wedge J03$$

$$\wedge J04$$

$$\wedge J05$$

$$\wedge J06$$

$$\wedge J07$$

$$\wedge J10$$

$$\wedge J12$$

$$\wedge J13$$

$$\wedge J14$$

$$\wedge J15$$

$$\wedge J16$$

$$\wedge J17$$

$$Fairness \triangleq$$

$$\wedge WF_{pc0}(L01)$$

$$\wedge WF_{pc0}(L12)$$

$$\wedge WF_{pc0}(L23)$$

$$\wedge WF_{pc0}(L34)$$

$$\wedge WF_{pc0}(L56)$$

$$\wedge WF_{pc0}(L67)$$

$$\wedge WF_{pc0}(L70)$$

$$\wedge WF_{pc1}(M01)$$

$$\wedge WF_{pc1}(M12)$$

$$\wedge WF_{pc1}(M23)$$

$$\wedge WF_{pc1}(M34)$$

$\wedge \text{WF}_{pc1}(M56)$

$\wedge \text{WF}_{pc1}(M67)$

$\wedge \text{WF}_{pc1}(M70)$

$vars \triangleq \langle pc0, pc1, turn, intr0, intr1 \rangle$

$FairSpec \triangleq$

$\wedge Init$

$\wedge \Box[Next]_{vars}$

$\wedge Fairness$

$MUTUAL\_EXCLUSION \triangleq \Box \neg((pc0 = 5) \wedge (pc1 = 5))$

$NOSTARVE\_1 \triangleq \Box((pc0 = 4) \Rightarrow \Diamond(pc0 = 5))$

$NOSTARVE\_2 \triangleq \Box((pc1 = 4) \Rightarrow \Diamond(pc1 = 5))$

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\ \* Modification History  
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