SLIIT
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BSc (Hons) in Information Technology

Year 2

Data Structures and Algorithms – IT2070

Worksheet 9 – Heaps

Question1

a) What is a binary tree?

b) Show that the relationship between height (h) of a Full Binary Tree and the number of nodes (n) is given by $n = 2^{h+1} - 1$.

Question 2

a) The following are the algorithms for Heap sort, Max Build Heap and Max_Heapify.

```
HEAPSORT(A)
1.BUILD_HEAP[A]
2.for  i = A.length down to 2
3.        Exchange A[1] with A[i]
4.        A.heap_size = A.heap_size-1;
5.        MAX_HEAPIFY(A,1)
```

```
MAX_BUILD_HEAP (A)
1. A.heap_size = A.length
2.  for i = ⌊A.length/2⌋ downto 1
3.        MAX_HEAPIFY(A, i)
```

```
MAX_HEAPIFY (A,i)

1.        l = LEFT_CHILD (i);
2.        r = RIGHT_CHILD (i);
3.        if l ≤ A.heap_size and A[ l ] > A[ i ]
4.                then largest = l;
5.                else largest = i;
6.        if r ≤ A.heap_size and A[r] > A[largest]
7.                then largest = r;
8.        if largest ≠ i
9.                then exchange A[i] with A[largest]
10.                        MAX_HEAPIFY (A, largest)
```

Illustrate the operations of the Heap sort for the array **A** of elements given below. (For the purpose of illustration, assign the values only once to the given algorithm and use diagrammatic way to reach the answer.)

1	2	3	4	5	6	7	8
4	20	30	1	50	60	0	80

b) We can compute the upper bound on the running time of BUILD-HEAP as follows.

$$T(n) = \sum_{h=o}^{\lfloor \lg n \rfloor} \left\lceil \frac{n}{2^{h+1}} \right\rceil .O(h)$$ Briefly explain two components of the above equation.