The Time Period & Frequency of Trigonometric Functions

We can think of the angle of the trigonometric function increasing over time with a Frequency f and Time Period T as the radial vector rotates around the origin. The sin and cos function graph show how the x and y radial coordinates or components change with time, and we can use these to show how the Frequency & Time Period change when the angle θ increases at a faster or slow rate. We can parameterise the angle θ with a constant angular frequency ω and a time t. The definition of the frequency or the period is then the time it takes to complete 1 full circle of 360° or 2π radians and has the equation:

Eq. 1
$$f = \frac{1}{T}$$

Where the angular displacement θ is:

Eq. 2
$$\theta = \omega t = 2\pi f t = \frac{2\pi t}{T}$$

And ωt is the angular displacement of constant angular speed multiplied by time t for a constant frequency f. This gives the following equation for the y component of the radial vector:

$$y = \sin(\omega t) = \sin(2\pi f t)$$

In figure 1. Is the plot of two sin functions with different frequencies from t =0 to t=5. In the case where f = 0.5:

$$y = \sin(\pi t)$$

On the graph is labelled T for the period, which is the time for one cycle where the waveform returns to the same place. In the graph of f = 0.5 this is from 0.5 to 2.5 and T = 2. You can also use Eq. 1 where 1/T = 0.5:

In the second waveform is the plot of the equation:

$$y = \sin(5\pi t)$$

there are 2.5 cycles from 0 to 1 the period is therefor 1/2.5 = 0.4 seconds and the frequency 2.5.

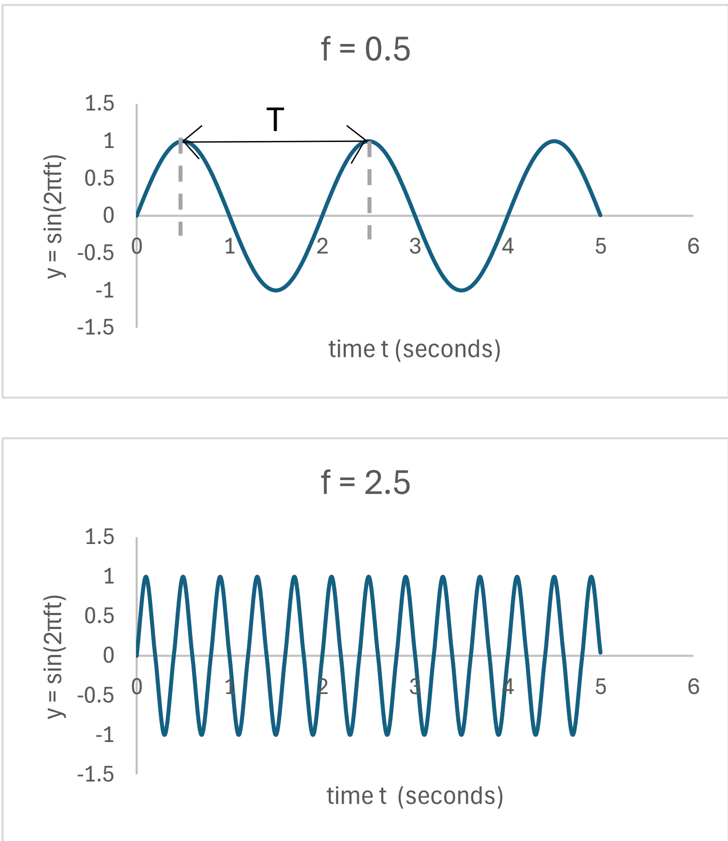

Figure 1.

Superposition of Trigonometric Functions

Superposition is a summation of trigonometric functions or waveforms changing the components and magnitude of a function, we can compose a waveform or function in the following ways:

$$y = A\sin(\omega_1 t) + Bcos(\omega_2 t)$$

Where A & B are the amplitude of the waveforms or the magnitude of the r vector, ω_1 & ω_2 are the angular frequencies of sin or cosine function. Figure 2 shows an example of the waveform with the equation from t=0 to t = 12:

Eq. 3

$$y = 4\sin(2\pi t) + 5\,cos(0.5\pi t)$$

In Eq. 3 the first sin function has an amplitude of 4 and a frequency of 1, the cos function has an amplitude of 5 and a frequency of 0.25. In figure 2. The plot of Eq.3 shows a maximum amplitude of the components and has two frequencies of f=1/T = 0.25Hz & 1Hz.

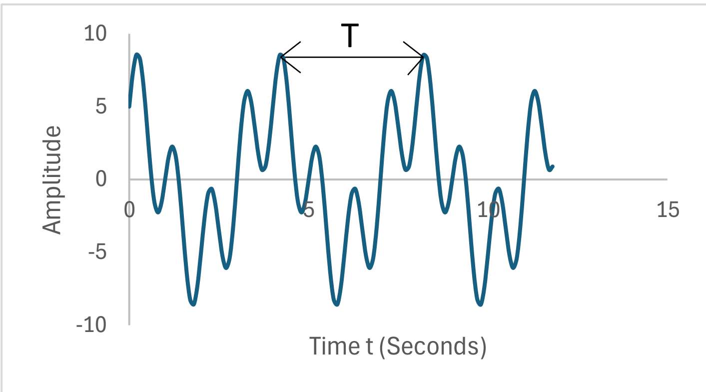

Figure 2.

Beat Frequency

The Beat frequency is the interference of more than one waveform with similar frequencies. The waveforms constructively interfere to a maximum amplitude and destructively interfere to a minimum amplitude, resulting in an interference pattern or envelope that has a different frequency to the angular frequency of the component waveforms.

Figure 3 shows the waveform of the equation:

$$y = 5\sin(5\pi t) + 3\,cos(5.5\pi t)$$

As you can see in the waveform there is constructive interference where the amplitudes add together to a maximum. The angular frequency of each function is 5π and 5.5π the frequencies of each are 2.5 and 2.75.

The interference frequency is then:

$$f_I = |f_1 - f_2| = 0.25\ Hz$$

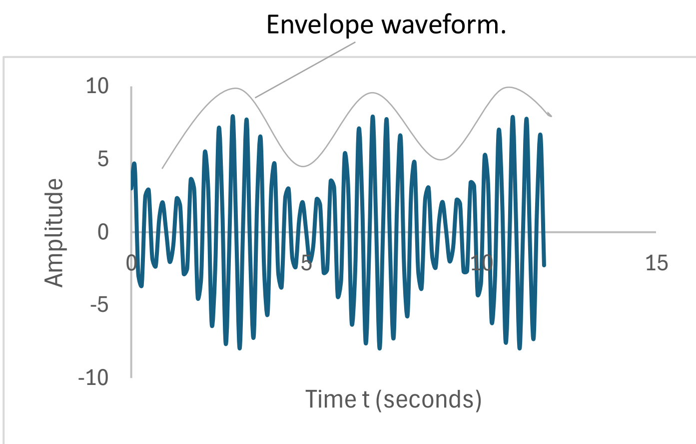

Figure 3.

Coupled Oscillator For A Spring Mass System.

We can analyse the dynamics of a system by the conservation of energy from potential energy to kinetic energy, for a closed system this value remains constant and applies to oscillatory systems where the energy is stored as a mechanical force such as tension. Using the mechanical definitions for potential energy PE & Kinetic energy KE the Lagrange equations is:

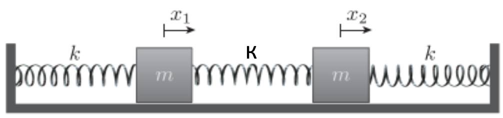

Figure 1.

In the case of the coupled oscillator, we can find the resultant force on each mass due the compression or extension to formulate the equations of motion.

$$F_{x1} = m\ddot{x}_1 = \ -k_1x_1 + \kappa x_2 - \kappa x_1 = -(k+\kappa)x_1 + \kappa x_2$$

$$F_{x2} = m\ddot{x}_2 = -kx_2 - \kappa x_2 + \kappa x_1 \ = -(k+\kappa)x_2 + \kappa x_1$$

$$\ddot{y}_S = F_{x1} + F_{x2} \ = \ddot{x}_1 + \ddot{x}_2 = \ -k(x_1 + x_2)$$

$$\ddot{y}_F = F_{x1} - F_{x2} \ = \ddot{x}_1 - \ddot{x}_2 = -(k + 2\kappa)(x_1 - x_2)$$

In the case of $\ddot{y}_S$ we have the function:

$$\omega_S = \sqrt{\frac{k}{m}}$$

$$x_1 + x_2 = \ A_s cos(\omega_S t + \varphi_s)$$

For $\ddot{y}_F$:

$$\omega_F = \sqrt{\frac{k + 2\kappa}{m}}$$

$$x_1 - x_2 = \ A_F cos(\omega_F t + \varphi_F)$$

The solutions for $\ddot{y}_S$ & $\ddot{y}_F$ are the normal modes of oscillations of the system where a general solution is a linear combination of the two modes in the form:

$$x_1 = \frac{1}{2}((x_1 + x_2) + (x_1 - x_2)) = \frac{1}{2}(\,A_s cos(\omega_S t + \varphi_s) + \,A_F cos(\omega_F t + \varphi_F))$$

$$x_2 = \frac{1}{2}((x_1 + x_2) - (x_1 - x_2)) = \frac{1}{2}(\,A_s cos(\omega_S t + \varphi_S) - \,A_F cos(\omega_F t + \varphi_F))$$

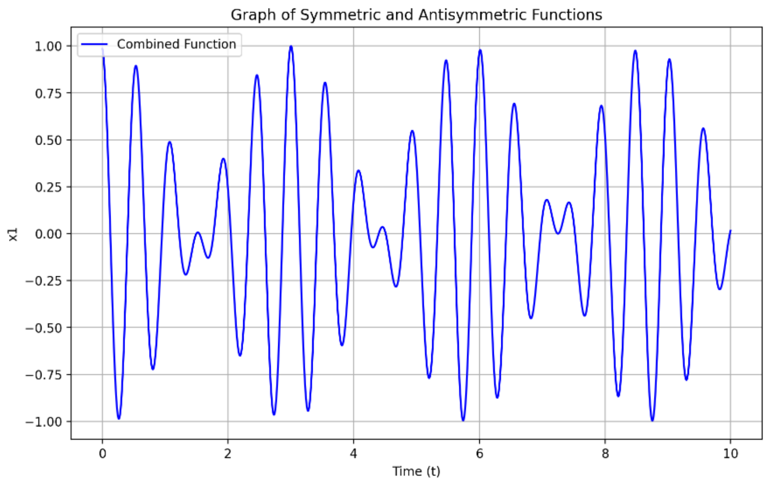

Figure 2. A plot of $\frac{1}{2}(\,A_s cos(\omega_S t + \varphi_s) + \,A_F cos(\omega_F t + \varphi_F))$ for $\omega_S = 3\pi$ $\omega_f = 3\pi$ $\varphi_s = 0.25$