Fourier Transform

For a non-periodic signal or pulse signal the Fourier Series is a Fourier Transform, which is an integral of trigonometric functions expressed as complex exponentials. The Fourier Transform converts the waveform from the time domain to the frequency domain, which results in a function of trigonometric frequencies and amplitudes for every term such that for every f(t) is defined by f(ω).

$$f(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\, dt$$

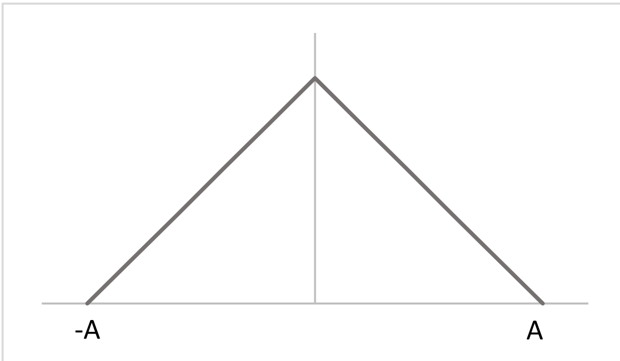
The Fourier integral can be used to represent a function in terms of THE Fourier Transform:

$$f(t) \ = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\, d\omega$$

In the case of the non-periodic triangle centred at 0, there is a discontinuity at t=0 so we can split the integral into two functions:

$$f(t) = t + A$$

$$f(t) = -t + A$$



This gives the following Fourier Transform:

$$f(\omega) = \int_{-A}^{0} (t + A)e^{-i\omega t}\, dt + \int_{0}^{A} (-t + A)e^{-i\omega t}\, dt$$

The integral using integration by parts is:

$$f(\omega) = \left[\frac{(t + A)e^{-i\omega t}}{-i\omega} + \frac{e^{-i\omega t}}{\omega^2}\right]_{-A}^{0} + \left[\frac{(-t + A)e^{-i\omega t}}{-i\omega} + \frac{e^{-i\omega t}}{\omega^2}\right]_{0}^{A}$$

$$f(\omega) = \left(\frac{A}{-i\omega} + \frac{1}{\omega^2} - \frac{e^{i\omega A}}{\omega^2}\right) + \left(\frac{A}{i\omega} + \frac{e^{-i\omega A}}{\omega^2} - \frac{1}{\omega^2}\right)$$

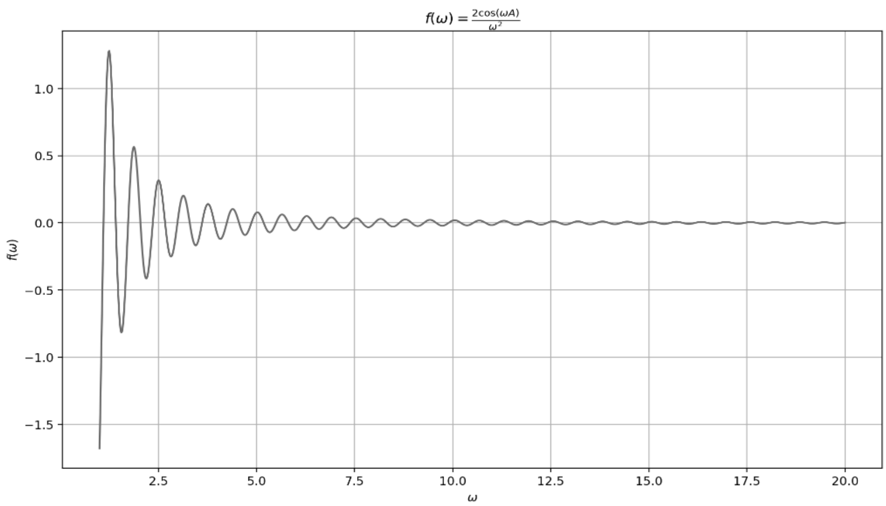
The integral simplifies to:

$$f(\omega) = \frac{1}{\omega^2}\left(e^{i\omega A} + e^{-i\omega A}\right)$$

And this function has a cosine identity which is:

$$f(\omega) = \frac{2\cos\,(\omega A)}{\omega^2}$$

Below is a plot of the above function f(ω) for a value of A=10.



Below is the function where f(t) is constant value from -τ to +τ also known as a top hat function, it has a well-known Fourier Transform called the sinc function where:

$$f(\omega) = \frac{2\sin\,(\omega\tau)}{\omega}$$