Fourier Series

Suppose we have an arbitrary periodic function f(t) that repeats itself every Time period T with a frequency 1/T. Under certain conditions the function can be expressed as the sum of an infinite number of sin & cosine functions known as a Fourier series. The Fourier equation is defined by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right)\right)$$

Where the amplitudes or coefficients of each term are defined as:

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(\frac{2\pi nt}{T})dt$$

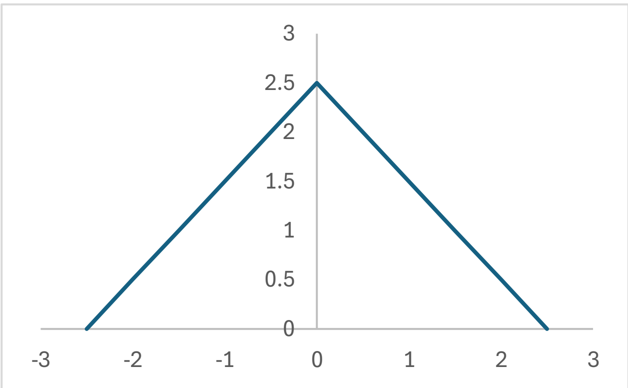
$$b_n = \frac{2}{T}\int_0^T f(t)\sin(\frac{2\pi nt}{T})dt$$

The coefficients of the Fourier series integrate an area of the function and factorise each area by the Time period T. The first coefficient a_0 calculates the area of the cyclic function that is being converted to trigonometric functions, the coefficients a_n (for an even function of f(t)) & b_n (for an odd function of f(t) calculates the amplitude & number of components of f(t) and represents each component with an area of a sin or cos term, each term is linearised by the period to a component value for f(t). The component values of f(t) are represented in the Fourier series as the amplitude of sin & cos terms added together to give the value of f(t) for any t from 0 to T. The sum of components is from n=1 to ∞ as each term is normalised by 1/n for each nth term.

In figure 1 is an example of a triangular wave function with a period of T =5:

$$f(t) = t + 2.5 \qquad -2.5 < t < 0$$

$$f(t) = -t + 2.5 \qquad 0 < t < 2.5$$



$$a_0 = \frac{2}{5}\left(\int_0^{2.5}(-t+2.5)dt + \int_{-2.5}^{0}(t+2.5)dt\right)$$

$$a_0 = \frac{2}{5}\left(\left[-\frac{t^2}{2}+2.5t\right]_0^{2.5} + \left[\frac{t^2}{2}+2.5t\right]_{-2.5}^{0}\right)$$

$$a_0 = \frac{5}{2}$$

$$a_n = \frac{2}{5}\left(\int_0^{2.5}(-t+2.5)\cos(\frac{2\pi nt}{T})dt + \int_{-2.5}^{0}(t+2.5)\cos(\frac{2\pi nt}{T})dt\right)$$

$$a_n = \frac{2}{5}\left(\left[\frac{(-t+2.5)\mathrm{T}\sin\left(\frac{2\pi nt}{T}\right)}{2\pi n} - \left(\frac{T}{2\pi n}\right)^2 cos(\frac{2\pi nt}{T})\right]_0^{2.5} + \left[\frac{(t+2.5)\mathrm{T}\sin\left(\frac{2\pi nt}{T}\right)}{2\pi n} + \left(\frac{T}{2\pi n}\right)^2 cos(\frac{2\pi nt}{T})\right]_{-2,5}^{0}\right)$$

$$a_n = \frac{8}{5}\left(\frac{T}{2\pi n}\right)^2$$

For the case of half range cosine expansion:

$$a_n = \frac{8}{5}\left(\frac{T}{(2n-1)\pi}\right)^2$$

$$f(t) = \frac{5}{4} + \sum_{n=1}^{\infty}\left(\frac{10}{\pi^2(2n-1)^2}\cos\left(\frac{(2n-1)\pi t}{5}\right)\right)$$