

## The Time Period & Frequency of Trigonometric Functions

We can think of the angle of the trigonometric function increasing over time with a Frequency  $f$  and Time Period  $T$  as the radial vector rotates around the origin. The sin and cos function graph show how the x and y radial coordinates or components change with time, and we can use these to show how the Frequency & Time Period change when the angle  $\theta$  increases at a faster or slow rate. We can parameterise the angle  $\theta$  with a constant angular frequency  $\omega$  and a time  $t$ . The definition of the frequency or the period is then the time it takes to complete 1 full circle of  $360^\circ$  or  $2\pi$  radians and has the equation:

$$\text{Eq. 1} \quad f = \frac{1}{T}$$

Where the angular displacement  $\theta$  is:

$$\text{Eq. 2} \quad \theta = \omega t = 2\pi f t = \frac{2\pi t}{T}$$

And  $\omega t$  is the angular displacement of constant angular speed multiplied by time  $t$  for a constant frequency  $f$ . This gives the following equation for the y component of the radial vector:

$$y = \sin(\omega t) = \sin(2\pi f t)$$

In figure 1. Is the plot of two sin functions with different frequencies from  $t=0$  to  $t=5$ . In the case where  $f = 0.5$ :

$$y = \sin(\pi t)$$

On the graph is labelled  $T$  for the period, which is the time for one cycle where the waveform returns to the same place. In the graph of  $f = 0.5$  this is from 0.5 to 2.5 and  $T = 2$ . You can also use Eq. 1 where  $1/T = 0.5$ :

In the second waveform is the plot of the equation:

$$y = \sin(5\pi t)$$

there are 2.5 cycles from 0 to 1 the period is therefor  $1/2.5 = 0.4$  seconds and the frequency 2.5.

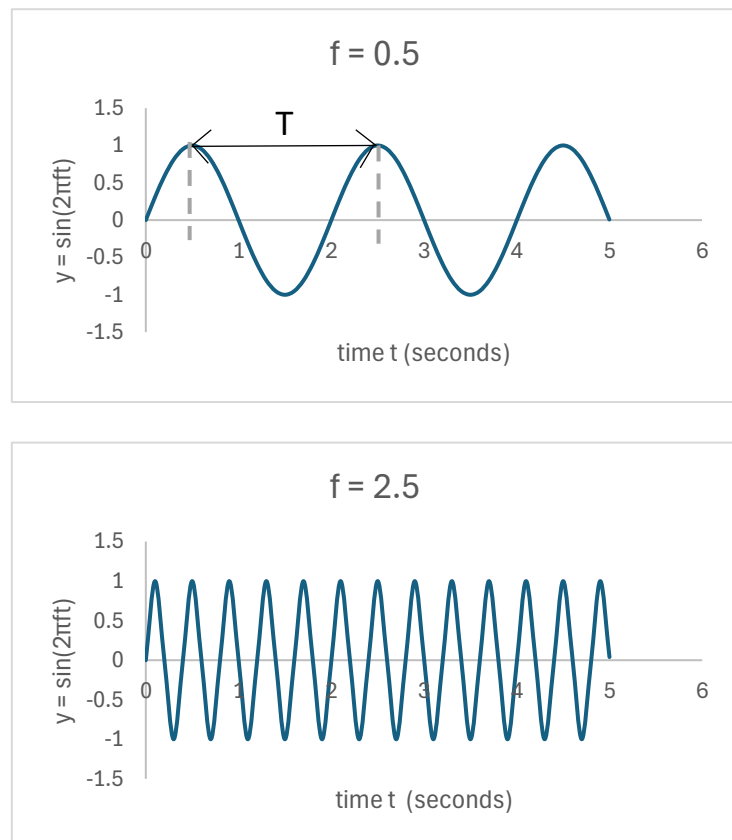


Figure 1.

### Superposition of Trigonometric Functions

Superposition is a summation of trigonometric functions or waveforms changing the components and magnitude of a function, we can compose a waveform or function in the following ways:

$$y = A \sin(\omega_1 t) + B \cos(\omega_2 t)$$

Where A & B are the amplitude of the waveforms or the magnitude of the r vector,  $\omega_1$  &  $\omega_2$  are the angular frequencies of sin or cosine function. Figure 2 shows an example of the waveform with the equation from  $t=0$  to  $t=12$ :

Eq. 3

$$y = 4 \sin(2\pi t) + 5 \cos(0.5\pi t)$$

In Eq. 3 the first sin function has an amplitude of 4 and a frequency of 1, the cos function has an amplitude of 5 and a frequency of 0.25. In figure 2. The plot of Eq.3 shows a maximum amplitude of 8.5 and a period T of 4 seconds, the same period as the cos function.

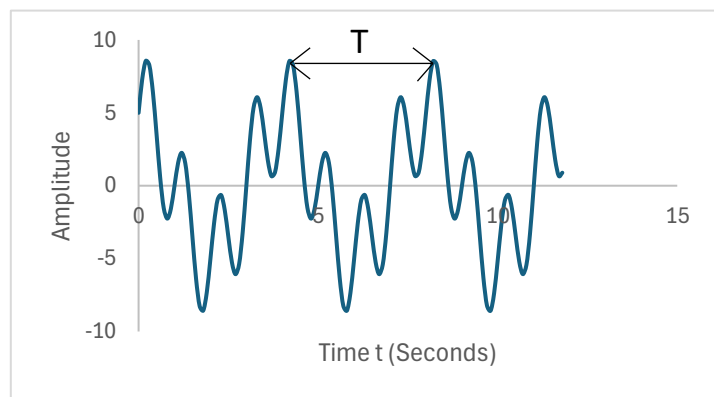


Figure 2.

### Beat Frequency

The Beat frequency is the interference of more than one waveform with similar frequencies. The waveforms constructively interfere to a maximum amplitude and destructively interfere to a minimum amplitude, resulting in an interference pattern or envelope that has a different frequency to the angular frequency of the component waveforms.

Figure 3 shows the waveform of the equation:

$$y = 5 \sin(5\pi t) + 3 \cos(5.5\pi t)$$

As you can see in the waveform there is constructive interference where the amplitudes add together to a maximum. The angular frequency of each function is  $5\pi$  and  $5.5\pi$  the frequencies of each are 2.5 and 2.75.

The interference frequency is then:

$$f_i = |f_1 - f_2| = 0.25 \text{ Hz}$$

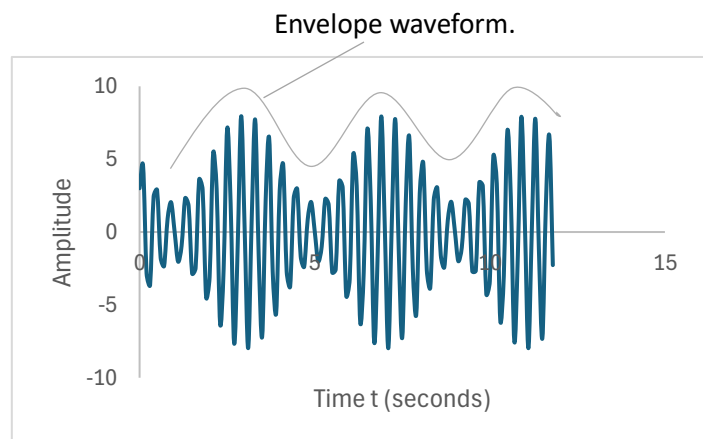


Figure 3.