

## Fourier Transform

For a non-periodic signal or pulse signal the Fourier Series is a Fourier Transform, which is an integral of trigonometric functions expressed as complex exponentials. The Fourier Transform converts the waveform from the time domain to the frequency domain, which results in a function of trigonometric frequencies and amplitudes for every term such that for every  $f(t)$  is defined by  $f(\omega)$ .

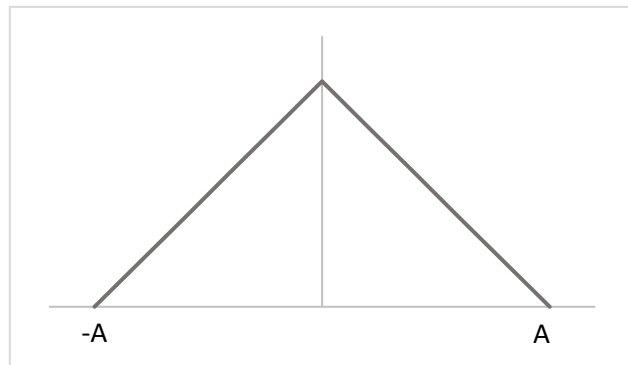
$$f(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$$

The Fourier integral can be used to represent a function in terms of The Fourier Transform:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i\omega t} d\omega$$

In the case of the non-periodic triangle centred at 0, there is a discontinuity at  $t=0$  so we can split the integral into two functions:

$$f(t) = t + A$$
$$f(t) = -t + A$$



This gives the following Fourier Transform:

$$f(\omega) = \int_{-A}^0 (t + A)e^{-i\omega t} dt + \int_0^A (-t + A)e^{-i\omega t} dt$$

The integral using integration by parts is:

$$f(\omega) = \left[ \frac{(t + A)e^{-i\omega t}}{-i\omega} + \frac{e^{-i\omega t}}{\omega^2} \right]_{-A}^0 + \left[ \frac{(-t + A)e^{-i\omega t}}{-i\omega} + \frac{e^{-i\omega t}}{\omega^2} \right]_0^A$$

$$f(\omega) = \left( \frac{A}{-i\omega} + \frac{1}{\omega^2} - \frac{e^{i\omega A}}{\omega^2} \right) + \left( \frac{A}{i\omega} + \frac{e^{-i\omega A}}{\omega^2} - \frac{1}{\omega^2} \right)$$

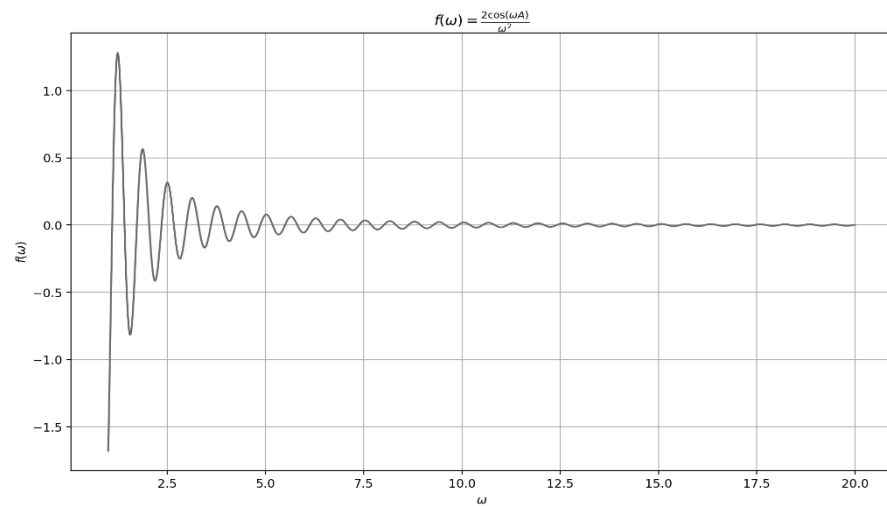
The integral simplifies to:

$$f(\omega) = \frac{1}{\omega^2} (e^{i\omega A} + e^{-i\omega A})$$

And this function has a cosine identity which is:

$$f(\omega) = \frac{2\cos(\omega A)}{\omega^2}$$

Below is a plot of the above function  $f(\omega)$  for a value of  $A=10$ .



Below is the function where  $f(t)$  is constant value from  $-\tau$  to  $+\tau$  also known as a top hat function, it has a well-known Fourier Transform called the sinc function where:

$$f(\omega) = \frac{2\sin(\omega\tau)}{\omega}$$

