2-D Transformations

Contents

This tutorial is about 2-D transformations.

```
clear all; close all;
```

The shape to be transformed

Let's begin by defining the shape of a rectangle. The rectangular shape can be described by four 2-D points, i.e., $\mathbf{p}_i = (x_i, y_i)^T$, for $i = 1, \ldots, 4$,. Their coordinates can be stored into a single matrix, X as follows:

$$X = [\,\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{p}_3 \quad \mathbf{p}_4\,]. \tag{1}$$

```
px = [0 2 2 0 0];   % x-coords
py = [0 0 2 2 0];   % y-coords
X  = [px;py]        % Matrix containing the 2-D points.
```

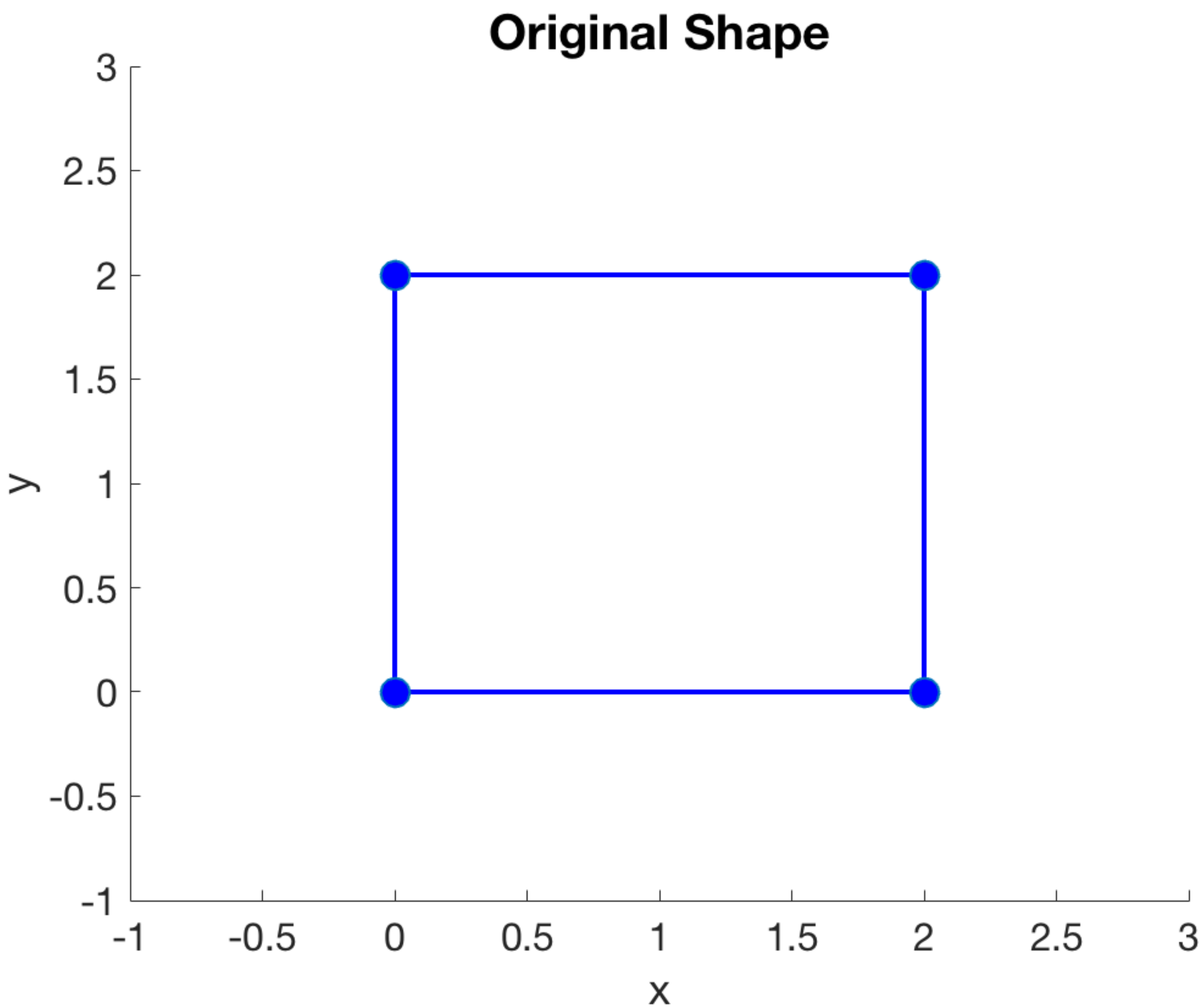
```
X =

     0     2     2     0     0
     0     0     2     2     0
```

Display the original shape

Note that the Matlab matrix that I use to store the points contains an additional point (5 points instead of 4 for a rectangle). This extra point at the end of the sequence makes the plot function ``close'' the shape. laternatively, we could use the command **patch** instead and the extra point is no longer needed.

Next, we display the original shape and the corner points using blue color lines:

```
figure;
hold on;
axis([ -1 3 -1 3 ] );
plot(X(1,:), X(2,:), 'b-', 'linewidth',2)
plot(X(1,:), X(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','b')
xlabel('x','FontSize',20);
ylabel('y','FontSize',20);
set(gca, 'fontsize',16);
set(gcf, 'color', 'w' );
title( 'Original Shape', 'FontSize',20 );
hold off;
```



Scaling transformation

The scaling (non-uniform) transformation is given by the following matrix:

$$S = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}. \tag{2}$$

An numerical example of a scaling transformation is given by:

$$S = \begin{bmatrix} .3 & 0 \\ 0 & .8 \end{bmatrix}. \tag{3}$$

```
S = [ 0.3 0.0; ...
      0.0 0.8 ];
```

To transform the rectangle shape, we multiply the transformation matrix by the matrix containing the points of our shape, i.e.:

$$X' = S\,X. \tag{4}$$

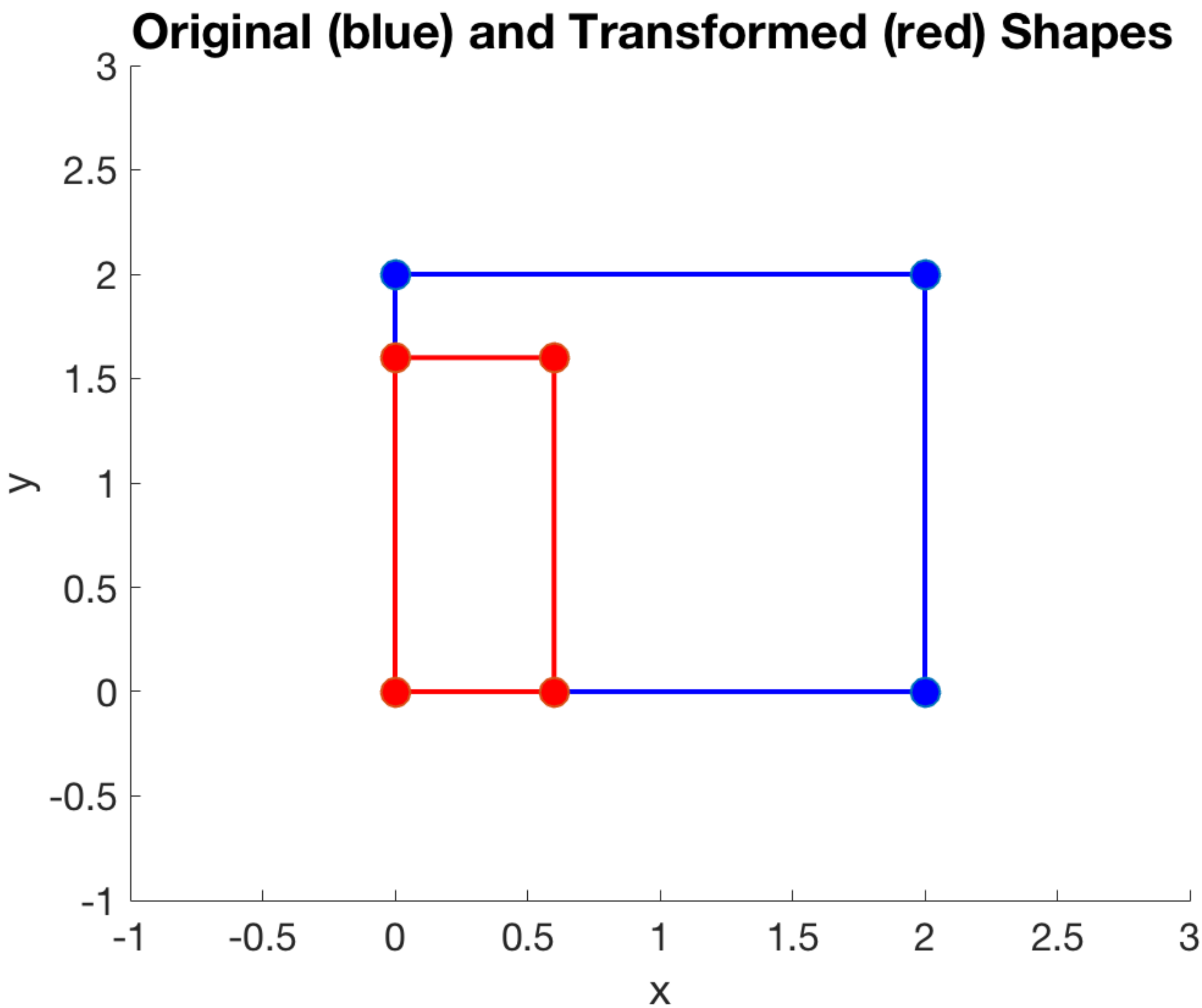
```
% Apply transformation to shape
X_t = S * X;
```

The common notation for transformation would be to multiply the transformation matrix and a single point, i.e.:

$$\mathbf{p}' = S\,\mathbf{p}. \tag{5}$$

Since matlab allows for direct matrix transformations, we can use matrix X to represent the whole shape so the multiplication SX transforms all points at once. The figure below shows the transformed shape superimposed on the same plot as the original shape.

```
% Display original points (again)
figure;
hold on;
axis([ -1 3 -1 3 ] );
plot(X(1,:), X(2,:), 'b-', 'linewidth',2)
plot(X(1,:), X(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','b')
xlabel('x','FontSize',20);
ylabel('y','FontSize',20);
set(gca, 'fontsize',16);
set(gcf, 'color', 'w' );
title( 'Original (blue) and Transformed (red) Shapes ', 'FontSize',20 );

% Show the transformed points
plot(X_t(1,:), X_t(2,:), 'r-', 'linewidth',2)
plot(X_t(1,:), X_t(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','r')
```



Rotation, Shear

Let's now consider the rotation and shear transformations. The matrix of the shear transformation is:

$$G = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}, \tag{6}$$

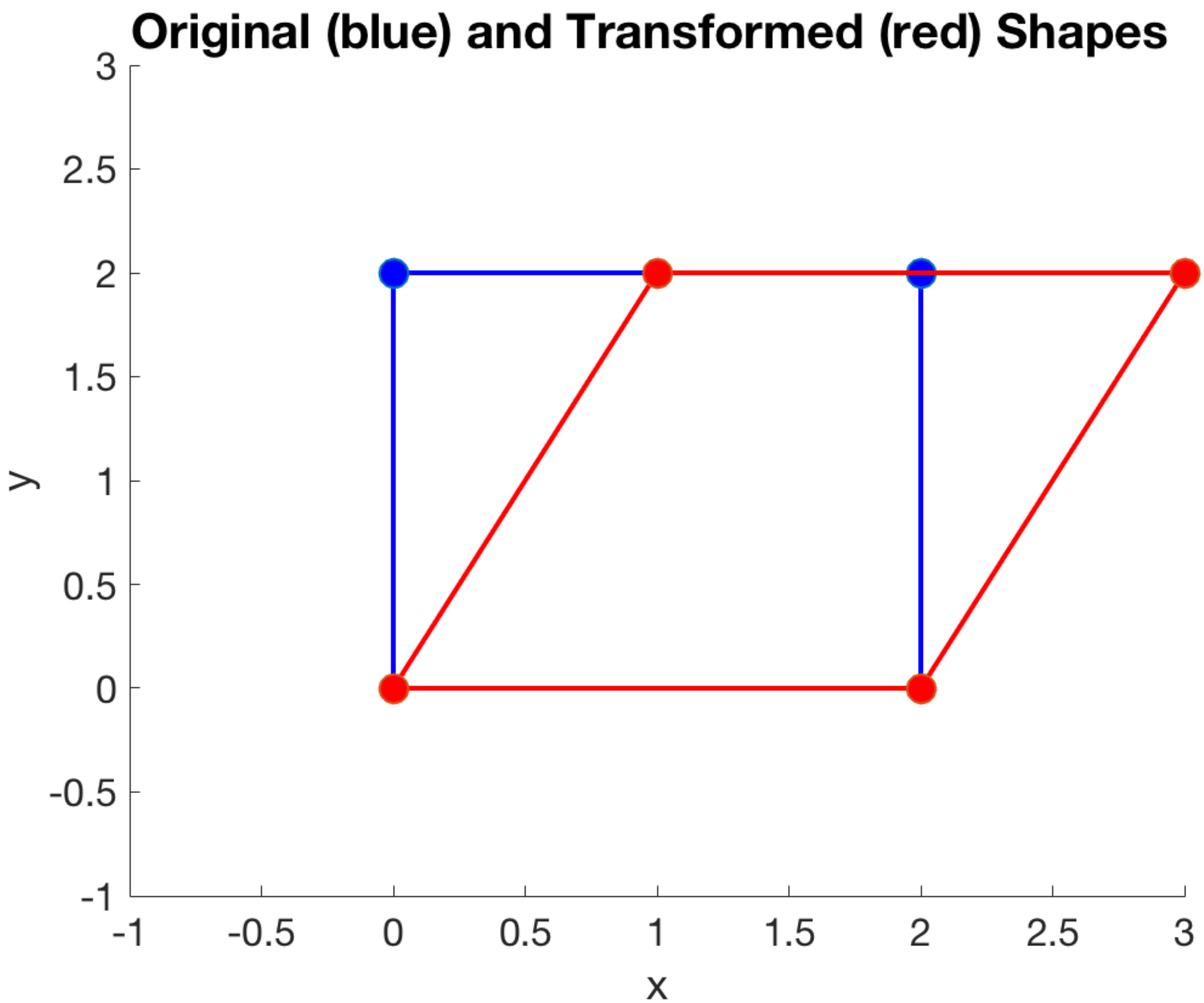
and the rotation matrix is given by:

$$R = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}. \tag{7}$$

Shear by .5 is:

$$G = \begin{bmatrix} 1 & .5 \\ 0 & 1 \end{bmatrix}, \tag{8}$$

```
% Shear
G = [ 1.0 0.5; ...
      0.0 1.0 ];

X2 = G * X;

% Display original points (again)
figure;
hold on;
axis([ -1 3 -1 3 ] );
plot(X(1,:), X(2,:), 'b-', 'linewidth',2)
plot(X(1,:), X(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','b')
xlabel('x','FontSize',20);
ylabel('y','FontSize',20);
set(gca, 'fontsize',16);
set(gcf, 'color', 'w' );
title( 'Original (blue) and Transformed (red) Shapes ', 'FontSize',20 );

% Show the transformed points
plot(X2(1,:), X2(2,:), 'r-', 'linewidth',2)
plot(X2(1,:), X2(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','r')
```



and a rotation of $\pi/8$ is given by:

$$R = \begin{bmatrix} \sin\frac{\pi}{8} & -\cos\frac{\pi}{8} \\ \cos\frac{\pi}{8} & \sin\frac{\pi}{8} \end{bmatrix}. \tag{9}$$

```
% Rotation
th = pi/8;
R = [ sin(th) -cos(th); ...
      cos(th)  sin(th) ];
X3 = R * X;


% Display original points (again)
```

```
figure;
hold on;
axis([ -3 3 -1 3 ] );
plot(X(1,:), X(2,:), 'b-', 'linewidth',2)
plot(X(1,:), X(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','b')
xlabel('x','FontSize',20);
ylabel('y','FontSize',20);
set(gca, 'fontsize',16);
set(gcf, 'color', 'w' );
title( 'Original (blue) and Transformed (red) Shapes ', 'FontSize',20 );

% Show the transformed points
plot(X3(1,:), X3(2,:), 'r-', 'linewidth',2)
plot(X3(1,:), X3(2,:), 'o', 'MarkerSize', 12, 'MarkerFaceColor','r')
```