$$\because \frac{\sum_{t=1}^{N}\sum_{i=1}^{K} r(z_i^t)}{\alpha} = 1 \quad \because \alpha = \sum_{i=1}^{K} N_i = N$$

$$\therefore \pi_i = \frac{N_i}{N}.$$

By binarizing $r(z_i^t)$ to $b_i^t = \begin{cases} 1 & i = \arg\max_j r(z_j^t) \\ 0 & \text{otherwise.} \end{cases}$

$$\mathcal{L}(\phi|X) = \sum_{t=1}^{N} \log \max\{\pi_i P(x^t|C_i)\}_{i=1\ldots k}$$

$$= \sum_{i=1}^{K} \sum_{t\in D_i} \log \pi_i P(x^t|C_i), \quad D_i = \{t \mid i = \arg\max_j r(z_j^t)\}$$

$$= \sum_{i=1}^{K} \sum_{t\in D_i} \log \pi_i \cdot \frac{1}{2b_i} \exp\left(-\frac{|x^t - u_i|}{b_i}\right)$$

$$= \sum_{i=1}^{K} \left[\left(\sum_{t\in D_i} \log \frac{\pi_i}{2b_i}\right) - \frac{1}{b_i} \sum_{t\in D_i} |x^t - u_i|\right]$$

$\therefore \mathcal{L}(\theta|X)$ will reach its maximum (w.r.t u_i) when:

$$u_i = \text{median}\{x^t\}, \quad t \in D_i$$

Complete log-likelihood:

$$p(z) = \prod_{i=1}^{K} \pi_i^{z_i} \quad p(x|z) = \prod_{i=1}^{K} p(x|C_i)^{z_i}$$

$$p(x, z) = p(z) \cdot p(x|z) = \prod_{i=1}^{K} (\pi_i P(x|C_i))^{z_i}$$

$$\mathcal{L}_c(\phi|X, z) = \sum_{t=1}^{N} \log p(x^t, z^t|\phi)$$

$$= \sum_{t=1}^{N} \log P(z^t|\phi) + \log P(x^t|z^t, \phi)$$

$$= \sum_{t=1}^{N} \sum_{i=1}^{K} z_i^t [\log \pi_i + \log p(x^t|\phi_i)]$$

$$= \sum_{t=1}^{N} \sum_{i=1}^{K} z_i^t \left[\log \pi_i - \log 2b_i - \frac{|x^t - u_i|}{b_i}\right]$$