HW3 Tao Ni

2019年3月28日 星期四 下午5:02

Q1: $P(x) = \sum_{i=1}^{K} P(x|C_i)\cdot P(C_i) = \sum_{i=1}^{K} \pi_i \frac{1}{2b_i} \exp\left(-\frac{|x-u_i|}{b_i}\right)$

$\sum_{i=1}^{K} \pi_i = 1$, $\phi = \{\pi_i, u_i, b_i\}_{i=1\ldots K}$, $X = \{x^1, x^2, \ldots, x^t, \ldots, x^N\}$

$$\mathcal{L}(\phi|X) = \sum_{t=1}^{N} \log \sum_{i=1}^{K} \pi_i P(x|C_i) = \sum_{t=1}^{N} \log \underbrace{\sum_{i=1}^{K} \pi_i \frac{1}{2b_i} \exp\left(-\frac{|x^t-u_i|}{b_i}\right)}_{=G}$$

$$\frac{\partial \mathcal{L}}{\partial b_i} = \sum_{t=1}^{N} \frac{\partial \mathcal{L}}{\partial G} \cdot \frac{\partial G}{\partial b_i^{-1}} \cdot \frac{\partial b_i^{-1}}{\partial b_i}$$

$$= \sum_{t=1}^{N} \frac{1}{G} \cdot \left(\frac{\pi_i}{2} \exp\left(-\frac{|x^t-u_i|}{b_i}\right) - \frac{\pi_i}{2} b_i^{-1} \cdot |x^t-u_i| \exp\left(-\frac{|x^t-u_i|}{b_i}\right)\right) \cdot \frac{1}{b_i^2}$$

$$= \sum_{t=1}^{N} \frac{\overbrace{\pi_i \frac{1}{2b_i} \exp\left(-\frac{|x^t-u_i|}{b_i}\right)}^{=r(z_i^t)} (1 - b_i^{-1}|x^t-u_i|) \cdot b_i^{-1}}{\sum_{j=1}^{K} \pi_j \frac{1}{2b_j} \exp\left(-\frac{|x^t-u_j|}{b_j}\right)}$$

$$= \sum_{t=1}^{N} r(z_i^t) \cdot (1 - b_i^{-1} \cdot |x^t - u_i|) \cdot b_i^{-1} = 0$$

$$\therefore b_i = \frac{\sum_{t=1}^{N} r(z_i^t) \cdot |x^t - u_i|}{N_i}, \quad N_i = \sum_{t=1}^{N} r(z_i^t)$$

$$\frac{\partial\left(\mathcal{L} + \alpha\left(1 - \sum_{i=1}^{K} \pi_i\right)\right)}{\partial \pi_i} = \left(\sum_{t=1}^{N} \frac{\partial \mathcal{L}}{\partial G} \cdot \frac{\partial G}{\partial \pi_i}\right) + \alpha$$

$$= \sum_{t=1}^{N} \frac{1}{G} \cdot \frac{1}{2b_i} \exp\left(-\frac{|x^t-u_i|}{b_i}\right) - \alpha = 0$$

$$\Longrightarrow \sum_{t=1}^{N} \overbrace{\frac{\pi_i \frac{1}{2b_i} \exp\left(-\frac{|x^t-u_i|}{b_i}\right)}{G}}^{r(z_i^t)} = \alpha \cdot \pi_i$$

$$\therefore \pi_i = \frac{\sum_{t=1}^{N} r(z_i^t)}{\alpha}, \text{ we have } \sum_{i=1}^{K} \pi_i = 1$$