

HW 0:

Problem 1:

$$1. \|Xw - y\|^2 = \|y\|^2 + \|Xw\|^2 - 2y^T Xw$$
$$= \|y\|^2 + w^T X^T X w - 2y^T Xw$$

$$\text{gradient} = 0 + 2X^T X w - 2X^T y = 0$$

$$\Rightarrow X^T X w = X^T y$$

$\therefore n \geq m$, if $X^T X$ is invertible

$$w = (X^T X)^{-1} X^T y$$

$$2. \|Xw - y\|^2 + \lambda \|w\|^2$$

$$= \|y\|^2 + w^T X^T X w - 2y^T Xw + \lambda w^T w$$

$$\text{gradient} = 0 + 2X^T X w - 2X^T y + 2\lambda w = 0$$

$$(X^T X + \lambda I) w = X^T y$$

if $(X^T X + \lambda I)$ is invertible, then

$$w = (X^T X + \lambda I)^{-1} X^T y$$

Problem 2:

$$1. \Pr(<H, H, T, T, H>) = p^3(1-p)^2 / \log \Pr(<H, H, T, T, H>) = \frac{3 \log p + 2 \log(1-p)}{2 \log(1-p)}$$

Given a particular sequence, i stands for the number of 'H' in seq

$$\Pr(\text{seq}) = p^i (1-p)^{5-i}$$

$$2. a. \Pr(\text{fair} \cap <H, H, T, T, H>) = \frac{1}{2} \times (\frac{1}{2})^3 \times (1 - \frac{1}{2})^2 = (\frac{1}{2})^6 = \frac{1}{64}$$

$$b. \Pr(\text{biased} \cap <H, H, T, T, H>) = \frac{1}{2} \times (\frac{2}{3})^3 \times (1 - \frac{2}{3})^2 = \frac{1}{2} \times \frac{8}{27} \times \frac{1}{9} = \frac{4}{243}$$

