Q2. d

$$\mathcal{L}(\phi|\mathcal{X}) = \sum_{t=1}^{N} \log \sum_{i=1}^{K} \pi_i P(x|C_i) - \frac{\lambda}{2} \sum_{i=1}^{K} \sum_{j=1}^{d} (\Sigma_i^{-1})_{jj}$$

$$\frac{\partial \mathcal{L}}{\partial \Sigma_i^{-1}} = \sum_{t=1}^{N} \frac{\pi_i P(x|C_i)}{\sum_{l=1}^{K} \pi_l P(x|C_i)} \cdot \frac{\partial \log P(x|C_i)}{\partial \Sigma_i^{-1}} - \frac{\lambda}{2} I$$

$$= \sum_{t=1}^{N} r(z_i^t) \cdot \frac{\partial \left(-\frac{d}{2}\log 2\pi - \frac{1}{2}\log|\Sigma| - \frac{1}{2}(x^t-u)^T \Sigma^{-1} (x^t-u)\right)}{\partial \Sigma^{-1}}$$

$$- \frac{\lambda}{2} I$$

$$= \left[\sum_{t=1}^{N} r(z_i^t) \cdot \left(\frac{\Sigma_i}{2} - \frac{1}{2}(x^t-u)(x^t-u)^T\right]\right. - \frac{\lambda}{2} I = 0$$

$$\Rightarrow \sum_{t=1}^{N} r(z_i^t) \Sigma_i = \sum_{t=1}^{N} r(z_i^t)(x^t-u_i)(x^t-u_i)^T + \frac{\lambda}{2} I$$

$$\therefore \Sigma_i = \frac{\sum_{t=1}^{N} r(z_i^t)(x^t-u_i)(x^t-u_i)^T + \frac{\lambda}{2} I}{N_i}$$