# Calculus

## Series Convergence Tests

| Test | Series | Converges | Diverges | Remarks |
|---|---|---|---|---|
| **For Divergence** (TFD) | $\sum_{n=1}^{\infty} a_n$ | CANNOT show convergence | $\lim_{n\to\infty} a_n \neq 0$ | always check first! |
| **Geometric** | $\sum_{n=1}^{\infty} ar^{n-1}$ | $\|r\| < 1$ | $\|r\| \geq 1$ | $\text{sum} = \frac{\text{first term}}{1-r}$ |
| **Telescoping** | $\sum_{n=1}^{\infty} (b_n - b_{n+k})$ | $\lim_{n\to\infty} b_{n+k} = L$<br>$L$ has to be finite | $\lim_{n\to\infty} b_{n+k}$<br>D.N.E. or inf | write out serval terms then cancel stuff to find paritial sum |
| **$P$-Series** | $\sum_{n=1}^{\infty} \frac{1}{n^p}$ | $p > 1$ | $p \leq 1$ | famous sum $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ |
| **Integral** | $\sum_{n=1}^{\infty} a_n$<br>$a_n = f(n) \geq 0$ | $\int_1^{\infty} f(x)\,dx$<br>Converges | $\int_1^{\infty} f(x)\,dx$<br>Diverges | $f(x)$ has to be positive, continuous & decreasing for $x \geq 1$ |
| **Direct Comparison** (DCT) | $\sum_{n=1}^{\infty} a_n$<br>$a_n > 0$ | $\sum_{n=1}^{\infty} a_n \leq$ a known convergent | $\sum_{n=1}^{\infty} a_n \geq$ a known divergent | try to use $p$-series or geometric series to compare |
| **Limit Comparison** (LCT) | $\sum_{n=1}^{\infty} a_n$<br>$a_n > 0$ | $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ &<br>$\sum_{n=1}^{\infty} b_n$ is known to be convergent | $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ &<br>$\sum_{n=1}^{\infty} b_n$ is known to be divergent | this version of LCT is inconclusive if $L = 0$ or $L = \infty$ |
| **Alternating** (AST) | $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$<br>$b_n \geq 0$ | (1.) $\lim_{n\to\infty} b_n = 0$<br>(2.) $b_{n+1} \leq b_n$ | use TFD<br>$\lim_{n\to\infty} (-1)^{n-1} b_n \neq 0$ | $(-1)^{n-1}$<br>$= \cos((n-1)\pi)$ |
| **Ratio** | $\sum_{n=1}^{\infty} a_n$ | $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = L < 1$ | $\lim_{n\to\infty} \left\|\frac{a_{n+1}}{a_n}\right\| = L > 1$ | inconclusive if $L = 1$<br>great for ! and $(\ )^n$ |
| **Root** | $\sum_{n=1}^{\infty} a_n$ | $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L < 1$ | $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L > 1$ | inconclusive if $L = 1$<br>great for $(\ )^n$ |

If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ is **absolute convergent** (which implies $\sum_{n=1}^{\infty} a_n$ also converges)

If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges, then $\sum_{n=1}^{\infty} a_n$ is **conditional convergent**