ES280 Problem Set 2
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Classmates consulted

I consulted Kaelyn, Sophie, and Daniel on this problem set.

1 Uniform colors

1.1 One-sided vs. two-sided tests

The researchers would likely use a two-sided test in this case because they do not appear to have a prior belief about one color having a higher probability of winning than the other. They state that the problem is to determine whether "one of the colors will win a majority of the time", which indicates that they are looking for any deviation from the expected probability of 0.5. One implication of using a two-sided test is that it will increase the p-value.

a. The null hypothesis H_0 is that the probability of a competitor wearing red winning is equal to 0.5. The alternative hypothesis H_a is that the probability of the competitor wearing red winning is greater than 0.5.

b. $H_0 : p = 0.5, \quad H_a : p > 0.5$

c. Since the number of samples is large ($n >> 40$) we will use the Z statistic.

$$Z = \frac{\hat{P} - p - \Delta_\mu}{\sqrt{p(1-p)/n}} = \frac{0.543 - 0.5 - 0}{\sqrt{0.5(1-0.5)/457}} = 1.82$$

d. We could use a binomial distribution (a series of coin flips) to simulate this same experiment. We could flip a fair coin ($p(heads) = 0.5$) 457 times and count the number of heads (successes). We could run this simulation thousands of times, creating an empirical PMF out of the proportion of successes observed. We could then sum the portion of the PMF that is as or more extreme than a success rate (rate of seeing heads) 54.3% of the time. Since the number of heads out of 457 flips is countably finite, we could use a discrete distribution.

e. Entering `1-normcdf(1.82)` into Matlab yields a p-value of 0.0341. My conclusion is that we can reject the null hypothesis with fairly good certainty. Within the context of our experiment, we would reject the null hypothesis that the likelihood of a competitor wearing red will win 50% of matches.

$$N \sim (0.5, 0.0234)$$

f. See Figure 1.

g. The null hypothesis H_0 is that the probability of a competitor wearing red winning is equal to 0.5. The alternative hypothesis H_a is that the probability of the competitor wearing red winning is not equal to 0.5.

$$H_0 : p = 0.5, \quad H_a : p \neq 0.5$$

h. From `2*(1-normcdf(Z))` in Matlab, we get a p-value of 0.0681 which is double the one-sided p-value.

i. See Figure 2.

j. Moving from a one-sided to a two-sided test increases the difficulty of rejecting the null hypothesis. This is because your p-value is doubled in a two-sided test.

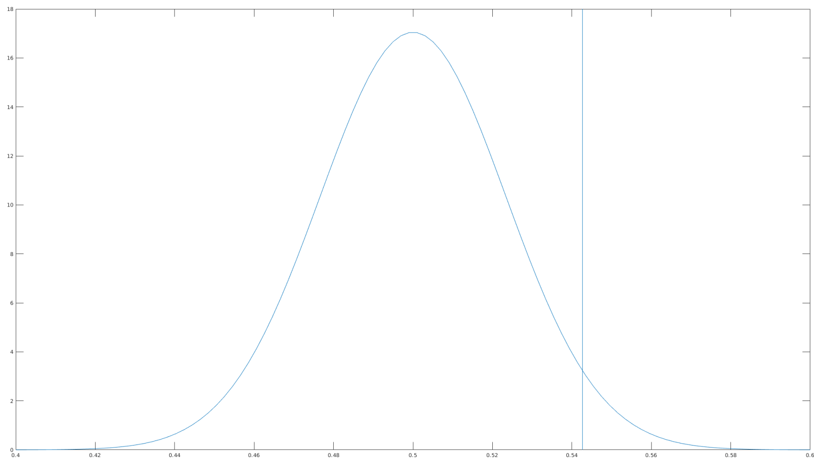

Figure 1: Plot of null distribution

1.2 Effect size

a. In this case, the test statistic is:

$$Z = \frac{\hat{P} - p - \Delta_\mu}{\sqrt{p(1-p)/n}} = \frac{0.57 - 0.5 - 0}{\sqrt{0.5(1-0.5)/457}} = 2.99$$

The new one-sided p-value is 0.0014 and the two-sided p-value is 0.0028.

b. They are smaller. The difference between the ratio $\hat{P}$ and the value of p under our null hypothesis has grown, so we should have more confidence in rejecting H_0.

c. The distance between the observed paramter $\hat{P}$ and the value specified under the null hypothesis, p, is in the numerator of the function which calculates our Z-score. This means that a larger value of $\hat{P} - p$ (assuming Δ_0 does not change) will result in a larger Z score, and a smaller p-value.

1.3 Sample size

a. The mean is still 0.5 but the standard deviation has increased by nearly 50% to 0.0303.

b. In this case, the test statistic is:

$$Z = \frac{\hat{P} - p - \Delta_\mu}{\sqrt{p(1-p)/n}} = \frac{0.551 - 0.5 - 0}{\sqrt{0.5(1-0.5)/272}} = 1.6977$$

The new one-sided p-value is 0.0448 and the two-sided p-value is 0.0896. These p-values are larger than the original case, which makes sense given the decrease in sample size.

c. Even though the effect size has actually increased, the decrease in the number of samples taken makes it more difficult for us to reject H_0.

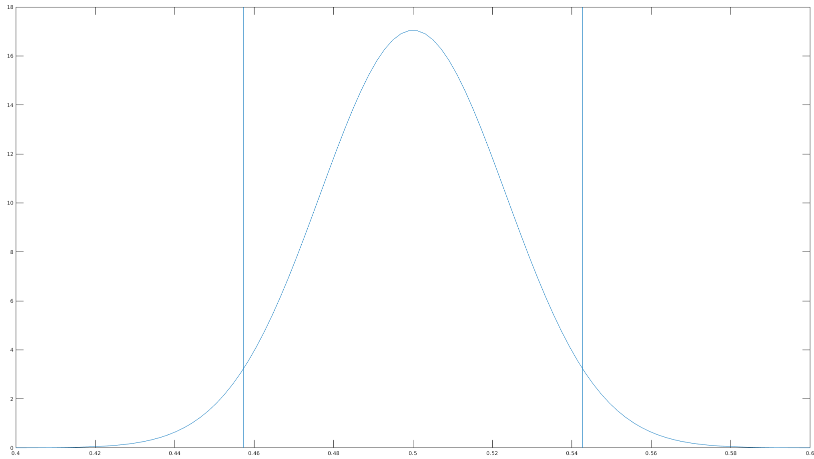

Figure 2: Plot of null distribution

2 Sampling distribution

a. We can calculate a Z-statistic using the standard error, which is $1.0/\sqrt{25} = 0.2$.

$$Z = \frac{x - \mu}{\sigma} = \frac{3.25 - 3.5}{0.2} = -1.25$$

From Matlab, `1- tcdf(-1.25, 24) = 0.8944`.

b. We can calculate a Z-statistic using the standard error, which is $1.0/\sqrt{25} = 0.2$.

$$Z = \frac{x - \mu}{\sigma} = \frac{3.25 - 3.5}{0.1} = -2.5$$

From Matlab, `1- tcdf(-1.25, 24) = 0.9938`.

c. The last two probabilities differ because increasing our sample size decreases our standard error.

3 Executive

a. Where x is defined as length of employment in years, $H_0 : x \geq 6, \quad H_a : x < 6$.

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{5.25 - 6}{1.5/\sqrt{6}} = -1.4697$$

We should fail to reject the null hypothesis because the T_{reject} value is -2.015.

b. The p-value from Matlab is `tcdf(-1.4697, 6-1) = 0.1008`.

4 Lottery

a. With x_1 as the array of states with 'For Education' and x_2 as the array of states with 'General Use', we can use Matlab to determine that we fail to reject the null hypothesis.

In Matlab, `ttest2(x1, x2, 'vartype', 'equal', 'Tail', 'right')` yields $p = 0.0481$.

5 Stomach flu

a. We can calculate the test statistic using:

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.43 - 0.36}{\sqrt{0.38(1-0.38)(\frac{1}{55} + \frac{1}{149})}} = 0.9645$$

The critical Z-value is 1.65. We therefore fail to reject the null hypothesis.

b. Matlab yields a p-value of `1-normcdf(0.9645)`$= 0.1674$.

6 Matlab problem

a. See Figure 3.

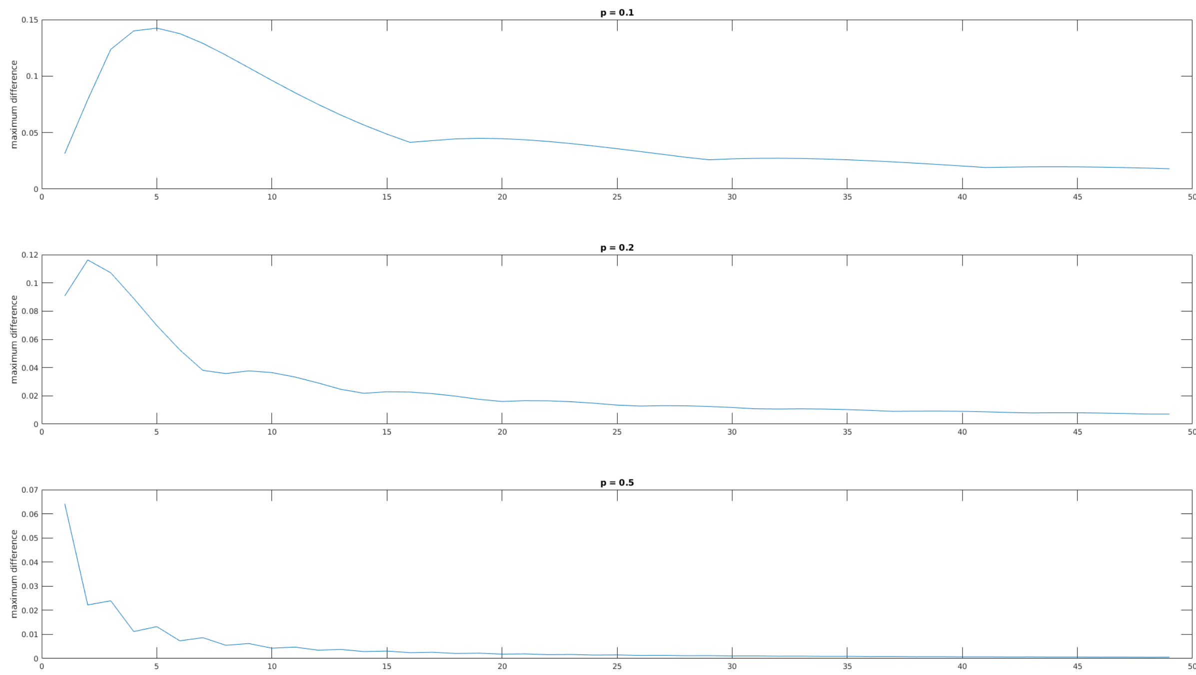

Figure 3: Plot of binomial and normal max differences

The maximum value when both $np > 5$ and $n(1-p) > 5$.

```
% Problem 1
n = 457
n_red = 248
p_hat = n_red / n
P = 0.5
delta = 0

% calculate S
sigma = sqrt(((P*(1-P))/n))

% calculate Z stat
Z = (p_hat - P) / sigma

% one-sided p value
p_one_sided = 1-normcdf(Z)

% two-sided p value
p_two_sided = 2*(1-normcdf(Z))

% create plot 1
x = linspace(0.4, 0.6, 100)
mu = 0.5;
sigma = 0.0234;
y = normpdf(x,mu,sigma)
plot(x, y)
hold on
% vertical line
line([p_hat, p_hat], [0, 18])

% new p-hat
```

```
Z_2 = (0.57 - P) / sigma

% create plot 2
x = linspace(0.4, 0.6, 100)
mu = 0.5;
sigma = 0.0234;
y = normpdf(x,mu,sigma)
plot(x, y)
hold on
% vertical line
line([p_hat, p_hat], [0, 18])
hold on
line([P-(p_hat-P), P-(p_hat-P)], [0, 18])


% one-sided p value
p_one_sided_2 = 1-normcdf(Z_2)

% two-sided p value
p_two_sided_2 = 2*(1-normcdf(Z_2))

% new sample size
n = 272
n_red = 150
p_hat = n_red / n
P = 0.5
delta = 0

% calculate S
sigma = sqrt(((P*(1-P))/n))
```

```
% calculate Z stat
Z = (p_hat - P) / sigma

% one-sided p value
p_one_sided = 1-normcdf(Z)

% two-sided p value
p_two_sided = 2*(1-normcdf(Z))


% problem 2
std_error_1 = 1 / sqrt(25)
z_1 = (3.25-3.5) / std_error_1
p_1 = 1- normcdf(z_1)

std_error_2 = 1 / sqrt(100)
z_2 = (3.25-3.5) / std_error_2
p_2 = 1- normcdf(z_2)


% problem 3
xbar = 5.25
mu = 6
n = 6
sigma = 1.25
std_err = sigma / sqrt(n)
T = (xbar - mu) / std_err
df = n-1
tinv(0.05, n-1)
```

```
p = tcdf(T, df)


%problem 4
x1 = [24 25 28 28 28 29 29 31 31 35 35 35]'
x2 =  [21 22 24 27 27 28 29 32 32]'

xbar1 = mean(x1)
xbar2 = mean(x2)

[h,p,ci,stats] = ttest2(x1, x2, 'vartype',↙
'equal', 'Tail', 'right')

%problem 5
n1 = 55
n2 = 149
phat = (24+54)/(55+149)
phat1 = 24/55
phat2 = 54/149
Z_num = (phat1-phat2)
Z_denom = sqrt(phat*(1-phat)*((1/n1) + (1/n2)))
Z = Z_num / Z_denom

Z_crit = norminv(0.95)

p = 1-normcdf(Z)

%problem 6
```

```
ps = [0.1 0.2 0.5]
ns=2:1:50
i = 1
for p = ps
    l=[]
    for n = ns
        x=0:1:n
        binos = binopdf(x, n, p)
        norms = normpdf(x, n*p,sqrt(n*p*(1-p)))
        maxval = max(abs(binos-norms))
        l = [l maxval]
    end
    subplot(3, 1, i)
    plot(l)
    title(['p = ' num2str(p)])
    ylabel('maximum difference')
    if i == 3
        xlabel('n')
    end
    i = i+1
end
```