

## 1 Stats

Distribution	$\mathbb{E}(x)$	$\mathbb{V}(x)$	$f(x)$	$F(x)$
Uniform	$(a+b)/2$	$(b-a)/12$	$1/(b-a)$	$\frac{x-a}{b-a}$
Binomial	$np$	$np(1-p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	
Bernoulli	$p$	$p(1-p)$	$p^x(1-p)^{1-x}$	$(1-p)^{1-x}$
Normal	$\mu$	$\sigma^2$		

- $Z = \frac{X-\mu}{\sigma}$ ,  $se = \frac{\sigma}{\sqrt{n}}$ ,
- **68, 95, 99.7** at 1, 2, 3  $\sigma$  in normal distribution
- CI widths are  $t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$  or  $Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$  or  $\sqrt{\hat{p}(1-\hat{p})/n}$
- for large population proportion, use  $\sim N(\hat{p}, \sqrt{p(1-p)/n})$
- Type I errors (false positives): incorrectly **reject** the null hypothesis. This is equivalent to  $\alpha$ .
- Type II errors (false negatives): incorrectly **fail to reject** the null hypothesis.

## 2 Matlab for stats

```
% MLR
data = readtable('...')
col = data(:,n)
X = table2array([cols])
mod = fitlm(X, y, 'VarNames', {'x', ..., 'y'})
coef_tbl = mod.Coefficients
size(X, 1) % get rows of X
[b, se, pval, inmodel, stats, nextstep, hst] = stepwisefit(X, y)
% sampling / stats
y = binornd(10, 0.1, 100, 1)
Z = (p_hat - P) / sigma % for prop test
p_one_sided = 1 - normcdf(Z); p_two_sided = 2*(1 - normcdf(Z))
% using binocdf for discrete hypothesis test
% ex: 22 of 32 successes
binocdf(21, 32, 0.5) % -> 97.49
% important inverses
tinv(alpha, n-1); norminv(alpha)
% pdfs
normpdf(x, mu, sigma); binopdf(x, n, p)
% approximation of norm w/ binomial
normpdf(x, n*p, sqrt(n*p*(1-p)))
% t-test helper fn
[h, p, ci, stats] = ttest2(x1, x2, 'vartype', 'equal',
    'Tail', 'right')
% golden rectangle example
n = 20, x = 0.66, s = 0.093, mu = 0.618
tcrit = tinv(1-(.05/2), n-1) % -> 2.093
t = (x-mu) / (n / sqrt(s)) % -> 2.019, DNR
%other
plotmatrix(X); corrplot(X)
```

## 3 LR / MLR

$$S_{xx} = \sum (x_i - \bar{x})^2, \quad S_{xy} = \sum y_i (x_i - \bar{x})^2$$

$$\beta_1 = \frac{S_{xy}}{S_{xx}}, \quad \beta_0 = \bar{y} - \beta_1 \bar{x}$$

$$SST = SSR + SSE, \quad R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

$$SSE = \sum (y_i - \bar{y})^2, \quad SSR = \sum (\hat{y}_i - \bar{y}_i)^2, \quad SST = \sum (y_i - \hat{y}_i)^2$$

$$R_{adj}^2 = 1 - \frac{SSE/(n-p)}{SST/(n-1)}$$

- if a single  $\beta_i$  is significant, regression is significant
- t-tests for individual  $\beta_i$  terms, F-test for overall regression

## 4 Diff eqs

- *Accumulation* =  $F_i - F_o + \text{generation}$
- For tank example with valve (with constant  $R$ ),  $F_o = \frac{h}{R}$ .
- Standard form is  $\tau_p \frac{dh}{dt} + h(t) K_p F_i(t)$  where  $\tau_p$  and  $K_p = R$  are the time constant and static gain.
- Laplace of standard form is  $h(s) = \frac{K_p}{\tau_p s + 1} F_i(s)$  where  $G_p = \frac{K_p}{\tau_p s + 1}$
- transfer functions can be multiplied.  $G_1(s) \rightarrow G_2(s) = G_1(s)G_2(s)$  where  $G_1(s) = h_1(s)/F_i(s)$  and  $G_2(s) = h_2(s)/h_1(s)$ .
- For a 2nd order ODE we have time constant  $\tau_p$ , damping factor  $\zeta$ , static gain  $K$ .

$$G_0(s) = \frac{k_1 k_2}{\tau_1 \tau_2 s^2 + 2\zeta \tau s + 1} \rightarrow \frac{K}{\tau^2 s^2 + 2\zeta \tau s + 1 = 0}$$

damping factor	type	behavior
$\zeta = 1$	critically damped	desireable, stable
$\zeta > 1$	overdamped	slow to reach step change
$\zeta < 1$	underdamped	initial overshoot, oscillations

- if all poles (roots of polynomial in denominator of  $G(s)$ ) are real and negative, system is stable. imaginary components of roots cause oscillatory behavior. positive real root components cause exponential growth.
- Laplace of impulse is  $s$ , step is  $1/s$ . For temporary step (eg from  $t = 2$  to  $t = 5$ ):

$$f_i(t) = u(t-2) - u(t-5) \rightarrow L[f(t)] = e^{-2s} - e^{-5s}/s$$

- adding in proportional control (PC), we get  $G_c = K_c$  (a constant) and  $G(s) = K_c K_p / (\tau_p s + 1 + K_c K_p)$ .

$$y(s) = \frac{G_p G_c}{1 + G_p G_c} y_{sp}(s) + \frac{G_d}{1 + G_p G_c} d(s) = \frac{k'_p}{\tau'_p s + 1} y_{sp}(s) + \frac{k'_d}{\tau'_p s + 1} d(s)$$

- We define  $k'_p = K_p K_c / (1 + K_p K_c)$ ,  $k'_d = K_d / (1 + K_p K_c)$ , and  $\tau'_p = \tau_p / (1 + K_p K_c)$ . Note that adding PC doesn't change order,  $\tau'_p < \tau_p$ , and  $k'_p < k_p$ .
- With  $d(s) = 0$ , a step change in  $y_{sp}$  yields  $y(t) = k'_p (1 - e^{-t/\tau'_p})$
- Final value theorem:  $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s f(s)$

## 5 Matlab for diff eqs

```
syms s, y(t) % make symbolic variables
ilaplace((1/15)/((10/3)*s^2 + s)) % -> 1/15 - exp(-(3*t)/10)/15
laplace(2*diff(y(t), 2) + 8*y(t) - 3)
[r, p, k] = residue([0.2], [10, 1, 0]) % -> [-0.2, 0.2], [-0.1, 0]
% interpret residue as -.2/(s+1) + .2/s
G = tf(2, [1 3]); stepplot(G); pole(G)
```