1 Stats

Distribution	$\mathbb{E}(x)$	$\mathbb{V}(x)$	$f(x)$	F(x)
Uniform	$(a+b)/2$	$(b-a)/12$	$1/(b-a)$	$\frac{x-a}{b-a}$
Binomial	np	$np(1-p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	
Bernoulli	p	$p(1-p)$	$p^x(1-p)^{1-x}$	$(1-p)^{1-x}$
Normal	μ	σ^2		

- $Z = \frac{X-\mu}{\sigma}, \quad se = \frac{\sigma}{\sqrt{n}},$
- **68, 95, 99.7** at 1, 2, 3 σ in normal distribution
- CI widths are $t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$ or $Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or $\sqrt{\hat{p}(1-\hat{p})/n}$
- for large population proportion, use $\sim N(\hat{p}, \sqrt{p(1-p)/n})$
- Type I errors (false positives): incorrectly **reject** the null hypothesis. This is equivalent to α.
- Type II errors (false negatives): incorrectly **fail to reject** the null hypothesis.

2 Matlab for stats

```
% MLR
data = readtable('...')'
col = data(:,n)
X = table2array([cols])
mod = fitlm(X, y, 'VarNames', {'x',..., 'y'})
coef_tbl = mod.Coefficients
size(X, 1) % get rows of X
[b,se,pval,inmodel,stats,nextstep,hst] = stepwisefit(X,y)
% sampling / stats
y = binornd(10, 0.1, 100, 1)
Z = (p_hat - P) / sigma  % for prop test
p_one_sided = 1-normcdf(Z); p_two_sided = 2*(1-normcdf(Z))
% using binocdf for discrete hypothesis test
% ex: 22 of 32 successes
binocdf(21, 32, 0.5) % -> 97.49
% important inverses
tinv(alpha, n-1); norminv(alpha)
% pdfs
normpdf(x, mu, sigma); binopdf(x, n, p)
% approximation of norm w/ binomial
normpdf(x, n*p,sqrt(n*p*(1-p))
% t-test helper fn
[h,p,ci,stats] = ttest2(x1, x2, 'vartype', 'equal',
    'Tail', 'right')
% golden rectange example
% n = 20, x = 0.66, s = 0.093, mu = 0.618
tcrit = tinv(1-(.05/2),n-1) % -> 2.093
t = (x-mu) / (n / sqrt(s)) % -> 2.019, DNR
%other
plotmatrix(X); corrplot(X)
```

3 LR / MLR

$$S_{xx} = \sum (x_i - \bar{x})^2, \quad S_{xy} = \sum y_i(x_i - \bar{x})^2$$

$$\beta_1 = \frac{S_{xy}}{S_{xx}}, \quad \beta_0 = \bar{y} - \beta_1 x$$

$$SST = SSR + SSE, \quad R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

$$SSE = \sum (y_i - \bar{y})^2, \quad SSR = \sum (\hat{y_i} - \bar{y}_i)^2, \quad SST = \sum (y_i - \hat{y}_i)^2$$

$$R^2_{adj} = 1 - \frac{SSE/(n-p)}{SST/(n-1)}$$

- if a single β_i is significant, regression is significant
- t-tests for individual β_i terms, F-test for overall regression

4 Diff eqs

- $Accumulation = F_i - F_o + generation$
- For tank example with valve (with constant R), $F_o = \frac{h}{R}$.
- Standard form is $\tau_p \frac{dh}{dt} + h(t) K_p F_i(t)$ where τ_p and $K_p = R$ are the time constant and static gain.
- Laplace of standard form is $h(s) = \frac{K_p}{\tau_p s+1} F_i(s)$ where $G_p = \frac{K_p}{\tau_p s+1}$
- transfer functions can be multiplied. $G_1(s) \to G_2(s) = G_1(s)G_2(s)$ where $G_1(s) = h_1(s)/F_i(s)$ and $G_2(s) = h_2(s)/h_1(s)$.
- For a 2nd order ODE we have time constant τ_p, damping factor ζ, static gain K.

$$G_0(s) = \frac{k_1 k_2}{\tau_1\tau_2 s^2 + 2\zeta\tau s + 1} \to \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1 = 0}$$

damping factor	type	behavior
$\zeta = 1$	critically damped	desireable, stable
$\zeta > 1$	overdamped	slow to reach step change
$\zeta < 1$	underdamped	initial overshoot, oscillations

- if all poles (roots of polynomial in denominator of $G(s)$) are real and negative, system is stable. imaginary components of roots cause oscillatory behavior. positive real root components cause exponential growth.
- Laplace of impulse is s, step is $1/s$. For temporary step (eg from $t = 2$ to $t = 5$):

$$f_i(t) = u(t-2) - u(t-5) \to \mathrm{L}[f(t)] = e^{-2s} - e^{-5s}/s$$

- adding in proportional control (PC), we get $G_c = K_c$ (a constant) and $G(s) = K_c K_p/(\tau_p s + 1 + K_c K_p)$.

$$y(s) = \frac{G_p G_c}{1 + G_p G_c} y_{sp}(s) + \frac{G_d}{1 + G_p G_c} d(s) = \frac{k'_p}{\tau'_p s + 1} y_{sp}(s) + \frac{k'_d}{\tau'_p s + 1} d(s)$$

- We define $k'_p = K_p K_c/(1 + K_p K_c)$, $k'_d = K_d/(1 + K_p K_c)$, and $\tau'_p = \tau_p/(1 + K_p K_c)$. Note that adding PC doesn't change order, $\tau'_p < \tau_p$, and $k'_p < k_p$.
- With $d(s) = 0$, a step change in y_{sp} yields $y(t) = k'_p(1 - e^{-t/\tau'_p})$
- Final value theorem: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sf(s)$
- adding PC to 2nd order system define: $K'_p = \frac{K_c K_p}{1+K_c K_p}$, $\tau' = \frac{\tau}{\sqrt{1+K_p K_c}}$, $\zeta' = \frac{\zeta}{\sqrt{1+K_p K_c}}$.

$$y(s) = \frac{k'_p}{\tau 2' s^2 + 2\zeta' \tau' s + 1}$$

- transfers for controls; IC: $\frac{K_c}{\tau_i s}$, PIC: $K_c(1 + \frac{1}{\tau_i s})$, DC: $K_c \tau_d s$, PIDC: $K_c(1 + (1/\tau_i) + \tau_d s)$

5 Matlab for diff eqs

```
syms s, y(t)% make symbolic variables
ilaplace((1/15)/((10/3)*s^2 +s)) % -> 1/15 - exp(-(3*t)/10)/15
laplace(2*diff(y(t), 2) + 8*y(t) -3)
[r,p,k] = residue([0.2], [10, 1, 0]) %-> [-0.2, 0.2], [-0.1, 0]
% interpret residue as -.2/(s+1) + .2/s
G = tf(2, [1 3]); stepplot(G); pole(G)
```