On my honor as a KAIST student, I pledge to take this exam honestly.

You should write down all your work for full credits.

Session 1, December 14, 2021, 19:00–20:15, Four problems: 1–4

1. (10 pts) Mark True or False for each of the following statements. 2 **pts for correct answer**, -1 **pt for wrong answer**, and 0 **pt for no response**. You do not need to justify your answers.

   (a) If a series $\sum_{n=0}^{\infty} a_n x^n$ converges on $[-\mathbf{1}, \mathbf{1}]$, then the series $\sum_{n=1}^{\infty} n a_n x^{n-1}$ converges on $(-\mathbf{1}, \mathbf{1})$.

   (b) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors of three dimension, then
   $$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}\,.$$

   (c) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are nonzero vectors of three dimension and $(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c} = 0$, then $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are in the same plane.

   (d) Assume f is infinitely differentiable at 0 (i.e. $f^{(n)}(0)$ exists for all $n = 1, 2, 3, \ldots$). Then there exists $\epsilon > 0$ such that $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ for all $-\epsilon < x < \epsilon$.

   (e) Let $A(x) = \sum_n a_n x^n$ and $B(x) = \sum_n b_n x^n$ be power series with radii of convergences R_1 and R_2, respectively. Then $A(x) \times B(x)$ is again a power series with a radius of convergence $\min\{R_1, R_2\}$.

2. (15 pts) Determine the radius and interval of convergence of the following series:

   (a) $\sum_{n=1}^{\infty} \frac{1}{n}\, x^n$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n!}\, x^n$

   (c) $\sum_{n=0}^{\infty} (-1)^n\, x^{2^n}$

3. (15 pts) How many domains are separately enclosed by the following polar equations:

   (a) $r = 4\cos(2\theta)$

   (b) $r = \frac{4}{2-\cos\theta}$

   (c) $r = 2(1+\cos\theta)$

4. (10 pts) Let $L_1 : x = 1 + t, y = 3 - t, z = 2t$ and $L_2 : x = 0, y = -2s, z = 5s$. Find the minimum of $|\overrightarrow{P_1P_2}|$ for $P_1 \in L_1$ and $P_2 \in L_2$.