20210808
TREEPAT

① Evaluate $\int \frac{1}{x^2-4x+8}\,dx$

$$I = \int \frac{1}{(x-2)^2+2^2}\,dx$$

$u = x-2 \rightarrow du = dx$

$$I = \int \frac{1}{u^2+2^2}\,du = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right) + C$$

$$I = \frac{1}{2}\tan^{-1}\left(\frac{x-2}{2}\right) + C \quad \#$$

② a.) $\cosh x = \frac{e^x+e^{-x}}{2}$, $\sinh x = \frac{e^x-e^{-x}}{2}$

$$\cosh^2 x - \sinh^2 x = \frac{\cancel{e^{2x}}+2+\cancel{e^{-2x}}}{4} - \frac{\cancel{e^{2x}}-2+\cancel{e^{-2x}}}{4}$$

$$= \frac{2-(-2)}{4} = 1 \quad \#$$

b.) $y = f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$

$$2y = e^x - e^{-x}$$

$$\cancel{\ln(2y) = \ln(e^x - e^{-x})}$$

$$2y(e^x) = (e^x - e^{-x})e^x$$

$$2ye^x = e^{2x} - 1$$

$$e^{2x} - 2ye^x - 1 = 0$$

$$e^x = \frac{2y \pm \sqrt{4y^2+4}}{2} = y \pm \sqrt{y^2+1}$$

$$x = \ln\left(y+\sqrt{y^2+1}\right), \ln\left(y-\sqrt{y^2+1}\right)$$

$$f(x) = \ln\left(x+\sqrt{x^2+1}\right) \quad \#$$