TDFEEPAT 4

MAS101 Midterm Exam, October 19, 2021 Name: TREEPAT. 2

$\ln(\frac{1}{2}) + \ln(\frac{1}{2}) \ln(\frac{1}{2})$

On my honor as a KAIST student, I pledge to take this exam honestly.

This page is a solution to problem # 1 ← *Write down the problem number.*

(1) True

(2)

(3) True

(4)

(5) True ✓

(6) False ✓

(7) False ✓

(8) True —?

(9) True

(10) True ✓

$-\frac{1}{2}\int_{-\infty}^{\infty} \operatorname{sech}^2 x$

$\frac{x}{\ln x^{2022}}$ $= \frac{x^{2022}}{x^{2022}} \frac{2022x^{2021}}{+\ln x^{22}}$ $\frac{}{0.0001 \times 0.0001}$

$f^{-1}(a) = f^{1.001}$

$\ln|x-4| \Big|^{1} = \ln\left|\frac{-3}{5}\right|$

$\frac{1}{e^x + e^{-x}}$ even

$\frac{x}{x+10^{2022}}$ $\frac{1}{1}$ $\frac{}{\frac{1}{x}}$

$\frac{-2x}{x^2+1}$ odd

$\int \frac{1}{x \ln x} dx$ $u = \ln x$ $dx = e^u du$

$= \int \frac{1}{\ln x} d\ln x = \int \frac{1}{u} du = \ln u$ (7) $= \ln(\ln x)\Big|_1^2$ $= \ln(\ln 2) - \ln(\ln 1)$

$\frac{x}{x+10^{2022}}$ $\frac{1}{1}$ $\frac{}{\frac{1}{x}}$

$\frac{1 - \frac{1}{2x}}{1+\sqrt{x}}$

$\int \frac{1e^u du}{u} = \int u^{-1} de^u = u^{-1}e^u - \int e^u(\frac{-1}{u^2})du$

$\int_2^{\infty} \frac{1}{\ln x} dx$ $\frac{1}{x^2}$ $\frac{\frac{1}{x^2}}{\ln x}$ $\frac{\frac{-2}{x^3}}{\frac{1}{x}} = \frac{-2}{x^4}$ 0 $n \times n-1$

$\frac{1}{\ln x}$

$y = \frac{1}{\ln x}$

$\ln x = \frac{1}{y}$

$e^{1/y}$

$x =$

$u = \ln x \rightarrow x = e^u$

$du = \frac{1}{x} dx$

$dx = x du$

$= e^u du$

$\int \frac{1}{\ln x} dx$ $\int u^{-1} de^u = u^{-1}e^u - \int e^u d(\frac{1}{u})$

$= u^{-1}e^u$

(6)

$\frac{x}{\ln x} - \int x \frac{1}{x} dx = \frac{x}{\ln x} - x \Big|_2^{\infty}$

$\frac{x - x\ln x}{\ln x} \overset{L'H}{=} \frac{1 - \frac{x}{x} - \ln x}{1/x} = \frac{-\ln x \cdot x}{\frac{1}{x}}$

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This page is a solution to problem # $\boxed{2}$ ← *Write down the problem number.*

Consider $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$

$\int_{x=f(a)}^{f(b)} f^{-1}(x)dx$ then sub. $u = f^{-1}(x) \rightarrow f(u) = x$

$dx = f'(u)du$

$$L_s = \int_{u=a}^{b} u f'(u) du$$

$$= \int_{u=a}^{b} u \, df(u)$$

by parts; $= u f(u)\Big|_a^b - \int_{u=a}^{b} f(u) du$

Combining the whole thing;

$$\int_{f(a)}^{f(b)} f^{-1}(x) dx + \int_a^b f(x)dx = x f(x)\Big|_a^b - \cancel{\int_a^b f(x)dx} + \cancel{\int_a^b f(x)dx}$$

$$= bf(b) - af(a)$$

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(a) *This page is a solution to problem #* $\boxed{3}$ ← *Write down the problem number.*

$\frac{d}{dx}\tanh^{-1}(x)$

$y = \tanh^{-1}(x)$

$\tanh y = x$

diff. both side;

$\operatorname{sech}^2 y \, dy = dx$

$\frac{dy}{dx} = \cosh^2 y$

$= \cosh^2(\tanh^{-1} x)$

~~$y = \tan^{-1}(x) \to \tan y = x$~~

~~$\left(\frac{e^y - e^{-y}}{e^y + e^{-y}}\right) = (x) e^y$~~

~~$\frac{e^{2y}-1}{e^{2y}+1} = x e^y$~~

~~$x e^{3y} - e^{2y} + x e^y + 1 = 0$~~

~~$x e^y (e^{2y}+1) - (e^{2y}-1) = 0$~~

(b) $\int x \tanh^{-1}(x) dx = I$

by parts;

$I = \int \tanh^{-1}(x) \, d\frac{x^2}{2}$

$= \frac{\tanh^{-1} x \cdot x^2}{2} - \frac{1}{2}\int x^2 \left(\frac{1}{1-x^2}\right) dx$

Since $\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$;

$\int \frac{x^2}{1-x^2} dx = \int \frac{1}{1-x^2} - 1 \, dx = \tanh^{-1}(x) - x$

Therefore;

$I = \frac{x^2 \tanh^{-1}(x)}{2} - \frac{1}{2}\tanh^{-1}(x) + \frac{1}{2}x + C$