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TREEPAT

9.2 (20) $\frac{dy}{dx} + xy = x \quad , y(0) = -6$

$\frac{dy}{dx} = x(1-y) \rightarrow \frac{dy}{1-y} = x\,dx$

$\int \frac{1}{1-y}\,dy = \int x\,dx$

$-\ln|1-y| = \frac{x^2}{2} + c$

$1-y = e^{-x^2/2 - c}$

$y = 1 - e^{-x^2/2 - c}$

$-6 = 1 - e^{-c}$

$e^{-c} = 7$

$c = \ln\left(\frac{1}{7}\right)$

$\therefore y = 1 - e^{-x^2/2 + \ln 7}$

10.1 (64) $a_n = \frac{(-4)^n}{n!}$

$a_n = (-1)^n \underbrace{\left(\frac{4}{n!}\right)}_{b_n}$ from the alternative test, $b_n = \frac{1}{n!} > \frac{1}{(n+1)!} = b_{n+1}$ and $\lim_{n\to\infty} a_n = 0$

$\therefore \sum a_n$ converges to 0

10.2 (58) $\sum_{n=0}^{\infty} \frac{1}{x^n} \; ; \; |x| > 1$

$= \frac{1}{x^0} + \frac{1}{x^1} + \frac{1}{x^2} + \frac{1}{x^3} + \cdots = \frac{x}{x-1}$

$\therefore \sum_{n=0}^{\infty} \frac{1}{x^n}$ converges to $\frac{x}{x-1}$

10.3 (6) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

by integral test, $\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{x(\ln x)^2}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{1}{u^2}\,du$

$= \lim_{t\to\infty} -\frac{1}{u}\Big|_{\ln 2}^{\ln t}$

$= \lim_{t\to\infty} -\frac{1}{\ln t} + \frac{1}{\ln 2}$

$= \frac{1}{\ln 2}$ converges

10.3 (34) $\sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right)$

by limit compare with $\sum_{n=1}^{\infty} 1$ which diverges,

$= \lim_{n\to\infty} \frac{n\tan\left(\frac{1}{n}\right)}{1} = \lim_{n\to\infty} \frac{\tan\left(\frac{1}{n}\right)}{\frac{1}{n}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\sec^2\left(\frac{1}{n}\right)\cancel{\cdot -\frac{1}{n^2}}}{\cancel{-\frac{1}{n^2}}}$

$= \lim_{n\to\infty} \sec^2\left(\frac{1}{n}\right) = \sec^2(0) = 1 > 0 \rightarrow$ both diverge

$\therefore \sum_{n=1}^{\infty} n\tan\left(\frac{1}{n}\right)$ diverges

10.4 (15) $\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$

limit compare with $\sum \frac{1}{n^2}$ which converges

$= \lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n^2}\right)}{\frac{1}{n^2}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{n^2}{n^2+1}\left(\cancel{\frac{-2}{n^3}}\right)}{\cancel{\frac{-2}{n^3}}} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = 1 > 0$ both converges

10.4 (60) Suppose $a_n > 0$, $\lim_{n\to\infty} n^2 a_n = 0$, Prove $\sum a_n$ converges

by limit compare with $\sum \frac{1}{n^2}$ which converges

$\lim_{n\to\infty} \frac{a_n}{\frac{1}{n^2}} = \lim_{n\to\infty} n^2 a_n = 0 \rightarrow \sum a_n$ converges