20210808
TREEPAT

10.9 (20) Find Taylor series at $x=0$ of $x\ln(1+2x)$

$$\ln(1+x)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}\rightarrow \ln(1+2x)=4\sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}$$

$$x\ln(1+2x)=4\sum_{n=1}^{\infty}\frac{(-1)^{n}x^{n+1}}{n}$$

10.9 (52 a) $f(x)=\sum_{n=0}^{\infty}a_nx^n$ converges for all $x\in(-R,R)$. Show that if f is even, then $a_1=a_3=a_5=\dots=0$ i.e., Taylor series for f at $x=0$ contains only even power of x.

$f(x)$ is even $\rightarrow$ $\sum_{n=0}^{\infty}a_nx^n=f(x)=f(-x)=\sum_{n=0}^{\infty}a_n(-x)^n$ converges as well

$$a_0+a_1x+a_2x^2+a_3x^3+\dots=a_0-a_1x+a_2x^2-a_3x^3+\dots$$

$$\therefore a_1=a_3=a_5=\dots=0$$

10.10 (8) Find first four terms of $(1+x^2)^{-1/3}$

$$(1+x)^n=\sum_{k=0}^{\infty}\binom{n}{k}x^k=1+nx+\frac{n(n-1)x^2}{2}+\frac{n(n-1)(n-2)x^3}{6}+\dots$$

$$(1+x^2)^{-1/3}=1-\frac{1}{3}x^2+\frac{4}{3^2}\frac{x^4}{2!}-\frac{28}{3^3}\frac{x^6}{3!}+\dots$$

10.10 (66) Derive the series $\tan^{-1}(x)=\frac{\pi}{2}-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\dots$; $x>1$ and
$\tan^{-1}(x)=-\frac{\pi}{2}-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\dots$; $x<-1$

$$\lim_{y\to\infty}\int_x^y\frac{1}{1+t^2}dt=\lim_{y\to\infty}\int_x^y\left(\frac{1}{t^2}-\frac{1}{t^4}+\frac{1}{t^6}-\frac{1}{t^8}+\dots\right)dt$$

$$\frac{\pi}{2}-\tan^{-1}(t)=\frac{1}{x}-\frac{1}{3x^3}+\frac{1}{5x^5}-\frac{1}{7x^7}+\dots$$

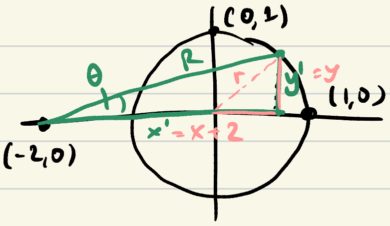
$\therefore$ for $x>1$, $\tan^{-1}(x)=\frac{\pi}{2}-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\dots$

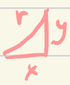
$$\lim_{y\to-\infty}\int_y^x\frac{1}{1+t^2}dt=\lim_{y\to-\infty}\int_y^x\left(\frac{1}{t^2}-\frac{1}{t^4}+\frac{1}{t^6}-\frac{1}{t^8}+\dots\right)dt$$

$$\tan^{-1}(x)-\left(-\frac{\pi}{2}\right)=-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\dots$$

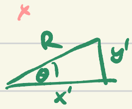
$\therefore$ for $x<-\frac{\pi}{2}$, $\tan^{-1}(x)=-\frac{\pi}{2}-\frac{1}{x}+\frac{1}{3x^3}-\frac{1}{5x^5}+\dots$

11.1 (34) Find parametrization for $x^2+y^2=1^2$ start at $(1,0)$ moving counterclockwise to terminal point $(0,1)$ using θ as in the figure.





$x^2+y^2=r^2$



$x'=R\cos\theta \rightarrow x=x'-2=R\cos\theta-2$
$y'=R\sin\theta \rightarrow y=y'=R\sin\theta$

$$(R\cos\theta-2)^2+(R\sin\theta)^2=r^2=1$$

$$R^2(\sin^2\theta+\cos^2\theta)-4R\cos\theta+4=1$$

$$R^2-4R\cos\theta+3=0$$

$$R=\frac{4\cos\theta\pm\sqrt{16\cos^2\theta-12}}{2}=2\cos\theta\pm\sqrt{4\cos^2\theta-3}$$

at $\theta=0\rightarrow R=3\rightarrow R=2\cos\theta+\sqrt{4\cos^2\theta-3}$

$$\therefore\ x=\left(2\cos\theta+\sqrt{4\cos^2\theta-3}\right)\cos\theta-2$$
$$y=\left(2\cos\theta+\sqrt{4\cos^2\theta-3}\right)\sin\theta$$