On my honor as a KAIST student, I pledge to take this exam honestly.

You should write down all your work for full credits.

Session 1, October 19, 2021, 19:00–20:15, Three problems: 1–3

1. (20 pts) Is each of the following statements true of false? Mark **T** or **F**. 2 **pts for correct answer**, -1 **pt for wrong answer**, and 0 **pt for no response**. You do not need to justify your answers.

   (1) $f = O(g)$ implies $f = o(g)$ for functions f and g that are positive for x sufficiently large.

   (2) $x \times \left(\ln x\right)^{2022} = o(x^{1.001})$

   (3) If $f'(a) \neq 0$, then there exists a $\delta > 0$ such that $f(x) \neq f(a)$ for all $x \neq a$ and $a - \delta < x < a + \delta$.

   (4) $\displaystyle\int_2^\infty \frac{1 - 2^{-x}}{1 + \sqrt{x}} dx$ converges.

   (5) $\displaystyle\int_1^6 \frac{1}{x - 4} dx$ converges.

   (6) $\displaystyle\int_2^\infty \frac{1}{\ln x} dx$ converges.

   (7) $\displaystyle\int_1^2 \frac{dx}{x \ln x}$ converges.

   (8) $\displaystyle\int_{-\infty}^\infty \frac{dx}{e^x + e^{-x}}$ converges.

   (9) $\displaystyle\int_{-\infty}^\infty \frac{2x}{x^2 + 1} dx$ converges.

   (10) $\displaystyle\lim_{x \to \infty} \frac{\ln(x + 10^{2022})}{\ln x} = 1.$

2. (10 pts) Let $f(x)$ be a continuously differentiable (i.e. $f(x)$ is differentiable and $f'(x)$ is continuous) one-to-one function defined on $[a, b]$. Assume that $f'(x) \neq 0$ on $[a, b]$. Compute the following sum of two integrals,

$$\int_{f(a)}^{f(b)} f^{-1}(y) dy + \int_a^b f(x) dx\,.$$

3. (15 pts)

   (a) (5 tps) Derive $\frac{d}{dx}(\tanh^{-1} x)$ for $|x| < 1$.

   (b) (10 pts) Find $\int x \tanh^{-1} x\, dx$.