MAS101 Calculus I – Midterm Exam

October 19, 2021 19:00–22:00

Rules for Online Midterm Exam

- **This is an online, supervised, closed-book exam. You should behave as you would in an in-class exam.**
- All answers should be handwritten. Typed or tablet-written answer will NOT be accepted.
- You should obey the directions of your TA strictly during the exam.
- Any academic dishonesty related to this exam will be handled as it is done in an in-class exam.
- Any attempt to cheat or failure to follow this instruction leads to serious disciplinary actions not limited to failing the exam or the course.

Honor Code Pledge

On my honor as a KAIST student, I hereby pledge to take this exam honestly and not engage in any form of cheating, including taking an exam for someone else. I sign this pledge based on the understanding that any breach of this honor code not only questions my ethics and morality but also threatens the survival of KAIST community and that I may face strong disciplinary action under the school regulations.

Please copy the sentence "***I have read the above rules, and I pledge that I will abide by them.***" and sign.

Write the sentence below I have read the above rules, and I pledge that I will abide by them

Student ID: 20210808 **Date:** 19 October 2021

Name: TREEPAT CHANTAURAI **Signature:** Treepat. C

Class Section: A☐ B☐ C☑ D☐

Do not write below this line.

Problem	Score	Problem	Score	Problem	Score
1		2		3	
4		5		6	
7		8		9	
10		11		12	
13		14		15	
Recitation TA Name :				**Total**	

Exam consists of two sessions, 75 minutes each. Total number of problems is at most 15. Solutions to problems of the first session should be submitted at the end of the first session. Later submission will not be accepted.

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This page is a solution to problem # 4 ← *Write down the problem number.*

(c)

$\cancel{w' + kw - \rho} = 0$

$\frac{dw(t)}{dt} = -kw + \rho$

$\frac{dw(t)}{-kw(t)+\rho} = 1\,dt$

integrate both side:

$-\int \frac{1}{kw-\rho}\,dw = \int 1\,dt$

$-\ln|kw-\rho| = t + C$

$|kw-\rho| = e^{-t-C}$

$w = \frac{\rho + e^{-t-C}}{k}$

sub $w(0) = V_0$; $V_0 = \frac{\rho + e^{-C}}{k}$

$e^{-C} = V_0 k - \rho$

Therefore;

$w(t) = \frac{\rho + (V_0 k - \rho)e^{-t}}{k}$

$\frac{dy}{dx} + ay - b = 0$

$w = \frac{\rho - e^{-t-C}}{k}$

$kw - \rho = -e^{-t-C}$

$w = \frac{\rho + e^{-t-C}}{k}$

$\frac{\rho + e^{-C}}{k} = V_0$

e^{-C} $\frac{1}{k}(V_0 k -$

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This page is a solution to problem # $\boxed{5}$ ← *Write down the problem number.*

$$\int_1^\infty 2\sqrt{x^2-1} - 2x + \frac{1}{2}x = \lim_{t\to\infty} \int_1^t 2\sqrt{x^2-1} - 2x + \frac{1}{2}x \, dx$$

$$= \lim_{t\to\infty} \int_1^t 2\sqrt{x^2-1}\, dx - \lim_{t\to\infty} \int_1^t -\frac{3}{2}x \, dx$$

Consider $\int \sqrt{x^2-1}\,dx$, sub $x = \sec\theta \to dx = \sec\theta\tan\theta\, d\theta$

$\hookrightarrow \sqrt{x^2-1} = \tan\theta$

$$\int \sqrt{x^2-1}\,dx = \int \tan\theta \cdot \sec\theta\tan\theta\, d\theta = \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \int \sec^3\theta\, d\theta - \int \sec\theta\, d\theta$$

$$- \ln|\sec\theta + \tan\theta|$$

$=$

$u = 2x^2 - 1 \quad dx = \frac{du}{\sqrt{u^2+1}}$

$\int \sqrt{x^2-1}\,dx$

$\int \sqrt{u} \frac{du}{\sqrt{u^2+1}}$

$u^2 = x^2 - 1$

$2u\,du = 2x\,dx$

$dx = \frac{u\,du}{\sqrt{u^2+1}}$

$I = \int \frac{u^2}{\sqrt{u^2+1}}\, du$

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This page is a solution to problem # $\boxed{6}$ ← *Write down the problem number.*

(a) Knowing that $\int_0^\infty e^{-x}\,dx$ converges

compare $\dfrac{\cancel{e^{-x}}}{x^{n-1}\cancel{e^{-x}}} = x^{1-n}$

(b)

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This page is a solution to problem # $\boxed{7}$ ← *Write down the problem number.*

(a) $xy' + 2y = \dfrac{8x}{(x-1)(x+1)(x+2)}$

$\dfrac{dy}{dx} + \left(\dfrac{2}{x}\right)y = \dfrac{8}{(x-1)(x+1)(x+2)}$

$p(x) = \dfrac{2}{x} \rightarrow v(x) = e^{\int \frac{2}{x}dx}$

$v(x) = e^{2\ln x}$

$v(x) = x^2$

$y = \dfrac{1}{v(x)}\int v(x)Q(x)dx$; $Q(x) = \dfrac{8}{(x-1)(x+1)(x+2)}$

$y = \dfrac{1}{x^2}\int \dfrac{8x^2}{(x-1)(x+1)(x+2)}dx$

onsider $\dfrac{8x^2}{(x-1)(x+1)(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{x+2}$

$8x^2 = A(x^2+3x+2) + B(x^2+x-2) + C(x^2-1)$

$\therefore A+B+C = 8 \longrightarrow -2A+C = 8$

$3A + B = 0$

$2A - 2B + C = 0 \longrightarrow 8A + C = 0$

$A = -\dfrac{4}{5}$

$B = \dfrac{12}{5}$

$C = \dfrac{32}{5}$

$\therefore y = \dfrac{1}{x^2}\int\left(\dfrac{-4/5}{x-1} + \dfrac{12/5}{x+1} + \dfrac{32/5}{x+2}\right)dx$

$= \dfrac{1}{x^2}\left(-\dfrac{4}{5}\ln|x-1| + \dfrac{12}{5}\ln|x+1| + \dfrac{32}{5}\ln|x+2|\right)$

$\frac{1}{x}dx$ x^{-1}

$-8-24+32$ 99

$12-4+x=8x^2$

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This page is a solution to problem # $\boxed{7}$ ← *Write down the problem number.*

b

$$\frac{dy}{dx} = 2x\sqrt{1+y^2}$$

$$\frac{dy}{\sqrt{1+y^2}} = 2x\,dx$$

integrate both side:

$$\int \frac{1}{\sqrt{1+y^2}}\,dy = \int 2x\,dx$$

$$\sinh^{-1}(y) = \frac{2x^2}{2} + C = x^2 + C$$

$$\sinh(x^2 + C) = y$$

$(0,0)$: $\sinh(C) = 0$

$$\frac{e^C - e^{-C}}{2} = 0$$

$$e^C = e^{-C}$$

$$C = 0$$

$$\therefore\ y = \sinh(x^2)$$

#

$e^x = e^{-x}$ $e^{2x} = 0$