On my honor as a KAIST student, I pledge to take this exam honestly.

You should write down all your work for full credits.

Session 2, December 14, 2021, 20:40–21:55, Four problems: 5–8

5. (10 pts) Let $\{\mathbf{x}_n\}$ be a sequence of vectors, where $\mathbf{x}_n = \langle x_n^{(1)}, x_n^{(2)}, x_n^{(3)} \rangle$. Let $\mathbf{x}$ be a vector. Prove or disprove:
   (a) $|\mathbf{x}_n - \mathbf{x}| \to 0$ if $\sum_{k=1}^{n} |\mathbf{x}_k - \mathbf{x}| \le 1$ for all n.
   (b) $|\mathbf{x}_n - \mathbf{x}| \to 0$ if $\lim_{n\to\infty} \frac{|\mathbf{x}_{n+1} - \mathbf{x}|}{|\mathbf{x}_n - \mathbf{x}|} = \rho < 1$.

6. (15 pts) Consider the power series
$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^{2n+1}}{n2^n}.$$
   (a) (7 pts) Find its interval of convergence.
   (b) (8 pts) At each point in the interval of convergence, discuss its mode of convergence as absolute or conditional convergence together with proper reasoning.

7. (10 pts) Your eye is at $(4, 0, 0)$. You are looking at a triangular plate whose vertices are at $(1, 0, 1)$, $(1, 1, 0)$, and $(-2, 2, 2)$. The line segment from $(1, 0, 0)$ to $(0, 2, 2)$ passes through the plate. What portion of the line segment is hidden from your view by the plate?

8. (15 pts) Consider the curve $r = \cos 3\theta$ for $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$.
   (a) Find the slopes of the curve at the origin.
   (b) Find the length of the curve, using the fact that $\int_0^{\frac{\pi}{3}} \sqrt{1 + 8\sin^2 3\theta}\, d\theta \approx 2.23$.
   (c) Find the area enclosed by the curve.