20210808
TREEPAT

11.2 (22) Area enclosed by y-axis curve $x = t - t^2,\ y = 1 + e^{-t}$

$$A = \int_0^1 x\,dy \quad , \quad dy = -e^{-t}\,dt$$

$$= -\int_0^1 (t - t^2) e^{-t}\,dt = \int_0^1 t^2 e^{-t}\,dt - \int_0^1 t e^{-t}\,dt$$

$$A = -e^{-t}(t^2 + 2t + 2) + e^{-t}(t+1) \Big]_0^1$$

$$= -\frac{3}{e} - (-2 + 1)$$

$$A = 1 - \frac{3}{e}$$

11.2 (26) Length of curve $x = t^3,\ y = \frac{3t^2}{2}$ $\quad 0 \le t \le \sqrt{3}$

$$l = \int_0^{\sqrt{3}} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt = \int_0^{\sqrt{3}} \sqrt{9t^2 + 9t^4}\,dt$$

$$= 3\int_0^{\sqrt{3}} t\sqrt{1+t^2}\,dt = 3\int_0^{\sqrt{3}} t\sqrt{t^2+1}\,\frac{dt^2+1}{2t}$$

$$= \cancel{\frac{3}{2}} \cdot \cancel{\frac{2}{3}} (t^2+1)^{3/2} \Big|_0^{\sqrt{3}} = 4^{3/2} - 1$$

$$l = 7$$

11.3 (38) Replace polar with Cartesian eq.

Describe identity of the graph: $r^2 \sin(2\theta) = 2$

$x = r\cos\theta, \quad y = r\sin\theta$

$$r^2 \sin(2\theta) = 2r^2 \sin\theta\cos\theta \Rightarrow 2xy = 2 \quad \boxed{y = \frac{1}{x}}$$

11.3 (60) Replace cartesian with polar eq. : $xy = 2$

$x = r\cos\theta, \quad y = r\sin\theta$

$$2r^2\cos\theta\sin\theta = 2 \cdot 2$$

$$r^2 \sin(2\theta) = 4$$

11.5 (6) Find area inside three-leaved rose $r = \cos(3\theta)$

$$A = \frac{1}{2}\int_{-\pi/6}^{\pi/6} r^2\,d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta = 2 \cdot \frac{1}{2}\int_0^{\pi/6} \left(\frac{1 + \cos(6\theta)}{2}\right) d\theta$$

$$A = \frac{1}{2}\left[\theta + \frac{\sin(6\theta)}{6}\right]\Big|_0^{\pi/6} = \frac{1}{2}\left[\frac{\pi}{6} + \frac{\sin\pi}{6}\right]$$

$$A = \pi/12$$

11.5 (24) Find length $r = a\sin^2\left(\frac{\theta}{2}\right)$, $0 \le \theta \le \pi,\ a > 0$

$$L = \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

$$= \int_0^{\pi} \sqrt{a^2\sin^4\left(\frac{\theta}{2}\right) + \left[a\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]^2}\,d\theta$$

$$= a\int_0^{\pi} \left|\sin\frac{\theta}{2}\right| \sqrt{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}\,d\theta$$

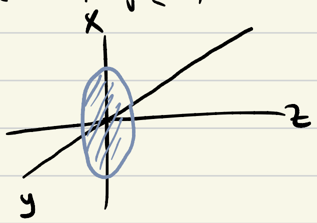
for $0 \le \theta \le \pi$, $\sin\frac{\theta}{2} \ge 0$;

$$L = a\int_0^{\pi} \sin\theta\,d\theta = a\cos\theta\Big|_{\pi}^{0} \Rightarrow L = 2a$$

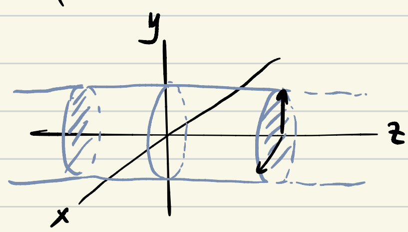
12.1 (20) Describe sets of point in equation

(a) $x^2 + y^2 \le 1,\ z = 0$

since $x^2 + y^2 = 1$ is a circle on xy plane radius $= 1$, origin-centered



(c) $x^2 + y^2 \le 1$, no restriction on z



This is like a cylinder

(b) $x^2 + y^2 \le 1,\ z = 3$: The value of x, y are the same. The value of z shift to +3.