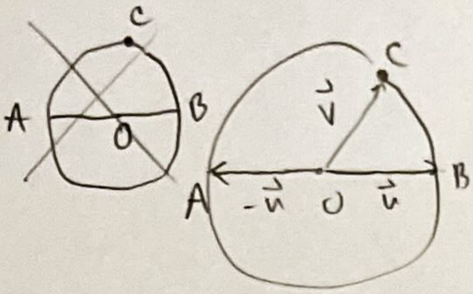
20210808
TREEDAT

①



$$\vec{AC} = \vec{AO} + \vec{OC} = -(-\vec{u}) + \vec{v} = \vec{u} + \vec{v}$$

$$\vec{BC} = \vec{BO} + \vec{OC} = -\vec{u} + \vec{v} = \vec{v} - \vec{u}$$

$$\vec{AC} \cdot \vec{BC} = (\vec{v} + \vec{u})(\vec{v} - \vec{u}) = v^2 - u^2$$

$$= r^2 - r^2$$ ← the radius

$$\vec{AC} \cdot \vec{BC} = 0$$

Therefore, $\vec{AC}$ and $\vec{BC}$ are orthogonal

②(a). $\vec{PQ} = 2\hat{i} - 1\hat{j} + 0\hat{k}$

$\vec{QR} = -\hat{i} + 1\hat{j} - 1\hat{k}$

$$\vec{PQ} \times \vec{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 0 \\ -1 & 1 & -1 \end{vmatrix} = \hat{i} + 2\hat{k} - \hat{k} + 2\hat{j} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\text{unit vector} = \frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{1^2 + 2^2 + 1^2}} = \frac{\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{6}}$$ #

(b). $$A = \frac{1}{2} \| \vec{PQ} \times \vec{QR} \| = \frac{1}{2}\sqrt{1^2 + 2^2 + 1^2} = \frac{\sqrt{6}}{2}$$