20210808
TREEPAT

(1)

it can be rearrangend into

$$\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots\right)-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\cdots\right)$$

$$X = \sum_{n=1}^{\infty}\frac{1}{3^n} - \sum_{n=1}^{\infty}\frac{1}{2^n}$$

which both of $\sum_{n=1}^{\infty}\frac{1}{a^n}$ when $a>1$ converge

therefore, $\sum_{n=1}^{\infty}\frac{1}{3^n}-\sum_{n=1}^{\infty}\frac{1}{2^n}$ should converge

since $\sum_{n=1}^{\infty}\frac{1}{a^n}$ as $a>1$ converges to $\frac{1/a}{1-\frac{1}{a}} = \frac{1}{a-1}$

$$\therefore X = \frac{1}{3-1}-\frac{1}{2-1} = -0.5 \quad \#$$

(2) By ratio test,

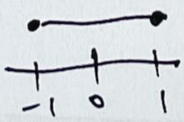
$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)\ln(n+1)}\frac{n\ln(n)}{x^n}\right| \leq 1 \text{ for conversenee}$$

$$1 \geq \lim_{n\to\infty}\left|x\,\frac{n}{n+1}\,\frac{\ln(n)}{\ln(n+1)}\right| = |x|\lim_{n\to\infty}\left[\frac{n}{n+1}\frac{\ln(n)}{\ln(n+1)}\right]$$

$$= |x|\left(\lim_{n\to\infty}\frac{n}{n+1}\cdot\lim_{n\to\infty}\frac{\ln(n)}{\ln(n+1)}\right) = |x|\left(1\times\lim_{n\to\infty}\frac{\frac{1}{n}}{\frac{1}{n+1}}\right)$$

$$= |x|(1\times 1)$$

$$\therefore |x| \leq 1$$



∴ Radius of convergence = 2 #