

Quiz 4

20210808
TREPAT

(1) $\int \sec x dx$

$u = \sec x$

$du = \sec x \tan x dx$

$\sec^2 x + \tan^2 x = 1$; $du = u \sqrt{u^2 - 1} dx$
 $dx = \frac{1}{u \sqrt{u^2 - 1}} du$

$\int \sec x dx = \int u \frac{1}{u \sqrt{u^2 - 1}} du$
 $= \int (u^2 - 1)^{-1/2} du$
 $= \tan^{-1}(u) + C$
 $= \tan^{-1}(\sec x) + C$

(2) $\int \frac{2x^2 - 4x}{(x^2 + 1)(x - 1)^2} dx$

$= \int \frac{2x(x - 2)}{(x^2 + 1)(x - 1)^2} dx$

$= \int \left(\frac{A}{x^2 + 1} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2} \right) dx$

Now solve for A, B, C

$A(x^2 - 2x + 1) + B(x^3 - x^2 + x - 1) + C(x^2 + 1) = 2x^2 - 4x$

Compare the coefficients: $\downarrow B = 0$
 $A = 2, C = -2$

$= \int \frac{2}{x^2 + 1} dx + \int \frac{-2}{(x - 1)^2} dx$

$= 2 \sin^{-1}(x) + \frac{2}{x - 1} + C$