MAS 101 Calculus I : Midterm Exam

April 16, 2019 7:00 PM ~ 10:00 PM

Full Name : ______________________________

Student Number: ____________________

Do not write in this box.

Instruction

- No items other than pen, pencil, eraser and your student ID card are allowed. You must leave all of your belongings that are not allowed at the designated area in the front of the classroom.
- Before you start, fill out the identification section on the title page and on the header of each page with an inerasable pen.
- Show your work for full credit and write the final answer to each problem in the box provided. You are encouraged to write in English but some Korean is acceptable. You can use the back of each page as scratch spaces but anything written there will not be graded.
- The exam is for three hours in length. Ask permission by raising your hand if you have any question or need to go to toilet. You are not allowed to go to toilet for the first 30 minutes and the last 30 minutes.
- Any attempt to cheat or a failure to follow this instruction lead to serious disciplinary actions not limited to failing the exam or the course.

Do not write in this table.

Problem	Score	Problem	Score	Problem	Score
1	/ 10	2	/ 10	3(a)	/ 10
3(b)	/ 10	4(a)	/ 10	4(b)	/ 10
5(a)	/ 10	5(b)	/ 10	5(c)	/ 10
6(a)	/ 10	6(b)	/ 15	7(a)	/ 10
7(b)	/ 10	8	/ 15	9(a)	/ 10
9(b)	/ 10	10(a)	/ 15	10(b)	/ 15
				Total	**/ 200**

1. Compute $\lim_{x \to 0} \dfrac{\sin^{-1} x^2}{x \tan^{-1} x}$.

Ans:

2. Show that $\lim_{n \to \infty} \left(-\dfrac{1}{2}\right)^n = 0$ via the definition of limits of sequences.

3. Let $y = f(x)$ for a differentiable real-valued function f defined on $(0, \infty)$ such that

$$xy' + 2y = xyy' \quad \text{and} \quad f(1) = 2.$$

(a) Show that f is one-to-one on $(0, \infty)$.

(b) Find the inverse function f^{-1} of f.

Ans:

4. Evaluate the following integrals:

(a) $\displaystyle\int \frac{\tan^{-1}\sqrt{x}}{\sqrt{x}(x+1)}\,dx.$

Ans:

(b) $\displaystyle\int (\cos^{-1} x)^2\,dx.$

Ans:

5. For each of the following statements, prove if it is true or give a counterexample if it is false.

   (a) If $f = O(x^p)$ for all $p > 1$, then $f = O(x)$.

   ☐ True ☐ False

   (b) If $\displaystyle\int_1^\infty f(x)\,dx$ converges for a nonnegative, continuous function f defined on $[1, \infty)$, then $\displaystyle\int_1^\infty \frac{\sqrt{f(x)}}{x}\,dx$ converges.

   ☐ True ☐ False

(c) If $\sum_{k=1}^{\infty} a_k$ converges conditionally, then $\sum_{k=1}^{\infty} a_k^2$ converges.

☐ True ☐ False

6. Find all real number a that makes the following converge.

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{a}{n+2} - \frac{1}{n+4} \right)$

Ans:

(b) $\displaystyle\int_0^\infty \frac{1}{\sqrt{x + x^a}}\,dx$

Ans:

7. Let $\{a_n\}$ be the sequence defined recursively by

$$a_1 = 2 \quad \text{and} \quad a_n = \sqrt{6 + \sqrt{a_{n-1}}} \quad \text{for } n \geq 2.$$

(a) Find an upper bound and a lower bound for $\{a_n\}$.

Ans:

(b) Prove that $\{a_n\}$ converges.

8. Test the improper integral $\displaystyle\int_1^\infty \sin x^2 \, dx$ for convergence.

9. How many terms are required in a partial sum to estimate the following series within the given error bound.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{4^{n-1}}$ within the error $1/50$

   Ans:

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{5^n}$ within the error 10^{-3} (Use the approximated value $\ln 5 \approx 1.6$)

   Ans:

10. Decide whether the following series converge absolutely, conditionally or diverge:

(a) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{(-2)^n+3}{n!}+\frac{(-1)^{n+1}\ln n}{n^2}\right)$

☐ Converge Absolutely ☐ Converge Conditionally ☐ Diverge

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^{n-1}}{n^{1/\ln\ln(n)}}$

☐ Converge Absolutely ☐ Converge Conditionally ☐ Diverge