20210808
TREEPAT

① $x = t - \sin t$, $y = 1 - \cos t$

[find equation for tangent line to the curve @ $t = \frac{3}{2}\pi$
[find length of curve in $0 \le t \le \pi$

(a) ~~d~~ slope $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{1-\cos t} = m$

in an equation $y = mx + c$, substitution leads to,

$$y = (1-\cos t) = \left(\frac{\sin t}{1-\cos t}\right)(t - \sin t) + c$$

$t = \frac{3}{2}\pi$, $\cos t = 0$, $\sin t = -1$,

$$1 = -1\left(\frac{3}{2}\pi + 1\right) + c$$

$$c = 2 + \frac{3}{2}\pi$$

$$\text{slope} = \frac{\sin t}{1-\cos t} = \frac{-1}{1} = -1$$

$\therefore$ tangent line $\Rightarrow$ $\boxed{y = -x + \frac{3}{2}\pi + 2}$

(b) $L = \int_0^{\pi} \sqrt{(y'(t))^2 + (x'(t))^2}\,dt = \int_0^{\pi} \sqrt{\sin^2 t + \cos^2 t - 2\cos t + 1}\,dt = \int_0^{\pi} \sqrt{2 - 2\cos t}\,dt$

$= \sqrt{2}\int_0^{\pi} \sqrt{1-\cos t}\,dt = \sqrt{2}\left(-2\sqrt{1+\cos t}\,\Big|_0^{\pi}\right) = -2\sqrt{2}\left(\sqrt{0} - \sqrt{2}\right) = 4$

$\therefore L = 4$

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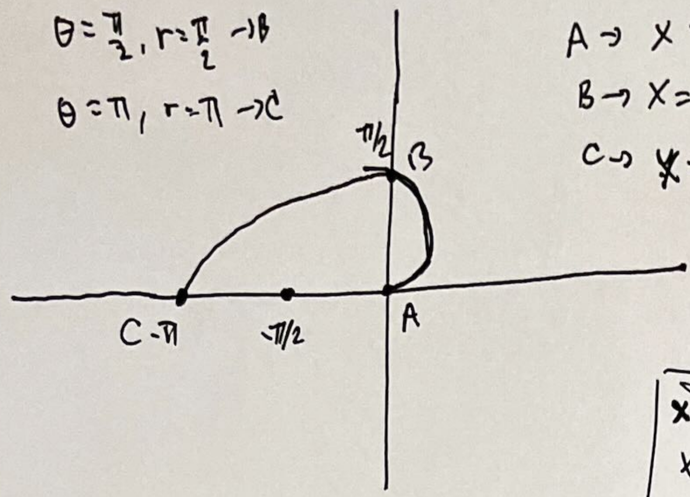
② (a) $\theta = 0, r = 0 \rightarrow A$

$\theta = \frac{\pi}{2}, r = \frac{\pi}{2} \rightarrow B$

$\theta = \pi, r = \pi \rightarrow C$



$A \rightarrow x = 0, y = 0$

$B \rightarrow x = 0, y = \pi/2$

$C \rightarrow x = -\pi, y = 0$

~~NOTE~~

~~$x = r\cos\theta,\ y = r\sin\theta$~~

~~$x = \theta\cos\theta,\ y = \theta\sin\theta$~~

~~$\frac{y}{x} = \tan\theta$~~

(b) $A = \frac{1}{2}\int_0^{\pi} r^2 \, d\theta$

$= \frac{1}{2}\int_0^{\pi} \theta^2 \, d\theta \quad \leftarrow r = \theta$

$= \frac{1}{2} \frac{\theta^3}{3} \Big|_0^{\pi}$

$\text{Area} = \frac{\pi^3}{6}$