

Assignment 8

Task 2 :

Let T be the event of temperature recorded $< 80^\circ$
 M be the event of sensor placed in Maine.

Given :

In Maine, the probability of daily high temp > 80 is 20%.

$$\therefore P\left(\frac{T}{M}\right) = 0.8$$

In Sahara, the probability of daily high temp > 80 is 90%.

$$\therefore P\left(\frac{T}{M'}\right) = 0.1$$

Prior probability that sensor placed in Maine is 5%.

$$\therefore P(M) = 0.05$$

Sensor not placed in Maine $P(M') = 0.95$

a) Using Bayes' theorem,

$$\text{then, } P\left(\frac{M}{T}\right) = \frac{P(M) \cdot P\left(\frac{T}{M}\right)}{P(M) \cdot P\left(\frac{T}{M}\right) + P(M') \cdot P\left(\frac{T}{M'}\right)}$$

$$= \frac{0.05 \times 0.8}{0.05 \times 0.8 + 0.95 \times 0.1} = 0.2963$$

\therefore probability that sensor placed in Maine with temp recorded $< 80^\circ$ is 0.2963.

Part b) From the part a we can see that the probability of sensor recording daily high less than 80 is 0.2963 in Maine.

$$P\left(\frac{M}{T}\right) = 0.2963$$

$$\therefore P\left(\frac{M'}{T}\right) = 0.7037$$

By the law of total probability,

$$P(T) = P\left(\frac{M}{T}\right) \cdot 0.8 + P\left(\frac{M'}{T}\right) \cdot 0.1$$

$$= 0.2963 \times 0.8 + 0.7037 \times 0.1$$

$$= 0.30741$$

\therefore the probability that second email also indicates a daily high under 80° is 0.31.

Part c :

L_1, L_2, L_3 be the first three emails all indicate daily highs under 80 degrees.

$$P(L_1, L_2, L_3) = P(L_3, L_2, L_1)$$

$$= P(L_3 | L_2, L_1) * P(L_2 | L_1) * P(L_1)$$

$$= P(L_3 | L_2, L_1) * 0.3074 * 0.135$$

$$P(L_3 | L_2, L_1) = P(L_3 | L_2, L_1, M) * P(M | L_2, L_1) \\ + P(L_3 | L_2, L_1, M') * P(M' | L_2, L_1)$$

$$= P(L_3 | M) * P(L_2, L_1 | M) * P(M) / P(L_2, L_1) \\ + P(L_3 | M') * P(L_2, L_1 | M') * P(M') / P(L_2, L_1)$$

$$P(L_3 | L_2, L_1) = 0.63977$$

$$P(L_1, L_2, L_3) = 0.63977 * 0.3074 * 0.135 \\ = 0.02654$$

Task 3

Given

11 variables $A, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9, B_{10}$.

Variable A has 5 values.

B_1, B_2, \dots, B_{10} has 7 values.

(a) Total numbers - that will stored in joint probability distribution table is
 $= (7^{10} \times 5)$ values in the table.

(b) Total numbers needed to store these 11 variables is 350, because in this case we can consider the independence of variables i.e. B_i is independent of B_j so we have $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 5 = 350$.