Exercise set 4

Monday, 16 October 2023 16.32

a) $\log p(x|\theta_{t+1}) - \log p(x|\theta_t) = a - b = \Delta$

b) $E_{z|x,\theta_t}[a-b] = a-b$

c) $\log p(x|\theta_{t+1}) - \log p(x|\theta_t)$

$= \log \frac{p(x|\theta_{t+1})\, p(z|x,\theta_{t+1})}{p(z|x,\theta_{t+1})} - \log \frac{p(x|\theta_t)\, p(z|x,\theta_t)}{p(z|x,\theta_t)}$

d) $L(x) = \sum p(z|x,\theta_t) \log \frac{p(x|\theta_{t+1})\, p(z|x,\theta_{t+1})}{p(z|x,\theta_{t+1})} - \sum p(z|x,\theta_t) \log \frac{p(x|\theta_t)\, p(z|x,\theta_t)}{p(z|x,\theta_t)}$

$= \sum p(z|x,\theta_t) \log p(x|\theta_{t+1}) \cdot p(z|x,\theta_{t+1}) - \sum p(z|x,\theta_t) \log p(x|\theta_t)\, p(z|x,\theta_t)$

$+ \sum p(z|x,\theta_t) \log p(z|x,\theta_t) - \sum p(z|x,\theta_t) \log p(z|x,\theta_{t+1})$

$= Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) + a$

$a = \sum p(z|x,\theta_t) \log \frac{p(z|x,\theta_t)}{p(z|x,\theta_{t+1})}$

e) $L(x) = Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t) + a$

since $p>0$, $\log ax$ with $x>0$ is the condition for begining $\rightarrow a>0$

$\rightarrow L(x) \geq Q(\theta_{t+1}|\theta_t) - Q(\theta_t|\theta_t)$

Since $\theta_{t+1} = \arg\max_\theta Q(\theta|\theta_t) \Rightarrow Q(\theta_t|\theta_t)$

$\rightarrow Q(\theta_{t+1}|\theta) - Q(\theta_t|\theta_t) \geq 0$

$\Rightarrow L \geq 0$

$\rightarrow \Delta \geq 0$