# Optimization of Parameters for 3-D Reconstruction with Uncalibrated Monoplane Angiograms

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Abstract—A method about optimization of parameters for 3-D reconstruction with uncalibrated monoplane angiograms is presented in this paper. Based on the model of monoplane angiographic system, variables concerning about the precision of reconstruction are discussed. Distance between initial image and projection of reconstructed 3-D model is employed to formulate objective function. Adaptive simulated annealing algorithm is utilized to solve the problem of optimization. Experiments on angiograms are presented. The results show that the method greatly improves the accuracy of 3-D reconstruction.

Keywords-optimization; 3-D reconstruction; uncalibrated monoplane angiogram; adaptive simulated annealing algorithm

### I. INTRODUCTION

X-ray coronary angiography is the "gold standard" in the diagnosis of coronary artery disease [1]. The angiographic image is the projection of artery from 3-D space to 2-D space, so stereoscopic information may be lost in the imaging process, which could interfere the doctor's judgment. In clinical doctors visualize 3-D artery structure in mind by subjective experience. 3-D reconstruction of coronary arterial tree makes up the shortcoming of x-ray angiographic system. It provides more spatial information about patients' coronary artery and assists doctors effectively.

Monoplane angiographic system is commonly used in hospital at present. It generates image sequences of different perspectives and times during the rotation of the C-arm. Though synchronous ECG helps select images at same Cardiac motion phase from two sequences, we cannot obtain precise 3-D representation directly based on parameters recorded by the angiographic system considering various systematic errors. So optimization of parameters is essential [2]. Methods based on calibration [3] are complex. Special moldboard is needed and calibration should be carried out before each imaging operation [4]. Wang Guodong [5] and Yang Jian [6] put forward uncalibrated methods, which got fairly accurate results. But they just optimized the transformation matrix on behalf of the spatial relation between two images to reduce computation. Besides, it was hard to acquire optimized parameters conversely according to transformation matrix. So they cannot obtain the 3-D representation at angiographic system coordinate. In further research the reconstructed 3-D left and right coronary artery

trees should be put in the same coordinate system, their methods could not meet the requirement.

This paper analyzes various error factors according to the angiographic system model, including angiographic parameters and the movement of bed. Optimization of these variables improves the accuracy of reconstruction significantly.

### II. MODEL OF MONOPLANE ANGIOGRAPHIC SYSTEM

The imaging model of angiographic system is similar to pinhole camera. The x-ray emitter could be assumed as an ideal point emitter and locates at focal spot. The C-arm rotates around iso-center, and coronary artery tree is projected into different imaging planes, as Fig.1 denotes. Point O is isocenter, Point  $S_1$  and  $S_2$  stand for the x-ray emitter at two positions.  $O_1$  and  $O_2$  are centers of Image A and Image B respectively.  $S_1O_1$  and  $S_2O_2$  denote the perpendicular distance between the x-ray focal spot and imaging plane which is denoted by D.  $S_1O$  and  $S_2O$  denote the distance between the focal spot and iso-center which is denoted by l. Besides,  $\alpha_1$  and  $\beta_1$  denote the orientation to which C-arm rotates when Image A is acquired, and  $\alpha_2$  ,  $\beta_2$  correspond to Image B.  $\alpha_i$  is LAO/RAO angle and  $\beta_i$  is CRAN/CAUD angle. These parameters are recorded automatically including others such as field of view (FOV) not mentioned above but used in reconstruction.

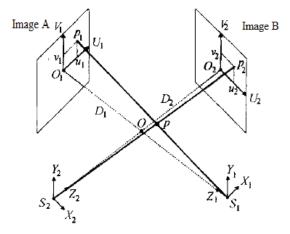


Figure 1 model of angiographic system

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The question of 3-D artery reconstruction is how to compute point P from  $p_1$  and  $p_2$ . Let  $(x_1,y_1,z_1)$  and  $(x_2,y_2,z_2)$  denote P in  $X_1Y_1Z_1S_1$  and  $X_2Y_2Z_2S_2$  coordinate system;  $(u_1,v_1)$  and  $(u_2,v_2)$  denote the projection of P in  $U_1O_1V_1$  and  $U_2O_2V_2$  coordinate system. According to the perspective theory, we get the following equation:

$$\begin{bmatrix} x_i, y_i, z_i \end{bmatrix}^T = z_i \cdot \begin{bmatrix} \xi_i, \eta_i, 1 \end{bmatrix}^T \tag{1}$$

Where  $\xi_i = u_i / D_i = x_i / z_i$ ,  $\eta_i = v_i / D_i = y_i / z_i$ .

The transformation between  $X_1Y_1Z_1S_1$  and  $X_2Y_2Z_2S_2$  coordinate system contains only rotation and translation, it can be defined as

$$[x_2, y_2, z_2]^T = R \cdot ([x_1, y_1, z_1]^T - \vec{t})$$
 (2)

R is a rotation matrix which is the function of four angles mentioned above.  $\vec{t}$  is translation vector which is the function of R,  $l_1$  and  $l_2$ .

$$R = R_{x}(\beta_{2})R_{y}(-\alpha_{2})R_{y}^{-1}(-\alpha_{1})R_{x}^{-1}(\beta_{1})$$
(3)

$$\vec{t} = [0, 0, l_1]^T - R^{-1}[0, 0, l_2]^T \tag{4}$$

Where  $R_X(\alpha)$  and  $R_Y(\beta)$  denote the rotation with respect to X-axis and Y-axis with  $\alpha$  and  $\beta$  angles respectively.

The location of P in  $X_1Y_1Z_1S_1$  coordinate system can be computed as follows:

$$\begin{bmatrix} 1 & 0 & -\xi_{1i} \\ 0 & 1 & -\eta_{1i} \\ r_{11} - r_{31} \cdot \xi_{2i} & r_{12} - r_{32} \cdot \xi_{2i} & r_{13} - r_{33} \cdot \xi_{2i} \\ r_{21} - r_{31} \cdot \eta_{2i} & r_{22} - r_{32} \cdot \eta_{2i} & r_{23} - r_{33} \cdot \eta_{2i} \end{bmatrix} \begin{bmatrix} x_{1i} \\ y_{1i} \\ z_{1i} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vec{a} \cdot \vec{t} \\ \vec{b} \cdot \vec{t} \end{bmatrix}$$
(5)

Where  $\vec{a} = \begin{bmatrix} r_{11} - r_{31} \cdot \xi_{2i} & r_{12} - r_{32} \cdot \xi_{2i} & r_{13} - r_{33} \cdot \xi_{2i} \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} r_{21} - r_{31} \cdot \eta_{2i} & r_{22} - r_{32} \cdot \eta_{2i} & r_{23} - r_{33} \cdot \eta_{2i} \end{bmatrix}$  and  $r_{ij}$  denotes the component of  $\mathbf{R}$ .

## III. OPTIMIZATION METHOD

# A. Analysis of Error Ssources

The precision of recorded parameters directly influences the accuracy of 3-D reconstruction. In uncalibrated monoplane angiographic system, parameters are not precise enough so optimization is necessary.

In clinical, the doctor used to adjust the bed randomly to ensure that the receptor cover the whole artery tree. In Fig.2, two images are acquired from a same sequence, while the motion between Fig.2(a) and Fig.2(b) is obvious. The

compensation for movement of image should be considered when reconstruction is carried out.

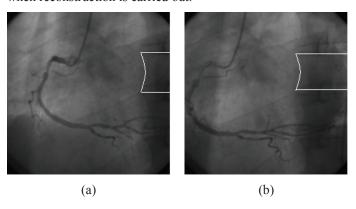


Figure 2 two images in a sequence .The white lines mark the backbone.(a) 20th frame;(b)43rd frame.

In summary, the optimization objects include not only recorded parameters, but also compensation vectors  $\vec{V}_1$ ,  $\vec{V}_2$  for image A and B respectively.  $\vec{V}_1 = (\Delta u_1, \Delta v_1)$ ,  $\vec{V}_2 = (\Delta u_2, \Delta v_2)$ . In total, 12 variables need optimization; they can be expressed by a vector:

$$\vec{V} = [\alpha_1, \beta_1, \alpha_1, \beta_2, l_1, D_1, l_2, D_2, \Delta u_1, \Delta v_1, \Delta u_2, \Delta v_2]^T$$
 (6)

## B. Objective Function of Optimization

The 3-D point is computed by Least Squares Method and just approximates to exact value. When an inaccurate 3-D point is projected with parameters used for reconstruction, two new 2-D points are obtained. The distance between initial 2-D point and back-projection point reflects the reconstruction error. Given the relationship between geometric parameters, reconstructed 3-D point and projection point, the distance is used as basis for the optimization.

Matched structural feature points such as bifurcation points in two images are selected. Points  $p_{11}, \ldots, p_{1n}$  in Image A correspond to points  $p_{21}, \ldots, p_{2n}$  in Image B. Calculate spatial points  $P_1, \ldots, P_n$  according to variables in (6). Projected points in two imaging planes are denoted as  $p'_{11}, \ldots, p'_{1n}$  and  $p'_{21}, \ldots, p'_{2n}$ . Objective function is defined as follows:

$$E(\vec{V}) = \sum_{i=1}^{n} (\|p_{1i}^{'} - p_{1i}\| + \|p_{2i}^{'} - p_{2i}\|)$$
 (7)

The optimal variables ensure the minimum cost of object function.

The object function is a nonlinear least square problem that can be solved by many nonlinear search algorithms. This paper utilizes adaptive simulated annealing method for its good rate of convergence and robustness [7]. The procedure is described as follows:

- 1) Initialize  $\vec{V}_0$  using recorded parameters,  $\Delta u_1, \Delta v_1, \Delta u_2, \Delta v_2$  are set as 0; calculate objective function,  $E = E(\vec{V}_0)$ ;
- 2) disturb current variables to generate new vector  $\vec{V}$ , compute corresponding E,  $\Delta E = E E$ ;
- 3) If  $\Delta E < 0$ , the new  $\vec{V}$  is accepted, else, whether the  $\vec{V}_n$  is accepted depends on :

$$P = [1 - (1 - h)\Delta E / T]^{1/(1 - h)}$$

Where T denotes temperature, h is a constant. Once accepted,  $\vec{V} = \vec{V}$ , E = E'

4) lower temperature slowly according to equation

$$T(k) = T_0 \exp(-C \cdot k^{1/N})$$

Where T(k) is the temperature after k times iteration,  $T_0$  is initial temperature, C is a constant and N denotes the number of variables.

5) Repeat procedure from step 2) to 4) until termination condition is satisfied.

## IV. EXPERIMENTS

Angiographic images used in the experiment were obtained from Philips Integris CV monoplane imaging system. Fig.3 shows images of right coronary artery whose imaging parameters are in Table I.

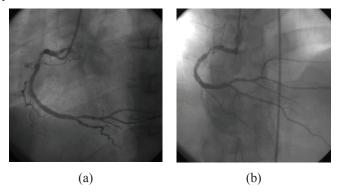


Figure 3 angiograms of right coronary artery for reconstruction.(a)image A,(b)image B.

TABLE I. COMPARISON OF PARAMETERS

Parameter	Initial		optimized		
	A	В	A	В	
LAO/RAO	39.0°	-3.8°	40.1°	-5.1°	
CRAN/CAUD	-2.7°	29.0°	-3.8°	31°	
l(mm)	755	865	776	840	
d(mm)	987	987	990	989	
∆ u(mm)	0	0	21.7	-4.8	
∆ v(mm)	0	0	44.1	-38.2	

Fig.4 shows the comparison between initial images and the corresponding projections of 3-D model reconstructed before and after optimization of variables. The solid line denotes the vessel centerline extracted from initial images and the dashed lines denotes the projection of computed 3-D centerline. Table II is the error statistic of distance between points in original image and their corresponding projections.

TABLE II. ERROR STATISTICS

Variable	Initial		Optimized	
variable	A	В	A	В
Max Error(mm)	18.06	16.83	5.43	5.58
Min Error(mm)	3.31	2.92	0.01	0.01
Mean Error(mm)	9.64	8.51	1.47	1.45
Standard Deviation(mm)	3.53	3.25	1.58	1.59

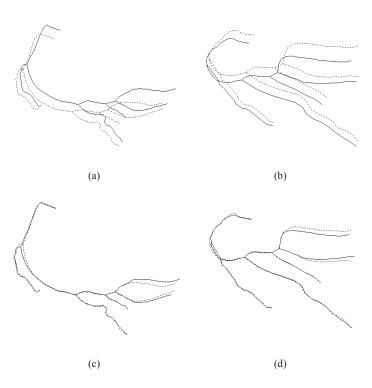


Figure 4 comparison of the centerlines in initial angiograms with their projection of reconstruction(a) and (b) use initial parameters, (c) and (d) use optimized parameters.

# V. CONCLUSION

This paper firstly researches the model of monoplane angiographic system. Then we analyze parameters which may affect reconstruction precision including those recorded by imaging system automatically and the ones hard to be assessed such as motion of bed. The distance between the projections of calculated 3-D points and initial images is employed to assess the precision of reconstruction. Adaptive simulated annealing method is applied to solve the object function of optimization. The result of experiment shows that the optimization method presented improves accuracy of 3-D reconstruction greatly.

The parameters could be optimized for higher precision and it is our further work.

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