PHYSICS I (Summer 2019)

Problem Set 7

Due: 4 July 2019, 12.30 p.m.

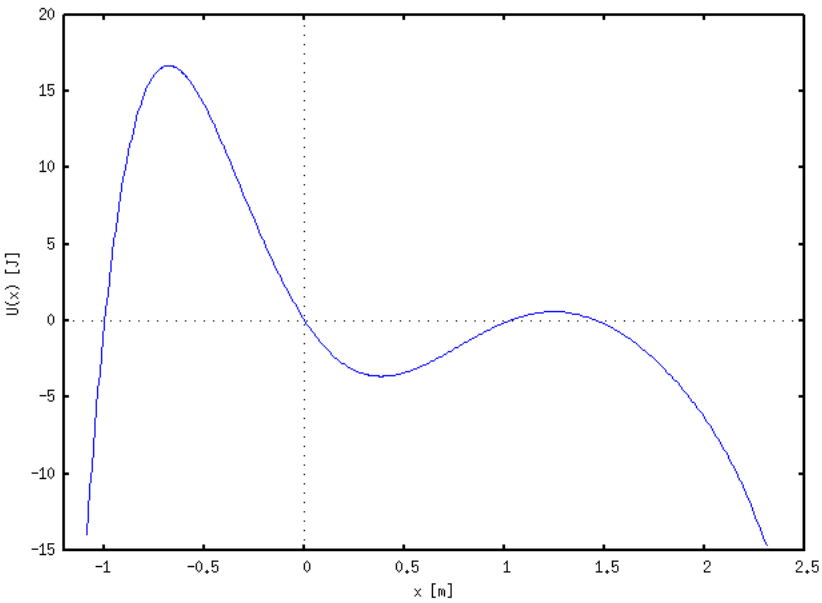
Problem 1. Given the potential energy $U(x,y) = xy^2 + yx^2$, (a) find the corresponding force field and visualize it (use a computer); (b) on the same graph sketch a few equipotential lines, i.e. lines defined by equation $U(x,y) = U_0$, for different values of U_0; (c) comment on the graph; (d) calculate work done by this force on a particle moving from (0,0) to (1,1) along a straight line; (e) same as (d) but along the parabola $y = x^2$.

Note. In (d) and (e) use the simplest possible method.

(*3/2 + 1 + 1 + 1 + 1 points*)

Problem 2. The figure below shows the graph of the potential energy $U = U(x)$ for a particle moving along the x axis in a certain force field. Identify all equilibrium positions and tell which of them are stable/unstable.

(*3/2 points*)



Problem 3. The Lennard–Jones potential energy $U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$, also called the L–J potential or 6–12 potential, is often used to approximate the potential energy associated with interaction between a pair of neutral atoms or molecules separated by distance r.

(a) Find the corresponding force (it is the force exerted by one atom/molecule on the other). Sketch the graphs of both the potential energy and the force as functions of r. Which term in the force is responsible for attraction and which for repulsion?

(b) What is the interpretation of the parameters U_0 and R_0 (both are positive)?

(c) Introducing a new variable $x = r - R_0$, find an approximate expression for the force in the regime $|x/R_0| \ll 1$. Interpret your result in terms of oscillations. Find their period.

Hint. The binomial theorem.

(d) What is oscillating here?

(*3/2 + 1/2 + 3 + 1 points*)

Problem 4. What is the period of *small* oscillations about the stable equilibrium position in the potential field with the potential energy $U(x) = U_0 \tan^2 \alpha x$, where U_0 and α are positive constants (what are their units)? The mass of the oscillating particle is m.

(*2 points*)