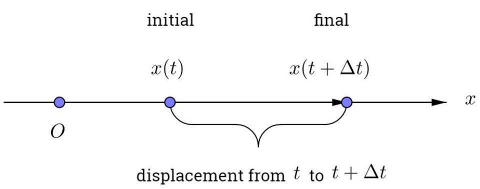
KINEMATICS

Describes motion quantitatively, does not discuss the cause of motion.

Motion along a straight line (1D kinematics)



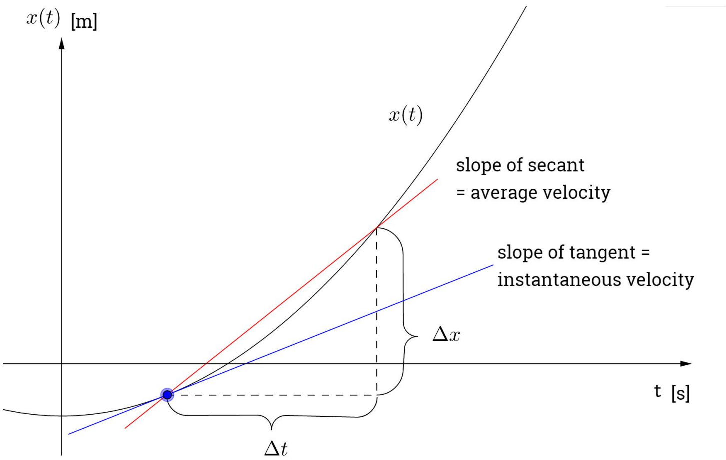
all vectors have only one component; choice of positive direction arbitary.

Average velocity (ove time interval $(t, t+\Delta t)$)

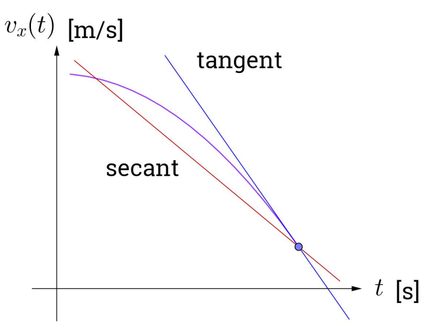
$$v_{av,x} = \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Instantaneous velocity (let $\Delta t \to 0$)

$$\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) \overset{def}{=} v_x(t) \; [m/s]$$



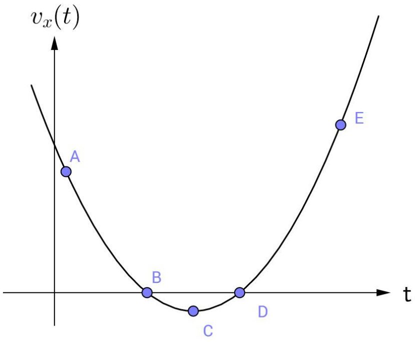
Analogously: acceleration



$$a_{av,x} = \frac{v_x(t+\Delta t) - v_x(t)}{\Delta t} \; [m/s^2] \qquad \text{average acceleration}$$

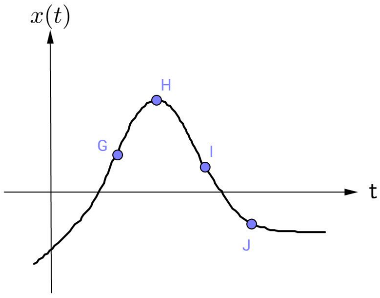
$$\begin{aligned} a_x(t) &= \lim_{\Delta t \to 0} \frac{v_x(t+\Delta t) - v_x(t)}{\Delta t} \\ &= \frac{\mathrm{d}v_x(t)}{\mathrm{d}t} = \dot{v}_x(t) \; [m/s^2] \qquad \text{instantaneous acceleration} \\ &= \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = \ddot{x}(t) \end{aligned}$$

Example: Analysis of $x(t)$ and $v_x(t)$ graphs



$$
\begin{array}{lll}
A: & v_x > 0,\ a_x < 0 & \text{moves to the right slows down} \\
B: & v_x = 0,\ a_x < 0 & \text{stops; about to move to the left} \\
C: & v_x < 0,\ a_x = 0 & \text{moves to the left; no instantaneous acceleration} \\
D: & v_x = 0,\ a_x > 0 & \text{stops; about to move to the right} \\
E: & v_x > 0,\ a_x > 0 & \text{moves to the right; accelerates/speeds up}
\end{array}
$$

(different particle)



$$
\begin{array}{lll}
G: & v_x > 0,\ a_x = 0 & \text{(inflection point)} \\
H: & v_x = 0,\ a_x < 0 & \\
I: & v_x < 0,\ a_x = 0 & \text{(inflection point)} \\
J: & v_x < 0,\ a_x > 0 &
\end{array}
$$

Note. Average speed vs. average velocity

$$
\text{average speed} = \frac{\text{distance traveled}}{\text{time interval}}
$$
$$
\text{average velocity} = \frac{\text{displacement}}{\text{time interval}}
$$

General remarks on mutual relation between x, v_x and a_x

position	$x(t)$
$\underset{\text{instantaneous}}{\text{velocity}}$	$v_x(t) = \dot{x}(t)$
$\underset{\text{instantaneous}}{\text{acceleration}}$	$a_x(t) = \dot{v}_x(t) = \ddot{x}(t)$

$x \to v_x \to a_x$	differentiate (easy)
$a_x \to v_x \to x$	?

Question: Given acceleration, how to find velocity and position?

acceleration $\quad a_x(t) = \frac{\mathrm{d}v_x}{\mathrm{d}t} \qquad$ (Leibnitz′s notaion)

separate... $\quad \mathrm{d}v_x = a_x(t)\mathrm{d}t$

and integrate $\quad \int \mathrm{d}v_x = \int a_x(t)\mathrm{d}t$

$$v_x(t) = \int a_x(t)\mathrm{d}t \ \rightarrow \text{ determined up to an additive constant}$$

velocity $\quad v_x(t) = \frac{\mathrm{d}x}{\mathrm{d}t}$

$$\mathrm{d}x = v_x(t)\mathrm{d}t \implies \int \mathrm{d}x = \int v_x(t)\mathrm{d}t \implies \underbrace{\boxed{x(t) = \textstyle\int v_x(t)\mathrm{d}t}}_{\text{determined up to a constant}}$$

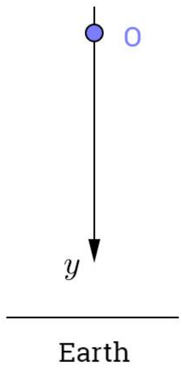
Note: The additive constants are found from the initial conditions.

Example: 1D motion with a constant acceleration (e.g. free fall near to Earth's surface, air drag neglected)

$$a_y(t) = g = \text{const}$$

$$\downarrow$$

plus sign (points in the positive direction of y − axis)



velocity

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = g \Rightarrow \mathrm{d}v_y = g\mathrm{d}t \Rightarrow \int \mathrm{d}v_y = \int g\mathrm{d}t$$

$$\Downarrow$$

$$\boxed{v_y(t) = gt + C_1 \ (*)}$$

Need to know v_y at some instant of time

$$\boxed{v_y(t_0) = v_{oy}} \rightarrow \text{initial condition}$$

On the other hand from (*)

$$v_y(t_0) = gt_0 + C_1$$

Combining both

$$v_{oy} = gt_0 + C_1 \ \Rightarrow \underline{C_1 = v_{oy} - gt_0}$$

Eventually

$$\boxed{v_y(t) = g(t - t_0) + v_{oy}}$$

Note. Usually we choose t_0 to be zero (we start measuring time from the initial instant)

position

$$\frac{\mathrm{d}y}{\mathrm{d}t} = g(t-t_0) + v_{oy}$$

$$\mathrm{d}y = \int [g(t-t_0) + v_{y,0}]dt$$

$$y(t) = \frac{1}{2}gt^2 - gt_0 t + v_{oy}t + C_2 = \underline{\frac{1}{2}gt^2 + (v_{oy} - gt_0)t + C_2} \; (**)$$

Need another initial condition

$$\boxed{y(t_0) = y_0}$$

Use it to combine with (**)

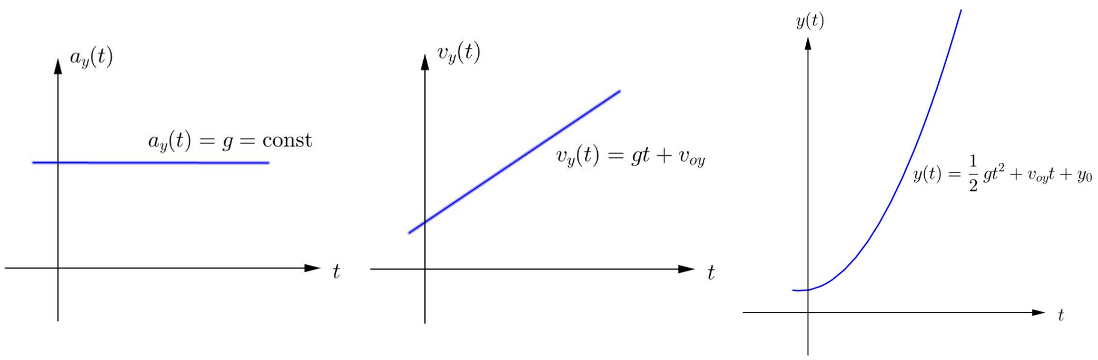
$$y(t_0) = \frac{1}{2}gt_0^2 + (v_{oy} - gt_0)t_0 + C_2 \;\Rightarrow\; \underline{C_2 = \frac{1}{2}gt_0^2 - v_{oy}t_0 + y_0}$$

$$\boxed{y(t) = \tfrac{1}{2}g(t^2 + t_0^2) + v_{oy}(t-t_0) - gt_0 t + y_0}$$

If we choose $t_0 = 0$

$$\boxed{\begin{aligned} v_y(t) &= gt + v_{oy} \\ y(t) &= \tfrac{1}{2}gt^2 + v_{oy}t + y_0 \end{aligned}}$$

Graphs



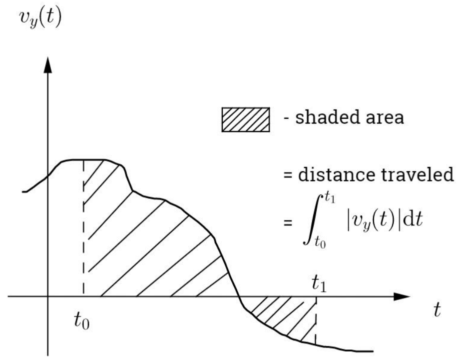
Comment: Initial conditions can be incorporated in definite integrals

$$v_y(t_0) = v_{oy}, \;\; y(t_0) = y_0$$

$$\int_{v_{oy}}^{v_y(t)} \mathrm{d}v_y = \int_{t_0}^{t} a_y(t)\mathrm{d}t \;\Rightarrow\; v_y(t) - v_{oy} = \int_{t_0}^{t} a_y(t)\mathrm{d}t$$

$$\int_{y_0}^{y(t)} \mathrm{d}y = \int_{t_0}^{t} v_y(t)\mathrm{d}t \;\Rightarrow\; y(t) - y_0 = \int_{t_0}^{t} v_y(t)\mathrm{d}t$$

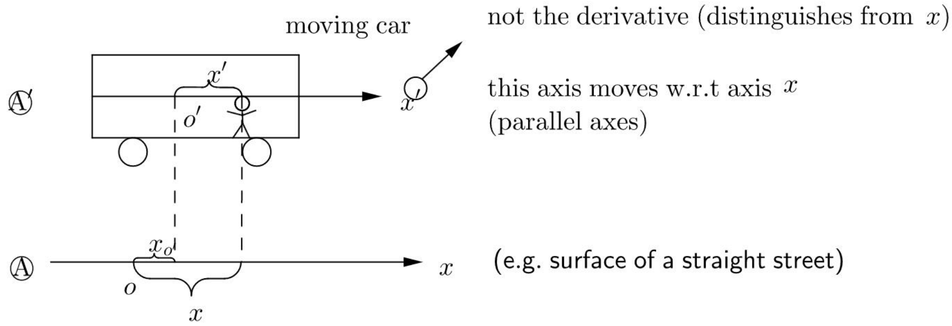
Recall interpretation of definite integral



Note

$$\int_{t_0}^{t_1} v_y(t) \mathrm{d}t = \text{not displacement}$$
$$= y(t_1) - y(t_0)$$

Relative motion (1D)



Axis with respect to which we describe motion - frame of reference (FoR)

Position in frame of reference A

$$x = x_{o'} + x'$$

position of the origin of axis x' | position of a particle in frame of ref. A'

Velocity

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x_{o'}}{\mathrm{d}t} + \frac{\mathrm{d}x'}{\mathrm{d}t}$$

$$\boxed{v_x = v_{ox} + v'_x}$$

velocity of the particle in FoR A | velocity of the origin O' of axis x' | velocity of a particle in FoR A' (watch the signs!)

Acceleration: analogously

$$\boxed{a_x = a_{o'x} + a'_x}$$

Special case:

$$\begin{array}{ll} v_{o'x} = \text{const, then} & a_x = a'_x \\ (x_{o'} = 0 \text{ at } t = 0) & v_x = v_{o'x} + v'_x \\ & x = \underset{\substack{\Downarrow \\ x_{o'}}}{\underline{v_{o'x}}} t + x' \qquad \underline{\text{Galilean transformation}} \end{array}$$

There is an implicit assumption in this discussion. Where?