1 Physical quantities, scalars and vectors

1. **Scalar quantities** – defined by a single number

   $\underset{\sim\sim\sim}{Examples}$: time, mass, length, volume, density of water, electric charge, potential energy, pressure

   Operational definitions– quantities obtained as a result of measurement operations.

   density of (bulk) matter = mass / $\underbrace{\text{volume}}_{(\text{length})^3}$

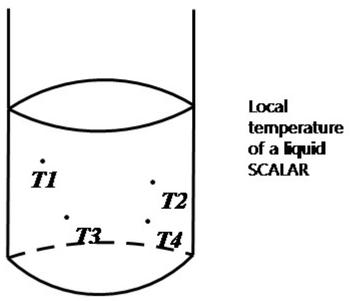
   $\underset{\sim\sim\sim}{Units}$ - metric system (Or SI - System international)

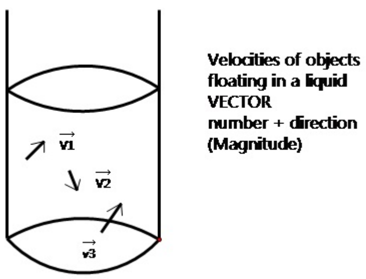
   Mathematical operations on physical quantities:

   - $+$ or $-$ only on compatible units
   - $*$ or $/$ can involve quantities with different units
   - sin(...), ln(...), ... only dimensionless arguments

2. **Vector quantities**

   $\underset{\sim\sim\sim}{Example}$:

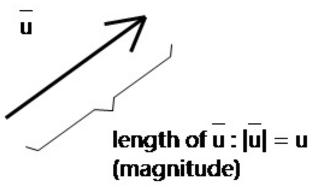




   $\underset{\sim\sim\sim}{Examples}$:

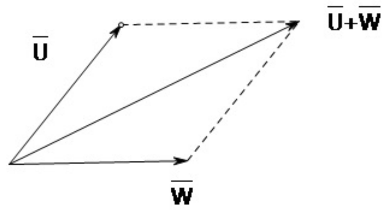
   velocity, force, momentum, electric/ magnetic field, angular velocity

   Notation: $\vec{u}, \bar{u}, \underline{u}$



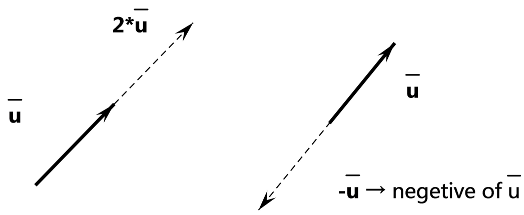
3. **Vectors**

- *Addition :*

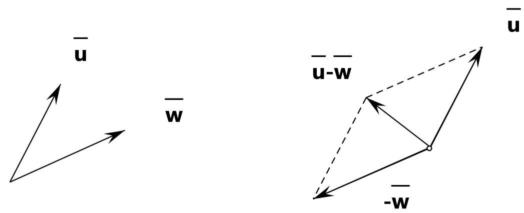


"Parallelogrem rule"

- *Multiplicationbyascalar :*

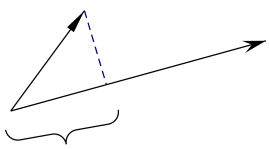


- *Subtraction :*



- *Multiplication :*

(1) Scalar (dot) product



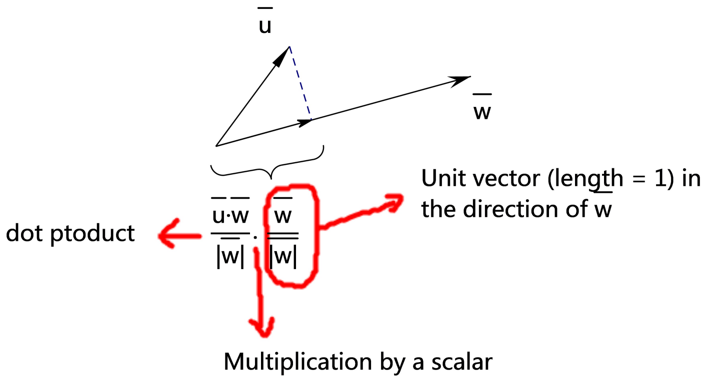
$$\frac{\bar{u}\cdot\bar{w}}{|\bar{w}|} = |\bar{u}|\cos\angle(\bar{u},\bar{w})$$

$\bar{u}\bullet\bar{w} = |\bar{u}|\bullet|\bar{w}|\cos\angle(\bar{u},\bar{w})$ → <u>number! (Scalar)</u>

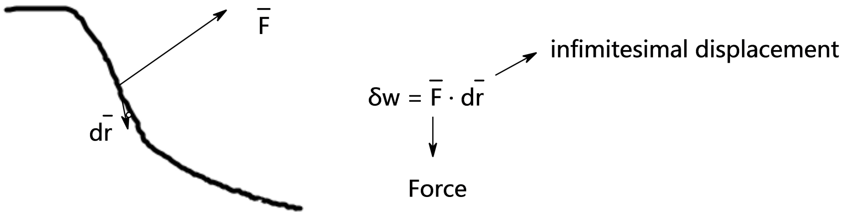
$\angle(\bar{u},\bar{w})$: Smaller angle between vectors $\bar{u}$ and $\bar{w}$

Note: if $u \perp w \Rightarrow u\bullet w = 0$, and vice-versa for non-zero vectors

Projection of Vector $\bar{u}$ on vector $\bar{w}$:



Example: elementary work

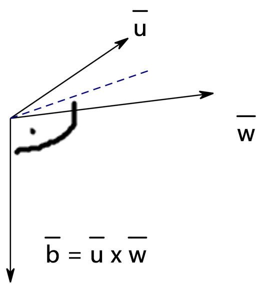


(2) Vector (cross) product

$\bar{b} = \bar{u} \times \bar{w}$ ($\bar{b}$ is a vector (precisely, psedovector))

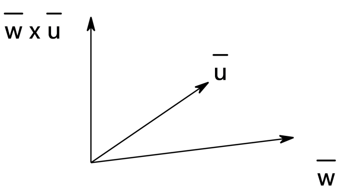
length: $|\bar{b}| = |\bar{u}| \cdot |\bar{w}| \cdot sin\angle(\bar{u}, \bar{w})$

direction: right-hand rule



Note:

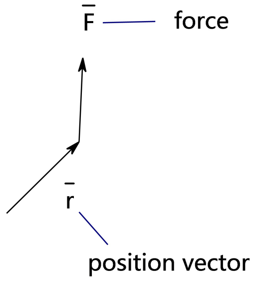
* description $\bar{u} \times \bar{w} = -\bar{w} \times \bar{u}$



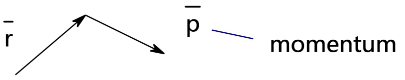
* if $\bar{u} \parallel \bar{w} \implies \bar{u} \times \bar{w} = \bar{0}$ (and vise-versa for non-zero vectors)

* $\bar{u} \times \bar{w} \perp \bar{u}$ and $\bar{u} \times \bar{w} \perp \bar{w}$

$Examples$:



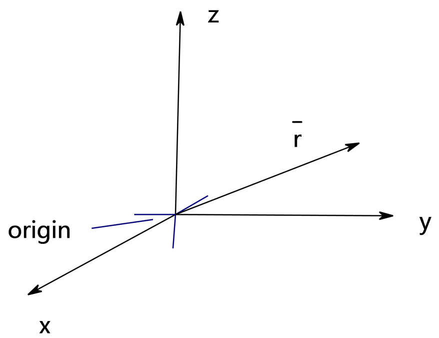
$\bar{r} \times \bar{F} = \bar{\tau}$ torque



angular momentum

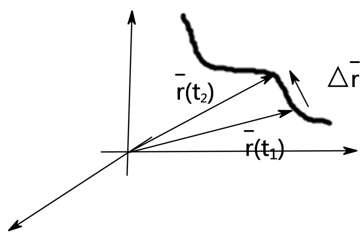
2 Position in space Coordinate systems and vectors

Cartician coordinate system

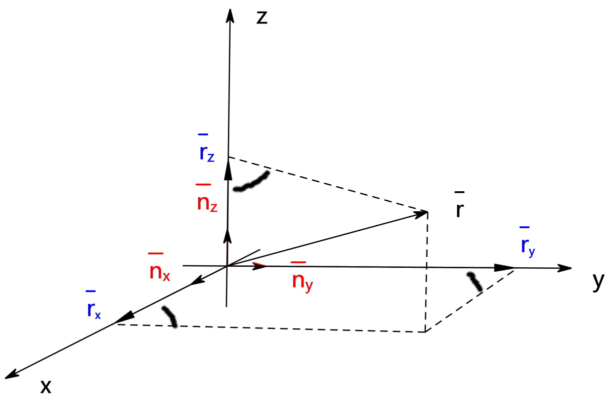


$\bar{r}$ - position vector (generally time dependant)

$\bar{r} = \bar{r}(t)$



$\Delta\bar{r} = \bar{r}(t_2) - \bar{r}(t_1)$ displacement vecrtot



$\bar{r} = \bar{r}_x + \bar{r}_y + \bar{r}_z$

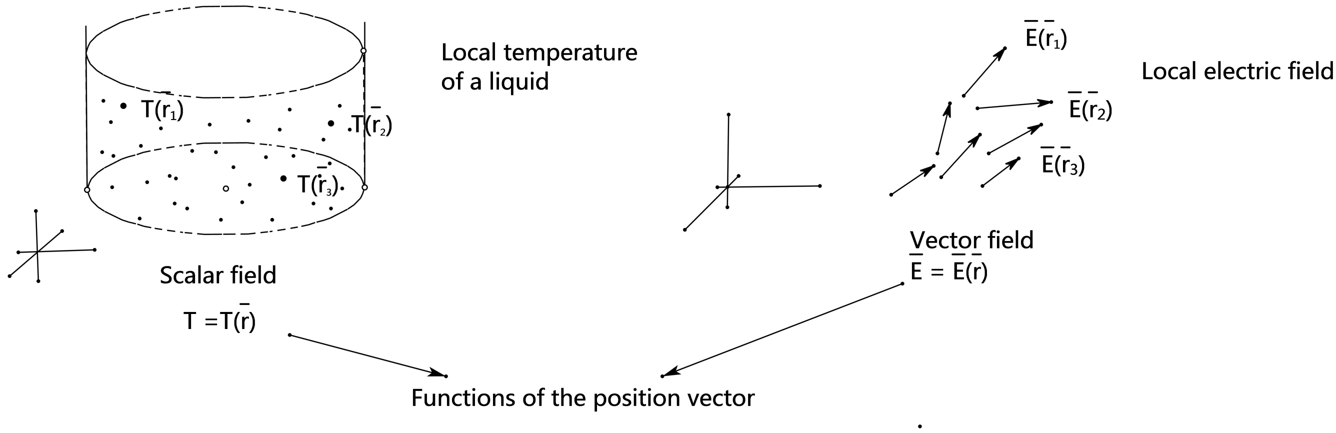
Component vectors along the axes

$\bar{r} = x \cdot \bar{n}_x + y \cdot \bar{n}_y + z \cdot \bar{n}_z$

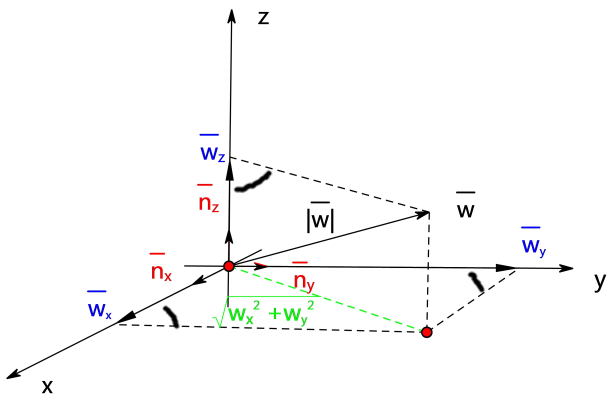
fixed unit vectors (vectors also denoted as $\bar{i}, \bar{j}, \bar{k}$, $\bar{x}, \bar{y}, \bar{z}$,)

Short notation: (x,y,z) or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Note: physical quantities may be position-dependent $\Longrightarrow$ fields



In general, for any vector



$\bar{w} = \bar{w}_x + \bar{w}_y + \bar{w}_z$
$\quad = w_x \bar{n}_x + w_y \bar{n}_y + w_z \bar{n}_z$

length (magnitude/modulus) of $\bar{w}$
$|\bar{w}| = w = \sqrt{w_x^2 + w_y^2 + w_z^2}$

unit vectors of the Cartesian system

- mutually perpendicular (orthogonal)

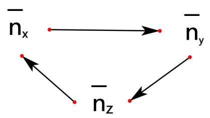
$$\widehat{n}_x \circ \widehat{n}_y = \widehat{n}_y \circ \widehat{n}_z = \widehat{n}_x \circ \widehat{n}_z = 0$$

- unit length

$$|\widehat{n}_x| = |\widehat{n}_y| = |\widehat{n}_z| = 1$$

- for right-handed system

$$\widehat{n}_x \times \widehat{n}_y = \widehat{n}_z$$
$$\widehat{n}_y \times \widehat{n}_z = \widehat{n}_x$$
$$\widehat{n}_z \times \widehat{n}_x = \widehat{n}_y$$



cyclic permutation

Differentiation and integration of vectors in Cartesian Coordinate system

$$\bar{u} = \bar{u}(t) \qquad \text{t:usually time}$$

$$\frac{d\bar{u}}{dt} = \dot{\bar{u}} = \frac{d}{dt}(u_x(t)\widehat{n}_x + u_y(t)\widehat{n}_y + u_z(t)\widehat{n}_z) = \dot{u_x}(t)\widehat{n_x} + \dot{u_y}(t)\widehat{n_y} + \dot{u_z}(t)\widehat{n}_z$$

$(\dot{\widehat{n_x}} = \dot{\widehat{n_y}} = \dot{\widehat{n_y}})$ because fixed unit vectors

Analogously,

$$\int_{t_0}^{t_1} = (\int_{t_0}^{t_1} u_x(t)dt)\widehat{n_x} + (\int_{t_0}^{t_1} u_y(t)dt)\widehat{n_y} + (\int_{t_0}^{t_1} u_z(t)dt)\widehat{n}_z$$

3 Scalar and vector products in Cartesian coordinate system

1. Dot product:

$$\bar{u} = u_x\widehat{n}_x + u_y\widehat{n}_y + u_z\widehat{n}_z = (u_x, u_y, u_z)$$
$$\bar{w} = w_x\widehat{n}_x + w_y\widehat{n}_y + w_z\widehat{n}_z = (w_x, w_y, w_z)$$
$$\hat{u} \circ \hat{w} = u_x w_x + u_y w_y + u_z w_z$$

Note:

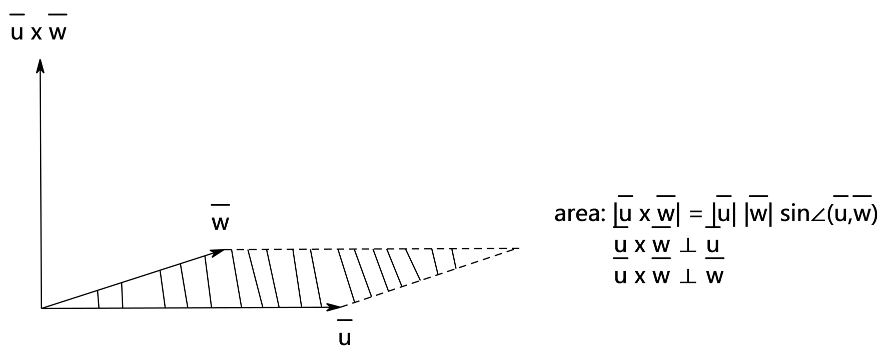
$$\hat{u} \circ \hat{u} = u_x^2 + u_y^2 + u_z^2 = u^2$$

2. Vector product (cross product)

$$\begin{aligned}\bar{u} \times \bar{w} &= (u_y w_z - u_z w_y)\widehat{n}_x + (u_z w_x - u_x w_z)\widehat{n}_y + (u_x w_y - u_y w_x)\widehat{n}_z \\ &= \begin{vmatrix} \hat{n}_x & \hat{n}_y & \hat{n}_z \\ u_x & u_y & u_z \\ w_x & w_y & w_z \end{vmatrix}\end{aligned}$$

Hint: Start with $\bar{u}$, $\bar{w}$ written as sums of components; then use the 3^{rd} property of unit vectors ($\hat{n}_x \times \hat{n}_y = \hat{n}_z$, ...) and $\hat{n}_x \times \hat{n}_x = 0$,...

Recall:



3. Other operations:

a)

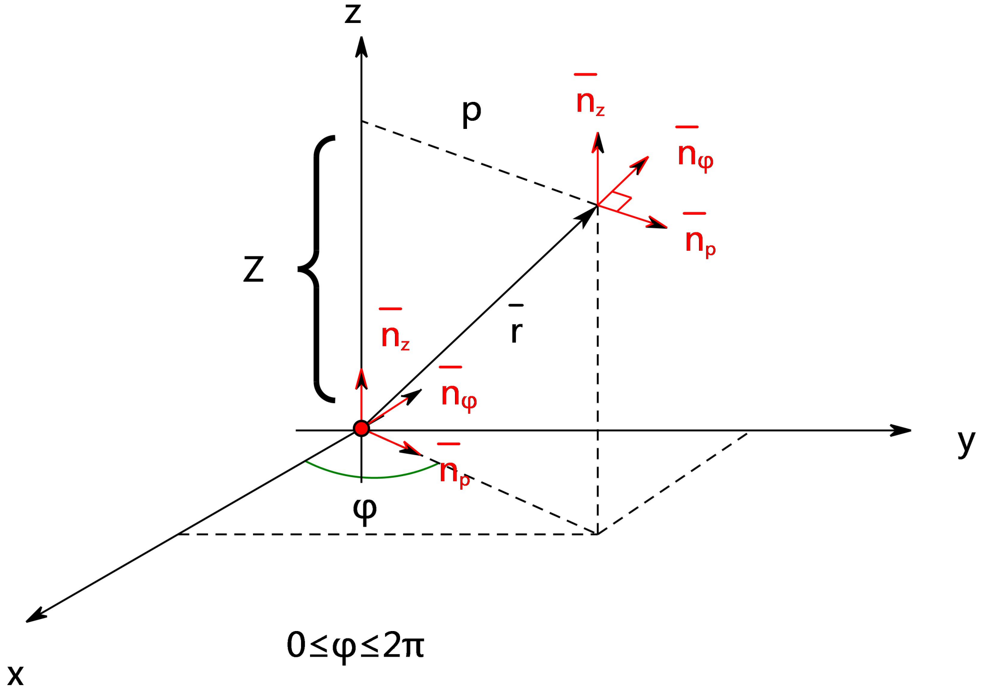
$$\begin{aligned}\bar{u} \pm \bar{w} &= (u_x\widehat{n}_x + u_y\widehat{n}_y + u_z\widehat{n}_z) \pm (w_x\widehat{n}_x + w_y\widehat{n}_y + w_z\widehat{n}_z) \\ &= (u_x \pm w_x)\widehat{n}x + (u_y \pm w_y)\widehat{n}_y + (u_z \pm w_z)\widehat{n}_z \\ &= (u_x \pm w_x, u_y \pm w_y, u_z \pm w_z)\end{aligned}$$

b)

$$\alpha\bar{u} = \alpha(u_x\widehat{n}_x + u_y\widehat{n}_y + u_z\widehat{n}_z) = ... = (\alpha u_x, \alpha u_y, \alpha u_z)$$

4 Alternative coordinate systems (curvilinear coordinates)

1. **Cylindrical coordinates**



Coordinates: ρ, φ, z

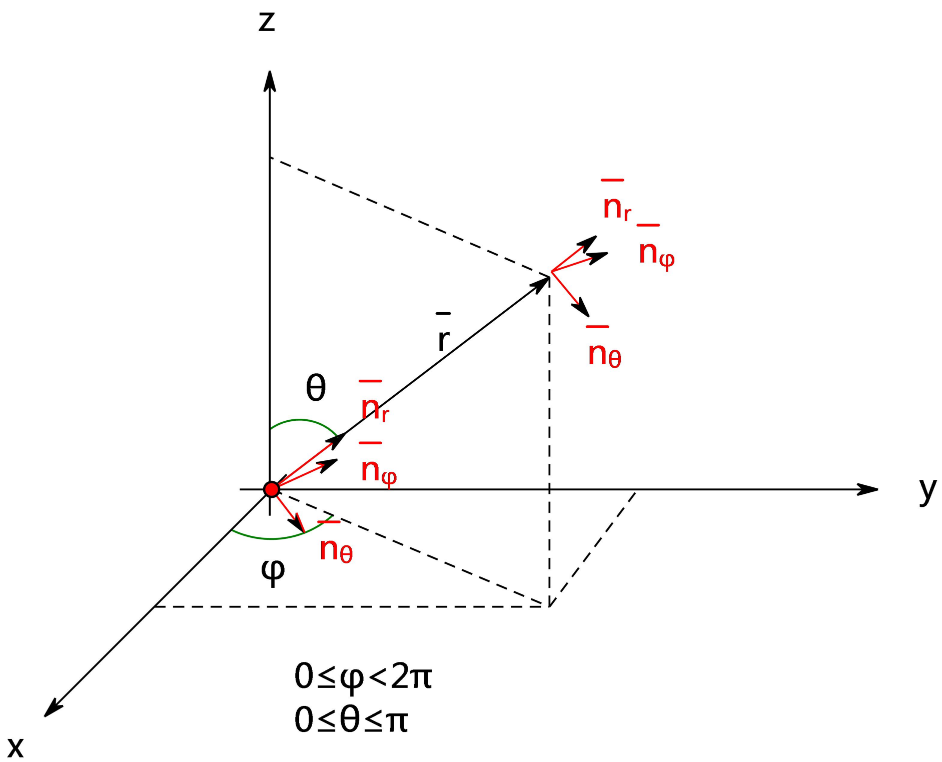
Unit vectors: $\widehat{n}_\rho$, $\widehat{n}_\varphi$, $\widehat{n}_z$ (usually orthogonal)

$$\begin{cases} \rho = & \sqrt{x^2+y^2} \\ \varphi = & arctan(y/x) \\ z = & z \end{cases} \qquad \begin{cases} x = & \rho cos\varphi \\ y = & \rho sin\varphi. \\ z = & z \end{cases}$$

Position vector

$$\hat{r} = \rho \underbrace{\widehat{n}_\rho}_{\text{carries information about the angle } \varphi} + z\widehat{n}_z$$

2. **Spherical Coordinates**



Coordinates: r, φ, θ

Unit vectors:$\widehat{n}_r$, $\widehat{n}_\varphi$, $\widehat{n}_\theta$ (mutually orthogonal)

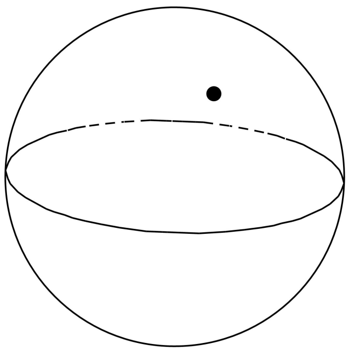
$$\begin{cases} r = & \sqrt{x^2+y^2+z^2} \\ \varphi = & arctan(y/x) \\ \theta = & arctan(\frac{\sqrt{x^2+y^2}}{z}) \end{cases} \qquad \begin{cases} x = & r \cdot sin\theta \cdot cos\varphi \\ y = & r \cdot sin\theta \cdot sin\varphi. \\ z = & r \cdot cos\theta \end{cases}$$

Position vector

$$\hat{r} = r\widehat{n}_r$$

When to use them?

Phenomenon with the corresponding symmetry
eg. particle moving on the surface of a ball



Note: Polar coordinates(2D) if z = 0 and $\theta = \frac{\pi}{2}$, respectively