

Summer_2019 VV255_Assignment 1: Vectors and surfaces

Deadline: 2019-05-29

Problem 1

For each surface defined by the following equations: use traces (cross sections) to identify the surface and sketch it by hand.

1. $4x^2 - y^2 - 16z^2 + 16 = 0$ 2. $x^2 + 4z = 0$ 3. $3x^2 + y^2 + 9z^2 - 9 = 0$

Problem 2

Determine the equations of surfaces obtained by the rotation of the following curves around the corresponding axis:

1. $y^2 = 2z, x = 0; Oz$ 2. $9y^2 + 4z^2 = 36, x = 0; Oy$

Sketch the geometric object bounded by the following surfaces:

$$y = x, x = 1, z = 0, z = xy.$$

Problem 3

Consider vectors $\vec{a} = -5\vec{m} - 4\vec{n}$ and $\vec{b} = -3\vec{m} - 6\vec{n}$, where $|\vec{m}| = 3, |\vec{n}| = 5$ and $\angle(\vec{m}, \vec{n}) = \frac{5\pi}{2}$.

Calculate

1. $\left(-2\vec{a} + \frac{1}{3}\vec{b}\right) \cdot (\vec{a} + 2\vec{b})$ 2. $\text{comp}_{\vec{b}}(\vec{a} + 2\vec{b})$ 3. $\cos(\vec{a}, 2\vec{b})$

Problem 4

Let $A(4, 6, 3), B(-5, 2, 6), C(4, -4, -3)$ and $\vec{a} = 4\vec{CB} - \vec{AC}, \vec{b} = \vec{AB}, \vec{c} = \vec{CB}$

Find 1. $|4\vec{CB} - \vec{AC}|$ 2. $(4\vec{CB} - \vec{AC}) \cdot \vec{AB}$ 3. $\text{comp}_{\vec{AC}}\vec{CB}$

4. The coordinates of the point M that divides the segment AB in the proportion 5:4.

Problem 5

The force $\vec{F} = (5, -3, 9)$ acts on a rigid body at the point $A(3, 4, -6)$. Find

1. The work done by the force pointing in the direction of the vector \vec{AB} .
2. The torque relative to the point $B(2, 6, 5)$.

Problem 6

A pyramid is defined by the points $A(3, 4, 5), B(1, 2, 1), C(-2, -3, 6), D(3, -6, -3)$. Find

1. The area of the face ACD .
2. The volume of the pyramid.