1 Outline

Particles in equilibrium

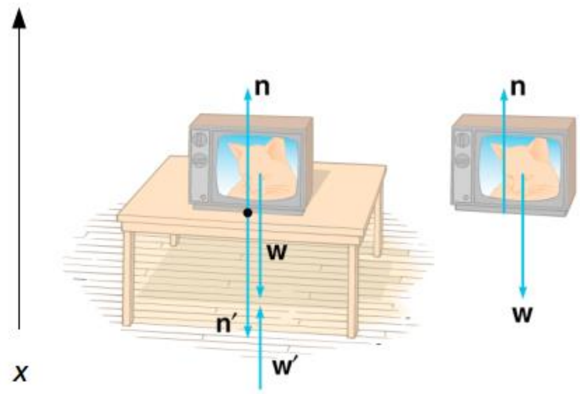
Accelerating particles

Motion with friction and air/uid resistance

2 Particles in Equilibrium

A body acted on by **zero net force** moves with **constant velocity**.

The mutual forces of **action** and **reaction** between two bodies are equal, opposite and collinear.

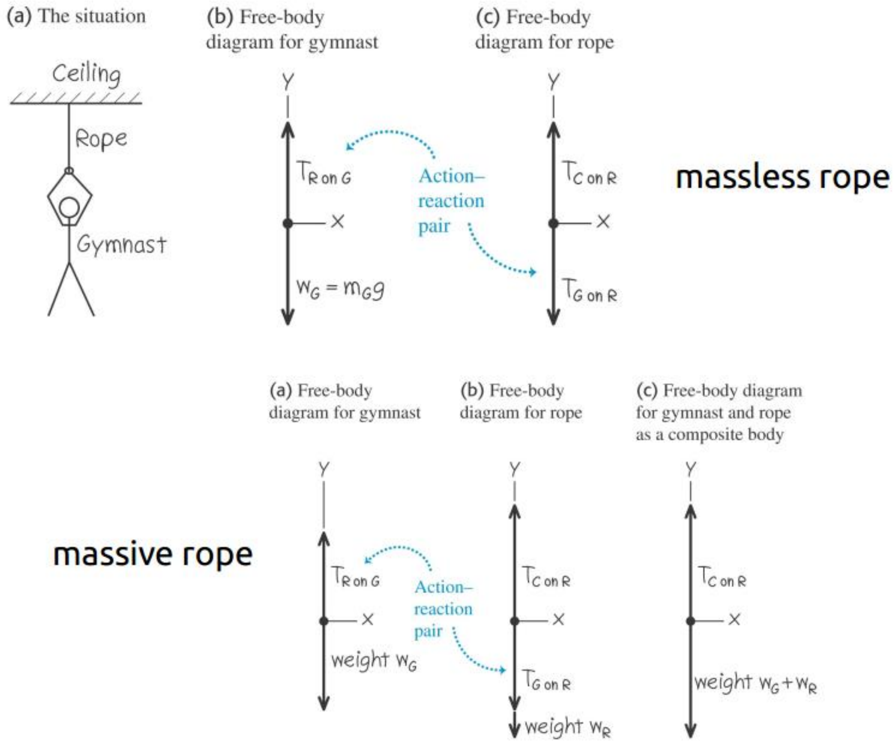


$$netforce = \sum \mathbf{F} = 0$$
$$\mathbf{n} + \mathbf{w} = 0$$

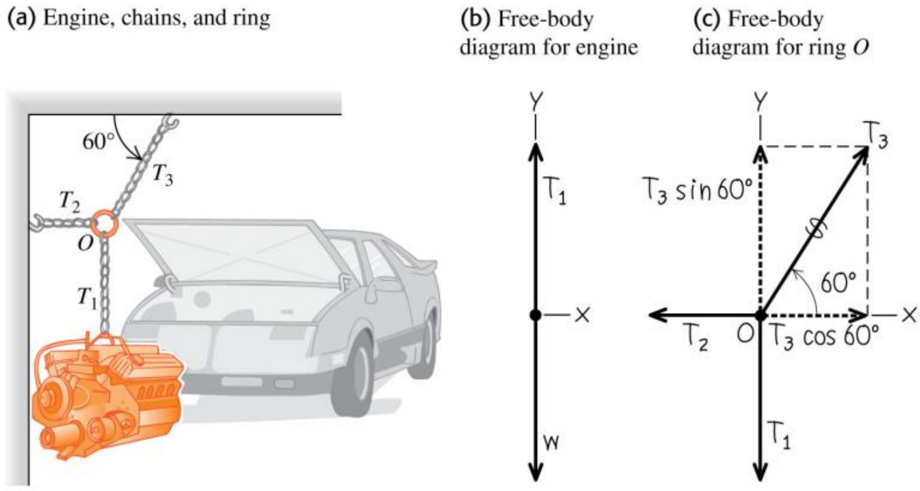
or operating with components and magnitudes **(watch the sign!)**

$$n - w = 0$$

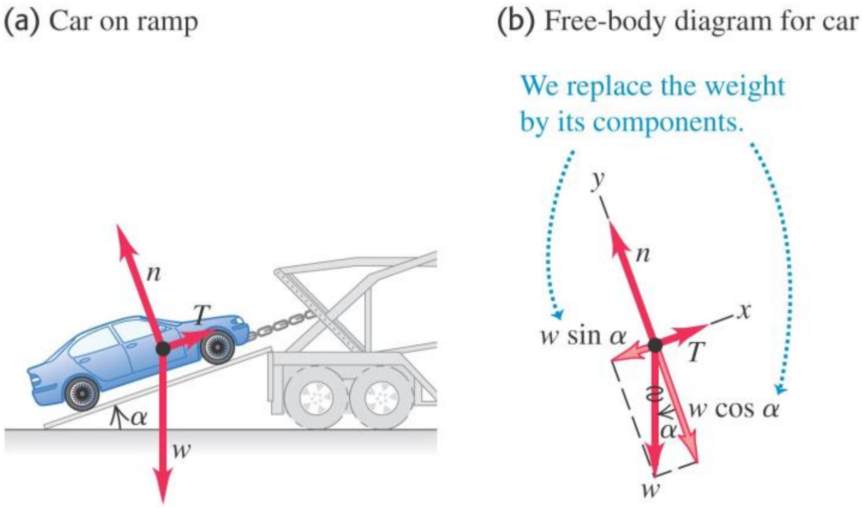
2.1 Example (equilibrium; collinear forces)



2.2 Example (equilibrium; non-collinear forces)

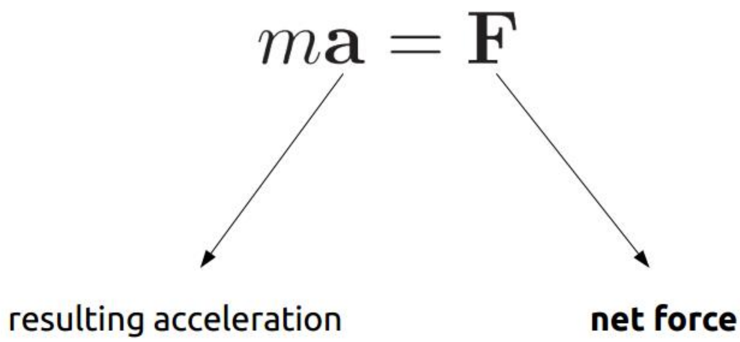


2.3 Example (equilibrium; object on an inclined plane)



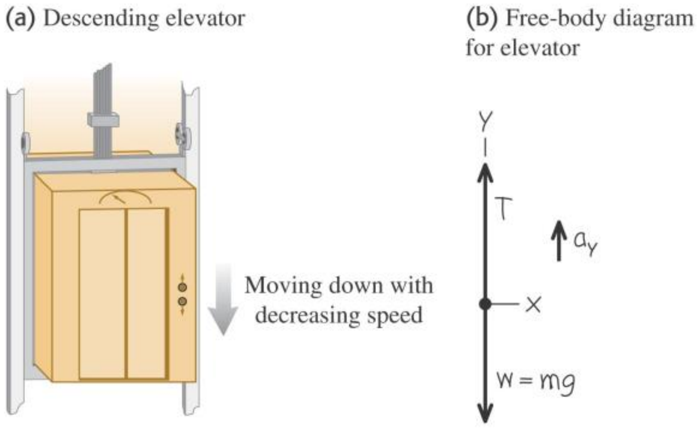
3 Particles in Motion

In an inertial frame of reference, the **acceleration** of an object is **directly proportional to the net force** acting on it, and **inversely proportional to the mass** of the object.



3.1 Example: Elevator (tension in a massless cable)

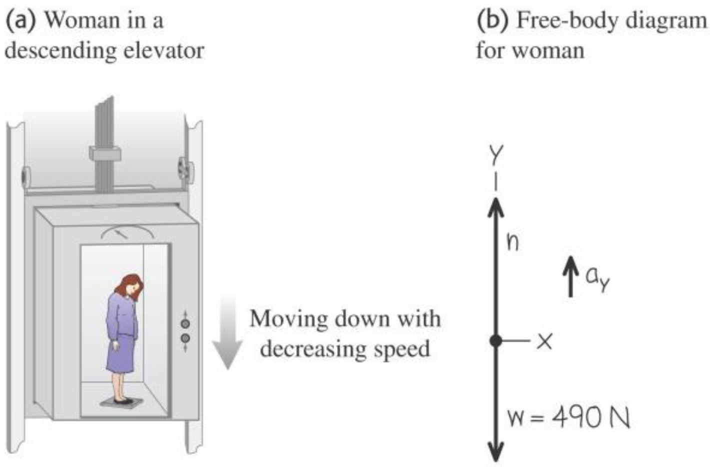
The elevator is moving downward but slowing to a stop. What is the tension in the supporting massless cable?



$$ma_y = T - mg \Rightarrow T = m(a_y + g)$$

3.2 Example: Elevator (apparent weight)

A woman inside the elevator of the previous example is standing on a scale. How will the acceleration of the elevator aect the scale reading?

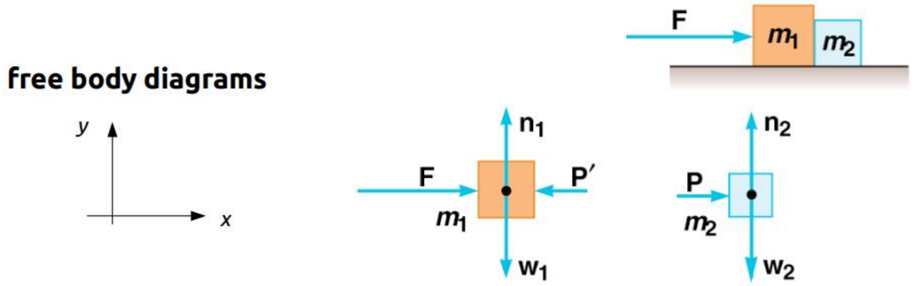


inertial frame of reference (e.g. the elevator shaft)

$$ma_y = n - mg \Rightarrow n = m(a_y + g)$$

3.3 Example: two objects in direct contact

two objects in contact acted upon an external force, frictionless surface



no motion along y direction (net forces have zero y component), **Newton's second law for motion along x direction** (operate with magnitudes, but watch the signs!)

$$m_1 a = F - P'$$

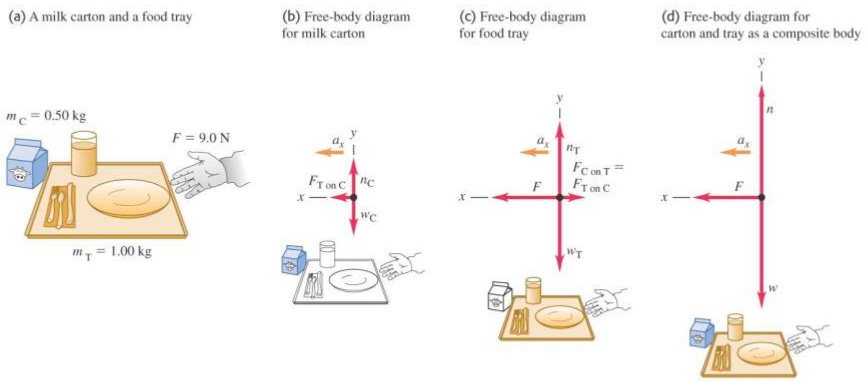
$$m_2 a = P$$

Newton's third law for the pair of forces between the blocks

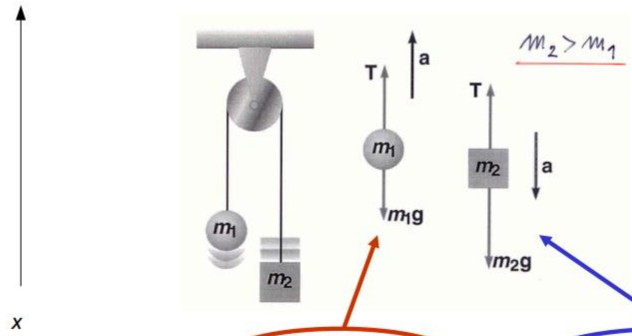
$$P = P'$$

$$\textbf{result: } a = \frac{F}{m_1 + m_2}$$

3.4 Example: two objects in contact and Newton's third law



3.5 Example: Atwood's machine



$$m_1 a = T - m_1 g \qquad -m_2 a = T - m_2 g$$

$$\textbf{solution:}\ a = (\frac{m_2 - m_1}{m_1 + m_2})g = const$$

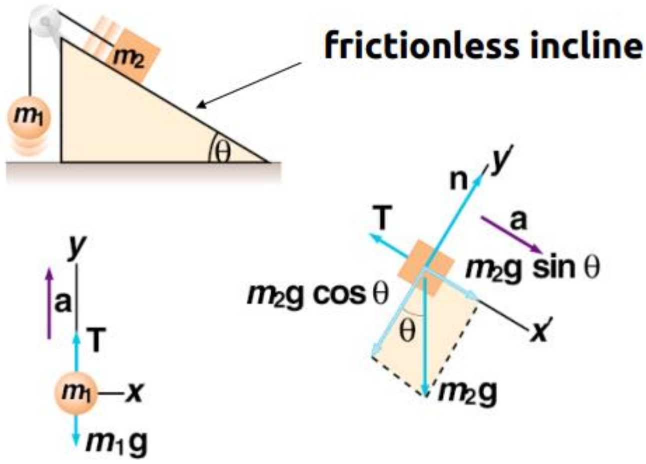
$$T = (\frac{2m_1 m_2}{m_1 + m_2})g$$

$$\textbf{special cases:}\ m_1 = m_2 \Rightarrow a = 0, T = m_1 g$$

$$m_2 \gg m_1 \Rightarrow a \approx g, T \approx 2m_1 g$$

3.6 Example: incline

Two objects of dierent masses connected by a massless cord that passes over a frictionless pulley of negligible mass.



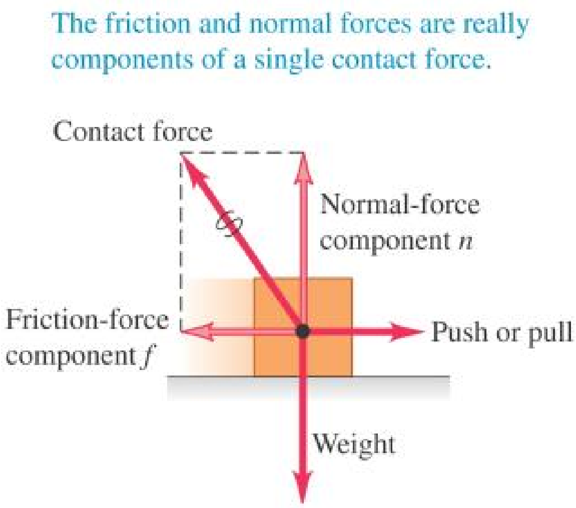
$$\textbf{solution: } a = (\frac{m_2 sin\theta - m_1}{m_1 + m_2})g = const$$

$$T = \frac{m_1 m_2}{m_1 + m_2}(1 + sin\theta)g$$

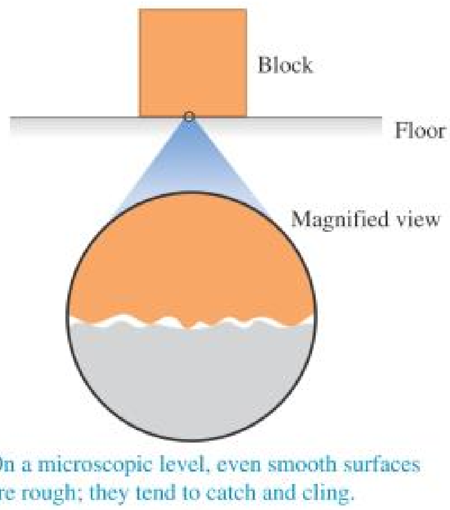
4 Friction and Fluid/Air Resistance

4.1 Frictional Forces

When a body rests or slides on a surface, the **friction force** is parallel to the surface.



Friction between two surfaces arises from **interactions between molecules** on the surfaces.



4.1.1 Kinetic vs Static Friction

Kinetic friction appears when a body slides over a surface. The magnitude of the **kinetic friction force** is

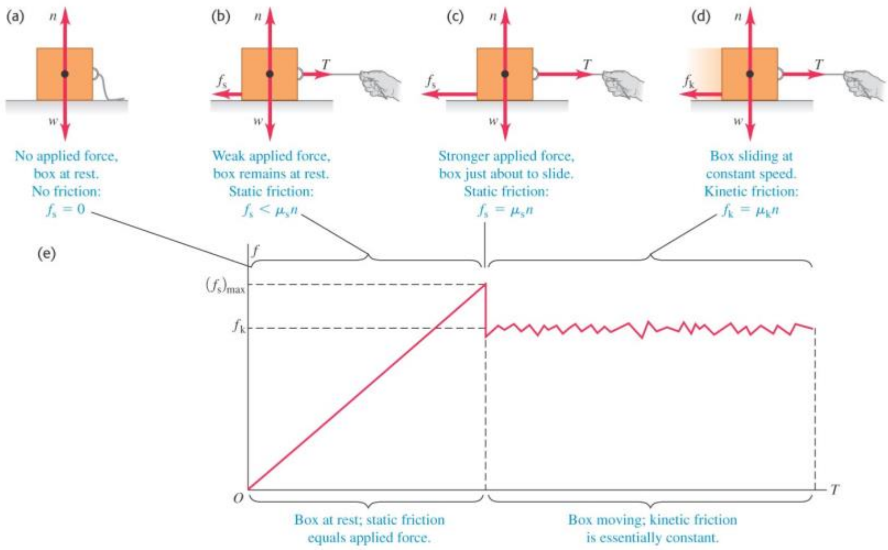
$$f_k = \mu_k n$$

Static friction force acts when there is no relative motion between bodies.

The magnitude of the **static friction force** can vary between zero and its maximum value:

$$f_s \leq \mu_s n$$

Before the box slides, static friction acts. But once it starts to slide, it turns into kinetic friction.

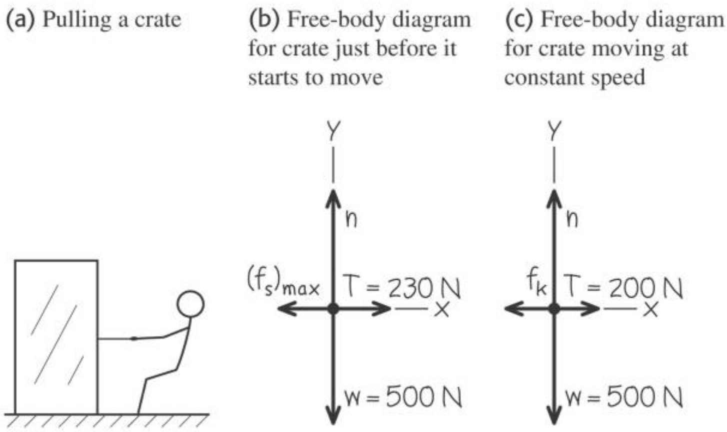


4.1.2 Values of Coefficients of Friction

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

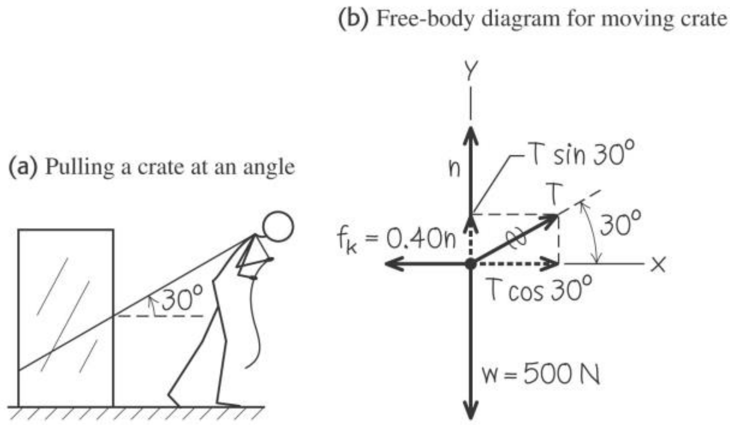
4.1.3 Example (static vs kinetic friction)

Before the crate moves, static friction acts on it. After it starts to move, kinetic friction acts.



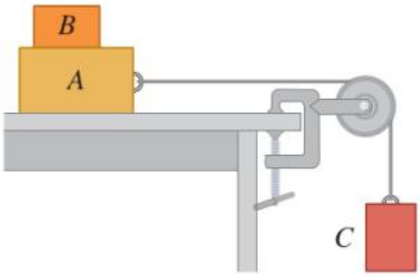
4.1.4 Example (inclined pull)

The angle of the pull aects the normal force, which in turn aects the friction force.



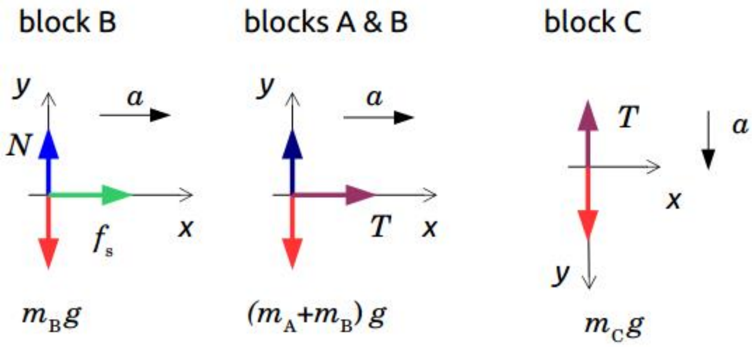
4.1.5 Example (static friction)

There is no friction between block A and the tabletop, but the coefficient of static friction between block A and block B is $\mu_s \neq 0$. A light string attached to block A passes over a frictionless, massless pulley, and block C is suspended from the other end of the string. Masses of blocks A and B are given.



What is the maximum mass that block C can have, so that blocks A and B still slide together when the system is released from rest?

If A and B move together, all three blocks have the same acceleration.



$$(m_A + m_B)a = T$$

$$m_C a = m_C g - T$$

$$\textbf{solution:} \quad a = \frac{m_C}{m_A + m_B + m_C} g$$

Block B moves acted upon the (static) frictional force.

$$m_B a = f_s$$

$$f_s \leq \mu_s N = \mu_s m_B g = f_{s,max}$$

Hence it will not slide as long as $a \leq \mu_s g$

case 1: $\mu_s \geq 1$

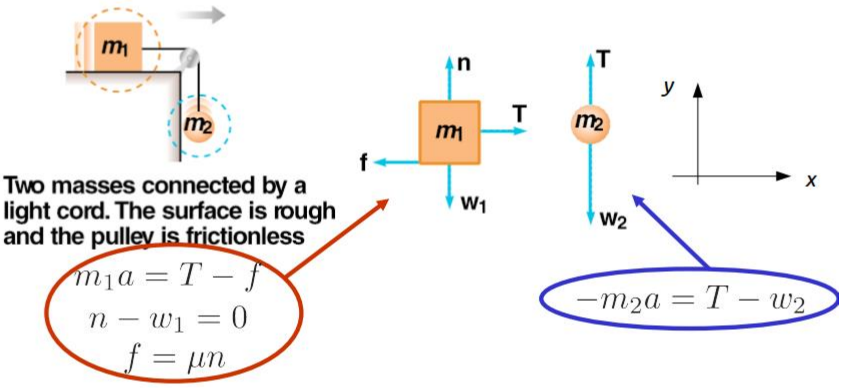
Since $a \leq g$ for this system, the inequality $a \leq \mu_s g$ always holds, irrespective of the value of mass m_C. (Maximum frictional force that can be provided is greater than that required for the block to move with acceleration a.) For any mass m_C block B moves together with block A.

case 2: $\mu_s < 1$

$$\frac{m_C}{m_A + m_B + m_C} g \leq \mu_s g$$

$$m_C \leq \frac{(m_A + m_B)\mu_s}{1 - \mu_s}$$

4.1.6 Example (two objects on a rough surface, connected by a massless cord)

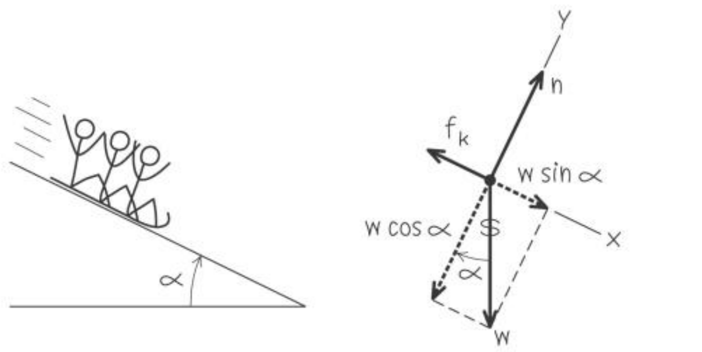


$$\textbf{solution:}\ a = \frac{m_2 - \mu m_1}{m_1 + m_2} g = const$$

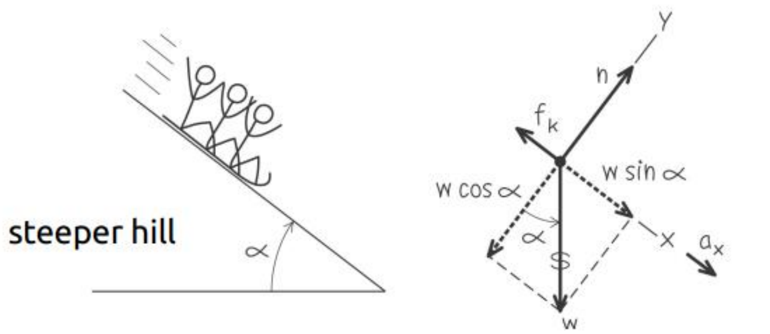
$$T = \frac{m_1 m_2}{m_1 + m_2}(1 + \mu) g$$

4.1.7 Example (motion on a rough incline)

(a) The situation (b) Free-body diagram for toboggan

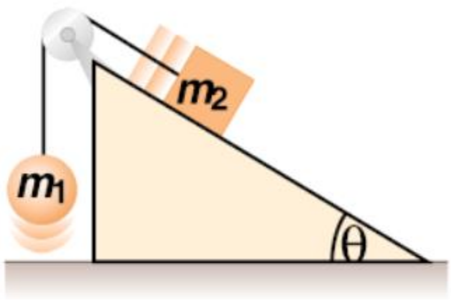


(a) The situation (b) Free-body diagram for toboggan

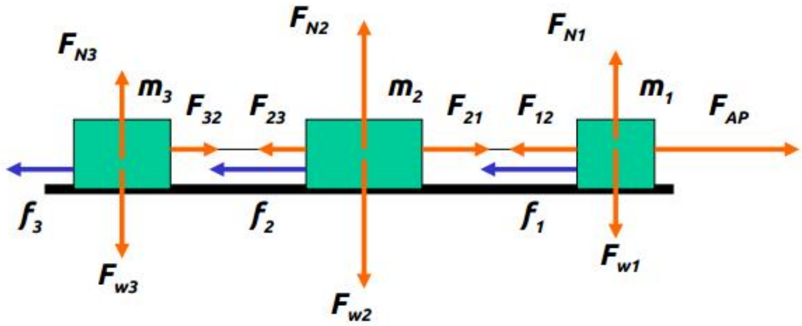


4.1.8 More Examples (DIY)...

two objects connected by a massless cord, rough incline, frictionless pulley



"train"



4.2 Fluid/Air Resistance

The **fluid resistance (drag) force f** on a body depends on the speed of the body. Usually $\mathbf{f} \propto -v^p \frac{\mathbf{v}}{v}$ with p=1 or 2.

A falling body reaches its **terminal speed** when the resisting force equals the weight of the body.