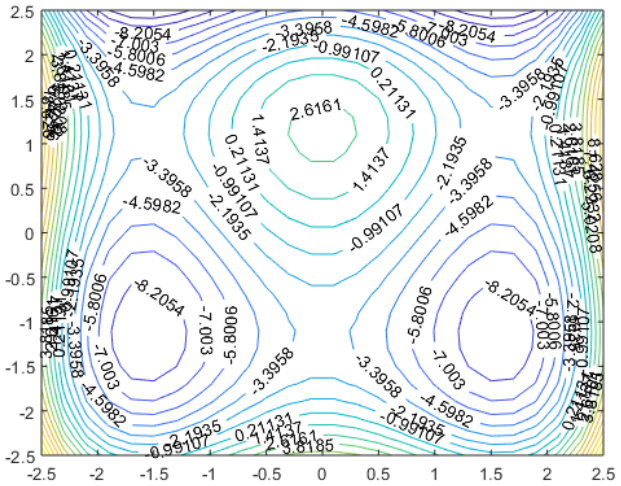
**Summer_2019 VV255_Assignment 4: Extreme Values. Basic Integration.**

Deadline: 2019-06-24

**Problem 1**

The level curves of $f(x,y) = 4y - y^3 - 5x^2 + x^4$ are shown below. Use the counter plot to identify possible critical points of the function $f$ and their type. Use the second derivative test to confirm your guess.



**Problem 2**

Use the method of Lagrange multipliers to find the points on the ellipse

$$\frac{x^2}{9} + \frac{y^2}{16} = 2$$

that are closest to and furthest from the straight line $x + 2y = 18$ in the $xy$ plane.

**Problem 3**

Find the global maximum and the global minimum of the function

$$f(x,y) = xy + x + y$$

defined in the closed triangular region with the vertices $(0,0), (0,3), (5,0)$.

**Problem 4**

Find the global maximum and the global minimum of the function

$$f(x,y) = e^{-x^2-y^2}(2x^2 + 3y^2)$$

in the region $D = \{(x,y): x^2 + y^2 \leq 1\}$

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**Problem 5**

Find the maximum and minimum of $x_1 + \cdots + x_n$ subject to $x_1^2 + \cdots + x_n^2 = 1$.

**Problem 6**

Find any extrema of the function

$$f(x, y, z) = x^2 + y^2 + z^2$$

Subject to the constraints

$$3x + y + z = 5, \qquad x + y + z = 1$$

**Problem 7**

a. Consider a reasonable partition of the region $[0,2] \times [0,2]$ and compute upper and lower Darboux sums for the function $f(x, y) = xy$ defined in the domain $[0,2] \times [0,2]$. How do you estimate the value of the Darboux integral?
b. Compute the Riemann integral of $f(x, y)$ over the region $[0,2] \times [0,2]$.

**Problem 8**

Evaluate

$$\iint \sin \pi x \cos \pi y \, dxdy$$

over $\left[0, \frac{\pi}{3}\right] \times [\frac{\pi}{6}, \frac{\pi}{3}]$.

**Problem 9**

Calculate the iterated integral

$$\int_0^1 \int_x^1 e^{y^2} dy \, dx.$$

**Problem 10**

Evaluate the integral

$$\int_0^1 \int_0^1 f(x, y) dx \, dy, \qquad f(x, y) = \begin{cases} y, & y > x^2, \\ x^2, & y \le x^2. \end{cases}$$

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