VE230 Homework 4

2021 Summer

P1 The upper and lower conducting plates of a large parallel-plate capacitor are separated by a distance d and maintained at potentials V_0 and 0 , respectively. A dielectric slab of dielectric constant 6.0 and uniform thickness $0.8d$ is placed over the lower plate. Assuming negligible fringing effect, determine

a) the potential and electric field distribution in the dielectric slab,

b) the potential and electric field distribution in the air space between the dielectric slab and the upper plate,

c) the surface charge densities on the upper and lower plates.

d) Compare the results in part (b) with those without the dielectric slab.

P2 Prove that the scalar potential V in

$$V = \frac{1}{4\pi\epsilon_0}\int_{V'}\frac{\rho}{R}dv'$$

satisfies Poisson's equation,

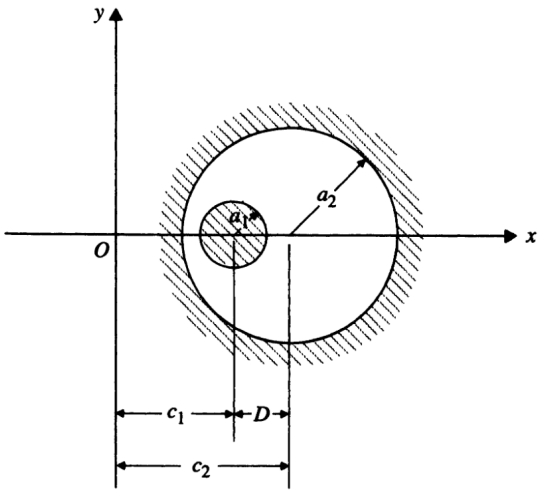
$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

P3 A point charge Q exists at a distance d above a large grounded conducting plane. Determine

a) the surface charge density ρ_s,

b) the total charge induced on the conducting plane.

P4 A very long two-wire transmission line, each wire of radius a and separated by a distance d, is supported at a height h above a flat conducting ground. Assuming both d and h to be much larger than a, find the capacitance per unit length of the line.



(a) A cross-sectional view.

Figure 1: Fig. 1 An Off-center Wire within a Tunnel (Problem 5)

P5 A long wire of radius a_1 lies inside a conducting circular tunnel of radius a_2, as shown in Fig. 1. The distance between their axes is D.

a) Find the capacitance per unit length.

b) Determine the force per unit length on the wire if the wire and the tunnel carry equal and opposite line charges of magnitude ρ_ℓ.

P6 Two dielectric media with dielectric constants ϵ_1 and ϵ_2 are separated by a plane boundary at $x = 0$, as shown in Fig. 1. A point charge Q exists in medium 1 at distance d from the boundary.

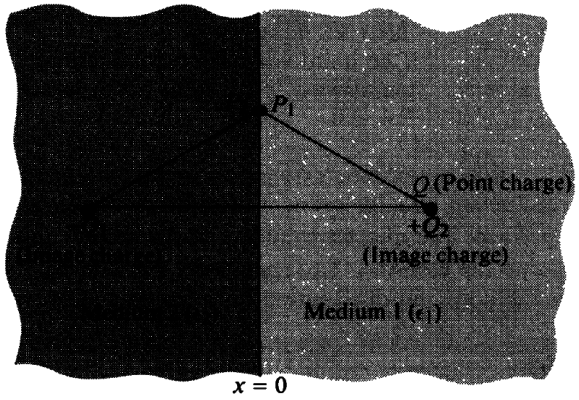


Figure 2: Fig. 2 Image charges in dielectric media (Problem 6)

a) Verify that the field in medium 1 can be obtained from Q and an image charge $-Q_1$, both acting in medium 1 .

b) Verify that the field in medium 2 can be obtained from Q and an image charge $+Q_2$ coinciding with Q, both acting in medium 2 .

c) Determine Q_1 and Q_2. (Hint: Consider neighboring points P_1 and P_2 in media 1 and 2, respectively, and require the continuity of the tangential component of the E-field and of the normal component of the D-field.)

P7 An infinite conducting cone of half-angle α is maintained at potential V_0 and insulated from a grounded conducting plane, as illustrated in Fig. 2. Determine

a) the potential distribution $V(\theta)$ in the region $\alpha < \theta < \pi/2$

b) the electric field intensity in the region $\alpha < \theta < \pi/2$

c) the charge densities on the cone surface and on the grounded plane.

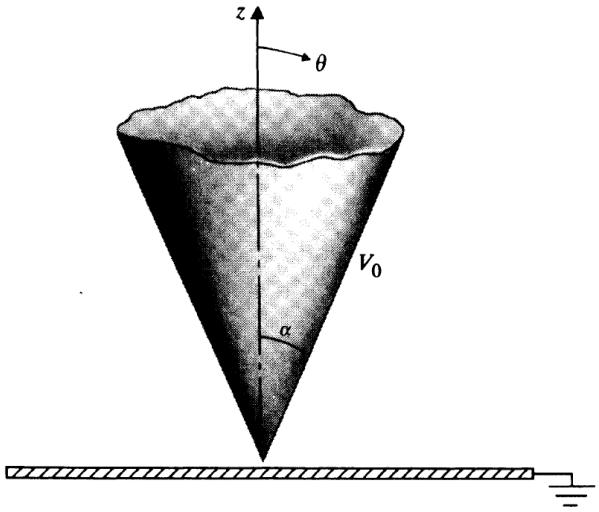


Figure 3: Fig. 3 An infinite conducting cone and a grounded conducting plane (Problem 7).