
Question1 (4 points)

Find an antiderivative for each of the following functions

(a) (1 point) $y = \pi + \dfrac{x^2 + x^3}{\sqrt{x}}$

(b) (1 point) $y = 3\sin x - 2\sec^2 x$

(c) (1 point) $y = \dfrac{1}{2\sqrt{1-x^2}} - \dfrac{3}{1+x^2}$

(d) (1 point) $y = x^x(\ln x + 1)$

Question2 (1 points)

Find the indefinite integral of

$$f(x) = x^x(\ln x + 1)$$

Question3 (3 points)

(a) (1 point) For the function below,

$$f(x) = x^2 - x^3$$

find a formula for the Riemann sum obtained by dividing the interval $[-1, 0]$ into n equal subintervals and using the right-hand endpoint for each x_k^*. Then take a limit of the sum as $n \to +\infty$ to calculate the area under the curve $y = f(x)$ over $[-1, 0]$.

(b) (1 point) Find two different tagged partitions

$$P$$

such that the following limit has two different values, one for each tagged partition.

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$

where

$$f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap \mathbb{Q}, \\ 0 & \text{if } x \in [0,1] \cap \mathbb{Q}^{\complement}. \end{cases}$$

(c) (1 point) The following function is continuous on $[0, 4]$, thus integrable on this interval.

$$g(x) = \sqrt{x}$$

Using the definition of definite integral to evaluate

$$\int_0^4 g(x)\,dx$$

Question4 (3 points)

Use the fundamental theorem of calculus to evaluate the following definition integrals.

(a) (1 point) $\displaystyle\int_0^{2\pi} \sin x\,dx$

(b) (1 point) $\displaystyle\int_0^1 \sqrt{x}\,dx$

(c) (1 point) $\displaystyle\int_{-1}^4 f(x)\,dx$, where $f(x) = \begin{cases} x & x < 2, \\ x^2 & x \geq 2. \end{cases}$

Question5 (24 points)

Find the followings. Show all your workings.

(a) (1 point) $\displaystyle\int_4^9 2x\sqrt{x}\,dx$

(b) (1 point) $\displaystyle\int_{\sqrt{2}}^2 \frac{dx}{x\sqrt{x^2-1}}$

(c) (1 point) $\displaystyle\int \frac{x^2}{x+1}\,dx$

(d) (1 point) $\displaystyle\int_0^1 xe^{-x^2/2}\,dx$

(e) (1 point) $\displaystyle\int x\ln x\,dx$

(f) (1 point) $\displaystyle\int \sin(\ln x)\,dx$

(g) (1 point) $\displaystyle\int_1^3 \sqrt{x}\arctan\sqrt{x}\,dx$

(h) (1 point) $\displaystyle\int \frac{x^2}{\sqrt{16-x^2}}\,dx$

(i) (1 point) $\displaystyle\int \frac{dx}{(4x^2-9)^{3/2}}$

(j) (1 point) $\displaystyle\int_1^3 \frac{dx}{x^4\sqrt{x^2+3}}$

(k) (1 point) $\displaystyle\int \frac{dx}{x^2-3x-4}$

(l) (1 point) $\displaystyle\frac{d}{dx}\int_1^x \sin\left(t^2\right)\,dt$

(m) (1 point) $\displaystyle\frac{d}{dx}\int_x^0 t\sec t\,dt$

(n) (1 point) $\displaystyle\int \sin x\cos(x/2)\,dx$

(o) (1 point) $\displaystyle\int_0^{\pi/3} \sin^4 3x\cos^3 3x\,dx$

(p) (1 point) $\displaystyle\int \tan^4 x\sec x\,dx$

(q) (1 point) $\displaystyle\int \frac{2x^2+3}{x(x-1)^2}\,dx$

(r) (1 point) $\displaystyle\int \frac{x^3-2x^2+2x-2}{x^2+1}\,dx$

(s) (1 point) $\displaystyle\int \frac{xe^{2x}}{(1+2x)^2}\,dx$

(t) (1 point) $\displaystyle\int_{\pi/3}^{\pi/2} \frac{dx}{1+\sin x-\cos x}$

(u) (2 points) $\displaystyle\lim_{x\to\infty}\frac{1}{\sqrt{x}}\int_1^x \frac{dt}{\sqrt{t}}$

(v) (2 points) $\displaystyle\frac{d^2}{dx^2}\int_0^x\left(\int_1^{\sin t}\sqrt{1+u^4}\,du\right)dt$

Question6 (0 points)

(a) (1 point (bonus)) Find the indefinite integral

$$\int \frac{\sqrt{x^2-3}}{x^2}\,dx, \qquad \text{where} \quad x>0$$

(b) (1 point (bonus)) Use u-substitution to show

$$\int_{-1}^1 \frac{x^2}{x^2+1}\,dx = \int_1^2 \frac{\sqrt{x-1}}{x}\,dx$$

(c) (1 point (bonus)) Compute the definite integral

$$\int_0^1 \frac{x^2-1}{\ln x}\,dx$$