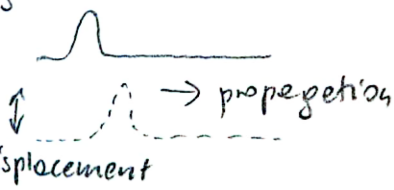
Mechanical waves

Wave – disturbance propagating through space
(mechanical waves need a medium to propagate)

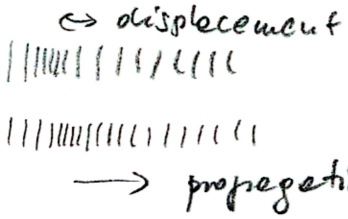
transverse wave
(direction of displacement of medium's particles perpendicular to the direction of wave propagation)

e.g. wave on a cord



longitudinal waves
(direction of displacement parallel to the direction of propagation

e.g. sound



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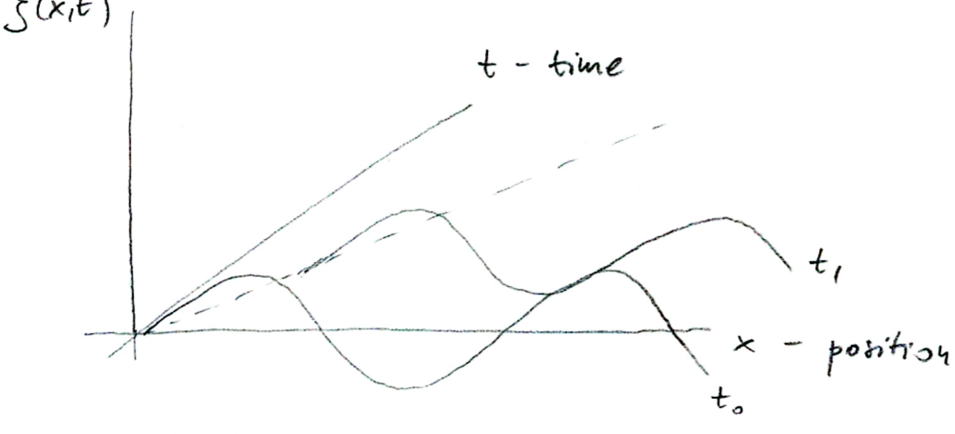
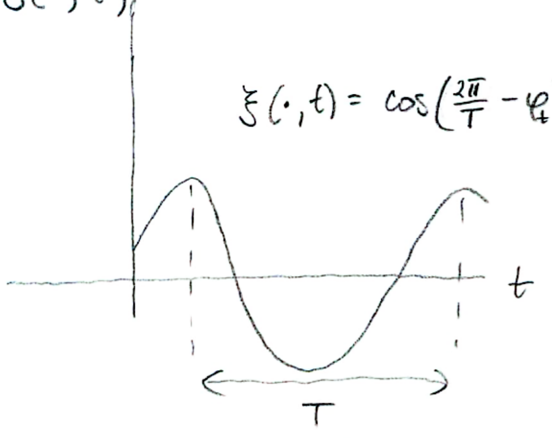
Sinusoidal (harmonic) waves (1D)



$\xi(\cdot,t)$ → fixed point (single particle of medium)

$$\xi(\cdot,t) = \cos\left(\frac{2\pi}{T} - \varphi_t\right)$$



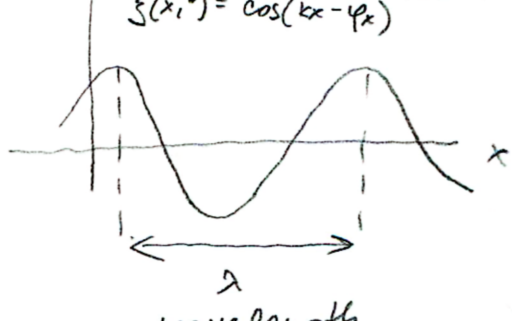
period

particle in harmonic motion about $\xi=0$

$\xi(x,\cdot)$

fixed instant of time "snapshot" of the wave

$$\xi(x,\cdot) = \cos(kx - \varphi_x)$$



wavelength

$v_{ph} = \frac{\lambda}{T}$ – phase speed

Propagating (traveling) harmonic wave

$$\xi(x,t) = \xi_0 \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) =$$
$$= \xi_0 \cos(kx - \omega t)$$

where $k = \frac{2\pi}{\lambda}$ (wave number)

$\omega = \frac{2\pi}{T}$ (angular frequency)

$$\boxed{\xi(x,t) = \xi_0 \cos(kx - \omega t)}$$

harmonic (sinusoidal) wave traveling to the right with phase speed $v_{ph} = \frac{\omega}{k}$

$kx - \omega t = const \Rightarrow k\dot{x} - \omega = 0$

$\dot{x} = \frac{\omega}{k}$

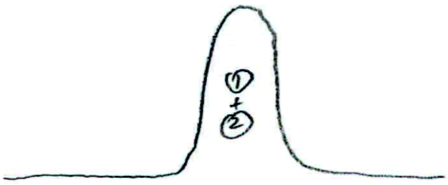
Interference of waves

Idea: two impulses $\xi(x,t) = \xi_1(x,t) + \xi_2(x,t)$

(a)

$t = 0$

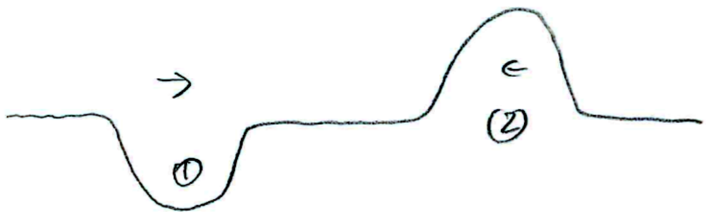
$t = t_1 > 0$



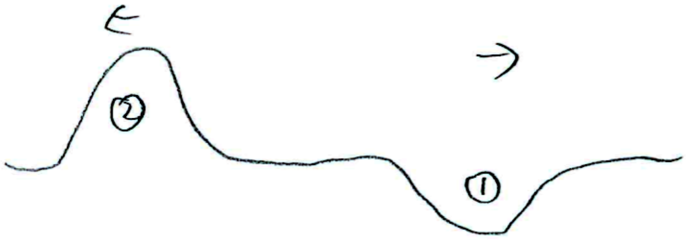
"constructive"

$t = t_2 > t_1$

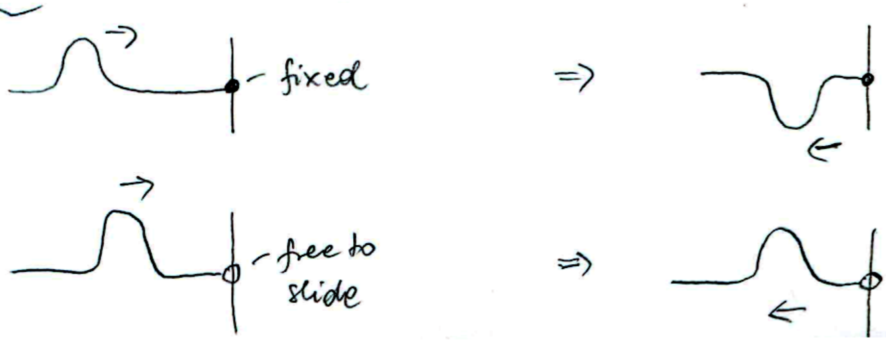
(b)



"destructive"



Digression: reflection of waves – wave pulse on a rope



different boundary conditions!

Interference of sinusoidal waves – special case: standing waves

Suppose: two sinusoidal waves with the same wavelength propagating in opposite directions

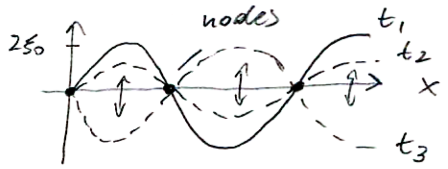
$$\xi_1(x,t) = -\xi_0 \cos(kx + \omega t) \qquad \leftarrow$$
$$\xi_2(x,t) = \xi_0 \cos(kx - \omega t) \qquad \rightarrow$$

Superposition

$$\xi(x,t) = \xi_1(x,t) + \xi_2(x,t) = \xi_0 \left[-\cos(kx + \omega t) + \cos(kx - \omega t) \right] =$$

$$= -2\xi_0 \sin \frac{kx - \cancel{\omega t} + kx + \cancel{\omega t}}{2} \sin \frac{\cancel{kx} - \omega t - \cancel{kx} - \omega t}{2} =$$

$$= 2\xi_0 \underbrace{\sin kx}_{\text{space-dependence}} \underbrace{\sin \omega t}_{\text{time-dependence}}$$



nodes are <u>fixed</u> : $x_{node} = \frac{n\pi}{k} = \lambda \frac{n}{2}$

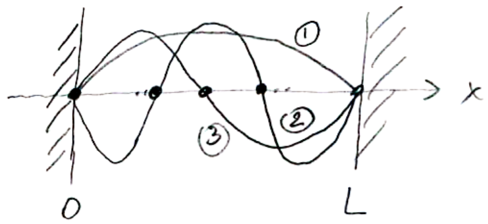
$n = 0, 1, 2, \ldots$

Standing wave

Example Standing wave on a string of length L clamped at both ends.

What are the possible wavelengths?

Boundary conditions: $\xi(0,t) = \xi(L,t) = 0$ for all t



Possible wavelengths:

$L = n \cdot \frac{\lambda}{2}$ (length of the string accomodates multiples of $\frac{\lambda}{2}$)

$\lambda = \frac{2L}{n} = \lambda_n$

1st harmonic: $\lambda_1 = 2L$

2nd harmonic: $\lambda_2 = L$

3rd harmonic: $\lambda_3 = \frac{2}{3}L$ ...