

VE230 Homework 7

2021 Summer

P.7-24 Derive the general wave equations for \mathbf{E} and \mathbf{H} in a nonconducting simple medium where a charge distribution ρ and a current distribution \mathbf{J} exist. Convert the wave equations to Helmholtz's equations for sinusoidal time dependence. Write the general solutions for $\mathbf{E}(\mathbf{R}, t)$ in terms of ρ and \mathbf{J} .

P.7-27 It is known that the electric field intensity of a spherical wave in free space is

$$\mathbf{E} = \mathbf{a}_\theta \frac{E_0}{R} \sin \theta \cos(\omega t - k\mathbf{R}).$$

Determine the magnetic field intensity \mathbf{H} and the value of k .

P.7-29 For a source-free polarized medium where $\rho = 0$, $\mathbf{J} = 0$, $\mu = \mu_0$, but where there is a volume density of polarization \mathbf{P} , a single vector potential π_e may be defined such that

$$\mathbf{H} = j\omega\epsilon_0 \nabla \times \pi_e.$$

- a) Express electric field intensity \mathbf{E} in terms of π_e and \mathbf{P} .
- b) Show that π_e satisfies the nonhomogeneous Helmholtz's equation

$$\nabla^2 \pi_e + k_0^2 \pi_e = -\frac{\mathbf{P}}{\epsilon_0}.$$

The quantity π_e is known as the **electric Hertz potential**.

P.8-7 Show that a plane wave with an instantaneous expression for the electric field

$$\mathbf{E}(z, t) = \mathbf{a}_x E_{10} \sin(\omega t - kz) + \mathbf{a}_y E_{20} \sin(\omega t - kz + \psi).$$

is elliptically polarized. Find the polarization ellipse.

P.8-9 Derive the following general expressions of the attenuation and phase constants for conducting media:

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right]^{\frac{1}{2}} \quad (Np/m)$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right]^{\frac{1}{2}} \quad (rad/m)$$

P.8-14 Assume the ionosphere to be modeled by a plasma region with an electron density that increases with altitude from a low value at the lower boundary toward a value N_{max} and decreases again as the altitude gets higher. A plane electromagnetic wave impinges on the lower boundary at angle θ_i with the normal. Determine the highest frequency of the wave that will be turned back toward the earth. (Hint: Imagine the ionosphere to be stratified into layers of successively decreasing constant permittivities until the layer containing N_{max} . The frequency to be determined corresponds to that for an emerging angle of $\pi/2$.)

P.8-15 Prove the following relations between group velocity u_θ and phase velocity u_p in dispersive medium:

a) $u_\theta = u_p + \beta \frac{du_p}{d\beta}.$

b) $u_\theta = u_p - \lambda \frac{du_p}{d\lambda}.$