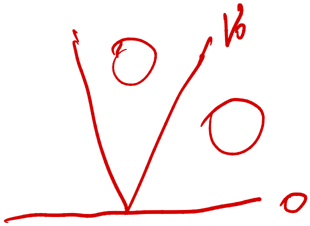
poisson's equation $\nabla^2 V = -\frac{\rho}{\varepsilon}$ when $\rho=0$, $\nabla^2 V = 0$

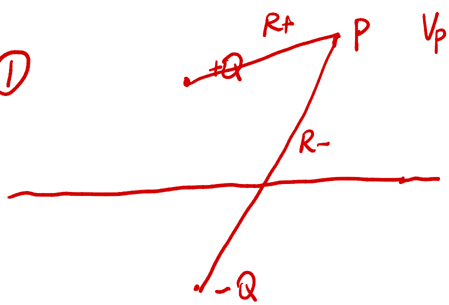
直角 $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

圆柱 $\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$

球 $\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

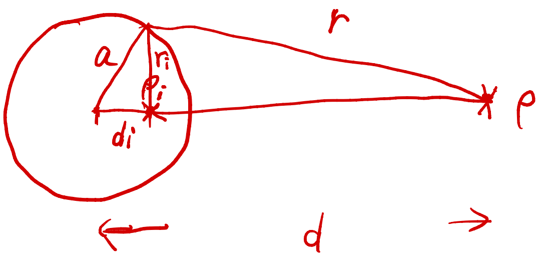


①



$V_P = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$

② 圆柱

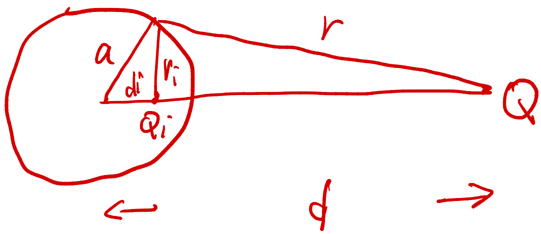


$$\boxed{\rho_i = -\rho}$$

$$d_i d = a^2$$

$$\frac{r_i}{r} = \frac{d_i}{a} = \frac{a}{d}$$

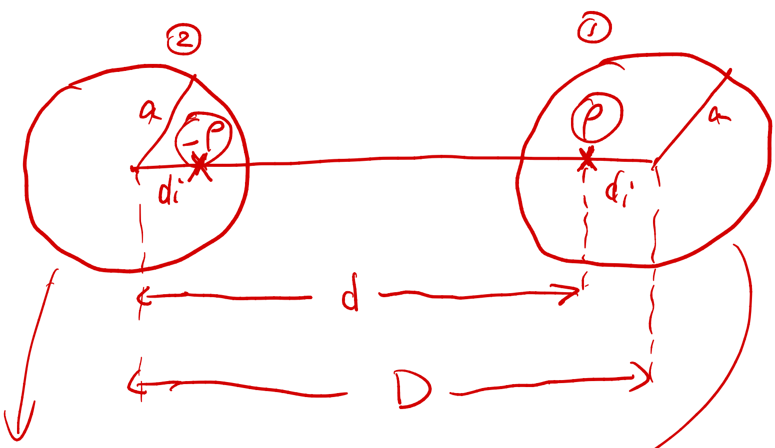
③ 球



$$\boxed{Q_i = -\frac{a}{d} Q}$$

$$d_i = \frac{a^2}{d}$$

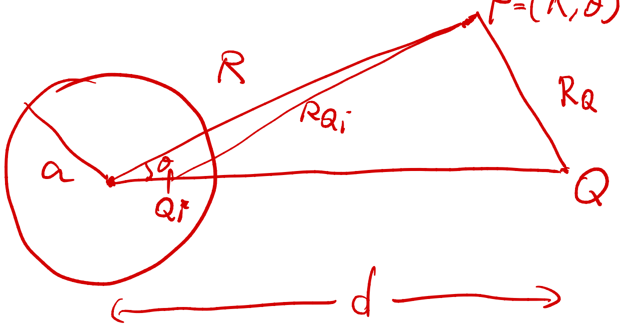
④



$$V_2 = \frac{\rho}{2\pi\varepsilon_0} \ln\frac{a}{d} = -V_1$$

③ extension

let $R = a$



$$V_P = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_Q} - \frac{a}{d R_{Qi}}\right)$$

$$R_Q = \sqrt{R^2 + d^2 - 2Rd\cos\theta}$$

$$R_{Qi} = \sqrt{R^2 + \left(\frac{a^2}{d}\right)^2 - 2R\left(\frac{a^2}{d}\right)\cos\theta}$$