RC_week9

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Antiderivative

- $F'(x) = f(x)$
- A set
- Indefinite integral: set of antiderivatives
- C!!!!
- Properties
- NOTE: the indefinite integral seems to be the same, but they have totally different definitions!

Riemann sum

- Tagged partition
- Norm
- Riemann sum: no limit
- Area: limit → definite integral
- Integrable: limit exists
- Properties

FTC

- Part I: relate finite integral with indefinite integral
- Part II: differentiate definite integral → original function
- Substitution:

Easier way to understand and memorize:

Because $u=g(x)$

So $du/dx=g'(x)$ → $du=g'(x)dx$ (Actually we can't do this)

Mean-value theorem (integrals)

- $f(c) = \frac{1}{a-b}\int_a^b f(x)dx$

If f(c) is continuous on [a,b]

Exercise

- Prove the function

$$f(x) = \begin{cases} 1, x\ is\ rational \\ 0, x\ is\ irrational \end{cases}$$

is not integrable

Exercise 1.

Starting from definition

$$\int_a^b f(x)\cdot dx = \lim_{\Delta x_k \to 0} \sum f(x_k^*)\cdot \Delta x_k$$

(1). if every x_k^* is rational

$\rightarrow$ limit is $b-a$

(2). if every x_k^* is irrational

$\rightarrow$ limit is 0

conflict.

Exercise

- $\int_0^1 \sqrt{x}dx$

Exercise 2.

$$\int_0^1 \sqrt{x}\, dx = \lim_{\Delta x \to 0} \sum_{k=1}^{n} f(x_k^*)\, \Delta x_k$$

Let $x_k = \left(\frac{k}{n}\right)^2$, $k = 0, 1, 2, \cdots, n$.

$x_k^* = \left(\frac{k}{n}\right)^2$

$$\therefore \lim_{\Delta x_k \to 0} \sum_{k=1}^{n} \frac{k}{n} \times \left(\frac{k^2}{n} - \frac{(k-1)^2}{n}\right) \longrightarrow \text{可解}$$

Exercise

- $\int_a^b x^m dx \;\; 0 < \mathrm{a} < \mathrm{b} \text{ and } \mathrm{m} \neq -1$

Exercise 3.

$$\int_a^b x^m \cdot dx$$

$$\xi_k = a q^k, \quad k = 0, 1, 2, \cdots, n$$

$$q = \left(\frac{b}{a}\right)^{\frac{1}{n}}$$

$$\therefore \int_a^b x^m \cdot dx = \lim\left(\sum \xi_k^m \cdot \Delta \xi_k\right)$$

$$= a^{m+1}(q-1)\sum_{i=0}^{n-1} q^{(m+1)i}$$

可解