ECE4880J: Computer Vision July 22, 2022

Homework 6: Graph neural networks

Instructor: Siheng Chen **YOUR NAME**

Instruction

- This homework is due at 11:59:59 p.m. on ** Aug. 1st, 2022.
- The write-up must be a soft copy *.pdf* file edited by LaTeX.
- The overall submission should be a *.pdf* file named by xxx(student id)-xxx(name)-Assignment6.pdf

Q1. Permutation-equivalent property of PointNet [50 points]

This problem revisits PointNet, a fundamental tool for 3D point cloud processing. You will show the permutation-equivalent property of PointNet.

Let $X \in \mathbb{R}^{N\times 3}$ be a 3D point cloud with N points. As we mentioned in the lecture, PointNet works as follows

$$y = \text{PointNet}(X) = \text{maxpool}_1\left(\text{ReLU}\left(\text{ReLU}(XW_1)W_2\right)\right),$$

where $W_1 \in \mathbb{R}^{3\times d_1}$ and $W_2 \in \mathbb{R}^{d_1\times d_2}$ are trainable parameters and $\text{maxpool}_1(\cdot)$ is a maximum pooling operation that downsamples the input along its first dimension by taking the maximum value.

a) What is the dimension of the output y?

b) Let $P \in \{0,1\}^{N\times N}$ be a permutation matrix, where each row and each column P sums up to one; see `https://en.wikipedia.org/wiki/Permutation_matrix`. Let $y' = \text{PointNet}(Px)$ Please show the relationship between y and y' and explain why.

c) Does the training of W_1 and W_2 effect the relationship between y and y'? Please also explain why.

Q2. Oversmoothing issue of GCNs [50 points]

This problem revisits graph convolutional networks (GCNs). You will show the oversmoothing issue of GCNs.

Let $G(\mathcal{V},\mathcal{E})$ be a graph, where $\mathcal{V}$ is the set with N nodes. The graph adjacency matrix is $A \in \mathbb{R}^{N\times N}$, where each element $A_{i,j}$ reflects the connectivity relationship between the ith node and the jth node. Let $X \in \mathbb{R}^{N\times F}$ be a node feature matrix. One graph convolutional layer works as follows

$$Y = \text{GCL}(X) = \text{ReLU}\left(AXW\right),$$

where $W \in \mathbb{R}^{F\times D}$ is a matrix of trainable parameters.

a) What is the dimension of the output Y?

b) Suppose that we remove $\mathrm{ReLU}(\cdot)$ and the trainable matrix W_i in the ith layer is independent, yet has the same dimension. Please write down the most compact form of K layers of graph convolution and explain why. Note that each W_i is trainable and the multiplication of two trainable matrices is also trainable.

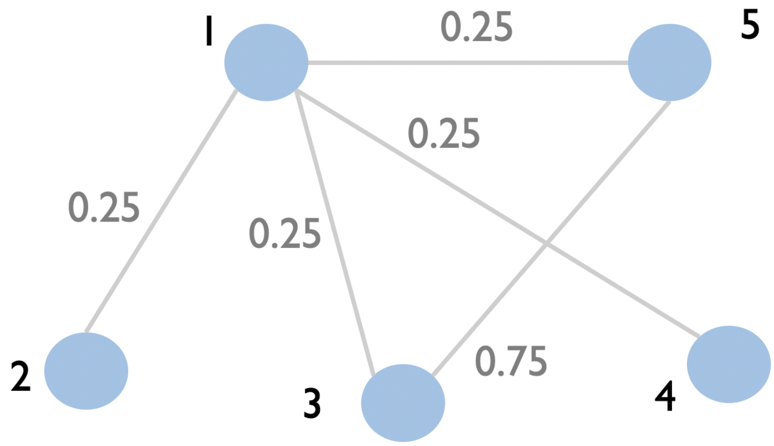

Figure 1: A 5-node graph with undirected, yet weighted edges.

c) Please write down the graph adjacency matrix A in Figure 1.

d) What are A^{10}, A^{100} and A^{1000}? What is the trend you observe? Also please explain why.

e) Let $x_1 = [1000, 0, 0, 0, 0]^T$ and $x_2 = [0, 0, 0, 1000, 0]^T$. What are $A^{1000}x_1$ and $A^{1000}x_2$? What is the trend you observe? Also please explain why.

Gaussian Mixture Model using 2D Multivariate Gaussians each of as N components: Cholesky decompesition and affine transformation for sampling: