

Question1 (1 points)

Find the point on the parabola

$$x = t, \quad y = t^2 \quad \text{for} \quad -\infty < t < \infty$$

closest to the point $(2, 1/2)$.

Question2 (1 points)

Find the volume swept out by revolving the region bounded by one arch of the curve

$$x = t - \sin t, \quad y = 1 - \cos t$$

and the x -axis about the x -axis.

Question3 (1 points)

Find the length of the curve

$$x = 8 \cos t + 8t \sin t, \quad y = 8 \sin t - 8t \cos t, \quad 0 \leq t \leq \pi/2$$

Question4 (1 points)

Find the coordinates of the centroid of the curve

$$x = \cos t, \quad y = t + \sin t, \quad 0 \leq t \leq \pi$$

Question5 (1 points)

Find the area of the region bounded by the spiral

$$r = \theta \quad \text{for} \quad 0 \leq \theta \leq \pi$$

Question6 (1 points)

Find the area of the region shared by two curves defined by polar equations

$$r = 2 \quad \text{and} \quad r = 2(1 - \cos \theta).$$

Question7 (3 points)

Evaluate the integral. Show all your workings.

(a) (1 point) $\int_0^\infty \frac{dv}{(1+v^2)(1+\arctan v)}$

(b) (1 point) $\int_{-1}^4 \frac{dx}{\sqrt{|x|}}$

(c) (1 point) $\int_{-1}^\infty \frac{dx}{x^2 + 5x + 6}$

Question8 (2 points)

Testing for Convergence

(a) (1 point)

$$\int_0^{\pi/2} \tan \theta \, d\theta$$

(b) (1 point)

$$\int_1^{\infty} \frac{dx}{x \left(\sqrt{\ln(x)} + \ln^2(x) \right)}$$

Question9 (1 points)

Suppose the following improper integral is convergent for all $x > 0$.

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt, \quad x > 0$$

Prove that if n is a natural number, then

$$\Gamma(n+1) = n!$$

Question10 (1 points)

Find the values of p for which each integral converges.

$$\int_1^2 \frac{dx}{x(\ln x)^p}$$

Question11 (1 points)

Let

$$S(x) = \int_0^x |\cos t| dt$$

Find

$$\lim_{x \rightarrow \infty} \frac{S(x)}{x}$$

Question12 (1 points)

Find

$$\int_0^{\infty} \frac{dx}{(1+x^2)^n}$$

where n is a positive integer.

Question13 (1 points)

Let

$$x = 0.9999 \dots$$

Determine whether $x < 1$, $x = 1$ or $x > 1$. Justify your answer.

Question14 (3 points)

(a) (1 point) Determine whether the series with partial sum $s_n = \frac{n}{3n-1}$ is convergent.

(b) (1 point) Determine whether the series $\sum_n \frac{n}{3n-1}$ is convergent.

(c) (1 point) Find all values of x for which the series converges, and what it converges to.

$$1 - e^{-x} + e^{-2x} - e^{-3x} + e^{-4x} - e^{-5x} + e^{-6x} - \dots$$

Question15 (2 points)

Determine whether the series converges. If so, find the sum.

(a) (1 point) $\ln \frac{1}{2} + \ln \frac{2}{3} + \ln \frac{3}{4} + \cdots + \ln \frac{k}{k+1} + \cdots$

(b) (1 point) $\ln \left(1 - \frac{1}{4}\right) + \ln \left(1 - \frac{1}{9}\right) + \ln \left(1 - \frac{1}{16}\right) + \cdots + \ln \left(1 - \frac{1}{(k+1)^2}\right) + \cdots$

Question16 (6 points)

Use appropriate tests or theorems to determine convergence or divergence.

(a) (1 point) $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$

(b) (1 point) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

(c) (1 point) $\sum_{n=1}^{\infty} \sqrt{\frac{n+3}{n^4+4}}$

(d) (1 point) $\sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!3^{2n}}$

(e) (1 point) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$

(f) (1 point) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

Question17 (3 points)

Prove the Ratio test.

$$\begin{aligned} \text{If } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= L < 1 \\ &\implies \text{Absolutely convergent} \\ &\implies \text{Convergent} \\ \text{If } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= L > 1 \\ &\implies \text{divergent} \\ \text{If } \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= 1 \\ &\implies \text{Inconclusive} \end{aligned}$$

Question18 (0 points)

(a) (10 points (bonus)) Complete the course evaluation for Vv156 before Tuesday 7th.

Link: [Vv156](#)

I will be able to tell whether you have submitted, but the content is surely anonymous!