10 Traffic Network Control

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Recap

- Smart highways
  - Sensing technology
  - Control technology
- Traffic flow model
  - Flow-density relation
  - Cell transmission model
- Ramp metering
  - Flow stabilization
  - Throughput maximization
  - Delay minimization

Outline

- Network systems
  - Transportation
  - Electricity
  - Communications
- Min-cost flow problem
  - Formulation
  - Solution
- Further applications
  - Shortest path problem
  - Max flow problem

Transportation: Planning

- Transportation planning is the process of defining future policies, goals, investments, and spatial planning designs to prepare for future needs to move people and goods to destinations.
  - Should we build a bridge here?
  - Should we build a shopping mall here?
  - Should we build subway here?
- To do the planning, we need to predict and optimize the consequent traffic flow.



Transportation: Traffic control

- During morning/evening peak hours, traffic managers may need to intervene people's route choice.
  - Navigation tool
  - Traveler information
  - Congestion pricing
- Objective: distribute traffic flow to minimize congestion

Transportation: Logistics

- Logistic companies receive packages at stations.
- Packages flow between stations.
- Limited/costly resource for package transportation.
  - How to allocate delivery trucks?
  - How to allocate station capacities?
  - Road or rail or air?

Electricity distribution

- Power stations generate electricity
- Transformers adjust voltage
- Distribution lines transmit power
- Consumers receive power
- Flow of power/current



Communications network

- Flow of data packets between terminals
- Cabled or wireless connectivity
- Routers distribute data flow between terminals
- Latency, bandwidth

Min-cost flow problem

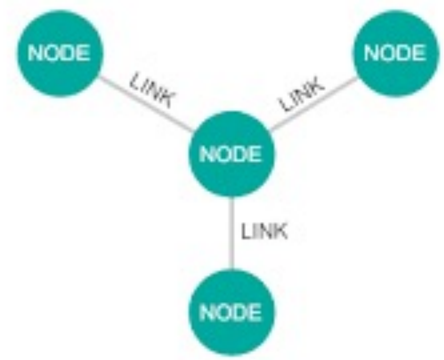
- Formulation
- Solution

Min-cost flow problem

- Data:
  - Demand
  - Cost function
  - Capacity
- Decision variables:
  - Link flows
- Constraints:
  - Mass conservation
  - Link capacity
- Objective:
  - Minimize total flow cost

Network model

- Consider a network with nodes N and links E
  - Network also called graph
  - Nodes also called vertices (singular: vertex)
  - Links also called edges/arcs
- We use integers to label nodes
  - Node 1, 2,…
- We use pairs of integers to label links
  - Link (1,2), (2,3),…
- Directed link (i,j)
- Undirected link (i,j)



Demand & flow

- Each node i is associated with demand b_i
  - $b_i > 0$: traffic flows in (origin)
  - $b_i < 0$: traffic flows out (destination)
  - $b_i = 0$: traffic flows through (transmission)
- Each link (i, j) is associated with flow f_{ij}
  - $f_{ij} > 0$: flow from i to j
  - $f_{ij} < 0$: flow from j to i
- Mass conservation: inflow = outflow

$$b_i + \sum_{j \in In(i)} f_{ji} = \sum_{j \in Out(i)} f_{ij}$$

Cost: linear

- When we assign flows, some patterns are better than the others.
- Let's start with the simplest case: linear cost
- Suppose that link (i,j) has a flow of f_{ij}
- Then, the cost on link (i,j) is $c_{ij}f_{ij}$, where $c_{ij} > 0$ is the cost per unit flow
- This is a rather simplistic model that ignores any congestion effect.

Cost: nonlinear

- Let $C_{ij}(f_{ij})$ be the cost function

$$C_{ij}(f_{ij}) = \frac{c_{ij}}{\bar{f}_{ij} - f_{ij}}$$

- $\bar{f}_{ij}$ = link capacity
- $C_{ij}(f_{ij}) \rightarrow \infty$ as $f_{ij} \rightarrow \bar{f}_{ij}$
- This is a more realistic cost function, since high inflow leads to congestion and deterioration of travel time

Capacity

- Sometimes we impose an upper bound on flows
$$f_{ij} \leq \bar{f}_{ij}$$
- Capacity $\bar{f}_{ij}$ depends on
  - Road: # of lanes, type of surface, weather
  - Electricity: transmission link capacity
  - Communications: bandwidth
- $\bar{f}_{ij} < \infty$: capacitated problem (harder)
- $\bar{f}_{ij} = \infty$: uncapacitated problem (easier)

Math: linear cost

- min $\quad \sum_{(i,j)\in E} c_{ij} f_{ij}$
- s.t. $\quad b_i + \sum_{j\in In(i)} f_{ji} = \sum_{j\in Out(i)} f_{ij}$ for all $i \in N$

  $0 \le f_{ij} \le \bar{f}_{ij}$ for all $(i,j) \in E$
- Linear programming!
- Can be solved very efficiently
- If $\bar{f}_{ij} = \infty$ for each link (i,j), i.e. if the problem is uncapacitated, we simply allocate demands to the "nearest" destinations. #

Solution: linear cost*

- For uncapacitated problems, min-cost flow problem is essentially finding the shortest paths between origins and destinations.
- Thus, all flows concentrate on the shortest paths.
- For capacitated problems, there is interaction between multiple origin-destination (OD) pairs.
- More sophisticated algorithms are needed, e.g. network simplex method.

Math: nonlinear cost

- min $\quad \sum_{(i,j)\in E} C_{ij}(f_{ij})$
- s.t. $\quad b_i + \sum_{j\in In(i)} f_{ji} = \sum_{j\in Out(i)} f_{ij}$ for all $i \in N$

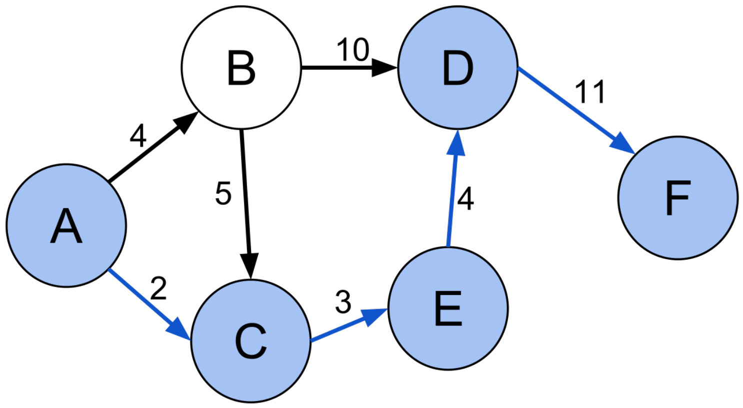
  $0 \le f_{ij} \le \bar{f}_{ij}$ for all $(i,j) \in E$
- Nonlinear optimization problem
- But typically convex
- Requires nonlinear solver
- A simple illustration: two parallel routes #

Further applications

- Shortest path problem
- Max flow problem

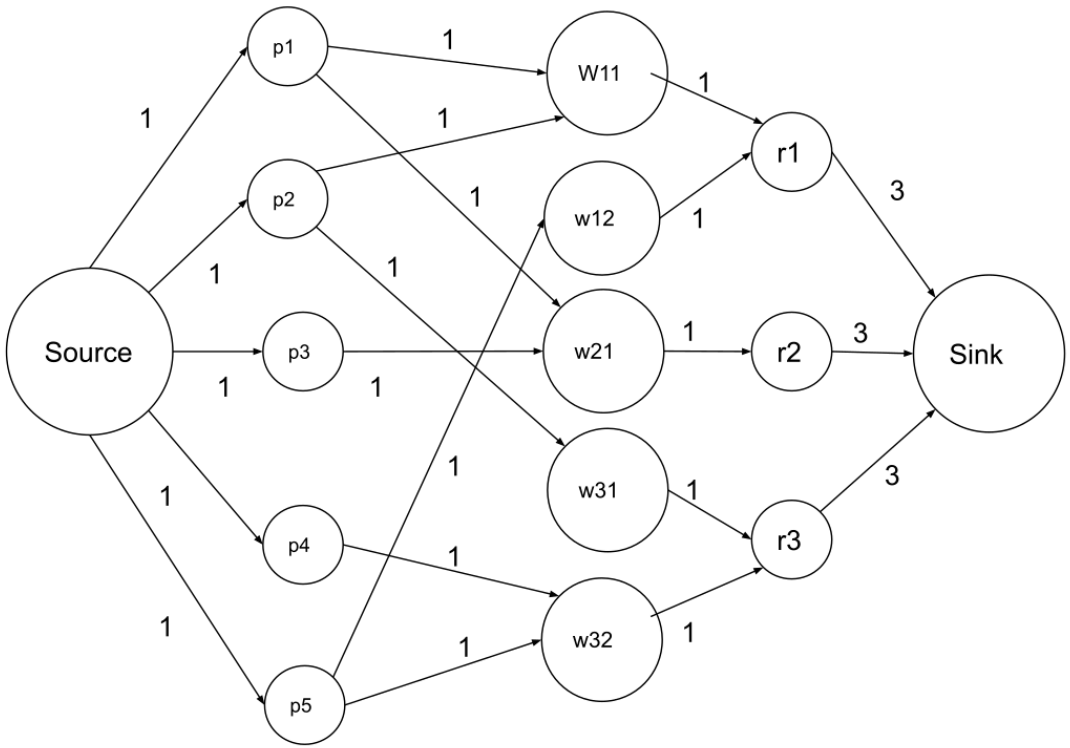
Shortest path problem

- Shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized.
- Can be formulated as a min-cost flow problem with unit demand and unit cost



Max flow problem

- Finding a feasible flow through a flow network that obtains the maximum possible flow rate.
- Single origin, single destination



Summary

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  - Electricity
  - Communications
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  - Formulation
  - Solution
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Next time

- Quiz 1