Shanghai Jiao Tong University ECE4530J

Homework 2: Due 2021.6.7

Instructions

1. Please clearly label the problem numbers in your response.
2. Attach any codes at the end of your response.
3. If you hand-write, please keep your response neat and readable.

Problem 1: Speed tracking

Coordination of 10 autonomous vehicles at a centralized intersection.

Part a: Randomly generate initial condition. That is, randomly generate two vectors

$$x^1(0) = [x_1^1(0), \dots, x_5^1(0)]^T$$
$$x^2(0) = [x_1^2(0), \dots, x_5^2(0)]^T$$

$x_i^k(0)$ takes values in 0-100 m; $x_i^k = 100$ means vehicle i has left the intersection.

Part b: We consider vehicles as points. The safe spacing between two vehicles is 6 m. Is your initial condition safe? If not, modify the initial condition so that the safe spacing constraint is satisfied.

Part c: Our decision variables are the speeds at various times. Suppose that the one-step fuel cost induced on vehicle i is given by $\left(v_i^k(t)\right)^2$. Our objective is to discharge all vehicles with the minimal fuel consumption. Formulate the trajectory planning problem in the four-stage representation, viz. data, decision variables, constraints, and objective function. Clearly define each notation you use.

Part d: Assume zero initial speeds, i.e., $v_i^k(0) = 0$ for each i and each k. Suppose maximal speed $\bar{v} = 10\,m/s$ and maximal acceleration (i.e., speed increment) is $5\,m/s^2$. Construct a feasible solution. Report the total fuel consumption.

Part e: Let T be the time at which all 10 vehicles are discharged by the intersection. Provide an upper bound on T. Your bound does not have to be tight.

Problem 2: Optimization & sensitivity analysis

Part a: Code the optimization problem in MATLAB/Python/C. Note that you can use your result in problem 1e to define the dimension of the decision variables.

- https://cvxopt.org/examples/tutorial/qp.html
- https://www.mathworks.com/help/optim/ug/quadprog.html

Part b: Compute the optimal solution and report the fuel consumption. Compare with your result in problem 1.

Part c: Gradually change the value for the safe spacing and plot the corresponding optimal fuel cost. You plot should be fuel cost vs. safe spacing.

Problem 3: Impact of noise

Part a: Add a noise term to the system dynamics, i.e.

$$x_i^k(t+1) = x_i^k(t) + v_i^k(t)\Delta t + \epsilon.$$

Then, implement the speed commands you generated in problem 2b.

Part b: Two vehicles i and j are said to interfere if $\left|x_i^k(t) - x_j^\ell(t)\right| < 6\,m$ and if $x_i^k, x_j^k < 100$. (Recall that "Manhattan distance" is used for vehicles on different orbits.) Let $N(t)$ be the number of interferences at time t. Plot $N(t)$ vs. t.