

VE230 Homework 1

2021 Summer

1.EXAMPLE 2-3 Prove the back-cab rule of vector triple product.

Back-cab rule:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

2.EXAMPLE 2-4 Given three vectors \mathbf{A} , \mathbf{B} , and \mathbf{C} , obtain the expressions of (a) $\mathbf{A} \cdot \mathbf{B}$, (b) $\mathbf{A} \times \mathbf{B}$, and (c) $\mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$ in the orthogonal curvilinear coordinate system (u_1, u_2, u_3) , and demonstrate the expressions explicitly in the cartesian, cylindrical and spherical coordinates.**3.EXAMPLE 2-19** Given $\mathbf{A} = \mathbf{a}_x x^2 + \mathbf{a}_y xy + \mathbf{a}_z yz$, verify the divergence theorem over a cube one unit on each side. The cube is situated in the first octant of the Cartesian coordinate system with one corner at the origin.**4.EXAMPLE 2-21** Show that $\nabla \times \mathbf{A} = 0$ ifa) $\mathbf{A} = \mathbf{a}_\phi(k/r)$ in cylindrical coordinates, where k is a constant, orb) $\mathbf{A} = \mathbf{a}_R f(R)$ in spherical coordinates, where $f(R)$ is any function of the radial distance R .**5.** Prove null identities in curvilinear coordinates.

Null identity I:

$$\nabla \times (\nabla V) \equiv 0$$

Null identity II:

$$\nabla \cdot (\nabla \times \mathbf{A}) \equiv 0$$

6.R 2-21 State the divergence theorem in words.**7.R 2-25** State Stokes's theorem in words.**8.R 2-27** State Helmholtz's theorem in words.