JOINT INSTITUTE
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vv214_SU2020_Open Book Quiz 1 **2020/05/22**

Problem 1

Write out the definition of a linear space and show that additive inverse condition can be replaced with the condition

$$0_{\mathbb{F}} * v = 0_V \quad \forall v \in V$$

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In $\mathbb{F}$: $1+(-1)=0_{\mathbb{F}}$
$\Rightarrow \forall v \in V$ $(1+(-1)) * v = 0_{\mathbb{F}} * v = 0_V$
On another hand,
$(1+(-1)) * v = 1 * v + (-1) * v = v + (-1) * v$
So, $v + (-1) * v = 0_V$
$\Rightarrow (-1) * v$ is the additive inverse $\forall v \in V$

Problem 2

M_1, M_2

Give an example of a nonempty subset M of $\mathbb{R}^2$ that IS NOT a subspace of $\mathbb{R}^2$ and

1. M_1 is closed under addition and under taking additive inverses. 10
2. M_2 is closed under scalar multiplication. 10

$M_1 = \{(x_1, x_2) \in \mathbb{Z}^2\}$
$M_2 = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \cdot x_2 = 0\}$

Problem 3

$$A = \begin{pmatrix} 2 & -4 & -8 & 6 & 3 \\ 0 & 1 & 3 & 2 & 3 \\ 3 & -2 & 0 & 0 & 8 \end{pmatrix}$$

1. Find $rref(A)$ and $rank(A)$. Show your work. 10
2. Represent columns of $rref(A)$ without pivots as linear combinations of pivot columns. 5

$\begin{pmatrix}2\\3\\0\end{pmatrix} = 2\begin{pmatrix}1\\0\\0\end{pmatrix} + 3\begin{pmatrix}0\\1\\0\end{pmatrix} + 0\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} \Rightarrow \begin{pmatrix}-8\\3\\0\end{pmatrix} = 2\begin{pmatrix}2\\0\\3\end{pmatrix} + 3\begin{pmatrix}-4\\1\\-2\end{pmatrix}$; $\begin{pmatrix}4\\2\\1/2\end{pmatrix} = 4\begin{pmatrix}1\\0\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\\0\end{pmatrix} + 1/2\begin{pmatrix}0\\0\\1\end{pmatrix}$

3. Let data be stored in columns of A. What conclusion would you make from the previous result? 5

The non-pivot columns of A represent redundant information that can always be expressed in terms of the data in the pivot columns

$$A \sim \begin{pmatrix} \boxed{1} & -2 & -4 & 3 & 3/2 \\ 0 & 1 & 3 & 2 & 3 \\ 3 & -2 & 0 & 0 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & -4 & 3 & 3/2 \\ 0 & \boxed{1} & 3 & 2 & 3 \\ 0 & 4 & 12 & -9 & 7/2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 & 7 & 15/2 \\ 0 & 1 & 3 & 2 & 3 \\ 0 & 0 & 0 & \boxed{-17} & -17/2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 & 7 & 15/2 \\ 0 & 1 & 3 & 2 & 3 \\ 0 & 0 & 0 & \boxed{1} & 1/2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 & 0 & 4 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1/2 \end{pmatrix}$$

$rank(A) = 3$

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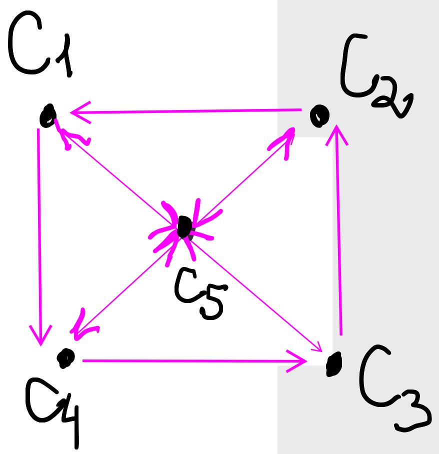
Problem 4

Let data centers located at the places, say C_1, C_2, C_3, C_4, C_5; be connected as shown:



$$A = \begin{array}{c|ccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \hline C_1 & 0 & 0 & 0 & 1 & 1 \\ C_2 & 1 & 0 & 0 & 0 & 1 \\ C_3 & 0 & 1 & 0 & 0 & 1 \\ C_4 & 0 & 0 & 1 & 0 & 1 \\ C_5 & 1 & 1 & 1 & 1 & 0 \end{array}$$

1. Write out the adjacency matrix $A = (a_{ij})$ with $a_{ij} = \begin{cases} 1 & if\ C_i is\ linked\ to\ C_j \\ 0 & otherwise \end{cases}$; $a_{ii} = 0$ and find $A^2, A^3, A + A^2 + A^3$.

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2. How would you interpret the elements $(A^3)_{25}, (A^4)_{34}, (A + A^2 + A^3)_{23}$?

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$= 4$ - the number of links $C_2 \to C_3$ with no more than 3 steps

$(A^3)_{25} = 5 \Rightarrow$ there are 5 3-step links from C_2 to C_5

Problem 5

Find a solution to the system. Show your work.

A linear system consistent
$\operatorname{rank}(A) = \operatorname{rank}(A|\vec{b})$

$$\begin{aligned} x_1 + 2x_2 + x_3 + 2x_4 &= 3 \\ 2x_1 + 4x_2 + x_3 + 3x_4 &= 4, \\ 3x_1 + 6x_2 + x_3 + 4x_4 &= 5 \end{aligned}$$

$$(A|\vec{b}) = \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 1 & 3 & 4 \\ 3 & 6 & 1 & 4 & 5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 2 & 3 \\ 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & -2 & -2 & -4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

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$\operatorname{rank}(A) = 2 = \operatorname{rank}(A|\vec{b})$

$x_1 + 2x_2 + x_4 = 1 \Rightarrow x_1 = 1 - 2x_2 - x_4$

$x_3 + x_4 = 2 \Rightarrow x_3 = 2 - x_4$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$$

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