VE215 Final Review

VE215 TA Group

SJTU Joint Institute

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2. Maximum Average Power Transfer
3. Effective or RMS Value
4. Apparent Power and Power Factor
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25. Passive Filters
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Instantaneous Power

Instantaneous Power

- $p(t) = v(t)i(t)$
- $p(t) = V_m cos(\omega t + \theta_v) I_m cos(\omega t + \theta_i)$
  $= V_m I_m cos(\omega t + \theta_v) cos(\omega t + \theta_i)$
  $= \frac{1}{2} V_m I_m cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m cos(2\omega t + \theta_v + \theta_i)$

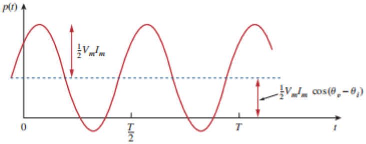


Figure 11.2
The instantaneous power $p(t)$ entering a circuit.

Average Power

- $P_{avg} = \frac{1}{T}\int_0^T p(t)dt$
  $= \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m cos(\theta_v - \theta_i)dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m cos(2\omega t + \theta_v + \theta_i)dt$
- $\int_0^T \frac{1}{2}V_m I_m cos(2\omega t + \theta_v + \theta_i)dt = 0$
- $P_{avg} = \frac{1}{2}V_m I_m cos(\theta_v - \theta_i)$

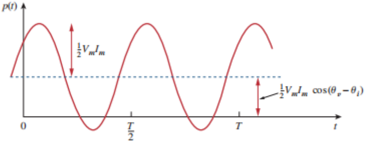


Figure 11.2
The instantaneous power $p(t)$ entering a circuit.

- $P_{avg} = \frac{1}{2}V_m I_m cos(\theta_v - \theta_i)$
  $= \frac{1}{2}Re(V_m I_m e^{j(\theta_v - \theta_i)})$
  $= \frac{1}{2}Re(V_m e^{j\theta_v} I_m e^{(} - j\theta_i))$
  $= \frac{1}{2}Re(\tilde{V}\tilde{I}^*)$

Average Power

- When $\theta_v = \theta_i, P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}\frac{V_m^2}{R}$, showing that a purely resistive load absorbs power at all times.
- When $\theta_v - \theta_i = \pm 90°, P = \frac{1}{2}V_m I_m cos(\pm 90°) = 0$, showing that a purely reactive load absorbs no average power.
- For Z=R+jX,
  $P = \frac{1}{2}Re(\tilde{V}\tilde{I}^*) = \frac{1}{2}Re(\tilde{I}(R + jX)\tilde{I}^*) = \frac{1}{2}Re(I^2R + jI^2X) = \frac{1}{2}I^2R$
  showing that only resistance contributes to the average power.

Maximum Average Power Transfer

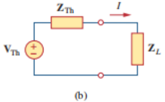


Figure 11.7
Finding the maximum average power transfer: (a) circuit with a load, (b) the Thevenin equivalent.

- $P = \frac{1}{2}I^2R_L = \frac{1}{2}\frac{V_{Th}^2R_L}{(R_{Th}+R_L)^2+(X_{Th}+X_L)^2}$
- If the load is not purely resistive($X_L \neq 0$), when $R_L = R_{Th}, X_L = -X_{Th}$, $P_{max} = \frac{V_{Th}^2}{8R_{Th}}$
- If the load is purely resistive, when $R_L = \sqrt{R_{Th}^2 + X_{Th}^2}$, $P_{max} = I^2R_L$ where $I = \frac{V_{Th}}{Z_{Th}+R_L}$

Maximum Average Power Transfer

Example 11.6

In the circuit in Fig. 11.11, find the value of R_L that will absorb the maximum average power. Calculate that power.

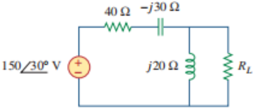


Figure 11.11
For Example 11.6.

Solution:
We first find the Thevenin equivalent at the terminals of R_L.

$$\mathbf{Z}_{\text{Th}} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

By voltage division,

$$\mathbf{V}_{\text{Th}} = \frac{j20}{j20 + 40 - j30}(150\underline{/30^\circ}) = 72.76\underline{/134^\circ}\ \text{V}$$

The value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_{\text{Th}}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + R_L} = \frac{72.76\underline{/134^\circ}}{33.66 + j22.35} = 1.8\underline{/100.42^\circ}\ \text{A}$$

The maximum average power absorbed by R_L is

$$P_{\max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29\ \text{W}$$

Effective or RMS Value

- The effective value of an ac current i is the dc current I_{eff} that delivers the same average powere to a resistor as the ac current.
- $P = \frac{1}{T}\int_0^T i^2 R dt = (\frac{1}{T}\int_0^T i^2 dt)R = I_{eff}^2 R$
  $I_{eff} = I_{rms}, V_{eff} = V_{rms}$
- For the sinusoid $i(t) = I_m cos\omega t$, $I_{rms} = \frac{I_m}{\sqrt{2}}$

Apparent Power and Power Factor

- $P = V_{rms} I_{rms} cos(\theta_v - \theta_i)$
- $S = V_{rms} I_{rms}$ is the apparent power. $unit : V \cdot A$
- Power Factor, $pf = cos(\theta_v - \theta_i) = \frac{P}{|S|}$
- $Z = \frac{\tilde{V}}{\tilde{I}} = \frac{V_m}{I_m} \angle(\theta_v - \theta_i)$
- $\theta_v - \theta_i < 0$, pf is leading because the current is leading the voltage.
  $\theta_v - \theta_i > 0$, pf is lagging.

Example 11.9

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10^\circ)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20^\circ)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Solution:

The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20^\circ - 10^\circ) = 0.866 \qquad \text{(leading)}$$

The pf is leading because the current leads the voltage. The pf may also be obtained from the load impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle{-20^\circ}}{4\angle{10^\circ}} = 30\angle{-30^\circ} = 25.98 - j15\ \Omega$$

$$\text{pf} = \cos(-30^\circ) = 0.866 \qquad \text{(leading)}$$

The load impedance $\mathbf{Z}$ can be modeled by a 25.98-Ω resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C}$$

or

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

Complex Power

- $S = \frac{1}{2}\tilde{V}\tilde{I}^* = \tilde{V_{rms}}\tilde{I_{rms}^*} = V_{rms}I_{rms}\angle(\theta_v - \theta_i) = |S|\angle(\theta_v - \theta_i) = V_{rms}I_{rms}cos(\theta_v - \theta_i) + jV_{rms}I_{rms}sin(\theta_v - \theta_i) = Re(S) + jIm(S) = P + jQ$
- P is the average or real power in watts(W).
  $P = Re(S) = I_{rms}^2 R = |S|\,cos(\theta_v - \theta_i)$ Pay attention that P$\neq V_{rms}^2/R$
- Q is the reactive or quadrature power in $volt - amperes\ reactive$(VAR).
  $Q = Im(S) = I_{rms}^2 X = |S|\,sin(\theta_v - \theta_i)$ Pay attention that Q$\neq V_{rms}^2/X$

About Q

- Q represents a lossless power exchange between the load and the source.
- Q=0 for resistive loads.
- Q$<$ 0 for capacitive loads.
- Q$>$ 0 for inductive loads.

Power Factor Correction

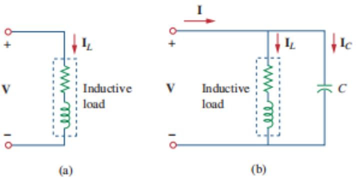


Figure 11.27
Power factor correction: (a) original inductive load, (b) inductive load with improved power factor.

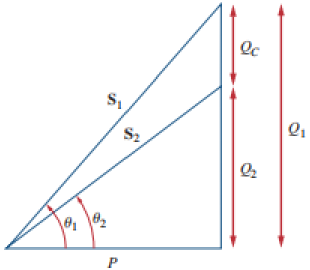


Figure 11.29
Power triangle illustrating power factor correction.

$$C = \frac{P(tan\theta_1 - tan\theta_2)}{\omega V^2}$$

Power Factor Correction

11.73 A 240-V rms 60-Hz supply serves a load that is 10 kW (resistive), 15 kVAR (capacitive), and 22 kVAR (inductive). Find:

(a) the apparent power

(b) the current drawn from the supply

(c) the kVAR rating and capacitance required to improve the power factor to 0.96 lagging

(d) the current drawn from the supply under the new power-factor conditions

Power Factor Correction

Chapter 11, Solution 73.

(a) $\mathbf{S} = 10 - j15 + j22 = 10 + j7$ kVA

$S = |\mathbf{S}| = \sqrt{10^2 + 7^2} = \underline{\mathbf{12.21\ kVA}}$

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* \longrightarrow \mathbf{I}^* = \dfrac{\mathbf{S}}{\mathbf{V}} = \dfrac{10{,}000 + j7{,}000}{240}$

$\mathbf{I} = 41.667 - j29.167 = \underline{\mathbf{50.86\angle{-35°}\ A}}$

(c) $\theta_1 = \tan^{-1}\left(\dfrac{7}{10}\right) = 35°, \qquad \theta_2 = \cos^{-1}(0.96) = 16.26°$

$Q_c = P_1[\tan\theta_1 - \tan\theta_2] = 10[\tan(35°) - \tan(16.26°)]$

$Q_c = \underline{\mathbf{4.083\ kVAR}}$

$C = \dfrac{Q_c}{\omega V_{rms}^2} = \dfrac{4083}{(2\pi)(60)(240)^2} = \underline{\mathbf{188.03\ \mu F}}$

(d) $\mathbf{S}_2 = P_2 + jQ_2, \qquad P_2 = P_1 = 10$ kW

$Q_2 = Q_1 - Q_c = 7 - 4.083 = 2.917$ kVAR

$\mathbf{S}_2 = 10 + j2.917$ kVA

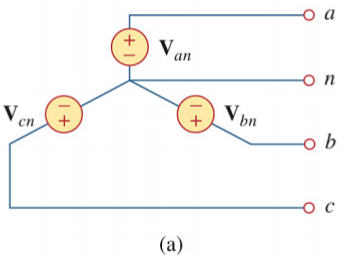
But $\mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^*$

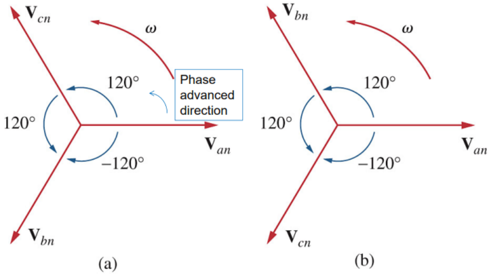
$\mathbf{I}_2^* = \dfrac{\mathbf{S}_2}{\mathbf{V}} = \dfrac{10{,}000 + j2917}{240}$

$\mathbf{I}_2 = 41.667 - j12.154 = \underline{\mathbf{43.4\angle{-16.26°}\ A}}$

Balanced Three-Phase Voltages

Phase Sequence

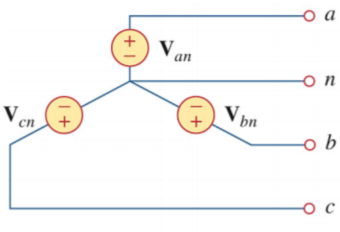




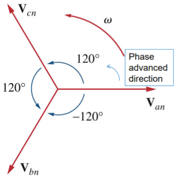
Phase Sequence

- (a). abc or positive sequence. (a leads b, b leads c)
- (b). acb or negative sequence.

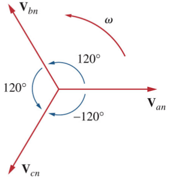
Balanced Three-Phase Voltages



(a)



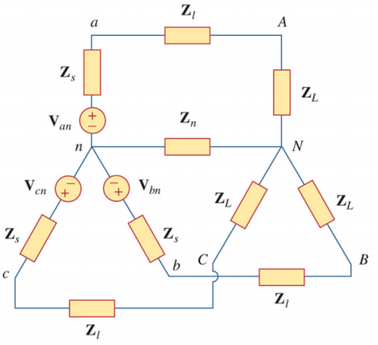
(a)

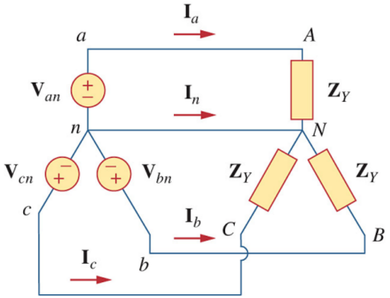


(b)

- abc sequence:
  $\tilde{V_{an}} = V_p\angle 0^\circ, \tilde{V_{bn}} = V_p\angle -120^\circ, \tilde{V_{an}} = V_p\angle -240^\circ = V_p\angle 120^\circ$,
  where V_p is the effective or rms value of the phase voltages.
- acb sequence:
  $\tilde{V_{an}} = V_p\angle 0^\circ, \tilde{V_{bn}} = V_p\angle -240^\circ = V_p\angle 120^\circ, \tilde{V_{cn}} = V_p\angle -120^\circ$

Balanced Wye-Wye Connection





- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balancd Y-connected load.

Balanced Wye-Wye Connection

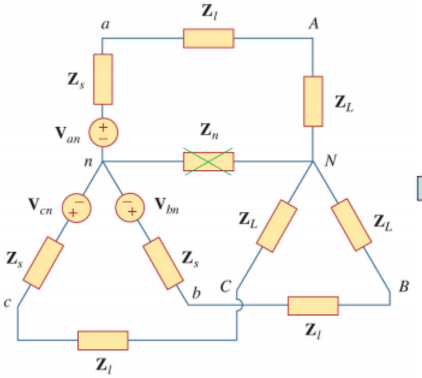


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

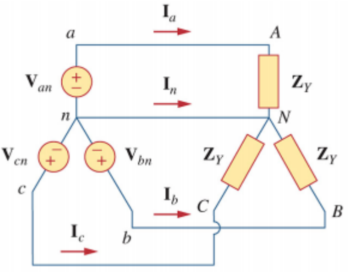


Figure 12.10 Balanced Y-Y connection.

$$\boxed{Z_Y = Z_S + Z_l + Z_L}$$

- If balanced, the system can be simplified.

Balanced Wye-Wye Connection

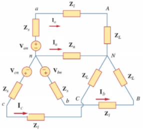


Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

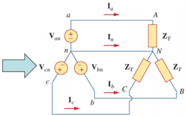


Figure 12.10 Balanced Y-Y connection.

$$\tilde{V}_{an} = \tilde{I}_a(Z_s + Z_l + Z_L) - \tilde{I}_n Z_n$$

$$\tilde{V}_{bn} = \tilde{I}_b(Z_s + Z_l + Z_L) - \tilde{I}_n Z_n$$

$$\tilde{V}_{cn} = \tilde{I}_c(Z_s + Z_l + Z_L) - \tilde{I}_n Z_n$$

$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn}$$

$$= (\tilde{I}_a + \tilde{I}_b + \tilde{I}_c)(Z_s + Z_l + Z_L) - 3\tilde{I}_n Z_n$$

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but $\tilde{I}_a + \tilde{I}_b + \tilde{I}_c + \tilde{I}_n = 0$, i.e., $\tilde{I}_a + \tilde{I}_b + \tilde{I}_c = -\tilde{I}_n$

$$\tilde{V}_{an} + \tilde{V}_{bn} + \tilde{V}_{cn} = -\tilde{I}_n(Z_s + Z_l + Z_L) - 3\tilde{I}_n Z_n$$

$$= -\tilde{I}_n(Z_s + Z_l + Z_L + 3Z_n) = 0$$

Balanced source

$$\tilde{I}_n = 0$$

$$\tilde{V}_{nN} = \tilde{I}_n Z_n = 0$$

so the neutral line can be replaced with an open circuit or a short circuit.

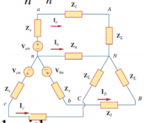

Figure 12.9 A balanced Y-Y system, showing the source, line, and load impedances.

- Therefore, we can igore Z_{nN}.

Phase Voltages and Line Voltages

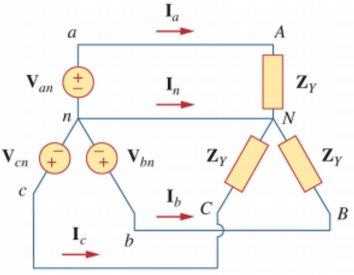

Figure 12.10 Balanced Y-Y connection.

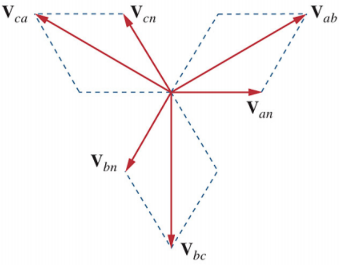

Figure 12.11(b) Phasor diagram illustrating the relationship between line voltages and phase voltages.

- Phase Voltages: $\tilde{V}_{an} = V_p\angle 0°, \tilde{V}_{bn} = V_p\angle -120°, \tilde{V}_{an} = V_p\angle -240°$
- Line Voltages: $\tilde{V}_{ab} = \tilde{V}_{an} - \tilde{V}_{bn} = \sqrt{3}V_p\angle 30°, \tilde{V}_{bc} = \sqrt{3}V_p\angle -90° = \tilde{V}_{ab}\angle -120°, \tilde{V}_{ca} = \sqrt{3}V_p\angle -210° = \tilde{V}_{ab}\angle -240°$

Phase Voltages and Line Voltages

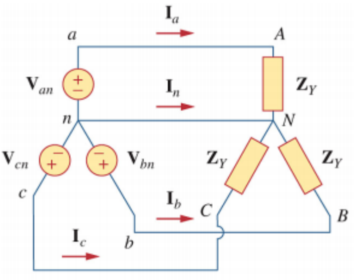

Figure 12.10 Balanced Y-Y connection.

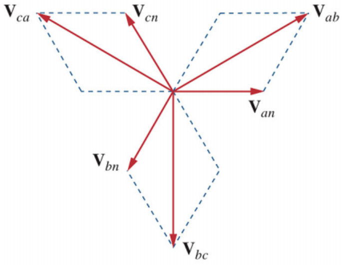

Figure 12.11(b) Phasor diagram illustrating the relationship between line voltages and phase voltages.

- The magnitude of the line voltages is $\sqrt{3}$ times the magnitude of the phase voltages. $V_L = \sqrt{3}V_p$
- The line voltages lead their corresponding phase voltages by 30°.

Line Currents and Phase Currents

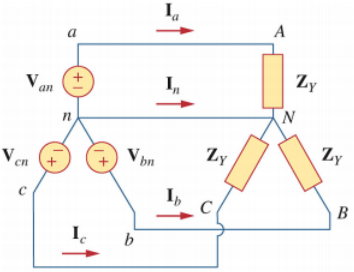

Figure 12.10 Balanced Y-Y connection.

- $\tilde{I_a} = \frac{\tilde{V_{an}}}{Z_Y}, \ \tilde{I_{AN}} = \tilde{I_a}$
- $\tilde{I_b} = \frac{\tilde{V_{bn}}}{Z_Y} = \tilde{I_a}\angle - 120^\circ, \ \tilde{I_{BN}} = \tilde{I_b}$
- $\tilde{I_c} = \frac{\tilde{V_{cn}}}{Z_Y} = \tilde{I_a}\angle - 240^\circ, \ \tilde{I_{CN}} = \tilde{I_c}$

Balanced Wye-Wye Connection

Summary of balanced Y-Y

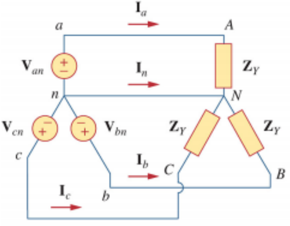

Figure 12.10 Balanced Y-Y connection.

If the source (phase voltages) are in abc sequence,

- Voltages: Magnitude Phase

$$V_L = \sqrt{3} V_p$$

$$\angle \tilde{V}_{ab} = \angle \tilde{V}_{an} + \angle 30°$$
$$\angle \tilde{V}_{bc} = \angle \tilde{V}_{bn} + \angle 30°$$
$$\angle \tilde{V}_{ca} = \angle \tilde{V}_{cn} + \angle 30°$$

Line voltages and phase voltages are in abc sequence

- Currents:
  Line currents and phase currents are same, and are in abc sequence

Balanced Wye-Delta Connection

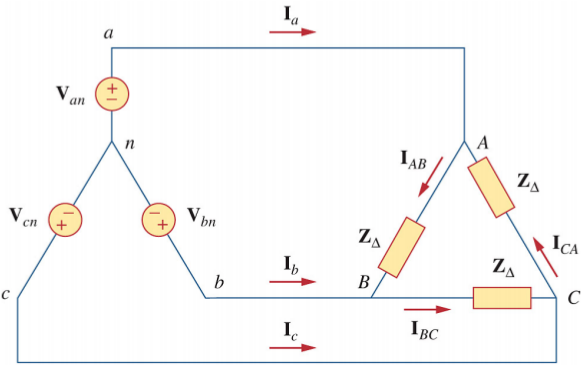

Figure 12.14 Balanced wye-delta connection.

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Phase Voltages and Line Voltages

Assume that

$$\tilde{V}_{an} = V_p \angle 0^\circ$$

$$\tilde{V}_{bn} = V_p \angle -120^\circ$$

$$\tilde{V}_{cn} = V_p \angle -240^\circ$$

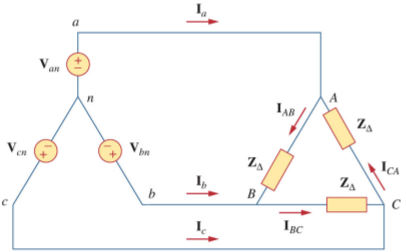

Figure 12.14 Balanced wye-delta connection.

The line voltages are

$$\left.\begin{aligned}
\tilde{V}_{ab} &= \sqrt{3} V_p \angle 30^\circ = \tilde{V}_{an} \sqrt{3} \angle 30^\circ = \tilde{V}_{AB} \\
\tilde{V}_{bc} &= \tilde{V}_{ab} \angle -120^\circ = \tilde{V}_{BC} \\
\tilde{V}_{ca} &= \tilde{V}_{ab} \angle -240^\circ = \tilde{V}_{CA}
\end{aligned}\right\}$$

Same as Y-Y case

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Line Currents and Phase Currents

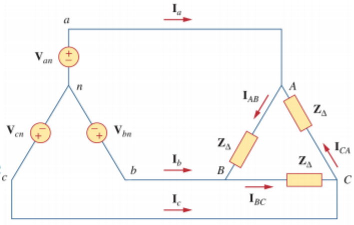

Figure 12.14 Balanced wye-delta connection.

The phase currents are

$$\tilde{I}_{AB}=\frac{\tilde{V}_{AB}}{Z_{\Delta}}$$

$$\tilde{I}_{BC}=\frac{\tilde{V}_{BC}}{Z_{\Delta}}=\tilde{I}_{AB}\angle-120°$$

$$\tilde{I}_{CA}=\frac{\tilde{V}_{CA}}{Z_{\Delta}}=\tilde{I}_{AB}\angle-240°$$

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Line Currents and Phase Currents

The line currents are

$$\tilde{I}_a = \tilde{I}_{AB} - \tilde{I}_{CA} = \tilde{I}_{AB}\sqrt{3}\angle -30°$$

$$\tilde{I}_b = \tilde{I}_{BC} - \tilde{I}_{AB} = \tilde{I}_{BC}\sqrt{3}\angle -30° = \tilde{I}_a\angle -120°$$

$$\tilde{I}_c = \tilde{I}_{CA} - \tilde{I}_{BC} = \tilde{I}_{CA}\sqrt{3}\angle -30° = \tilde{I}_a\angle -240°$$

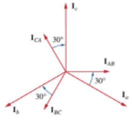

Figure 12.15 Phasor diagram illustrating the relationship between phase and line currents.

showing that

$$I_L = \sqrt{3} I_p$$

where

$$I_L = |\tilde{I}_a| = |\tilde{I}_b| = |\tilde{I}_c|, \; I_p = |\tilde{I}_{AB}| = |\tilde{I}_{BC}| = |\tilde{I}_{CA}|$$

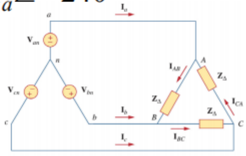

Figure 12.14 Balanced wye-delta connection.

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Line Currents and Phase Currents

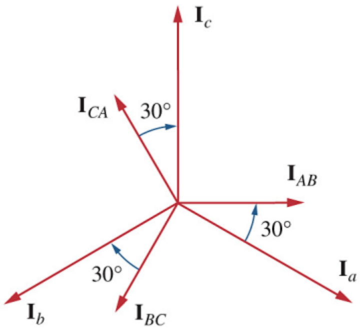

Figure 12.15 Phasor diagram illustrating the relationship between phase and line currents.

Balanced Wye-Delta Connection

Summary of balanced Y-Δ

If the source (phase voltages) are in abc sequence,

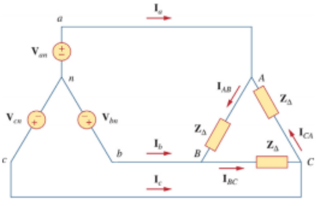

Figure 12.14 Balanced wye-delta connection.

- Voltages: <u>Magnitude</u> <u>Phase</u>

$$V_L = \sqrt{3} V_p \qquad \angle \tilde{V}_{ab} = \angle \tilde{V}_{an} + \angle 30^\circ$$

$$\angle \tilde{V}_{bc} = \angle \tilde{V}_{bn} + \angle 30^\circ$$

$$\angle \tilde{V}_{ca} = \angle \tilde{V}_{cn} + \angle 30^\circ$$

Line voltages and phase voltages are in abc sequence

- Currents: <u>Magnitude</u> <u>Phase</u>

$$I_L = \sqrt{3} I_p \qquad \tilde{I}_a = \tilde{I}_{AB} \sqrt{3} \angle -30^\circ$$

......

Line currents and phase currents are in abc sequence

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Balanced Delta-Delta Connection

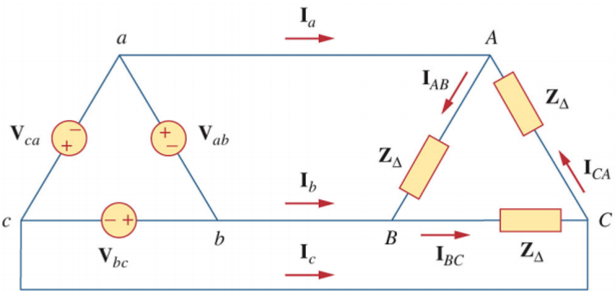

Figure 12.17 A balanced delta-delta connection.

Phase Voltages and Line Voltages

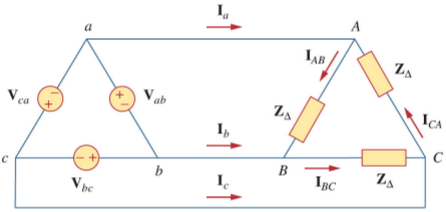

Figure 12.17 A balanced delta-delta connection.

Assume that

$$\widetilde{V}_{ab} = V_p \angle 0^\circ$$

$$\widetilde{V}_{bc} = V_p \angle -120^\circ$$

$$\widetilde{V}_{ca} = V_p \angle +120^\circ$$

The line voltages are

$$\widetilde{V}_{ab} = \widetilde{V}_{AB}, \ \widetilde{V}_{bc} = \widetilde{V}_{BC}, \ \widetilde{V}_{ca} = \widetilde{V}_{CA}$$

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Line Currents and Phase Currents

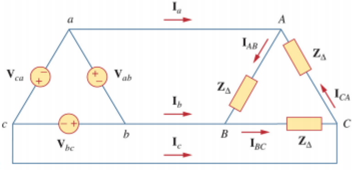

Figure 12.17 A balanced delta-delta connection.

The phase currents are

$$\tilde{I}_{AB} = \frac{\tilde{V}_{AB}}{Z_\Delta}$$

$$\tilde{I}_{BC} = \frac{\tilde{V}_{BC}}{Z_\Delta} = \tilde{I}_{AB} \angle -120°$$

$$\tilde{I}_{CA} = \frac{\tilde{V}_{CA}}{Z_\Delta} = \tilde{I}_{AB} \angle +120°$$

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Line Currents and Phase Currents

The line currents are

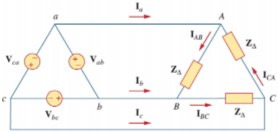

Figure 12.17 A balanced delta-delta connection.

$$\tilde{I}_a = \tilde{I}_{AB} - \tilde{I}_{CA} = \tilde{I}_{AB}\sqrt{3}\angle -30^\circ$$

$$\tilde{I}_b = \tilde{I}_{BC} - \tilde{I}_{AB} = \tilde{I}_{BC}\sqrt{3}\angle -30^\circ = \tilde{I}_a\angle -120^\circ$$

$$\tilde{I}_c = \tilde{I}_{CA} - \tilde{I}_{BC} = \tilde{I}_{CA}\sqrt{3}\angle -30^\circ = \tilde{I}_a\angle -240^\circ$$

showing that

$$I_L = \sqrt{3} I_p$$

where

$$I_L = |\tilde{I}_a| = |\tilde{I}_b| = |\tilde{I}_c|,\ I_p = |\tilde{I}_{AB}| = |\tilde{I}_{BC}| = |\tilde{I}_{CA}|$$

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Balanced Delta-Delta Connection

Summary of balanced Δ-Δ

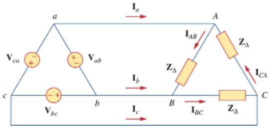


Figure 12.17 A balanced delta-delta connection.

If the source (phase voltages) are in abc sequence,

- Voltages:
  Line voltages and phase voltages are same, and are in abc sequence

- Currents: Magnitude Phase

$$I_L = \sqrt{3} I_p \qquad \tilde{I}_a = \tilde{I}_{AB} \sqrt{3} \angle -30°$$

......

Line currents and phase currents are in abc sequence

Balanced Delta-Wye Connection

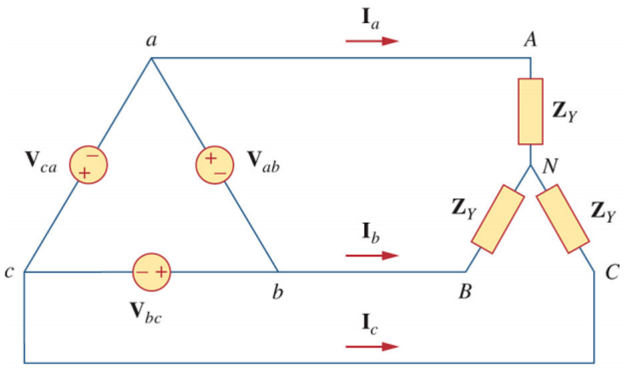


Figure 12.18 A balanced delta-wye connection.

Phase Voltages and Line Voltages

Assume that

$$\widetilde{V}_{ab} = V_p \angle 0°$$

$$\widetilde{V}_{bc} = V_p \angle -120°$$

$$\widetilde{V}_{ca} = V_p \angle +120°$$

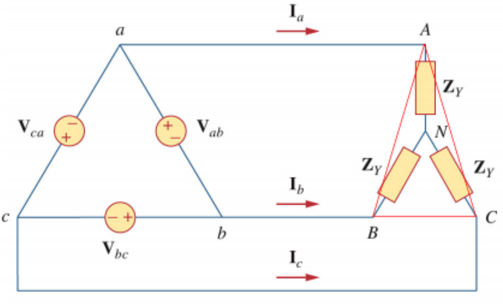


Figure 12.18 A balanced delta-wye connection.

Line Currents and Phase Currents

$$\tilde{I}_a = \tilde{I}_{AB}\sqrt{3}\angle -30^\circ = \frac{\tilde{V}_{AB}}{Z_\Delta}\sqrt{3}\angle -30^\circ$$

$$= \frac{\tilde{V}_{ab}}{3Z_Y}\sqrt{3}\angle -30^\circ = \frac{\tilde{V}_{ab}}{\sqrt{3}Z_Y}\angle -30^\circ$$

$$\tilde{I}_b = \frac{\tilde{V}_{bc}}{\sqrt{3}Z_Y}\angle -30^\circ = \tilde{I}_a\angle -120^\circ$$

$$\tilde{I}_c = \frac{\tilde{V}_{ca}}{\sqrt{3}Z_Y}\angle -30^\circ = \tilde{I}_a\angle +120^\circ$$

$$\boxed{\begin{array}{l} Z_\Delta = 3Z_Y \\ Z_Y = Z_\Delta/3 \end{array}}$$

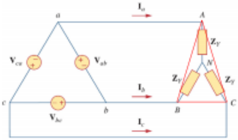


Figure 12.18 A balanced delta-wye connection.

Balanced Delta-Wye Connection

Summary of balanced Δ-Y

If the source (phase voltages) are in abc sequence,

- Voltages:
  Line voltages and phase voltages are same, and are in abc sequence
- Currents:
  Line currents and phase currents are same, and are in abc sequence

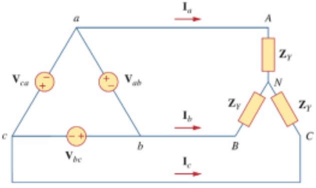


Figure 12.18 A balanced delta-wye connection.

Summary

How to deduce voltages and currents

- Y - Y: phase voltages $\rightarrow$ line voltages; phase voltages $\rightarrow$ line currents, phase currents
- Y - Δ: phase voltages $\rightarrow$ line voltages $\rightarrow$ phase currents $\rightarrow$ line currents
- $\Delta - \Delta$: phase voltages, line voltages $\rightarrow$ phase currents $\rightarrow$ line currents
- Δ - Y: phase voltages, line voltages $\rightarrow$ line currents, phase currents (Get line currents by transforming Δ - Y to $\Delta - \Delta$)

Summary

TABLE 12.1

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

Summary

Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$	Same as phase voltages
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$	Same as phase voltages
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

[1] Positive or abc sequence is assumed.

Power in a Balanced System

- For a Y-connect load, the phase voltages are $V_{AN} = \sqrt{2}V_p cos\omega t$, $V_{BN} = \sqrt{2}V_p cos(\omega t - 120^\circ)$, $V_{CN} = \sqrt{2}V_p cos(\omega t + 120^\circ)$
- If $Z_Y = Z\angle\theta$, the phase currents are $I_a = \sqrt{2}I_p cos(\omega t - \theta)$, $I_b = \sqrt{2}I_p cos(\omega t - \theta - 120^\circ)$, $I_c = \sqrt{2}I_p cos(\omega t - \theta + 120^\circ)$
- The total instantaneous power is $P = P_a+P_b+P_c = V_{AN}I_a+V_{BN}I_b+V_{CN}I_c = 2V_pI_p[cos\omega t cos(\omega t-\theta)+cos(\omega t-120^\circ)cos(\omega t-\theta-120^\circ)+cos(\omega t+120^\circ)cos(\omega t-\theta+120^\circ)] = 2V_pI_p[\frac{cos\theta cos(2\omega t-\theta)}{2}+\frac{cos\theta cos(2\omega t-\theta-240^\circ)}{2}+\frac{cos\theta cos(2\omega t-\theta+240^\circ)}{2}] = 3V_pI_p cos\theta$
- $Q = 3V_pI_p sin\theta$, $S_p = P_p+jQ_p = 3V_pI_p cos\theta+j3V_pI_p sin\theta = \sqrt{3}V_LI_L cos\theta+j\sqrt{3}V_LI_L sin\theta$

Problem 12.22

Chapter 12, Problem 22.

Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_\Delta = 12 - j15\Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_l = 2\ \Omega$.

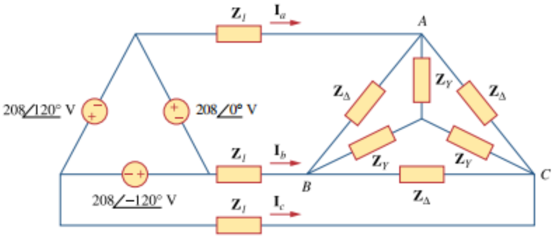


Figure 12.53
For Prob. 12.22.

Problem 12.22 Solution

Chapter 12, Solution 22.

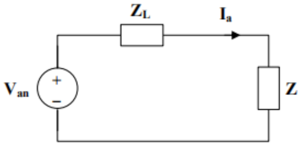
Convert the Δ-connected source to a Y-connected source.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ = \frac{208}{\sqrt{3}}\angle -30^\circ = 120\angle -30^\circ$$

Convert the Δ-connected load to a Y-connected load.

$$\mathbf{Z} = \mathbf{Z}_Y \,||\, \frac{\mathbf{Z}_\Delta}{3} = (4 + j6) \,||\, (4 - j5) = \frac{(4 + j6)(4 - j5)}{8 + j}$$

$$\mathbf{Z} = 5.723 - j0.2153$$



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}} = \frac{120\angle -30^\circ}{7.723 - j0.2153} = \underline{\mathbf{15.53\angle -28.4^\circ\ A}}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = \underline{\mathbf{15.53\angle -148.4^\circ\ A}}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = \underline{\mathbf{15.53\angle 91.6^\circ\ A}}$$

Advantage of Three-Phase System

- The total instantaneous power in a balanced three-phase system is constant.
- The material to deliver the same power and to tolerate the same loss needed is $\frac{3}{4}$ times less.

Problem 12.38

ps Given the circuit in Fig. 12.57 below, find the total complex power absorbed by the load.

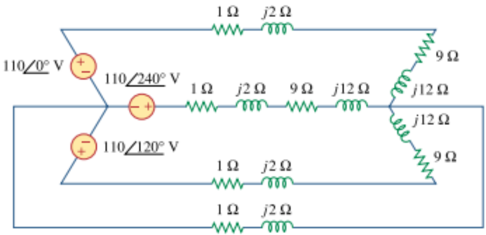


Figure 12.57
For Prob. 12.38.

Problem 12.38 Solution

Chapter 12, Solution 38.

As a balanced three-phase system, we can use the per-phase equivalent shown below.

$$\mathbf{I}_a = \frac{110\angle 0°}{(1+j2)+(9+j12)} = \frac{110\angle 0°}{10+j14}$$

$$\mathbf{S}_p = \frac{1}{2}|\mathbf{I}_a|^2 \mathbf{Z}_Y = \frac{1}{2}\cdot\frac{(110)^2}{(10^2+14^2)}\cdot(9+j12)$$

The complex power is

$$\mathbf{S} = 3\mathbf{S}_p = \frac{3}{2}\cdot\frac{(110)^2}{296}\cdot(9+j12)$$

$$\mathbf{S} = \underline{\mathbf{551.86 + j735.81\ VA}}$$

Unbalanced Three-Phase Systems

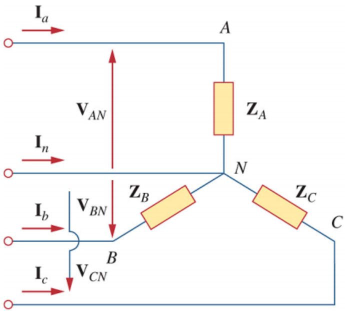
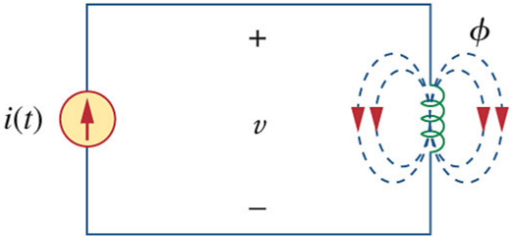


Figure 12.23 Unbalanced three-phase Y-connected load.

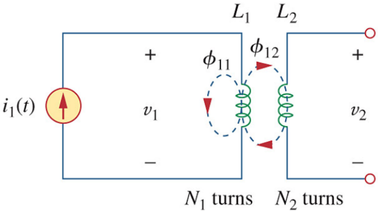
- Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis.

Self Inductance



- According to Faraday's Law, $v = N\frac{d\phi}{dt}$ (N turns)
- $v = N\frac{d\phi}{di}\frac{di}{dt} = L\frac{di}{dt}$, where $L = N\frac{d\phi}{di}$ is commonly called the *self-inductance* of the coil.

Mutual Inductance



- Assume coil 2 carries no current. The magnetic flux ϕ_1 emanating from coil 1 has two components. $\phi_1 = \phi_{11} + \phi_{12}$
- $v_1 = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$
- $v_2 = N_2 \frac{d\phi_{12}}{dt} = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$, where $M_{21} = N_2 \frac{d\phi_{12}}{di_1}$ is known as the mutual inductance of coil 2 with respect to coil 1, measured in henrys(H).

Dot Convention

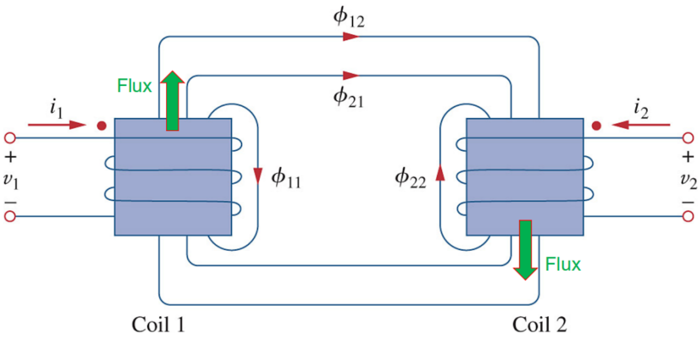


Figure 13.4 Illustration of the dot convention.

Current entering the dotted end of one winding produces flux in the same direction as the flux produced by current entering the dotted end of the other winding.

Dot Convention

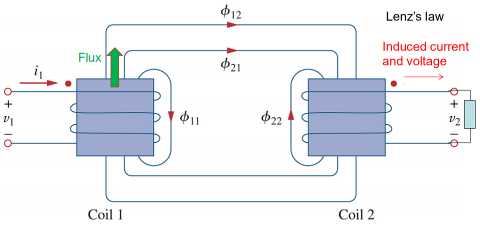
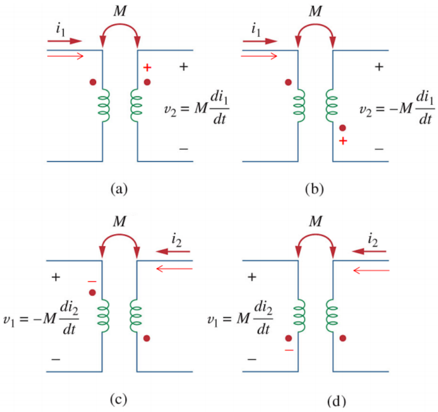


Figure 13.4 Illustration of the dot convention.

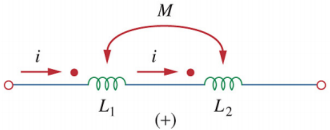
- Lenz's Law: The direction of any magnetic induction effect is such as to oppose the cause of the effect.
- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage of the second coil is positive at the dotted terminal of the second coil.

Mutual Inductance

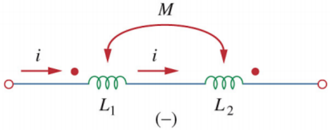


- Entering current in dot 1 $\rightarrow$ + voltage in dot 2
- Leaving current in dot 1 $\rightarrow$ - voltage in dot 2

Mutual Inductance

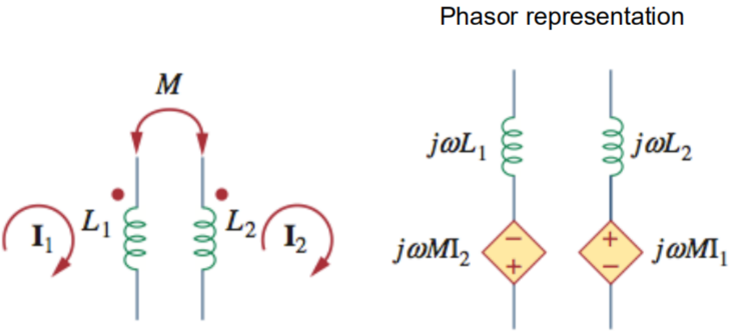


- $L = L_1 + L_2 + 2M$
- $V = (L_1\frac{di}{dt} + M\frac{di}{dt}) + (L_2\frac{di}{dt} + M\frac{di}{dt})$ Therefore, $L_{eq} = L_1 + L_2 + 2M$

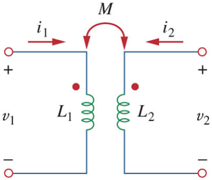


- $L = L_1 + L_2 - 2M$
- $V = (L_1\frac{di}{dt} - M\frac{di}{dt}) + (L_2\frac{di}{dt} - M\frac{di}{dt})$ Therefore, $L_{eq} = L_1 + L_2 - 2M$

Mutual Inductance

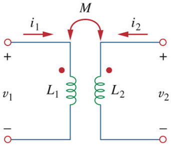


Deriving Energy Stored in a Coupled Circuit



- Step 1: i_1 from 0 to I_1. If we let i_1 increase from zero to I_1 while maintaining $i_2 = 0$, the power in the circuit is $p_1 = v_1 i_1 = L_1 \frac{di_1}{dt} i_1$ and the energy stored in the circuit is $w_1 = \int p_1 dt = L_1 \int_0^{I_1} i_1 di_1 = \frac{1}{2} L_1 I_1^2$
- Step 2: i_2 from 0 to I_2. If we now maintain $i_1 = I_1$ and increase i_2 from zero to I_2, the power in the coils is now$p_2 = (M_{12} \frac{di_2}{dt}) I_1 + (L_2 \frac{di_2}{dt}) i_2$ and the energy stored in the circuit is $w_2 = \int p_2 dt = M_{12} I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2 di_2 = M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2$

Deriving Energy Stored in a Coupled Circuit



- The total energy stored in the coils now $w = w_1 + w_2 = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{12} I_1 I_2$
- If we reverse the order by which the currents reach their final values, the total energy stored in the coils is $w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{21} I_1 I_2$
- Therefore, $M_{12} = M_{21} = M$ and $w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$
- If one current enters one dotted terminal while the other current leaves the other dotted terminal, the total energy is $w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$

Coefficient of Coupling

- $w = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 \pm MI_1I_2 \geq 0$ leads to $\frac{1}{2}(\sqrt{L_1 i_1} - \sqrt{L_2 i_2})^2 + i_1 i_2(\sqrt{L_1L_2} - M) \geq 0$. Therefore, $\sqrt{L_1L_2} \geq M$
- $k = \frac{M}{\sqrt{L_1L_2}}(0 \leq k \leq 1)$
- Coils are said to be loosely coupled when $k < 0.5$. If $k > 0.5$, they are tightly coupled. If $k = 1$, they are perfectly coupled.

Problem 13.20

13.20 Determine currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 13.89. Find the energy stored in the coupled **ML** coils at $t = 2$ ms. Take $\omega = 1{,}000$ rad/s.

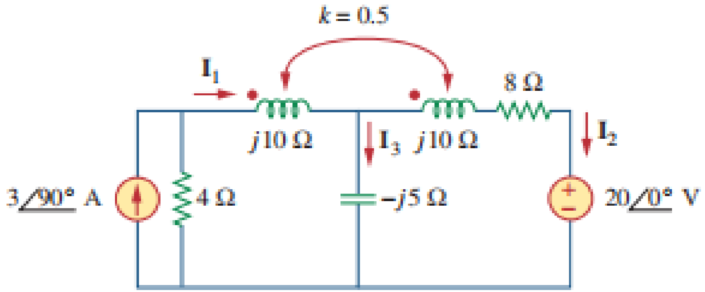
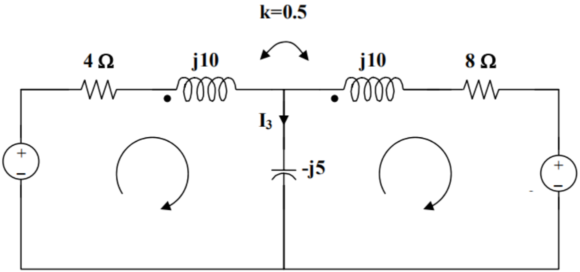


Figure 13.89
For Prob. 13.20.

Problem 13.20 Solution

Chapter 13, Solution 20.

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1 L_2} \quad \text{or} \quad M = k\sqrt{L_1 L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

Problem 13.20 Solution (Continued)

For mesh 1, $j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2$ (1)

For mesh 2, $0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$-20 = +j10I_1 + (8 + j5)I_2$ (2)

From (1) and (2), $\begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \underline{\mathbf{2.462\angle 72.18^\circ\ A}}$$

$$I_2 = \Delta_2/\Delta = \underline{\mathbf{0.878\angle -97.48^\circ\ A}}$$

$$I_3 = I_1 - I_2 = \underline{\mathbf{3.329\angle 74.89^\circ\ A}}$$

$$i_1 = 2.462\cos(1000t + 72.18^\circ)\ A$$

$$i_2 = 0.878\cos(1000t - 97.48^\circ)\ A$$

At $t = 2$ ms, $1000t = 2$ rad $= 114.6^\circ$

$$i_1 = 0.9736\cos(114.6^\circ + 143.09^\circ) = -2.445$$

$$i_2 = 2.53\cos(114.6^\circ + 153.61^\circ) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 - Mi_1i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(10)(-2.445)^2 + 0.5(10)(-0.8391)^2 - 5(-2.445)(-0.8391)$$

$\mathbf{w} = \underline{43.67\ \text{mJ}}$

Linear Transformers

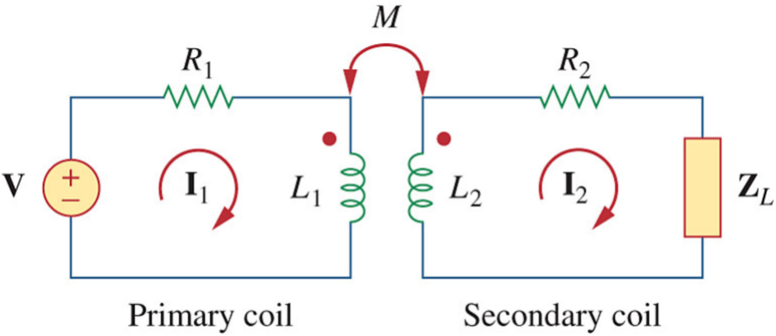


Figure 13.19 A linear transformer.

Input Impedance

For the linear transformer in Fig. 13.19,

$$\begin{cases} v_1 = L_1 \dfrac{di_1}{dt} - M \dfrac{di_2}{dt} \\ v_2 = M \dfrac{di_1}{dt} - L_2 \dfrac{di_2}{dt} \end{cases} \Rightarrow \begin{cases} \tilde{V}_1 = j\omega L_1 \tilde{I}_1 - j\omega M \tilde{I}_2 \\ \tilde{V}_2 = j\omega M \tilde{I}_1 - j\omega L_2 \tilde{I}_2 \end{cases}$$

where v_1 and v_2 denote the primary and secondary voltages, respectively.

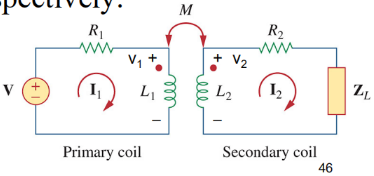


Figure 13.19 A linear transformer.

Input Impedance

$$\begin{cases} \tilde{V} = \tilde{I}_1 R_1 + \tilde{V}_1 = \tilde{I}_1 R_1 + j\omega L_1 \tilde{I}_1 - j\omega M \tilde{I}_2 \\ \tilde{V}_2 = -j\omega L_2 \tilde{I}_2 + j\omega M \tilde{I}_1 = \tilde{I}_2 R_2 + \tilde{I}_2 Z_L \end{cases}$$

Mesh Eq. 1

Mesh Eq. 2

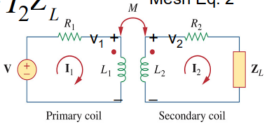


Figure 13.19 A linear transformer.

$$\tilde{I}_2 = \frac{j\omega M \tilde{I}_1}{R_2 + j\omega L_2 + Z_L}$$

$$\tilde{V} = \tilde{I}_1 R_1 + j\omega L_1 \tilde{I}_1 - j\omega M \frac{j\omega M \tilde{I}_1}{R_2 + j\omega L_2 + Z_L}$$

$$Z_{in} = \frac{\tilde{V}}{\tilde{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

Reflected Impedance

Notice that Z_{in} comprises two terms. The first term, $R_1 + j\omega L_1$, is the primary impedance. The second term, known as the *reflected impedance* Z_R, is due to the coupling between the primary and secondary windings.

$$Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

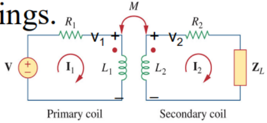


Figure 13.19 A linear transformer.

This equation is not affected by the location of the dots on the transformer.

$\boxed{(\pm M)^2 = M^2}$

$$\boxed{Z_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}$$

Equivalent T Circuit

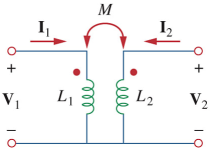


Figure 13.21 Determining the equivalent circuit of a linear transformer.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

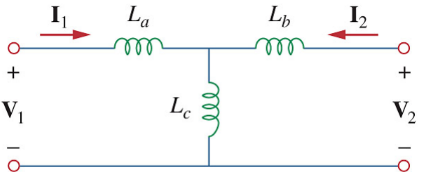


Figure 13.22 An equivalent T circuit.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

2 KVL equations

- $L_a = L_1 - M$
- $L_b = L_2 - M$
- $L_c = M$

Equivalent Π Circuit

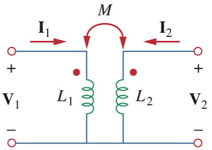


Figure 13.21 Determining the equivalent circuit of a linear transformer.

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

- $L_A = \frac{L_1 L_2 - M^2}{L_2 - M}$
- $L_B = \frac{L_1 L_2 - M^2}{L_1 - M}$
- $L_C = \frac{L_1 L_2 - M^2}{M}$

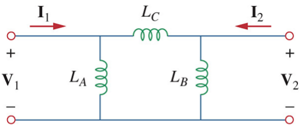


Figure 13.23 An equivalent Π circuit.

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$ 2 KCL equations

Problem 13.29

13.29 In the circuit of Fig. 13.98, find the value of the coupling coefficient k that will make the 10-Ω resistor dissipate 320 W. For this value of k, find the energy stored in the coupled coils at $t = 1.5$ s.

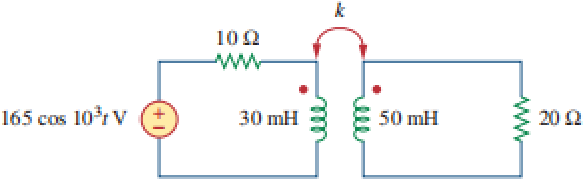


Figure 13.98
For Prob. 13.29.

Problem 13.29 Solution

Chapter 13, Problem 29.

In the circuit of Fig. 13.98, find the value of the coupling coefficient k that will make the 10-Ω resistor dissipate 320 W. For this value of k, find the energy stored in the coupled coils at $t = 1.5$ s.

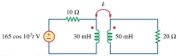


Figure 13.98
For Prob. 13.29.

Chapter 13, Solution 29.

$$30\text{ mH becomes } j\omega L = j30\text{x}10^{-3}\text{x}10^{3} = j30$$

$$50\text{ mH becomes } j50$$

$$\text{Let } X = \omega M$$

Using the concept of reflected impedance,

$$Z_{in} = 10 + j30 + X^2/(20 + j50)$$

$$I_1 = V/Z_{in} = 165/(10 + j30 + X^2/(20 + j50))$$

$$p = 0.5|I_1|^2(10) = 320 \text{ leads to } |I_1|^2 = 64 \text{ or } |I_1| = 8$$

$$8 = |165(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$$

$$= |165(20 + j50)/(X^2 - 1300 + j1100)|$$

$$\text{or} \qquad 64 = 27225(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$$

$$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}633$$

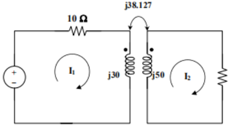
$$X = 33.86 \text{ or } 38.13$$

$$\text{If } X = 38.127 = \omega M$$

$$M = 38.127\text{ mH}$$

$$k = M/\sqrt{L_1L_2} = 38.127/\sqrt{30\text{x}50} = \underline{0.984}$$

Problem 13.29 Solution (Continued)



$$165 = (10 + j30)I_1 - j38.127I_2 \quad (1)$$

$$0 = (20 + j50)I_2 - j38.127I_1 \quad (2)$$

In matrix form, $\begin{bmatrix} 165 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 + j30 & -j38.127 \\ -j38.127 & 20 + j50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 154 + j1100 = 1110.73\angle 82.03^\circ,\ \Delta_1 = 888.5\angle 68.2^\circ,\ \Delta_2 = j6291$$

$$I_1 = \Delta_1/\Delta = 8\angle -13.81^\circ,\ I_2 = \Delta_2/\Delta = 5.664\angle 7.97^\circ$$

$$i_1 = 8\cos(1000t - 13.83^\circ),\ i_2 = 5.664\cos(1000t + 7.97^\circ)$$

$$\text{At } t = 1.5\text{ ms},\ 1000t = 1.5\text{ rad} = 85.94^\circ$$

$$i_1 = 8\cos(85.94^\circ - 13.83^\circ) = 2.457$$

$$i_2 = 5.664\cos(85.94^\circ + 7.97^\circ) = -0.3862$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(30)(2.547)^2 + 0.5(50)(-0.3862)^2 - 38.127(2.547)(-0.3862)$$

$$= \underline{\mathbf{130.51\ mJ}}$$

Ideal Transformers

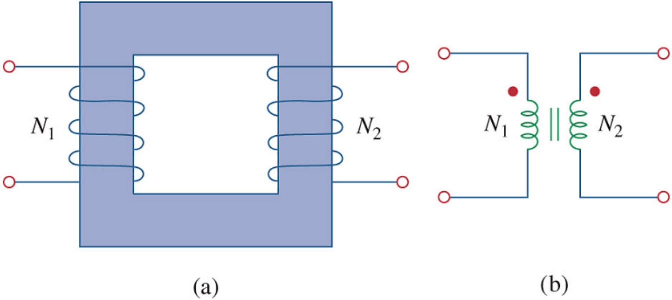


Figure 13.30 (a) Ideal transformer, (b) circuit symbol for ideal transformers.

Ideal Transformers

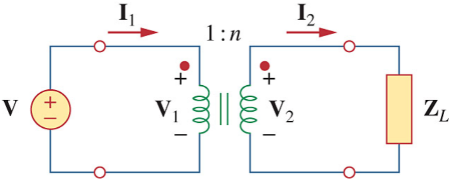


Figure 13.31 Relating primary and secondary quantities in an ideal transformer.

We have

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

where n is the *turns ratio* or *transformation ratio*. Using the phasor voltages,

$$\frac{\tilde{V}_2}{\tilde{V}_1} = \frac{N_2}{N_1} = n$$

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Ideal Transformers

Ideal transformer:

1. k=1
2. R_1=R_2=0
3. L_1, L_2, M → ∞

For the reason of power conservation,

$$v_1 i_1 = v_2 i_2$$

$$\frac{i_1}{i_2} = \frac{v_2}{v_1} = n$$

$$\frac{\tilde{I}_1}{\tilde{I}_2} = \frac{\tilde{V}_2}{\tilde{V}_1} = n$$

Ideal Transformers

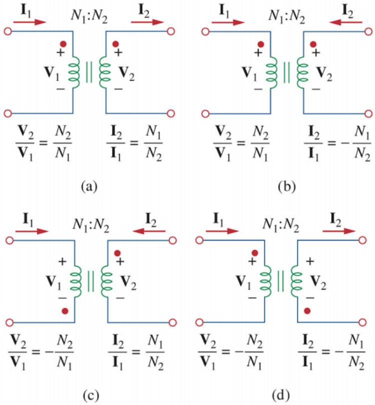

Figure 13.32 Typical circuits illustrating proper voltage polarities and circuit directions in an ideal transformer.

For V, same polarity at dotted terminals ➔ +n
For I, same flowing direction (entering/leaving) the dotted terminals ➔ –n

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Input Impedance of Ideal Transformers

The input impedance as seen by the source is

$$Z_{in} = \frac{\tilde{V}_1}{\tilde{I}_1} = \frac{\tilde{V}_2 / n}{n\tilde{I}_2} = \frac{1}{n^2}\frac{\tilde{V}_2}{\tilde{I}_2} = \frac{1}{n^2} Z_L$$

Reflecting Impedances and Sources

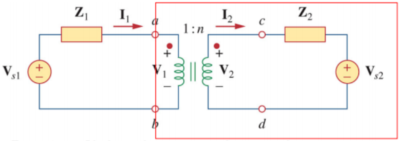


Figure 13.33 Ideal transformer circuit whose equivalent circuits are to be found.

Replace the circuit to the right of a-b with its Thevenin equivalent

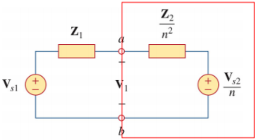


Figure 13.35 Equivalent circuit for Fig. 13.33 obtained by reflecting the secondary circuit to the primary side.

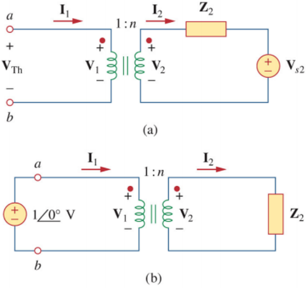


Figure 13.34 Obtaining the Thevenin equivalent for the circuit in Fig. 13.33.

$I_1=0=I_2$

$V_{Th}=V_{oc}$ $\quad \mathbf{V}_{\text{Th}} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{s2}}{n} \qquad (13.61)$

Z_{Th}: turn off the source $\quad \mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \quad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2 \qquad (13.62)$

Reflecting Impedances and Sources

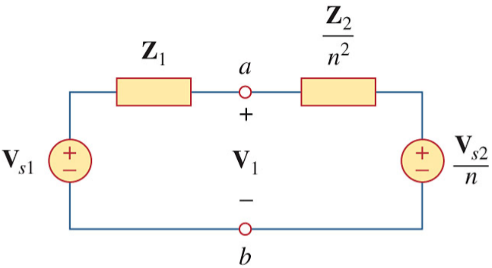


Figure 13.35 Equivalent circuit for Fig. 13.33 obtained by reflecting the secondary circuit to the primary side.

Reflecting Impedances and Sources

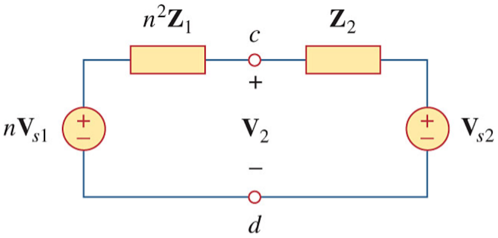
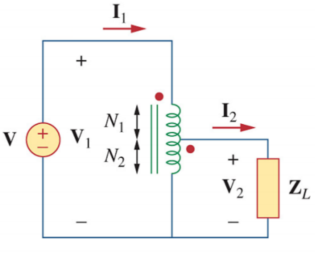


Figure 13.36 Equivalent circuit for Fig. 13.33 obtained by reflecting the primary circuit to the secondary side.

Ideal Autotransformers

- The autotransformer is a type of power transformer.
- Its major advantage over the two-winding transformer is its ability to transfer larger apparent power.
- Its major disadvantage is the lack of electrical isolation between the primary and secondary sides.

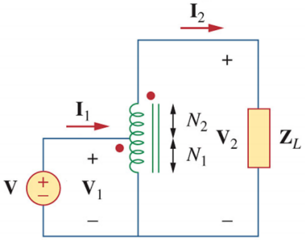
Step-Down Autotransformers



(a)

$$\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{N_1 + N_2}{N_2} = \frac{\tilde{I}_2}{\tilde{I}_1} \text{ or } \frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{I_2}{I_1}$$

Step-Up Autotransformers



(b)

$$\frac{\tilde{V}_1}{\tilde{V}_2} = \frac{N_1}{N_1 + N_2} = \frac{\tilde{I}_2}{\tilde{I}_1} \text{ or } \frac{V_1}{V_2} = \frac{N_1}{N_1 + N_2} = \frac{I_2}{I_1}$$

Problem 13.70

13.70 In the ideal transformer circuit shown in Fig. 13.133, determine the average power delivered to the load.

ML

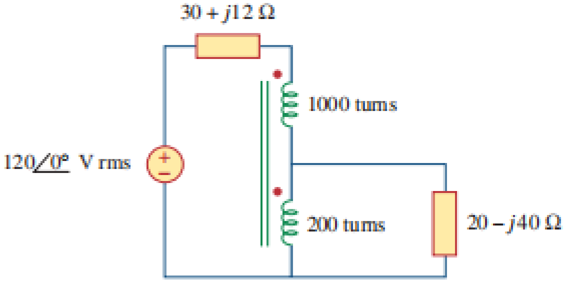
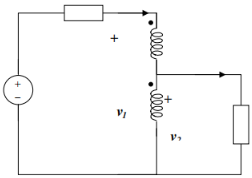


Figure 13.133
For Prob. 13.70.

Problem 13.70 Solution

Chapter 13, Solution 70.

This is a step-down transformer.



$$I_1/I_2 = N_2/(N_1 + N_2) = 200/1200 = 1/6, \text{ or } I_1 = I_2/6 \qquad (1)$$

$$v_1/v_2 = (N_2 + N_2)/N_2 = 6, \text{ or } v_1 = 6v_2 \qquad (2)$$

For the primary loop, $120 = (30 + j12)I_1 + v_1$ (3)

For the secondary loop, $v_2 = (20 - j40)I_2$ (4)

Substituting (1) and (2) into (3),

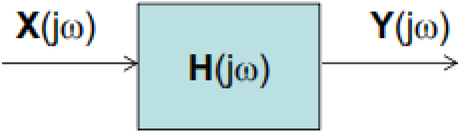
$$120 = (30 + j12)(I_2/6) + 6v_2$$

and substituting (4) into this yields

$$120 = (49 - j38)I_2 \text{ or } I_2 = 1.935\angle 37.79°$$

$$p = |I_2|^2(20) = \underline{\textbf{74.9 watts}}.$$

Transfer Function H($j\omega$)



- $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ (Voltage Gain)
- $H(j\omega) = \frac{I_{out}(j\omega)}{I_{in}(j\omega)}$ (Current Gain)
- $H(j\omega) = \frac{V_{out}(j\omega)}{I_{in}(j\omega)}$ (Transfer impedance)
- $H(j\omega) = \frac{I_{out}(j\omega)}{V_{in}(j\omega)}$ (Transfer admittance)

Transfer Function H($j\omega$)

The frequency response can be expressed in terms of its numerator polynomial $N(j\omega)$ and denominator polynomial $D(j\omega)$ as

$$H(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{\sum_{m=0}^{M} b_m (j\omega)^m}{\sum_{n=0}^{N} a_n (j\omega)^n}$$

Transfer Function H($j\omega$)

Proof :

A linear network can be described by a linear constant-coefficient differential equation

$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

Recall Chaps. 7 and 8:
First-order circuit
Second-order circuit

Transform the equation to the phasor domain,

$$\sum_{n=0}^{N} a_n (j\omega)^n \mathbf{Y}(j\omega) = \sum_{m=0}^{M} b_m (j\omega)^m \mathbf{X}(j\omega)$$

Recall Chap. 9:
d/dt → jω

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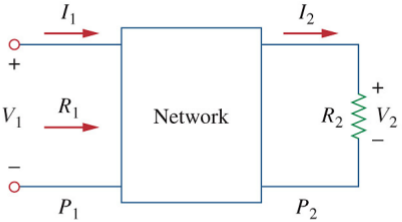
$$\mathbf{Y}(j\omega)\sum_{n=0}^{N} a_n (j\omega)^n = \mathbf{X}(j\omega)\sum_{m=0}^{M} b_m (j\omega)^m$$

$$H(j\omega) = \frac{\mathbf{Y}(j\omega)}{\mathbf{X}(j\omega)} = \frac{\sum_{m=0}^{M} b_m (j\omega)^m}{\sum_{n=0}^{N} a_n (j\omega)^n}$$

Decibel Scale

- G=Number of bels=$log_{10}\frac{P_2}{P_1}$
- $G_{dB} = 10log_{10}\frac{P_2}{P_1}$
- When $R_1 = R_2$, $G_{dB} = 10log_{10}(\frac{V_2}{V_1})^2 = 20log_{10}\frac{V_2}{V_1} = 20log_{10}\frac{I_2}{I_1}$



- If $P_2 = 2P_1$, $G_{dB} = 10log_{10}2 = 3dB$

Bode Plots

Construct the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{200\,j\omega}{(j\omega + 2)(j\omega + 10)}$$

Solution:

We first put $\mathbf{H}(\omega)$ in the standard form by dividing out the poles and zeros. Thus,

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{10\,j\omega}{(1 + j\omega/2)(1 + j\omega/10)} \\ &= \frac{10|j\omega|}{|1 + j\omega/2||1 + j\omega/10|}\underline{/90^\circ - \tan^{-1}\omega/2 - \tan^{-1}\omega/10}\end{aligned}$$

Hence, the magnitude and phase are

$$\begin{aligned}H_{\text{dB}} &= 20\log_{10}10 + 20\log_{10}|j\omega| - 20\log_{10}\left|1 + \frac{j\omega}{2}\right| \\ &\quad - 20\log_{10}\left|1 + \frac{j\omega}{10}\right|\end{aligned}$$

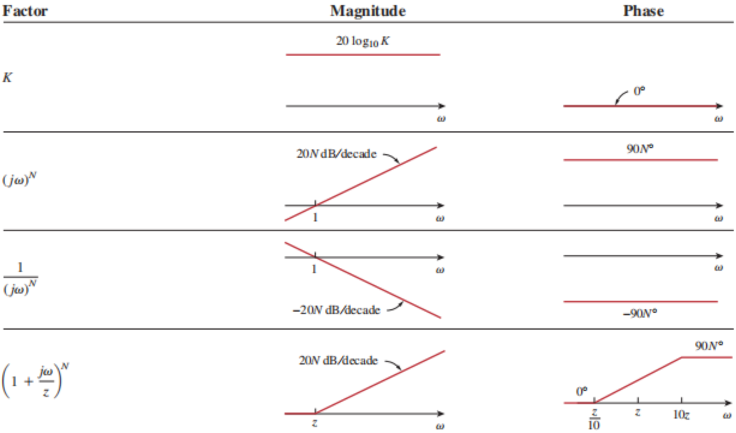
$$\phi = 90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

- Step1: Find trasnfer function $H(j\omega)$.
- Step2: Find magnitude H_{dB} and phase ϕ.
- Step3: Draw bode plots according to the following table.

Bode Plots

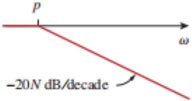
TABLE 14.3

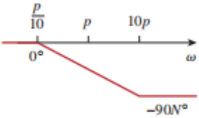
Summary of Bode straight-line magnitude and phase plots.



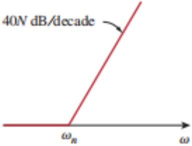
Bode Plots

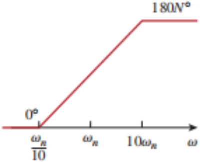
$$\frac{1}{(1 + j\omega/p)^N}$$



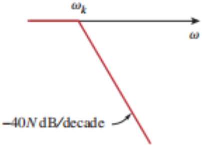


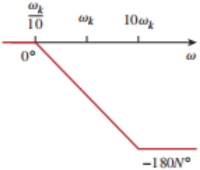
$$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$$





$$\frac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$$





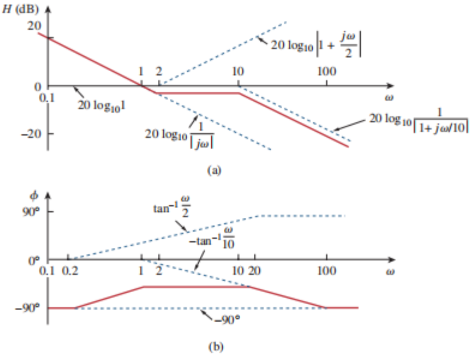
Practice Problem 14.3

Practice Problem 14.3

Draw the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$

Answer: See Fig. 14.14.



Series Resonance

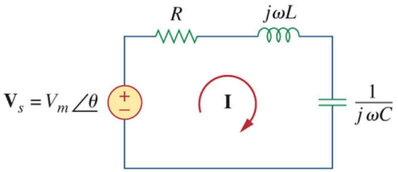
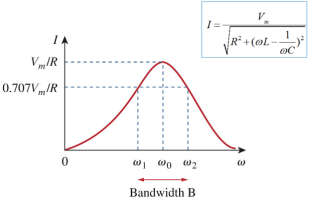


Figure 14.21 The series resonant circuit.

- The input impedance is $Z = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$
- Resonance results when $Im(Z) = 0$. Thus, the resonance condition is $\omega_0 L = \frac{1}{\omega_0 C}$

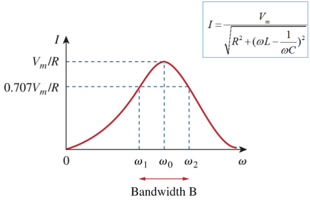
ω_1 ω_2

- The average power dissipated by the RLC circuit is $P(\omega) = \frac{1}{2}I^2R = \frac{1}{2}\frac{V_m^2}{R^2+(\omega L-1/(\omega C))^2}R$
- The highest power dissipated occurs at resonance. $P(\omega_0) = \frac{1}{2}I_{max}^2R = \frac{1}{2}\frac{V_m^2}{R}$



- At ω_1 and ω_2, the dissipated power is half the maximum value. ω_1 and ω_2 are called the *half − power frequencies*. $I(\omega_1) = I(\omega_2) = \frac{I_{max}}{\sqrt{2}}$

ω_1 ω_2 BW Q



- $\frac{1}{2}\frac{V_m^2}{R^2+(\omega L-1/(\omega C))^2}R = \frac{1}{2}P(\omega_0) = \frac{V_m^2}{4R} \to \omega_{1,2} = \pm\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$
- $\omega_0 = \sqrt{\omega_1\omega_2} = \sqrt{\frac{1}{LC}}$
- The $half - power\ bandwidth\ BW = \omega_2 - \omega_1 = \frac{R}{L}$.
- The sharpness $Q = 2\pi\frac{E_s}{E_d} = 2\pi\frac{\frac{1}{2}LI_{max}^2}{\frac{1}{2}I_{max}^2R(1/f_0)} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$
- $BW = \frac{R}{L} = \frac{\omega_0}{Q} \to Q = \frac{\omega_0}{BW}$

BW Q

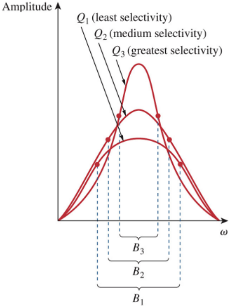


Figure 14.23 The higher the circuit Q, the smaller the bandwidth.

- $BW = \frac{R}{L} = \frac{\omega_0}{Q} \rightarrow Q = \frac{\omega_0}{BW}$
- For same ω_0, $Q \uparrow \leftrightarrow BW \downarrow \leftrightarrow sharpness \uparrow$

Parallel Resonance

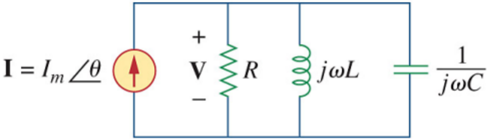


Figure 14.25 The parallel resonant circuit.

ω_0 ω_1 ω_2 BW Q

- $\omega_0 = \frac{1}{LC}$
- $\omega_1 = -\frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$
- $\omega_2 = \frac{1}{2RC} + \sqrt{(\frac{1}{2RC})^2 + \frac{1}{LC}}$
- $BW = \omega_2 - \omega_1 = \frac{1}{RC}$
- $Q = \frac{\omega_0}{BW} = \omega_0 RC = \frac{R}{\omega_0 L}$

Lowpass Filter

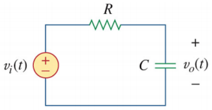


Figure 14.32 A lowpass filter.

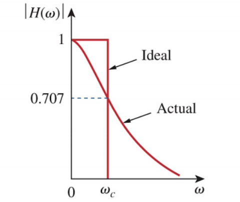


Figure 14.32 Ideal and actual frequency response of a lowpass filter.

- $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega} = \frac{1}{1+j\omega RC}$
- $\left|H(j\omega_c) = \frac{1}{\sqrt{2}}\right| \rightarrow \omega_c = \frac{1}{RC}$

Highpass Filter

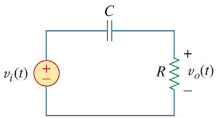


Figure 14.34 A highpass filter.

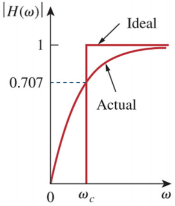


Figure 14.34 Ideal and actual frequency response of a highpass filter.

- $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega} = \frac{j\omega RC}{1+j\omega RC}$
- $\left| H(j\omega_c) = \frac{1}{\sqrt{2}} \right| \rightarrow \omega_c = \frac{1}{RC}$

Bandpass Filter

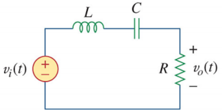


Figure 14.35 A bandpass filter.

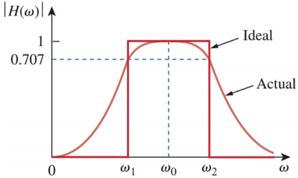


Figure 14.36 Ideal and actual frequency response of a bandpass filter.

- $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega} = \frac{R}{R+j(\omega L - \frac{1}{\omega C})}$

Bandstop Filter

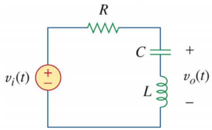


Figure 14.37 A bandstop filter.

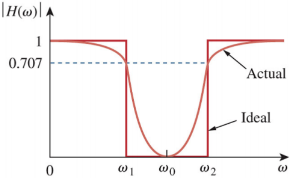


Figure 14.38 Ideal and actual frequency response of a bandstop filter.

- $H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$

First-Order Lowpass Filter

$$H(j\omega)=\frac{\tilde{V}_o(j\omega)}{\tilde{V}_i(j\omega)}=-\frac{R_f \parallel [1/(j\omega C_f)]}{R_i}$$

Inverting amplifier:
$H(j\omega)=-\mathbf{Z_f}/\mathbf{Z_1}$

$$=-\frac{R_f}{R_i}\frac{1}{1+j\omega R_f C_f}=-\frac{R_f}{R_i}\frac{1}{1+j\omega/\omega_c}$$

$$\omega_c=1/(R_f C_f)$$

Recall lowpass filter H(jω)

$$H(j\omega)=\frac{\tilde{V}_o(j\omega)}{\tilde{V}_i(j\omega)}=\frac{1}{1+j\omega RC}$$

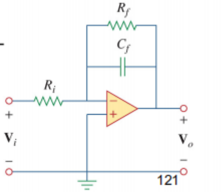


Figure 14.42 Active first-order lowpass filter.

First-Order Lowpass Filter

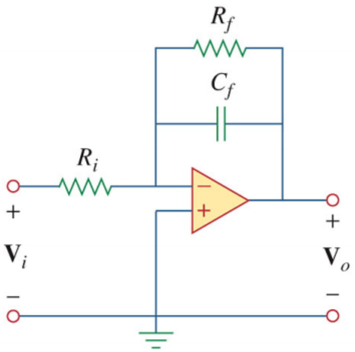


Figure 14.42 Active first-order lowpass filter.

First-Order Highpass Filter

$$H(j\omega)=\frac{\tilde{V}_o(j\omega)}{\tilde{V}_i(j\omega)}=-\frac{R_f}{R_i+1/(j\omega C_i)}$$

$$=-\frac{R_f}{R_i}\frac{1}{1+1/(j\omega R_i C_i)}=-\frac{R_f}{R_i}\frac{j\omega/\omega_c}{1+j\omega/\omega_c}$$

$$\omega_c=1/(R_i C_i)$$

Inverting amplifier:
$H(j\omega)=-\mathbf{Z}_f/\mathbf{Z}_1$

Recall highpass filter $H(j\omega)$

$$H(j\omega)=\frac{\tilde{V}_o(j\omega)}{\tilde{V}_i(j\omega)}=\frac{j\omega RC}{1+j\omega RC}$$

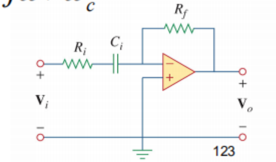


Figure 14.43 Active first-order highpass filter.

First-Order Highpass Filter

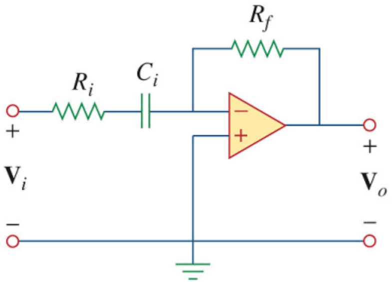


Figure 14.43 Active first-order highpass filter.

Bandpass Filter

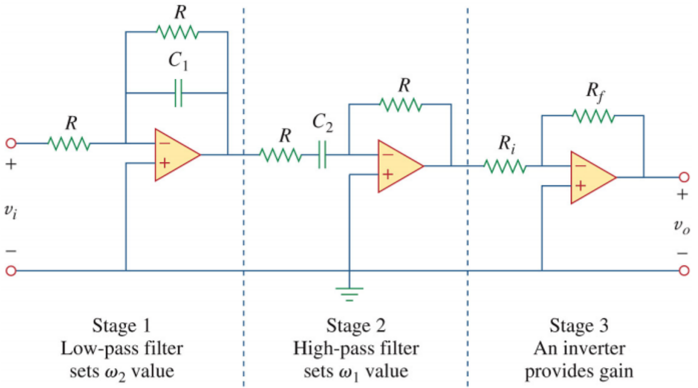


Figure 14.45 Active bandpass filter.

Bandpass Filter

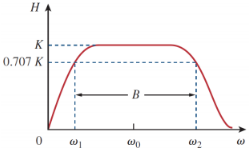
$$H(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}$$

$$=\left(-\frac{R}{R}\frac{1}{1+j\omega RC_1}\right)\left(-\frac{R}{R}\frac{j\omega RC_2}{1+j\omega RC_2}\right)\left(-\frac{R_f}{R_i}\right)$$

$$=-\frac{R_f}{R_i}\frac{1}{1+j\omega RC_1}\frac{j\omega RC_2}{1+j\omega RC_2}$$

$$=-\frac{R_f}{R_i}\frac{1}{1+j\omega/\omega_2}\frac{j\omega/\omega_1}{1+j\omega/\omega_1}$$

$$=-\frac{R_f}{R_i}\frac{j\omega/\omega_1}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)}$$



(b)

- The lowpass section sets $\omega_2 = \frac{1}{RC_1}$.
- The highpass section sets $\omega_1 = \frac{1}{RC_2}$.

Bandreject Filter

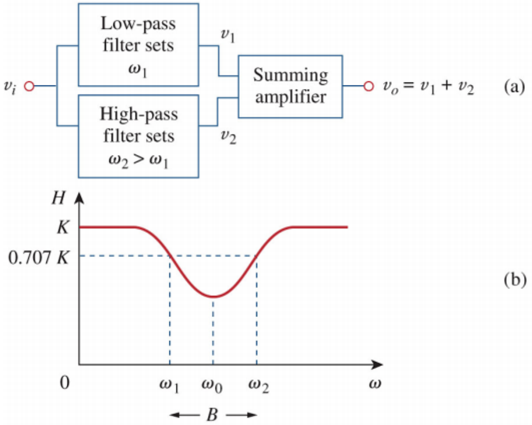


Figure 14.46 Active bandreject filter: (a) block diagram, (b) frequency response.

Bandreject Filter

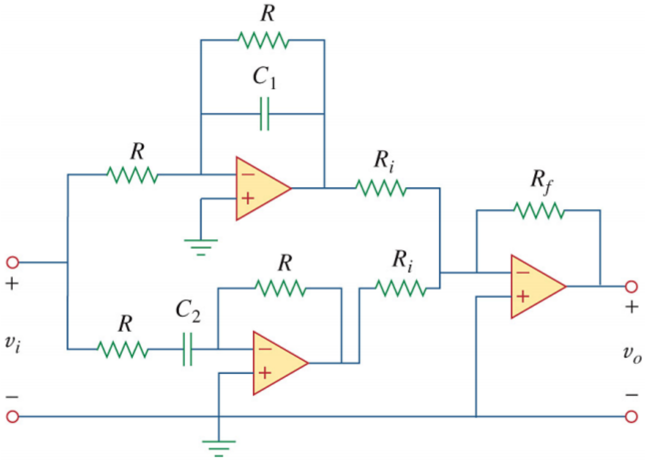


Figure 14.47 Active bandreject filter.

Bandreject Filter

$$H(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}$$
$$=\frac{R_f}{R_i}\left(\frac{1}{1+j\omega RC_1}+\frac{j\omega RC_2}{1+j\omega RC_2}\right)$$

$$H(j\omega)=\frac{R_f}{R_i}\left(\frac{1}{1+j\omega/\omega_1}+\frac{j\omega/\omega_2}{1+j\omega/\omega_2}\right)$$
$$=\frac{R_f}{R_i}\frac{1+2(j\omega)/\omega_2+(j\omega)^2/(\omega_1\omega_2)}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)}$$
$$=K\frac{1+2\zeta(j\omega)/\omega_0+(j\omega/\omega_0)^2}{(1+j\omega/\omega_1)(1+j\omega/\omega_2)}$$
$$K=\frac{R_f}{R_i},\ \omega_0=\sqrt{\omega_1\omega_2},\ \zeta=\sqrt{\frac{\omega_1}{\omega_2}}$$

- The lowpass section sets $\omega_1=\frac{1}{RC_1}$.
- The highpass section sets $\omega_2=\frac{1}{RC_2}$.

The End