6. High-Speed Signal-Free Intersections I

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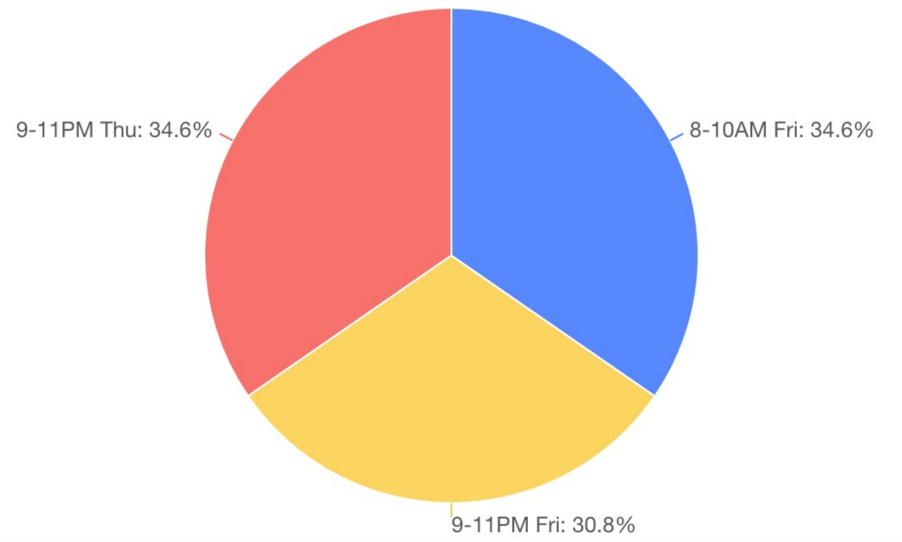
Logistics

- Recitation II: Friday 8AM via Feishu.
- HW1 graded
  - Good job! (Avg. 9.26/10)
  - Neat calligraphy
  - Clear labeling
  - Simulation required
  - Position tracking
  - Negative speed



- HW2 to be announced
  - Due on 6/7
  - Office hour: Fri 6/4 1:30-3PM, Longbin Lou 424
- Project teaming (1st round): 6/7

Grading

- University policy: class avg $\leq$ B+ (88/100)

Letter Grade	Percentage	Grade Point Average (GPA)
A	90%-100%	4.0
B	80%-89%	3.0
C	74%-79%	2.0
D	70%-73%	1.0
F	0%-69%	0.0

- HW avg (expected) = 9.0/10 (40%)
- Midterm avg (expected) = 85/100 (30%)
- Project avg (expected, teamwork) = 85/100 (30%)

Recap

- Technological basis
  - Autonomous driving
  - Vehicle-to-vehicle coordination
- Classical approach
  - Modeling
  - Decision making
- Learning-based approach
  - Objective
  - Design

Outline

- Background
  - Signalized & unsignalized intersections
  - Connected & autonomous vehicles
  - Vehicle-to-infrastructure connectivity
- Control in nominal setting
  - Centralized approach
  - Decentralized approach
  - Hierarchical control
  - **HW2**
- Control in face of disruptions
  - How to address latency
  - How to address packet loss
  - How to address malicious attacks

Background

- Signalized & unsignalized intersections
- Connected & autonomous vehicles
- Vehicle-to-infrastructure connectivity

Signalized intersection

Signalized intersection: Fixed cycle design

- Data:
  - Traffic demand in each direction
  - Saturation rate & response time
- Decision variables
  - Green ratio/time in each direction
- Constraint
  - Safety (no simultaneous greens)
  - Technical constraint (switching frequency)
- Objective
  - Ensure bounded waiting time #
  - Minimize average waiting time

Signalized intersection: Adaptive cycles

- Data:
  - Saturation rate & response time
  - Real-time traffic state on each lane
- Decision variables
  - Signaling in the next decision period
  - Or, policy for signaling
- Constraint
  - Safety (no simultaneous greens)
  - Technical constraint (switching frequency)
- Objective
  - Ensure bounded waiting time
  - Minimize expected waiting time

Unsignalized intersection

- Typically, vehicles are supposed to stop as they arrive at the intersection.
- Then, vehicles cross according to convention or rule.
- Could be chaotic…
- http://heze.dzwww.com/qx/yc/201908/t20190810_17039691.htm

High-speed signal-free intersections

Connected and Autonomous Vehicles (CAVs):

- Vehicle to vehicle/infrastructure (V2V/V2I) connectivity
- Low response time and high speed

https://www.bilibili.com/video/av503115801/

High-Speed Signal-Free intersections:

Latency? Packet loss?

Quality of communication → Contribute to → Safety and efficiency

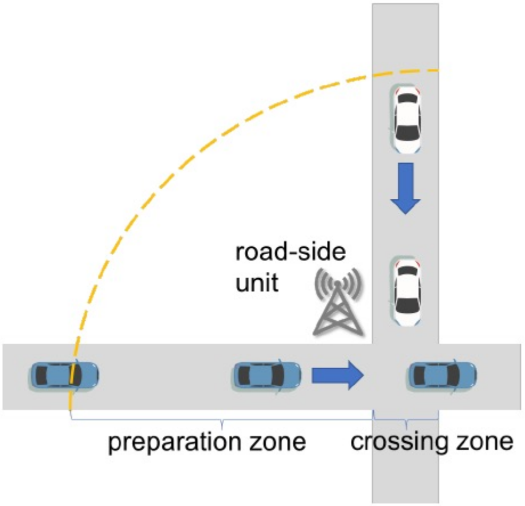
Control in nominal setting

- Centralized approach
- Decentralized approach
- Hierarchical control
- HW2

Modeling

The states of the intersection:

- Two orthogonal routes: 1 and 2;
- Positions: $x^k = \left[x_1^k(t)\ x_2^k(t) \cdots x_{n_k}^k(T)\right]^T$;
- Speeds: $v^k = \left[v_1^k(t)\ v_2^k(t) \cdots v_{n_k}^k(T)\right]^T$.

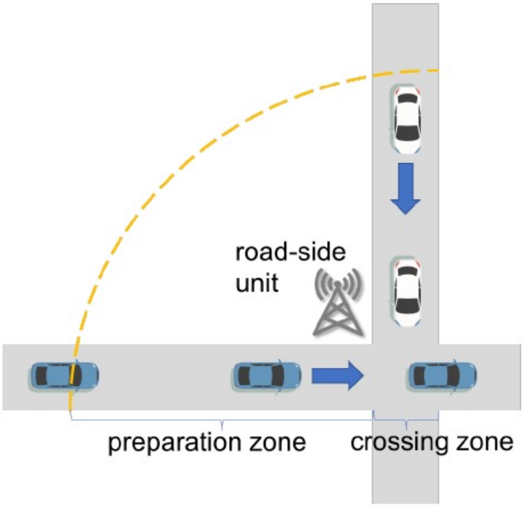


Task:

- Assume that the trajectories of all vehicles in front of vehicle i have been optimized and fixed;
- Optimize $x_i^j(t)$ and $v_i^j(t)$.

Centralized coordination

- Data:
  - Initial conditions $x_i^k(0), v_i^k(0)$
  - Maximal speed & acceleration $\bar{v}, \bar{a}$
  - Safe distance d (simplistic)
  - Control zone radius L
- Decision variables:
  - Time series of speed $v_i^k(t)$
- Constraints:
  - No collision (and no overtaking)
  - Saturation
- Objective function:
  - Minimize fuel cost $\sum_{t=0}^{T} \sum_k \sum_i \left(v_i^k(t)\right)^2$



Constraints

- For vehicles on the same orbit

$$x_i^k(t) - x_{i+1}^k(t) \geq d, \forall i, \forall t, \forall k$$

- For vehicles on different orbits, let's be simplistic #

$$\left(L - x_i^1(t)\right) + \left(L - x_j^2(t)\right) \geq d, \forall i, \forall j, \forall t$$

- A more realistic (and more complex) set of constraints:
    - For vehicles on the same orbit, the headway (rather than distance) between two vehicles should not exceed a certain threshold.
    - For vehicles on different orbits, their look-ahead positions do not interfere with each other.

Policy-free formulation

$$
\begin{aligned}
\min \quad & \sum_{t=0}^{T} \sum_k \sum_i \left(v_i^k(t) \right)^2 \\
\text{s.t.} \quad & x_i^k(t) - x_{i+1}^k(t) \geq d, \forall i, \forall t, \forall k \\
& \left(L - x_i^1(t) \right) + \left(L - x_j^2(t) \right) \geq d, \forall i, \forall j, \forall t \\
& 0 \leq v_i^k(t) \leq \bar{v}, \forall i, \forall t, \forall k \\
& -\bar{a} \leq v_i^k(t) - v_i^k(t-1) \leq \bar{a}, \forall i, \forall t, \forall k
\end{aligned}
$$

You are solving this. 

Homework 2

Problem 1: Coordination of 10 autonomous vehicles at a centralized intersection.

- **Part a**: Randomly generate initial condition. That is, randomly generate two vectors

$$x^1(0) = [x_1^1(0), \dots, x_5^1(0)]^T$$
$$x^2(0) = \left[x_1^2(0), \dots, x_5^2(0)\right]^T$$

$x_i^r(0)$ takes values in 0-100 m; $x_i^r = 100$ means vehicle i has left the intersection.

- **Part b**: We consider vehicles as points. The safe spacing between two vehicles is 6 m. Is your initial condition safe? If not, modify the initial condition so that the safe spacing constraint is satisfied.

Homework 2

- **Part c**: Our decision variables are the speeds at various times. Suppose that the one-step fuel cost induced on vehicle i is given by $\left(v_i^k(t)\right)^2$. Our objective is to discharge all vehicles with the minimal fuel consumption. Formulate the trajectory planning problem in the four-stage representation, viz. data, decision variables, constraints, and objective function.
- **Part d**: Assume zero initial speeds, i.e. $v_i^k(0) = 0$ for each i. Suppose maximal speed $\bar{v} = 10\ m/s$ and maximal acceleration (i.e. speed increment) is $5\ m/s^2$. Construct a feasible solution. Report the total fuel consumption.
- **Part e**: Let T be the time at which all 10 vehicles are discharged by the intersection. Provide an upper bound on T. Your bound does not have to be tight.

Homework 2

Problem 2: Optimization & sensitivity analysis.

- **Part a**: Code the optimization problem in MATLAB/Python/C. Note that you can use your result in problem 1e to define the dimension of the decision variables.
  - https://cvxopt.org/examples/tutorial/qp.html
  - https://www.mathworks.com/help/optim/ug/quadprog.html
- **Part b**: Compute the optimal solution and report the fuel consumption. Compare with your result in problem 1.
- **Part c**: Gradually change the value for the safe spacing and plot the corresponding optimal fuel cost. You plot should be fuel cost vs. safe spacing.

Problem 3: Impact of noise.

- **Part a**: Add a noise term to the system dynamics, i.e.

$$x_i^k(t+1) = x_i^k(t) + v_i^k(t)\Delta t + \epsilon.$$

Then, implement the speed commands you generated in problem 2b.

- **Part b**: Two vehicles i and j are said to interfere if $\left|x_i^r(t) - x_j^r(t)\right| < 6\ m$ and if $x_i^r, x_j^r < 100$. Let $N(t)$ be the number of interferences at time t. Plot $N(t)$ vs. t.

Policy-based formulation: centralized policy

$$\min \sum_{t=0}^{T} \sum_k \sum_i \left(v_i^k(t)\right)^2$$

s.t. $v(t+1) = \mu(x(t), v(t))$ (vector-valued function)

$$x_i^k(t) - x_{i+1}^k(t) \geq d, \forall i, \forall t, \forall k$$

$$\left(L - x_i^1(t)\right) + \left(L - x_j^2(t)\right) \geq d, \forall i, \forall j, \forall t$$

$$0 \leq v_i^k(t) \leq \bar{v}, \forall i, \forall t, \forall k$$

$$-\bar{a} \leq v_i^k(t) - v_i^k(t-1) \leq \bar{a}, \forall i, \forall t, \forall k$$

Policy-based formulation: decentralized policy

min $\sum_{t=0}^{T}\sum_k\sum_i\left(v_i^k(t)\right)^2$

s.t. $v_i^k(t+1)=\mu\left(x_i^k(t),v_i^k(t)\right)$ (scalar-valued function)

$$x_i^k(t)-x_{i+1}^k(t)\geq d,\forall i,\forall t,\forall k$$

$$\left(L-x_i^1(t)\right)+\left(L-x_j^2(t)\right)\geq d,\forall i,\forall j,\forall t$$

$$0\leq v_i^k(t)\leq\bar{v},\forall i,\forall t,\forall k$$

$$-\bar{a}\leq v_i^k(t)-v_i^k(t-1)\leq\bar{a},\forall i,\forall t,\forall k$$

Policy-based formulation: solution

- How to determine the optimal policy?
- In general, we need to use numerical methods.
- In very special cases, we can find an analytical solution. (Unfortunately, the intersection control problem does not belong to these "special cases".)
- What special cases?
  - Linear time-invariant (LTI) system
  - Quadratic cost function
  - No other constraints (e.g. no saturation)
- The optimal policy for the above cases is linear-quadratic regulator (LQR).*

A simple optimal control problem

- Consider an AV
$$\begin{bmatrix}\dot{x}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ u\end{bmatrix}$$
- x = position, v = speed (states)
- u = acceleration (control input)
- Initial state: $x(0) = -L$, $v(0) = 0$
- Required final state: $x(T) = 0$, $v(T) = 0$ (equilibrium)
- Cost to minimize: $J = \sum_{t=0}^{T} \frac{1}{2} v^2(t)$
  - Note that $\frac{1}{2}v^2 = \frac{1}{2}\begin{bmatrix}x & v\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ v\end{bmatrix}$
- We consider a linear controller $u = -K\begin{bmatrix}x\\ v\end{bmatrix}$

LQR optimal control*

- Consider an LTI system
$$\dot{x} = Ax + Bu$$
- Suppose that we want to drive the system from the initial state $x(0)$ to the target state $x(T) = 0$
- Instantaneous cost
$$L(t) = \frac{1}{2}x^T(t)Qx(t) + \frac{1}{2}u^T(t)Ru(t)$$
- The cost induced during the control process is given by
$$\frac{1}{2}x^T(T)Qx(T) + \int_{s=0}^{T} L(s)ds$$
- Linear feedback $u = -Kx$

LQR optimal control*

- Linear-quadratic regulator (LQR)
- Design task: obtain the optimal gain matrix K for $u = -Kx$
- Conclusion:
  - Let P be the solution matrix to the matrix Riccati equation
  $$PA + A^T P + Q - PBR^{-1}B^T P = 0$$
  - Then the optimal K is given by
  $$K = -R^{-1}B^T P$$
- Design task: pick Q and R (i.e. cost function)
- Reference: Zhou, K., Doyle, J. C., & Glover, K. (1996). *Robust and optimal control* (Vol. 40, p. 146). New Jersey: Prentice hall.

Hierarchical coordination

- In the previous slides, we were actually doing two things simultaneously: sequencing & trajectory tracking
- A more intuitive way is to decompose these two tasks
- A hierarchical decision-making framework
  - Upper level: scheduling/sequencing
  - Lower level: trajectory planning
  - Can you write the four-stage formulations of the above?
- Such a hierarchical framework makes more practical sense:
  - A centralized controller determines vehicle sequencing
  - Then, each vehicle determines its own trajectory to fulfill the designated sequencing

Control in face of disruptions

- How to address latency
- How to address packet loss
- How to address malicious attacks

How to address latency

Liu, Y., Nicolai-Scanio, Z., Jiang, Z.-P. and Jin, L. 2021, May. Latency-robust control of high-speed signal-free intersections. In *2021 American Control Conference.*

- **Latency** → delayed state observation
- Vehicle **trajectories** are subject to bounded uncertainty.

The main questions we ask here:

- How to design a vehicle coordination algorithm that is robust against communication latency?
- How to quantify the relation between communication latency and intersection capacity?

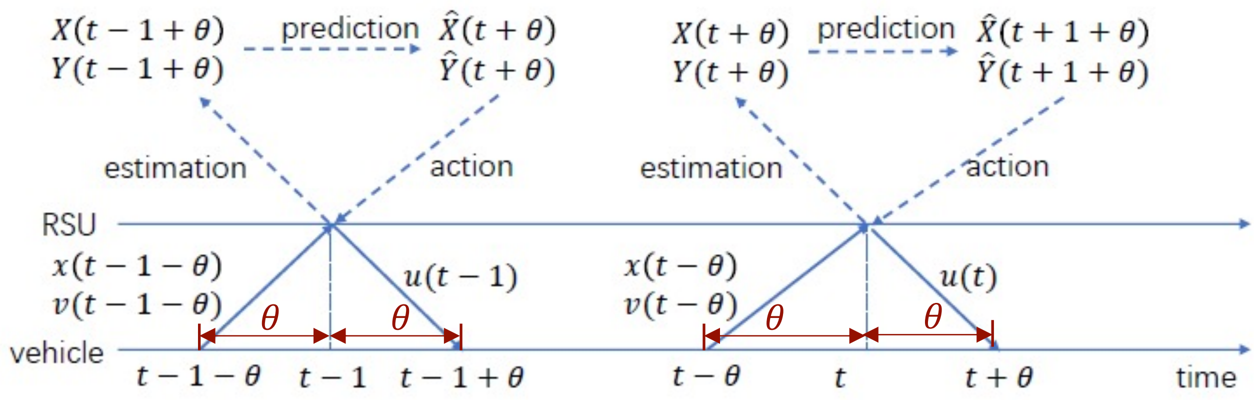
Communication under Latency

- Control input: the sequence of target speeds $\{u(t): t \in \mathbb{Z}_{\geq 0}\}$:

$$u_i(t) = \mu_i(\hat{x}_i(t), \hat{v}_i(t), \hat{x}_{i-1}(t), \hat{v}_{i-1}(t))$$

- Latency: θ
- The state $(\hat{x}(t), \hat{v}(t))$ that the road-side unit (RSU) receives at time t:

$$\hat{x}(t) = x(t-\theta);$$
$$\hat{v}(t) = v(t-\theta).$$



Problem Formulation

Propose a controller:

- Robust against communication latency
- Satisfy the safety constraint (lower-bounds the headway between vehicles):

(i) $i \neq j, k = 1, 2$:

$$|x_i^k(t) - x_j^k(t)| \begin{cases} \geq h v_j^k(t) & x_i^k(t) > x_j^k(t), \\ \geq h v_i^k(t) & x_i^k(t) < x_j^k(t), \end{cases}$$

The **distance** between two vehicles on the **same** lane $\geq$
Minimum headway for **same** lane
$\times$
the speed of the vehicle behind

(ii) $k_1 \neq k_2, x_i^{k_1} \leq -R, x_j^{k_2} \leq -R$:

$$|x_i^{k_1}(t) - x_j^{k_2}(t)| \begin{cases} \geq \bar{h} v_j^{k_1}(t) & x_i^{k_1}(t) > x_j^{k_2}(t), \\ \geq \bar{h} v_i^{k_1}(t) & x_i^{k_1}(t) < x_j^{k_2}(t), \end{cases}$$

The **distance** between two vehicles on **orthogonal** lanes $\geq$
Minimum headway for **orthogonal** lanes
$\times$
the speed of the vehicle behind

Evaluate the capacity of the intersection

Estimator Design

Estimator Design → Controller Design

Given:

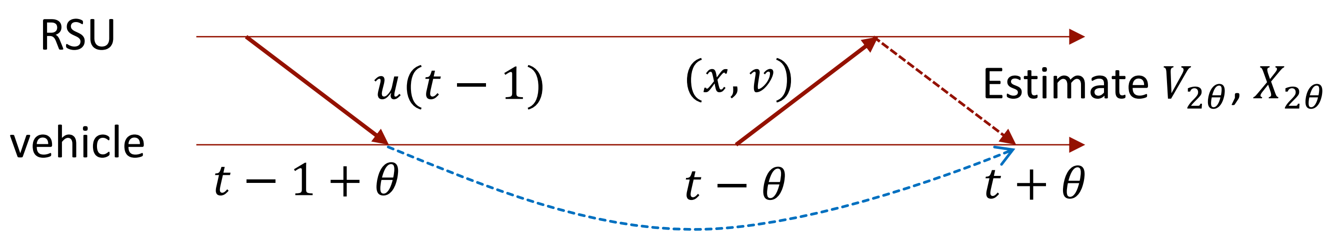
- **Model parameters:** θ (latency), ϵ (speed uncertainty)
- **Observed variables:** $x(t-\theta), v(t-\theta), u(t-1), u(t)$

Estimate:

- **Positions** $x(t+\theta)$ and **speeds** $v(t+\theta)$

Estimator:

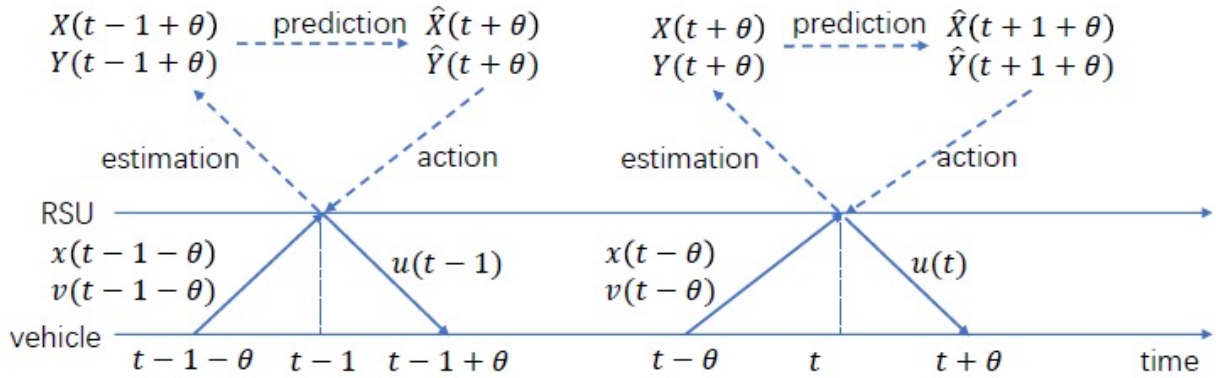
- $V_{2\theta}(x,v) = \{v' \in V: u(t-1) - \epsilon \le v' \le u(t-1) + \epsilon\}$
- $X_{2\theta}(x,v) = \left\{x' \in X: \begin{array}{c} x + \theta(v + u(t-1) - \epsilon) \le x' \le \\ x + \theta(v + u(t-1) + \epsilon) \end{array}\right\}$



Estimator Design

Predictor:

- $\tilde{V}(t+\theta+1) = \{v' \in V: u(t) - \epsilon \le v' \le u(t) + \epsilon\}$
- $\tilde{X}(t+\theta+1) = X_{2\theta}\big(\hat{x}(t), \hat{v}(t)\big) + \frac{\delta}{2}\Big(V_{2\theta}\big(\hat{x}(t), \hat{v}(t)\big) + \tilde{V}(t+\theta+1)\Big)$



- The interval of $\tilde{X}(t+\theta+1)$:

$$\Delta = 2\epsilon(\theta + \delta)$$

Controller Design

Given:

- **Model parameters:** θ (latency), ϵ (speed uncertainty), δ (time step size), $\bar{a}$ (maximum acceleration), h (minimum allowable headway)
- **Observed variables:** $x(t-\theta)$ (positions), $v(t-\theta)$ (speeds)

Determine:

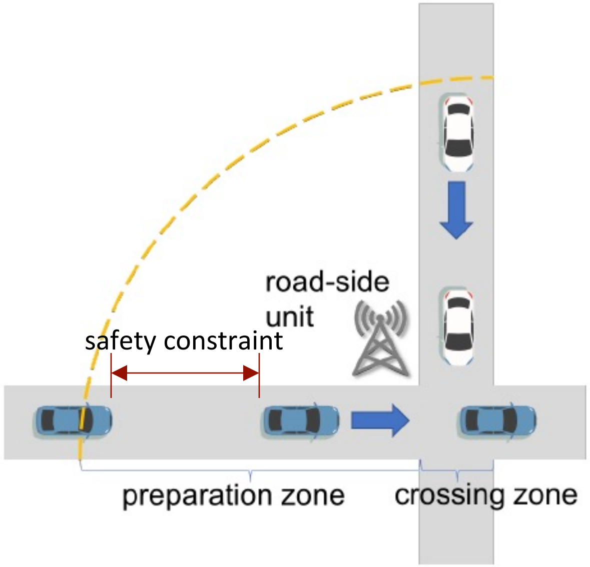
- **Target speeds:** $u_i(t)$, $i = 1,2, \ldots,$ for all $t \geq 0$

Controller Design

Safety Constraint:

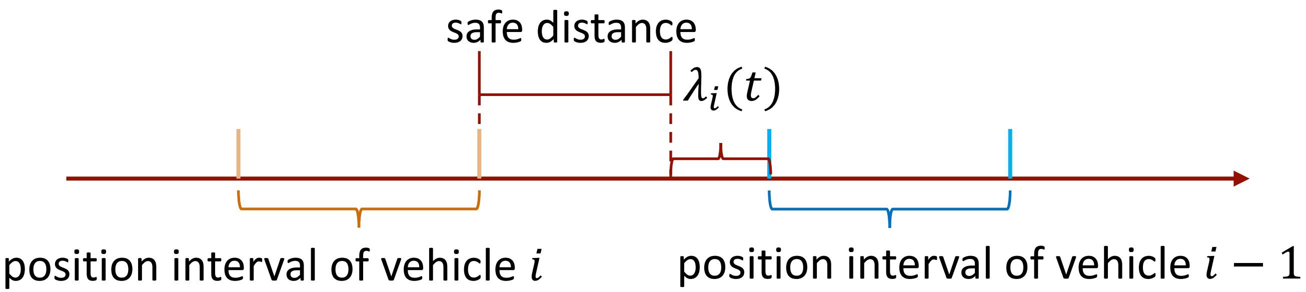
$$\left|\tilde{X}_i^{k_1}(t+1) - \tilde{X}_j^{k_2}(t+1)\right| \geq$$

$$\begin{cases} h\tilde{V}_j^{k_2}(t+1) & k_1 = k_2,\ \tilde{X}_i^{k_1}(t+1) > \tilde{X}_j^{k_2}(t+1), \\ h\tilde{V}_i^{k_1}(t+1) & k_1 = k_2,\ \tilde{X}_i^{k_1}(t+1) < \tilde{X}_j^{k_2}(t+1), \\ \bar{h}\tilde{V}_j^{k_2}(t+1) & k_1 \neq k_2,\ \tilde{X}_i^{k_1}(t+1) > \tilde{X}_j^{k_2}(t+1), \\ \bar{h}\tilde{V}_i^{k_1}(t+1) & k_1 \neq k_2,\ \tilde{X}_i^{k_1}(t+1) < \tilde{X}_j^{k_2}(t+1). \end{cases}$$



Objective Function:

$$\lambda_i(t) = \tilde{X}_{(i-1),min}(t+\theta+1) - \tilde{X}_{i,max}(t+\theta+1) - hu_i(t)$$



Controller Design

Control Law:

- **Information needed:**

state information of vehicle i

state information of vehicle $i-1$

- **Feasibility:**

$$U_{feasible}(t) = [u(t-1) - \bar{a}\delta + \epsilon, u(t-1) + \bar{a}\delta - \epsilon]$$

- **Controller:**

$$\begin{aligned} u_i(t) &= \mu_i(x_i(t-\theta), v_i(t-\theta), x_{i-1}(t-\theta), v_{i-1}(t-\theta)) \\ &= \operatorname*{arg\,min}_{u_i(t)\in U_{feasible}(t)} \{\lambda_i(t) | \lambda_i(t) \geq 0\}. \end{aligned}$$

Linear w.r.t $u_i(t)$!

Safety Criteria for One Time Step*

Theorem 1:

- The existence of the safety control input depends on the following conditions:

1. There exists a safe control input $u(t)$ if and only if it satisfies the following condition:

$$\hat{x}_i(t) - \hat{x}_{i-1}(t) + \theta\big(\hat{v}_i(t) - \hat{v}_{i-1}(t)\big) + (\theta + \delta + h)u_i(t-1) - \left(\theta + \frac{\delta}{2}\right)u_{i-1}(t-1) - \frac{\delta}{2}u_{i-1}(t) + 2\epsilon\theta + \frac{3}{2}\delta\epsilon - \frac{1}{2}\bar{a}\delta^2 - h\bar{a}\delta + h\epsilon \leq 0.$$

(condition 1)

Vehicle i must be able to **keep the safe distance** from vehicle $i-1$ when it **takes the maximum deceleration**.

Safety Criteria for One Time Step*

Theorem 1:

- The existence of the safety control input depends on the following conditions:

2. There exists a safe control input $u(t)$ such that vehicle i is able to keep the minimal allowable distance from the leading vehicle if and only if condition 1 and the following condition hold:

$$\hat{x}_i(t) - \hat{x}_{i-1}(t) + \theta\left(\hat{v}_i(t) - \hat{v}_{i-1}(t)\right) + (\theta + \delta + h)u_i(t-1) - \left(\theta + \frac{\delta}{2}\right)u_{i-1}(t-1) - \frac{\delta}{2}u_{i-1}(t) + 2\epsilon\theta + \frac{1}{2}\delta\epsilon + \frac{1}{2}\bar{a}\delta^2 + h\bar{a}\delta - h\epsilon \geq 0.$$

(condition 2)

While ensuring safety (condition 1 holds), vehicle i also must be able to keep the distance from vehicle $i-1$ which is **smaller than the minimal allowable distance** when it takes the **maximum acceleration**.

Safety Criteria for Time Series*

Theorem 2:

- Consider the observed initial state $\hat{x}(1)$, $\hat{v}(1)$ and initial control input $u(0)$:

1. There exists a safe input for all subsequent times if the following conditions hold:

(a) For $t = 1$ and for any $i = 1, \ldots, n$,

Condition 1 holds for initial states.

$$\hat{x}_i(1) - \hat{x}_{i-1}(1) + \theta\left(\hat{v}_i(1) - \hat{v}_{i-1}(1)\right) + (\theta + \delta + h)u_i(0) - \left(\theta + \frac{\delta}{2}\right)u_{i-1}(0) - \frac{\delta}{2}u_{i-1}(1) + 2\epsilon\theta + \frac{3}{2}\delta\epsilon - \frac{1}{2}\bar{a}\delta^2 - h\bar{a}\delta + h\epsilon \leq 0.$$

(b) For $t = 2,3, \ldots$ and for any $i = 1, \ldots, n$,

$$\left(\frac{3}{2}\delta + h\right)u_i(t) - \left(\frac{\delta}{2} + h\right)u_i(t-1) - \delta u_{i-1}(t) + \frac{\delta}{2}(\bar{a}\delta + \epsilon) \leq 0.$$

If for time t condition 1 holds, then for time t+1 condition 1 also holds.

(condition 3)

Safety Criteria for Time Series*

Theorem 2:

- Consider the observed initial state $\hat{x}(1)$, $\hat{v}(1)$ and initial control input $u(0)$:

2. There exists a safe input such that vehicle i is able to keep the minimal allowable distance from vehicle $i-1$ at all times if condition 3 and the following condition hold:

(a) For $t = 1$,

Condition 2 holds for initial states

$$\hat{x}_i(1) - \hat{x}_{i-1}(1) + \theta\left(\hat{v}_i(1) - \hat{v}_{i-1}(1)\right) + (\theta + \delta + h)u_i(0) - \left(\theta + \frac{\delta}{2}\right)u_{i-1}(0) - \frac{\delta}{2}u_{i-1}(1) + 2\epsilon\theta + \frac{1}{2}\delta\epsilon + \frac{1}{2}\bar{a}\delta^2 + h\bar{a}\delta - h\epsilon \leq 0.$$

(b) For $t = 2,3, \ldots$ and for any $i = 1, \ldots, n$,

$$\left(\frac{3}{2}\delta + h\right)u_i(t) - \left(\frac{\delta}{2} + h\right)u_i(t-1) - \delta u_{i-1}(t) - 4\epsilon\theta - \frac{3}{2}\epsilon\delta - \frac{1}{2}\bar{a}\delta^2 \geq 0.$$

If for time t condition 2 holds, then for time t+1 condition 2 also holds.

(condition 4)

Controller*

Controller:

$$
\begin{aligned}
u_i(t) &\in U_{feasible}(t) \\
&= \mu_i(x_i(t-1), v_i(t-1), x_{i-1}(t-1), v_{i-1}(t-1)) \\
&= \begin{cases}
\theta\Big(\hat{v}_{i-1}(t) - \hat{v}_i(t) + u_{i-1}(t-1) - u_i(t-1)\Big) \\
-2\delta\epsilon - 2\theta\epsilon + \hat{x}_{i-1}(t) - \hat{x}_i(t) + \frac{1}{2}\delta\Big(u_{i-1}(t-1) \\
-u_i(t-1) + u_{i-1}(t)\Big)/(\frac{1}{2}\delta + h), \text{ condition } 1 \\
\text{and condition } 2, \\
\arg\min \lambda_i(t), \text{ condition } 1 \text{ and not condition } 2, \\
not\ safe, \text{ else.}
\end{cases}
\end{aligned}
$$

Be able to obtain the minimal allowable distance

Be able to keep safe distance

Throughput Evaluation*

Capacity:

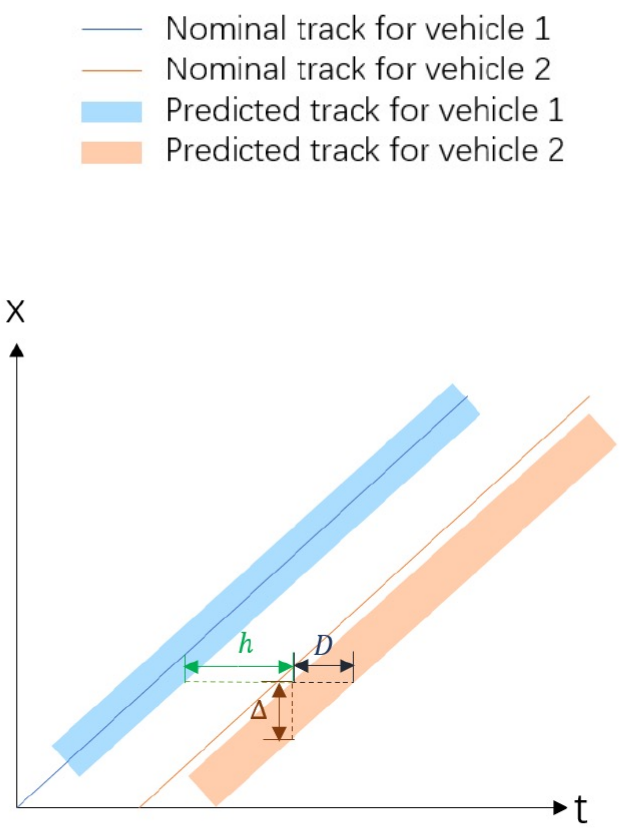
- Crossing time interval: D ;
- Minimal headway: h
- The capacity F of the intersection is lower bounded by:

$$F \geq \frac{1}{D+h}$$

Proposition 1:

- Under the specific situation when all the vehicles are able to satisfy the condition 1 and 2 in Theorem 1, capacity F is lower bounded by:

$$F \geq \frac{\bar{v}}{2\epsilon(\theta+\delta)+h\bar{v}}$$



How to address packet loss

- Suppose that a signal-free intersection requires synchronization of each vehicle's kinematic information every 100ms.
- Every synchronization involves 100 quantities (position, speed, route, etc.).
- However, due to disruption in wireless communication, some information may not be synced.
- This can be problematic… #
- How to address this? (similar to latency-robust control)
    - Estimate: try to recover the lost information
    - Control: insert safety margin for uncertainty

How to address packet loss

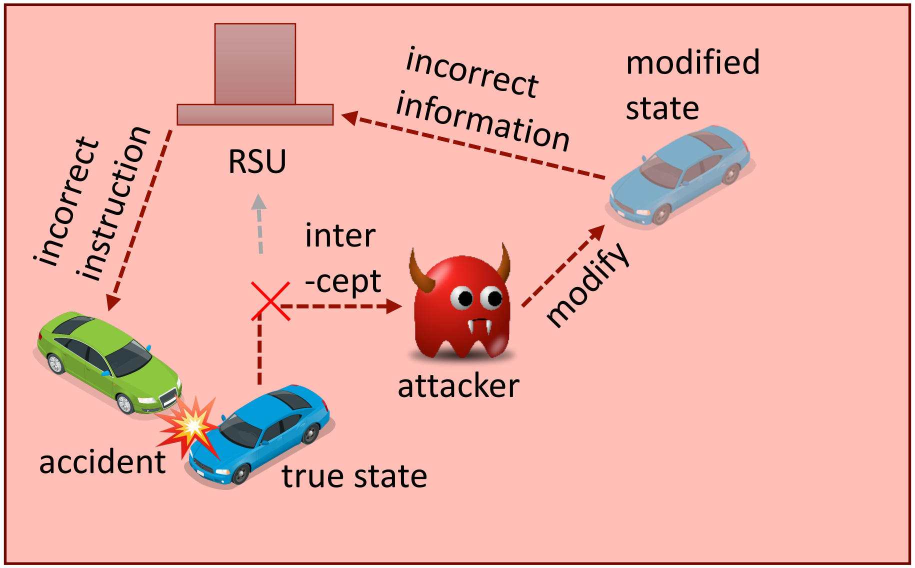
- Stochastic approach
  - Suppose that every piece of information is lost with probability α (e.g. 0.01)
  - Design the **safety margin** so that the probability of interference is no greater than β (e.g. 10^{-4})
- Robust approach
  - Suppose that no more than N pieces of information can be lost per cycle
  - Design the **safety margin** so that the worst combination of the lost information does not lead to interference
- Also, the intersection can invoke onboard sensing to augment its decision making.

How to address malicious attacks

- Suppose that an attacker can cut off A pieces of information per unit time.
- The attacker wants to create as many interference as possible.
- If interference is indeed impossible, the attacker wants to force the intersection to set a safety margin as large as possible (i.e. maximal efficiency loss).
- The system operator (defender) should design the information network so that it is the hardest to hack.

How to address malicious attacks

- An even nastier attacker: can modify correct information
- A strategic attacker may be aware of the defender's diagnosis & response method and can design its attack accordingly…



Summary

- Background
  - Signalized & unsignalized intersections
  - Connected & autonomous vehicles
  - Vehicle-to-infrastructure connectivity
- Control in nominal setting
  - Centralized approach
  - Decentralized approach
  - Hierarchical control
  - **HW2**
- Control in face of disruptions
  - How to address latency
  - How to address packet loss
  - How to address malicious attacks

Next time