Vv557 Methods of Applied Mathematics II

Green Functions and Boundary Value Problems

Assignment 3

Date Due: 12:55 PM, Wednesday, the 17$^{\text{th}}$ of March 2021

Discussion Class Preparation

Please (re-)view Video files 20-22 and/or finish reading the section "Differential operators and Types of Solutions" in the lecture slides. You should be able to answer the following questions:

i) Given an ordinary differential operator, explain what the formal adjoint, the conjunct, Green's formula and Lagrange's identity are.

ii) Explain what classical solutions, weak solutions and distributional solutions to a differential equation are. Give examples.

iii) What is a fundamental solution to a differential equation? Are fundamental solutions unique?

iv) What is a causal fundamental solution to a differential equation that involves a time variable? How is a causal fundamental solution found?

Exercises (25 Marks)

Exercise 3.1
Calculate the Fourier transforms of the following elements in $L^1(\mathbb{R})$ (the theory of distributions is not needed):

i) $e^{-a|x|}$, $a > 0$.

ii) e^{-ax^2}, $a > 0$.

iii) $\cos(x)e^{-x^2}$.

iv) $\cos(2x)/(4 + x^2)$.

v) the convolution of xe^{-x^2} and e^{-x^2}.

(10 Marks)

Exercise 3.2
Calculate the Fourier transforms of the following elements in $\mathcal{S}'(\mathbb{R})$:

i) $\begin{cases} e^{-\varepsilon x} & x \geq 0, \\ 0 & x < 0, \end{cases} \qquad \varepsilon > 0,$

ii) $\sin(3x - 2)$,

iii) $x^2 \cos(x)$,

iv) $xH(x - 2)$,

v) $x^2\delta(x - 1)$.

(10 Marks)

Exercise 3.3
Using primitives such as `Exp`, `Cos`, `DiracDelta`, `HeavisideTheta`, etc., as well as the commands `Convolve` and `FourierTransform` repeat all of the above calculations with Mathematica.
(5 Marks)