Shanghai Jiao Tong University ECE4530J

Homework 1: Due 5/24

Problem 1: Speed tracking

Consider the speed tracking problem in the slides of lecture 3.

Part a:

Reformulate the problem in discrete time (so that you can code). Suppose that you have an ODE

$$\frac{d}{dt}x(t) = f\big(x(t)\big).$$

You can discretize it as

$$\frac{x(t+\Delta t) - x(t)}{\Delta t} = f\big(x(t)\big).$$

So, the discrete dynamic equation is

$$x(t+\Delta t) = x(t) + f\big(x(t)\big)\Delta t.$$

This is how you can update the state in Python/Matlab/C++…

Hint: your response should be in this form:

$$v(t+\Delta t) = v(t) + f\big(v(t)\big)\Delta t.$$

Part b:

Find a controller T_e such that

$$e(t)\dot{e}(t) = -\frac{1}{2}\big(e(t)\big)^2.$$

Part c:

Assume $v_{ref}(t) = 1$ for all $t \geq 0$. Assume zero initial condition, i.e., $v(0) = 0$. Simulate the ODE

$$\dot{v}(t) = \alpha(T_e(t) - \beta - \gamma v(t)^2)$$

with the controller that you found in part 2. Plot $v(t)$ vs. t and label v_{ref}; please also label the axes!

Part d:

Select a time-varying reference speed $v_{ref}(t)$ and see if your controller still works. (Everyone should have his/her unique reference speed profile.) Plot $v(t)$ vs. t and label v_{ref}.

Problem 2: trajectory tracking

Assume the same setting as problem 1.

Part a:

Select a **linear** position profile, generate a corresponding speed profile

Part b:

Simulate the position tracking process. (Everyone should have his/her unique reference speed profile.) Plot the position $x(t)$ vs. t and label the reference position $x_{ref}(t)$.

Part c:

Select a **nonlinear** position profile, generate a corresponding speed profile

Part d:

Simulate the position tracking process. (Everyone should have his/her unique reference speed profile.) Plot the position $x(t)$ vs. t and label the reference position $x_{ref}(t)$.

Problem 3: Saturation

Assume the same setting as problem 1. For the torque profile $T_e(t)$ that you generated in problem 2, let

$$\bar{T}_e = 0.8 \max_t T_e(t).$$

Suppose now that the torque cannot exceed $\bar{T}_e$. Simulate the trajectory again and discuss the difference from the result in problem 2.

Problem 4: Noise

Assume the same setting as problem 1. Suppose that the update equation is

$$v(t + \Delta t) = v(t) + f\big(v(t)\big)\Delta t + \epsilon,$$

where f is your response to problem 1a and ϵ is a white noise, i.e., a normally distributed random variable with zero mean; select the variance of ϵ on your own. Simulate the trajectory again and discuss the difference from the result in problem 2.