
Question1 (1 points)

Find the interval of convergence for the power series

$$\sum_{n=10}^{\infty} \frac{(3x+2)^n}{n^2}$$

Question2 (1 points)

Find the Taylor series of the following function around $x = 0$

$$f(x) = \frac{2}{3x-5}$$

and determine the radius of convergence.

Question3 (1 points)

Find the Taylor series of

$$f(x) = \cos x$$

and show it converges to $f(x)$ for all $x \in \mathbb{R}$.

Question4 (1 points)

Find the value of the following series if it is convergent. If not, justify why it is divergent.

$$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

Question5 (1 points)

Let $f \in \mathcal{C}^3[a,b]$, that is, $f^{(3)}$ is continuous on $[a,b]$. Show there exists $c \in (a,b)$ such that

$$f(b) = f(a) + (b-a)f'\left(\frac{a+b}{2}\right) + \frac{1}{24}(b-a)^3 f^{(3)}(c)$$

Question6 (0 points)

Given that

$$\frac{1}{\sqrt{1-2xt+x^2}} = \sum_{n=0}^{\infty} P_n(t)x^n, \qquad \text{for} \quad |x| < 1$$

(a) (1 point (bonus)) Suppose $-1 < t < 1$, show that $P_0(t) = 1$ and $P_1(t) = t$, and

$$P_{n+1}(t) = \frac{2n+1}{n+1} t P_n(t) - \frac{n}{n+1} P_{n-1}(t), \qquad \text{for} \quad n \geq 1$$

(b) (1 point (bonus)) Show P_n is a polynomial of degree n, and

$$\frac{1}{\sqrt{1-2xt+x^2}}$$

is the generating function of the sequence $\{P_n\}$. P_n is known as the

Legendre polynomial