Vv156 Lecture 1

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UM-SJTU Joint Institute

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- Instructor:

  Jing Liu

- Lectures:

Monday	(08:00 – 09:40)	in F-215	Odd weeks only
Tuesday	(08:00 – 09:40)	in A-115	
Thursday	(08:00 – 09:40)	in A-215	

- Office Hours:

Tuesday/Thursday	(12:30pm – 03:30pm)	in JI-Building 441A
Tuesday/Thursday	(05:40pm – 06:40pm)	in E2-204

- Email:

  stephen.liu@sjtu.edu.cn

  When sending an email related to this course please include the tag [vv156] in the subject e.g. Subject: [vv156] Special Request

- Teaching Assistant/s:

  See Canvas for his/her contact information

- Assignment:

25%	Assignments will be given in the form of problem sets, and may require extra reading and the use of Matlab
Bonus	Assignments will have bonus questions. Solutions to the bonus questions will not be provided. Bonus points obtained from an assignment can only be credited to that assignment.
Location	Assignments need to be submitted at the beginning of class on the day indicated on the assignment.
Canvas	Assignments are strongly recommended to be submitted online through Canvas on time as well. Be reasonable in terms of size, quality and format.
Late	Please plan your time accordingly, late assignments will be severely penalised.

- Exam:

  75% There will be two exams:

Midterm	35%
Final	40%

  The final is not accumulative.

- Bonus:

  When 1% is all you needed to have a better grade, it may be given based on "class participation".

  Your TAs will recommend whether to give you this bonus based on their observation during recitation classes and the effort you put into your assignments.

  This bonus cannot be used to obtain "A^+".

- For this course, the grade will be curved to achieve a median grade of "B^+".

Honour Code

- Honesty and trust are important. Students are responsible for familiarising themselves with what is considered as a violation of honour code.
- Assignments/projects are to be solved by each student individually. You are encouraged to discuss problems with other students, but you are advised not to show your written work to others. Copying someone else's work is a very serious violation of the honour code.
- Students may read resources on the Internet, such as articles on Wikipedia, Wolfram MathWorld or any other forums, but you are not allowed to post the original assignment question online and ask for answers. It is regarded as a violation of the honour code.
- Since it is impossible to list all conceivable instance of honour code violations, the students has the responsibility to always act in a professional manner and to seek clarification from appropriate sources if their or another student's conduct is suspected to be in conflict with the intended spirit of the honour code.

- James STEWART, Calculus (7th edition).

Week	Topics	Textbook Sections
1	**Orientation day I**	
	Orientation day II	
	Real Numbers and Sets	Appendix A;
2	Sequences of Numbers	Ch–11.1;
	The limit of a sequence	Ch–11.1; Ch–2.6;
3	**Mid-Autumn Festival**	
	The limit of a function	Ch–$2.1 \sim 2.2$;
	Limit laws	Ch–$2.3 \sim 2.4$;
4	**National day**	
5	Continuity	Ch–2.5;
	Rates of Change; Derivatives	Ch–$2.7 \sim 2.8$;
	Techniques of Differentiation I	Ch–$3.1 \sim 3.2$;
6	Techniques of Differentiation II	Ch–$3.3 \sim 3.6$;

7	Mean-Value Theorem Applications of Differentiation I Applications of Differentiation I	Ch–$4.1 \sim 4.2$; Ch–$4.3 \sim 4.4$; Ch–$4.7 \sim 4.8$;
8	**Midterm Exam** Integral	 Ch–4.9; Ch–$5.1 \sim 5.2$;
9	Fundamental Theorem of Calculus Techniques of Integration	Ch–$5.3 \sim 5.4$; Ch–5.5; Ch–$7.1 \sim 7.4$;
10	Applications of Integration I Applications of Integration II	Ch–$6.1 \sim 6.5$; Ch–$8.1 \sim 8.3$;
11	Parametric Equations Polar Coordinates Improper Integral	Ch–$10.1 \sim 10.2$ Ch–$10.3 \sim 10.4$ Ch–7.8
12	Series Convergence Test	Ch–11.2 Ch–$11.3 \sim 11.7$
13	Power Series Talyor Series Differential Equations (optional)	Ch–$11.8 \sim 11.9$ Ch–$11.10 \sim 11.11$ Ch–9
14	**Final Exam**	

- To warm up we will go over some elementary notions in mathematics.

Definition

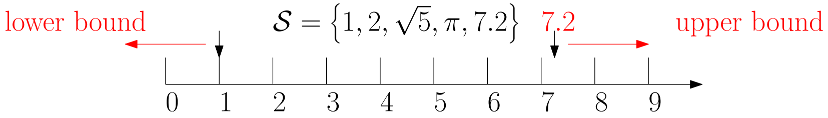
Let $\mathcal{S} \subset \mathbb{R}$ be a set of real numbers.

- A real number $M \in \mathbb{R}$ is said to be an upper bound of $\mathcal{S}$ if

$$x \leq M \text{ for every } x \in \mathcal{S}$$

- A real number $m \in \mathbb{R}$ is said to be a lower bound of $\mathcal{S}$ if

$$x \geq m \text{ for every } x \in \mathcal{S}$$



- A set is said to be bounded from above if it has an upper bound, bounded from below if it has a lower bound, and bounded if it has both.
- For example, the set of natural numbers

$$\mathbb{N}_1 = \{1, 2, 3, 4, \ldots\}$$

is bounded from below by any $m \leq 1$.

- However, the set $\mathbb{N}_1$ is clearly not bounded from above, so $\mathbb{N}_1$ is unbounded.

Q: Is the set of reciprocals of the natural numbers bounded?

$$\mathcal{S} = \left\{ \frac{1}{n} \mid n \in \mathbb{N}_1 \right\}$$

Q: Can you think of an equivalent condition for a set $\mathcal{S}$ to be bounded?

Q: Can we define the notion of boundedness using $W = \max\{|m|, |M|\}$?

Definition

Let $\mathcal{S} \subset \mathbb{R}$ be a set of real numbers.

- If $M \in \mathbb{R}$ is an upper bound of $\mathcal{S}$ such that

$$M \leq M^* \text{ for every upper bound } M^* \text{ of } \mathcal{S},$$

  then M is called the supremum or least upper bound of $\mathcal{S}$, denoted

$$M = \sup(\mathcal{S})$$

- If $m \in \mathbb{R}$ is a lower bound of $\mathcal{S}$ such that

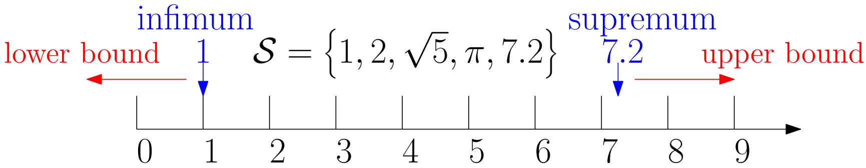
$$m \geq m^* \text{ for every lower bound } m^* \text{ of } \mathcal{S},$$

  then m is called the infimum or greatest lower bound of $\mathcal{S}$, denoted

$$m = \inf(\mathcal{S})$$

Q: Find the supremum and infimum of $\mathcal{S} = \left\{1, 2, \sqrt{5}, \pi, 7.2\right\}$.

- Notice the difference between upper/lower bounds and supremum/infimum.



Theorem

Every finite set of $\mathcal{S} \subset \mathbb{R}$ is bounded. Its supremum is the greatest element

$$\sup(\mathcal{S}) = \max(\mathcal{S})$$

and its infimum is the smallest element in the set.

$$\inf(\mathcal{S}) = \min(\mathcal{S})$$

Q: Find the supremum and infimum of the following set $\mathcal{S}$, do they belong to $\mathcal{S}$?

$$\mathcal{S} = \left\{ \frac{1}{n} \mid n \in \mathbb{N}_1 \right\}$$

Q: Find the supremum of the set of real numbers

$$\mathcal{S} = \{x \in \mathbb{R} \mid x \leq \sqrt{2}\}$$

Q: Find the supremum of the set of rational numbers

$$\mathcal{S} = \{x \in \mathbb{Q} \mid x \leq \sqrt{2}\}$$

- I will assume everyone is familiar with open, closed and half-closed intervals

$$(a, b), \qquad [a, b], \qquad (a, b], \qquad \text{and} \qquad [a, b)$$

Definition

A set $\mathcal{V} \subset \mathbb{R}$ is a neighbourhood of a point $x \in \mathbb{R}$ if there exists some $\delta > 0$ s.t.

$$(x - \delta, x + \delta) \subset \mathcal{V}$$

The open interval $(x - \delta, x + \delta)$ is called a δ-neighbourhood of x.

Q: Can a closed interval be a neighbourhood?

- For example, suppose

$$a < x < b$$

then the closed interval

$$\mathcal{V} = [a, b] \quad \text{is a neighbourhood of } x$$

since $\mathcal{V}$ contains the interval $(x - \delta, x + \delta)$ for sufficiently small $\delta > 0$.

Definition

A set $\mathcal{S} \subset \mathbb{R}$ is open in $\mathbb{R}$ if for every $x \in \mathcal{S}$ there exists a $\delta > 0$ such that

$$(x-\delta, x+\delta) \subset \mathcal{S}$$

A set $\mathcal{S} \subset \mathbb{R}$ is open if every $x \in \mathcal{S}$ has a neighbourhood $\mathcal{V}$ such that $\mathcal{V} \subset \mathcal{S}$.

Q: Are rational numbers an open set in $\mathbb{R}$?

Q: Is the union of open sets open?

Q: Is the intersection of open sets open?

- For example, the interval

$$\mathcal{I}_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$$

is open for every $n \in \mathbb{N}$, but

$$\bigcap_{n=1}^{\infty} \mathcal{I}_n = \{0\}$$

is clearly not open.

- Another way to state the definition of open set is in terms of interior points.

Definition

Let $\mathcal{S} \subset \mathbb{R}$ be a set of real numbers. A point $x \in \mathcal{S}$ is an interior point of $\mathcal{S}$ if there is a $\delta > 0$ such that $(x-\delta, x+\delta) \subset \mathcal{S}$

A point $x \in \mathbb{R}$ is a boundary point of $\mathcal{S}$ if every neighbourhood

$$(x-\delta, x+\delta)$$

contains at least one point in $\mathcal{S}$ and at least one point not in $\mathcal{S}$.

- Hence a set is open if and only if every point in the set is an interior point.

Q: Find the interior point/s of a set of irrational numbers.

Q: Find the boundary point/s of $\mathcal{S} = [1, 2)$.

Q: Is $\mathbb{R}$ open in $\mathbb{R}$? Is the empty set $\emptyset$ open in $\mathbb{R}$?

- Closed sets are complements of open sets.

Definition

A set $\mathcal{S} \subset \mathbb{R}$ is closed if $\mathcal{S}^{\complement} = \{x \in \mathbb{R} \mid x \notin \mathcal{S}\}$ is open.

Q: Is the interval $[a, \infty)$ closed?

Q: Is the intersection of closed sets closed?

Q: Is the union of closed sets closed?

- For example, suppose
$$\mathcal{I}_n = \left[\frac{1}{n}, 1 - \frac{1}{n}\right]$$
then the union of the $\mathcal{I}_n$ is an open interval
$$\bigcup_{n=2}^{\infty} \mathcal{I}_n = (0, 1)$$

Q: Is $\mathbb{R}$ closed in $\mathbb{R}$? Is the empty set $\emptyset$ closed in $\mathbb{R}$?

Definition

A point $x \in \mathbb{R}$ is a limit point of $\mathcal{S} \subset \mathbb{R}$ if for every neighbourhood $(x-\delta, x+\delta)$ contains a point in $\mathcal{S}$ other than x itself.

- A limit point of a set $\mathcal{S}$ is a point that has points in $\mathcal{S}$ arbitrarily close to it.

Q: Is every point of every open set $\mathcal{S} \subset \mathbb{R}$ a limit point of $\mathcal{S}$?

Q: Is every limit point an interior point?

- For example, consider the closed set

$$\mathcal{S} = [0, 1]$$

points 0 and 1 are limit points but not interior points of $\mathcal{S}$.

Q: Is the point 0 a boundary point of $\{0\}$? Is the point 0 a limit point of $\{0\}$?

Q: Is the point 0 a boundary point of $\mathcal{I} = [-1, 1]$? Is it a limit point of $\mathcal{I}$?

- A limit point of a set $\mathcal{S}$ is either an interior point or a boundary point of $\mathcal{S}$.

Q: Find the limit point/s of the set $\mathbb{N}$ of natural numbers.

Definition

Let $\mathcal{S} \subset \mathbb{R}$. A point $x \in \mathcal{S}$ is an isolated point of $\mathcal{S}$ if there exists $\delta > 0$ such that x is the only point belonging to $\mathcal{S}$ in the neighbourhood $(x - \delta, x + \delta)$.

- An isolated point of a set is a point in the set that does not have other points in the set arbitrarily close to it.
- Unlike limit points, isolated points are required to belong to the set.
- Every point $x \in \mathcal{S}$ is either a limit point or an isolated point.
- Clearly the set of natural numbers $\mathbb{N}$ contains only isolated points.

Q: Is it true that every interval has no isolated points?

Q: Find the isolated points for

$$\mathcal{S} = \{0\} \cup \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$$

Definition

A subset of $\mathbb{R}$ is compact if and only if it is closed and bounded.