

VE475

Introduction to Cryptography

Homework 3

Manuel — UM-JI (Summer 2021)

Non-programming exercises:

- Write in a neat and legible handwriting, or use \LaTeX
- Clearly explain the reasoning process
- Write in a complete style (subject, verb and object)

Programming exercises:

- Write a README file for each program
- Upload an archive with all the programs onto Canvas

Ex. 1 — Finite fields

1. Show that $X^2 + 1$ is irreducible in $\mathbb{F}_3[X]$.
2. Why does the multiplicative inverse of $1 + 2X \bmod X^2 + 1$ exist in $\mathbb{F}_3[X]$?
3. Find the multiplicative inverse of $1 + 2X \bmod X^2 + 1$, in $\mathbb{F}_3[X]$.

Ex. 2 — AES

The goal of this exercise is to reshape the decryption of AES such that it has the same structure as encryption.

1. First we determine the inverse of each layer.
 - a) Describe *InvShiftRows* the inverse operation of *ShiftRows*.
 - b) What is the inverse of the layer *AddRoundKey*?
 - c) Explain why the transformation *InvMixColumns* is given by the multiplication by the matrix

$$\begin{pmatrix} 00001110 & 00001011 & 00001101 & 00001001 \\ 00001001 & 00001110 & 00001011 & 00001101 \\ 00001101 & 00001001 & 00001110 & 00001011 \\ 00001011 & 00001101 & 00001001 & 00001110 \end{pmatrix}.$$

We call *InvSubBytes* the lookup table inverse of *SubBytes*.

2. Describe the decryption process using the previous transformations.
3. Why can *InvShiftRows* and *InvSubBytes* be applied on reverse order?
4. Similarly we want to inverse the order of application of *AddRoundKey* and *InvMixColumns*.
 - a) Why is it not possible to reverse the order of application of *AddRoundKey* and *InvMixColumns*?
 - b) Calling the initial matrix $(a_{i,j})$, the *MixColumns* matrix $(m_{i,j})$, and the *AddRoundKey* matrix $(k_{i,j})$, what is the result of applying first *MixColumns* and then *AddRoundKey*?
 - c) Show that the inverse operation is given by

$$(e_{i,j}) \rightarrow (m_{i,j})^{-1}(e_{i,j}) \oplus (m_{i,j})^{-1}(k_{i,j}).$$

- d) Define a new operations *InvAddRoundKey* such that “*AddRoundKey* then *InvMixColumns*” can be replaced by “*InvMixColumns* then *InvAddRoundKey*”.

5. Conclude on how to process the decryption.
6. What are the advantages of this strategy over simply reversing the order of application of the transformations?

Ex. 3 — DES

1. Research and explain how the DES cryptosystem works.
2. Quickly explain linear and differential cryptanalysis.
3. What is triple DES and why was it used instead of double DES?
4. Explain how passwords are stored on Unix systems? Is it a problem?

Hint: check `man passwd` and then follow the suggested man pages

Ex. 4 — Programming

In the AES, choose two layers to implement in C. The 128 bits should be looked at as an `unsigned char` pointer. Operations should be implemented using logical operators (and, or, and xor).

A bonus will be given for each extra layer implemented, and the generation of the S-Box. A big bonus will reward a complete implementation of the AES.