Chapter 24

Capacitance and Dielectrics

Goals for Chapter 24

- To understand capacitors and calculate capacitance
- To analyze networks of capacitors
- To calculate the energy stored in a capacitor
- To examine dielectrics and how they affect capacitance

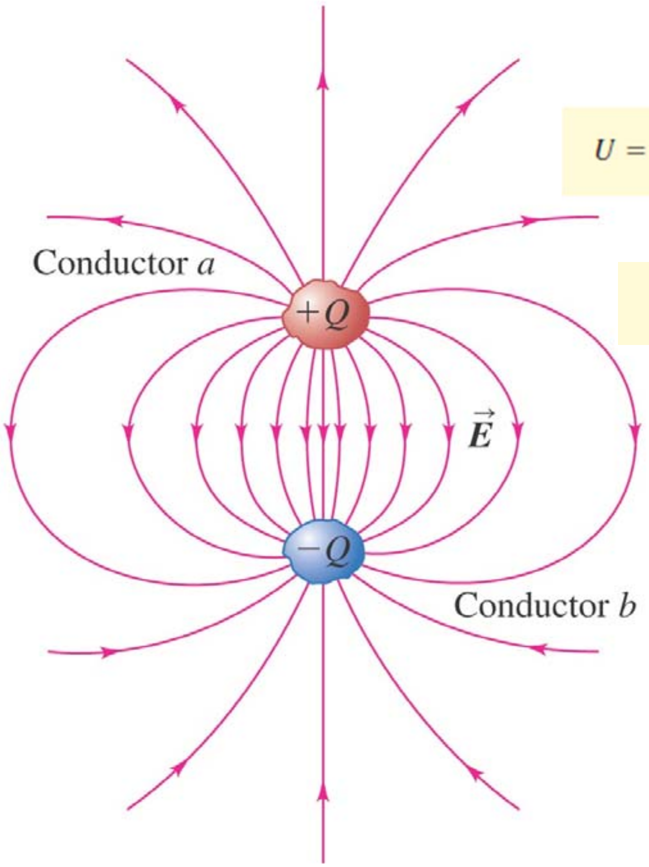
Introduction

- How does a camera's flash unit store energy?
- Capacitors are devices that store electric potential energy.
- The energy of a capacitor is actually stored in the electric field.

Capacitors and capacitance

- Any two conductors separated by an insulator form a *capacitor*, as illustrated in Figure 24.1 below.

- The definition of capacitance is $\boldsymbol{C = Q/V_{ab}}$.

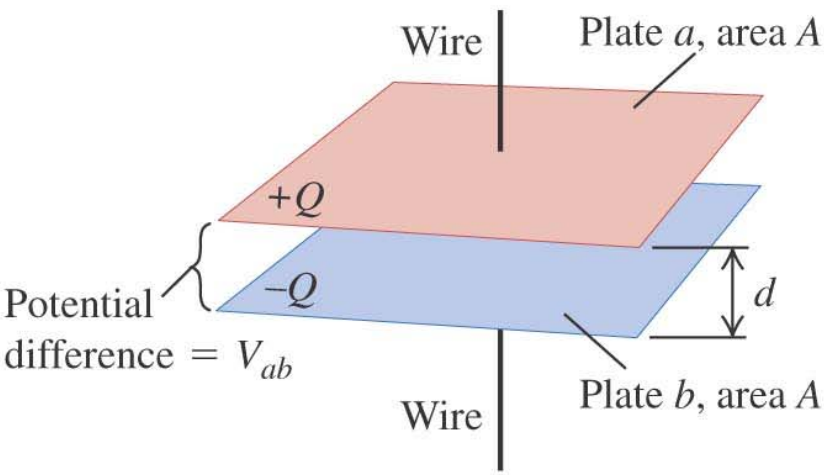


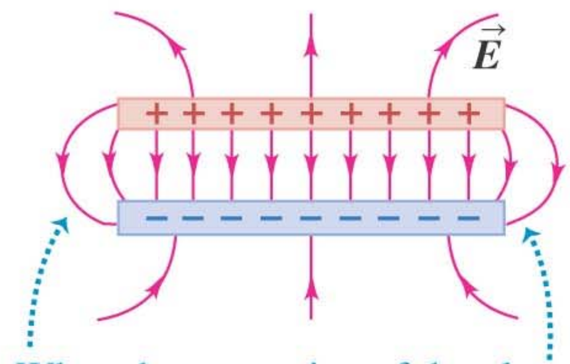
$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$ (electric potential energy of two point charges q and q_0)

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$ (potential due to a point charge)

Parallel-plate capacitor

- A *parallel-plate capacitor* consists of two parallel conducting plates separated by a distance that is small compared to their dimensions. (See Figure 24.2 below.)



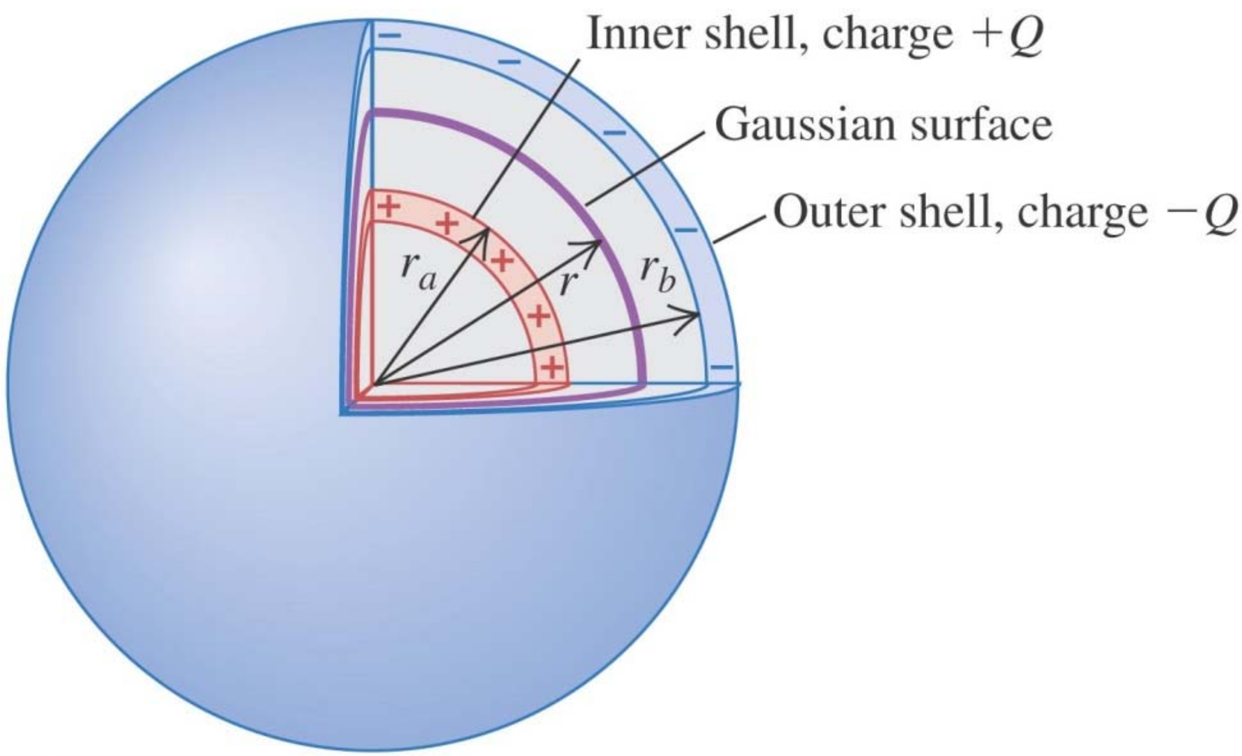


$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \quad \text{(capacitance of a parallel-plate capacitor in vacuum)}$$

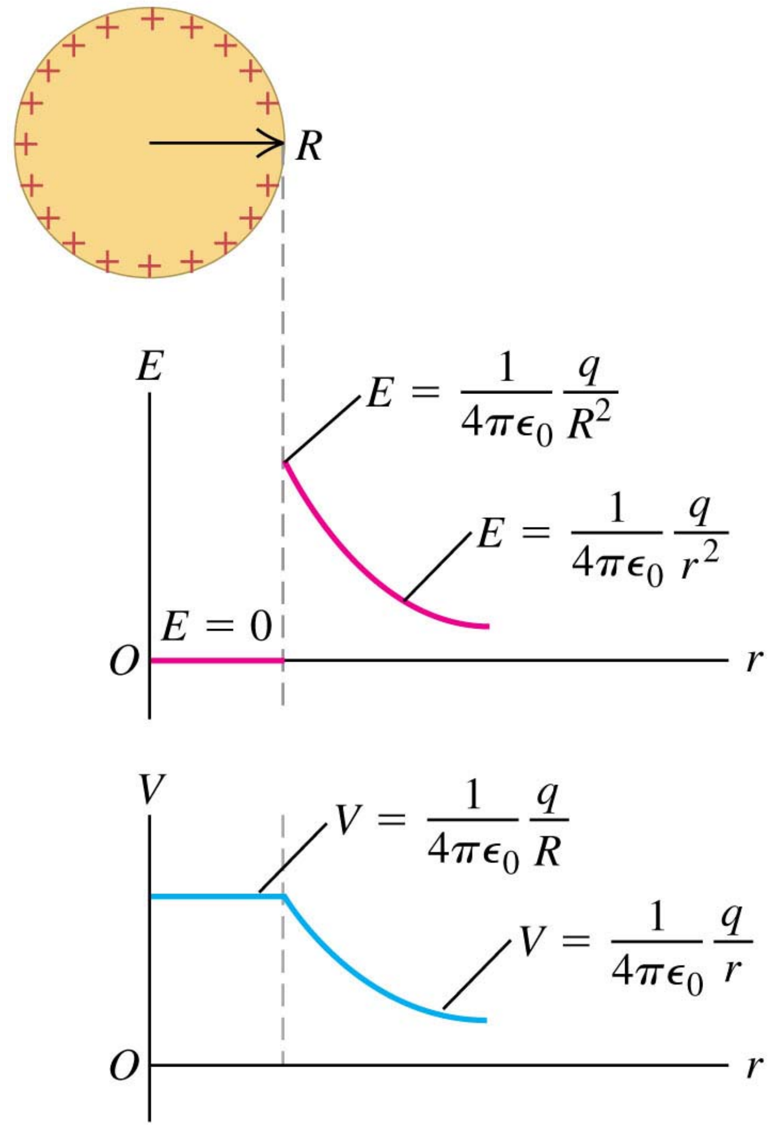
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A}$$

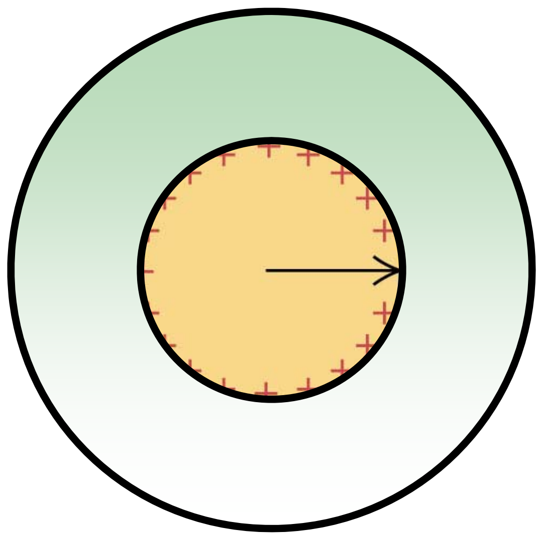
A spherical capacitor



A spherical capacitor



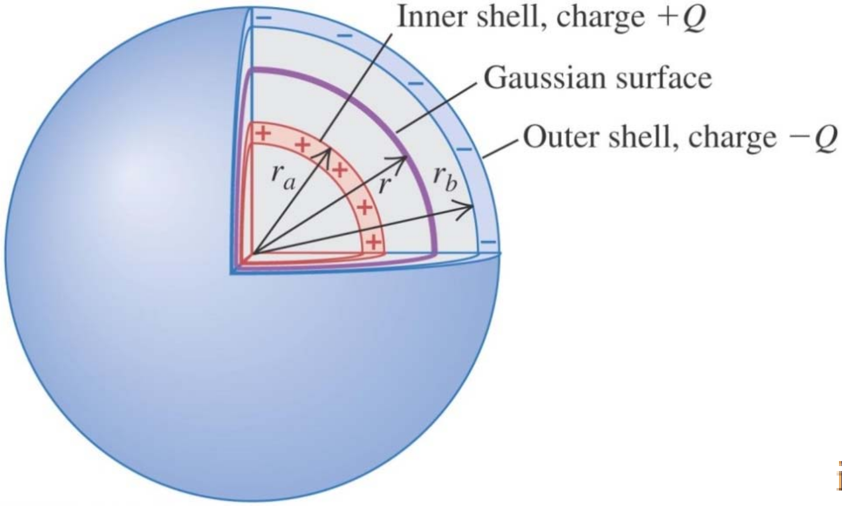
A spherical capacitor



$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos\phi \, dl$$ (potential difference as an integral of $\vec{E}$)

A spherical capacitor



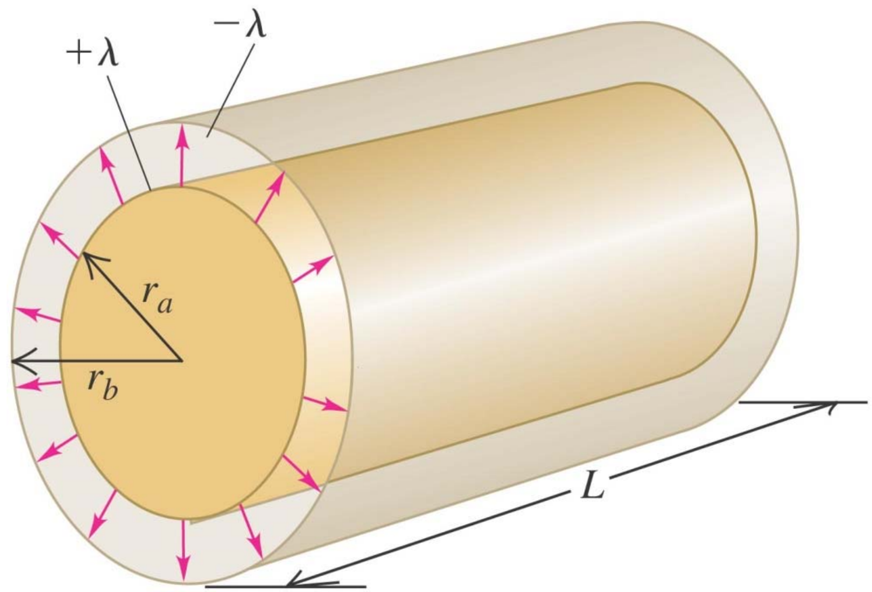
$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

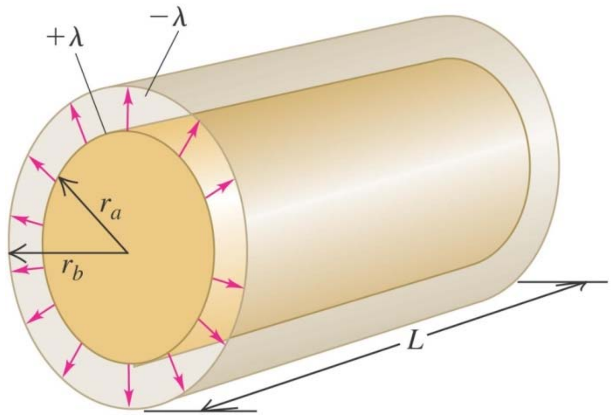
itance is then

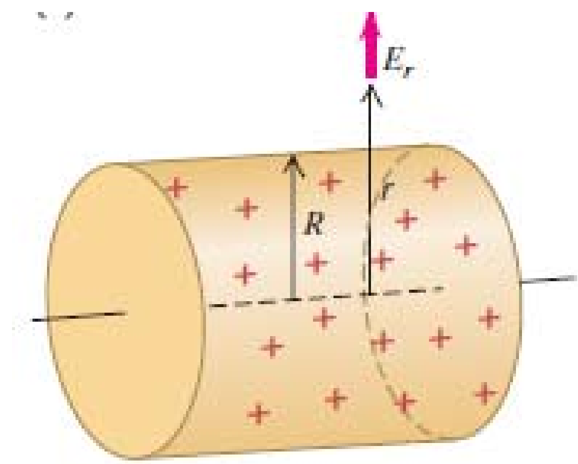
$$C = \frac{Q}{V_{ab}} = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

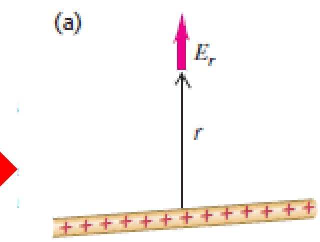
A cylindrical capacitor



A cylindrical capacitor







$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} \quad \text{(field of an infinite line of charge)}$$

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\dfrac{\lambda}{2\pi\epsilon_0}\ln\dfrac{r_b}{r_a}} = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

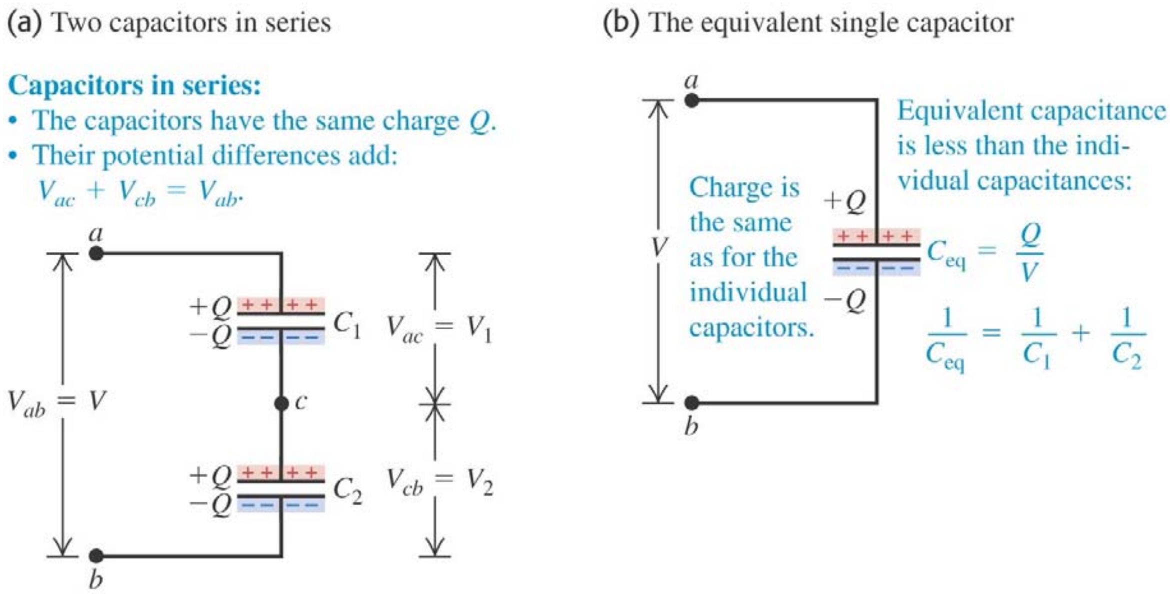
$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E_r\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

The capacitance per unit length is

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$$

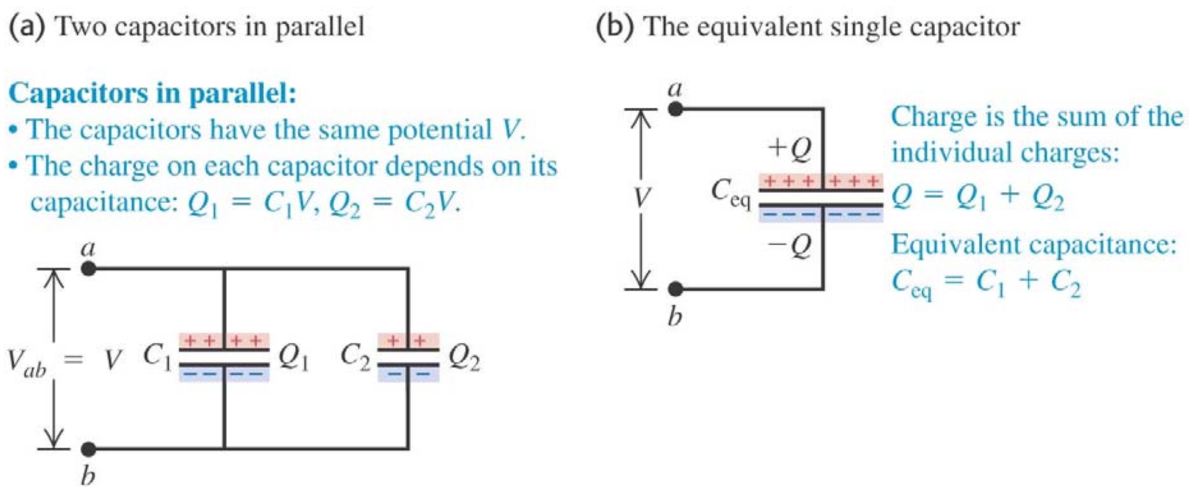
Capacitors in series

- Capacitors are in *series* if they are connected one after the other, as illustrated in Figure 24.8 below.
- The *equivalent capacitance* of a series combination is given by $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$



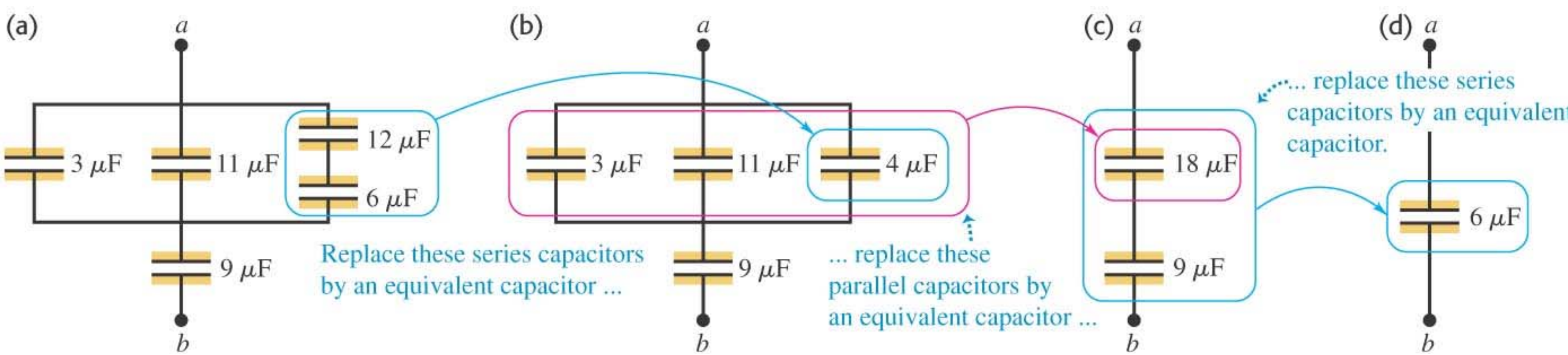
Capacitors in parallel

- Capacitors are connected in *parallel* between *a* and *b* if the potential difference V_{ab} is the same for all the capacitors. (See Figure 24.9 below.)
- The *equivalent capacitance* of a parallel combination is the *sum* of the individual capacitances: $C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots$.



Calculations of capacitance

- Refer to Problem-Solving Strategy 24.1.
- Follow Example 24.5.
- Follow Example 24.6, a capacitor network, using Figure 24.10 below.



Energy stored in a capacitor

- The potential energy stored in a capacitor is

$$V = \frac{Q}{C}$$

$$dW = v\,dq = \frac{q\,dq}{C}$$

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

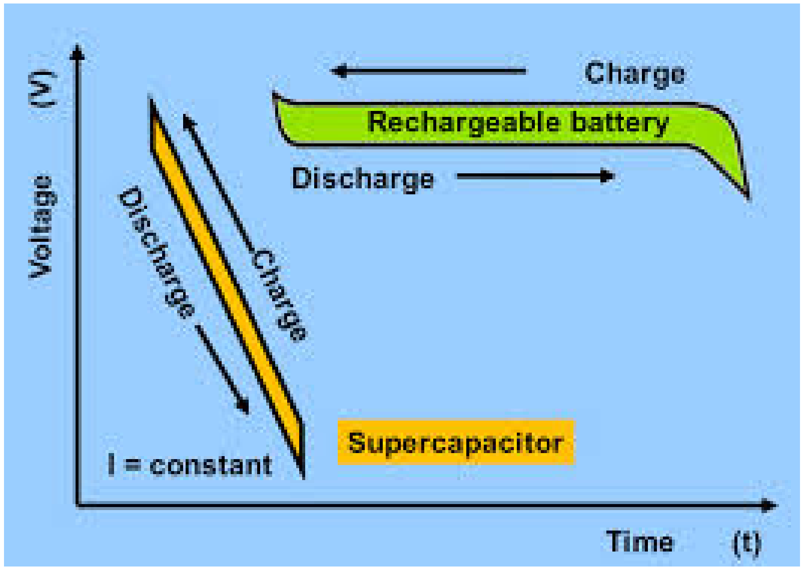
$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \quad \text{(potential energy stored in a capacitor)}$$

- The *energy density* is $u = 1/2\ \varepsilon_0 E^2$.

$$u = \text{Energy density} = \frac{\frac{1}{2}CV^2}{Ad} \qquad C = \epsilon_0 A/d. \qquad V = Ed.$$

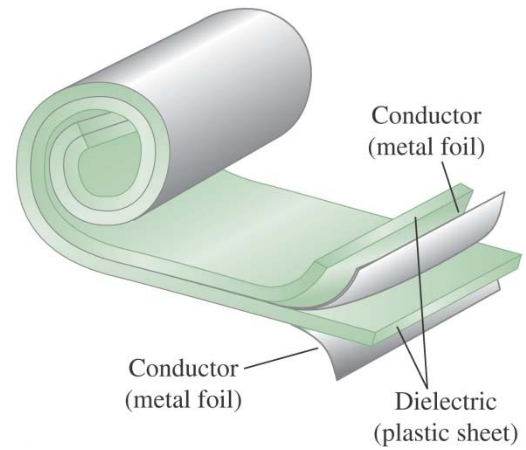
Battery vs. Capacitor

Available Performance	Lead Acid Battery	Ultracapacitor	Conventional Capacitor
Charge Time	1 to 5 hrs	0.3 to 30 s	10^{-3} to 10^{-6} s
Discharge Time	0.3 to 3 hrs	0.3 to 30 s	10^{-3} to 10^{-6} s
Energy (Wh/kg)	10 to 100	1 to 10	< 0.1
Cycle Life	1,000	>500,000	>500,000
Specific Power (W/kg)	<1000	<10,000	<100,000
Charge/discharge efficiency	0.7 to 0.85	0.85 to 0.98	>0.95
Operating Temperature	-20 to 100 C	-40 to 65 C	-20 to 65 C

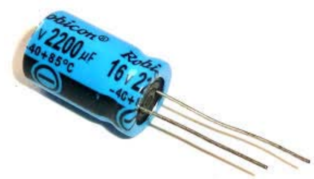


Dielectrics

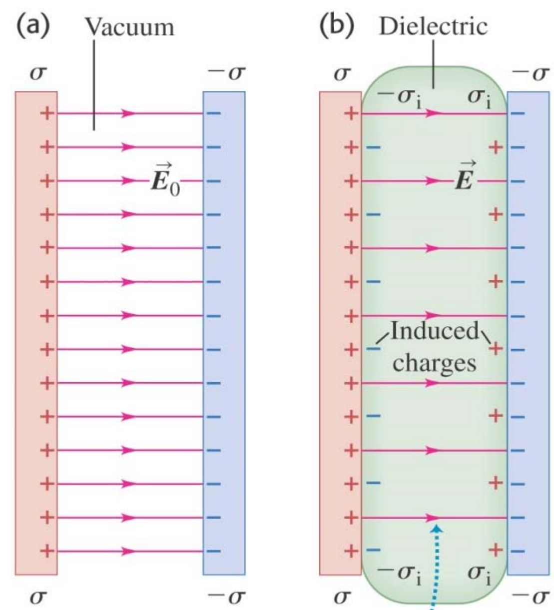
- A *dielectric* is a nonconducting material. Most capacitors have dielectric between their plates. (See Figure 24.13 at upper right.)



- The *dielectric constant* of the material is $K = C/C_0 > 1$.



- Dielectric *increases* the capacitance and the energy density by a factor K.
- Figure 24.15 (lower right) shows how the dielectric affects the electric field between the plates.
- Table 24.1 on the next slide shows some values of the dielectric constant.



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

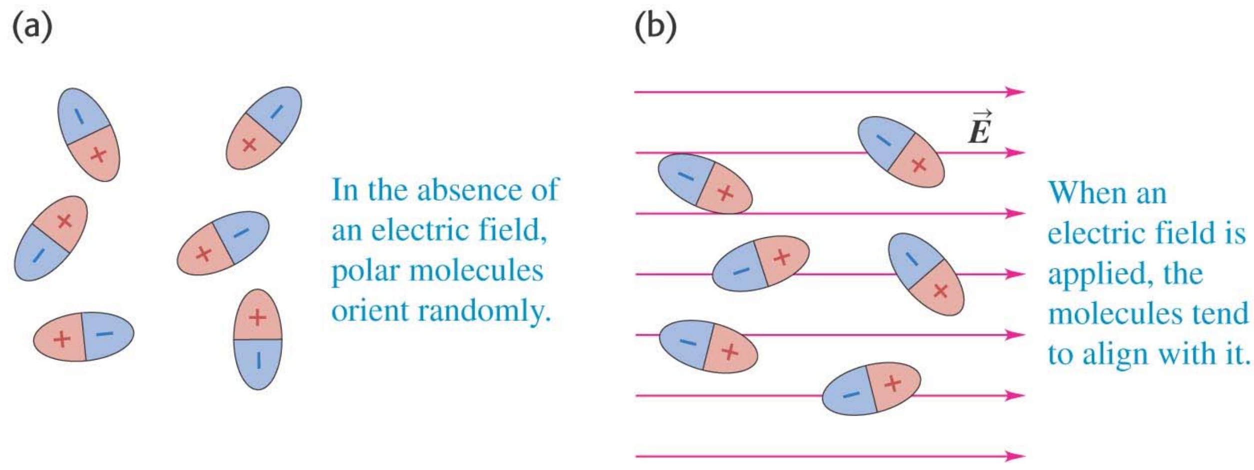
Table 24.1—Some dielectric constants

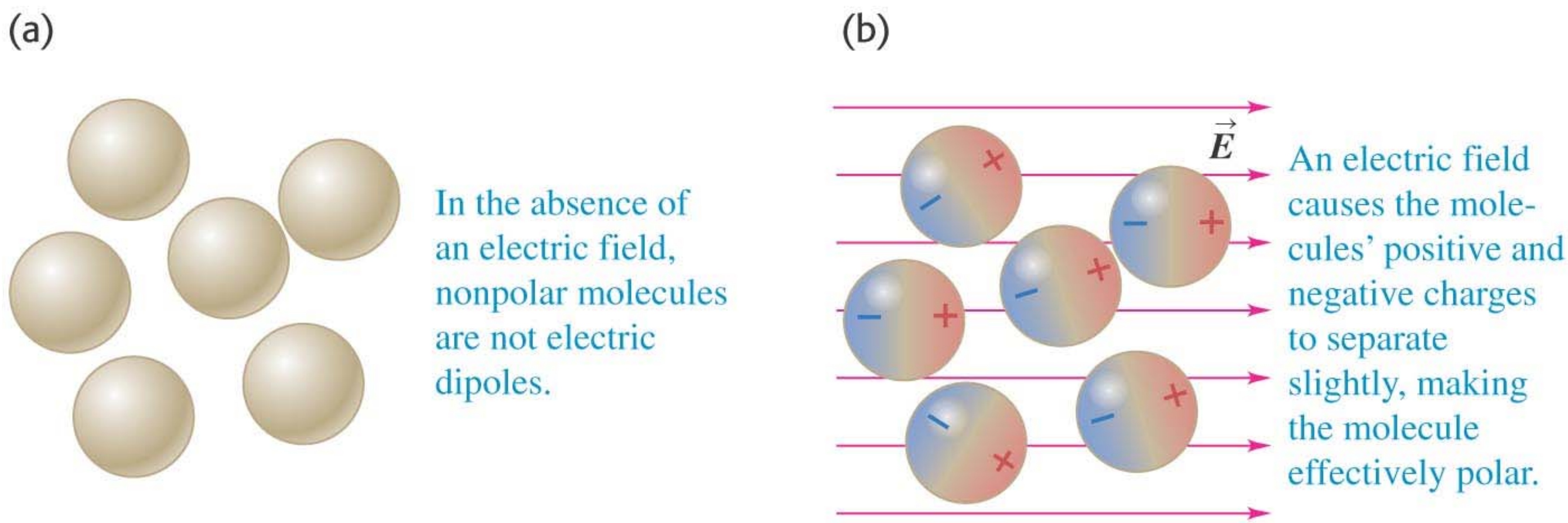
Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Molecular model of induced charge - I

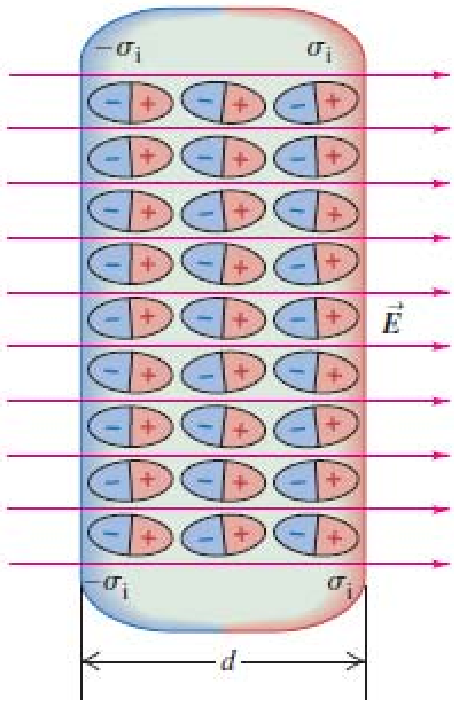
- Figures 24.17 (right) and 24.18 (below) show the effect of an applied electric field on polar and nonpolar molecules.





Molecular model of induced charge - II

- Figure 24.20 below shows *polarization* of the dielectric and how the induced charges reduce the magnitude of the resultant electric field.



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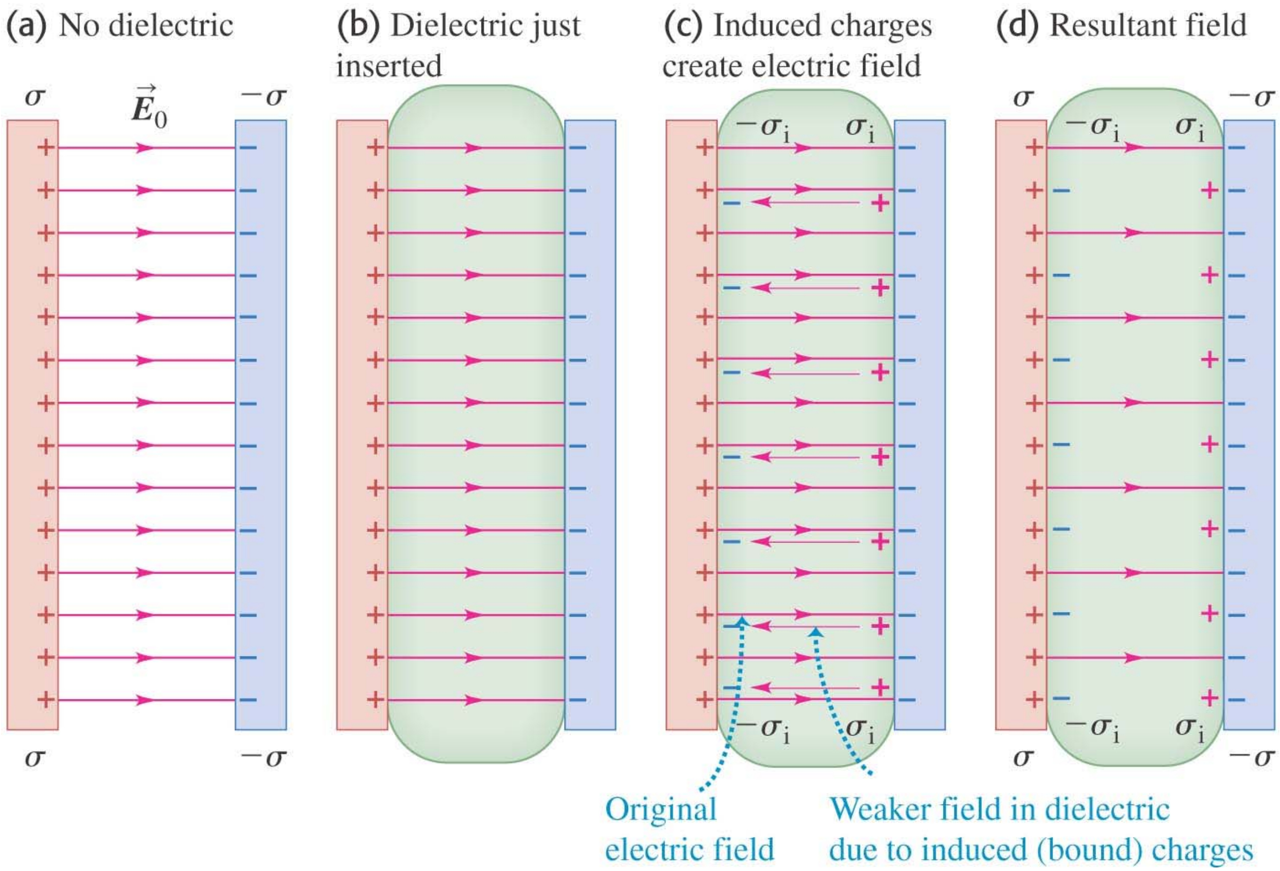

Molecular model of induced charge - II

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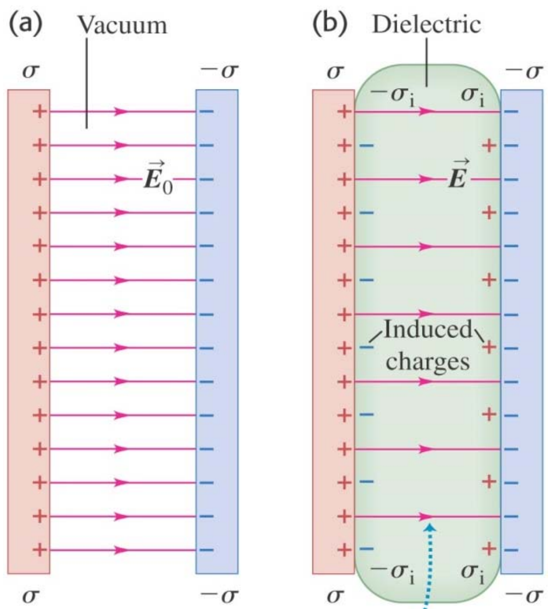


For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A}$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

$$E = \frac{E_0}{K} \qquad \text{(when } Q \text{ is constant)}$$

$$E_0 = \frac{\sigma}{\epsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \qquad \text{(induced surface charge density)}$$

The product $K\epsilon_0$ is called the **permittivity** of the dielectric,

$$\epsilon = K\epsilon_0 \qquad \text{(definition of permittivity)}$$

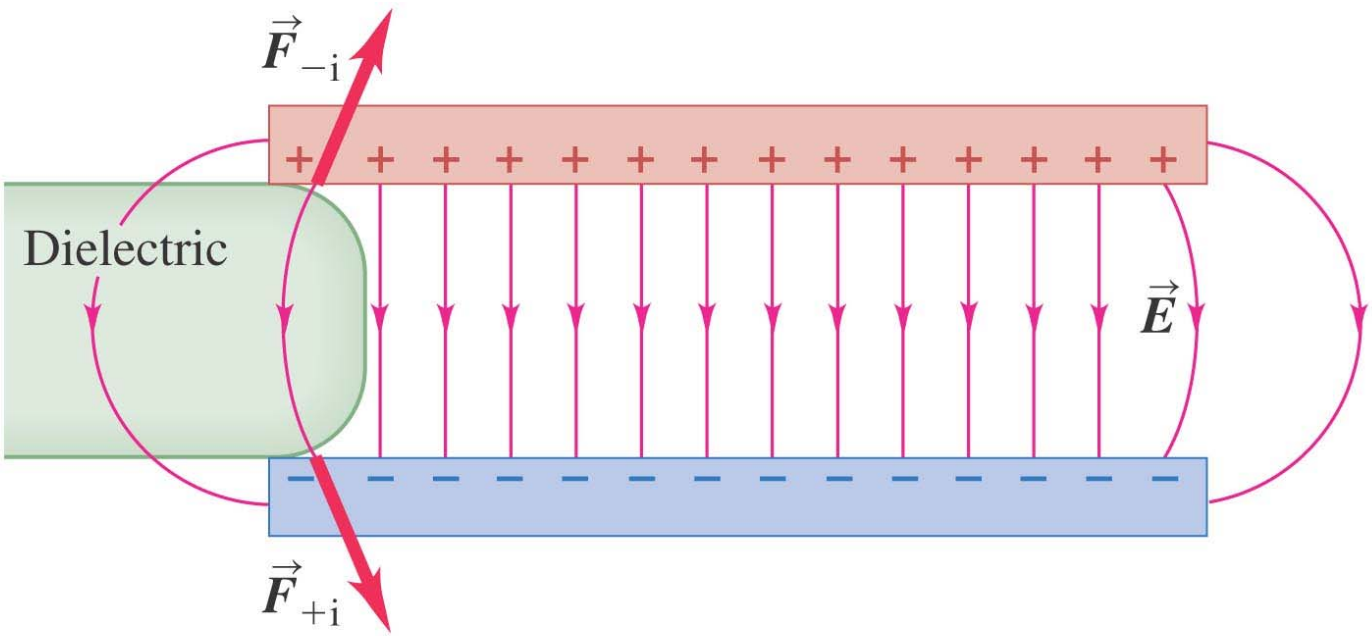
$$\boxed{E = \frac{\sigma}{\epsilon}}$$

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \qquad \text{(parallel-plate capacitor, dielectric between plates)} \qquad (24.19)$$

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \qquad \text{(electric energy density in a dielectric)} \qquad (24.20)$$

Examples with and without a dielectric

- Refer to Problem-Solving Strategy 24.2.
- Follow Example 24.10 to see the effect of the dielectric.
- Follow Example 24.11 to see how the dielectric affects energy storage. Use Figure 24.16 below.



Dielectric breakdown

- If the electric field is strong enough, *dielectric breakdown* occurs and the dielectric becomes a conductor.
- The *dielectric strength* is the maximum electric field the material can withstand before breakdown occurs.
- Table 24.2 shows the dielectric strength of some insulators.

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$$

Gauss's law in dielectrics

- Follow the text discussion of Gauss's law in dielectrics, using Figure 24.22 at the right.

$$EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \quad \text{or} \quad \sigma - \sigma_i = \frac{\sigma}{K}$$

$$EA = \frac{\sigma A}{K\epsilon_0} \quad \text{or} \quad KEA = \frac{\sigma A}{\epsilon_0}$$

$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0} \quad \text{(Gauss's law in a dielectric)}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon} \quad \text{(Gauss's law in a dielectric)}$$