VE 320 Fall 2021

Introduction to Semiconductor Devices

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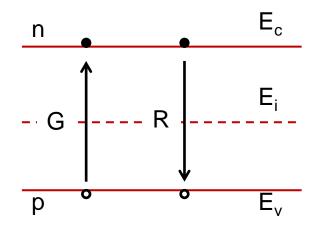
Lecture 6

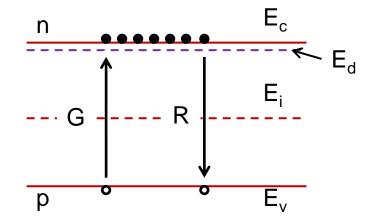
Nonequilibrium excess carriers (Chapter 6)

Nonequilibrium

- Chapter 4: thermal equilibrium
- Chapter 5: there are voltages applied, so it should be nonequilibrium, but assumed that the equilibrium is not heavily perturbed
- Chapter 6: External excitation applied, excess carriers

Direct recombination





Intrinsic: n=p=n_i

n type: $n>>n_i>>p$

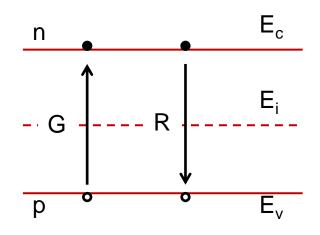
Generation rate G or Recombination rate R:

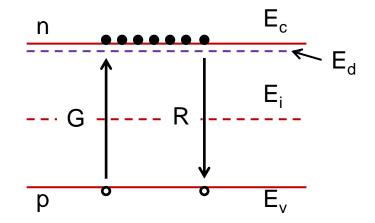
The number of charge carriers generated or recombined per unit volume per second

Thermal equilibrium:

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

Unit: #/cm³-s





Intrinsic: n=p=n_i

n type: $n>>n_i>>p$

Excess electrons and excess holes

Excess electron and hole generation rates $g_n' = g_p'$

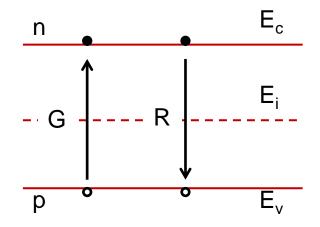
No longer thermal equilibrium

$$n = n_0 + \delta n \qquad p = p_0 + \delta p$$

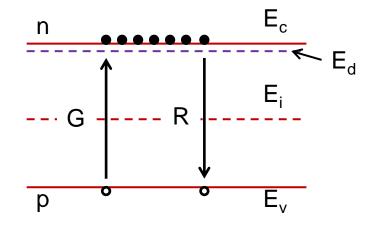
$$np \neq n_0 p_0 = n_i^2$$



• Direct recombination



Intrinsic: n=p=n_i



n type: $n > n_i > p$

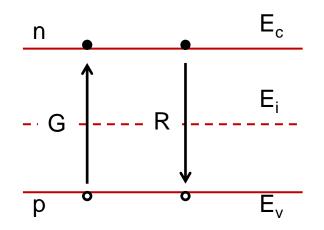
First, look at R

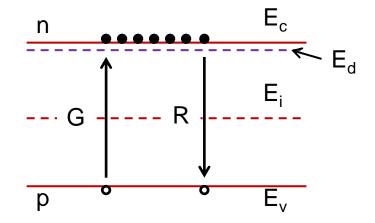
$$R \sim n$$
, $R \sim p$

$$\Rightarrow R \sim np$$

 $\Rightarrow R = \alpha_r np \text{ where } \alpha_r \text{ is a constant}$

• Direct recombination





Intrinsic: n=p=n_i

First, look at R

n type: n>>n_i>>p

Assume: the probability of an electron and hole recombining is constant with time

$$R \sim n$$
,

 $R \sim p$

$$\Rightarrow R \sim np$$

 $\Rightarrow R = \alpha_r np$ where α_r is a constant

Then, look at G

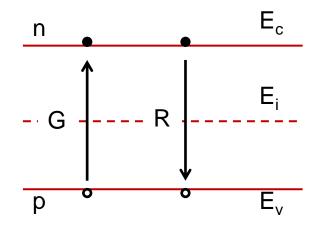
Can we write similar equations?

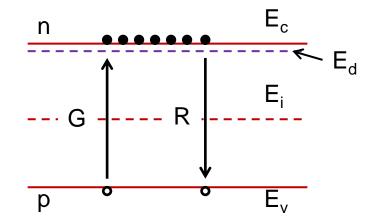
G~*n* ?

R~p?

No! G₀ is intrinsic and only a function of T

Direct recombination





Intrinsic: n=p=n_i

n type: $n>n_i>p$

At equilibrium, we must have
$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$G_0 = \alpha_r \cdot n_0 \cdot p_0 = \alpha_r \cdot n_i^2$$
 where α_r is a constant

Boltzmann approximation

At non-equilibrium

- 1 Light illumination
- (2) current injection: forward biased pn junction

Direct recombination

At non-equilibrium

- 1 Light illumination
- (2) current injection: forward biased pn junction

$$n = n_0 + \delta n$$

$$R = \alpha_r \cdot n \cdot p = \alpha_r \cdot (n_0 + \delta n) \cdot (p_0 + \delta p)$$

$$Q = p_0 + \delta p$$

$$G = G_0 = R_0 = \alpha_r \cdot n_i^2$$

Net recombination rate:

$$R'_n = R - G_0 = \alpha_r \cdot (n_0 + \delta n) \cdot (p_0 + \delta p) - \alpha_r \cdot n_i^2$$

$$= \alpha_r \cdot (n_0 \cdot p_0 + \delta n \cdot p_0 + n_0 \cdot \delta p + \delta n \cdot \delta p) - \alpha_r \cdot n_i^2$$

$$= \alpha_r \cdot [n_i^2 + \delta n \cdot (p_0 + n_0) + (\delta n)^2] - \alpha_r \cdot n_i^2$$

$$= \alpha_r \cdot \delta n \cdot (p_0 + n_0) + \alpha_r \cdot (\delta n)^2$$

Low-level injection

Net recombination rate:

$$R'_n = \alpha_r \cdot \delta n \cdot (p_0 + n_0) + \alpha_r \cdot (\delta n)^2$$

p-type material: $\delta n \ll \rho_0$

$$R_n' = \alpha_r \cdot \delta n \cdot p_0$$

$$\frac{d(\delta n(t))}{dt} = -\alpha_r p_0 \delta n(t)$$

Solution:

$$\delta n(t) = \delta n(0)e^{-\alpha_r p_0 t} = \delta n(0)e^{-t/\tau_{n0}}$$

$$\tau_{n0} = \frac{\delta n}{R_n'} = \frac{1}{\alpha_r p_0}$$

 $\tau_{n0} = \frac{\delta n}{R'_n} = \frac{1}{\alpha_n n_0}$ Excess minority carrier lifetime

$$R'_n = R'_p = rac{\delta n(t)}{ au_{n0}}$$

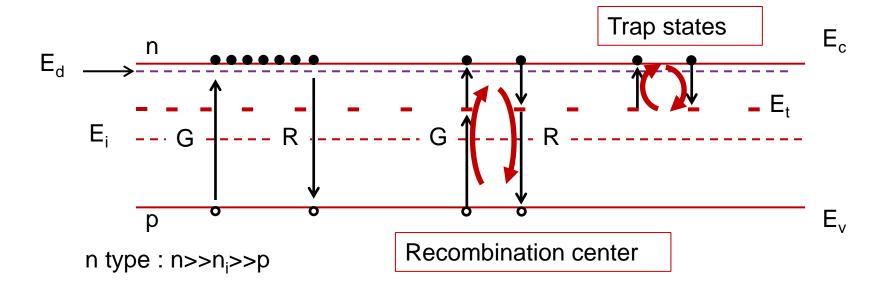
n-type material: $\delta n \ll n_0$

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

$$\tau_{\rm p0} = \frac{1}{\alpha_r n_0}$$

In general
$$au = \frac{1}{\alpha_r(n_0 + p_0)}$$

Indirect recombination



Indirect recombination

$$R'_{n} = \frac{np - n_{i}^{2}}{\tau_{p} \left[n + n_{i} \exp\left(\frac{E_{t} - E_{i}}{kT}\right) \right] + \tau_{n} \left[p + n_{i} \exp\left(\frac{E_{i} - E_{t}}{kT}\right) \right]}$$

where
$$au_p = rac{1}{N_t lpha_{rp}}$$
 , $au_n = rac{1}{N_t lpha_{rn}}$

Surface recombination

Bulk recombination rate:

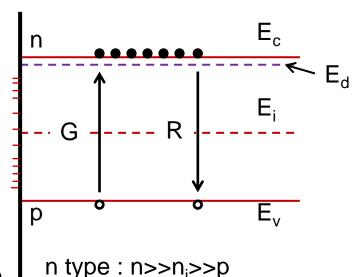
number of recombined carriers in a unit volume at a give unit time

$$\tau = \frac{\delta n}{R'_n} \Rightarrow R'_n = \delta n \frac{1}{\tau}$$

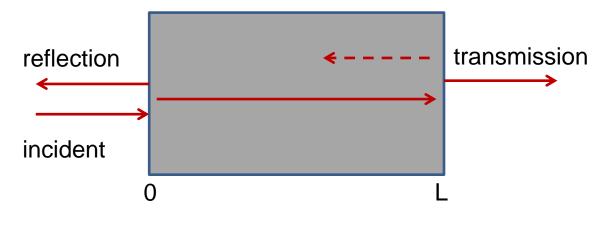
Surface recombination rate:

number of recombined carriers in a unit surface area at a give unit time

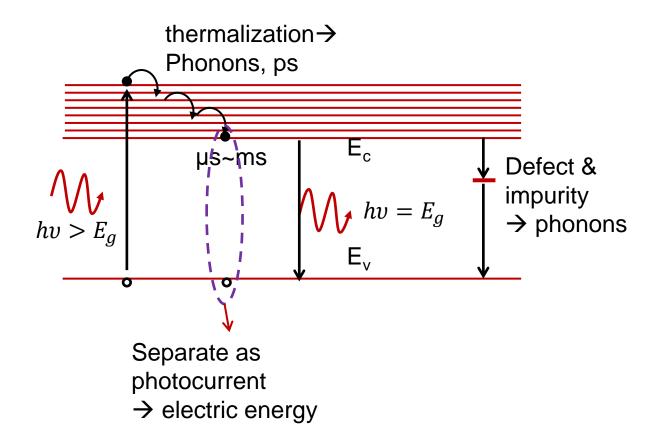
$$R'_{ns} = R'_n \cdot (\Delta x)_s = \delta n \frac{\Delta x}{\tau} = \delta n \cdot s$$

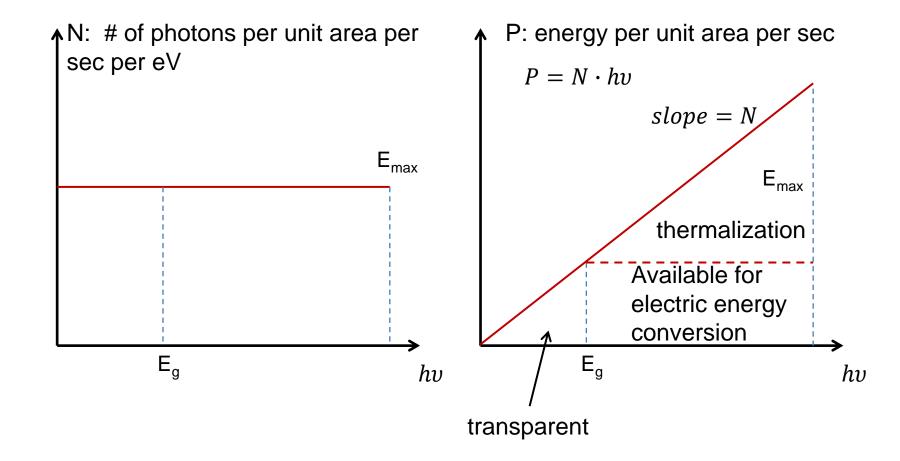


Absorption coefficient a

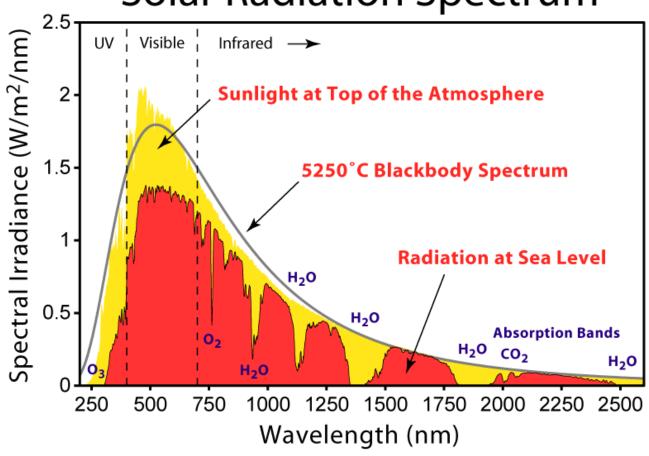


$$I = I_0 e^{-\alpha x}$$







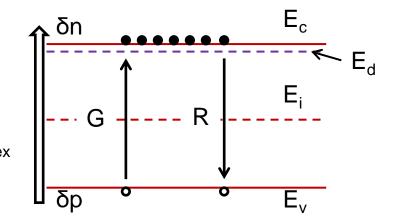


Time dependent

Once light illumination is switched off:

$$t_1$$
: $\delta p(t)$

$$t_2$$
: $\delta p(t + \Delta t)$



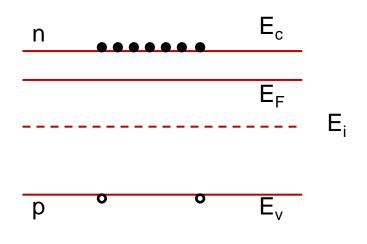
n type: $n>>n_i>>p$

$$|\lim_{\Delta t \to 0} \frac{\delta p(t) - \delta p(t + \Delta t)}{t - (t + \Delta t)}| = recombination \ rate = \frac{\delta p(t)}{\tau}$$

$$\Rightarrow \frac{d\delta p(t)}{dt} = -\frac{\delta p(t)}{\tau}$$

Quasi Fermi level

Excess carriers in a semiconductor: no thermal Equilibrium, Fermi energy is no longer defined



n type: n>>n_i>>p

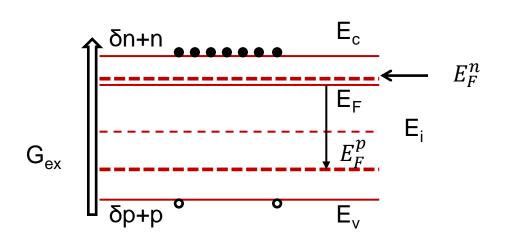
$$n_0 \times p_0 = n_i^2$$

$$n_0 = N_c \exp(\frac{E_F - E_c}{kT})$$
 $p_0 = N_v \exp(\frac{E_v - E_F}{kT})$

$$n_0 = n_i \exp(\frac{E_F - E_i}{kT})$$
 $p_0 = n_i \exp(\frac{E_i - E_F}{kT})$

Quasi Fermi level

Excess carriers in a semiconductor: no thermal Equilibrium
Strictly, Fermi energy is no longer defined



$$(n_0 + \delta n) \times (p_0 + \delta n) = n_i^2 \exp(\frac{E_F^n - E_F^p}{kT})$$

$$n_0 + \delta n = N_c \exp(\frac{E_F^n - E_c}{kT})$$
 $p_0 + \delta p = N_v \exp(\frac{E_v - E_F^p}{kT})$

$$n_0 + \delta n = n_i \exp(\frac{E_F^n - E_i}{kT})$$
 $p_0 + \delta p = n_i \exp(\frac{E_i - E_F^p}{kT})$

$$N_c \approx 10^{19} cm^{-3}$$

$$N_{v} \approx 10^{19} cm^{-3}$$

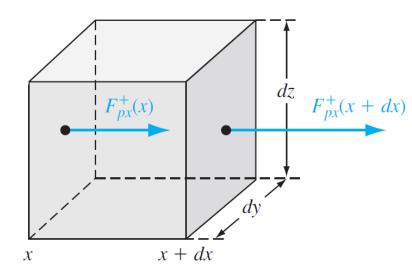
$$n_i \approx 10^{10} cm^{-3}$$

Hole particle flux

Taylor expansion, for small dx

$$F_{px}^{+}(x + dx) = F_{px}^{+}(x) + \frac{\partial F_{px}^{+}}{\partial x} \cdot dx$$

The net increase in the number of holes per unit time within the differential volume due to the hole flux



$$\frac{\partial p}{\partial t} dx dy dz = -\frac{\partial F_p^+}{\partial x} dx dy dz + g_p dx dy dz - \frac{p}{\tau_{pt}} dx dy dz$$

where
$$-\frac{\partial F_p^+}{\partial x} dx = F_{px}^+(x) - F_{px}^+(x + dx)$$

recombination rate for holes is given by $p/\tau_{\rm pt}$ where $\tau_{\rm pt}$ includes both the thermal-equilibrium carrier lifetime and the excess carrier lifetime

Continuity equation for holes
$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}}$$

Continuity equation for electrons
$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}}$$

$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_n}$$



Time dependent diffusion equation

For diffusion:

Previously we have current density

$$J_p = e\mu_p pE - eD_p \frac{\partial p}{\partial x}$$

$$J_n = e\mu_n nE + eD_n \frac{\partial n}{\partial x}$$

Then the flux for electron and hole

$$\frac{J_p}{(+e)} = F_p^+ = \mu_p \, pE - D_p \frac{\partial p}{\partial x}$$

$$\frac{J_n}{(-e)} = F_n^- = -\mu_n n E - D_n \frac{\partial n}{\partial x}$$

Remember we have
$$\frac{\partial p}{\partial t} = -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}}$$
 $\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}}$

$$\frac{\partial n}{\partial t} = -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}}$$

Take the derivative of the flux
$$\frac{\partial p}{\partial t} = -\mu_p \frac{\partial (pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial n}{\partial t} = +\mu_n \frac{\partial (nE)}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$

Time dependent diffusion equation

$$\frac{\partial p}{\partial t} = -\mu_p \frac{\partial (pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}}$$

$$\frac{\partial n}{\partial t} = +\mu_n \frac{\partial (nE)}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}$$
We always have
$$\frac{\partial (pE)}{\partial x} = E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x}$$

Time-dependent diffusion equations for holes and electrons

$$D_{p} \frac{\partial^{2} p}{\partial x^{2}} - \mu_{p} \left(E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_{p} - \frac{p}{\tau_{pt}} = \frac{\partial p}{\partial t}$$

$$D_n \frac{\partial^2 n}{\partial x^2} + \mu_n \left(E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial n}{\partial t}$$

Homogeneous semiconductor: n_0 and p_0 are not a function of time or space coordinate

$$D_{p} \frac{\partial^{2}(\delta p)}{\partial x^{2}} - \mu_{p} \left(E \frac{\partial(\delta p)}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_{p} - \frac{p}{\tau_{pt}} = \frac{\partial(\delta p)}{\partial t}$$

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \left(E \frac{\partial(\delta n)}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\partial(\delta n)}{\partial t}$$

Drift & diffusion current

for holes

for electrons

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$$\overrightarrow{J}_{p} = e \overrightarrow{F}_{p} = -eD_{p} \frac{d\delta p}{dx} \overrightarrow{x}$$

$$(\overrightarrow{J_p})_{drift} = ep\mu_p \vec{E}$$
, where $p = p_0 + \delta p$

$$\overrightarrow{J}_{n} = -e\overrightarrow{F}_{n} = eD_{n} \frac{d\delta n}{dx} \overrightarrow{x}$$

$$(\overrightarrow{J_n})_{drift} = -en\mu_n \vec{E},$$

where $n = n_0 + \delta n$

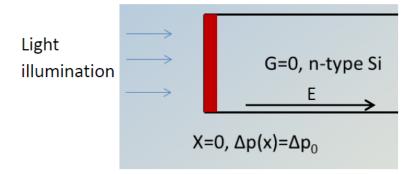
$$J = J_p + J_n$$

$$J_p = (J_p)_{diff} + (J_p)_{dift}$$

$$J_n = (J_n)_{diff} + (J_n)_{diff}$$

Examples

A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface (x=0). The wafer is placed in a constant electric field with a known intensity E. We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface (x=0). Small injection condition is always maintained and the wafer is uniformly doped as N_d .

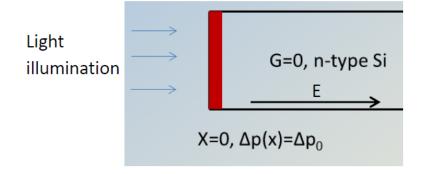


We can usually cancel the generation and recombination rate for the thermal equilibrium part, because they are the same

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p \mu_p \frac{\partial E}{\partial x} + -\frac{p}{\tau_{pt}} + g_p$$

Examples

A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface (x=0). The wafer is placed in a constant electric field with a known intensity E. We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface (x=0). Small injection condition is always maintained and the wafer is uniformly doped as N_d .



$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p \mu_p \frac{\partial E}{\partial x} + -\frac{\delta p}{\tau} + g_p''$$

Examples

$$D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p E \frac{\partial \delta p}{\partial x} - \frac{\delta p}{\tau} = 0$$

The solution likely looks like this:

$$\delta p = Aexp(\lambda x) + C$$

$$\frac{\partial \delta p}{\partial x} = A\lambda exp(\lambda x) \qquad \qquad \frac{\partial^2 \delta p}{\partial x^2} = A\lambda^2 \exp(\lambda x)$$

$$D_p[A\lambda^2 \exp(\lambda x)] - \mu_p E[A\lambda exp(\lambda x)] - \frac{Aexp(\lambda x)}{\tau} - \frac{C}{\tau} = 0$$

$$\operatorname{Aexp}(\lambda x) \left(D_p \lambda^2 - \mu_p E \lambda - \frac{1}{\tau} \right) - \frac{C}{\tau} = 0 \qquad \qquad \mathsf{True \ for \ all \ } \mathbf{x}$$

$$D_p \lambda^2 - \mu_p E \lambda - \frac{1}{\tau} = 0, C = 0$$



Examples

$$D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p E \frac{\partial \delta p}{\partial x} - \frac{\delta p}{\tau} = 0$$

$$\tau D_p \lambda^2 - \tau \mu_p E \lambda - 1 = 0 \qquad L_p = \sqrt{\tau D_p} \qquad L_p(E) = \tau \mu_p E$$

$$L_p = \sqrt{\tau D_p}$$

$$L_p(E) = \tau \mu_p E$$

$$\lambda = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}$$

$$\delta p = (\delta p)_0 exp \left(\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x \right)$$

Examples

$$D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p E \frac{\partial \delta p}{\partial x} - \frac{\delta p}{\tau} = 0$$

$$\delta p = (\delta p)_0 exp \left(\frac{L_p(E) - \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x \right) \qquad L_p = \sqrt{\tau D_p} \qquad L_p(E) = \tau \mu_p E$$

$$\delta p = (\delta p)_0 exp\left(-\frac{x}{L_p}\right)$$

Examples

A n-type semiconductor wafer is uniformly doped and uniformly illuminated by light. There is no electric field. At equilibrium, the excess minority carrier concentration is $(\delta p)_0$. Start from zero time t=0, the illumination is cut off. Find how does the concentration of the excess minority carriers change over time. Small injection condition is always maintained.

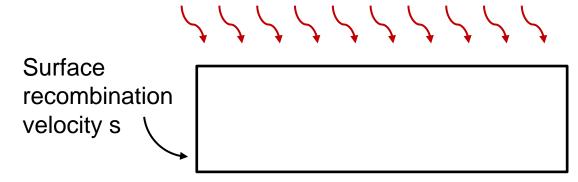
Examples

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$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p \mu_p \frac{\partial E}{\partial x} - \frac{\delta p}{\tau} + g_p'$$

Examples

A n-type semiconductor wafer is <u>uniformly doped</u> and <u>uniformly illuminated</u> by light. There is no <u>electric field</u>. The illumination generation rate is g and the minority carrier lifetime is τ . The semiconductor is long enough on the right side. On the left side edge, the surface recombination velocity is s. Find how does the concentration of the excess minority carriers change along x coordinate. Small injection condition is always maintained.



Examples

A n-type semiconductor wafer <u>is uniformly doped and uniformly illuminated</u> by light. There is no electric field. The illumination generation rate is g and the minority carrier lifetime is τ . The semiconductor is long enough on the right side. On the left side edge, the surface recombination velocity is s. Find how does the concentration of the excess minority carriers change along x coordinate at equilibrium. Small injection condition is always maintained.

Surface recombination velocity s

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p \mu_p \frac{\partial E}{\partial x} - \frac{\delta p}{\tau} + g_p'$$

Examples

A n-type semiconductor wafer is <u>uniformly doped</u> and <u>uniformly illuminated</u> by light. There is <u>no electric field</u>. The illumination generation rate is g and the minority carrier lifetime is τ . The semiconductor is long enough on the right side. On the left side edge, the surface recombination velocity is s. Find how does the concentration of the excess minority carriers change along x coordinate at equilibrium. Small injection condition is always maintained.

Surface recombination velocity s

$$0 = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau} + g_p'$$

The solution likely looks like this:

$$\delta p = Aexp(\lambda x) + C$$



$$0 = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau} + g_p'$$

The solution likely looks like this:

$$\delta p = Aexp(\lambda x) + C$$

$$\frac{\partial \delta p}{\partial x} = A\lambda exp(\lambda x) \qquad \qquad \frac{\partial^2 \delta p}{\partial x^2} = A\lambda^2 \exp(\lambda x)$$

$$D_p[A\lambda^2 \exp(\lambda x)] - \frac{Aexp(\lambda x)}{\tau} - \frac{C}{\tau} + g = 0$$

$$A\exp(\lambda x) \left(D_p \lambda^2 - \frac{1}{\tau}\right) - \frac{C}{\tau} + g = 0$$

$$D_p \lambda^2 - \frac{1}{\tau} = 0; \quad -\frac{C}{\tau} + g = 0 \implies \lambda = \pm \sqrt{D_p \tau}; \quad C = g \tau$$

$$0 = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau} + g$$

The solution likely looks like this:

$$\delta p = Aexp(\lambda x) + g\tau$$

$$D_p \lambda^2 - \frac{1}{\tau} = 0; \quad -\frac{C}{\tau} + g = 0 \implies \lambda = \pm \frac{1}{\sqrt{D_p \tau}}; \quad C = g\tau$$

Boundary conditions:

(1)
$$x \to \infty$$
, $\delta p \ limited \Rightarrow \delta p = Aexp\left(-\frac{x}{\sqrt{D_p \tau}}\right) + g\tau$

(2)
$$x = 0$$
, $-F_p = s \cdot \delta p(x = 0) \Rightarrow D_p \frac{\partial \delta p(x)}{\partial x}|_{x=0} = s \cdot \delta p(x = 0)$

$$0 = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau} + g$$

The solution likely looks like this:

$$\delta p = Aexp(\lambda x) + g\tau$$

(1)
$$x \to \infty$$
, $\delta p \ limited \Rightarrow \delta p = Aexp\left(-\frac{x}{\sqrt{D_p \tau}}\right) + g\tau$

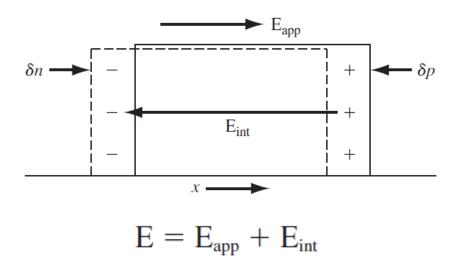
(2)
$$x = 0$$
, $-F_p = s \cdot \delta p(x = 0) \Rightarrow D_p \frac{\partial \delta p(x)}{\partial x}|_{x=0} = s \cdot \delta p(x = 0)$

$$\Rightarrow -D_p \frac{Aexp\left(-\frac{x}{\sqrt{D_p\tau}}\right)}{\sqrt{D_p\tau}}|_{x=0} = s \cdot (A + g\tau)$$

$$\Rightarrow -D_p \frac{A}{\sqrt{D_p \tau}} = s \cdot (A + g\tau) \Rightarrow A = -\frac{g\tau s}{s + D_p/L_p}$$



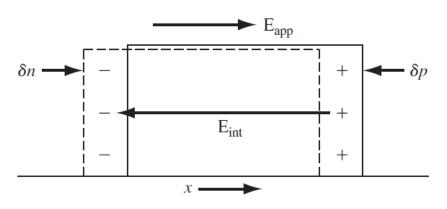
Ambipolar transport



 $E_{\rm app}$ is the applied electric field and $E_{\rm int}$ is the induced internal electric field

The negatively charged electrons and positively charged holes then will drift or diffuse together with a single effective mobility or diffusion coefficient, which are those of the minority carriers. This phenomenon is called ambipolar diffusion or ambipolar transport.

Ambipolar transport



How to simplify problems

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate)	$D_n \frac{\partial^2 (\delta n)}{\partial x^2} = 0, D_p \frac{\partial^2 (\delta n)}{\partial x^2} = 0$
Zero electric field	$E \frac{\partial (\delta n)}{\partial x} = 0$, $E \frac{\partial (\delta p)}{\partial x} = 0$
No excess carrier generation	g'=0
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}}=0, \frac{\delta p}{\tau_{p0}}=0$