Name and ID: ____________________

1. Beta and Gamma function.

   Question1 (0 points)
   Why use Beta and Gamma (exponential form) to substitute $\sin^n x$ and $\cos^n x$?

   Question2 (0 points)
   List the definition form for Beta and Gamma function.

   Question3 (0 points)
   Specify the transformation between Beta and Gamma function.

   Question4 (0 points)
   Specify the other useful formula for applying beta and gamma funcrion

   Question5 (0 points)
   Use Beta and Gamma function to represent

$$\int_0^{\pi/2} \sin^{a-1}\varphi \cdot \cos^{b-1}\varphi \, d\varphi, \quad (a, b > 0)$$

2. Integration by I don't know why but anyway useful.

   Question1 (1 point)
   Prove: If

$$(a-c)^2 + b^2 \neq 0$$

   then

$$\int \frac{a_1 \sin x + b_1 \cos x}{a \sin^2 x + 2b \sin x \cos x + c \cos^2 x} dx = A \int \frac{du_1}{k_1 u_1^2 + \lambda_1} + B \int \frac{du_2}{k_2 u_2^2 + \lambda_2}$$

   where λ_1, λ_2 is the root of the equation

$$\begin{vmatrix} a-\lambda & b \\ b & c-\lambda \end{vmatrix} = 0, (\lambda_1 \neq \lambda_2)$$

   and

$$u_i = (a - \lambda_i)\sin x + b \cos x, \;\; k_i = \frac{1}{a - \lambda_i} \;\; (i = 1, 2)$$

   A and B are constants to be determined.
   Hint: Try first to reform the denominator according to the given determinate and then the numerator.

   Question2 (1 point)
   Change variables and find the area of regions bounded by the following curves:

$$\left(x^3 + y^3\right)^2 = x^2 + y^2, \quad x \geq 0, \quad y \geq 0$$

   (Hint: Apply polar transformation and utilize the conclusion in question 1)

3. Double Integral

   Question1 (1 point)
   Let $R : 0 \leq x \leq t, 0 \leq y \leq t$ and

$$F(t) = \iint_R e^{-\frac{tx}{y^2}} dxdy$$

   Compute $F'(t)$
   Hint: Try to write down $F'(t)$ in terms of $F(t)$ And the transformation T:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} tu \\ tv \end{pmatrix}$$

Question2 (1 point)

Use Generalized Polarize, compute

$$\left(\frac{x}{a}+\frac{y}{b}\right)^4=\frac{x^2}{h^2}+\frac{y^2}{k^2}\ (x>0,y>0)$$

Question3 (1 point)

Change variables to find the area of regions bounded by the following curves:

$$\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1,\quad \sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=2,\quad \frac{x}{a}=\frac{y}{b},\quad 4\frac{x}{a}=\frac{y}{b}\quad (a>0,b>0)$$

Hint: consider

$$\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=u,\quad \frac{x}{y}=v$$