VE 320 Fall 2021

Introduction to Semiconductor Devices

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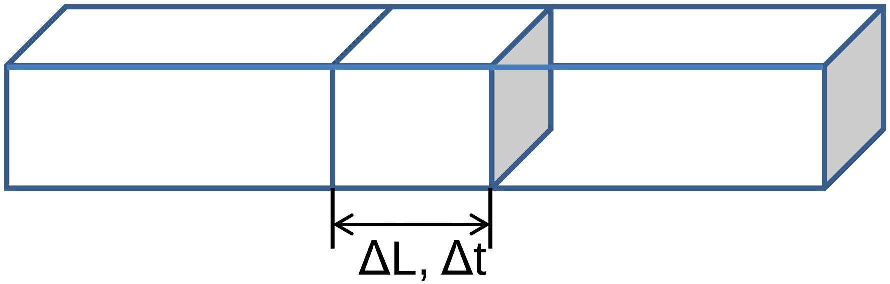
Lecture 5

Carrier Transport
(Chapter 5)

Carrier transport

$n\ type\ semiconductor$

$$I = \frac{\Delta Q}{\Delta t} = \frac{neA_c\Delta L}{\Delta t} = neA_c v$$



Current density

$$J = \frac{I}{A_c} = nev$$

v: average drift velocity, *e*: electron charge, *n*: electron density

$$\mathrm{F} = -eE = m^*a \ \rightarrow a = -\frac{eE}{m^*} \ \rightarrow v = at$$

m^*: conductivity effective mass of electrons

$$v = -\frac{et}{m^*}E$$

If effective mass and *t* are constant, then will *v* linearly increase with *E*?

True for low electric field

$$\boxed{v = -\mu_n E}$$

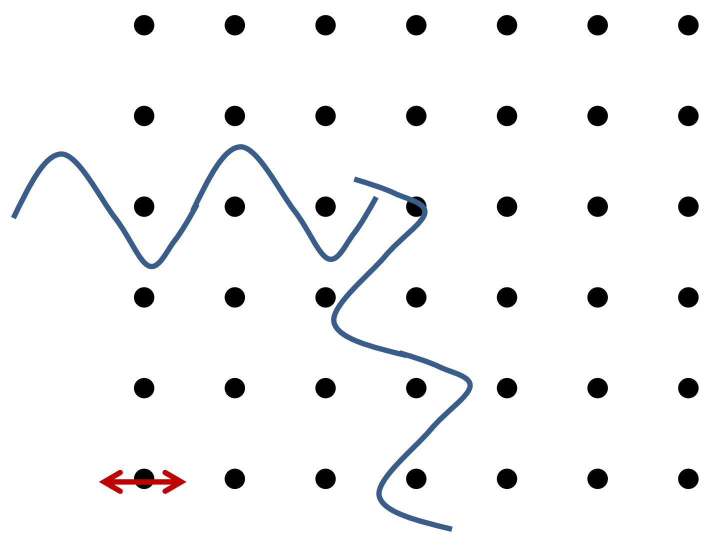
μ_n is the electron mobility – Very important for semiconductors! Unit: $cm^2/(V \cdot s)$

Drift current density due to electrons $J_{n|drf} = (-en)(-\mu_n \mathrm{E}) = e\mu_n n\mathrm{E}$

Total drift current density: $J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E}$

Carrier transport

Electron:



Resistor heating up by current

Thermal vibration ←→ phonon

Ionized impurity

$$scattering \rightarrow average < v > does\ not\ increase\ as\ time$$

$$\mu_n = \frac{et}{m^*}$$

What is t here? Time? Give more time, then the mobility increases?

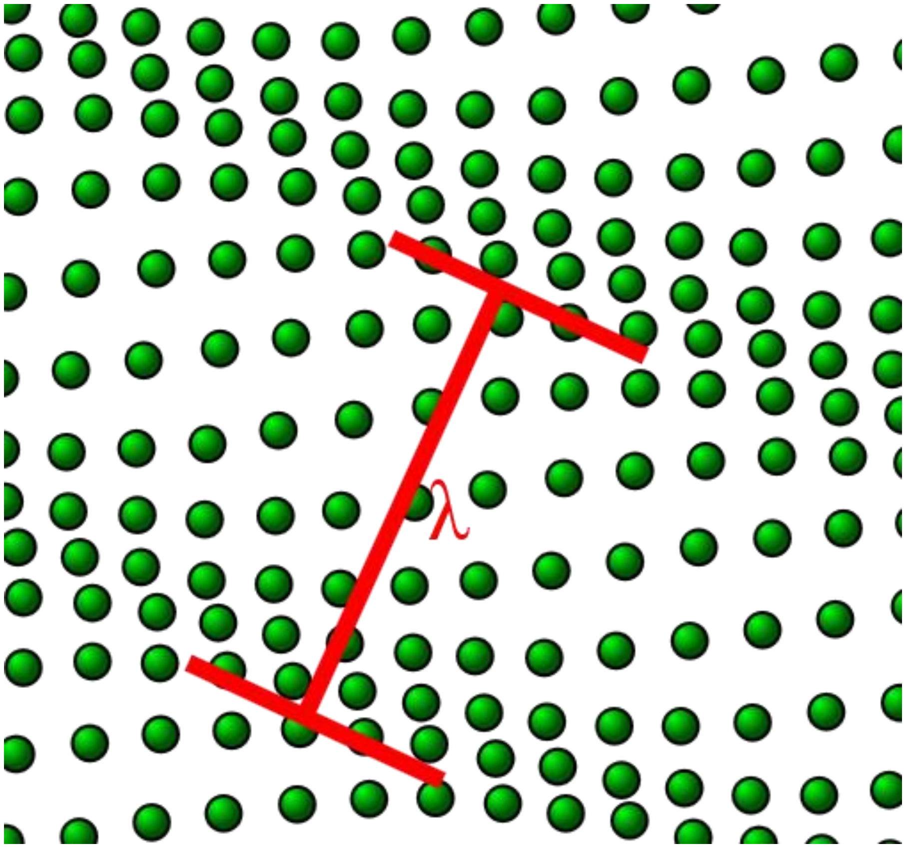
$$\mu_n = \frac{e\tau_{cn}}{m^*_{cn}}$$

τ_{cn} is the mean time between collisions for electron

Hole: $$\mu_p = \frac{e\tau_{cp}}{m^*_{cp}}$$

Phonons

Basic facts:



Atom displacement exaggerated.

Lattice scattering, or phonon scattering

- Perfect periodic lattice: no scattering
- Thermal vibration: potential function disruption, scattering

$$\mu_L \propto T^{-3/2}$$

μ_L: mobility only due to phonon scattering

Phonon scattering

Basic facts:

- Quantum mechanic description of atom vibration at a frequency
- A wave and also a particle
- Two kinds: optical phonons and acoustic phonons
- Optical phonons:

  short wavelength → high energy

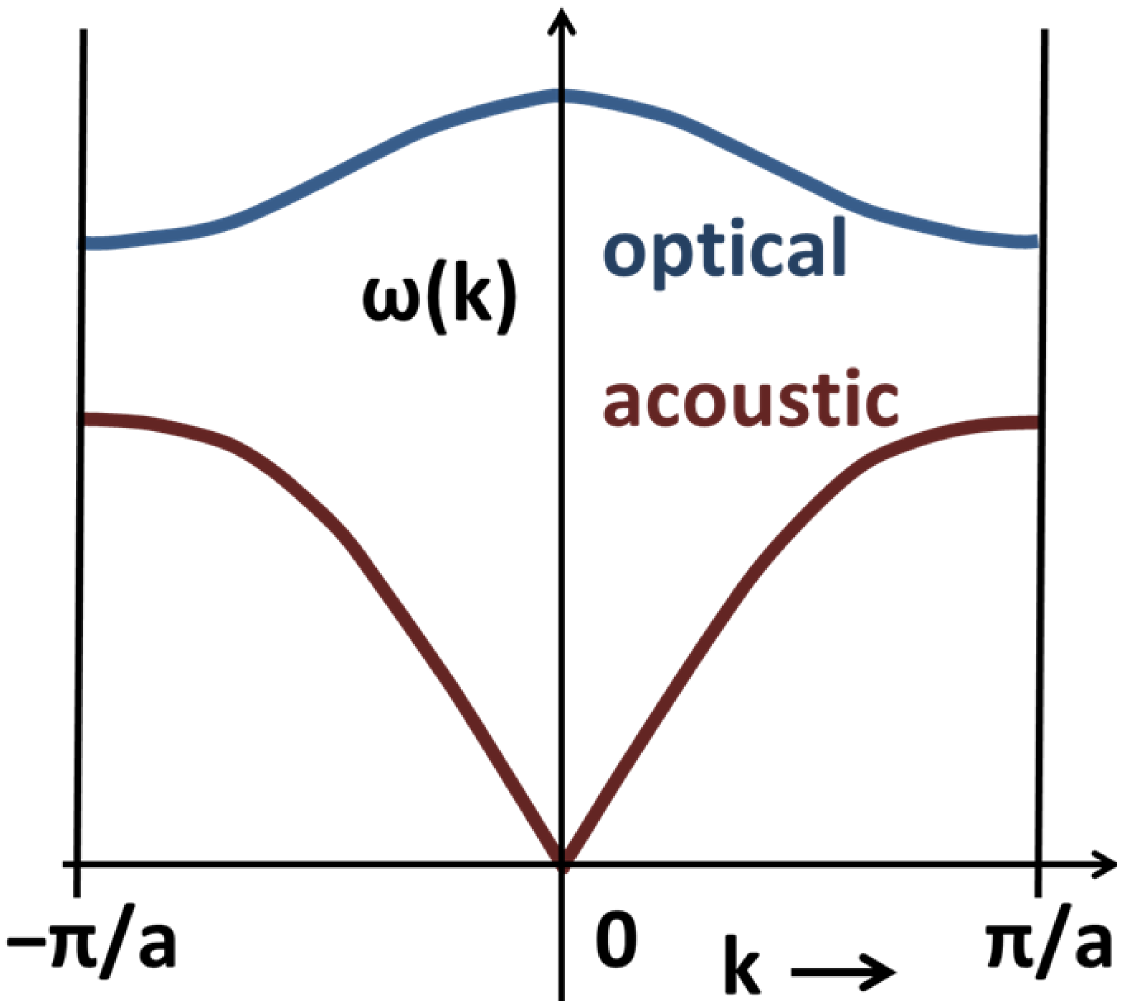
  electron-phonon collision: inelastic scattering (energy E change)
- Acoustic phonons:

  long wavelength (tens of lattice constant) → low energy

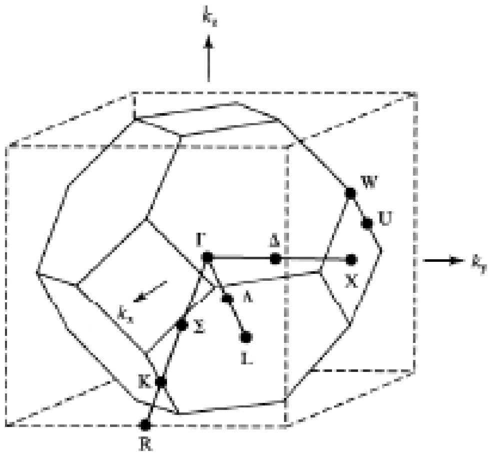
  electron-phonon collision: elastic scattering (momentum k change)
- Acoustic: coherent movement of atoms, like sound wave
- Optical: out-of-phase movements of atoms, lattice basis consists of two or more atoms, can be excited by infrared light. Raman scattering-> useful

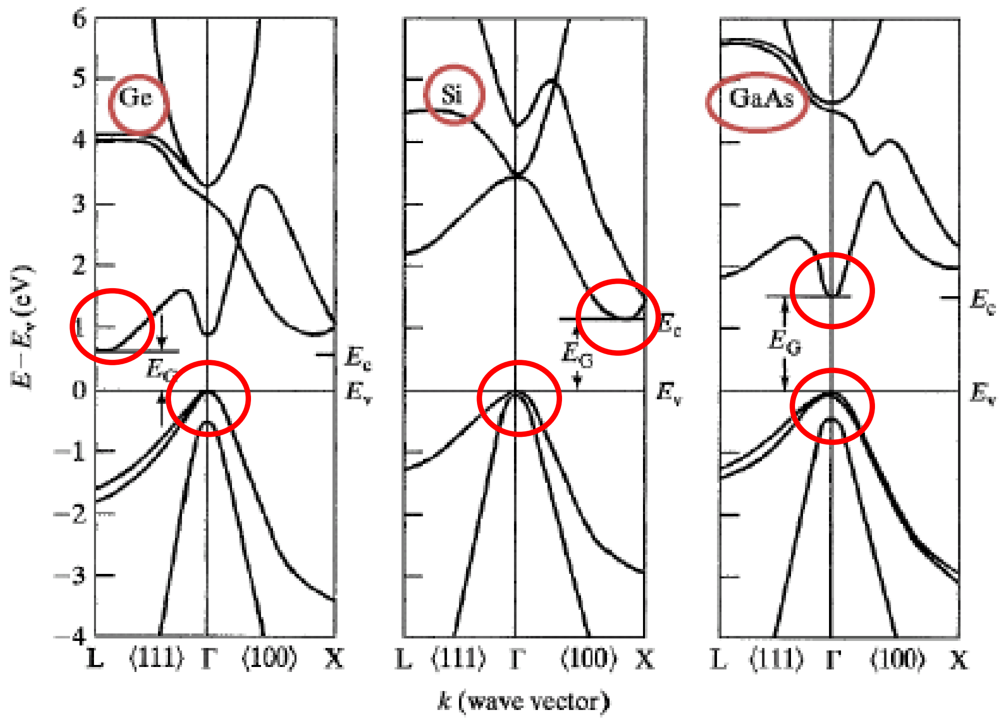
Phonons

Basic facts:



Previously: band structure in 3D k-space





- Direct bandgap

  (electron excited, k constant, E ↑ → f ↑, v = f λ ↑)

- Indirect bandgap (P(absorbing photon) * P(phonon scattering))

Ionized impurity scattering

Coulomb interaction

$$\mu_I \propto \frac{T^{+3/2}}{N_I}$$

μ_I: mobility only due to ionized impurity scattering

$N_I = N_d^+ + N_a^-$ is the total ionized impurity concentration

If the number of ionized impurity centers increases, then the probability of a carrier encountering an ionized impurity center increases, implying a smaller value of μ_I

Total scattering and mobility

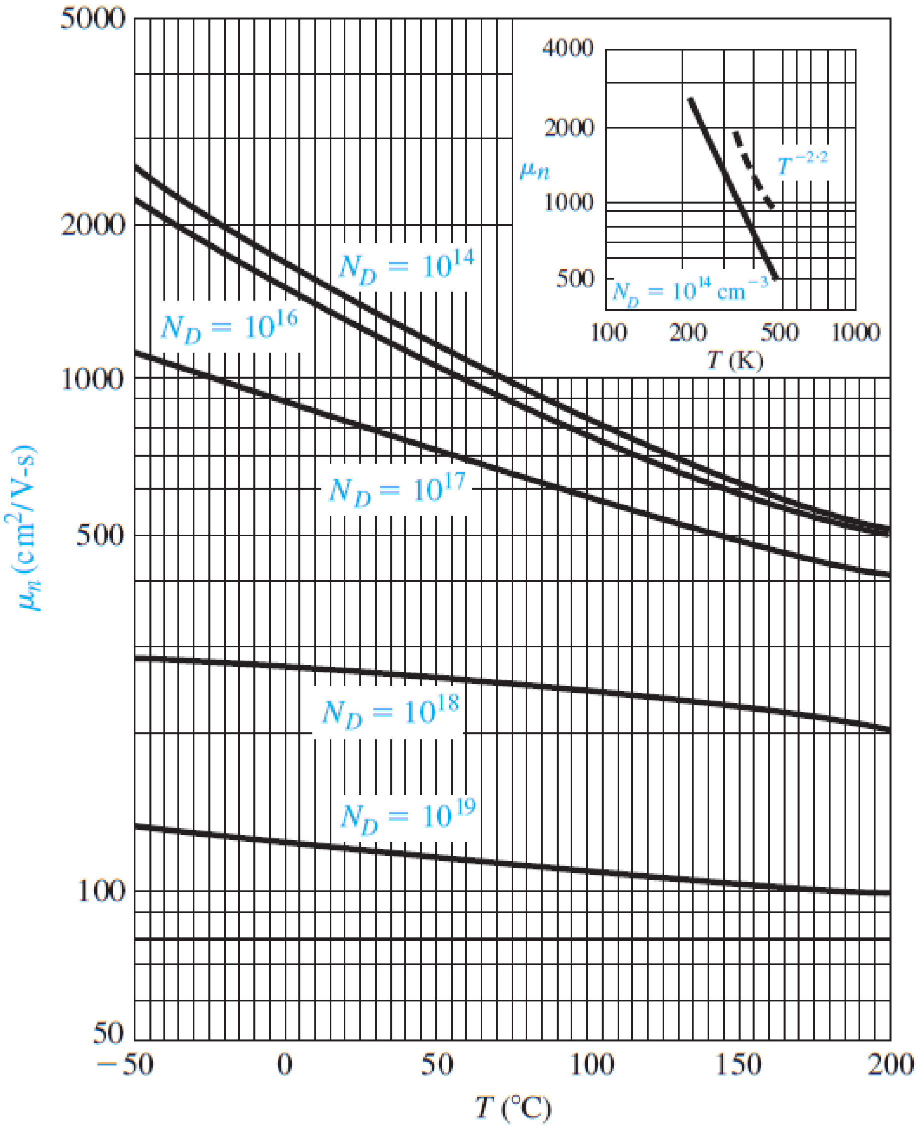
Total probability of a scattering event occurring in the differential time dt

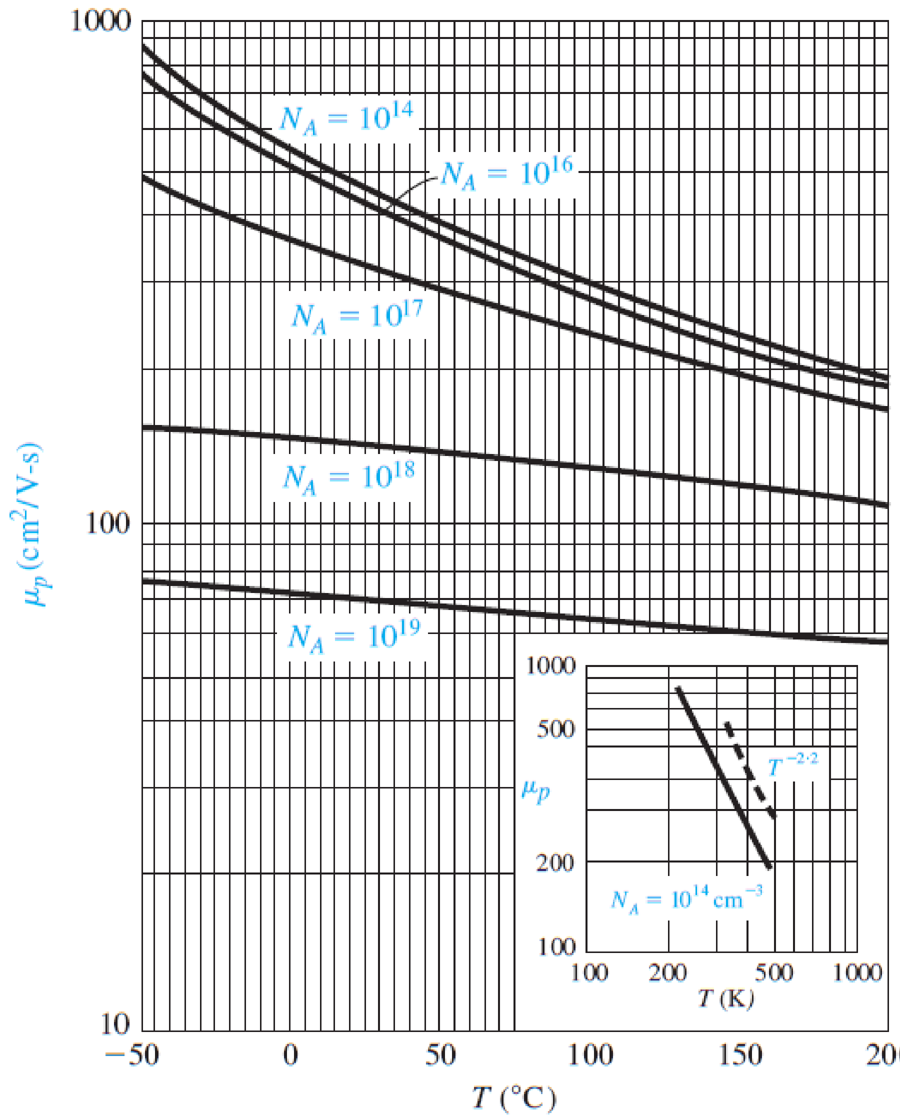
$$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L}$$

Total mobility

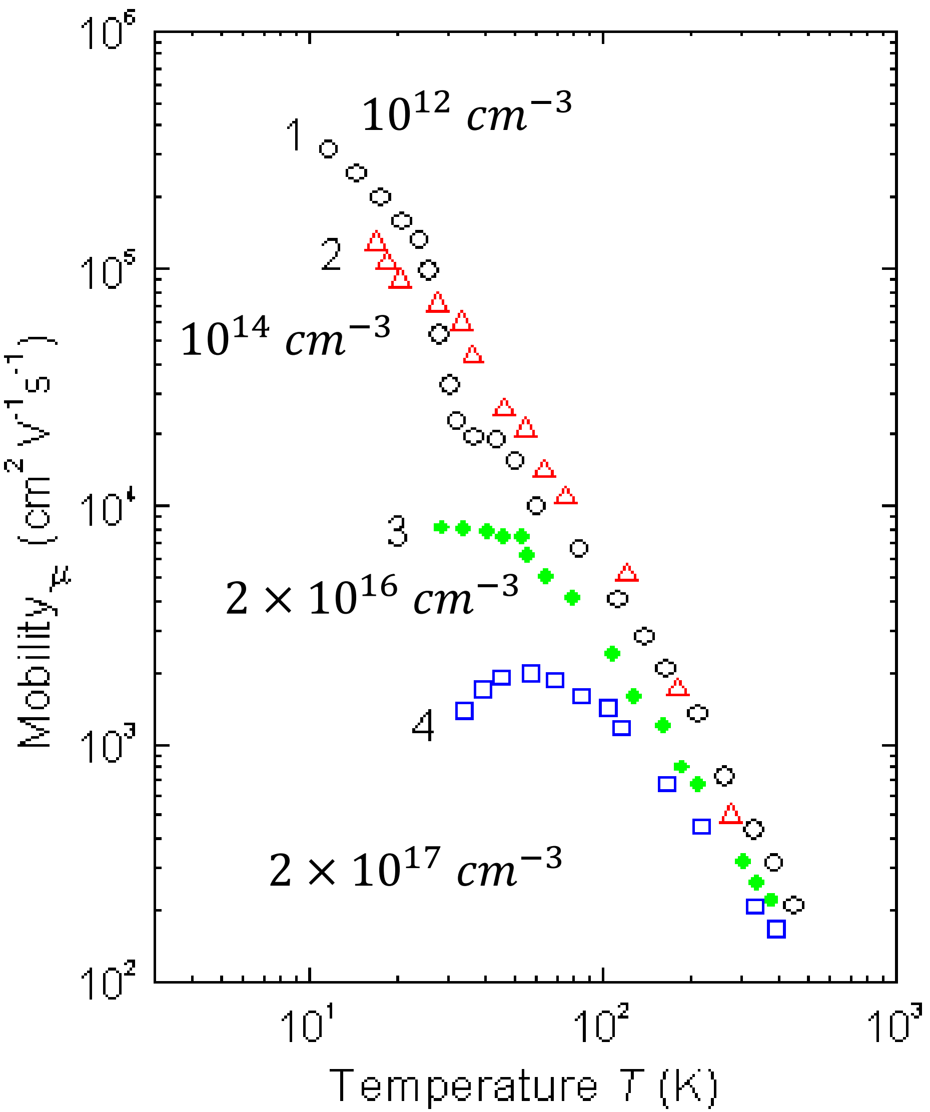
$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

Mobility vs. T at different doping





Mobility vs. *T*



Drift current density

$$J_{drf} = e(\mu_n n + \mu_p p)\mathrm{E} = \sigma \mathrm{E}$$

σ is the conductivity of the semiconductor material

Resistivity $\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$

For p-type semiconductor with an acceptor doping N_a (N_d=0) in which $N_a >> n_i$, and if electron and hole mobilities are similar, then

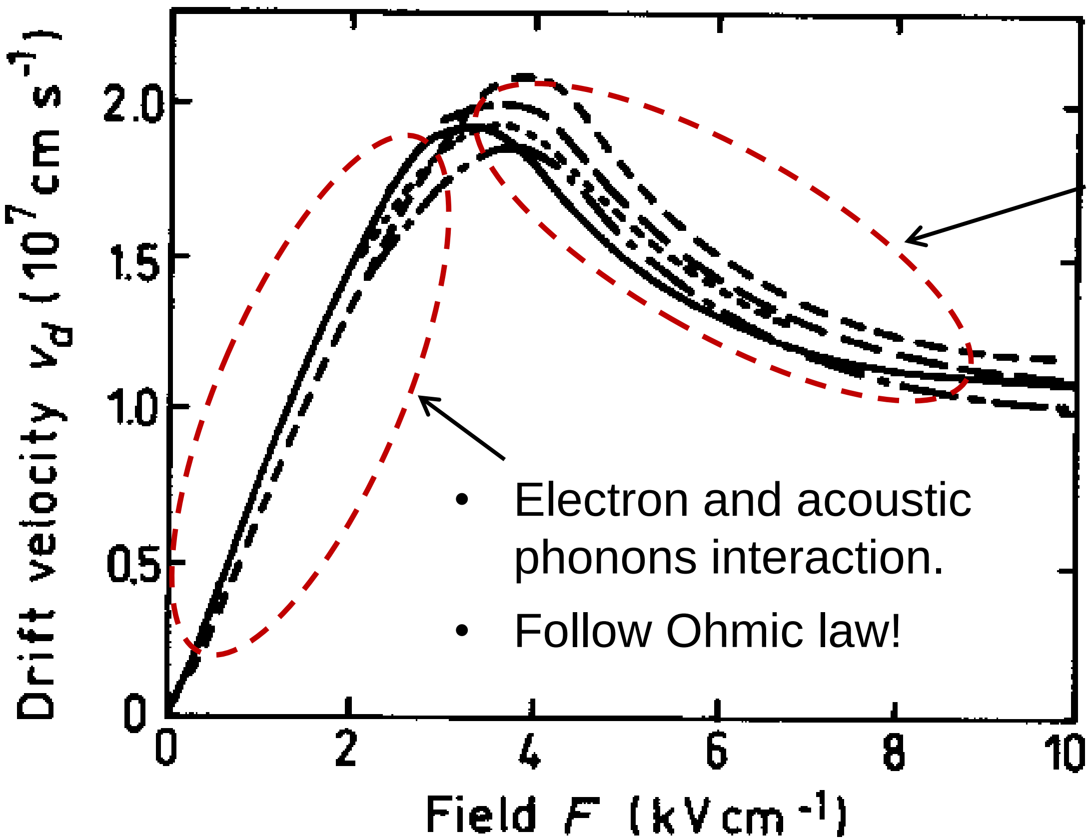
$$\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p$$

Assume complete ionization:

$$\sigma \approx e\mu_\rho N_a$$

Velocity saturation

$\lambda_{ac\ phon}\ long;$ $\lambda_{op\ phon}\ short;$

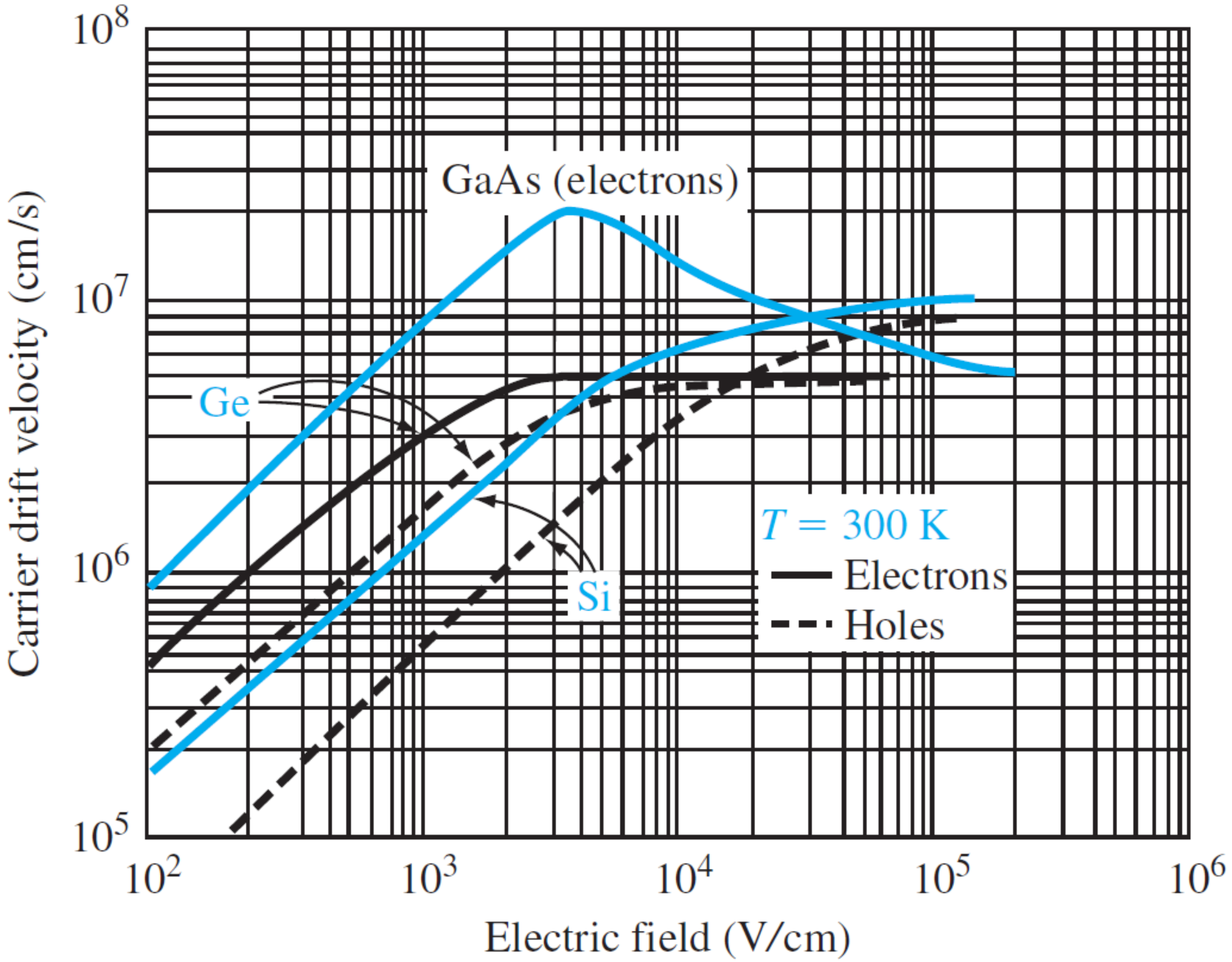


- Electron and optical phonon interaction
- Not follow Ohmic law

- Electron and acoustic phonons interaction.
- Follow Ohmic law!

$\lambda_{electron}\ decreases$

Velocity saturation



Velocity saturation

Carrier drift velocity versus electric field for electrons in silicon

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{1/2}}$$

v_s: saturation velocity ~ 10^7 cm/s at 300K, E_{on}= 7×10^3 V/cm

Small electric field:

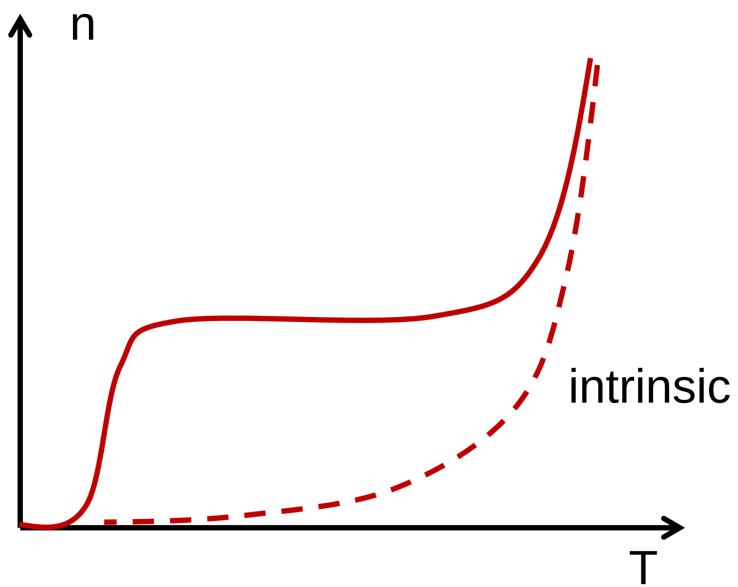
$$v_n \cong \left(\frac{E}{E_{on}}\right) \cdot v_s$$

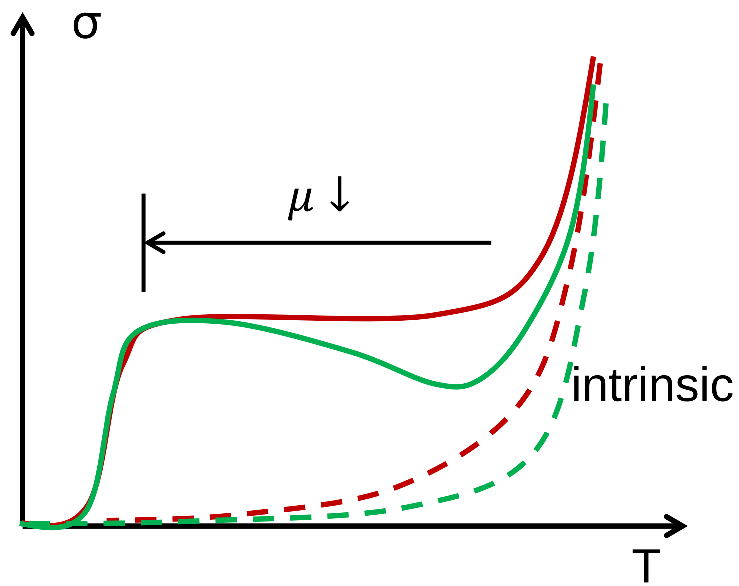
Holes, small electric field:

$$v_p \cong \left(\frac{E}{E_{op}}\right) \cdot v_s$$

Mobility vs T

Semiconductor:



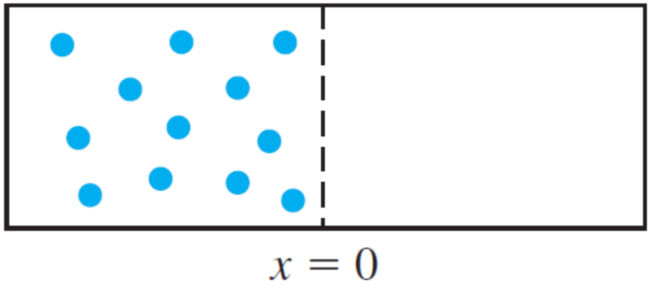


metal: conductivity ↘

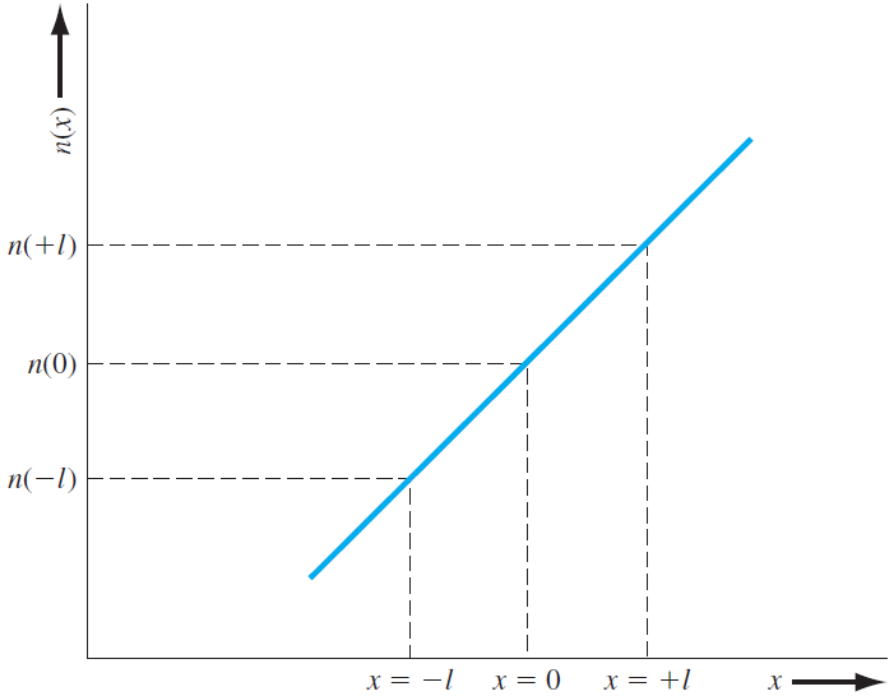
Carrier diffusion

2 mechanisms that can induce current in a semiconductor: drift, diffusion

Diffusion is the process whereby particles flow from a region of high concentration toward a region of low concentration.



One half of the electrons at $x=-l$ will be traveling to the right and one half of the electrons at $x=l$ will be traveling to the left, then the net rate of electron flow, F_n in the $+x$ direction at $x=0$ is



$$F_n = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(+l)v_{th} = \frac{1}{2}v_{th}[n(-l) - n(+l)]$$

Carrier diffusion

Electron: $$F_n = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(+l)v_{th} = \frac{1}{2}v_{th}[n(-l) - n(+l)]$$

Taylor expansion of *n* at *x*=0 (*l* is very small)

$$F_n = \frac{1}{2}v_{th}\left\{\left[n(0) - l\frac{dn}{dx}\right] - \left[n(0) + l\frac{dn}{dx}\right]\right\}$$

$$F_n = -v_{th}l\frac{dn}{dx}$$

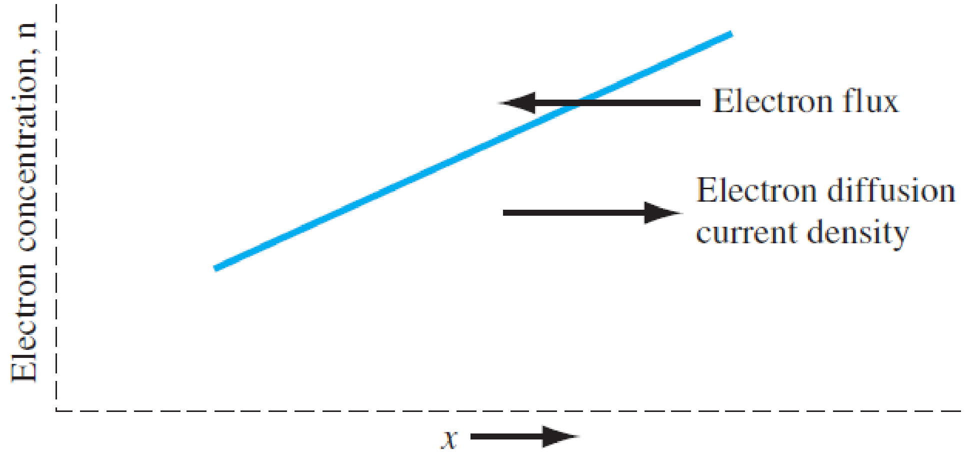
Current density $$J = -eF_n = +ev_{th}l\frac{dn}{dx}$$

$$J_{nx|dif} = eD_n\frac{dn}{dx}$$

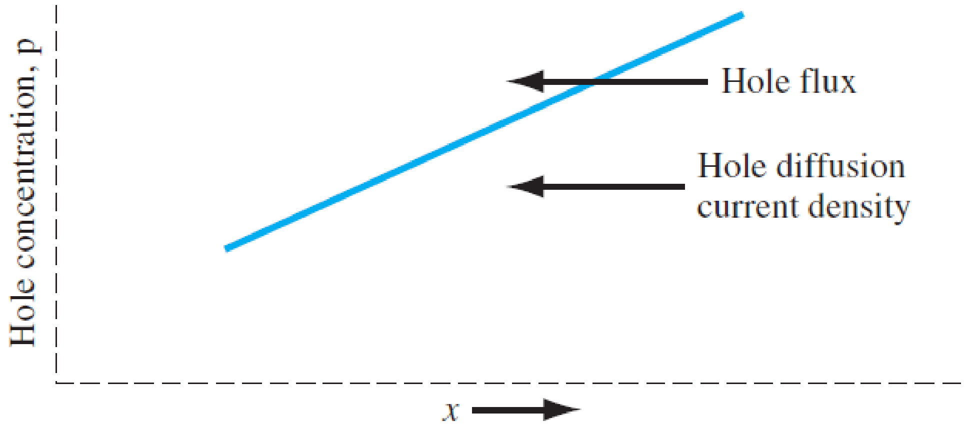
where $D_n = v_{th}l$ is called the electron diffusion coefficient, has units of cm^2/s

Hole: $$J_{px|dif} = -eD_p\frac{dp}{dx}$$

Carrier diffusion



(a)



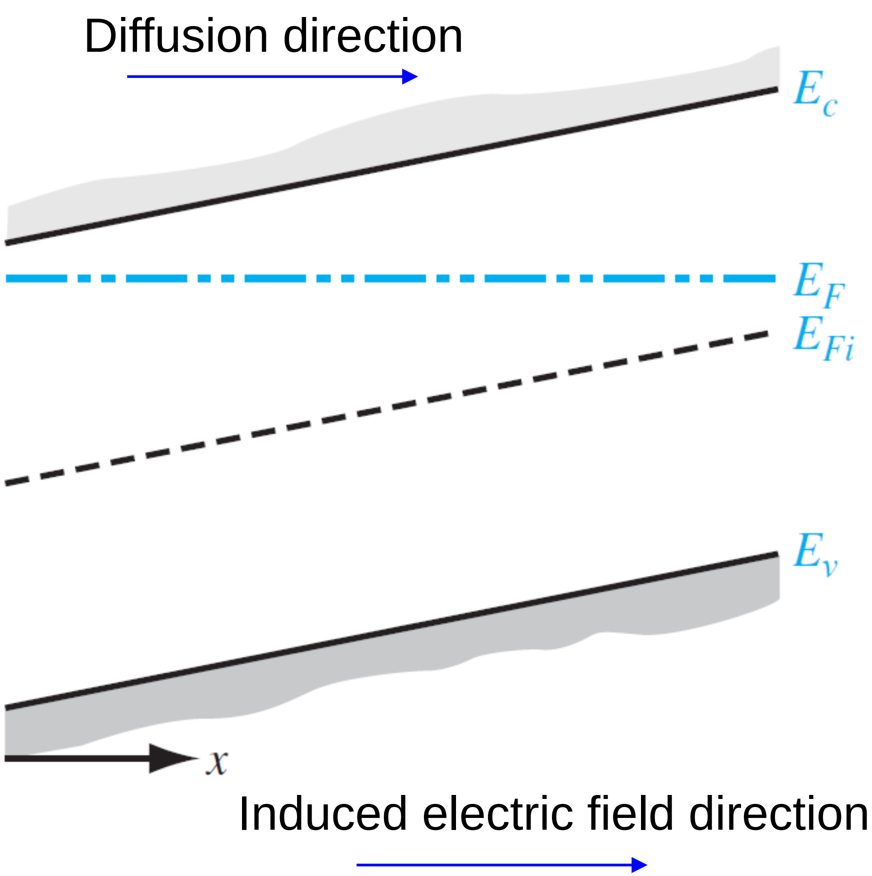
Total current density with both drift and diffusion

$$J = en\mu_n \mathrm{E}_x + ep\mu_p \mathrm{E}_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

4 terms: but usually some certain term will dominate for certain cases

Nonuniform doping

Fermi level: always constant at thermal equilibrium



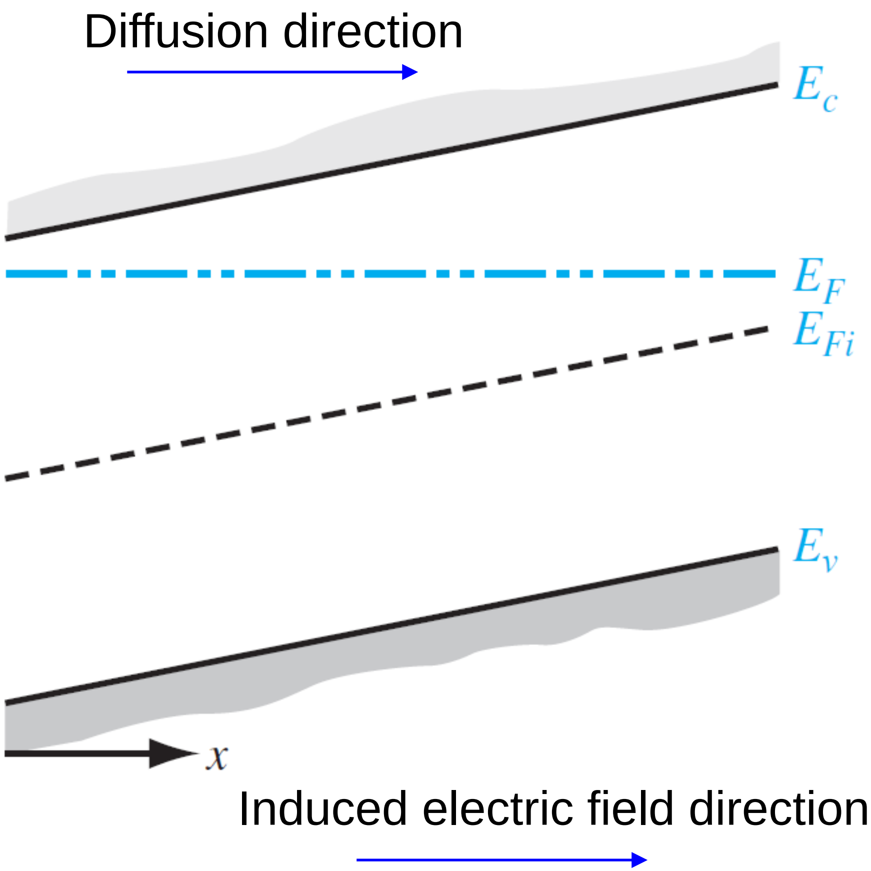
- The space charge induced by this diffusion process is a small fraction of the impurity concentration
- Induced electric field: prevents any further diffusion, counter the diffusion

$$\mathrm{E}_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

Intrinsic Fermi-level changes-> there is an electric field

Nonuniform doping

Fermi level: always constant at thermal equilibrium



$$\mathrm{E}_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$$

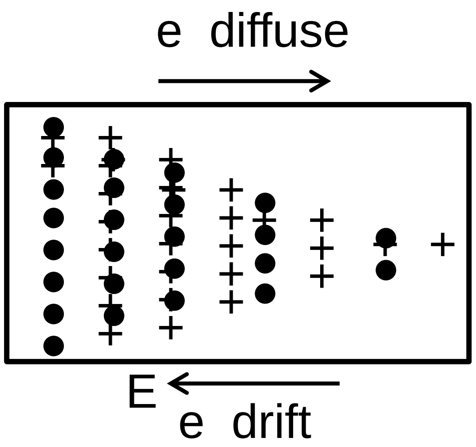
$$E_F - E_{Fi} = kT \ln\left(\frac{N_d(x)}{n_i}\right)$$

$$-\frac{dE_{Fi}}{dx} = \frac{kT}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$\mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

Einstein relation

$\boxed{\text{Drift current = diffusion current}}$



Nonuniformly doped semiconductor with no electrical connection

Drift from induced electric field balances the diffusion

Electrons:

$$J_n = 0 = en\mu_n \mathrm{E}_x + eD_n \frac{dn}{dx}$$

Assume $n=N_d$

$$J_n = 0 = e\mu_n N_d(x)\mathrm{E}_x + eD_n \frac{dN_d(x)}{dx}$$

$$\mathrm{E}_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

$$0 = -e\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n \frac{dN_d(x)}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{e}$$

Einstein relation $$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

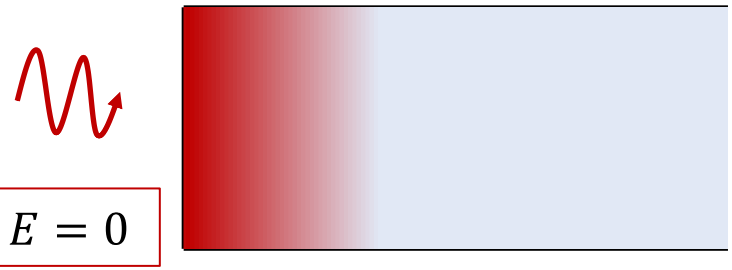
Carrier concentration

- Carrier drift & diffusion

$$carrier\ drift\text{: } electric\ field\ E \neq 0$$

$$driving\ force\text{: } electric\ field$$

$flux$:
$\#\ of\ charges\ passing\ a\ unit\ area\ at\ a\ give\ unit\ time$



$$carrier\ diffusion\text{: } electric\ field\ E = 0$$

$$driving\ force\text{: } thermal\ dynamics$$

$flux$:
$\#\ of\ carriers\ passing\ a\ unit\ area\ at\ a\ give\ unit\ time$