Chapter 23

Electric Potential

Goals for Chapter 23

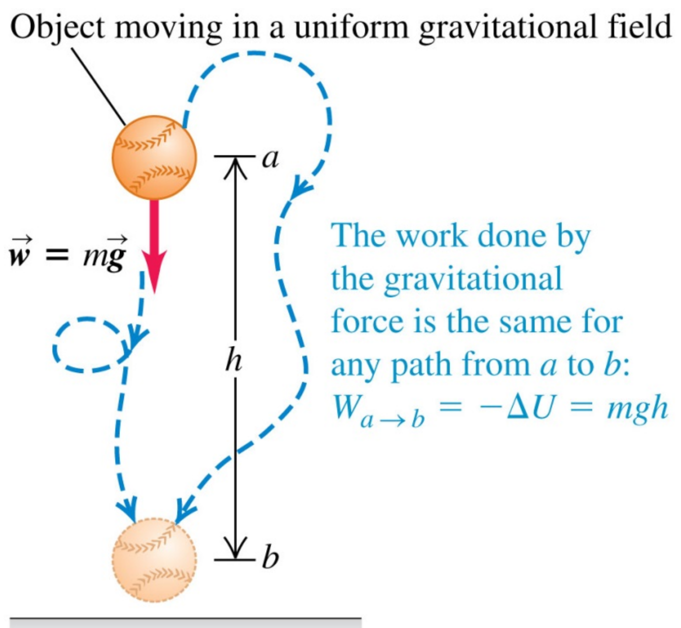
- To calculate the electric potential energy of a group of charges
- To know the significance of electric potential
- To calculate the electric potential due to a collection of charges
- To use equipotential surfaces to understand electric potential
- To calculate the electric field using the electric potential

Introduction

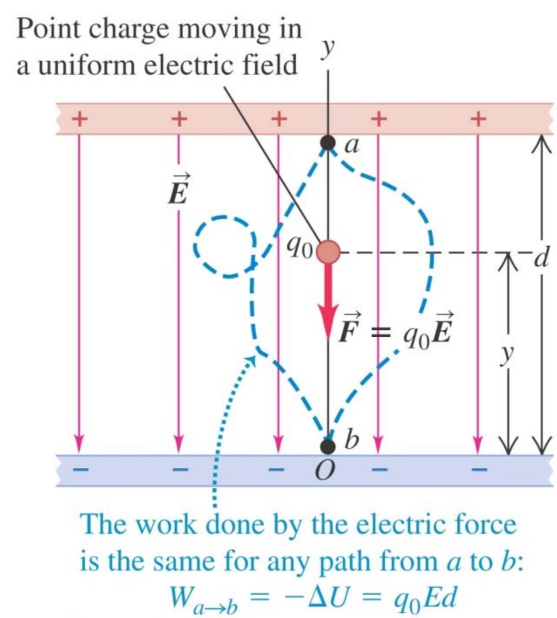
- How is electric potential related to welding?
- Electric potential energy is an integral part of our technological society.
- What is the difference between electric potential and electric potential energy?
- How is electric potential energy related to charge and the electric field?

Electric potential energy in a uniform field

- The behavior of a point charge in a uniform electric field is analogous to the motion of a baseball in a uniform gravitational field



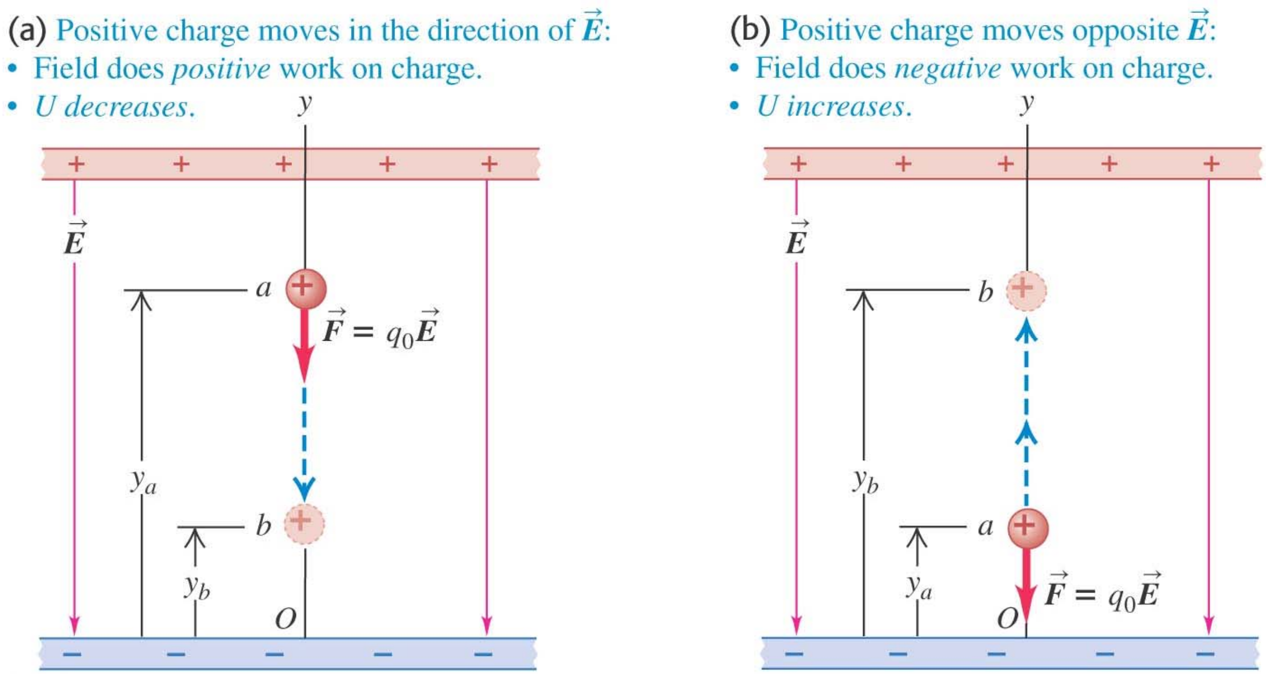
$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$ (work done by a conservative force)



$\boxed{W_{a\to b} = Fd = q_0Ed}$

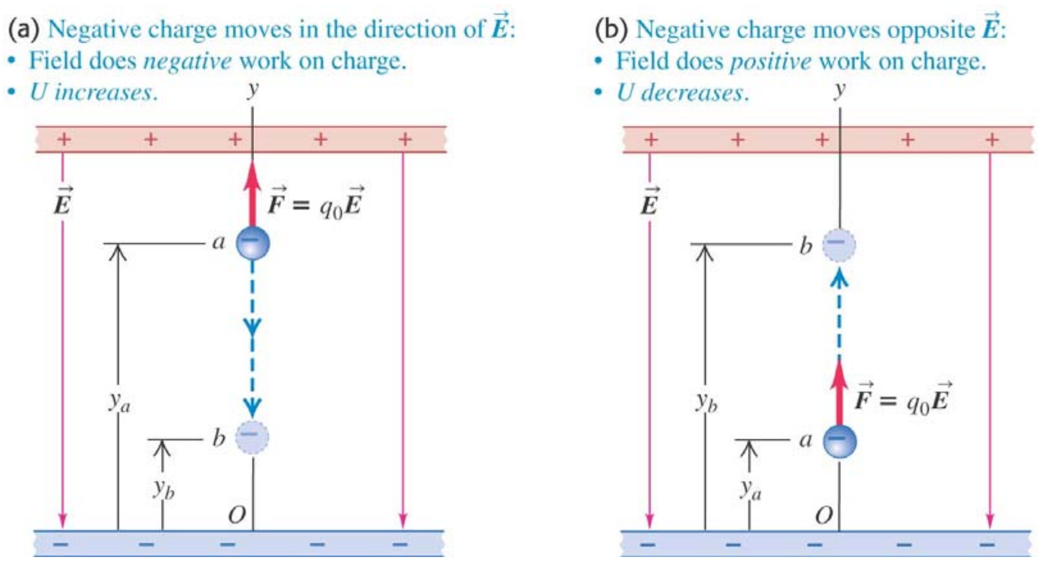
A positive charge moving in a uniform field

- If the positive charge moves in the direction of the field, the potential energy *decreases*, but if the charge moves opposite the field, the potential energy *increases*.



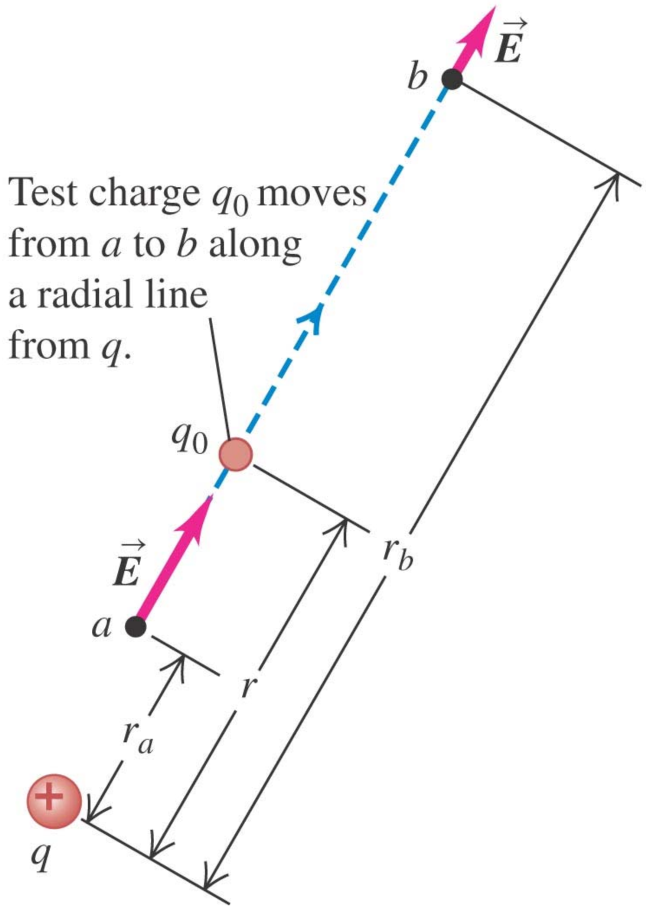
A negative charge moving in a uniform field

- If the negative charge moves in the direction of the field, the potential energy *increases*, but if the charge moves opposite the field, the potential energy *decreases*.
- Figure 23.4 below illustrates this point.



Electric potential energy of two point charges

- Follow the discussion of the motion of a test charge q_0 in the text.
- The electric potential is the same whether q_0 moves in a radial line (left figure) or along an arbitrary path (right figure).



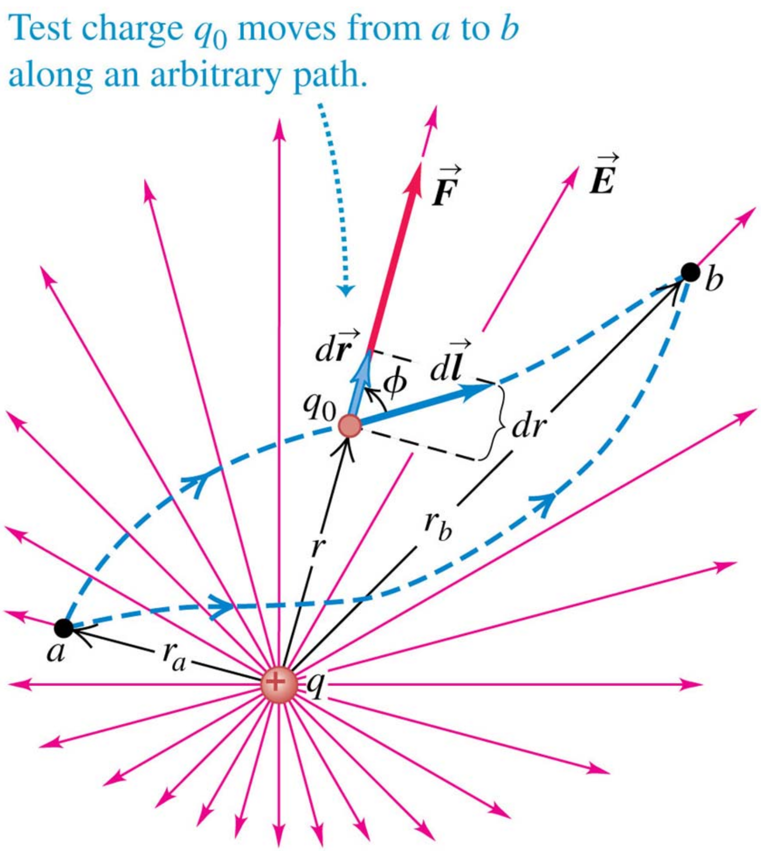
Coulomb's law, and its radial component is

$$F_r = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Electric potential energy of two point charges

- Follow the discussion of the motion of a test charge q_0 in the text.
- The electric potential is the same whether q_0 moves in a radial line (left figure) or along an arbitrary path (right figure).



$$W_{a \to b} = \int_{r_a}^{r_b} F \cos\phi \, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \cos\phi \, dl$$

Fig. 23.6 shows that $\cos\phi \, dl = dr$.

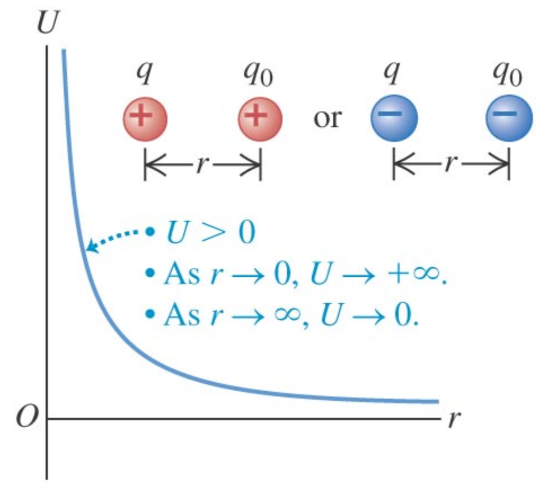
Graphs of the potential energy

- The sign of the potential energy depends on the signs of the two charges.

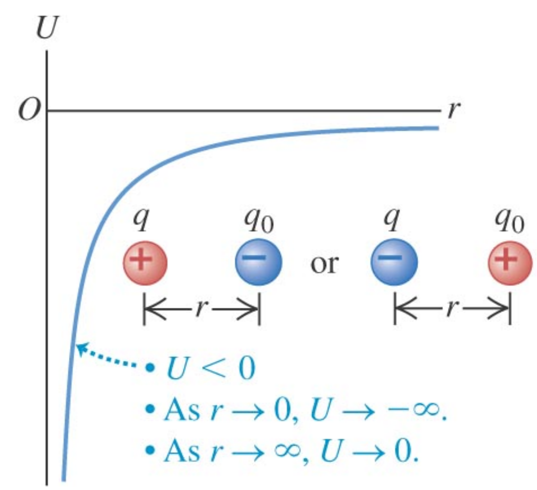
$$W_{a\to b} = \int_{r_a}^{r_b} F_r\, dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\, dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$ (electric potential energy of two point charges q and q_0)

(a) q and q_0 have the same sign.

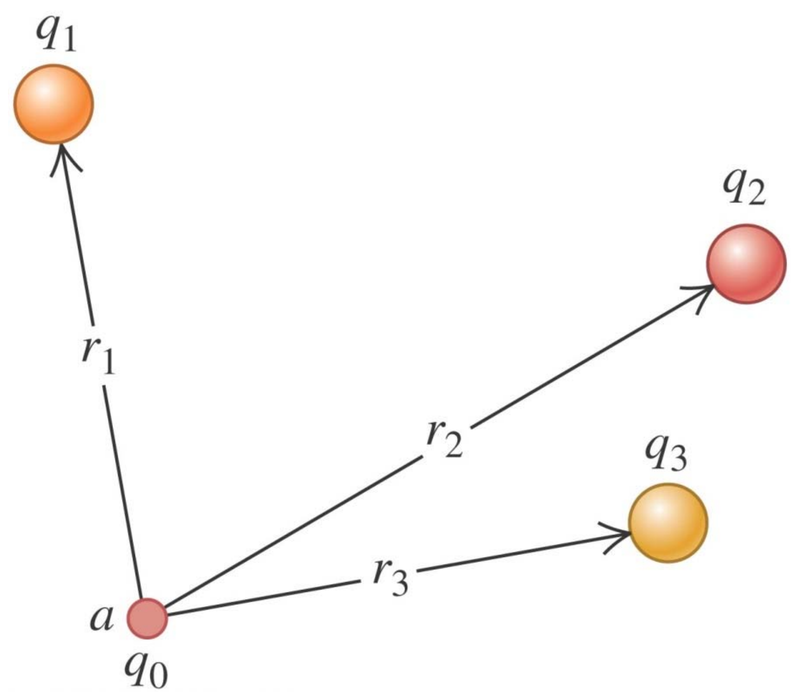


(b) q and q_0 have opposite signs.



Electrical potential with several point charges

- The potential energy associated with q_0 depends on the other charges and their distances from q_0, as shown in Figure 23.8 at the right.
- Follow the derivation in the text of the formula for the total potential energy U.
- Follow Example 23.2.



$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

(point charge q_0 and collection of charges q_i)

Electric potential

- *Potential* is *potential energy per unit charge*.
- We can think of the potential difference between points a and b in either of two ways. The potential of a with respect to b ($V_{ab} = V_a - V_b$) equals:
    - ✓ the work done by the electric force when a *unit* charge moves from a to b.
    - ✓ the work that must be done to move a *unit* charge slowly from b to a against the electric force.
- Follow the discussion in the text of how to calculate electric potential.

Electric potential

- *Potential* is *potential energy per unit charge*.

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(potential due to a point charge)}$$

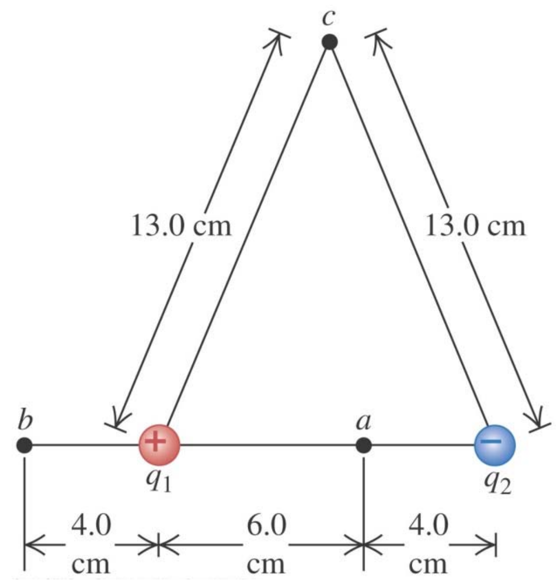
$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad \text{(potential due to a collection of point charge)}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad \text{(potential due to a continuous distribution of charge)}$$

Potential zero at ∞.

Potential due to two point charges

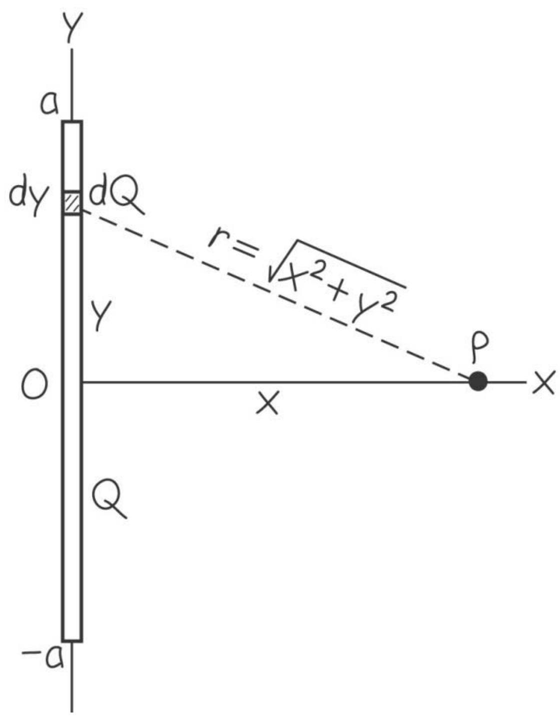
- Follow Example 23.4 using Figure 23.13 at the right.
- Follow Example 23.5.



$$V_a = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$

A finite line of charge

- Follow Example 23.12 using Figure 23.21 below.



EXECUTE: As in Example 21.10, the element of charge dQ corresponding to an element of length dy on the rod is $dQ = (Q/2a)dy$. The distance from dQ to P is $\sqrt{x^2 + y^2}$, so the contribution dV that the charge element makes to the potential at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2 + y^2}}$$

To find the potential at P due to the entire rod, we integrate dV over the length of the rod from $y = -a$ to $y = a$:

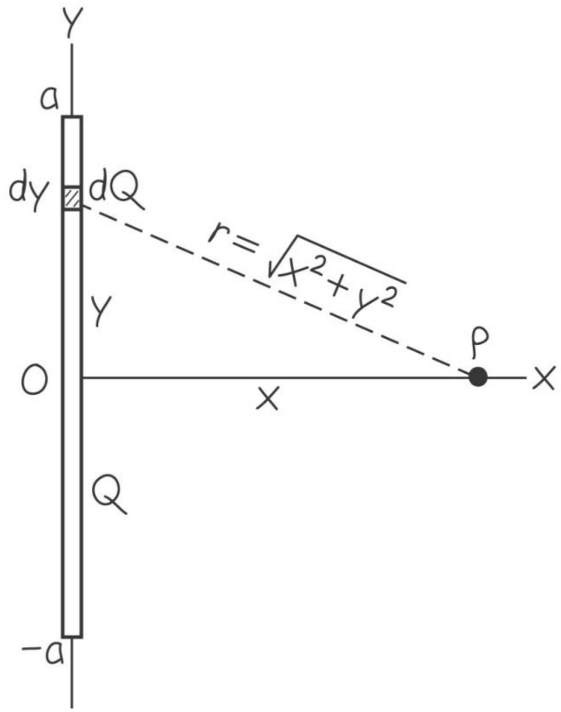
$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2 + y^2}}$$

You can look up the integral in a table. The final result is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)$$

A finite line of charge

- Follow Example 23.12 using Figure 23.21 below.



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{\imath} \qquad (21.9)$$

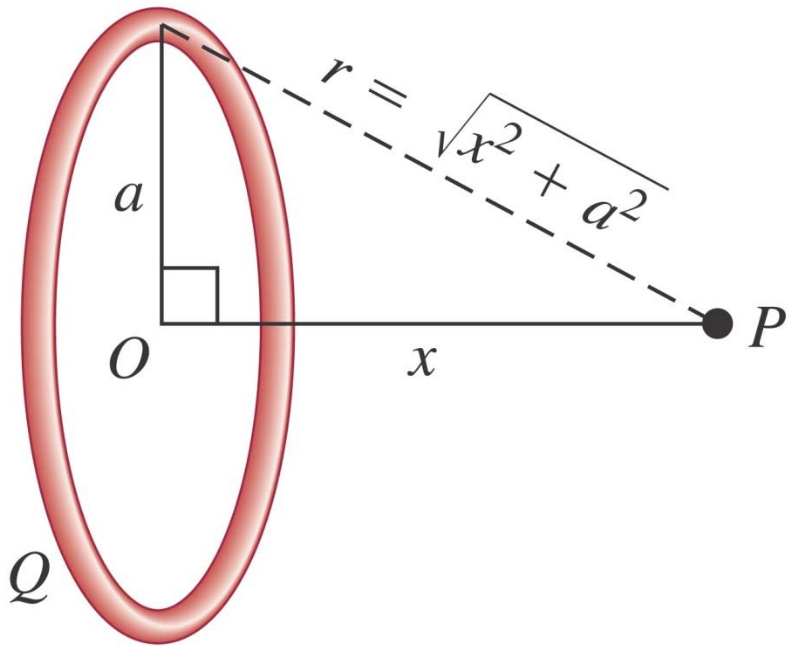
$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

$$W_{a\rightarrow b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b}\frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$ (electric potential energy of two point charges q and q_0)

A ring of charge

- Follow Example 23.11 using Figure 23.20 below.



$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$ (potential due to a continuous distribution of charge)

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2 + a^2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2 + a^2}}$$

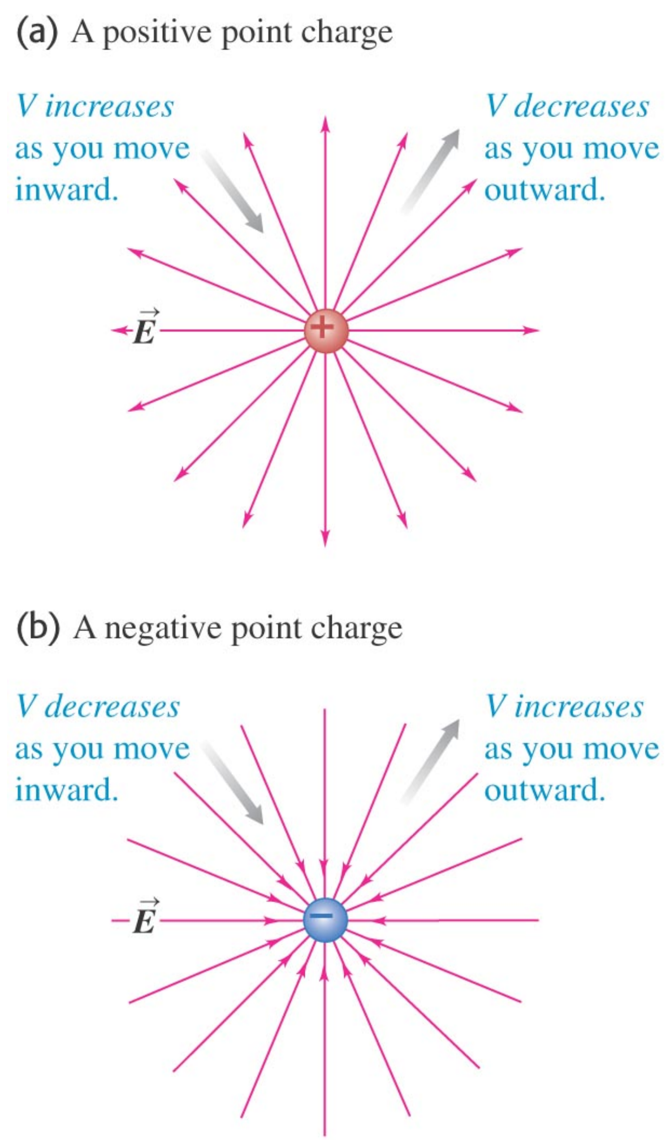
$$\vec{E} = E_x\hat{\imath} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}\hat{\imath} \qquad (21.8)$$

Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*. (See Figure 23.12 at the right.)

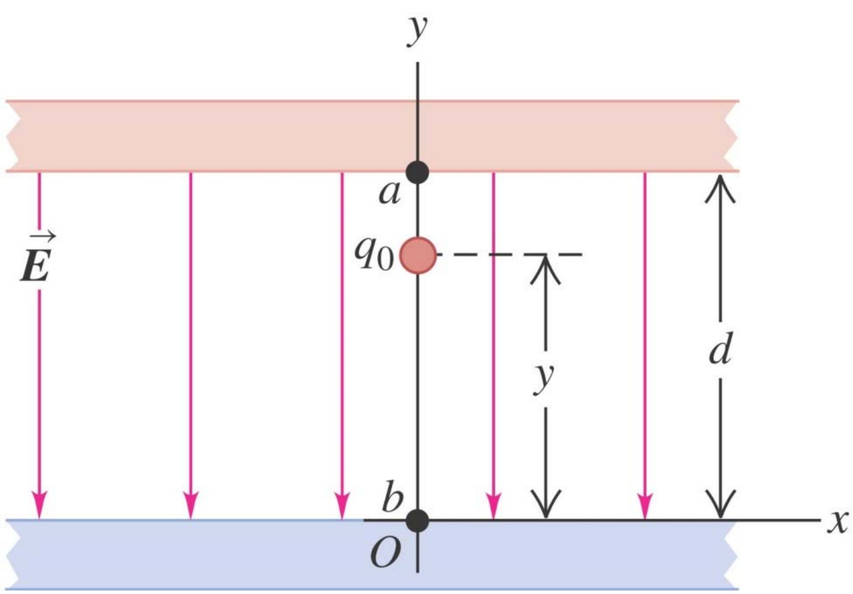
$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos\phi \, dl \quad \text{(potential difference as an integral of } \vec{E}\text{)}$$

$$\cos\phi \, dl = dr.$$



Oppositely charged parallel plates

- Follow Example 23.9 using Figure 23.18 below.



EXECUTE: The potential $V(y)$ at coordinate y is the potential energy per unit charge:

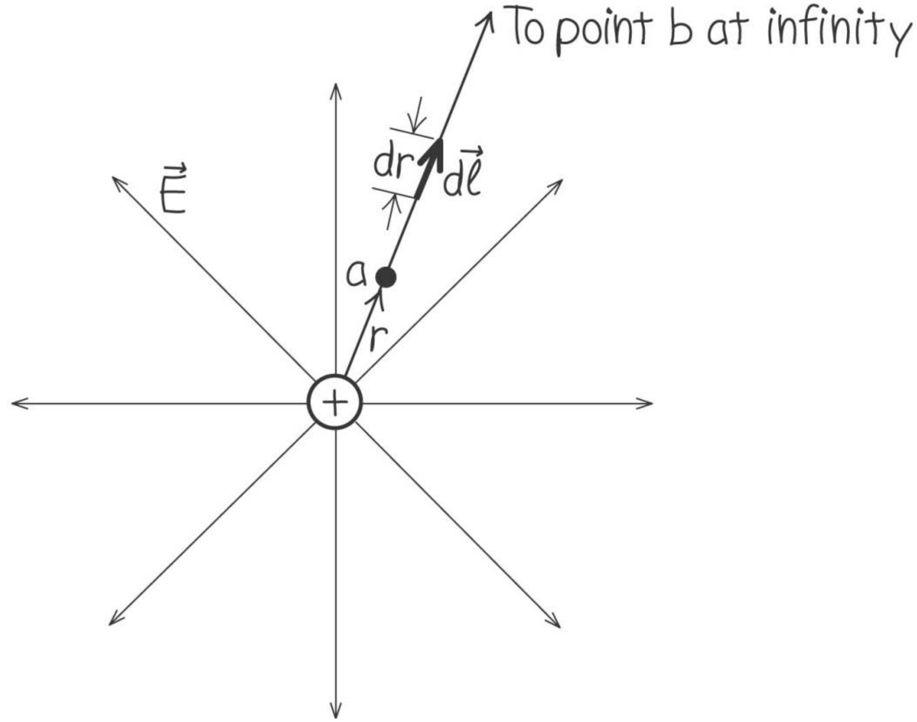
$$V(y) = \frac{U(y)}{q_0} = \frac{q_0 E y}{q_0} = Ey$$

The potential decreases as we move in the direction of $\vec{E}$ from the upper to the lower plate. At point a, where $y = d$ and $V(y) = V_a$,

$$V_a - V_b = Ed \quad \text{and} \quad E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}$$

Point Charge

- Example 23.6 shows how to find the potential by integration. Follow this example using Figure 23.14 at the right.



potential to be zero at an infinite distance from the charge q.

$$V - 0 = V = \int_r^\infty \vec{E} \cdot d\vec{l}$$

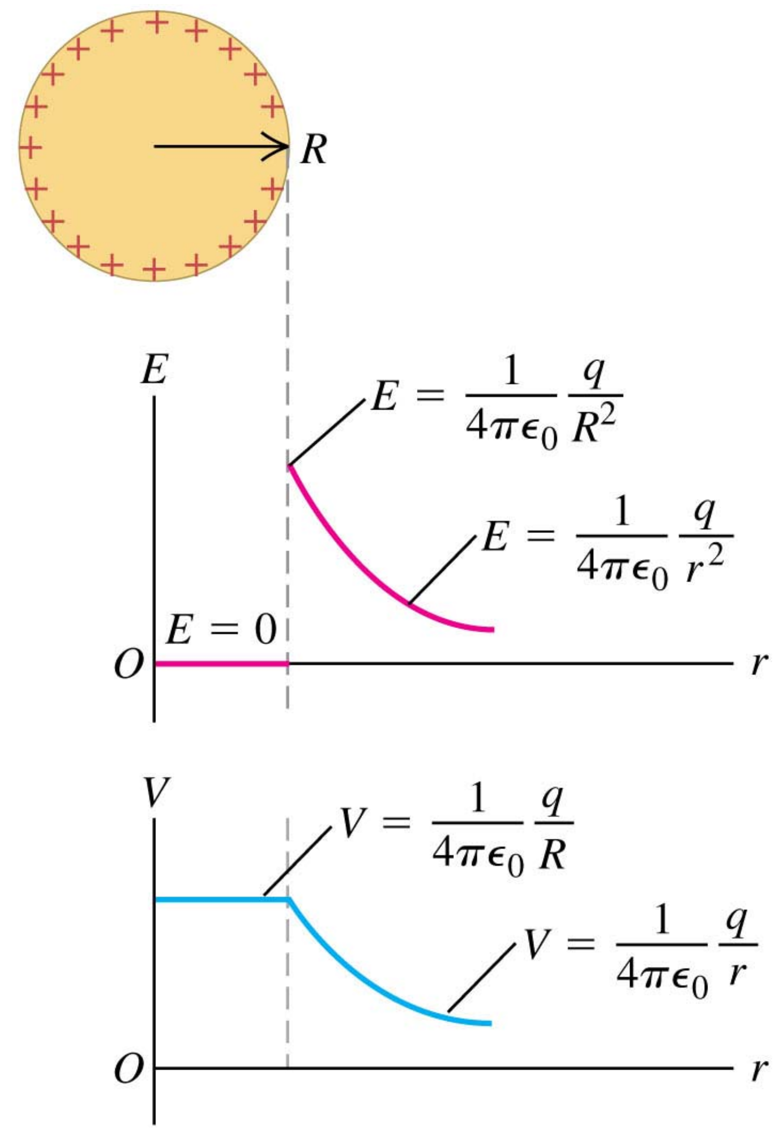
$$= \int_r^\infty \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot \hat{r} dr = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$= -\frac{q}{4\pi\epsilon_0 r}\bigg|_r^\infty = 0 - \left(-\frac{q}{4\pi\epsilon_0 r}\right)$$

$$V = \frac{q}{4\pi\epsilon_0 r}$$

Calculating electric potential

- Read Problem-Solving Strategy 23.1.
- Follow Example 23.8 (a charged conducting sphere) using Figure 23.16 at the right.



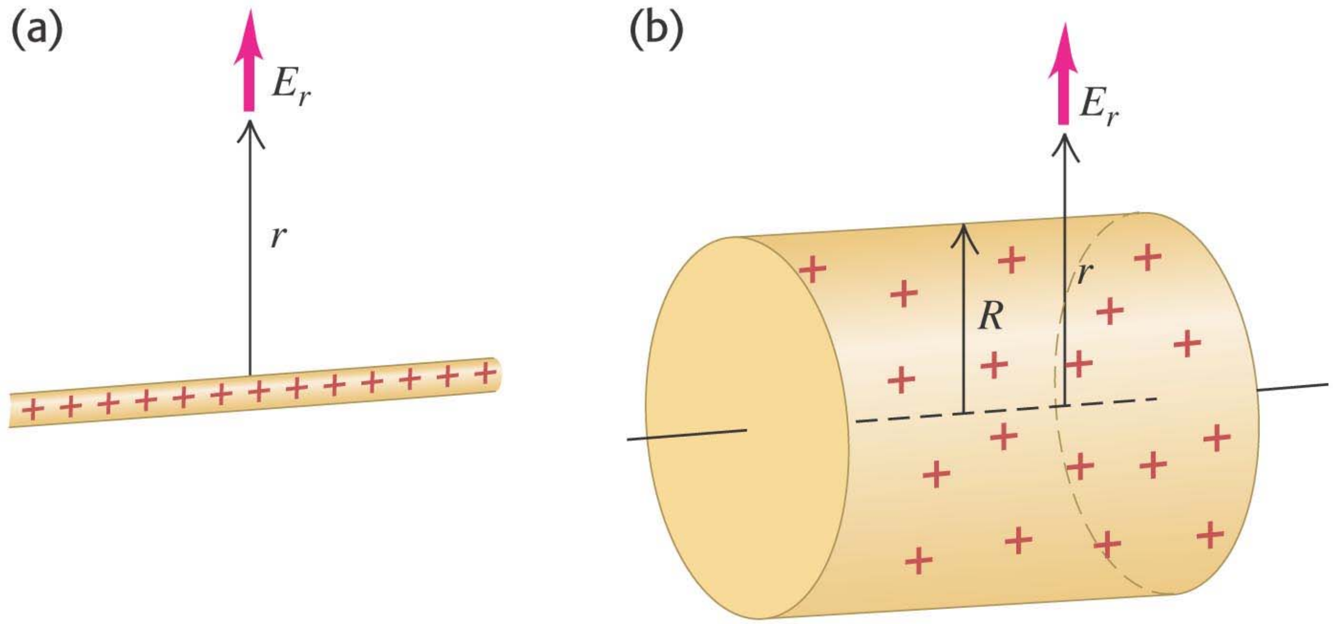
Outside: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

Inside: $\vec{E}$ is zero everywhere. I

"Zero potential" is arbitrary

An infinite line charge or conducting cylinder

- Follow Example 23.10 using Figure 23.19 below.

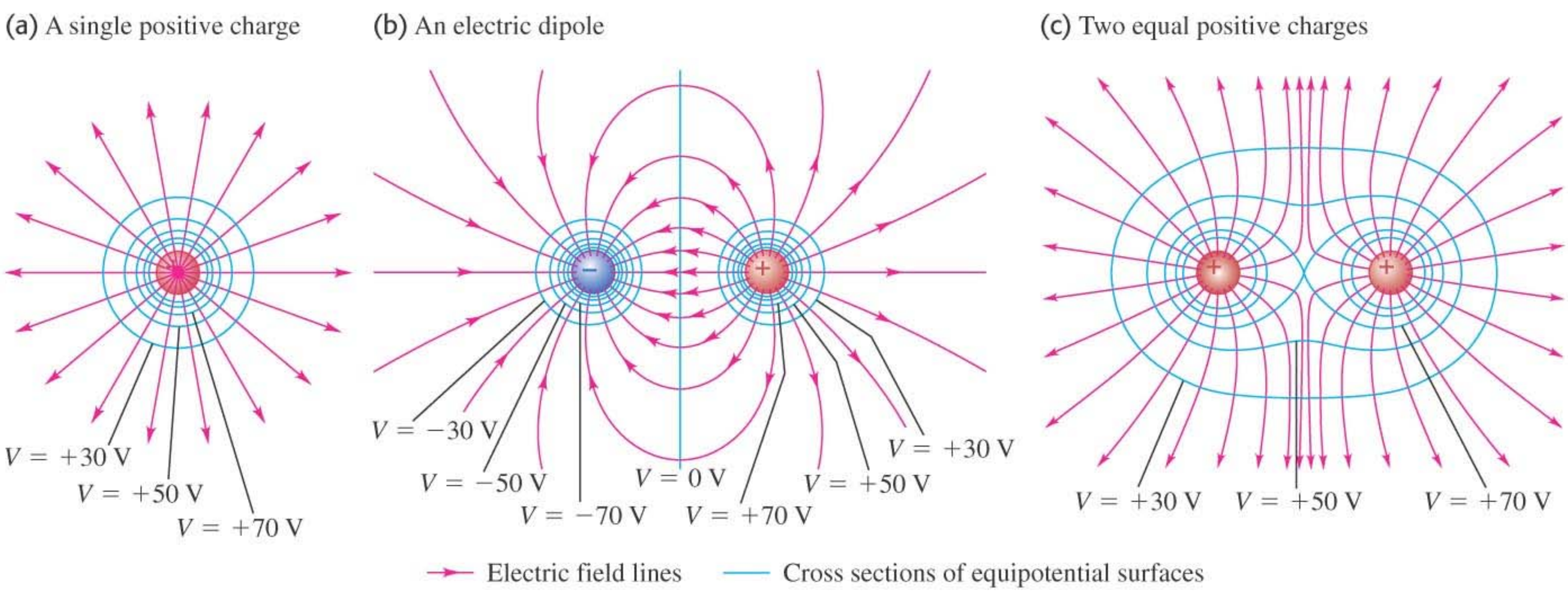


$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r} \quad \text{(field of an infinite line of charge)}$$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E_r\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

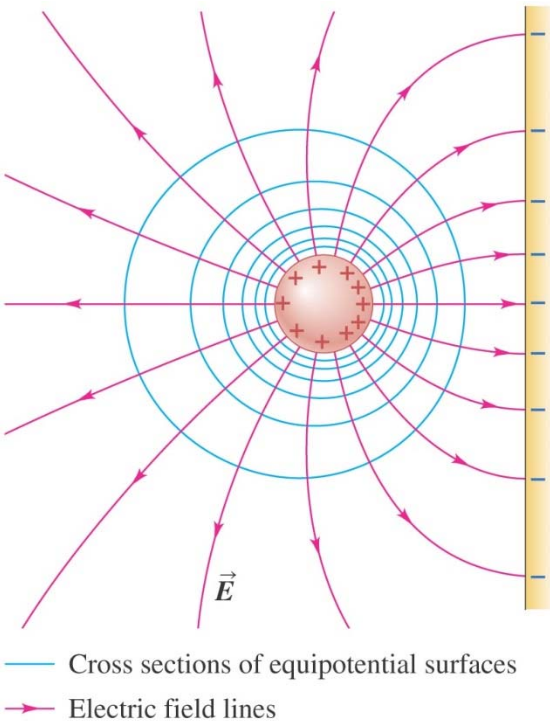
Equipotential surfaces and field lines

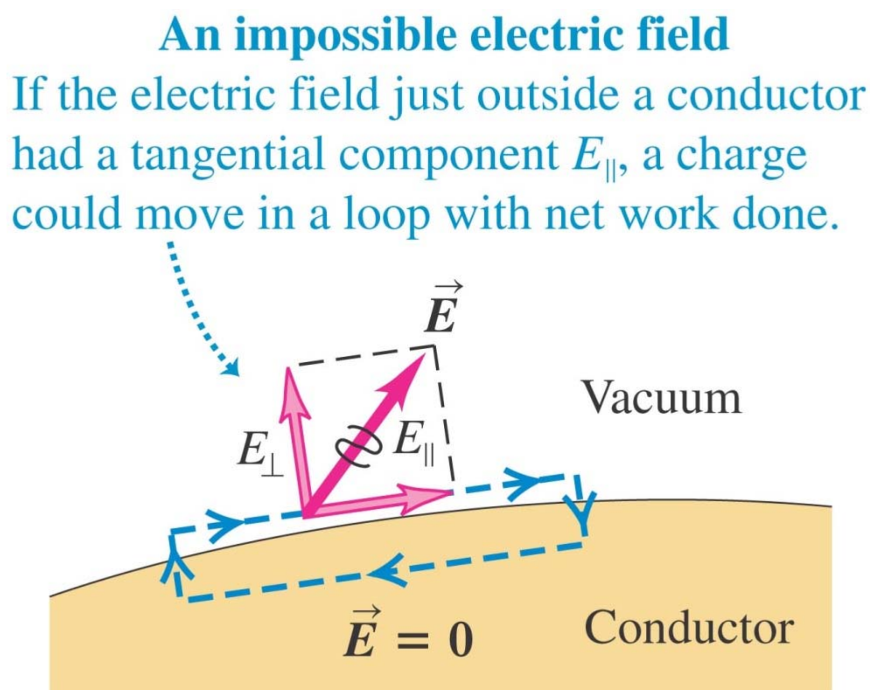
- An *equipotential surface* is a surface on which the electric potential is the same at every point.
- Figure 23.23 below shows the equipotential surfaces and electric field lines for assemblies of point charges.
- Field lines and equipotential surfaces are always mutually perpendicular.



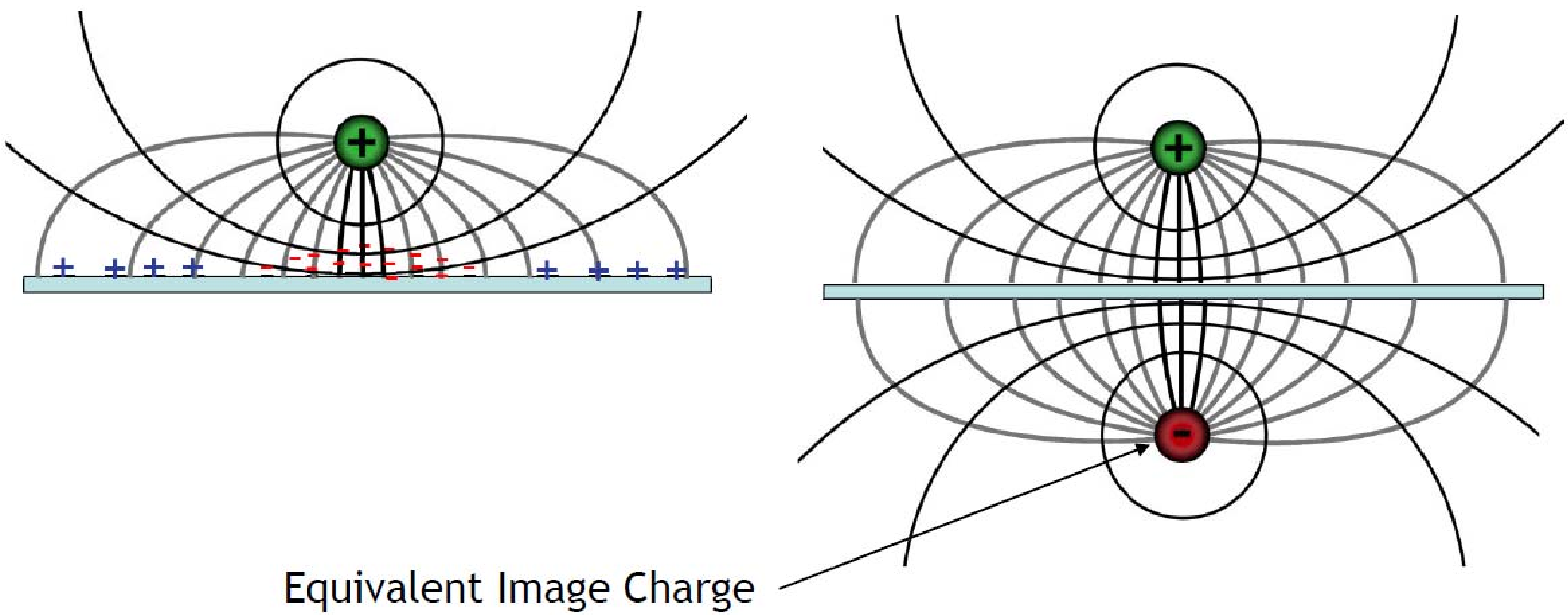
Equipotentials and conductors

- When all charges are at rest:
  - ✓ the surface of a conductor is always an equipotential surface.
  - ✓ the electric field just outside a conductor is always perpendicular to the surface (see figures below).
  - ✓ the entire solid volume of a conductor is at the same potential.

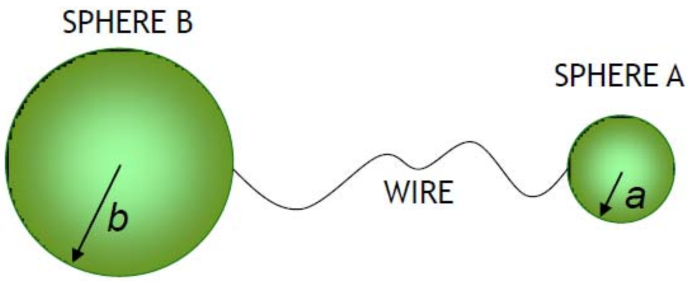




Equipotentials and conductors



Equipotentials and conductors



Because the two spheres are far apart, we can assume that charges are uniformly distributed across the surfaces of the two spheres, with charge q_a *on the surface of sphere A and* q_b *on the surface of sphere B*

$$q_a + q_b = q$$

$$V_b = \frac{q_b}{4\pi\epsilon b} \qquad V_a = \frac{q_a}{4\pi\epsilon a}$$

$$since \quad V_b = V_a \quad then$$

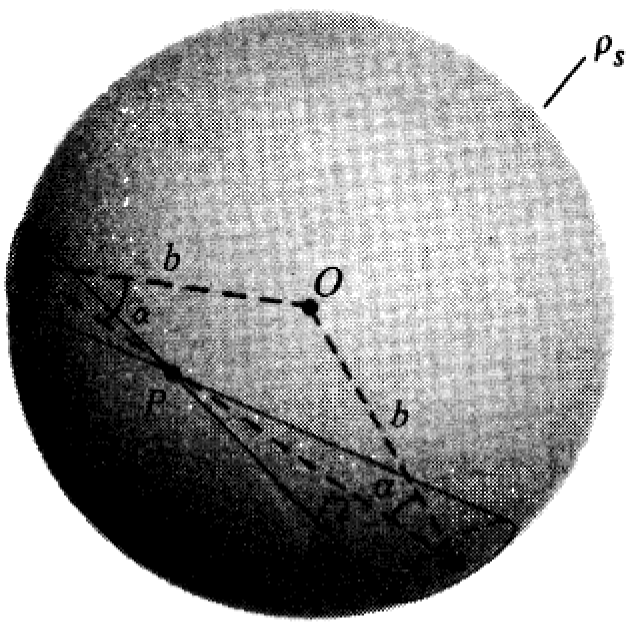
$$q_b = q\frac{b}{a+b} \qquad q_a = q\frac{a}{a+b}$$

… and the E-field on the surface of the spheres is:

$$E_b = \frac{q_b}{4\pi\epsilon b^2} = \frac{q}{4\pi\epsilon(a+b)b} \qquad E_a = \frac{q_a}{4\pi\epsilon a^2} = \frac{q}{4\pi\epsilon(a+b)a}$$

Note that $E_a >> E_b$ if $b >> a$

from Shen and Kong



:es ds_1 and ds_2 is, from Eq. (3–12),

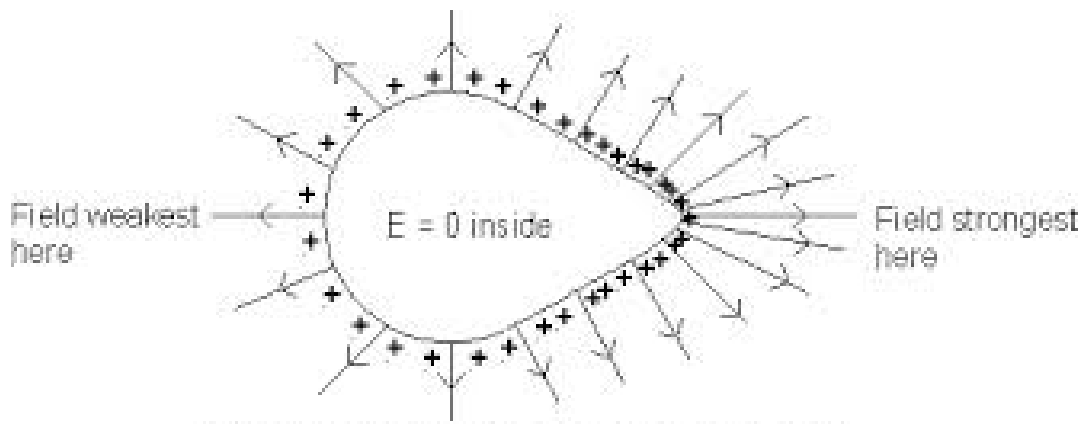
$$dE = \frac{\rho_s}{4\pi\epsilon_0}\left(\frac{ds_1}{r_1^2} - \frac{ds_2}{r_2^2}\right).$$

ıngle $d\Omega$ equals

$$d\Omega = \frac{ds_1}{r_1^2}\cos\alpha = \frac{ds_2}{r_2^2}\cos\alpha.$$

: expressions of dE and $d\Omega$, we find that

$$dE = \frac{\rho_S}{4\pi\epsilon_0}\left(\frac{d\Omega}{\cos\alpha} - \frac{d\Omega}{\cos\alpha}\right) = 0.$$



(d) Electric field & charge distribution around a pear-shaped conductor

Equipotentials and conductors