Summer_2019 VV255_Assignment 2: Lines and planes in 3D. Vector functions.

Deadline: 2019-06-03

Problem 1

Let $\bar{a} = 2\bar{\imath} - 3\bar{\jmath} + \bar{k}, \bar{b} = \bar{\jmath} + 4\bar{k}, \bar{c} = 5\bar{\imath} + 2\bar{\jmath} - 3\bar{k}$. Find

$$\bar{a} \cdot (3\bar{b} \times \bar{c}), \qquad |3\bar{a} \times 2\bar{c}|, \qquad \bar{b} \cdot (-4\bar{c})$$

Check if **a.** the vectors $\bar{a}, \bar{c}$ are perpendicular or parallel, **b.** the vectors $\bar{a}, 2\bar{b}, 3\bar{c}$ are coplanar.

Problem 2

Consider the points $A_1(3,1,4)$, $A_2(-1,6,1)$, $A_3(-1,1,6)$, $A_4(0,4,-1)$.

A. Find the equations of the following objects:

a. the plane $A_1A_2A_3$, **b.** the line A_1A_2, **c.** the line A_4M perpendicular to the plane $A_1A_2A_3$,

d. the line A_3N parallel to the line A_1A_2, **e.** the plane $\wp$: $A_4 \in \wp$, $\wp \perp (line\ A_1A_2)$.

B. Calculate:

a. $\sin\theta$, where θ is the angle between the line A_1A_4 and the plane $A_1A_2A_3$,

b. $\cos\varphi$, where φ is the angle between the coordinate plane $z = 0$ and the plane $A_1A_2A_3$.

Problem 3

A. Find the equation of the plane that passes through the lines

$$\frac{x-3}{2} = \frac{y}{1} = \frac{z-1}{2}, \qquad \frac{x+1}{2} = \frac{y-1}{1} = \frac{z}{2}.$$

B. Find the equation of the plane that passes through the origin and is perpendicular to the planes

$$2x - 3y + z - 1 = 0, \qquad x - y + 5z + 3 = 0.$$

C. Find the symmetric point of $M(4,3,10)$ about the line

$$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{5}$$

Problem 4

Find the vector function $\bar{r}(t)$ if $\bar{r}'(t) = t\bar{\imath} + e^t\bar{\jmath} + te^t\bar{k}$ and $\bar{r}(0) = \bar{\imath} + \bar{\jmath} + \bar{k}$.

Problem 5

The curve C in $\mathbb{R}^2$ is defined by a vector function $\bar{r}(t) = t^2\bar{\imath} + (t^3 - t)\bar{\jmath}$.

a. Find the point P at which the curve C intersects itself.

b. Does the curve C have more than one tangent line at P? If yes, find the angle between tangent lines.

Problem 6

Consider the curve $\bar{r}(t) = \cos t\,\bar{\imath} + \sin t\,\bar{\jmath} + \frac{2}{3}t^{\frac{3}{2}}\bar{k}$ and find

a. the length of the curve, **b.** the equation of the tangent line at the point $t = 0$,

c. the speed of the point moving along the curve at the point $t = 2\pi$.

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