Mid2-Grading Rubric

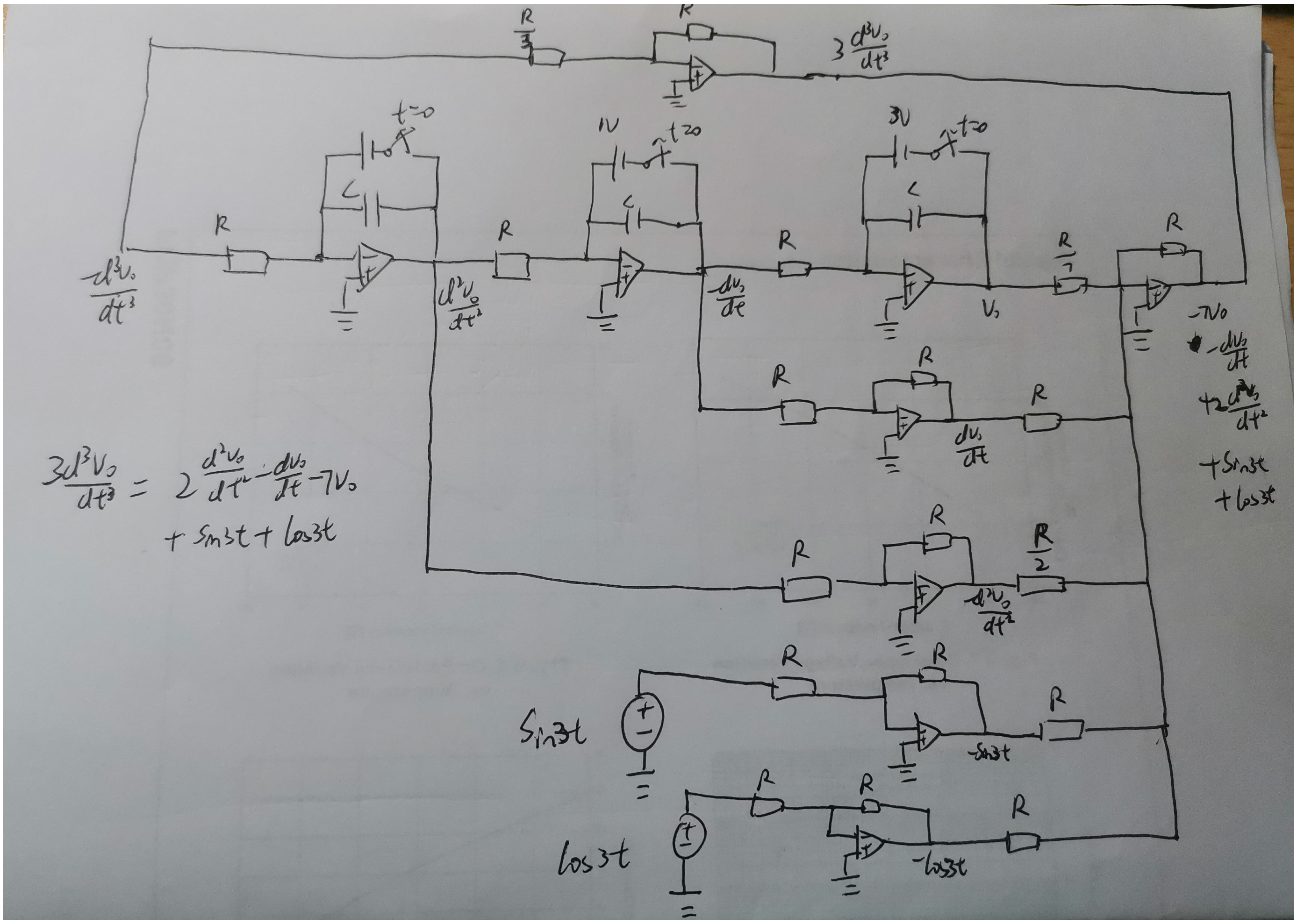
VE215 - Introduction to Circuits, Sung-Liang Chen Autumn 2019

1 Q1 (16pts)

1. Design an analog computer to simulate (16pts)

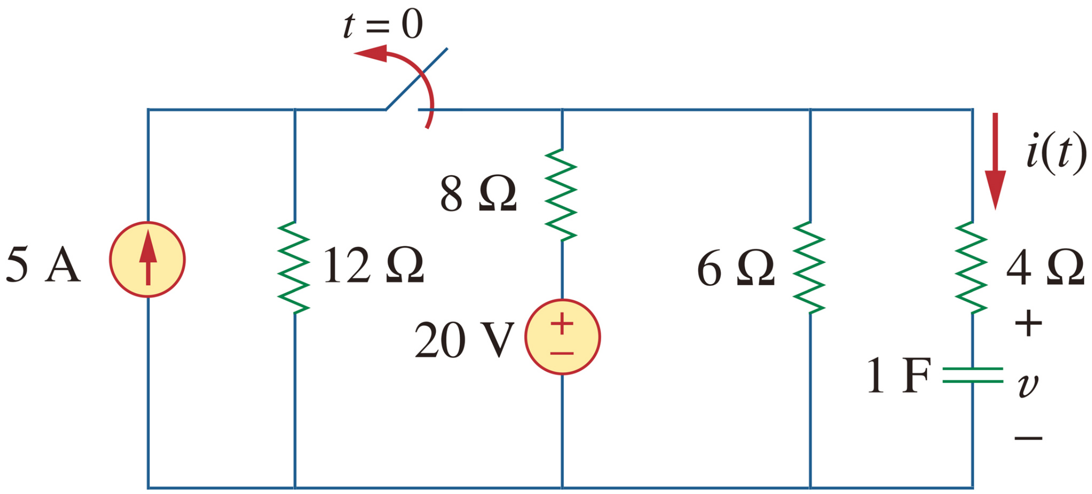
$$3\frac{d^3v_0}{dt^3} - 2\frac{d^2v_0}{dt^2} + \frac{dv_o}{dt} + 7v_0 = sin3t + cos3t$$

where $v_0^{''}(0) = -1$, $v_0^{'}(0) = 1$ and $v_0(0) = -3$



2 Q2 (16pts)

2. For the circuit below, find $i(t)$ for $t > 0$. (16 pts)



$v(0) : \frac{v}{12} + \frac{v-20}{8} + \frac{v}{6} = 5,\ v(0) = 20V$
$v(0^-) = v(0^+) = 20V$ (4pts)

$v(\infty) : \frac{v}{6} + \frac{v-20}{8} = 0,\ v(\infty) = \frac{60}{7}V \approx 8.57V$ (4pts)

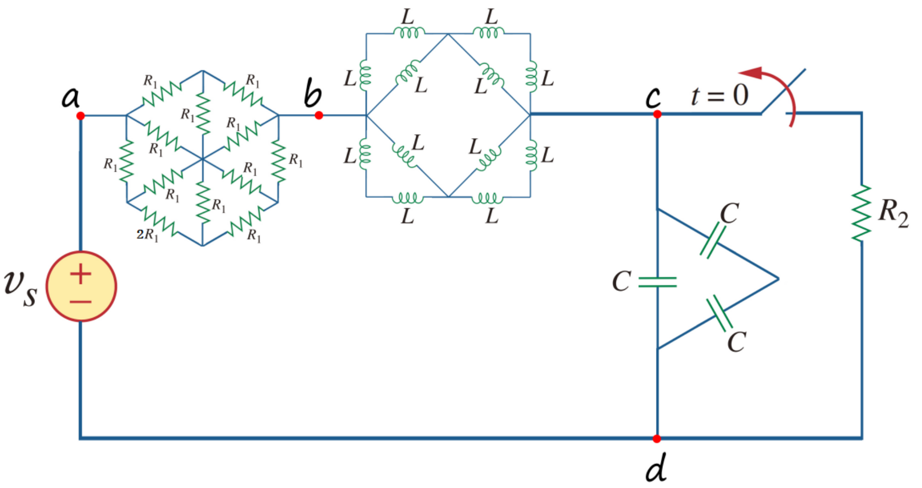
Find τ: (4pts)
I:
$R_{eq} = 8//6 + 4 = \frac{24}{7} + 4 = \frac{52}{7}\Omega$
$\tau = RC = \frac{52}{7} \times 1 = \frac{52}{7}$
II:
or by differential equation
$\frac{v+\frac{dv}{dt}\times 4}{6} + \frac{v+\frac{dv}{dt}\times 4-20}{8} + \frac{dv}{dt} = 0$
$(\frac{1}{6} + \frac{1}{8})v + (\frac{2}{3} + \frac{1}{2} + 1)\frac{dv}{dt} = 2.5$
$\frac{7}{24}v + \frac{13}{6}\frac{dv}{dt} = 2.5$

$v(t) = 8.57 + (20 - 8.57)e^{-\frac{t}{\frac{52}{7}}}$
$= 8.57 + 11.43e^{-\frac{7}{52}t}$ (2pts)

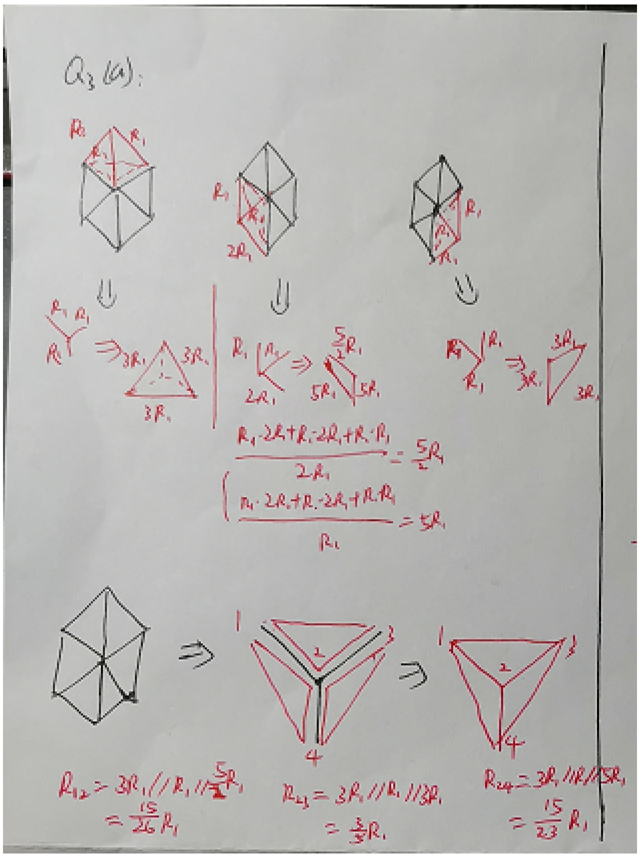
$i(t) = C\frac{dv}{dt} = -\frac{7}{52} \times 11.43 \times e^{\frac{7}{52}t}$
$= -1.54e^{-\frac{7}{52}t}$ (2pts)

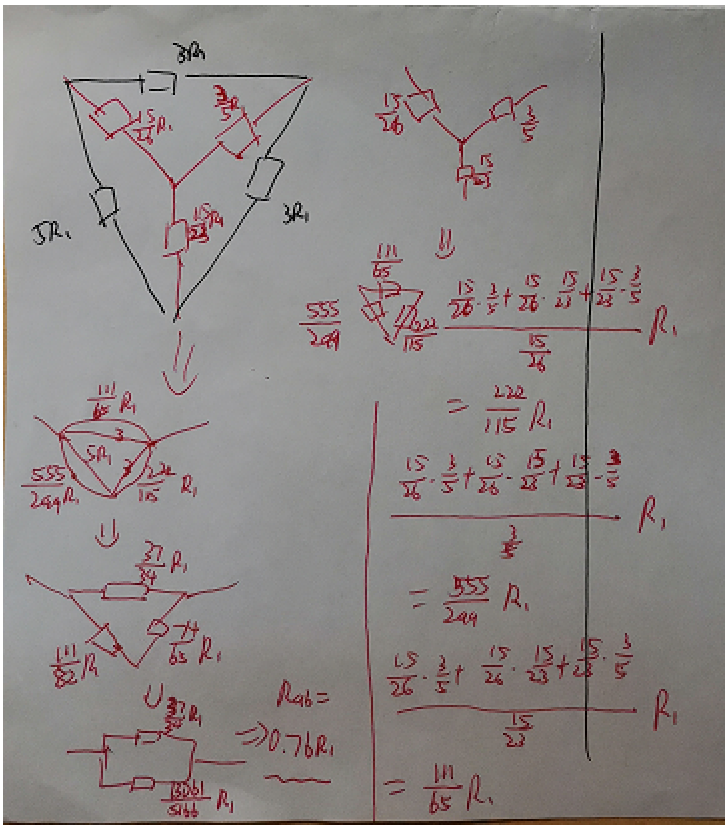
3 Q3 (17pts)

3.(a) For the circuit below, find resistance from a to b, R_{ab} in terms of R_1; (4pts)
(b) Find inductance from b to c L_{bc} in terms of L; (3pts)
(c) Find capacitance from c to d C_{cd} in terms of C; (2pts)
(d) When V_s=11V, R_2=1Ω, L=2H, C=0.5F, for what values(or range) of R_1 will V_{cd} reach steady state most quickly?
Find V_{cd} for $t > 0$ in this condition (select a value for R_1 if its in a range) and sketch V_{cd} for $t > 0$. (8pts)



3.1 a





$R_{ab} \approx 0.76R_1$

3.2 b

$L//2L = \frac{2}{3}L$
$\frac{2}{3}L + \frac{2}{3}L = \frac{4}{3}L$
L_{bc}=$\frac{4}{3}L//\frac{4}{3}L = \frac{2}{3}L$. (3pts)

3.3 c

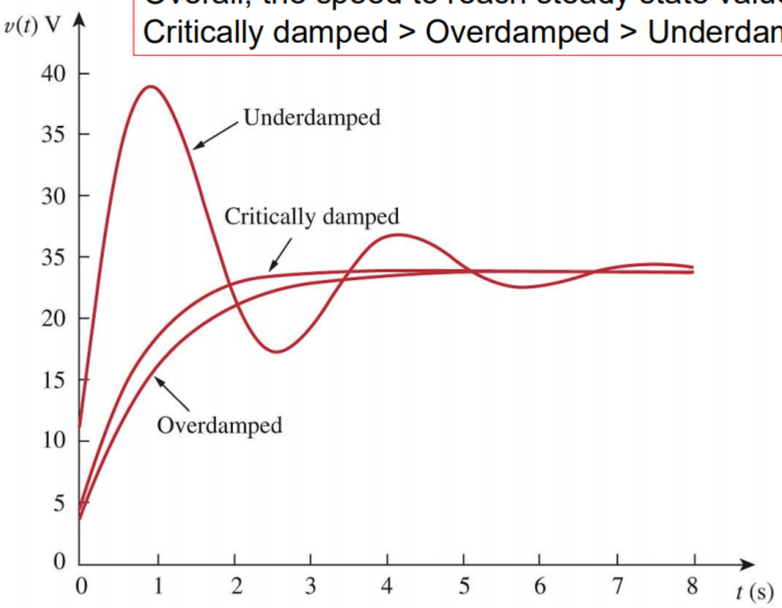
$\frac{C \times C}{C+C} = \frac{C}{2}$
C_{cd}=$\frac{C}{2}//C = \frac{3}{2}C$ (2pts)

3.4 d

This is a series RLC circuit. $L_{bc} = \frac{2}{3}L = \frac{4}{3}H$, $C_{cd} = \frac{3}{2}C = \frac{3}{4}F$
$\alpha = \frac{R}{2L}$, $\omega = \frac{1}{\sqrt{LC}}$
As V_{cd} reaches steady state most quickly, it should be critically damped.
Therefore, $\alpha^2 - \omega^2 = 0$. $R = 2\sqrt{\frac{L_{bc}}{C_{cd}}} = 2\sqrt{\frac{\frac{4}{3}H}{\frac{3}{4}F}} = \frac{8}{3}\Omega$
Since $R = 0.76R_1$, $R_1 = \frac{200}{57}\Omega \approx 3.509\Omega$ (2pts)

When $t < 0$, $V_{cd}(0^-) = \frac{11V}{\frac{8}{3}+1} \times 1 = 3V$
$V_{cd}(0^-)$=$V_{cd}(0^+)$=$3V$
$i_{cd}(0) = \frac{11V}{1\Omega + \frac{8}{3}\Omega} = 3A$
$V_{ss} = V_{cd}(\infty) = 11V$
$\alpha = \frac{\frac{8}{3}\Omega}{2 \times \frac{4}{3}H} = 1$
Let $V_{cd} = V_{ss} + (A_1 + A_2 t)e^{-\alpha t} = V_{ss} + (A_1 + A_2 t)e^{-\alpha t} = 11V + (A_1 + A_2 t)e^{-t}$
$i(0)$: $C\frac{dv}{dt} = i(0)$ so that C($-A_1 e^{-t} + A_2 e^{-t} - A_2 t e^{-t}$)=3 when t=0. Then $-A_1 + A_2$=4.
$v(0)$: 11+$A_1 = 3$. Then $A_1 = -8$
Thus $A_2 = -4$
So $V_{cd} = [11 + (-8 - 4t)e^{-t}]$V(3pts)

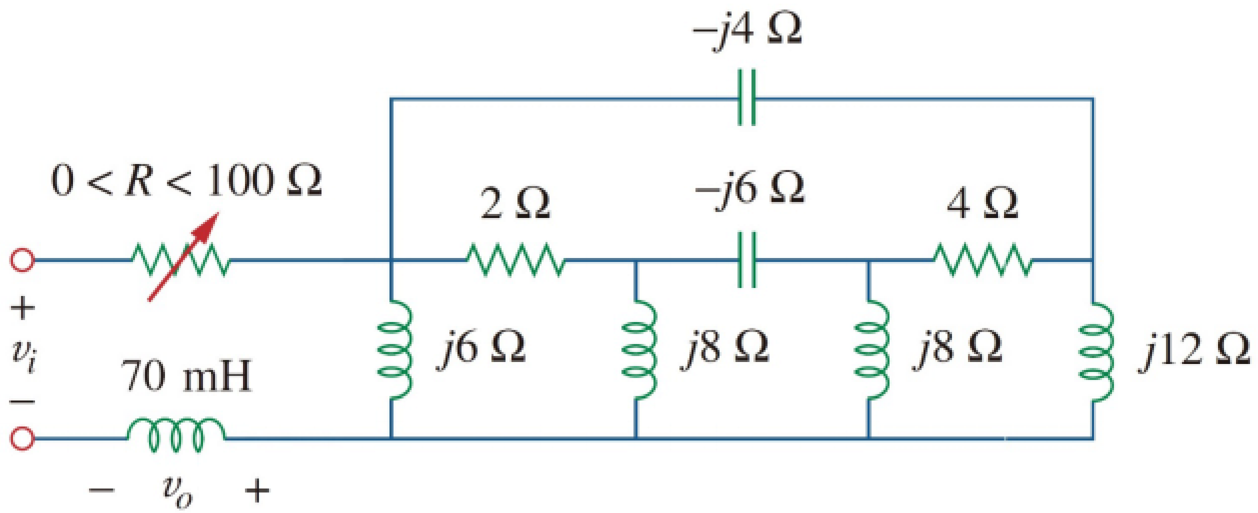
Overall, the speed to reach steady state value:
Critically damped > Overdamped > Underdamped



(3pts)

4 Q4 (17pts)

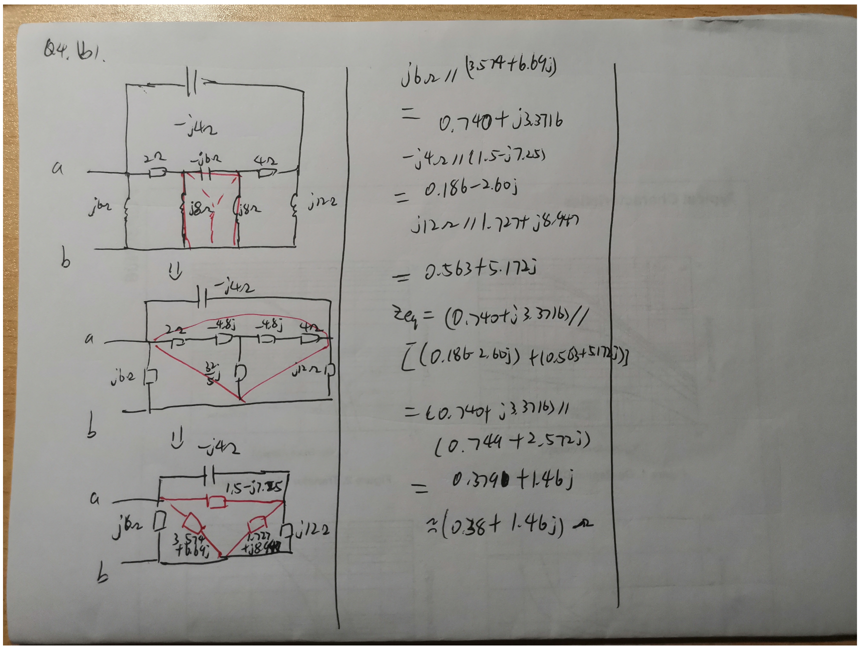
(a) Write down three forms of Z, mark each form by its name. (3 pts)
For the circuit below,
(b) Determine the equivalent impedance of the circuit between a and b (6pts)
(c) The frequency of V_i is 50Hz. Determine V_0 when R is maximum, state whether the phase side is leading or lagging (4pts)
(d) Find the value of R that will produce a phase shift of 75 of V_0 (4pts)



4.1 a

$Z = x + jy$ rectangular form (1')
$Z = r\angle\phi$ polar form (1')
$Z = re^{j\phi}$ exponential form (1')

4.2 b



$Z_{eq} = 0.38 + 1.46j\Omega$

4.3 c

For 70mH, $j\omega L = j \times 2\pi(50Hz) \times 0.07H \approx j22\Omega$ (2')
When $R = 100\Omega$, $V_0 = V_i\angle 0^\circ \times \frac{j22}{100+0.38+j1.46+j22} = \frac{V_i\angle 0^\circ \times j22}{100.38+j23.46} = \frac{22V_i\angle 90^\circ}{103.08\angle 13.14^\circ} = 0.213V_i\angle 76.86^\circ$ so that V_o leads V_i by 76.86° (2')

4.4 d

To produce a phase shift of 75°, the phase of $V_0 = 90^\circ + 0^\circ - \alpha = 75^\circ$
Hence $\alpha =$ phase of $(R + 0.38 + j23.46) = 15^\circ$ (2')
For α to be 45°, $\frac{R+0.38}{23.46} = tan75^\circ$
Therefore, $R + 0.38\Omega = 87.5539\Omega$, $R = 87.1743\Omega \approx 87.17\Omega$ (2')

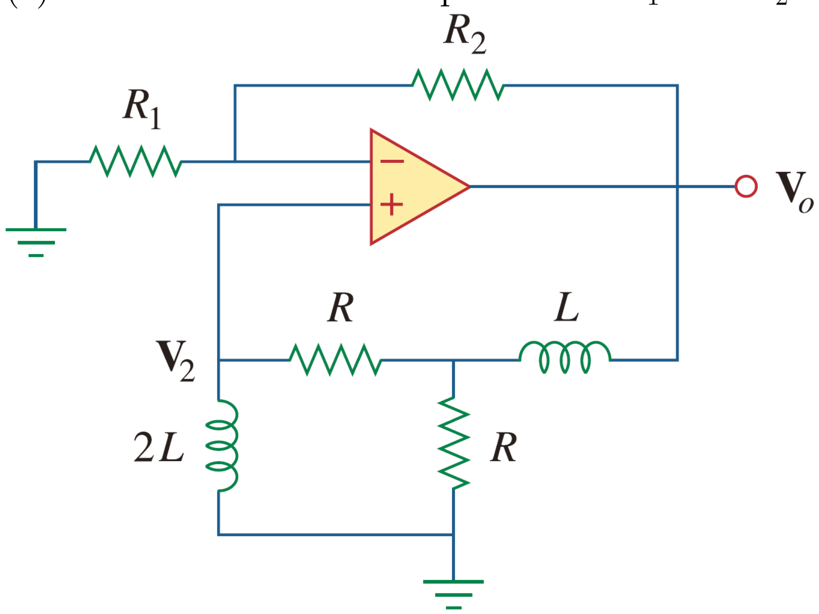
5 Q5 (17pts)

Refer to the oscillator below

(a). Determine $\frac{V_2}{V_0}$

(b). Determine the oscillation frequency f_0

(c). Obtain the relationship between R_1 and R_2 in order for oscillation to occur.



5.1 a

$\frac{V_2}{2j\omega L} = \frac{V_1}{2j\omega L+R}$ (1')

$V_2 = \frac{2j\omega L}{2j\omega L+R}V_1$ (2')

By applying KCL, $\frac{V_0-V_1}{j\omega L} = \frac{V_1}{R+2j\omega L} + \frac{V_1}{R}$ (2')

$\frac{V_2}{V_0} = \frac{2}{4+2j\omega L/R+R/j\omega L}$

$\frac{V_2}{V_0} = \frac{2}{4+j(2\omega L/R-R/\omega L)}$ (2')

5.2 b

$\frac{2\omega_0 L}{R} - \frac{R}{\omega_0 L} = 0$ (2')

$\omega_0 = 2\pi f_0 = \frac{\sqrt{2}R}{2L}$ (1')

$f_0 = \frac{\sqrt{2}R}{4\pi L}$ (2')

5.3 c

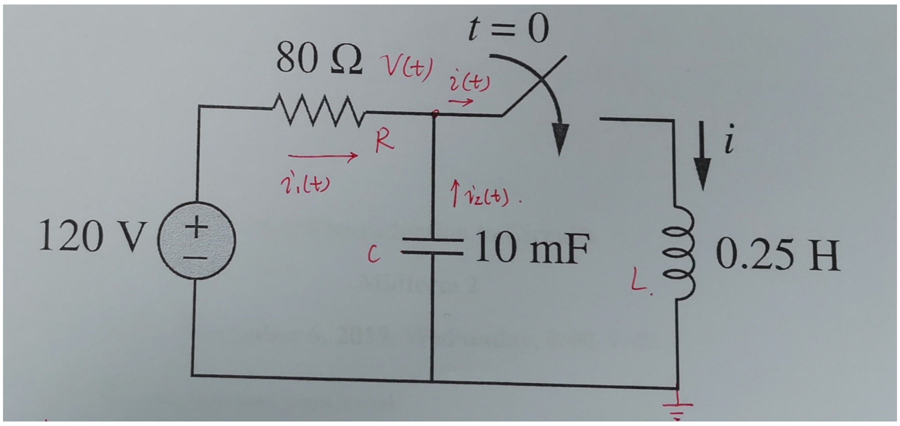
When $\omega = \omega_0$, $\frac{V_2}{V_0} = \frac{1}{2}$ (2')

Thus, $A_v = 1 + \frac{R_2}{R_1} = 2$ (1')

Therefore, $R_2 = R_1$ (2')

6 Q6 (17pts)

6.1 Using Differential Equations



$L\frac{di(t)}{dt} = v(t)$ (3')

KVL: $i_1(t) = [120 - v(t)] \cdot \frac{1}{R}$ (3')
-$C\frac{dv(t)}{dt} = i_2(t)$ (3')

KCL: $i(t) = i_1(t) + i_2(t) = [120 - L\frac{di(t)}{dt}] \cdot \frac{1}{R} - C \cdot L \cdot \frac{d^2i(t)}{dt^2}$ (3')

$i(t) = 1.5 - 3.125 \times 10^{-3} \cdot \frac{di(t)}{dt} - 2.5 \times 10^{-3} \cdot \frac{d^2i(t)}{dt^2}$ (3')

$i(t)$ (1') $= 1.5$ (1') $-3.125 \times 10^{-3}\frac{di(t)}{dt}$ (1') $-2.5 \times 10^{-3} \cdot \frac{d^2i(t)}{dt^2}$ (2')

or $i(t) = \frac{3}{2} - \frac{1}{320}\frac{di(t)}{dt} - \frac{1}{400} \cdot \frac{d^2i(t)}{dt^2}$

where $i_0(t) = 0$, $\frac{di_0(t)}{dt} = 480A/s$

6.2 Source Transformation + Formula for Parallel RLC Circuit

$1.5A$ current source calculated (5')
Figure for source transformation (5')
$\frac{d^2i}{dt^2} + \frac{1}{RC} \cdot \frac{di}{dt} + \frac{i}{LC} = \frac{1.5A}{LC}$
$\frac{d^2i}{dt^2} + \frac{4}{5}\frac{di}{dt} + 400i = 600$ (7')
$i(t) = \frac{3}{2} - \frac{1}{320} \cdot \frac{di(t)}{dt} - \frac{1}{400} \cdot \frac{d^2i(t)}{dt^2}$