vv255_Assignment 7: Surface Integrals. Stoke's Theorem
Due to: 2019-07-31 16:30 (use the mailbox to submit this assignment)

Problem 1: Find an equation for the plane tangent to the given surface at the specified point:

1. $x = 2u, \quad y = u^2 + v, \quad z = v^2, \ (0,1,1)$
2. $x = u^2, \ y = u \sin e^v, \ z = \frac{1}{3} u \cos e^v, \ (13, -2, 1)$

Problem 2

1. Find the area of the portion of the unit sphere that is cut out by the cone $z \geq \sqrt{x^2 + y^2}$.
2. Find the area of the helicoid with vector equation
$$r(u,v) = (u \cos v, u \sin v, v), \qquad 0 \leq u \leq 1, \qquad 0 \leq v \leq \pi$$

Problem 3: Express the surface area of the following graphs over the indicated region D as a double integral. Do not evaluate.

1. $xy^3 e^{x^2 y^2}$, $D =$ a unit circle centered at the origin,
2. $y^3 \cos^2 x$, $D =$ a triangle with vertices $(-1,1), (0,2)$, and $(1,1)$.

Problem 4: Evaluate the surface integrals:

1. $\iint_S (x^2 + y^2) dS$, S is the surface with vector equation
$$r(u,v) = (2uv, u^2 - v^2, u^2 + v^2), \qquad u^2 + v^2 \leq 1$$
2. $\iint_S xz\, dS$, S is the part of the plane $2x + 2y + z = 4$ that lies in the first octant.
3. $\iint_S xz\, dS$, S is the boundary of the region enclosed by $y^2 + z^2 = 9, x = 0, x + y = 5$

Problem 5

1. Let the temperature of a point in $\mathbb{R}^3$ be given by $u(x,y,z) = 3x^2 + 3z^2$.
Compute the heat flux across the surface $x^2 + z^2 = 2$, $0 \leq y \leq 2$, if $K = 1$.
2. Find the flux of $F(x,y,z) = (yz, xz, xy)$ across $S: z = x \sin y, 0 \leq x \leq 2, 0 \leq y \leq \pi$ with upward orientation.
3. Find the flux of $F(x,y,z) = (x, 2y, 3z)$ across the cube S with the vertices $(\pm 1, \pm 1, \pm 1)$ oriented outward.
4. Use the Divergence Theorem to calculate the surface integral to calculate the flux of F across S.
$$F(x,y,z) = (x^3 + y^3, y^3 + z^3, z^3 + x^3), \qquad S: x^2 + y^2 + z^2 = 4$$
5. Use the Divergence Theorem to calculate the surface integral to calculate the flux of F across S.
$$F(x,y,z) = (xy + 2xz, x^2 + y^2, xy - z^2), \qquad S:\ x^2 + y^2 = 4, \qquad z = y - 2, \qquad z = 0$$

Problem 6

1. Verify Stokes' theorem for the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$, $z > 0$, and the radial vector field $F(x,y,z) = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$.
2. Use Stokes' Theorem to evaluate $\iint_S F\, d\bar{S}, F = (xyz, xy, x^2yz)$, S consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$ oriented outward.