Ve 280

Programming and Introductory Data Structures

Abstract Data Types

Learning Objectives:

Understand what is an abstract data type (ADT)

Understand the usefulness of an ADT

Know how to define an ADT in C++

Outline

- Introduction to Abstract Data Types
- Class in C++: A Trivial Example
- More Details on Class
- Another Class Example: a Mutable Set of Integers (IntSet)
- Improve the Efficiency of IntSet

Types

- The role of a type:
 - The set of values that can be represented by items of the type
 - The set of operations that can be performed on items of the type.
- Example
 - C++ string values:

operations:

Struct Types

- Struct types have the following feature:
 - Every detail of the type is known to all users of that type.
 - This is sometimes called the **concrete implementation**.
- Example: the struct Grades talked before.

```
struct Grades {
  char name[9];
  int midterm;
  int final;
};
```

Struct Types

```
struct Grades {
  char name[9];
  int midterm;
  int final;
};
```

- Every function knows the details of exactly how Grades are represented.
- A change to the Grades definition (for example, change C-string for name to a C++-String) requires that we **make** changes throughout the program and recompile everything using this struct.

Introduction

- Contrast the property of struct types with that of the functions
 - A function written by others shows **what** the function does, but not **how** it does it
- For function, if we find a faster way to implement, we can just replace the old implementation with the new one
 - No other components of the program calling the function need to change

Introduction

- To solve the problem for struct type, we'll define **abstract** data types, or ADTs.
- An ADT provides an abstract description of values and operations.
- The definition of an ADT must combine **both** some notion of **what** values that type represents, and **what** operations on values it supports.
 - However, we can leave off the details of **how**.
- Example: mobile phone
 - Type: a portable telephone that can make and receive calls
 - Operations: turn on/off, make/receive call, text message

We don't know details!

Introduction

- Abstract data types provide the following two advantages:
- 1. <u>Information hiding</u>: we don't need to know the details of how the **object** is **represented**, nor do we need to know how the **operations on those objects** are **implemented**.
- 2. <u>Encapsulation</u>: the objects and their operations are defined in the same place; the ADT combines both data and operation in one entity.

Example

- list t:
 - <u>Information Hiding</u>: In the <code>list_t</code> data type, you never knew the precise implementation of the <code>list_t</code> structure (except by looking in <code>recursive.cpp</code>).
 - <u>Encapsulation</u>: The definitions of the operations on lists (list_print, list_make, etc.) were found in the same header file as the type definition of list t.

Benefits

- Abstract data types have several benefits like we had with functional abstraction:
 - ADTs are **local**: the implementation of other components of the program does not depend on the **implementation** of ADT.
 - To realize other components, you only need to focus <u>locally</u>.
 - ADTs are **substitutable**: you can change the implementation and no users of that type can tell.

Introduction

- Someone still needs to know/access the details of how the type is implemented.
 - I.e., how the values are represented and how the operations are implemented
 - This is referred to as the "concrete representation" or just the "representation"
- Question: Who can access the representation?
- <u>Answer</u>: **only** the <u>operations defined for that type</u> should have access to the representation.
 - Everyone else may access/modify this state only **through** operations.

On to Classes

- C++ "class" provides a mechanism to give **true** encapsulation.
- The basic idea behind a class is to provide a single entity that both defines:
 - The **value** of an object.
 - The **operations** available on that object. These operations are sometimes also called **member functions** or **methods**.

Outline

- Introduction to Abstract Data Types
- Class in C++: A Trivial Example
- More Details on Class
- Another Class Example: a Mutable Set of Integers (IntSet)
- Improve the Efficiency of IntSet

```
class anInt {
    // OVERVIEW: a trivial class to get/set a
    //
                 single integer value
    int v;
public:
    int get value();
          // EFFECTS: returns the current
                   value
    void set value(int newValue);
          // MODIFIES: this
          // EFFECTS: sets the current value
          // equal to newValue
```

```
class anInt {
       OVERVIEW: a trivial class to get/set a
                  single integer value
   int
          V;
 public:
   int get value();
         // EFFECTS: returns the current
                     value
   void set value(int newValue);
         // RME: Omitted for space
};
```

- There are a few things to notice about this definition:
 - There is a single OVERVIEW specification that describes the class as a whole.

```
class anInt {
   // OVERVIEW: Omitted for space
    int
           V;
  public:
    int get value();
       // EFFECTS: returns the current value
    void set value(int newValue);
      // RME: Omitted for space
};
```

- There are a few things to notice about this definition:
 - The declaration includes both data elements (int v) and member functions/methods (get_value and set value).

```
class anInt {
   // OVERVIEW: Omitted for space
   int.
          V;
 public:
   int
         get value();
              EFFECTS: returns the current
                       value
           set value(int newValue);
    void
             MODIFIES: this
              EFFECTS: sets the current value
              equal to arg
};
```

- There are a few things to notice about this definition:
 - Each function that is declared must have a corresponding specification.

```
class anInt {
   // OVERVIEW: Omitted for space
    int
          V;
 public:
   int get value();
          // EFFECTS: returns the current value
   void
          set value(int newValue);
           // MODIFIES: this
           // EFFECTS: sets the current value
           // equal to arg
};
```

- There are a few things to notice about this definition:
 - set_value says it MODIFIES this. This is the generic name for "this object".

Outline

- Introduction to Abstract Data Types
- Class in C++: A Trivial Example
- More Details on Class
- Another Class Example: a Mutable Set of Integers (IntSet)
- Improve the Efficiency of IntSet

Classes - More Details

- By default, every member of a class is **private**.
 - Members = data members + function members
- A private member is visible **only** to **other members** of this class.
 - int v was a private member in the class an Int.
 - "Private" hides the implementation of the type from the user.

Classes - More Details

- However, if everything were private, the class wouldn't be particularly useful!
- So, the **public** keyword is used to signify that some members are **visible** to anyone who sees the class declaration, not just visible to other members of this class.
 - Everything after the **public** keyword is **visible** to others.

```
class anInt {
    // OVERVIEW: a trivial class to get/set a
                 single integer value
    int v;
 public:
       get value();
    int
          // EFFECTS: returns the current
                     value
    void set value(int newValue);
          // MODIFIES: this
          // EFFECTS: sets the current value
          // equal to arg
```

Classes – A trivial example

This declaration, as it is, is incomplete. We have not yet defined the bodies of the member functions.

```
class anInt {
    // OVERVIEW: a trivial class to get/set a
    //
                 single integer value
    int v;
 public:
    int get value();
          // EFFECTS: returns the current
                     value
   void set value(int newValue);
          // MODIFIES: this
          // EFFECTS: sets the current value
          // equal to arg
```

Classes – Defining member functions

```
class anInt {
    // OVERVIEW: a trivial class to get/set a
    // single integer value
```

Note: You can actually define the functions within the class definition, but this "exposes" information, which is best left hidden!

```
int anInt::get_value() {
  return v;
}

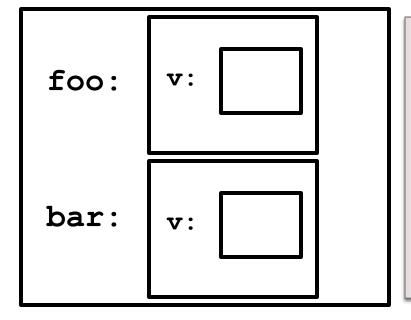
void anInt::set_value(int newValue) {
  v = newValue; The definitions of member functions are
  usually put in the .cpp file;
  You should include .h in the .cpp!
```

Classes - Declaring class objects

We can declare objects of type anInt as you would expect:

```
anInt foo;
anInt bar;
```

• This produces an environment with two objects:



These values are still undefined (i.e. there is no initial value). We'll see several ways to set an <u>initial</u> value for data members later.

Classes – Establishing data member values

• We can call the set_value member function to establish a value:

```
foo.set_value(1);
```

This calls foo's set_value() method.

foo:	v:	
bar:	v:	

Classes – Establishing data member values

- There is one very important difference between <u>normal</u> function calls and <u>member</u> function calls:
 - The **other** members of the object are **also visible** to the function members!
 - For example, v is visible to the function set_value()
 void anInt::set_value(int newValue) {
 v = newValue;
 }

Classes – Establishing data member values

• So, set value changes **foo**'s V:

```
foo.set_value(1);
```

foo:	v: 1
bar:	v:

Classes – Accessing data member values

- We can't access v directly:
 cout << foo.v; // Compile-time error
 because v is private!
- However, we can use the get_value() method to do so for us:
 cout << foo.get_value(); // OK.
 because get_value() is public!
- Finally, class objects can be passed just like anything else.
- Like everything else (except arrays), they are passed by value.

Class Example: Classes

• What is the result of the following?

```
void add one(anInt i) {
  i.set value(i.get value()+1);
int main() {
  anInt foo;
  foo.set value(0);
  add one (foo);
  cout << foo.get value() << endl;</pre>
  return 0;
```

Classes - Passing by reference

• To pass a class object by reference, you use either a pointer argument or a reference argument, i.e.:

```
void add_one(anInt *ip) {
   ip->set_value(ip->get_value() + 1);
}
```

• This version would change the class object passed to it!



Which Statements Are Correct?

- A. A C++ class can define a type.
- **B.** The information stored in an object of a class is accessible to any one.
- C. A class defines all the basic operations that are possible on objects of that class.
- **D.** All member functions of a class are accessible to any one.

Outline

- Introduction to Abstract Data Types
- Class in C++: A Trivial Example
- More Details on Class
- Another Class Example: a Mutable Set of Integers (IntSet)
- Improve the Efficiency of IntSet

Using Classes

- Suppose we wanted to build an abstraction that held a **mutable** set of integers.
- This is a **<u>set</u>** in the mathematical sense:
 - A collection of zero or more integers, with **no duplicates**.
- The set is "mutable" because we can insert values into and remove objects from the set.

Using Classes

- Suppose we wanted to build an abstraction that held a mutable set of integers.
- There are four **operations** on this set that we will define:
 - 1. Insert a value into the set.
 - 2. Remove a value from the set.
 - 3. Query to see if a value is in the set.
 - 4. Count the number of elements in the set.

Using Classes

 Here is an **incomplete** definition of a class implementing such an ADT: class IntSet { // OVERVIEW: a mutable set of integers public: void insert(int v); // MODIFIES: this // EFFECTS: this = this + {v} void remove(int v); // MODIFIES: this // EFFECTS: this = this - {v} bool query(int v); // EFFECTS: returns true if v is in this, false otherwise int size(); // EFFECTS: returns |this|.

```
class IntSet { // omitted OVERVIEW for space
  public:
    void insert(int v); // omitted RME for space
    void remove(int v); // omitted RME for space
    bool query(int v); // omitted RME for space
    int size(); // omitted RME for space
};
```

- The class is incomplete because we haven't chosen a representation for sets.
- Choosing a representation involves two things:
 - Deciding what **concrete** data elements will be used to **represent the values** of the set.
 - Providing an **implementation** for each **method**.

```
class IntSet { // omitted OVERVIEW for space
  public:
    void insert(int v); // omitted RME for space
    void remove(int v); // omitted RME for space
    bool query(int v); // omitted RME for space
    int size(); // omitted RME for space
};
```

- Despite not having a representation for a set, the (incomplete) definition above is all that a **customer** of the IntSet abstraction needs to know since it has:
 - The general overview of the ADT.
 - The specification of each method.

Using Classes

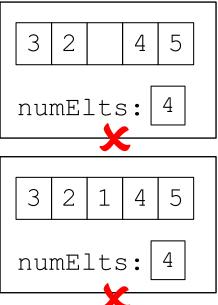
- Start with a representation for the set itself:
 - Use an array.
 - Represent a set of size N as an **unordered** array of integers with no duplicates, stored in the first N slots of the array.
 - int numElts: maintains the number of elements currently in the array.
- These last two statements are called **representation invariants** or **rep invariants** (more on this later).
- This invariant is a rule that the representation must obey both **immediately before** and **immediately after** any method's execution.

rep

invariant

Using Classes

- Start with a representation for the set itself:
 - Use an array.
 - Represent a set of size N as an **unordered** array of integers with no duplicates, stored in the first N slots of the array.
 - int numElts: maintains the number of elements currently in the array.



rep

invariant

Using Classes

• Since this is an array, and arrays have maximum sizes, we have to choose a maximum size and modify the OVERVIEW:

```
// OVERVIEW: a mutable set of
// integers, |set| <= 100</pre>
```

• We also have to change the EFFECTS clause of insert:

```
// EFFECTS: this = this + {v} if
// room available, throws int
// 100 otherwise
```

100 is arbitrary. What is a better way to specify such size?

```
const int MAXELTS = 100;
class IntSet {
    // OVERVIEW: a mutable set of integers ( |set | <= MAXELTS
              elts[MAXELTS]
    int
    int
              numElts;
                                   Use a global constant like we
 public:
    void insert(int v);
                                   have talked about.
      // MODIFIES: this
      // EFFECTS: this = this + {v} if room,
                 throws int MAXELTS otherwise
    void remove(int v);
      // MODIFIES: this
      // EFFECTS: this = this - {v}
   bool query(int v); // RME omitted for space
    int size();  // RME omitted for space
};
```

Using Classes

Given this representation, and the representation invariants, we can write the methods.

```
const int MAXELTS = 100;
class IntSet { // OVERVIEW omitted for space
  int     elts[MAXELTS];
  int     numElts;
public:
  void insert(int v); // RME omitted for space
  void remove(int v); // RME omitted for space
  bool query(int v); // RME omitted for space
  int size(); // RME omitted for space
};
```

```
int IntSet::size() {
  return numElts;
}
```

Because our rep invariant says that numElts is always the size of the set, we can return it directly.

- Next, consider the three final routines:
 - query: search the array looking for a specific number.
 - remove: search the array for a number; if it exists, remove it.
 - insert: search the array for a number; if it doesn't exist, add it.
- All three of these have "search" in common.
- One might be tempted to just write insert and remove in terms of query, will this work?
 - Hint: think about remove.
- query only tells us **whether** the element exists, not **where** we need one more method...

Using Classes

```
public:
    void insert(int v);
    void remove(int v);
    bool query(int v);
    int size();
};
```

Note: This member function must be private. This is because it exposes details about the concrete representation. It is inappropriate to expose these details to a user of this class.

```
const int MAXELTS = 100;
class IntSet { // OVERVIEW omitted for space
           elts[MAXELTS];
   int
           numElts;
   int
   int indexOf(int v); // RME omitted for space
 public:
   void insert(int v); void remove(int v); // RME omitted
   bool query(int v); int size();  // RME omitted
};
int IntSet::indexOf(int v) {
  for (int i = 0; i < numElts; i++) {</pre>
    if (elts[i] == v) return i;
  return MAXELTS;
```

Using Classes

```
const int MAXELTS = 100;
class IntSet { // OVERVIEW omitted for space
   int    elts[MAXELTS];
   int    numElts;
   int indexOf(int v); // RME omitted for space
   public:
    void insert(int v); void remove(int v); // RME omitted
   bool query(int v); int size(); // RME omitted
};
```

With indexOf, query is trivial...

```
bool IntSet::query(int v) {
  return (indexOf(v) != MAXELTS);
}
```



How to Implement insert (v)?

Select all the correct answers.

- A. We can first search v to check if it is already there with indexOf (v)
- **B.** If v is not present, we then add v
- C. If we add v, it should be added as elts [numElts-1] before we increment numElts
- D. If v is added, we need to increment numElts



- The code for insert is not much more difficult than query:
 - First look for the indexOf the element to insert.
 - If it doesn't exist, we need to add this element to the **end** of the array.
 - What is the index of the current "end"?

- Place the element in the next slot and update numElts.
- The only exception to this is if numElts already equals MAXELTS.

```
const int MAXELTS = 100;
class IntSet { // OVERVIEW omitted for space
   int
           elts[MAXELTS];
   int numElts;
   int indexOf(int v); // RME omitted for space
 public:
   void insert(int v); void remove(int v); // RME omitted
   bool query(int v); int size();  // RME omitted
};
void IntSet::insert(int v) {
  if (indexOf(v) == MAXELTS) {
    if (numElts == MAXELTS) throw MAXELTS;
    elts[numElts++] = v;
```

How about Remove?

- If the element (its index is called the victim) is in the array, we have to remove it leaving a "hole" in the array.
- What representation invariants are violated?
 - How can we fix them?

How about Remove?

- Instead of moving each element after the victim to the left by one position, pick up the current "last" element and move it to the hole.
- This also breaks the invariant on numElts, so we must fix it.



```
void IntSet::remove(int v) {
  int victim = indexOf(v);
  if (victim != MAXELTS) {
    elts[victim] = elts[numElts-1];
    numElts--;
  }
}
```

Using Classes

• Question: There is one problem with our implementation. What is it?

• <u>Hint</u>: Consider the newly-created set:

```
IntSet s;
```

What does the computer actually create when we declare S?

Using Classes

• <u>Question</u>: There is one problem with our implementation. What is it?

• Answer: On creation, S's data members are uninitialized!

• This means that the value of numElts could be a random value, but our representational invariant says it must be zero!

• How can we fix this?

Automatically Initializing Classes

- Using constructor!
- The constructor (really, the **default** constructor) has the following type signature:

```
class IntSet { // OVERVIEW omitted for space
    ...
    public:
        IntSet();
        // EFFECTS: creates an empty IntSet
        ...
};
```

Automatically Initializing Classes

```
IntSet();
  // EFFECTS: creates an empty IntSet
```

- The name of the function is the same as the name of the class.
- This function doesn't have a return type.
- It also does not take an argument in this case.
- It is guaranteed to be the **first** function called immediately after an object is created.
- It builds a "blank" uninitialized IntSet and makes it satisfy the rep invariant.

Automatically Initializing Classes

```
IntSet();
  // EFFECTS: creates an empty IntSet
```

• Here's how it's written:

```
IntSet::IntSet(): numElts(0)
{
}
```

Automatically Initializing Classes

```
IntSet::IntSet()
    : numElts(0)
{
}
```

```
Class_T::Class_T(): anInt(0),
     aDouble(1.2),
     aString("Yes")
{
}
```

- This syntax is called "initialization syntax".
- Each data member is initialized this way.
- <u>Note</u>: The order in which elements are initialized is the order they **appear in the definition**, NOT the order in the initialization list. It is a good practice to keep them in the same order to avoid confusion.

Automatically Initializing Classes

• Alternatively, we could write this function as follows, but this is not considered as a good way!

```
IntSet::IntSet()
{
   numElts = 0;
}
```



A Benefit of Classes

• Now, instead of writing this:

```
void add_one (int a[], int elts);
```

and having to worry about the number of elements in the array. All we have to write is this:

```
void add_one (IntSet& set);
```

and we no longer have to worry about the array and its count being separated.

• A slight change to the class definition: const int MAXELTS = 100; class IntSet { int elts[MAXELTS]; int numElts; int indexOf(int v) const; public: void insert(int v); void remove(int v); bool query(int v) const; int size() const; };

int size() const;

- Each member function of a class has an extra, implicit parameter named **this**.
 - "this" is a pointer to the current instance on which the function is invoked.
- **const** keyword modifies the implicit **this** pointer: **this** is now a pointer to a **const instance**.
 - <u>Means</u>: the member function **size()** cannot change the object on which **size()** is called.
 - By its definition, **size()** shouldn't change the object! Adding **const** keyword prevents any accidental change.
 - It is a good practice to add const keyword when possible!

• Implement size()
 int IntSet::size() const {
 return numElts;
}
The function body is the same as before.

• A **const** object can only call its **const** member functions!

```
const IntSet is;
cout << is.size(); ✓
is.insert(2); ✗</pre>
```

• If a const member function calls other **member** functions, they must be **const** too!

```
void A::g() const { f(); }
```







Outline

- Introduction to Abstract Data Types
- Class in C++: A Trivial Example
- More Details on Class
- Another Class Example: a Mutable Set of Integers (IntSet)
- Improve the Efficiency of IntSet

?

How many elements of the array must indexOf examine?

```
int IntSet::indexOf(int v) {
  for (int i = 0; i < numElts; i++) {
    if (elts[i] == v) return i;
  }
  return MAXELTS;
}</pre>
```

- A. In the best case, 1 element
- B. In the worst case, numElts elements
- C. In the worst case, MAXELTS elements
- **D.** None of the above



Improving Efficiency

- We say the time for indexOf grows **linearly** with the size of the set.
- If there are N elements in the set, we have to examine all N of them in the worst case. For large sets that perform lots of queries, this is too expensive!
- Luckily, we can replace this implementation with a different one that can be more efficient. The only change we need to make is to the **representation (implementation)** the abstraction can stay precisely the same.

Improving Efficiency

• Still use an array to store the elements of the set and the values will still occupy the first numElts slots.

• However, now we'll keep the elements in sorted order.

Question: Which Member Functions of the Class Should Be Changed?

```
const int MAXELTS = 100;
class IntSet {
    // OVERVIEW: a mutable set of integers
    int elts[MAXELTS];
    int numElts;
    int indexOf(int v) const;
 public:
    IntSet();
    void insert(int v);
    void remove(int v);
    bool query(int v) const;
    int size() const;
};
```

Improving Efficiency

• The constructor and size methods don't need to change at all since they just use the numElts field.

• query also doesn't need to change.

```
bool IntSet::query(int v) {
    return (indexOf(v) != MAXELTS);
}
```

- indexOf also doesn't need to change.
- However, insert and remove do need to change.

Improving Efficiency

• We'll start with the easiest one: remove.

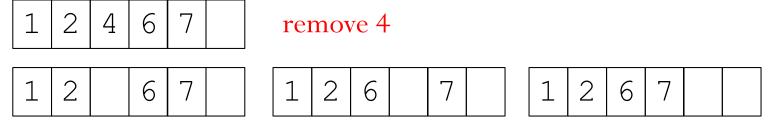
• Recall the old version that moved the last element from the end to somewhere in the middle, this will break the new "sorted" invariant.



• Instead of doing a swap, we have to "squish" the array together to cover up the hole.



- How are we going to do the "squish"?
 - Move the element next to the hole to the left leaving a new hole.
 - Keep moving elements until the hole is "off the end" of the elements.



- We'll reuse the variable victim as a loop variable.
- victim's invariant is that it always points at the hole in the array.

```
void IntSet::remove(int v) {
  int victim = indexOf(v);
  if (victim != MAXELTS) {
      // victim points at hole
    numElts--; // one less element
    while (victim < numElts) {</pre>
      // ..hole still in the array
      elts[victim] = elts[victim+1];
      victim++;
```

Improving Efficiency

• We also have to change insert since it currently just places the new element at the end of the array. This will also break the new "sorted" invariant.



- How are we going to do the insert?
 - Start by moving the last element to the right by one position.
 - Repeat this process until the correct location is found to insert the new element.
 - Stop if the start of the array is reached or the element is sorted.
 - We'll need a new loop variable called cand(idate) to track this movement.
 - It's invariant in that it always points to the next element that <u>might</u> have to move to the right.

```
void IntSet::insert(int v) {
  if (indexOf(v) == MAXELTS) { // duplicate not found
    if (numElts == MAXELTS) throw MAXELTS; // no room
    int cand = numElts-1; // last element
    while ((cand \geq 0) && elts[cand] \geq v) {
      elts[cand+1] = elts[cand];
      cand--;
    }
    // Now, cand points to the left of the "gap".
    elts[cand+1] = v;
    numElts++; // repair invariant
    insert 5
                                                        4 | 5 |
                                   2
                                                      2
                                      4
                                                   1
                       cand
                                       cand
                                                       cand
```

```
void IntSet::insert(int v) {
  if (indexOf(v) == MAXELTS) { // duplicate not found
    if (numElts == MAXELTS) throw MAXELTS; // no room
    int cand = numElts-1; // last element
    while ((cand >= 0) \& \& elts[cand] > v) {
      elts[cand+1] = elts[cand];
      cand--;
                                 Note: We are using the
    // Now, cand points to the "short-circuit" property
                                 of &&. If cand is not
    elts[cand+1] = v;
    numElts++; // repair invar: greater than or equal to
                                 zero, we never evaluate
                                 the right-hand clause.
```

Improving Efficiency

```
void IntSet::insert(int v) {
  if (indexOf(v) == MAXELTS) { // duplicate not found
    if (numElts == MAXELTS) throw MAXELTS; // no room
    int cand = numElts-1; // largest (last) element
    while ((cand \geq 0) && elts[cand] \geq v) {
      elts[cand+1] = elts[cand];
      cand--;
    // Now, cand points to the left of the "gap".
    elts[cand+1] = v;
    numElts++; // repair invariant
         Question: What is the situation when the loop terminates due
```

to cand < 0? Is our implementation correct?

Improving Efficiency

• **Question**: Do we have to change indexOf?

```
int IntSet::indexOf(int v) {
  for (int i = 0; i < numElts; i++) {
    if (elts[i] == v) return i;
  }
  return MAXELTS;
}</pre>
```

- **Question**: Do we have to change indexOf?
- **Answer**: No, but it can be made more efficient with the new representation.
- How? Using binary search! (The array is sorted)

```
int IntSet::indexOf(int v) {
  for (int i = 0; i < numElts; i++) {
    if (elts[i] == v) return i;
  }
  return MAXELTS;
}</pre>
```

Complexity

	Unsorted	Sorted
query	O(N)	?
insert	?	?
remove	?	?

Complexity

	Unsorted	Sorted
query	O(N)	O(log N)
insert	O(N)	O(N)
remove	O(N)	O(N)

insert and remove are still **linear**, because they may have to "swap" an element to the beginning/end of the array.

Complexity

	Unsorted	Sorted
query	O(N)	O(log N)
insert	O(N)	O(N)
remove	O(N)	O(N)

- If you are going to do more searching than inserting/removing, you should use the "sorted array" version, because query is faster there.
- However, if query is relatively rare, you may as well use the "unsorted" version. It's "about the same as" the sorted version for insert and remove, but it's MUCH simpler!

References

- Problem Solving with C++ (8th Edition)
 - Chapter 10.3 Abstract Data Types
 - Chapter 10.2 Classes and constructors
- C++ Primer, 4th Edition
 - Chapter 7.7.1 const Member Function