Shanghai Jiao Tong University ECE4530J

Homework 3: Due 2021.6.28

Instructions

1. Please clearly label the problem numbers in your response.
2. Attach any codes at the end of your response.
3. If you hand-write, please keep your response neat and readable.

Problem 1: Single queuing system with state $X(t) \in \mathbb{Z}_{\geq 0}$

a) Suppose $\lambda = 0.5$ and $\mu = 0.7$. Draw the state transition diagram. You need to indicate the numerical values for the transition probabilities.

b) Write codes to simulate the queuing system. You can assume zero initial condition. Report the time-average queue size

$$\frac{1}{T}\sum_{t=0}^{T} X(t)$$

with $T = 360$.

c) Change the value for λ from 0.1 to 0.8 while keep $\mu = 0.7$. Repeat the simulation in part b. Plot the time-average queue size vs. demand λ. Interpret the trend of the curve.

Problem 2: Bernoulli routing for two parallel queues.

a) Suppose that $\lambda = 0.5$, $\mu_1 = \mu_2 = 0.5$. Assume that $b_1 = 0.2$ and $b_2 = 0.8$. Draw the state transition diagram with transition probabilities. Simulate the parallel queues with zero initial condition. Report the time-average queue size $\frac{1}{T}\sum_{t=0}^{T}\big(X_1(t) + X_2(t)\big)$.

b) Repeat the above simulation for $b_1 \in \{0, 0.1, \ldots, 1.0\}$ and find the optimal value.

c) Suppose that $\lambda = 0.5$, $\mu_1 = 0.1$ and $\mu_2 = 0.9$. Find the optimal b_1 by simulation (i.e. trying various values for b_1).

d) Compare and interpret the results in parts b and c.

Problem 3: JSQ routing for two parallel queues. Ties are broken uniformly at random.

a) Assume that $\lambda = 0.5$, $\mu_1 = \mu_2 = 0.5$. Draw the state transition diagram with transition probabilities (you can eliminate arrows associated with zero probabilities). Repeat 2b with JSQ routing. Compare and interpret the average queue size with that in 2b.

b) Repeat 2c with JSQ routing. Compare and interpret the average queue size with that in 2c.

c) Compare the interpret JSQ's performance in 3a and 3b.