Chapter 27

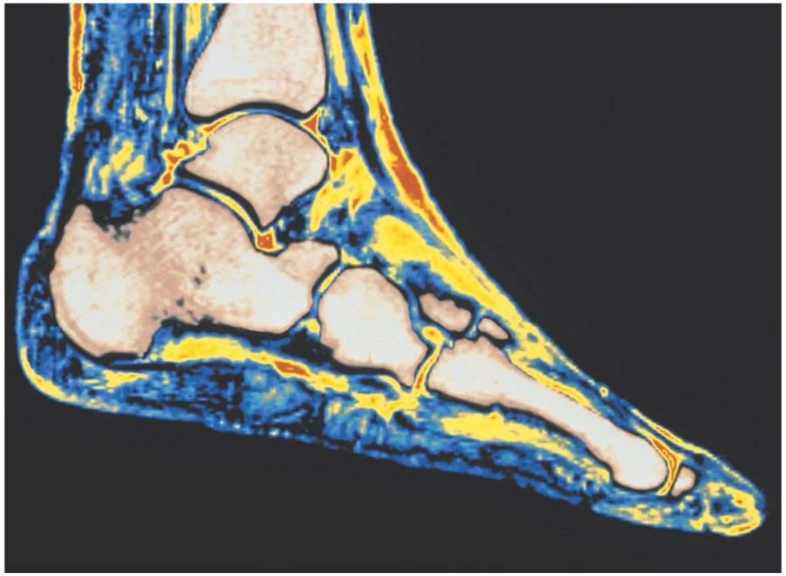
Magnetic Field and Magnetic Forces

Introduction

- How does magnetic resonance imaging (MRI) allow us to see details in soft nonmagnetic tissue?

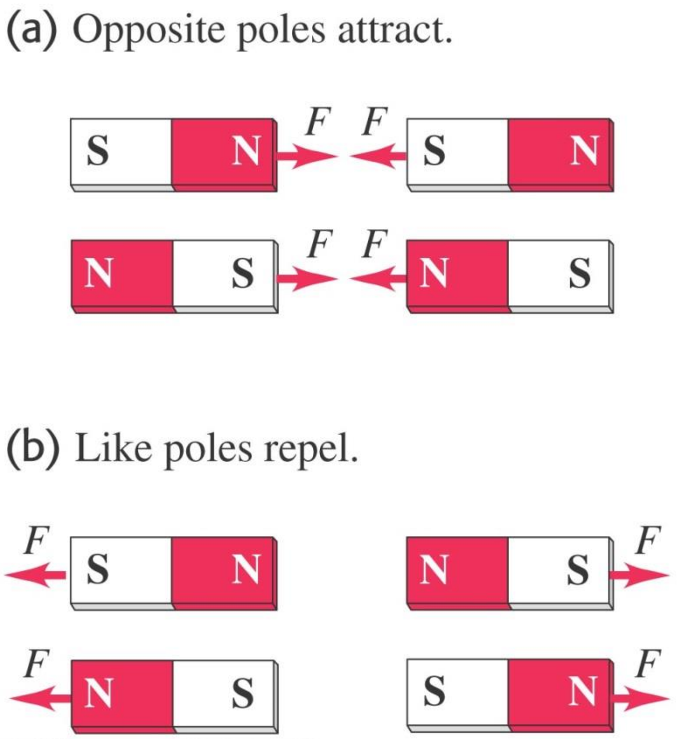
- How can magnetic forces, which act only on moving charges, explain the behavior of a compass needle?

- In this chapter, we will look at how magnetic fields affect charges.

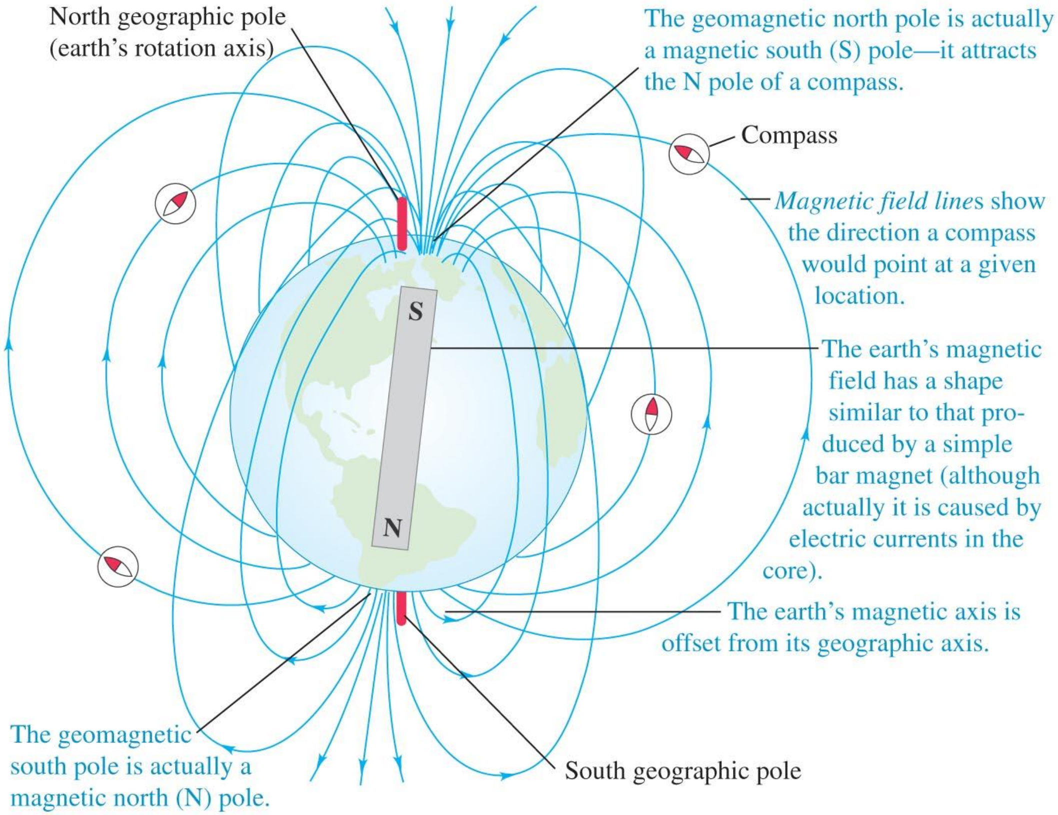


Magnetic poles

- Figure 27.1 at the right shows the forces between magnetic poles.



Compass : attractive force



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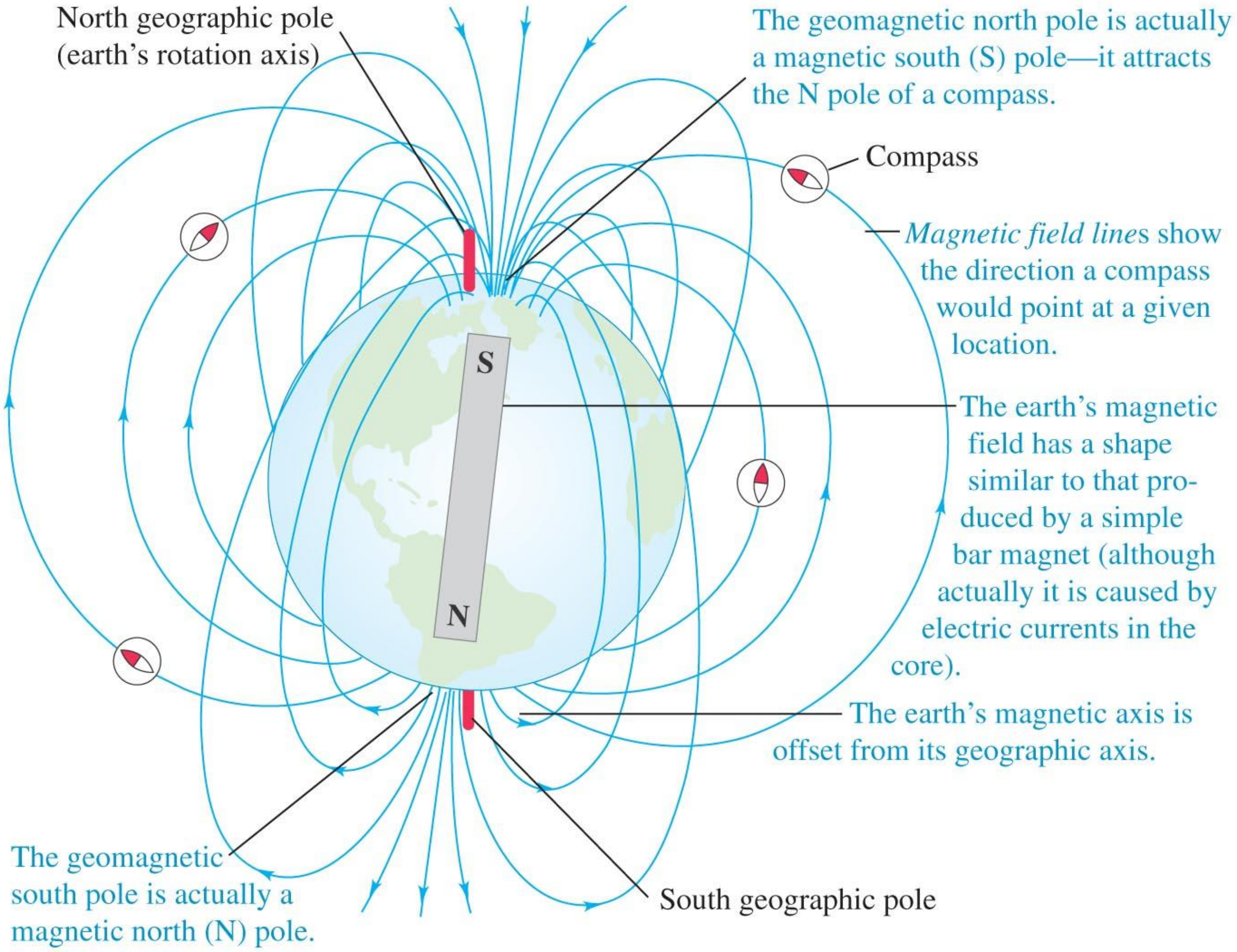
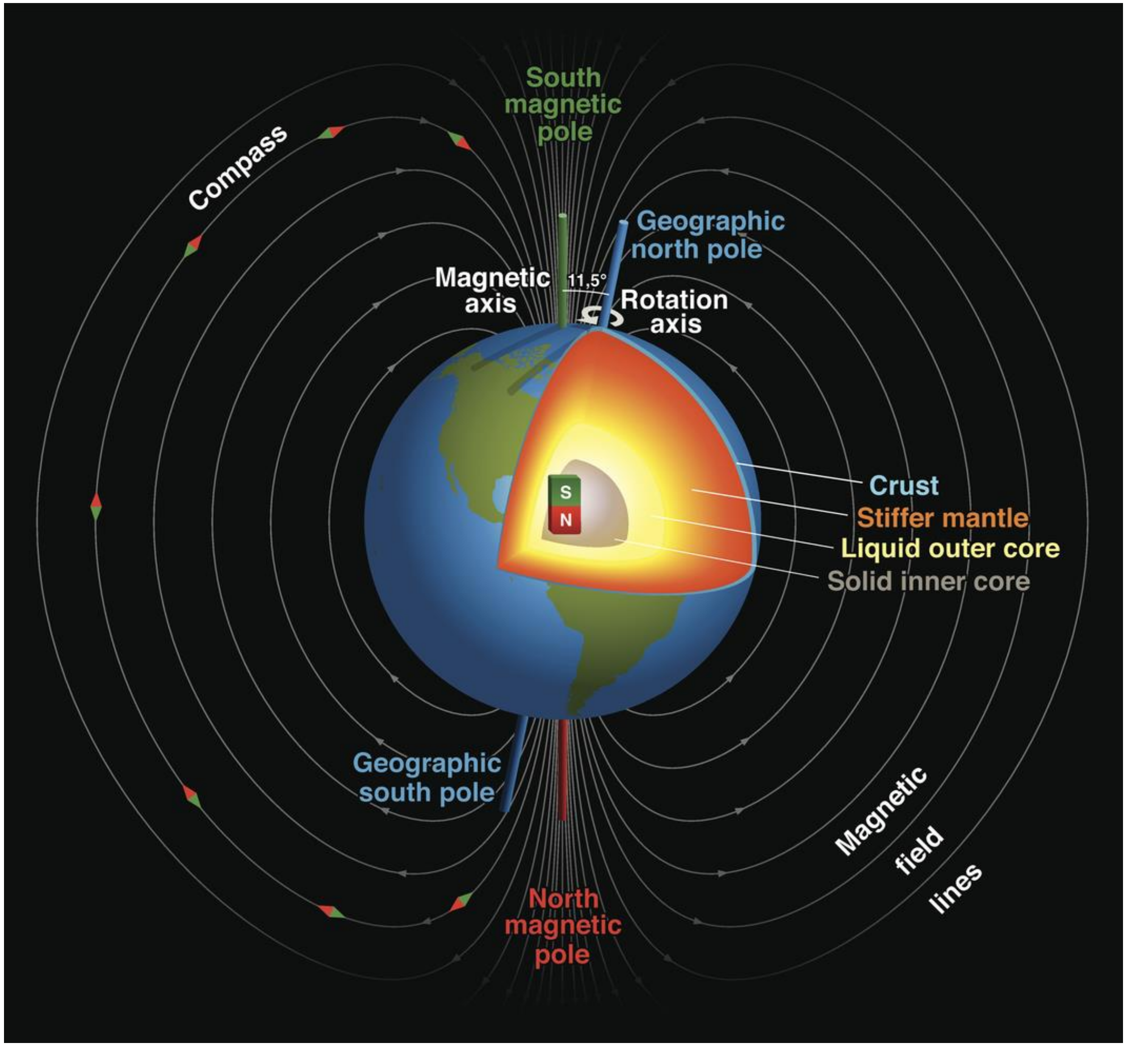
Magnetic field of the earth

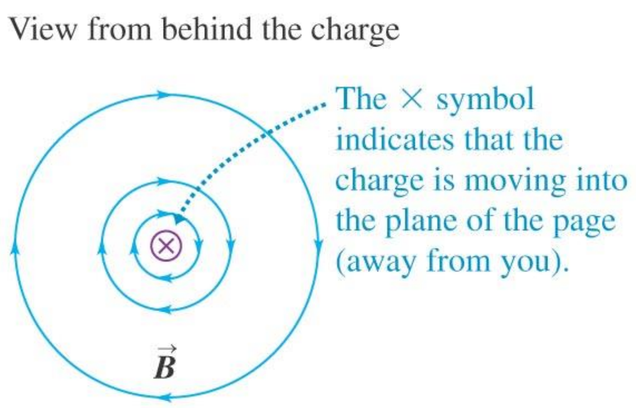
- The earth itself is a magnet. Figure 27.3 shows its magnetic field.

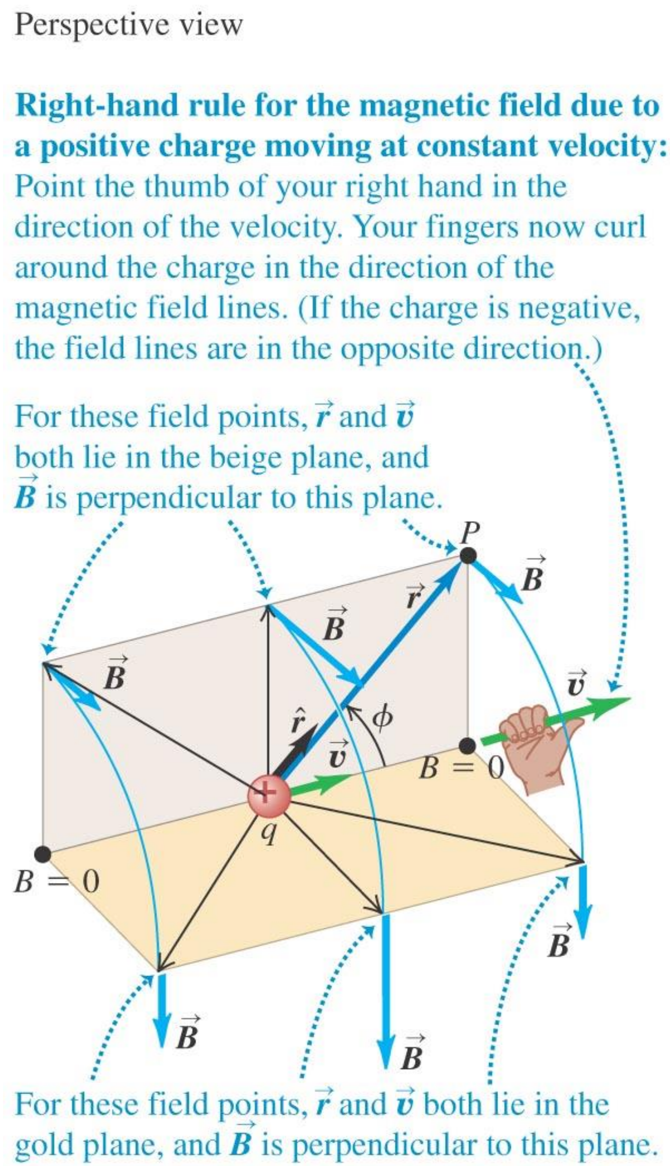



South magnetic pole
Compass
Geographic north pole
Magnetic axis
11.5°
Rotation axis
S
N
Crust
Stiffer mantle
Liquid outer core
Solid inner core
Geographic south pole
North magnetic pole
Magnetic field lines

The magnetic field of a moving charge

- **A moving charge generates a magnetic field** that depends on the velocity of the charge.
- Figure 28.1 shows the direction of the field.





The magnetic field of a moving charge

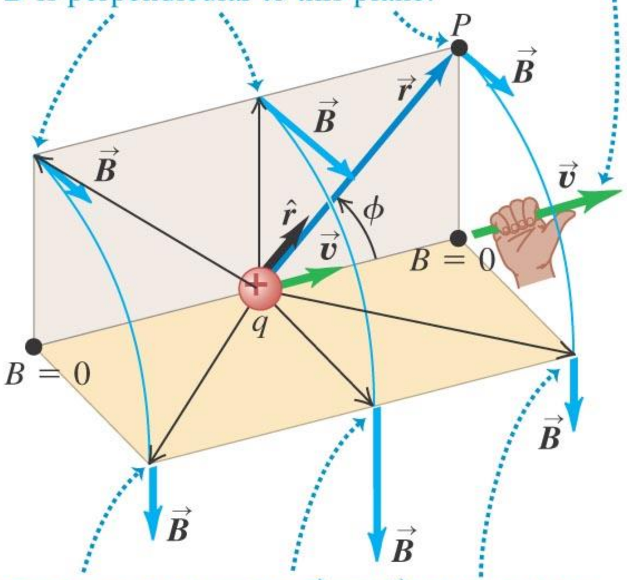
$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$ (magnetic field of a point charge with constant velocity)

Perspective view

Right-hand rule for the magnetic field due to a positive charge moving at constant velocity: Point the thumb of your right hand in the direction of the velocity. Your fingers now curl around the charge in the direction of the magnetic field lines. (If the charge is negative, the field lines are in the opposite direction.)

For these field points, $\vec{r}$ and $\vec{v}$ both lie in the beige plane, and $\vec{B}$ is perpendicular to this plane.



For these field points, $\vec{r}$ and $\vec{v}$ both lie in the gold plane, and $\vec{B}$ is perpendicular to this plane.

Magnetic field of a current element

- *law of Biot and Savart.*

$$dQ = nqA\,dl$$

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_{\mathrm{d}}\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_{\mathrm{d}}A\,dl\sin\phi}{r^2}$$

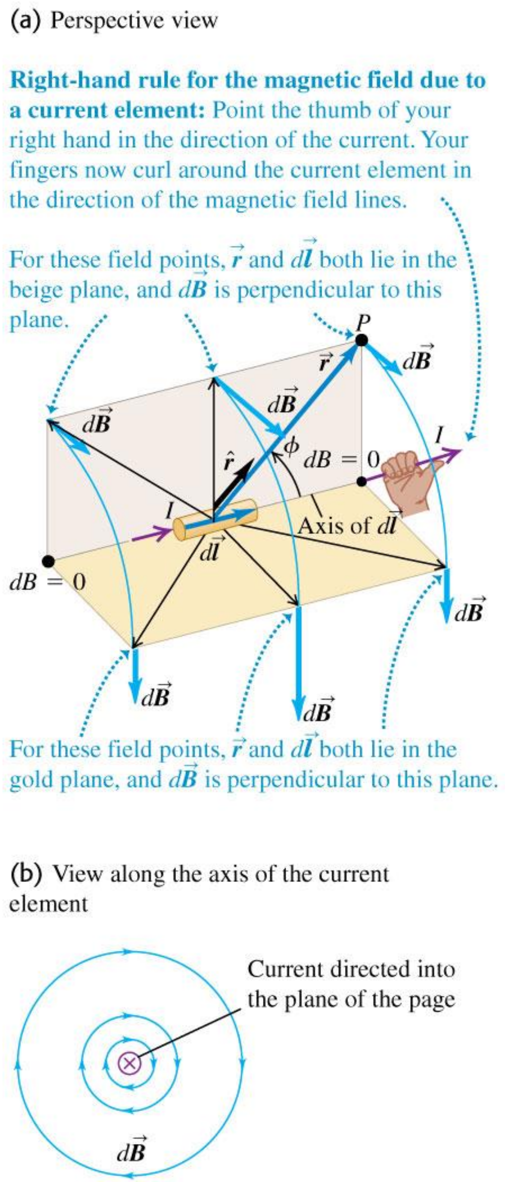
$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2} \qquad n|q|v_{\mathrm{d}}A$$

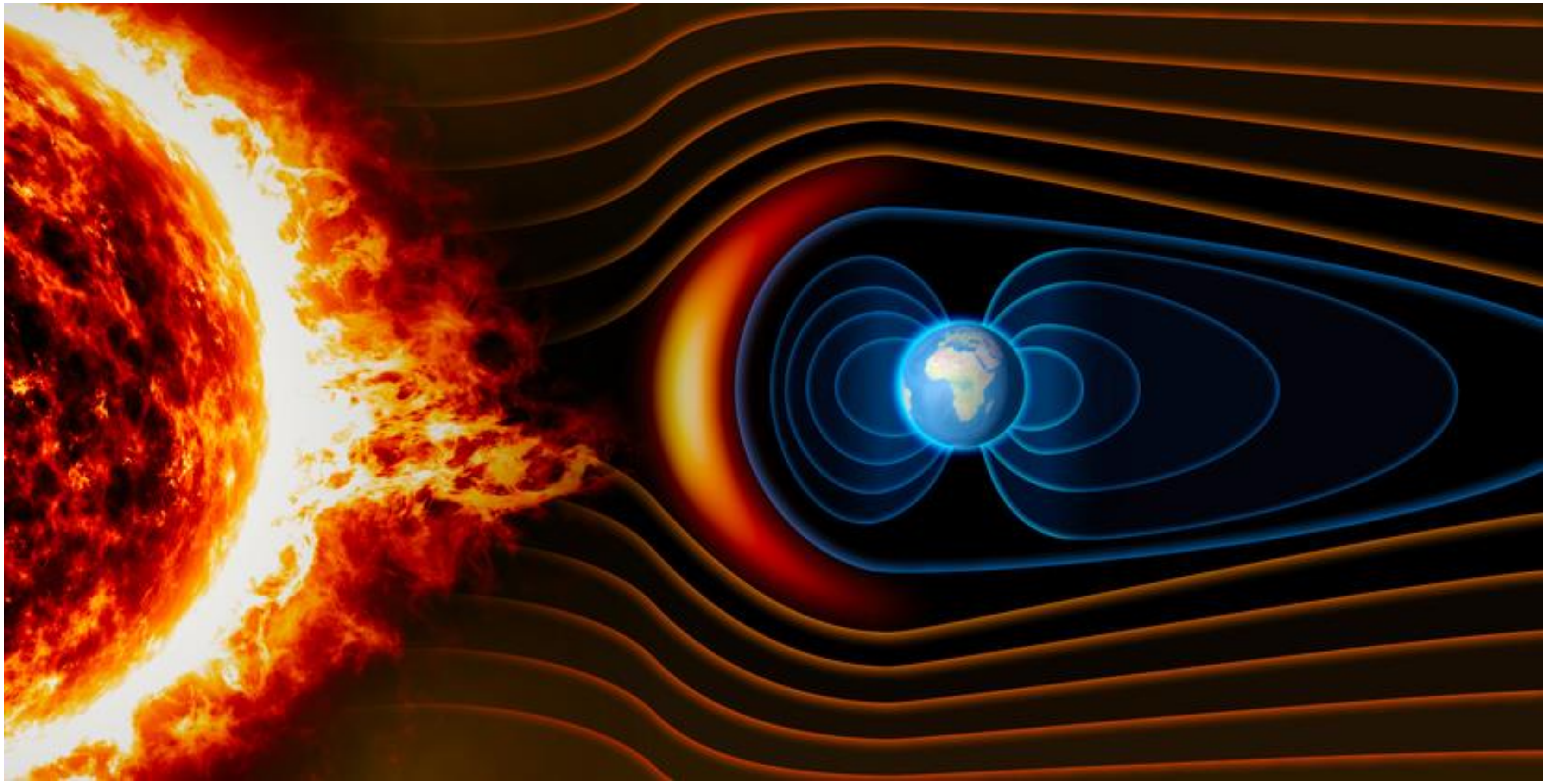
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{(magnetic field of a current element)}$$

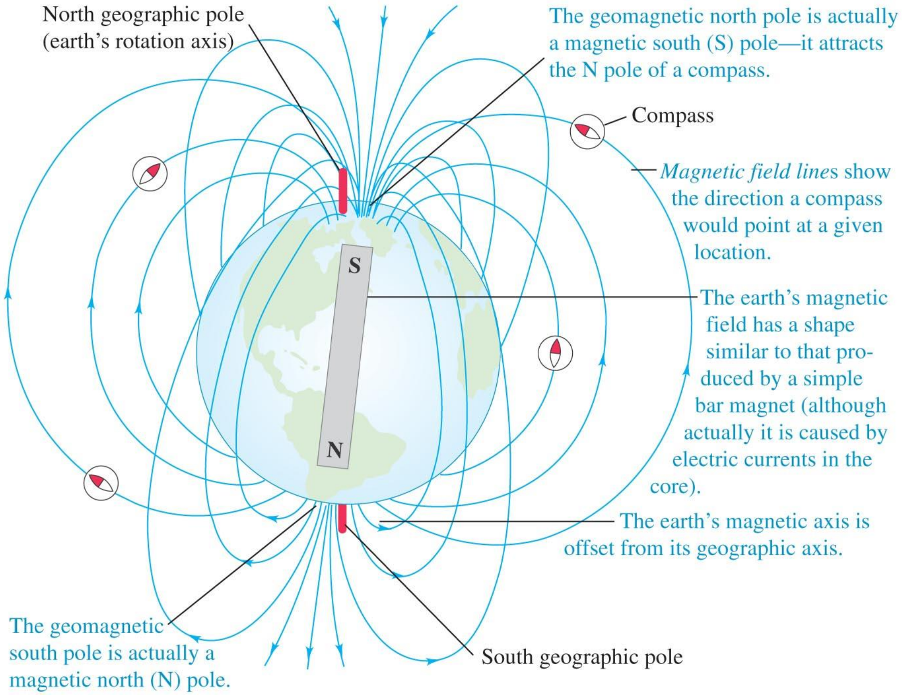
$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

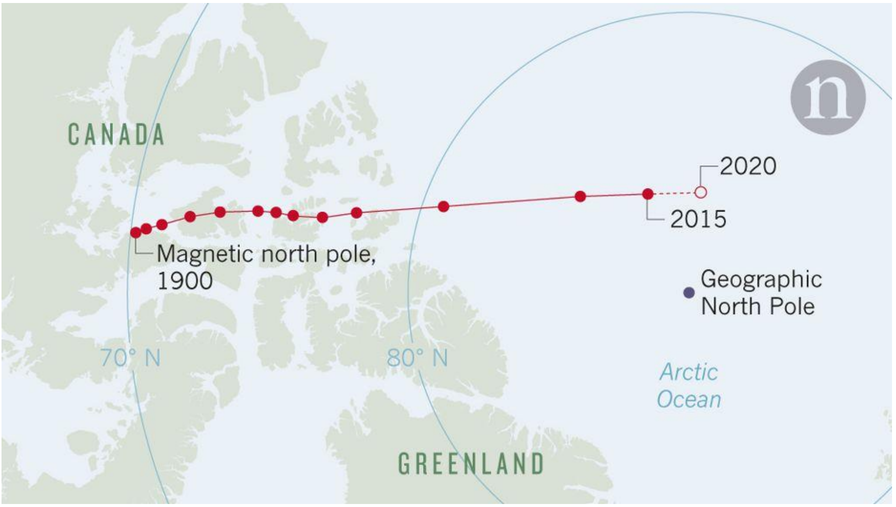
For surface and volume currents the Biot-Savart law becomes

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}(\mathbf{r}')\times\hat{\boldsymbol{\imath}}}{\imath^2}\,da' \quad \text{and} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')\times\hat{\boldsymbol{\imath}}}{\imath^2}\,d\tau',$$

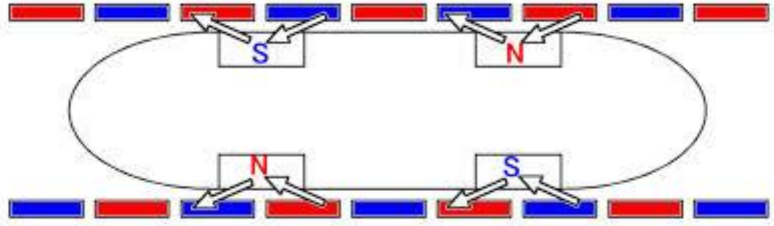


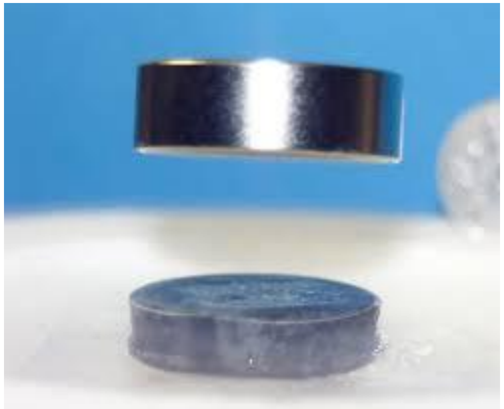



North geographic pole
(earth's rotation axis)
The geomagnetic north pole is actually a magnetic south (S) pole—it attracts the N pole of a compass.
Compass
Magnetic field lines show the direction a compass would point at a given location.
S
N
The earth's magnetic field has a shape similar to that produced by a simple bar magnet (although actually it is caused by electric currents in the core).
The earth's magnetic axis is offset from its geographic axis.
The geomagnetic south pole is actually a magnetic north (N) pole.
South geographic pole

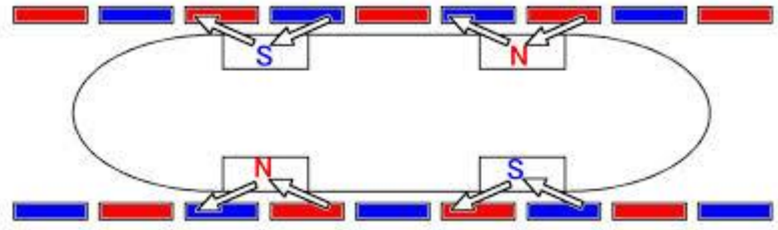

CANADA
2020
2015
Magnetic north pole, 1900
Geographic North Pole
70° N
80° N
Arctic Ocean
GREENLAND

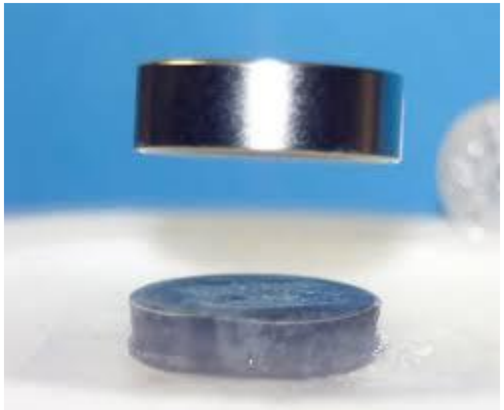
Maglev : repulsive force





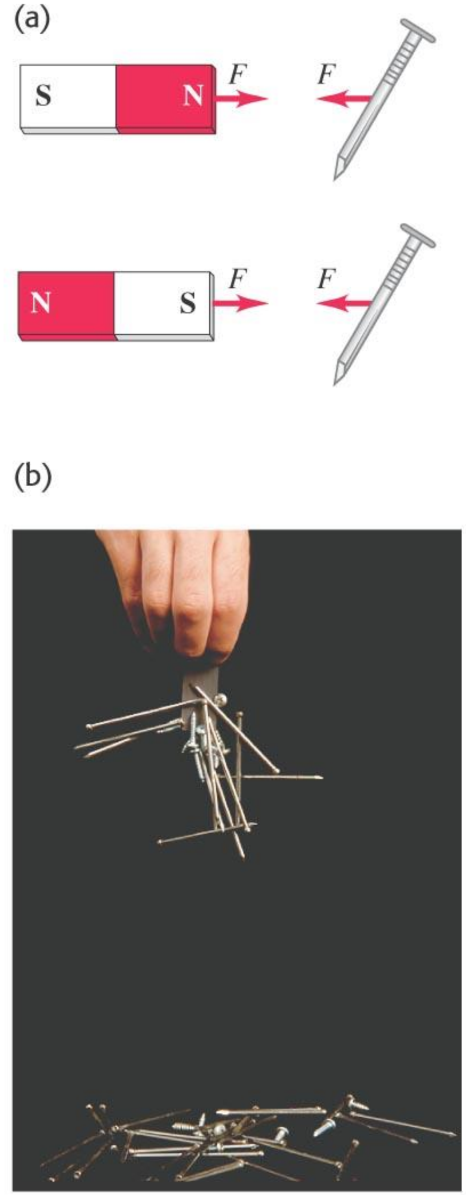
Maglev : repulsive force



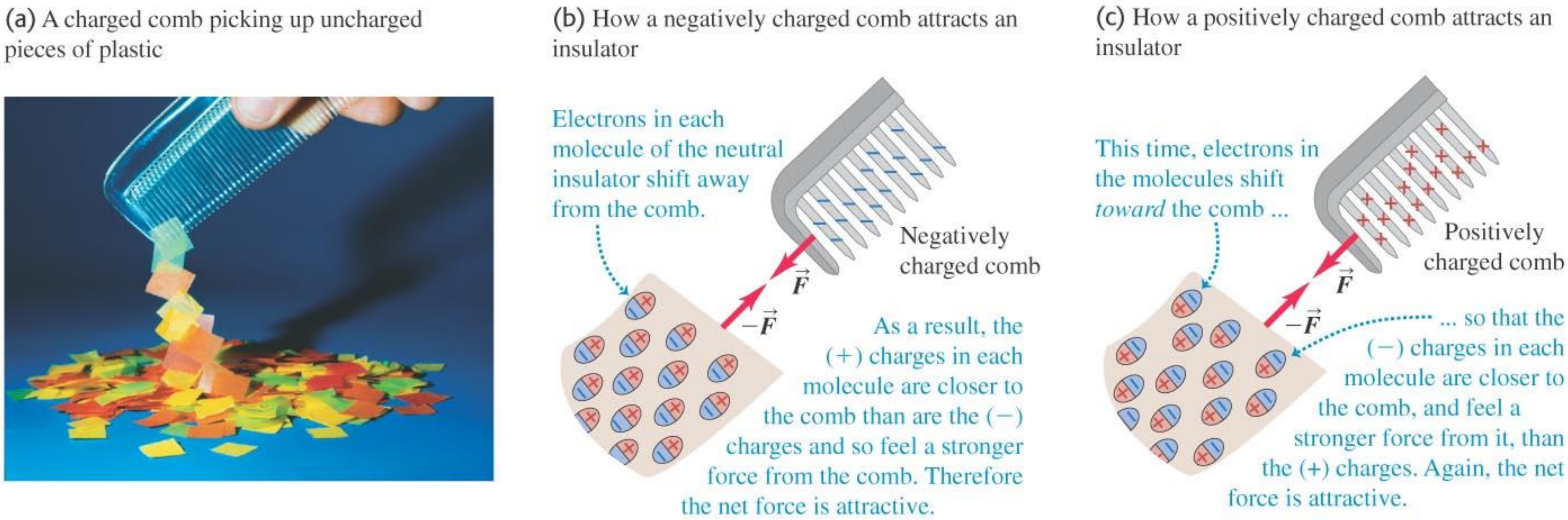


Magnetism and certain metals

- Either pole of a permanent magnet will attract a metal like iron, as shown in Figure 27.2 at the right.



Magnetism and certain metals



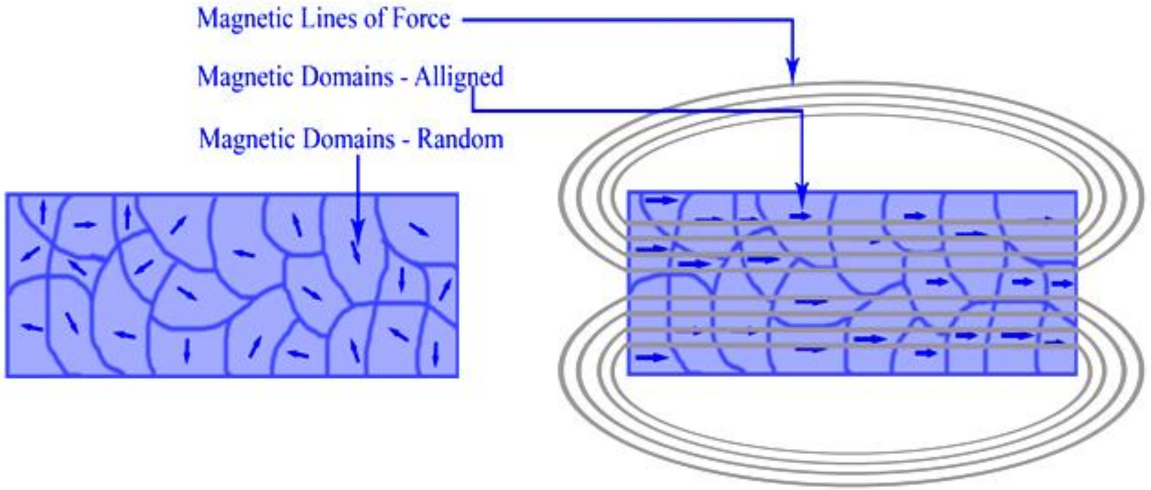
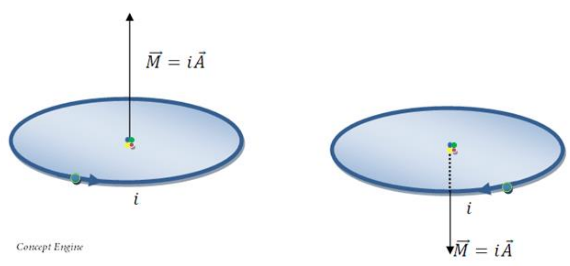


Fig. Ferromagnetism (a) Unmagnetized Material (b) Magnetized Material



Magnetism and certain metals

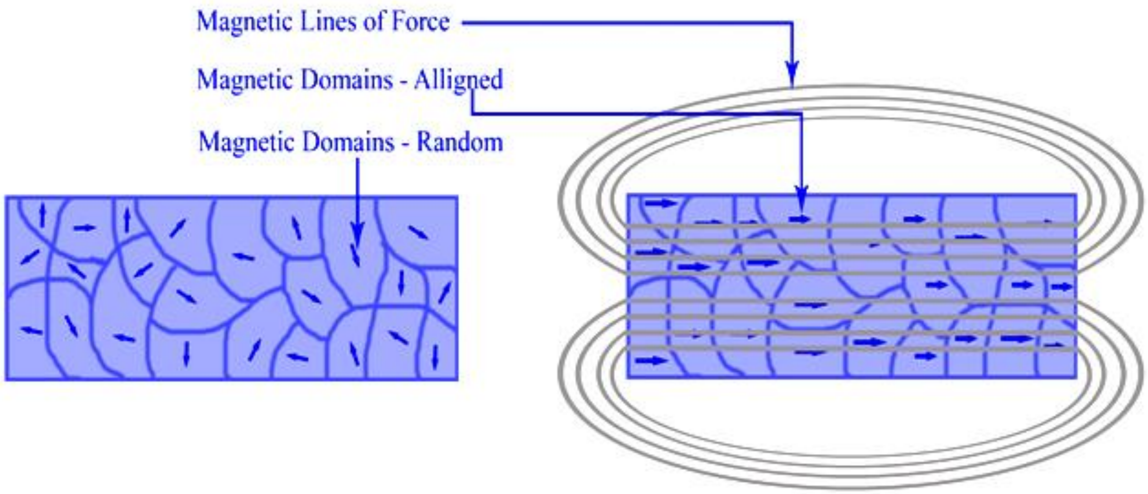
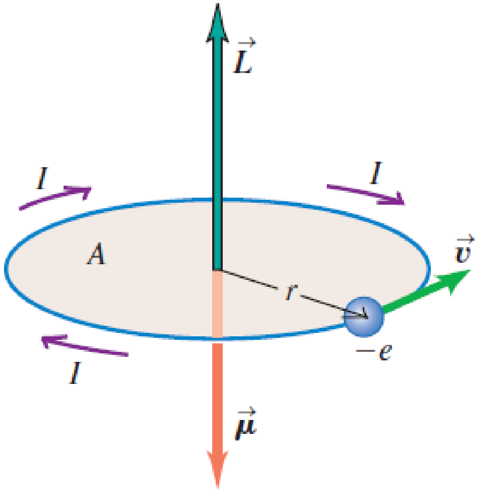


Fig. Ferromagnetism (a) Unmagnetized Material (b) Magnetized Material

The Bohr Magneton



Magnetism and certain metals

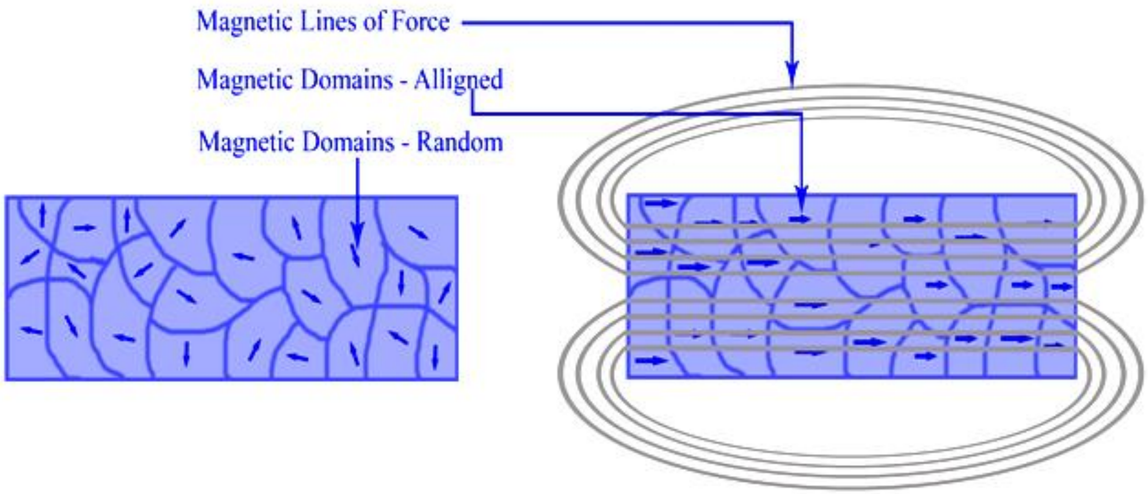
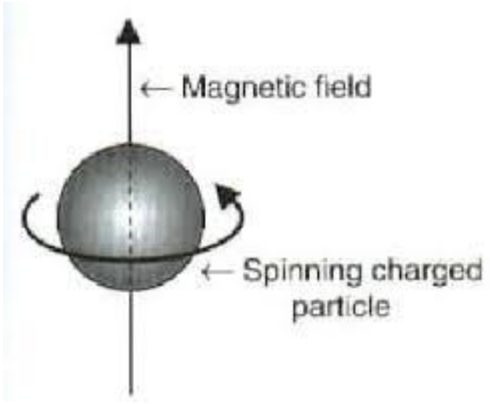
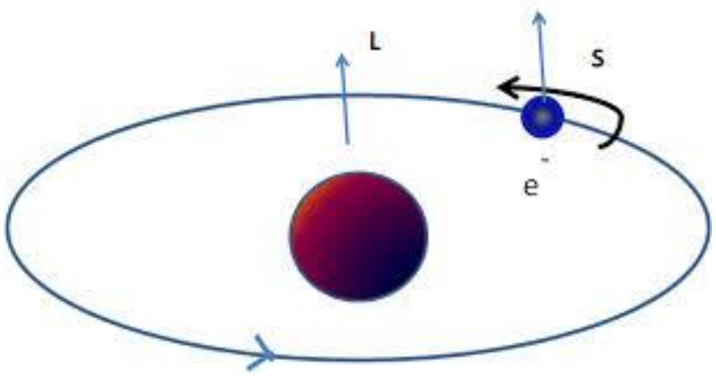


Fig. Ferromagnetism (a) Unmagnetized Material (b) Magnetized Material

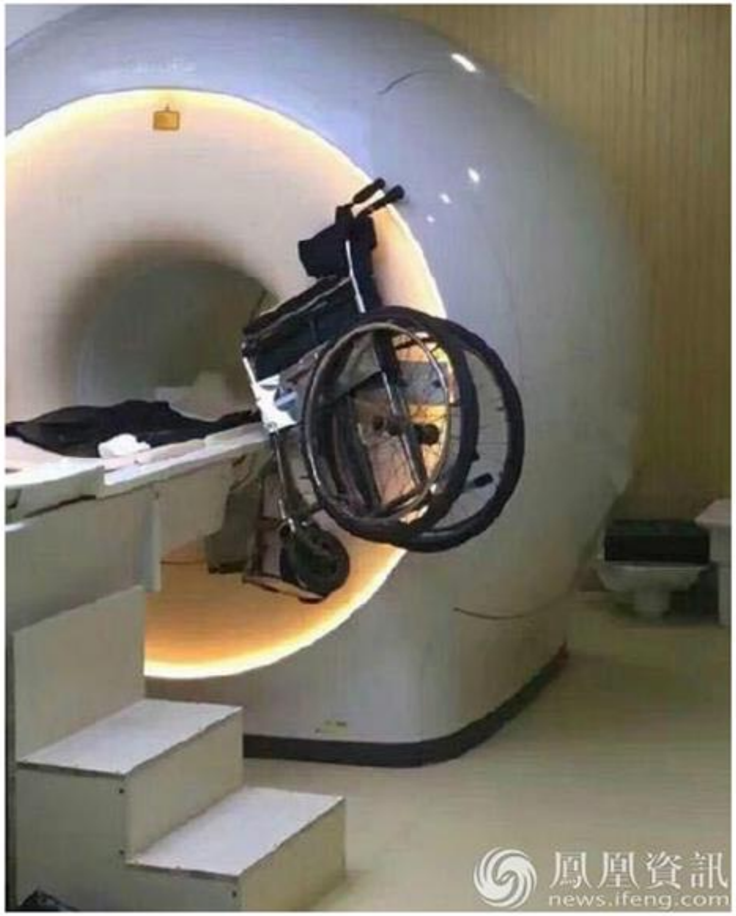




Superconductor

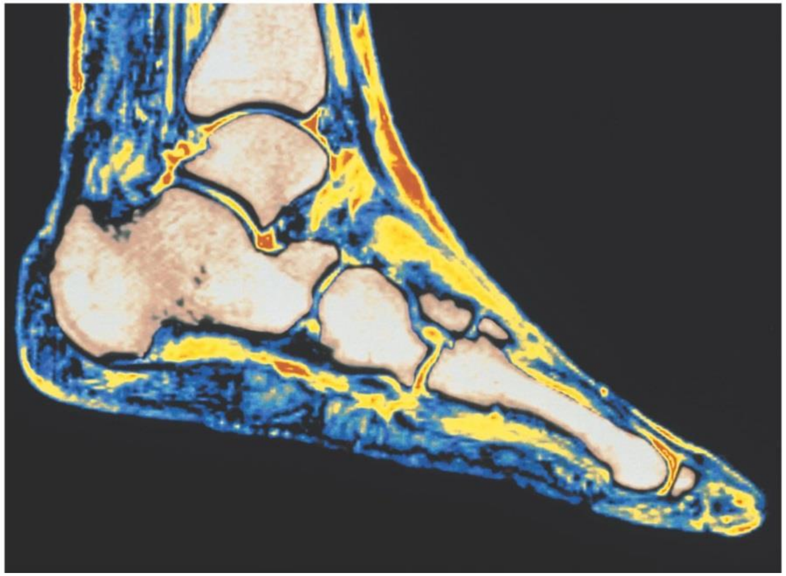
上海肺科医院核磁共振室惊人一幕：家属不听劝告把轮椅推进医院核磁共振房间被“亲吻”【3】

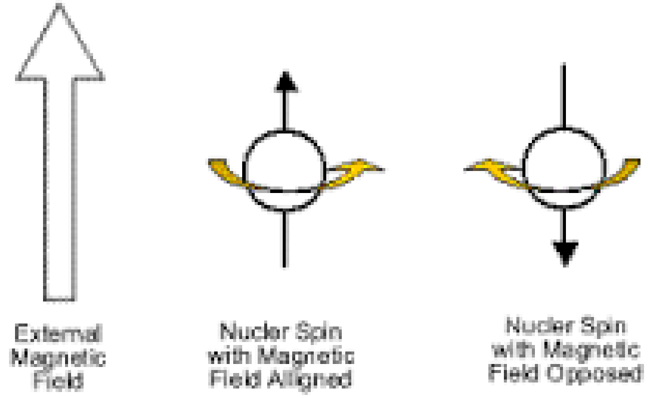
作者： 来源：凤凰网 2016年07月06日09:19

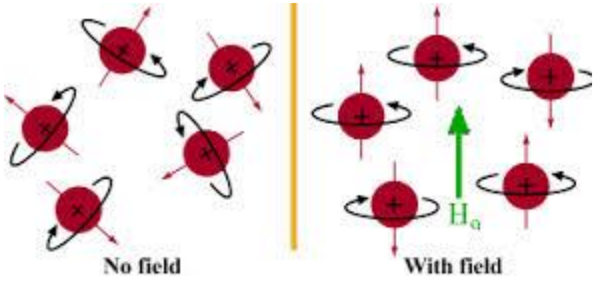


据东方网2016年7月5日报道：昨天，上海肺科医院核磁共振室内发生“惊人一幕”，一台轮椅“亲吻”上了核磁共振仪，网传仪器修理费将达到300万元。不过东方网记者今日获悉，300万元维修费的说法子虚乌有，这台仪器明后天就能继续使用。

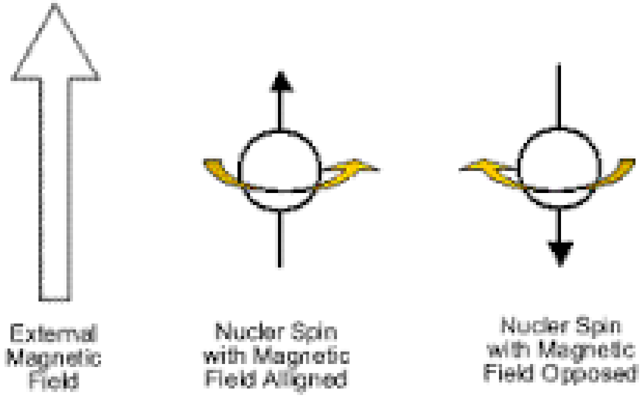
MRI – nuclear magnetic resonance

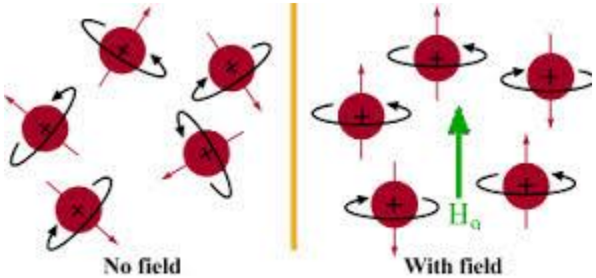




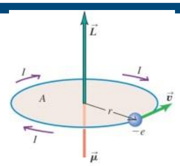


Diamagnetic





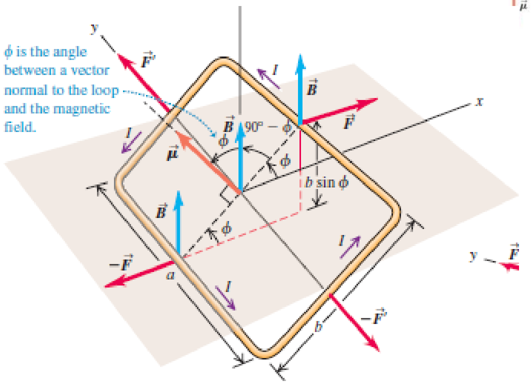
The Bohr magneton and paramagnetism



$$\mu = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi m}$$

e **Bohr magneton,** denoted b

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$



e **magnetization** of the material, denoted by $\vec{M}$:

$$\vec{M} = \frac{\vec{\mu}_{\text{total}}}{V}$$

, the total magnetic field $\vec{B}$ in the material is

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$$

as μ and is called the **permeability**

$$\mu = K_m \mu_0 \qquad (2$$

Table 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20°C$

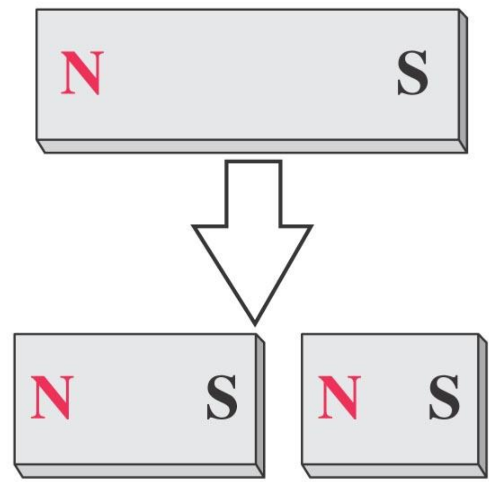
Material	$\chi_m = K_m - 1\ (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

Magnetic monopoles

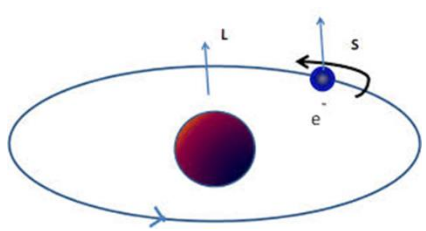
- Breaking a bar magnet does not separate its poles, as shown in Figure 27.4 at the right.
- There is no experimental evidence for *magnetic monopoles*.

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Breaking a magnet in two ...



... yields two magnets, not two isolated poles.

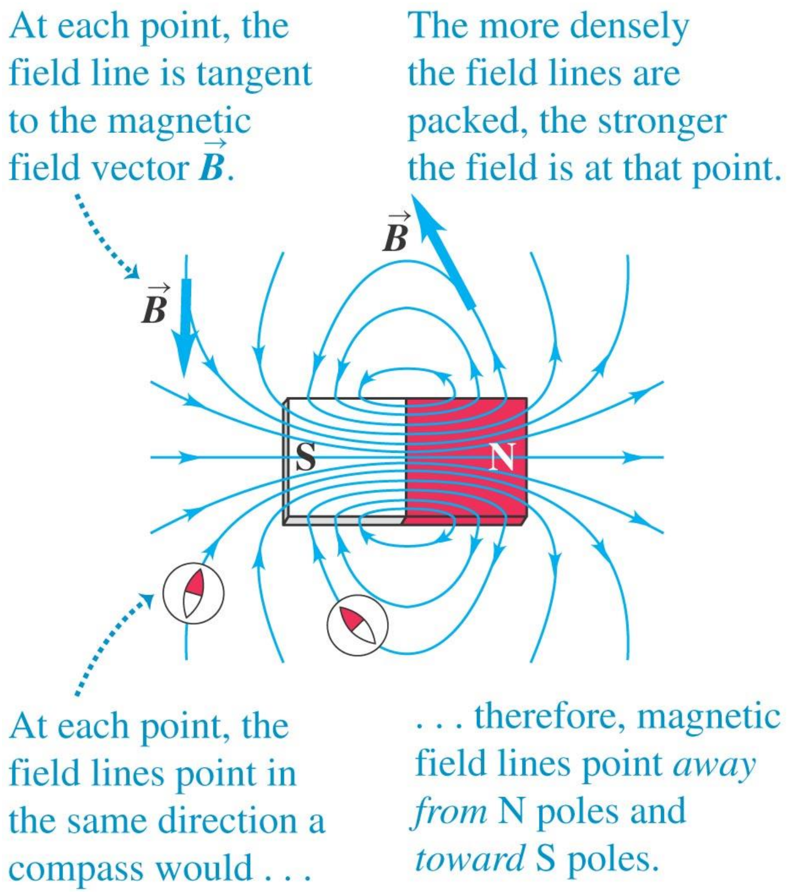


The magnetic field

- A moving charge (or current) creates a *magnetic field* in the surrounding space.
- The magnetic field exerts a force on any other moving charge (or current) that is present in the field.

Magnetic field lines

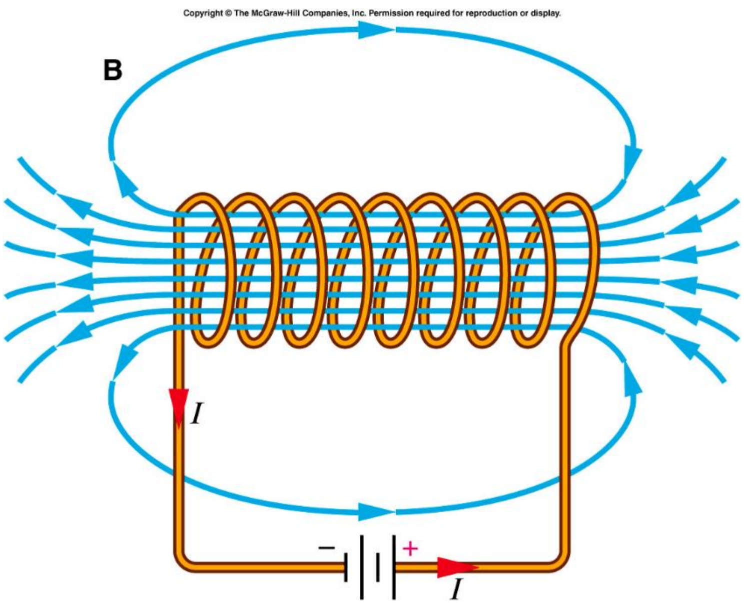
- Figure 27.11 below shows the *magnetic field lines* of a permanent magnet.

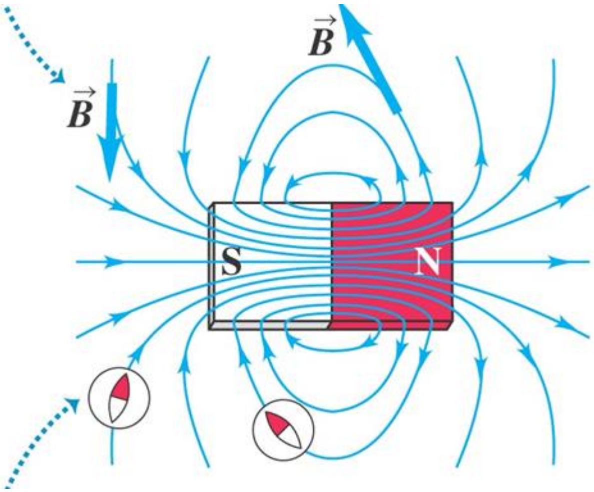


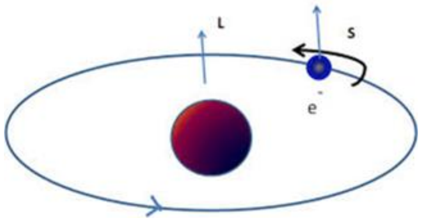
CAUTION **Magnetic field lines have no ends** Unlike electric field lines that begin and end on electric charges, magnetic field lines *never* have end points; such a point would indicate the presence of a monopole. You might be tempted to draw magnetic field lines that begin at the north pole of a magnet and end at a south pole. But as Fig. 27.11 shows, the field lines of a magnet actually continue through the interior of the magnet. Like all other magnetic field lines, they form closed loops.

The magnetic field of a moving charge

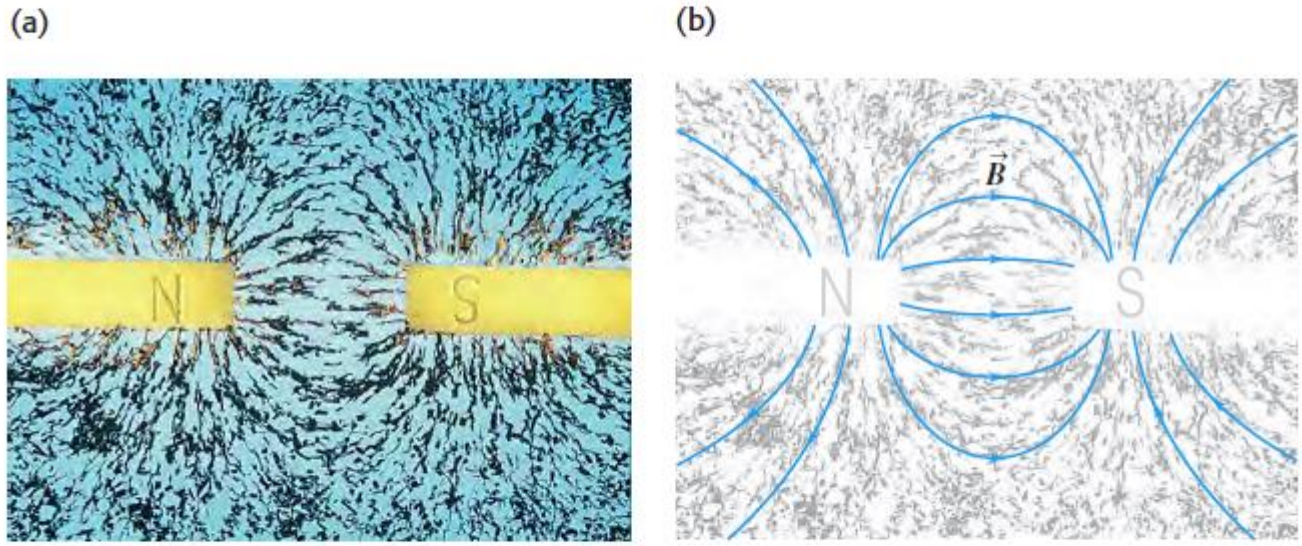
- **A moving charge generates a magnetic field** that depends on the velocity of the charge.



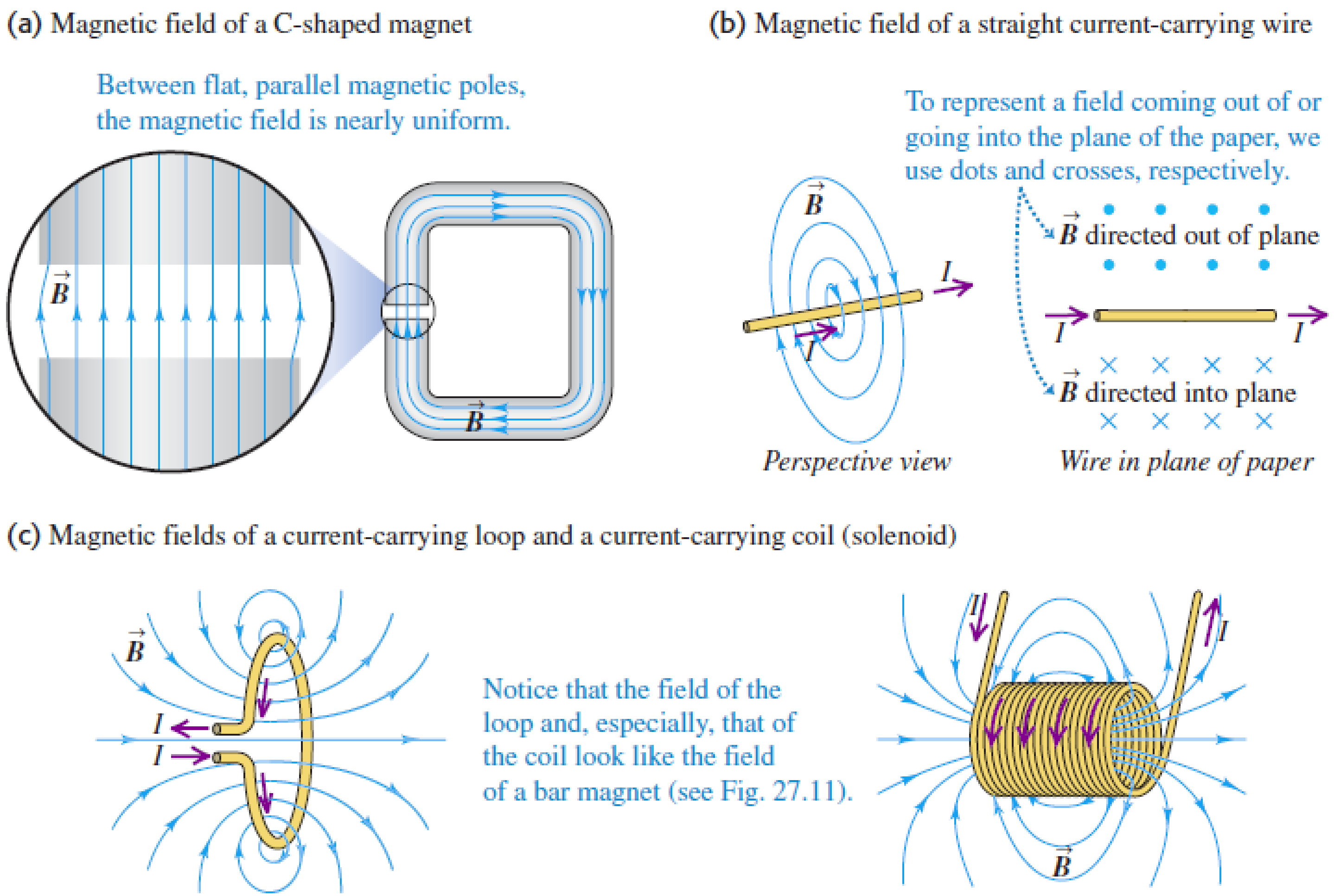




Magnetic flux



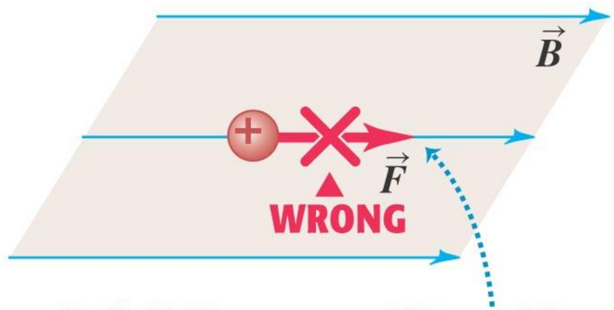
Magnetic flux : closed loop



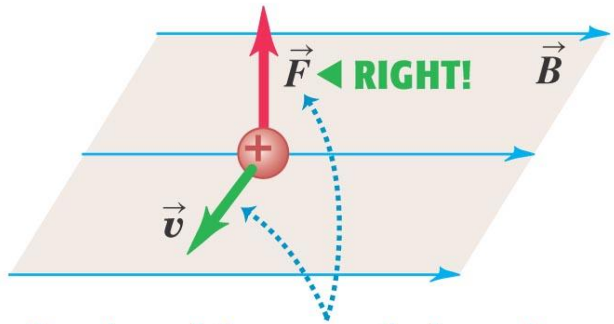
Magnetic field lines are *not* lines of force

- It is important to remember that magnetic field lines are *not* lines of magnetic force. (See Figure 27.12 below.)

Not for a charge
But for a small magnet



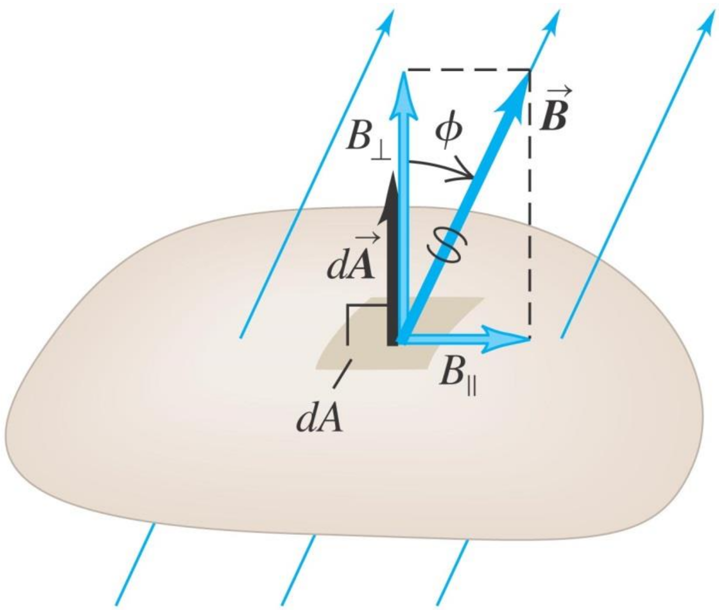
Magnetic field lines are *not* "lines of force." The force on a charged particle is not along the direction of a field line.



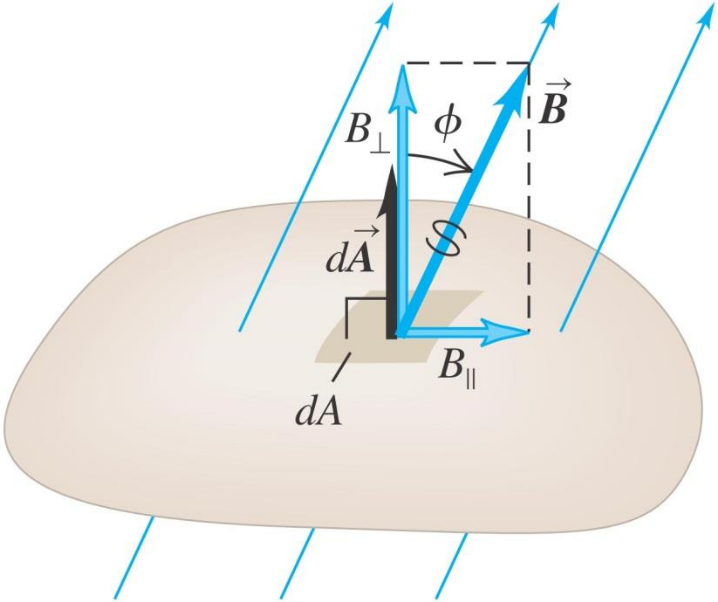
The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

Magnetic flux

- We define the *magnetic flux* through a surface just as we defined electric flux. See Figure 27.15 below.
- Follow the discussion in the text of magnetic flux and Gauss's law for magnetism.
- The magnetic flux through any closed surface is zero.



Magnetic flux

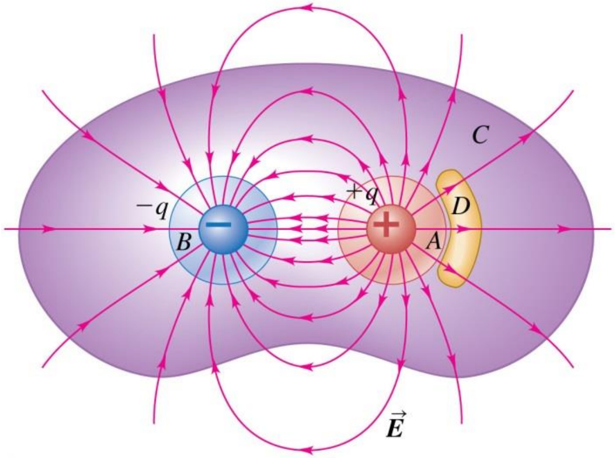


$$d\Phi_B = B_\perp \, dA = B\cos\phi \, dA = \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \int B_\perp \, dA = \int B\cos\phi \, dA = \int \vec{B} \cdot d\vec{A} \quad \text{(magnetic flux through a surface)}$$

CAUTION Magnetic field lines have no ends

No Magnetic monopole



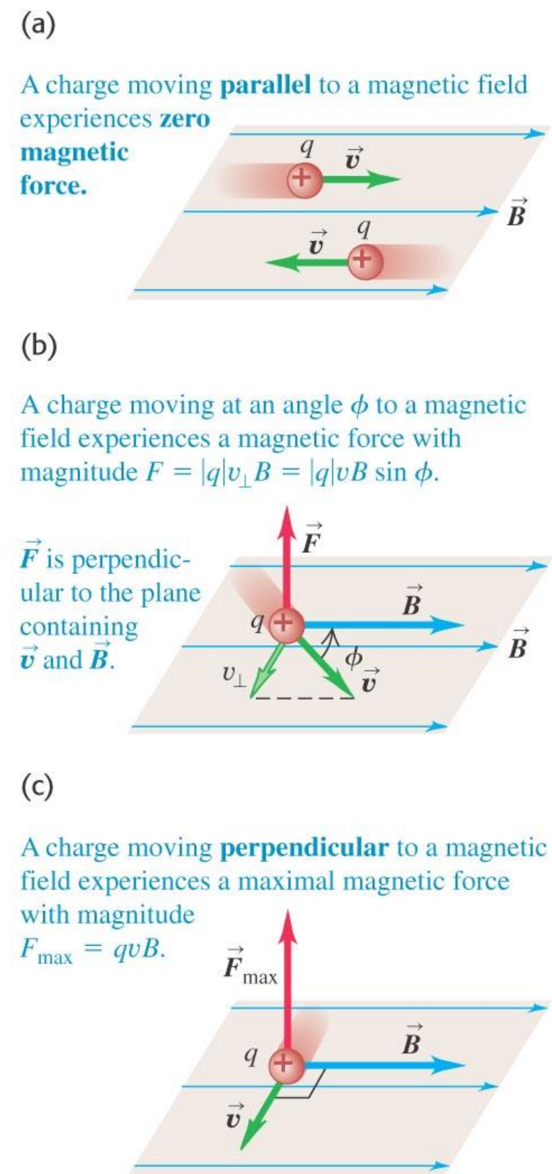
The total magnetic flux through a closed surface is always zero.

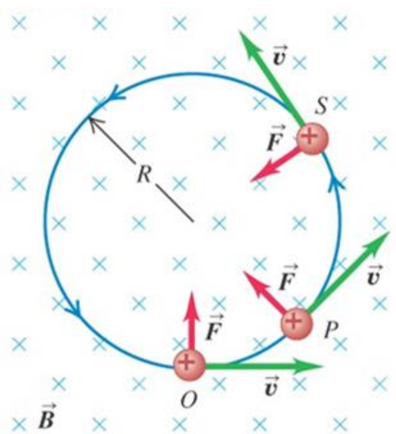
Symbolically,

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{(magnetic flux through any closed surface)}$$

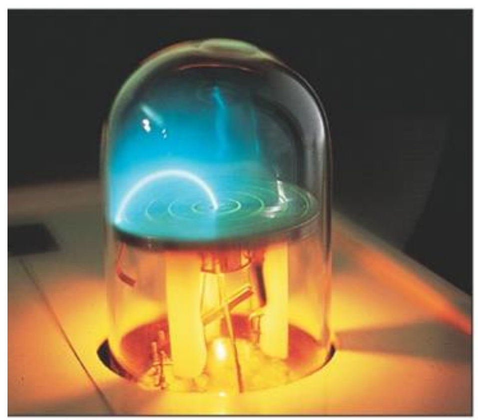
The magnetic force on a moving charge

- The magnetic force on q is perpendicular to *both* the velocity of q and the magnetic field. (See Figure 27.6 at the right.)
- The magnitude of the magnetic force is $F = |q|vB \sin\phi$.





(b) An electron beam (seen as a blue arc) curving in a magnetic field



$$\vec{F} = q\vec{v} \times \vec{B} \qquad \text{(magnetic force on a moving charged particle)} \quad (27.2)$$

The magnetic force on a moving charge

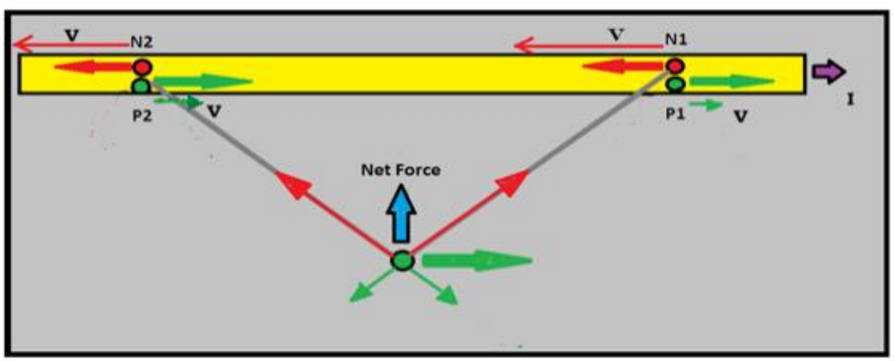
Relationship between electricity and magnetism [edit]

> One part of the force between moving charges we call the magnetic force. It is really one aspect of an electrical effect.
>
> —Richard Feynman[8]

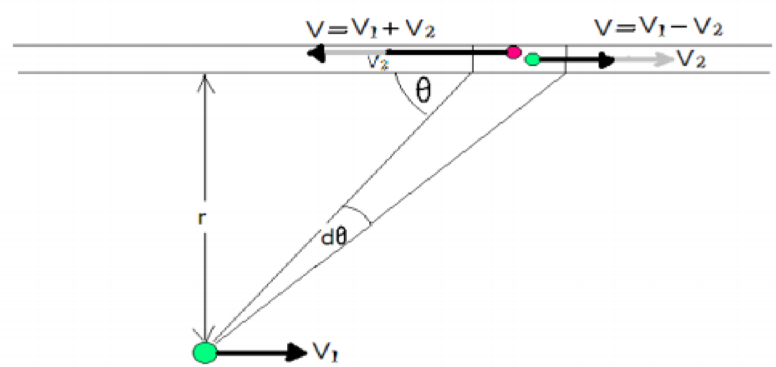
Deriving magnetism from electrostatics [edit]

Main article: Relativistic electromagnetism

The chosen reference frame determines if an electromagnetic phenomenon is viewed as an effect of electrostatics or magnetism. Authors usually derive magnetism from electrostatics when special relativity and charge invariance are taken into account. The Feynman Lectures on Physics (vol. 2, ch. 13-6) uses this method to derive the "magnetic" force on a moving charge next to a current-carrying wire. See also Haskell[9] and Landau.[10]



The magnetic force of an infinitely long straight wire for non-relativistic speeds can be obtained by applying the formulation:

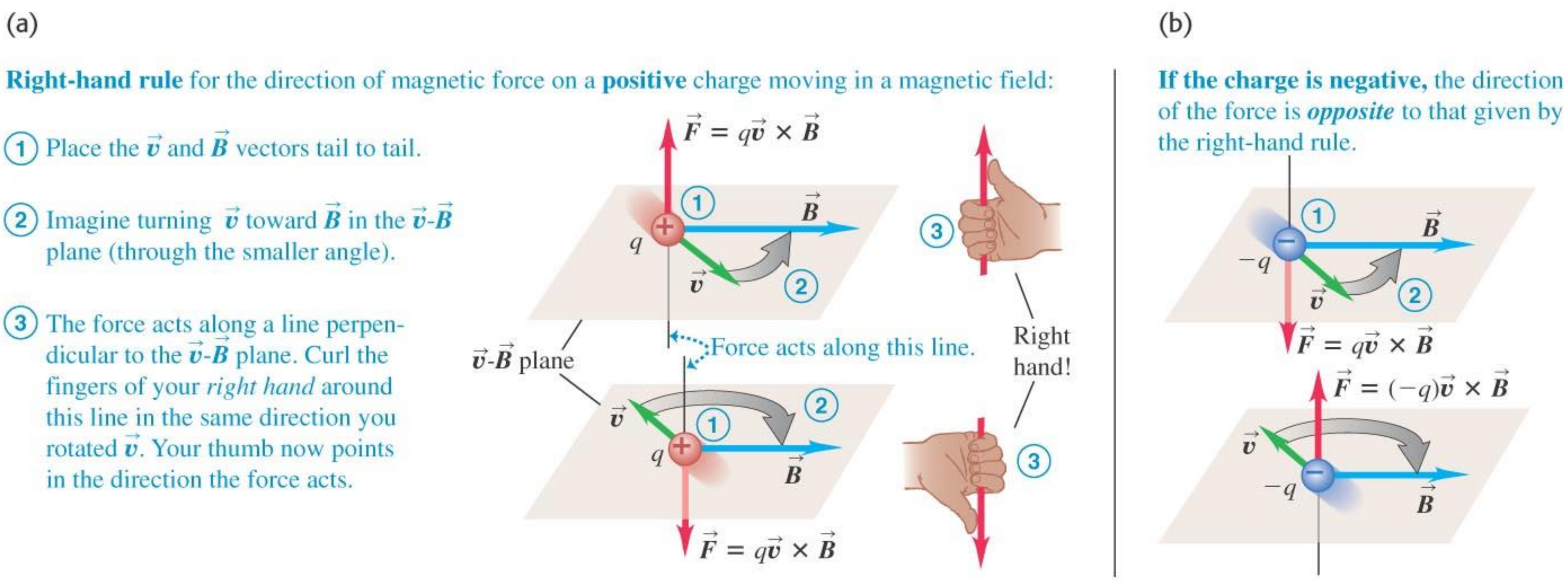


The force acting on the test charge is :

$$dF = k\,\frac{q\rho A}{r^2}\,2\sin\theta\,d\theta\left(\sqrt{\frac{c^2-(v_1+v_2)^2(\sin\theta)^2}{c^2-(v_1+v_2)^2}} - \sqrt{\frac{c^2-(v_1-v_2)^2(\sin\theta)^2}{c^2-(v_1-v_2)^2}}\right)$$

Magnetic force as a vector product

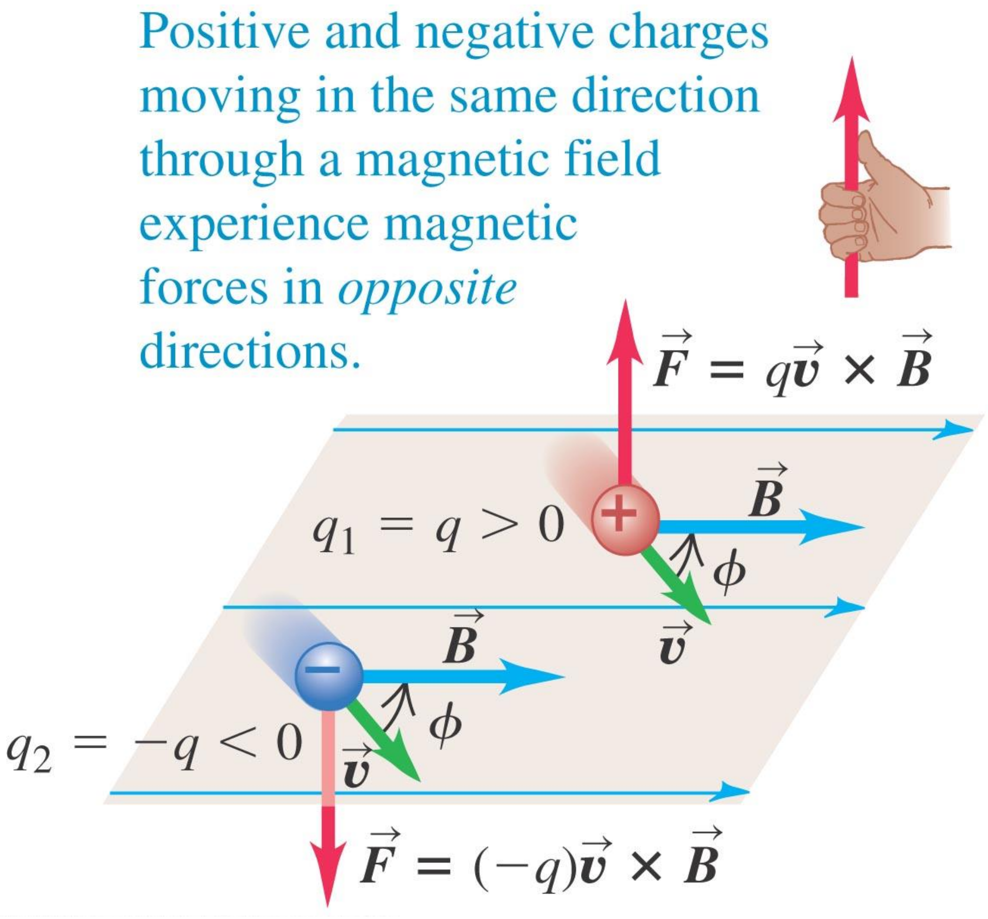
- We can write the magnetic force as a vector product (see Figure 27.7 below).
- The right-hand rule gives the direction of the force on a *positive* charge.



$$\vec{F} = q\vec{v} \times \vec{B} \qquad \text{(magnetic force on a moving charged particle)} \quad (27.2)$$

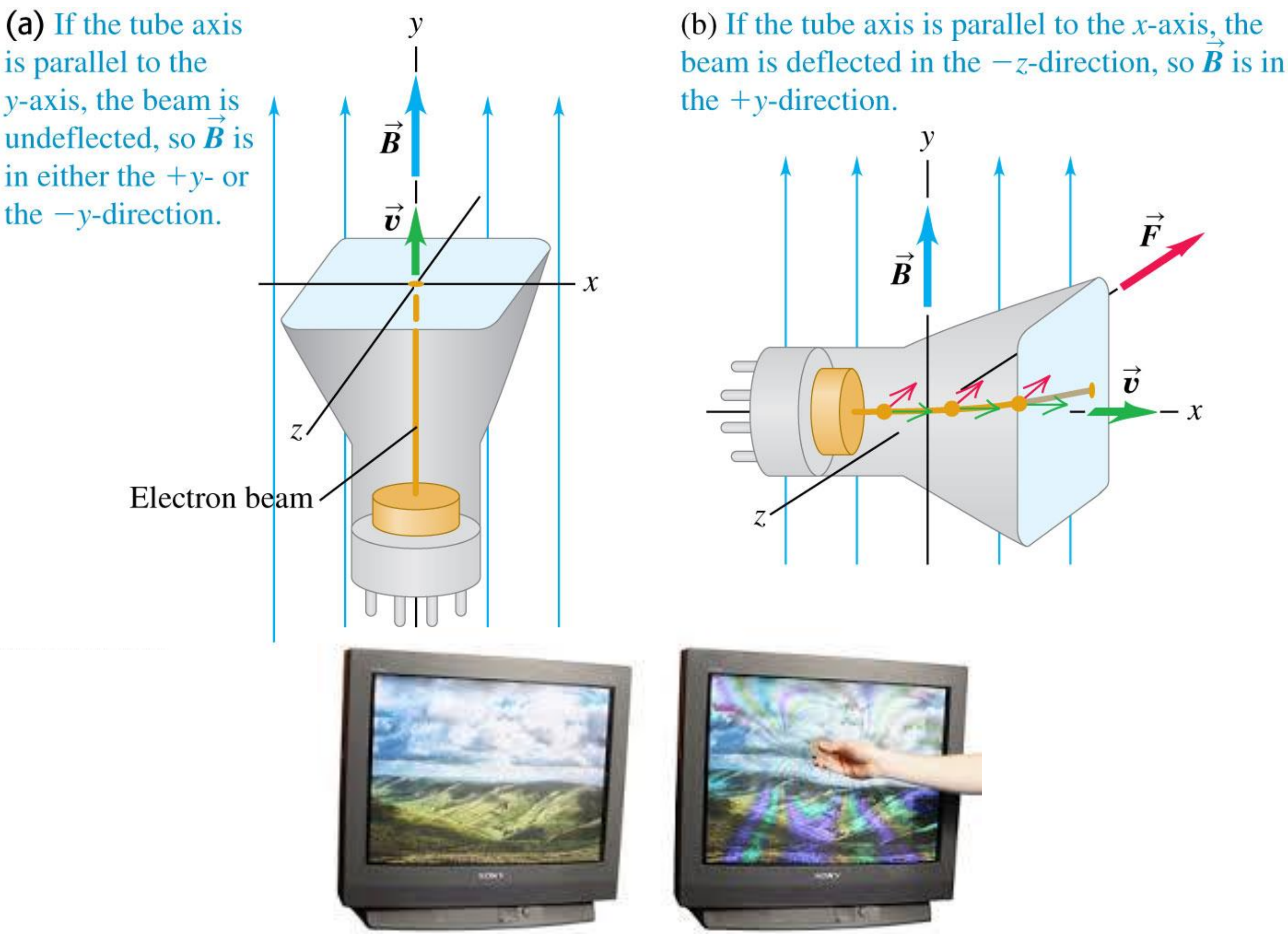
Equal velocities but opposite signs

- Two charges of equal magnitude but opposite signs moving in the same direction in the same field will experience magnetic forces in opposite directions. (See Figure 27.8 below.)



Determining the direction of a magnetic field

- A cathode-ray tube can be used to determine the direction of a magnetic field, as shown in Figure 27.9 below.



Motion of charged particles in a magnetic field

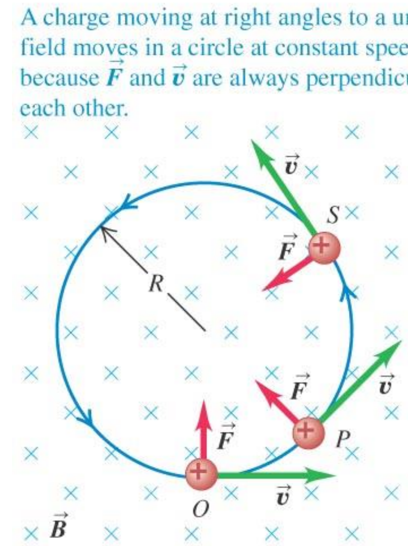
- A charged particle in a magnetic field always moves with constant speed.
- If the velocity of the particle is perpendicular to the magnetic field, the particle moves in a circle of radius $R = mv/|q|B$.

$$F = |q|vB = m\frac{v^2}{R}$$

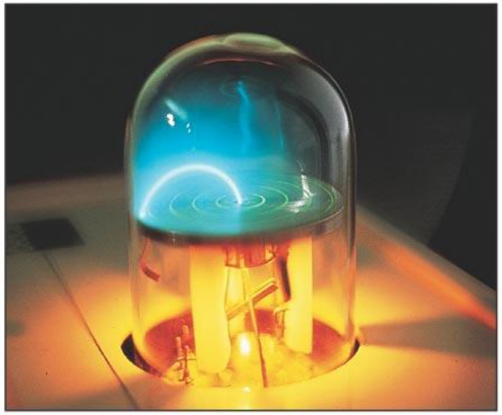
- The number of revolutions of the particle per unit time is the *cyclotron frequency*.

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}$$

(a) The orbit of a charged particle in a uniform magnetic field



(b) An electron beam (seen as a blue arc) curving in a magnetic field

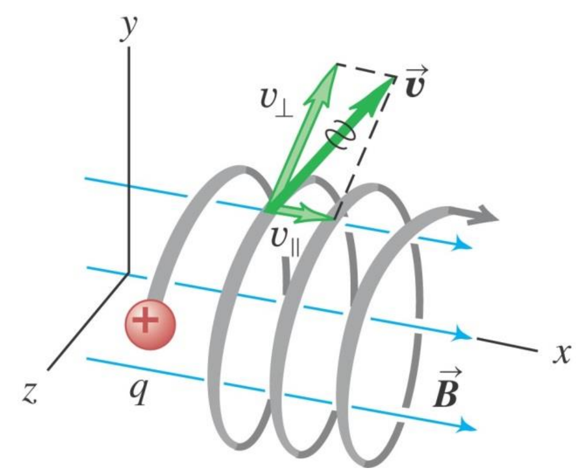


Helical motion

- If the particle has velocity components parallel to and perpendicular to the field, its path is a *helix*. (See Figure 27.18 at the right.)

- The speed and kinetic energy of the particle remain constant.

This particle's motion has components both parallel ($v_{\parallel}$) and perpendicular ($v_{\perp}$) to the magnetic field, so it moves in a helical path.



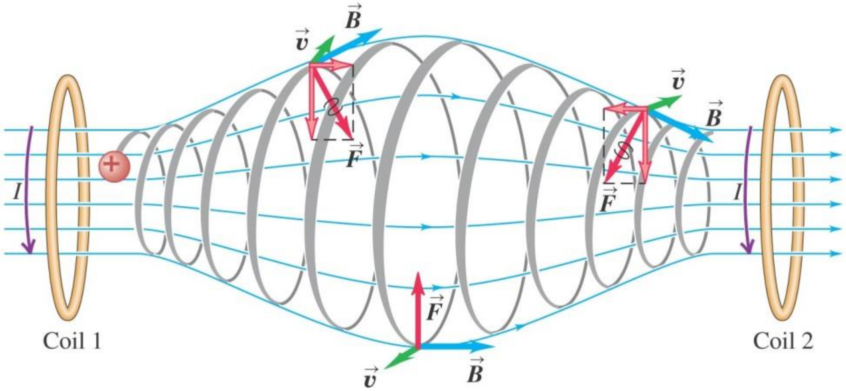
Lorentz force: $\boldsymbol{F} = -e\boldsymbol{v} \times \boldsymbol{B}$

$F = m_e v^2/r = evB$

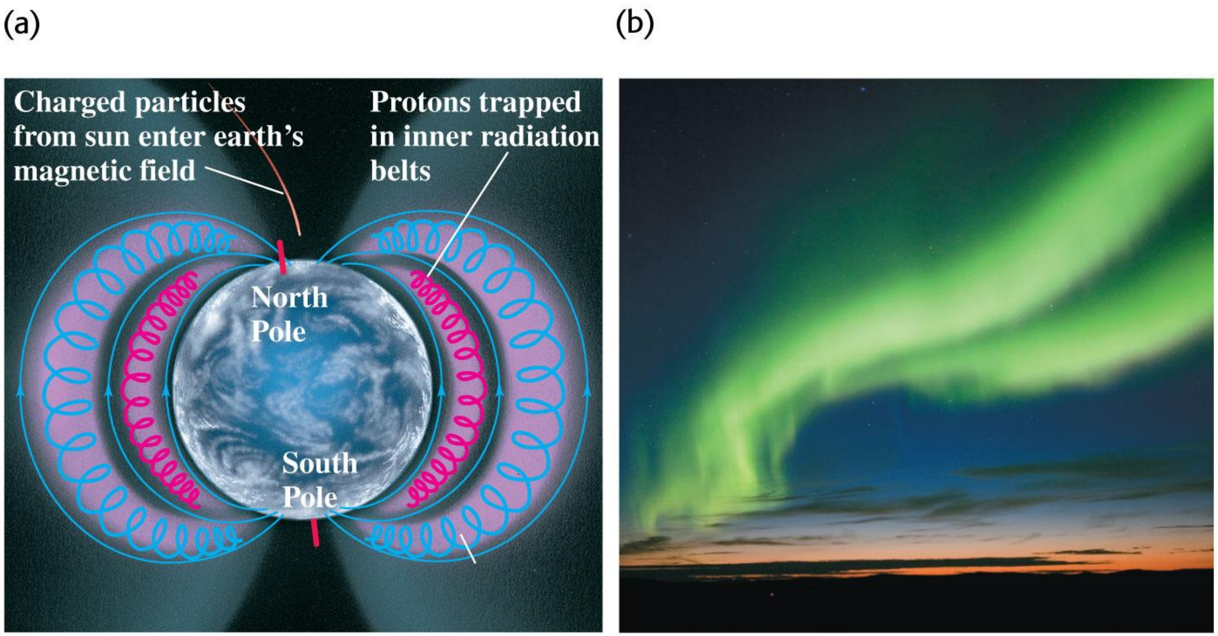
Cyclotron frequency $f_c = v/2\pi r = eB/ 2\pi m_e$

Cyclotron radiation

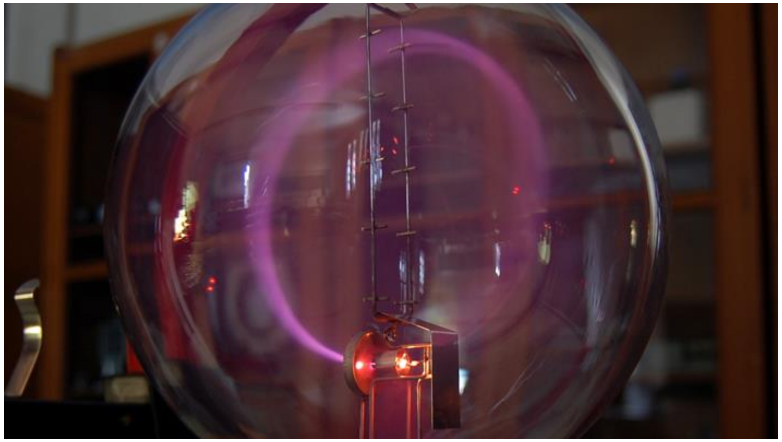
- Figure 27.19 at the right shows charges trapped in a *magnetic bottle*, which results from a nonuniform magnetic field.

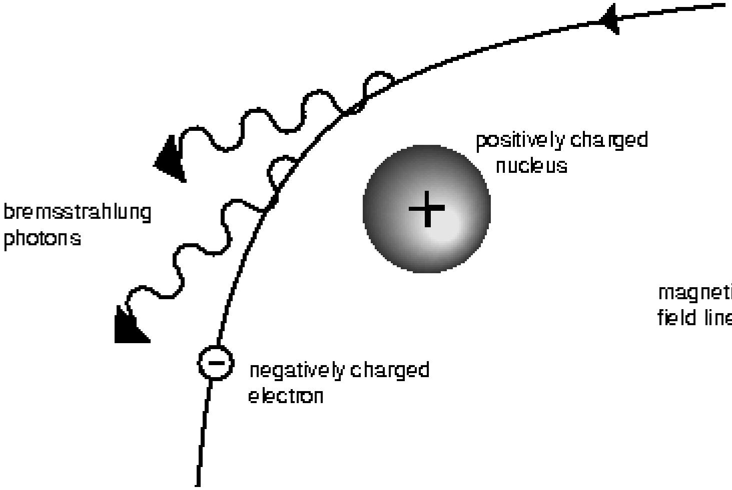


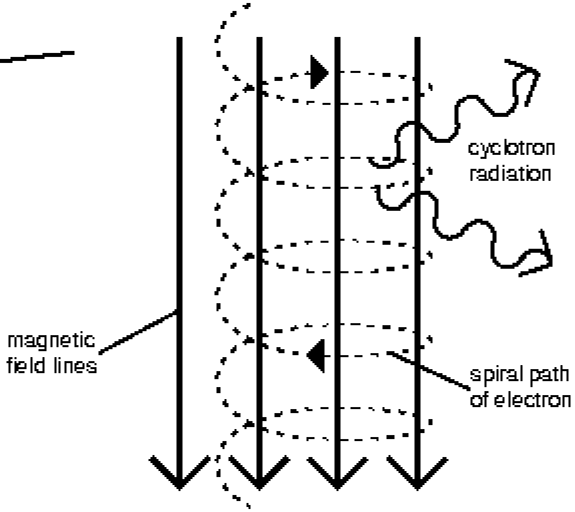
- Figure 27.20 below shows the Van Allen radiation belts and the resulting aurora. These belts are due to the earth's nonuniform field.



A nonuniform magnetic field







$$\frac{-dE}{dt} = \frac{\sigma_t B^2 V^2}{c\mu_o}$$

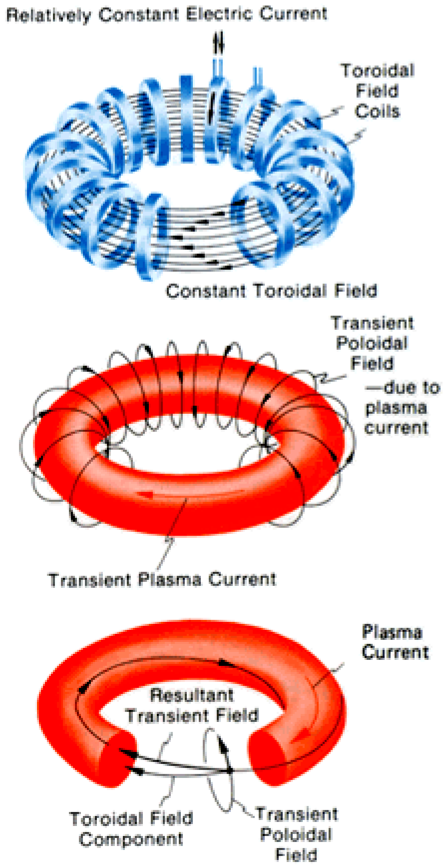
Cyclotron radiation

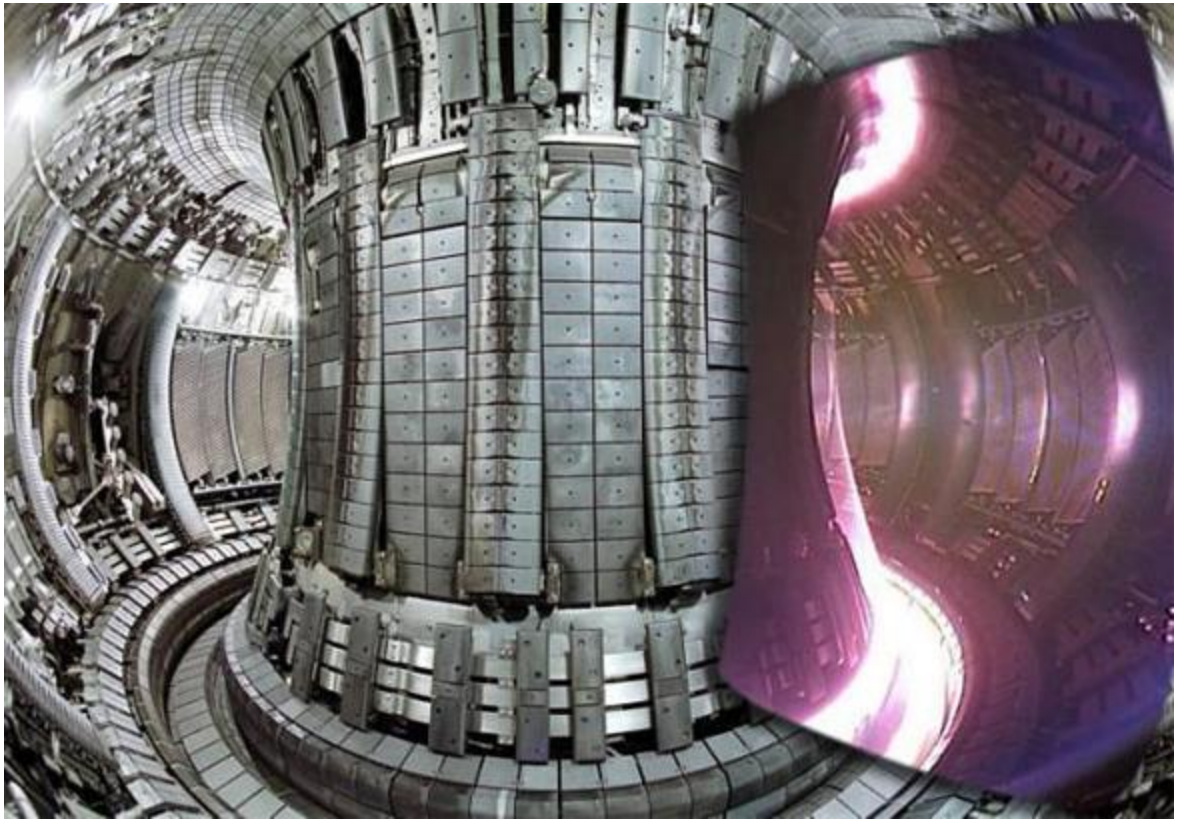
A nonuniform magnetic field

$$\frac{-dE}{dt} = \frac{\sigma_t B^2 V^2}{c\mu_o}$$

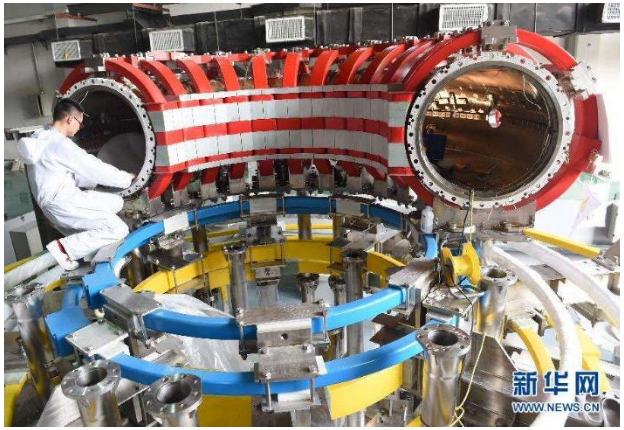
Cyclotron radiation

Nuclear Fusion: Tokamak Scheme



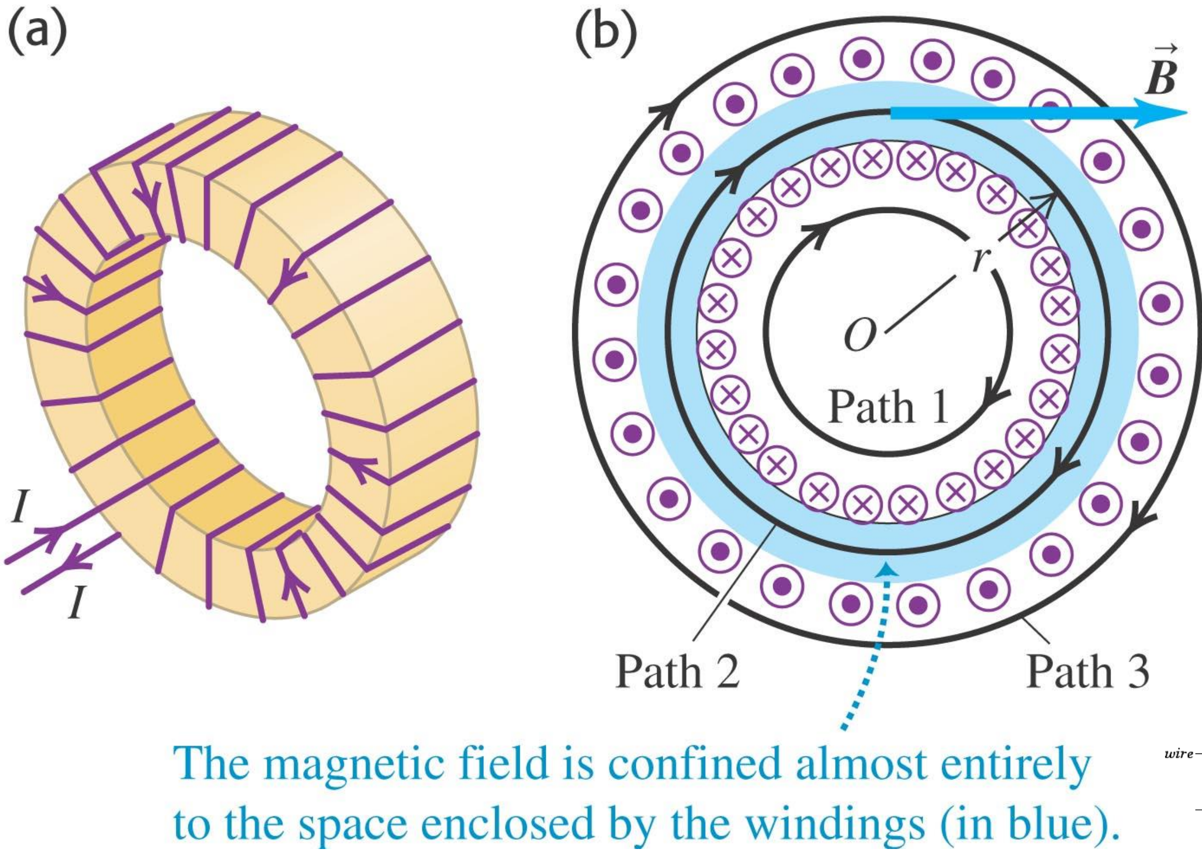


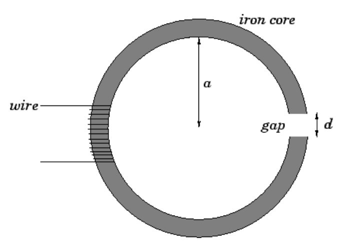
Nuclear Fusion: Tokamak Scheme

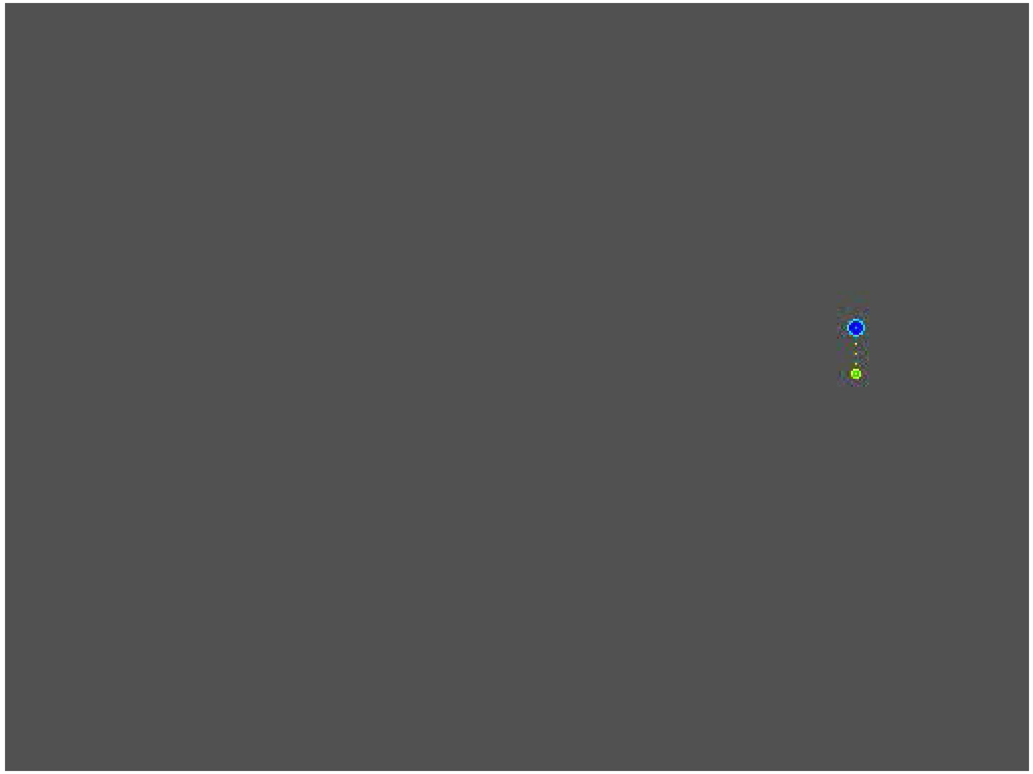


Field of a toroidal solenoid

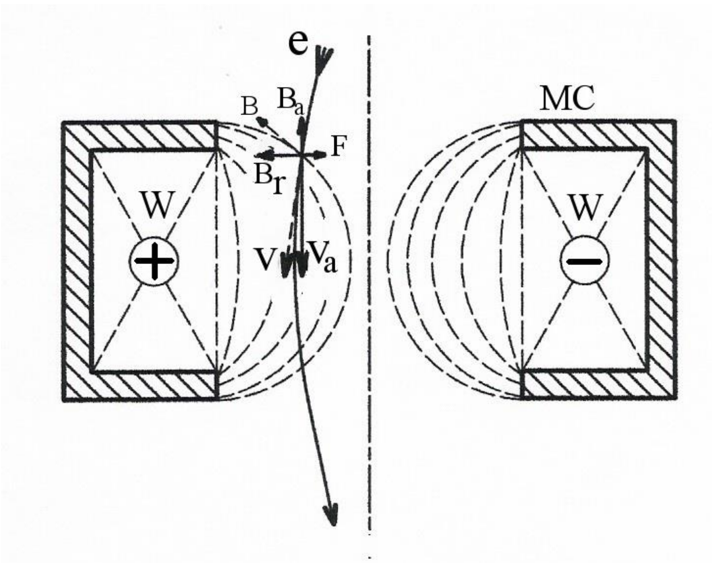
- A *toroidal solenoid* is a doughnut-shaped solenoid.
- Follow Example 28.10 using Figure 28.25 below.

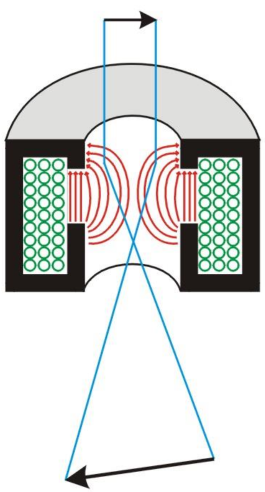


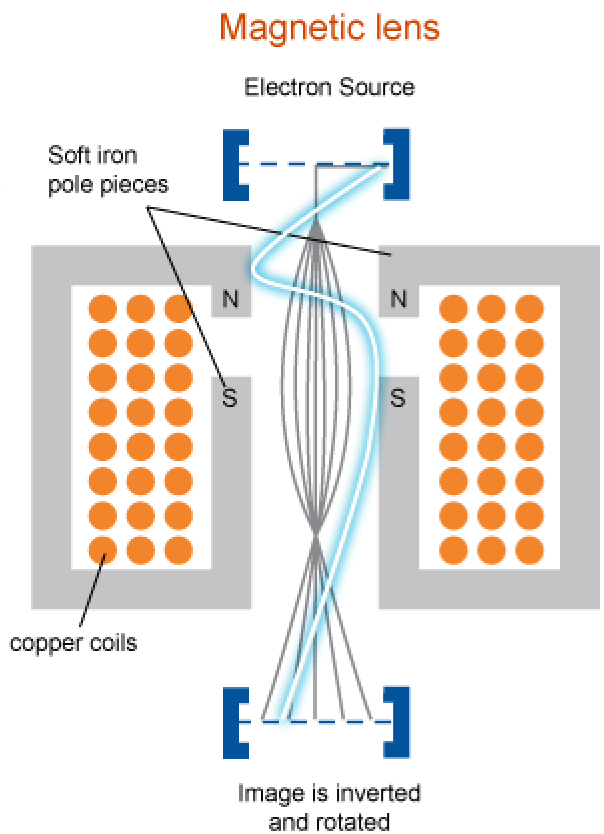


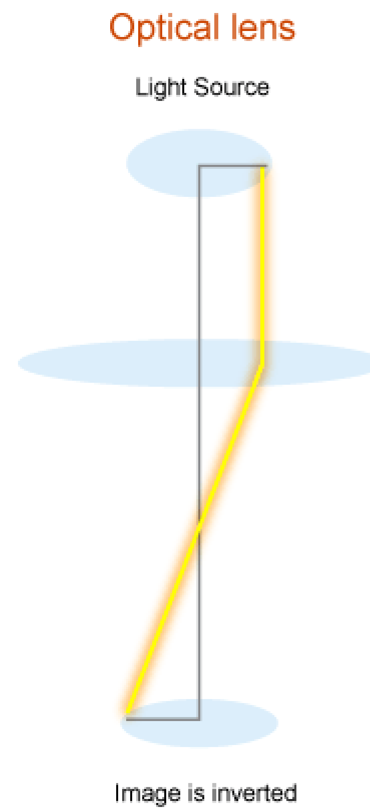


Magnetic Lens



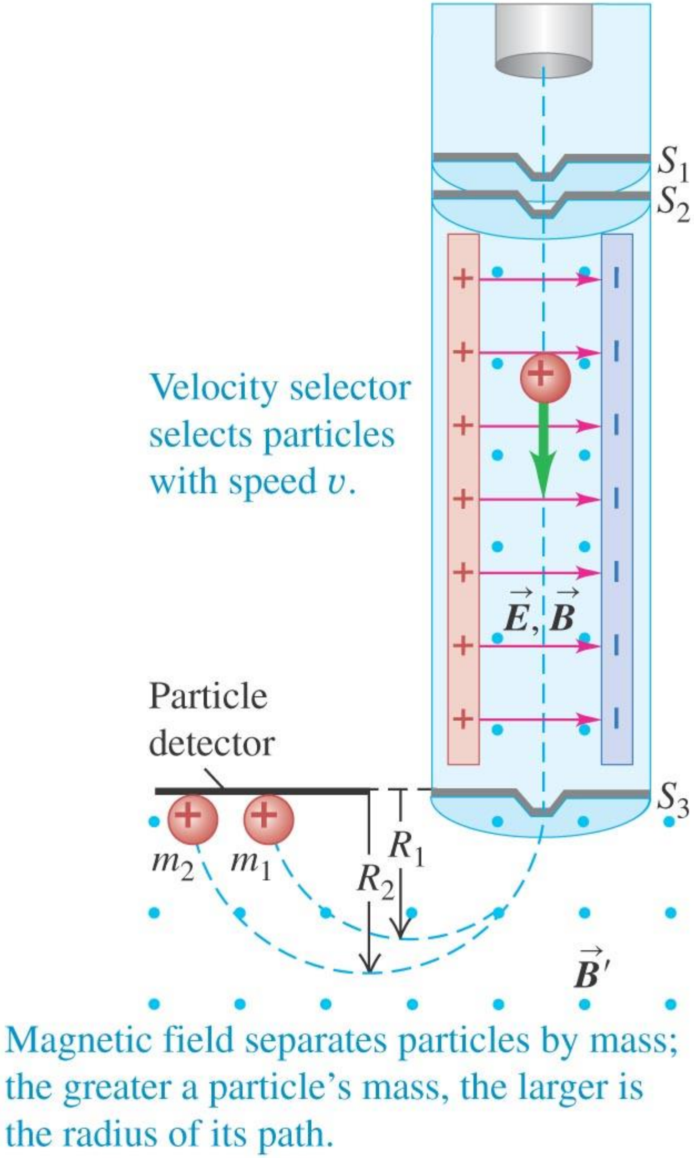






Mass spectrometer

- A *mass spectrometer* measures the masses of ions.
- The Bainbridge mass spectrometer (see Figure 27.24 at the right) first uses a velocity selector. Then the magnetic field separates the particles by mass.
- Follow Example 27.5.
- Follow Example 27.6.



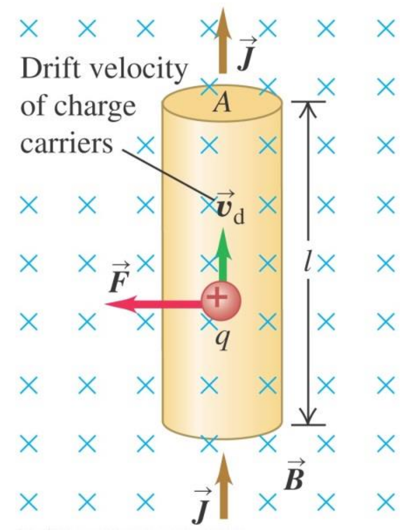
The magnetic force on a current-carrying conductor

- Figure 27.25 (top) shows the magnetic force on a moving positive charge in a conductor.
- Figure 27.26 (bottom) shows that the magnetic force is perpendicular to the wire segment and the magnetic field.
- Follow the discussion of the magnetic force on a conductor in the text.

$$F = (nAl)(qv_d B) = (nqv_d A)(lB)$$

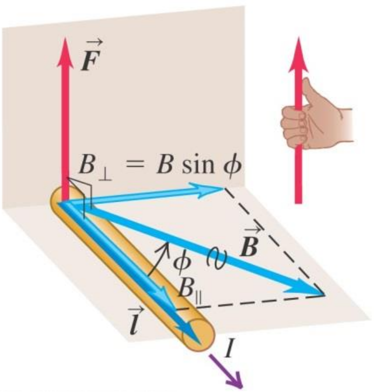
is $J = nqv_d$.

$$F = IlB$$



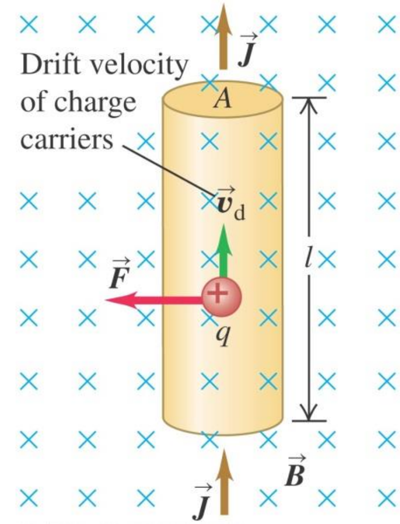
Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_\perp = IlB \sin \phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



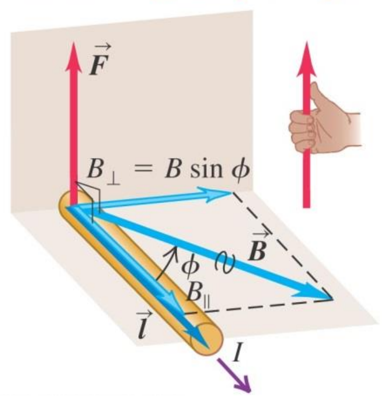
The magnetic force on a current-carrying conductor

- Figure 27.25 (top) shows the magnetic force on a moving positive charge in a conductor.
- Figure 27.26 (bottom) shows that the magnetic force is perpendicular to the wire segment and the magnetic field.
- Follow the discussion of the magnetic force on a conductor in the text.



Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_\perp = IlB \sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



$$F = (nAl)(qv_d B) = (nqv_d A)(lB)$$

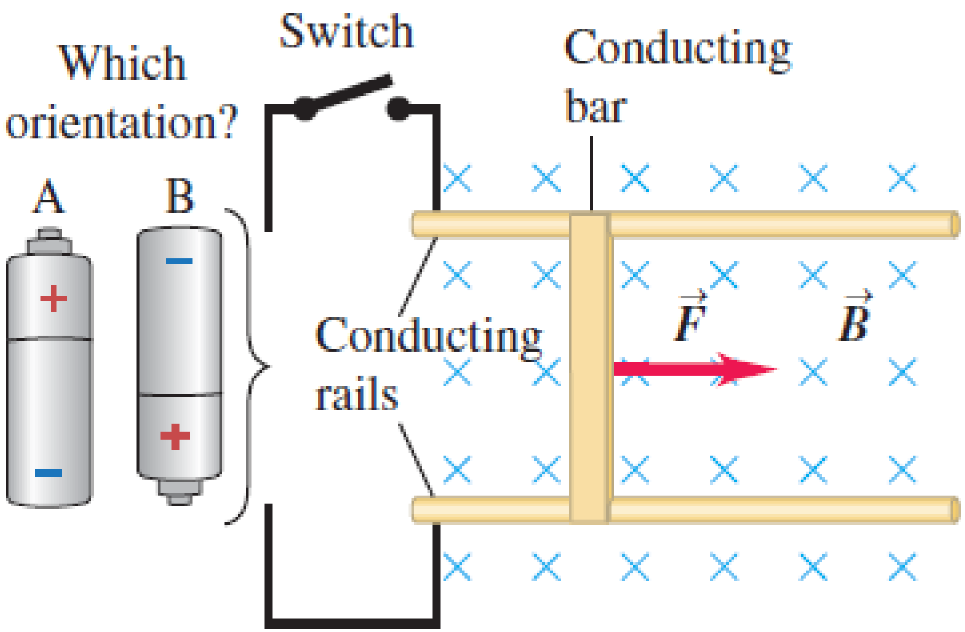
is $J = nqv_d$.

$$F = IlB$$

$$\vec{F} = I\vec{l} \times \vec{B} \qquad \text{(magnetic force on a straight wire segment)}$$

Magnetic force on a curved conductor

- Follow Example 27.8 using Figure 27.30 below.

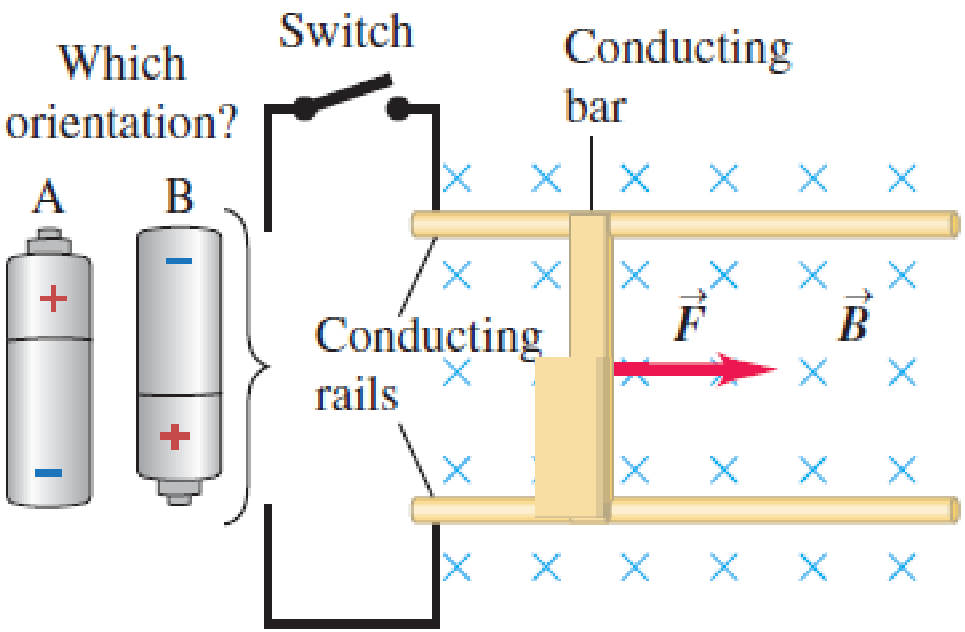


$$F = IlB$$

$$\vec{F} = I\vec{l} \times \vec{B} \qquad \text{(magnetic force on a straight wire segment)}$$

Magnetic force on a curved conductor

- Follow Example 27.8 using Figure 27.30 below.

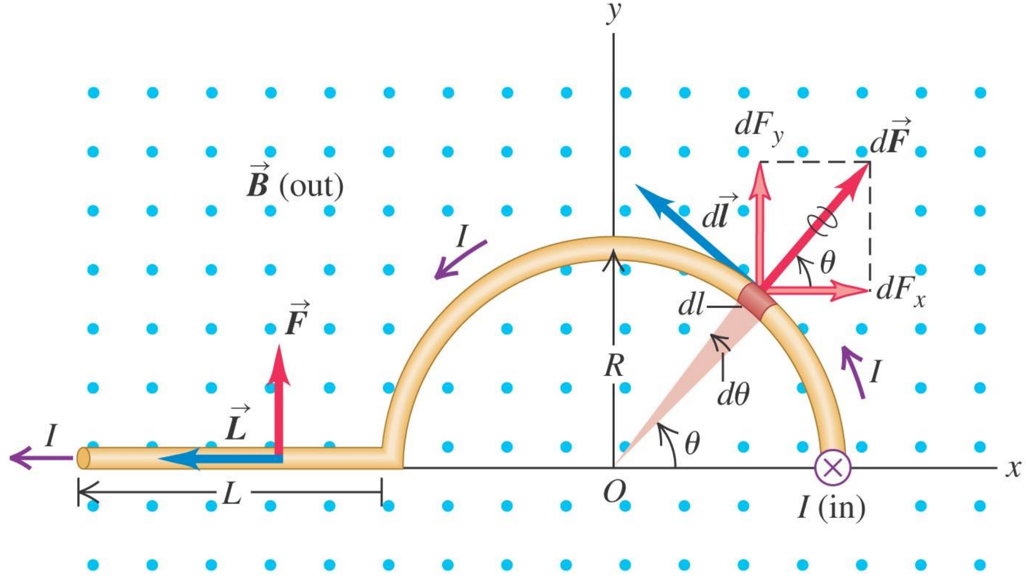


$$F = IlB$$

$\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment)

Magnetic force on a curved conductor

- Follow Example 27.8 using Figure 27.30 below.

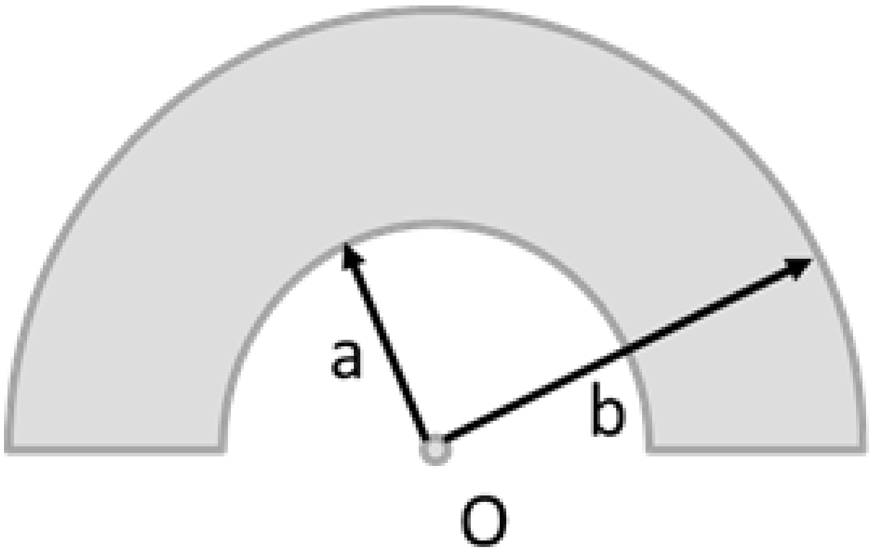


$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$

$$dF_{2x} = IR\,d\theta\,B\cos\theta \qquad dF_{2y} = IR\,d\theta\,B\sin\theta$$

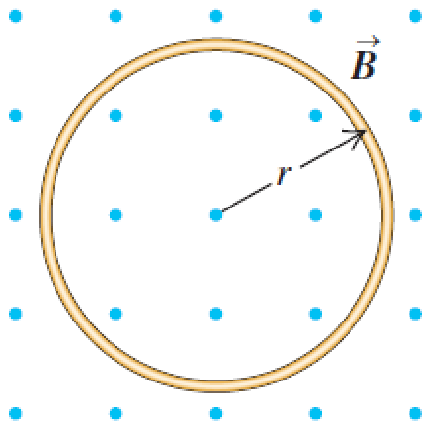
$$F_{2x} = IRB\int_0^{\pi}\cos\theta\,d\theta = 0$$

$$F_{2y} = IRB\int_0^{\pi}\sin\theta\,d\theta = 2IRB$$


a
b
O

Magnetic force on a curved conductor

- Follow Example 27.8 using Figure 27.30 below.



$$\boxed{d\vec{F} = I\,d\vec{l} \times \vec{B}}$$

$$dF_{2x} = IR\,d\theta\,B\cos\theta \qquad dF_{2y} = IR\,d\theta\,B\sin\theta$$

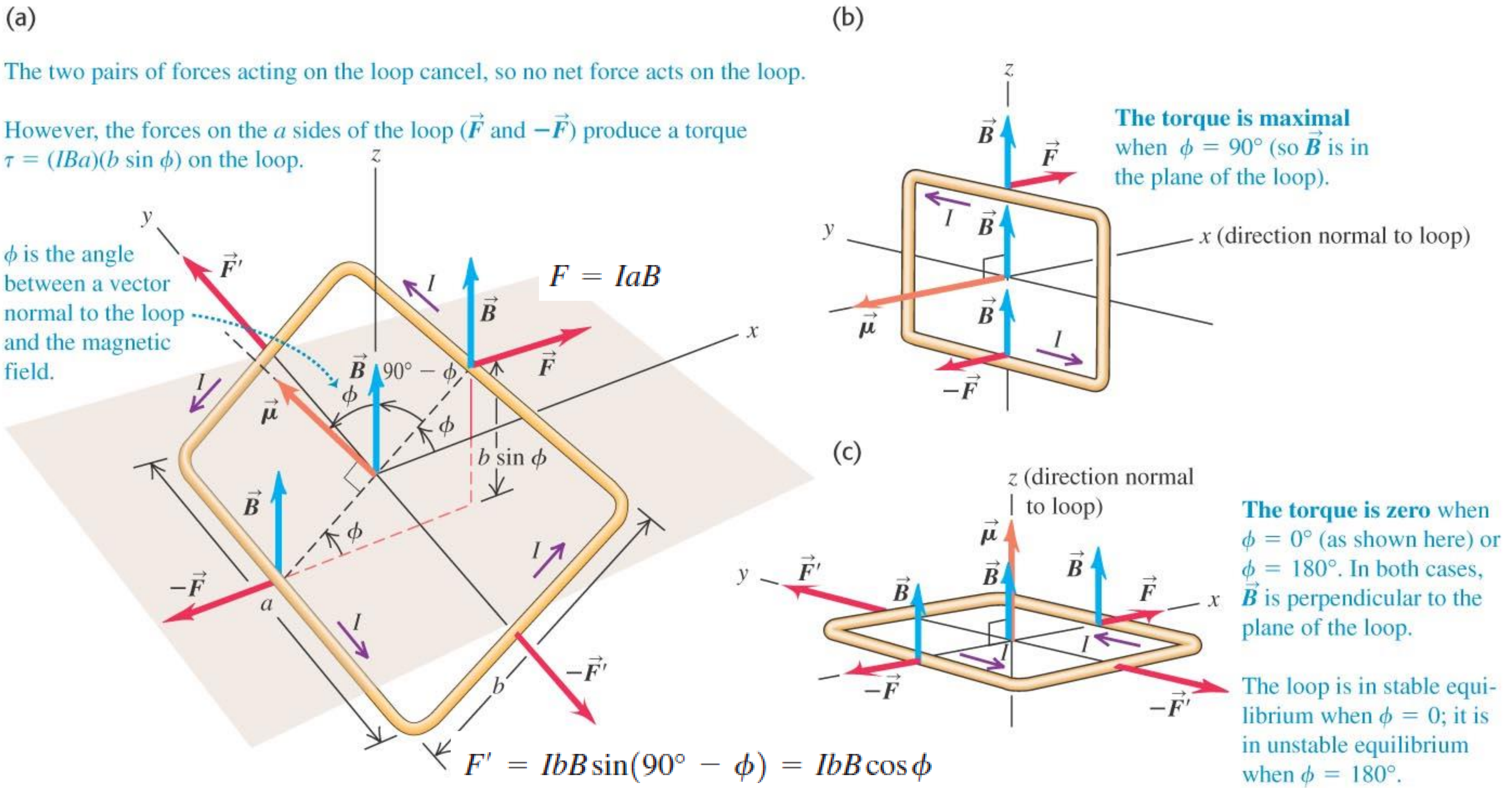
$$F_{2x} = IRB\int_0^{\pi}\cos\theta\,d\theta = 0$$

$$F_{2y} = IRB\int_0^{\pi}\sin\theta\,d\theta = 2IRB$$

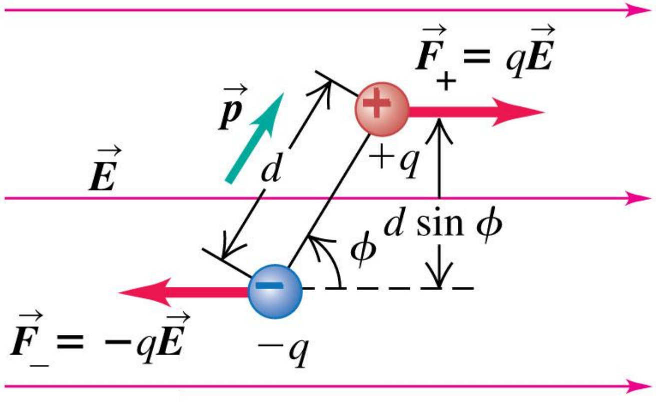
Force and torque on a current loop

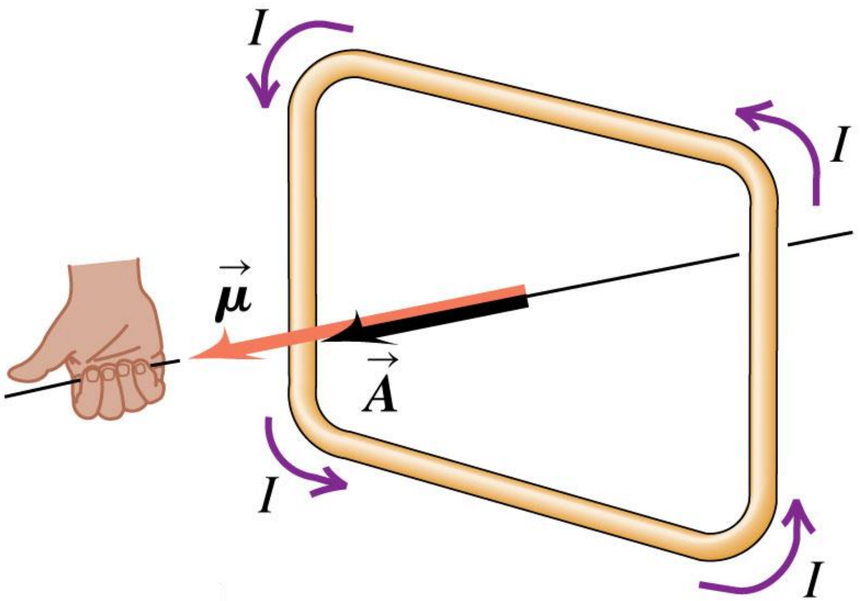
- The net force on a current loop in a uniform magnetic field is zero. But the net torque is not, in general, equal to zero.

- Figure 27.31 below shows the forces and how to calculate the torque.



Magnetic moment





: **electric dipole moment,** denoted by p:

$$p = qd \quad \text{(magnitude of electric dipole moment)}$$

$$\tau = pE\sin\phi \quad \text{(magnitude of the torque on an electric dipole)} \quad (21.15)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on an electric dipole, in vector form)} \quad (21.16)$$

$$F = IaB$$

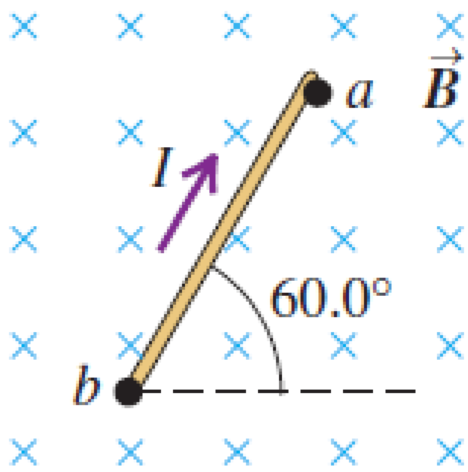
$$\tau = 2F(b/2)\sin\phi = (IBa)(b\sin\phi)$$

$$\tau = IBA\sin\phi \quad \text{(magnitude of torque on a current loop)}$$

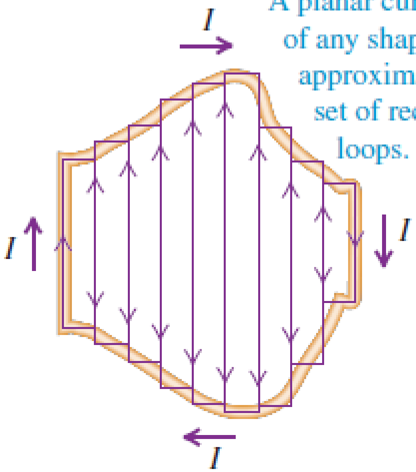
magnetic dipole moment $\quad \mu = IA$

$$\tau = \mu B\sin\phi$$

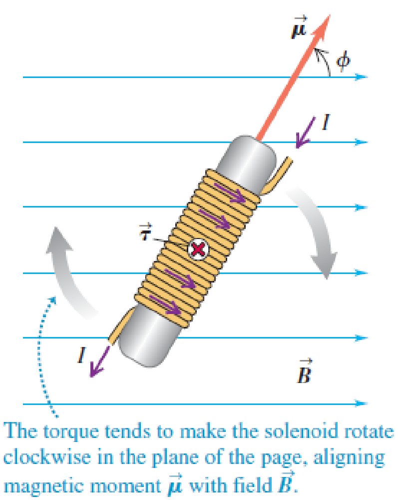
$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{(vector torque on a current loop)}$$


a
$\vec{B}$
I
60.0°
b

Magnetic Torque and coil

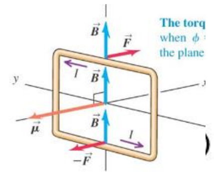


$$\vec{\mu} = I\vec{A}.$$



$$\mu = NIA$$

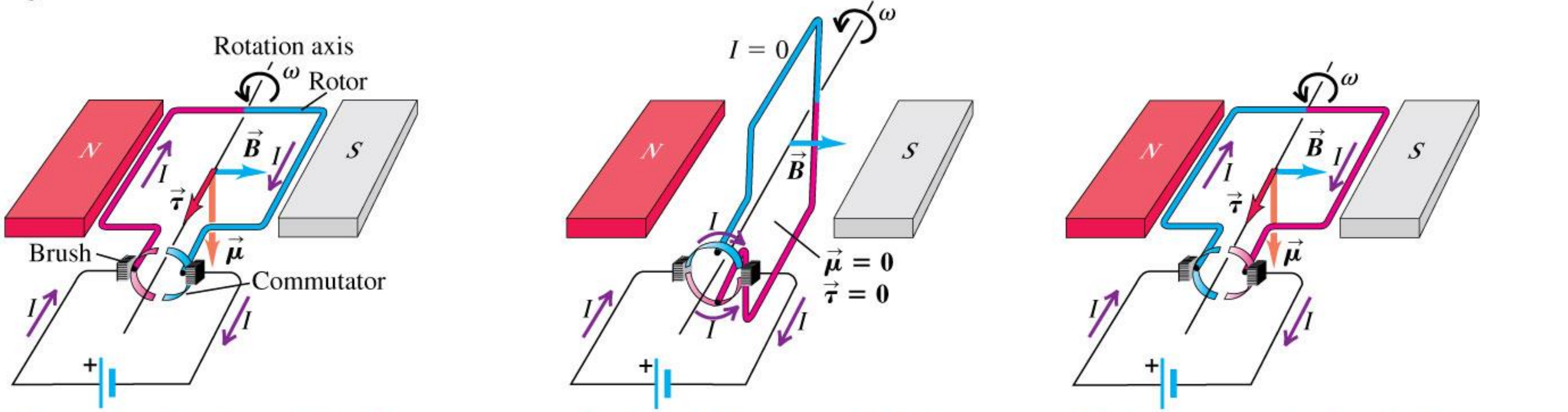
The direct-current motor



(a) Brushes are aligned with commutator segments.

(b) Rotor has turned 90°.

) Rotor has turned 180°.

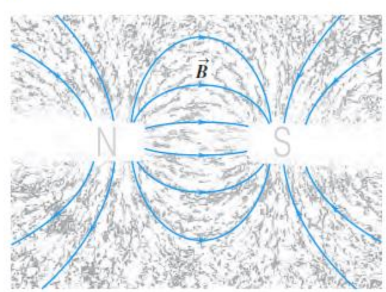


- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

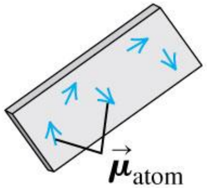
- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

(b)

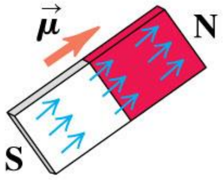


How magnets work

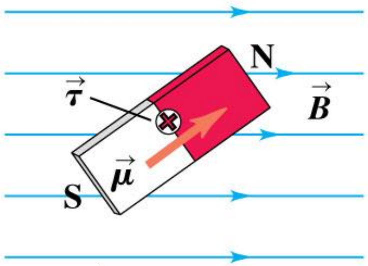
(a) Unmagnetized iron: magnetic moments are oriented randomly.

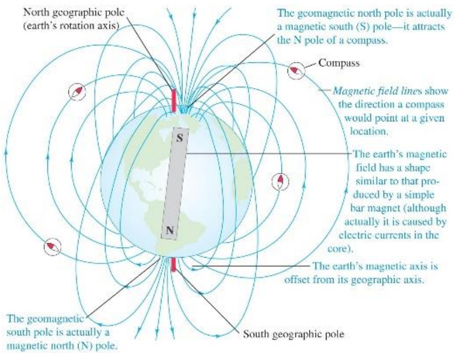


(b) In a bar magnet, the magnetic moments are aligned.



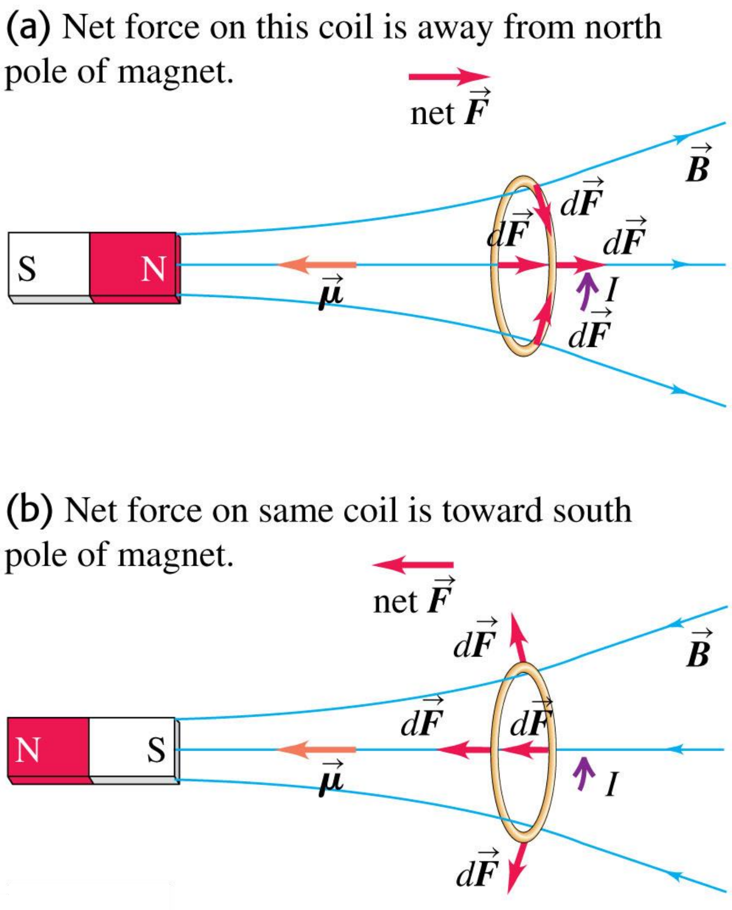
(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the $\vec{B}$ field.





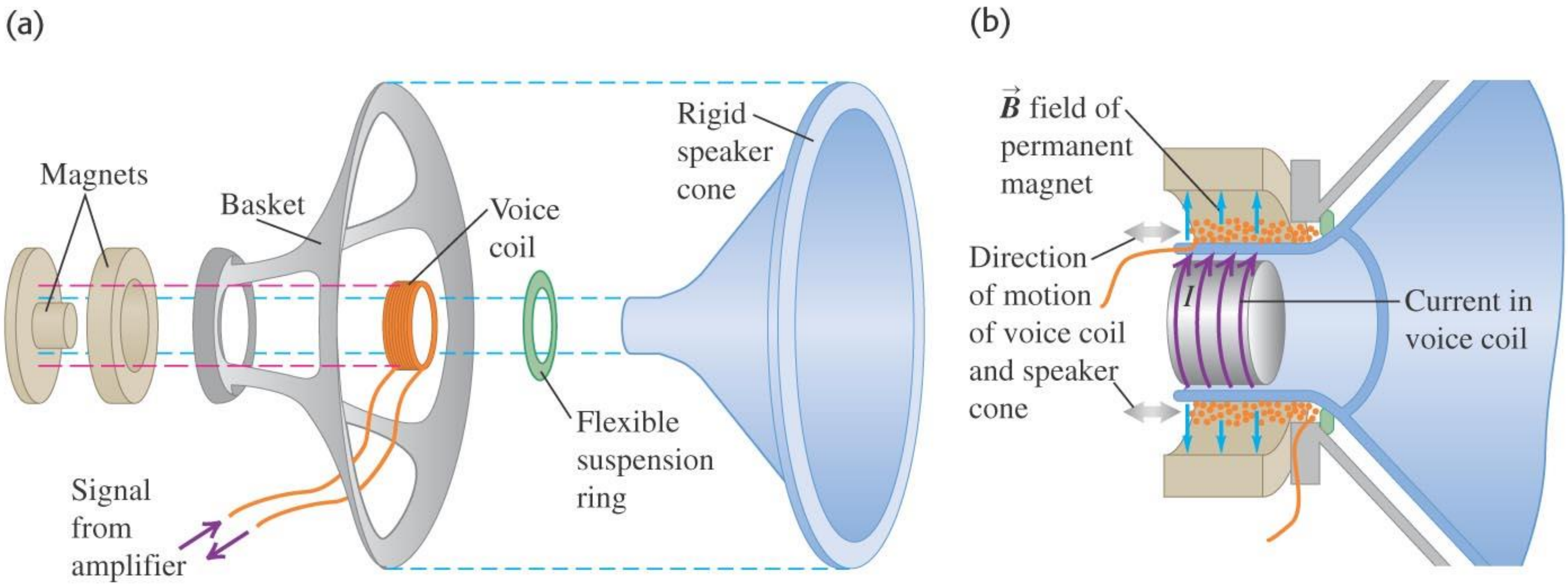
How magnets work

- Follow the discussion in the text of magnetic dipoles and how magnets work. Use Figures 27.36 (below) and 27.37 (right).



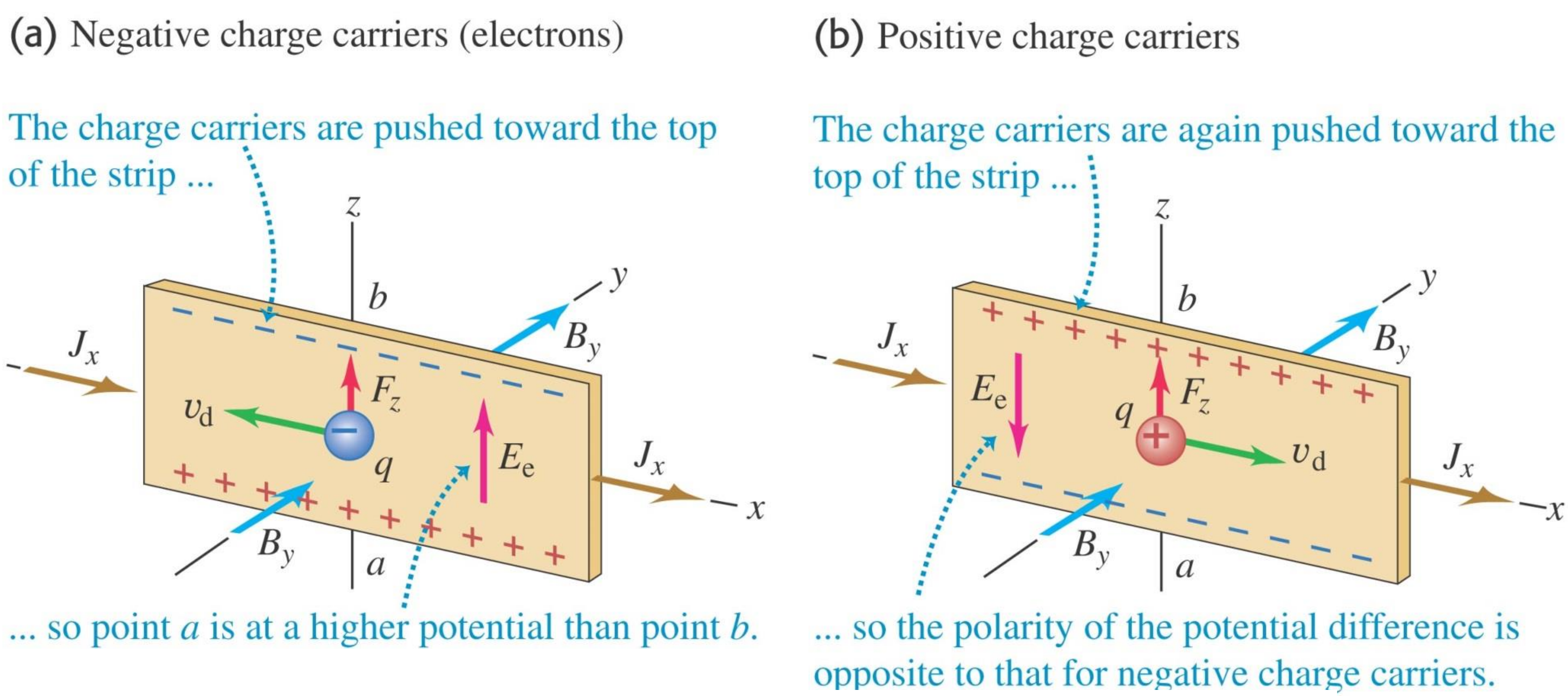
Loudspeaker

- Figure 27.28 shows a loudspeaker design. If the current in the voice coil oscillates, the speaker cone oscillates at the same frequency.



The Hall Effect

- Follow the discussion of the Hall effect in the text using Figure 27.41 below.
- Follow Example 27.12.

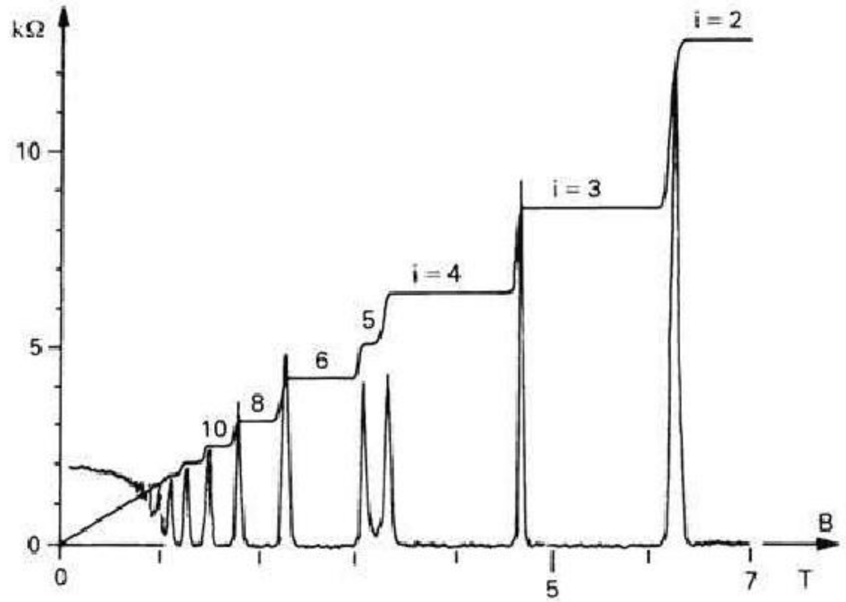


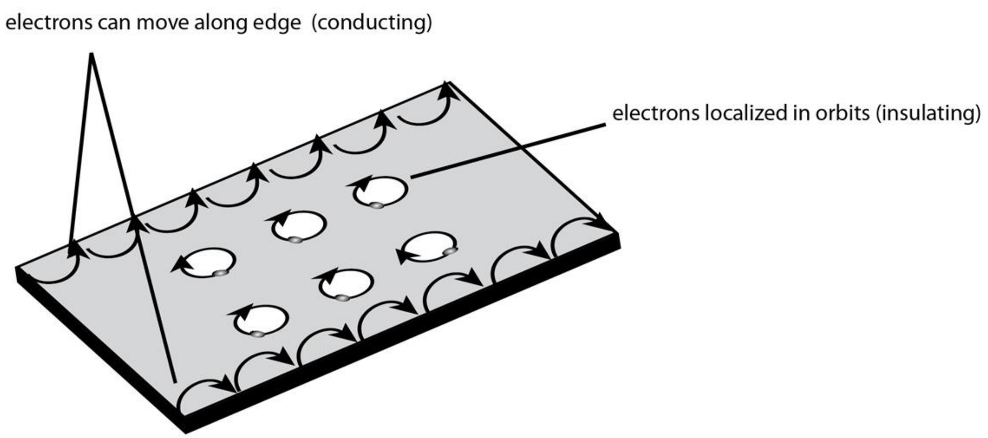
$$qE_z + qv_dB_y = 0$$

$$J_x = nqv_d$$

$$nq = \frac{-J_xB_y}{E_z} \qquad \text{(Hall effect)}$$

Quantum Hall effect





Giant Magnetoresistance