Problem 1
Consider a linear space $\mathbb{P}_2(\mathbb{R})$.

1. Prove that
$$(f_1, f_2) = f_1(-1)f_2(-1) + f_1(0)f_2(0) + f_1(1)f_2(1)$$
satisfies all the requirements for an inner product.
2. Choose any nonzero element $f \in \mathbb{P}_2(\mathbb{R})$ and calculate $||f||$.
3. Find a pair of orthonormal elements $u_1(t), u_2(t) \in \mathbb{P}_2(\mathbb{R})$. Calculate the distance between $u_1(t)$ and $u_2(t)$.
4. Let $V = span(u_1(t), u_2(t))$. Describe $V^\perp$, calculate $\dim V^\perp$ and find any nonzero element in $V^\perp$.
5. Find an element $f \in \mathbb{P}_2(\mathbb{R})$ such that $f \notin V \cup V^\perp$ and calculate its orthogonal projection onto V.

Problem 2
Find an orthonormal basis for $\mathbb{R}^3$ that contains the vector $\bar{v} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

Problem 3
Consider
$$V = span\left((1,1,1,1), (1,0,0,1), (0,2,1,-1)\right) \subset \mathbb{R}^4$$

1. Find an orthonormal basis $\mathcal{B}$ for V using the Gram-Schmidt process.
2. Find $proj_V(\bar{x}, \bar{x})$, $\bar{x} = (1,-1,0,2)$
3. Find an orthonormal basis for $V^\perp$
4. Extend the orthonormal basis $\mathcal{B}$ for V up to an orthonormal basis for $\mathbb{R}^4$.

Problem 4
Find the QR factorization of the following matrices:

$$1.\ \begin{pmatrix} 2 & 1 \\ 2 & 1 \\ 1 & 5 \end{pmatrix} \quad 2.\ \begin{pmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 7 \end{pmatrix} \quad 3.\ \begin{pmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Problem 5
Find the least-squares solutions of the following systems and determine the errors $||\bar{b} - A\bar{x}^\star||$:

$$1.\ \begin{pmatrix} 6 & 9 \\ 3 & 8 \\ 2 & 10 \end{pmatrix} \bar{x} = \begin{pmatrix} 0 \\ 49 \\ 0 \end{pmatrix} \qquad 2.\ \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \bar{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Problem 6

Bretscher 5.4.10, 5.4.31, 5.4.32, pp.230-231 (review pp.225-227 first)

Problem 7
Bretscher 5.5.32, 5.5.33, p.246