Problem Set 4

Due: 11 June 2019, 12.30 p.m.

Problem 1. We have seen in class that the general solution of the equation of motion of a simple (neither driven, nor damped) harmonic oscillator can be expressed in the form $x(t) = A\cos(\omega_0 t + \varphi)$, where A is the amplitude and φ is the phase shift of the oscillations to be determined from the initial conditions.

(a) The linear combination $x(t) = B\cos\omega_0 t + C\sin\omega_0 t$, where B and C are real constants, is another possible representation of the general solution. Check that these two forms are equivalent, i.e. express B and C in terms of A and φ or *vice–versa*.

(b) Show that given the initial conditions $x(0) = x_0$ and $v(0) = v_0$ the amplitude and the phase shift can be found as

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}, \qquad \varphi = \arctan\left(-\frac{v_0}{\omega_0 x_0}\right),$$

respectively.

(*2 + 2 points*)

Problem 2. Consider a cylinder of weight w and cross-sectional area S that is floating upright, partially submerged, in a liquid. If we push it a little deeper it will start to oscillate. Neglecting fluid viscosity, show that the cylinder follows simple harmonic motion. Find the density of the liquid, if the period of oscillations is T. Acceleration due to gravity g is given.

Hint. Archimedes' principle.

(*5 points*)

Problem 3. A horizontal platform oscillates in the vertical direction with amplitude A. Find the maximum angular frequency of oscillations at which a block placed on the platform is still in contact with the surface of the platform.

Clearly indicate the frame of reference you are solving the problem in.

(*4 points*)

Problem 4. For a critically damped harmonic oscillator show that the oscillating mass can pass through the equilibrium position at most once, regardless of initial conditions.

(*2 points*)