VE230 Homework 7

2021 Summer

P.7-2 The circuit in Fig.1 is situated in a magnetic field

$$\mathbf{B} = \mathbf{a}_z 3\cos\left(5\pi 10^7 t - \frac{2}{3}\pi x\right) \qquad (\mu T).$$

Assume $R = 15(\Omega)$, find the current i.

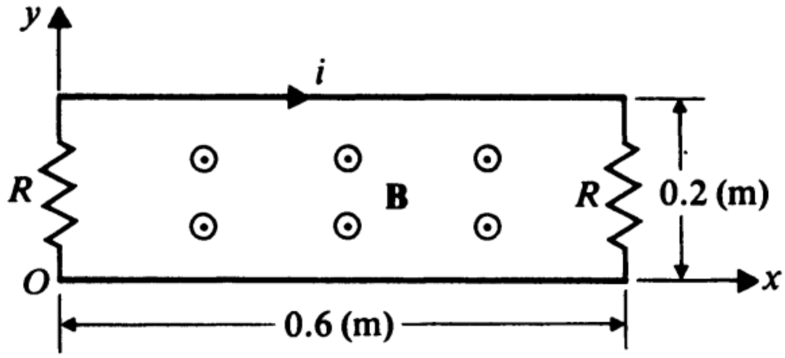

Figure 1: A circuit in a time-varying magnetic field.

P.7-6 A suggested scheme for reducing eddy-current power loss in transformer cores with a circular cross section is to divide the cores into a large number of small insulated filamentary parts. As illustrated in Fig.2, the section shown in part a) is replaced by that in part b). Assuming that $\mathbf{B(t)} = \mathbf{B}_0 \sin\omega t$ and that N filamentary areas fill 95% of the original cross-sectional area, find

a) the average eddy-current power loss in the section of core of height h in Fig.2(a),

b) the total average eddy-current power loss in the N filamentary sections in Fig.2(b).

The magnetic field due to eddy currents is assumed to be negligible. (Hint: First find the current and power dissipated in the differential circular ring section of height h and width dr at radius **r**.)

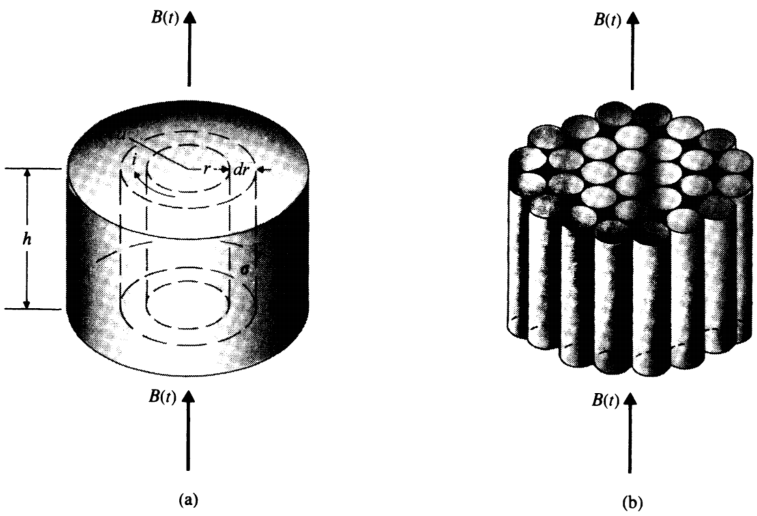

Figure 2: Suggested eddy-current power-less reduction scheme

P.7-11 Derive the two divergence equations, Eqs. (7-53c) and (7-53d) from the two curl equations, (7-53a) and (7-53b), and the equation of continuity, Eq. (7-48).

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{7–53a}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \tag{7–53b}$$

$$\nabla \cdot \mathbf{D} = \rho, \tag{7–53c}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{7–53d}$$

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}. \tag{7–48}$$

P.7-12 Prove that the Lorentz condition for potentials as expressed in Eq. (7-62) is consistent with the equation of continuity.

$$\boxed{\nabla \cdot \mathbf{A} + \mu\epsilon \frac{\partial V}{\partial t} = 0,} \tag{7–62}$$

P.7-14 Substitute Eqs. (7-55) and (7-57) in Maxwell's equations to obtain wave equations for scalar potential V **A** for a linear, isotropic but inhomogeneous medium. Show that these wave equations reduce to Eqs. (7-65) and (7-63) for simple media. (Hint: Use the following gauge condition for potentials in an homogeneous medium.

$$\nabla \cdot (\epsilon \mathbf{A}) + \mu\epsilon^2 \frac{\partial V}{\partial t} = 0.)$$

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A} \qquad \text{(T)}.} \tag{7–55}$$

$$\boxed{\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \qquad \text{(V/m)}.} \tag{7–57}$$

$$\boxed{\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J}.} \tag{7–63}$$

$$\boxed{\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon},} \tag{7–65}$$

P.7-17 Discuss the relations

1. between the boundary conditions for the tangential components of **E** and those for the normal components of **B**.
2. between the boundary conditions for the normal components of **D** and those for the tangential components of **H**.

P.7-20 Prove by direct substitution that any twice differentiable function of $(t - \mathbf{R}\sqrt{\mu\epsilon})$ or of $(t + \mathbf{R}\sqrt{\mu\epsilon})$ is a solution of the homogeneous wave equation of Eq. (7-73).

$$\frac{\partial^2 U}{\partial R^2} - \mu\epsilon \frac{\partial^2 U}{\partial t^2} = 0. \tag{7–73}$$