# Chapter 30

# Inductance

# Goals for Chapter 30

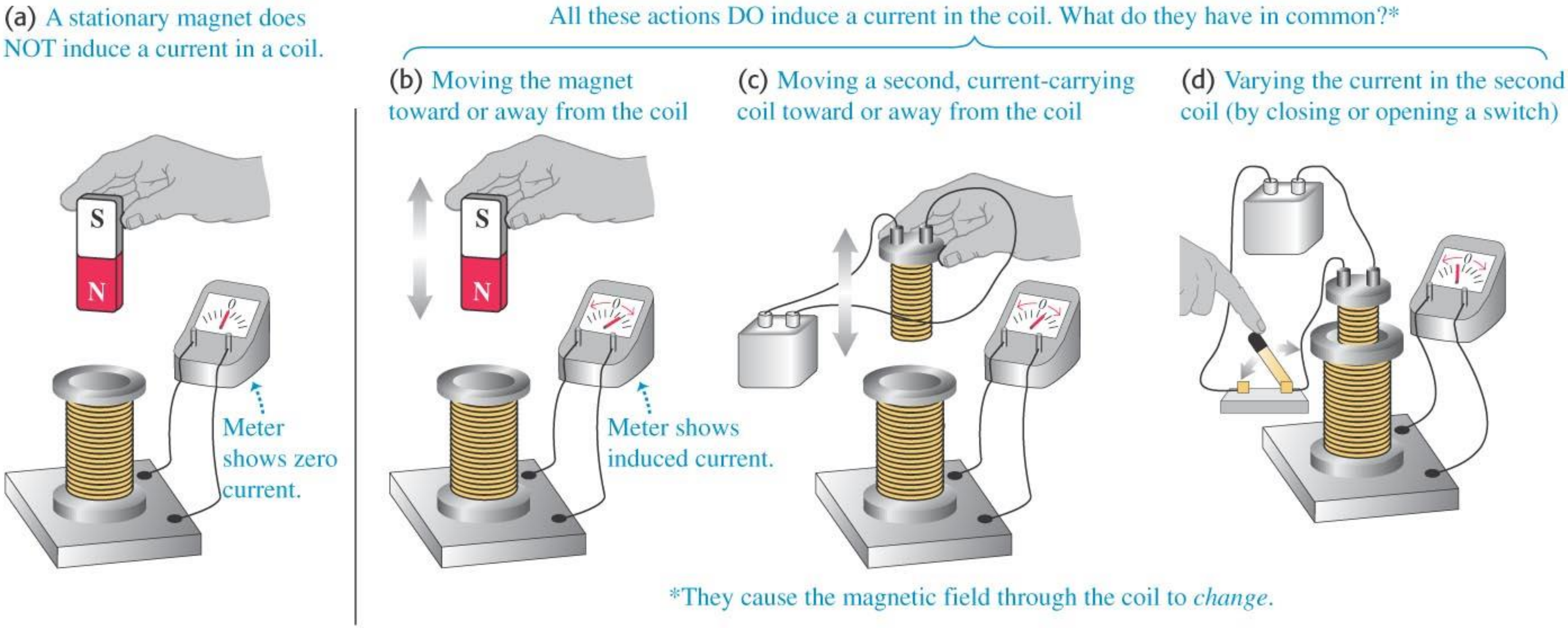
- To learn how current in one coil can induce an emf in another unconnected coil
- To relate the induced emf to the rate of change of the current
- To calculate the energy in a magnetic field
- To analyze circuits containing resistors and inductors
- To describe electrical oscillations in circuits and why the oscillations decay

# Introduction

- How does a coil induce a current in a neighboring coil.
- A sensor triggers the traffic light to change when a car arrives at an intersection. How does it do this?
- Why does a coil of metal behave very differently from a straight wire of the same metal?
- We'll learn how circuits can be coupled without being connected together.

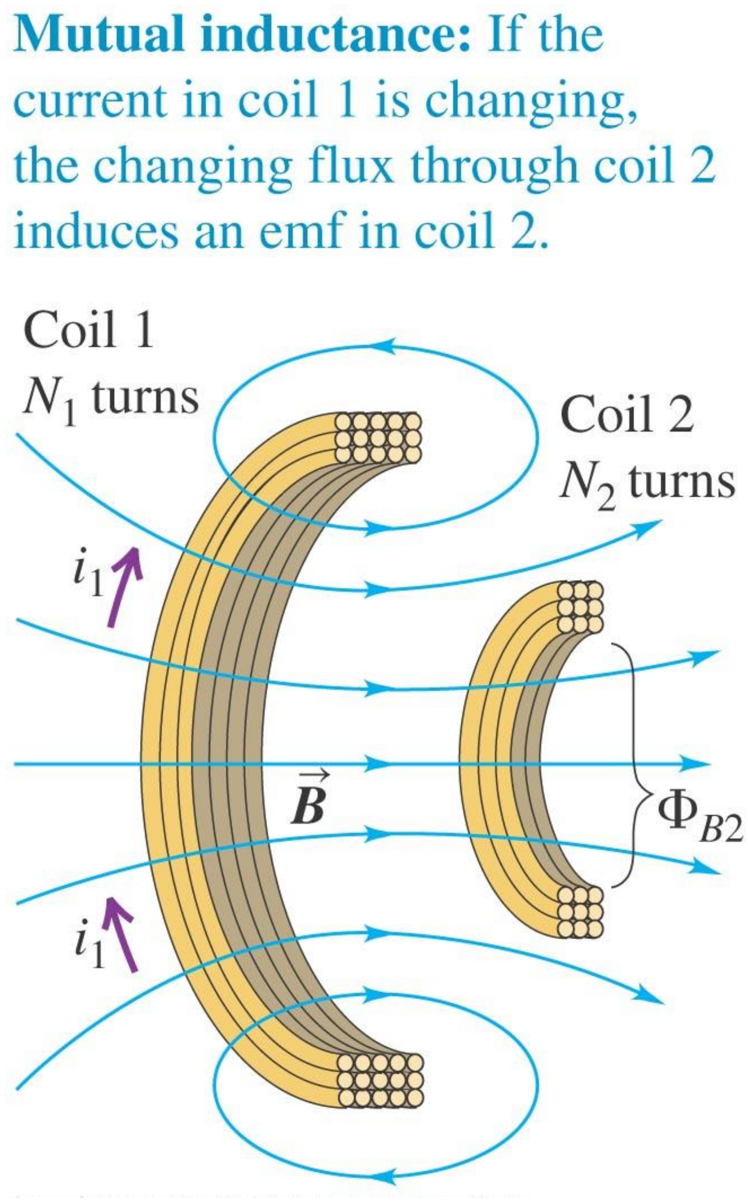
# Induced current

- A changing magnetic flux causes an *induced current*. See Figure 29.1 below.
- The *induced emf* is the corresponding emf causing the current.



# Mutual inductance

- *Mutual inductance*: A changing current in one coil induces a current in a neighboring coil. See Figure 30.1 at the right.
- Follow the discussion of mutual inductance in the text.



# Induced electric fields

$$\Phi_B = BA = \mu_0 n I A$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\mu_0 n A \frac{dI}{dt}$$

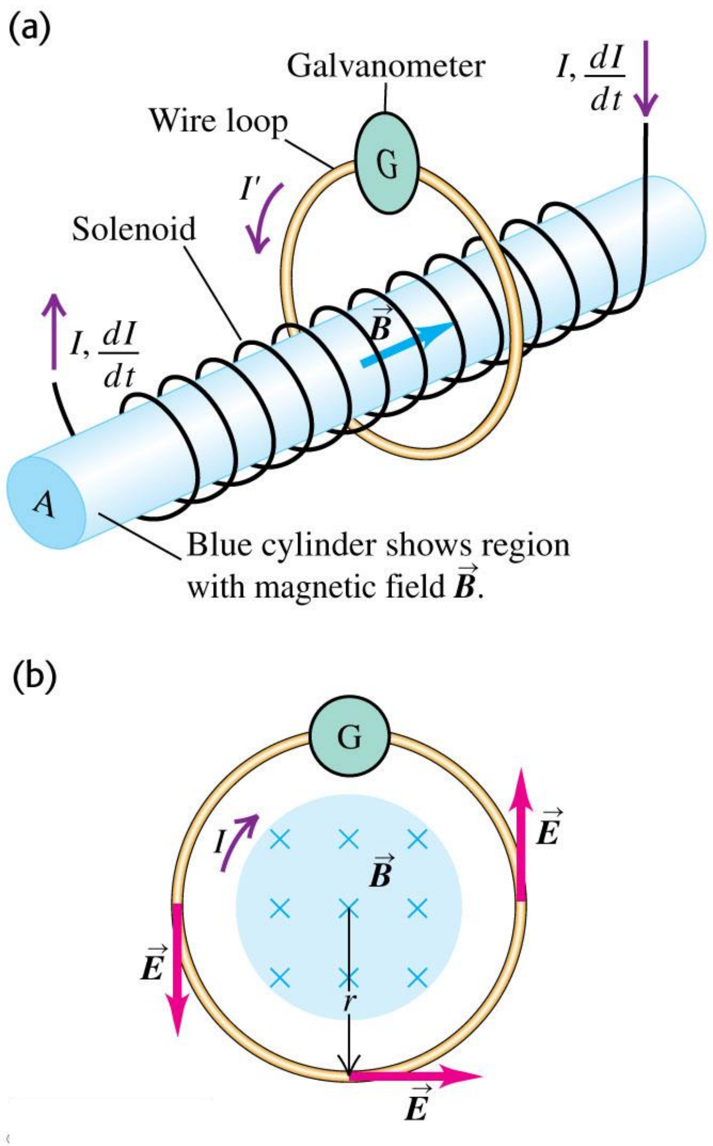
$V_{ab} = \mathcal{E}$ (ideal source of emf) ; $I' = \mathcal{E}/R$.

induced electric field $\oint \vec{E} \cdot d\vec{l} = \mathcal{E}$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \text{(stationary integration path)}$$

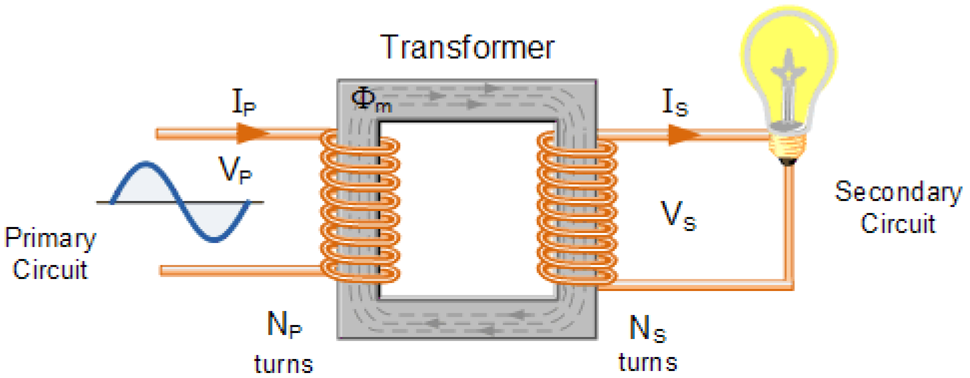
$\oint \vec{E} \cdot d\vec{l} = 2\pi r E$, and Eq. (29.10) gives

$$E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$



# Induced current

# Induced current




# Mutual inductance

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt} \tag{30.1}$$

We could represent the proportionality of $\Phi_{B2}$ and $i_1$ in the form $\Phi_{B2} = (\text{constant})i_1$, but instead it is more convenient to include the number of turns $N_2$ in the relationship. Introducing a proportionality constant $M_{21}$, called the **mutual inductance** of the two coils, we write

$$N_2 \Phi_{B2} = M_{21} i_1 \tag{30.2}$$

where $\Phi_{B2}$ is the flux through a *single* turn of coil 2. From this,

$$N_2 \frac{d\Phi_{B2}}{dt} = M_{21} \frac{di_1}{dt}$$

and we can rewrite Eq. (30.1) as

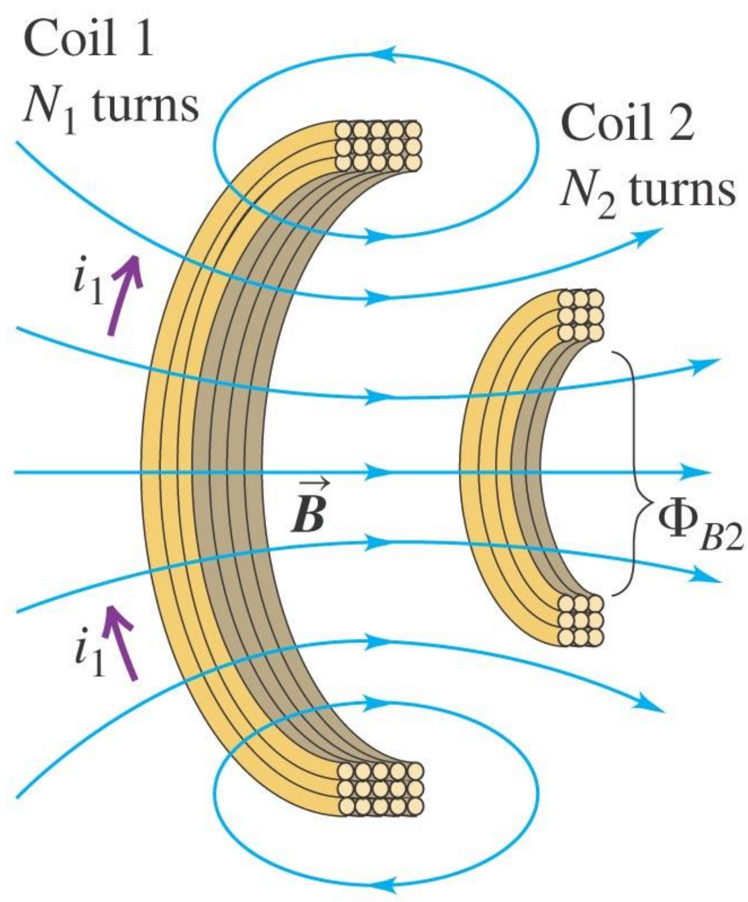
$$\mathcal{E}_2 = -M_{21} \frac{di_1}{dt} \tag{30.3}$$

That is, a change in the current $i_1$ in coil 1 induces an emf in coil 2 that is directly proportional to the rate of change of $i_1$ (Fig. 30.2).

We may also write the definition of mutual inductance, Eq. (30.2), as

$$M_{21} = \frac{N_2 \Phi_{B2}}{i_1}$$

**Mutual inductance:** If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



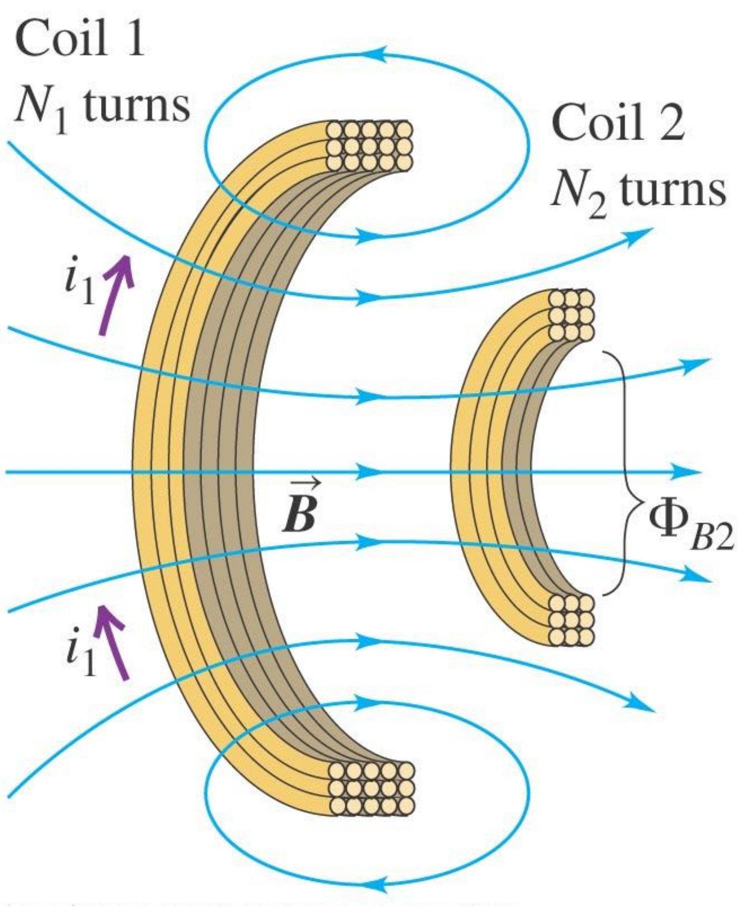
# Mutual inductance

B Flux is constant going through both coils

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt}$$

$$M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}$$

**Mutual inductance:** If the current in coil 1 is changing, the changing flux through coil 2 induces an emf in coil 2.



# Mutual inductance examples

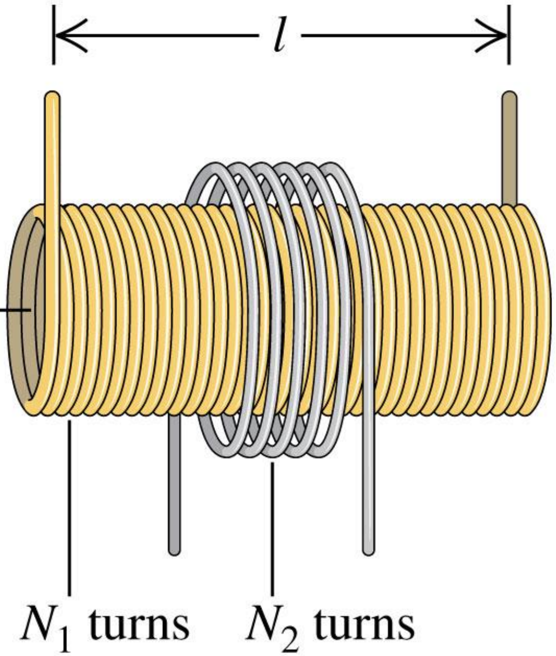
- Follow Example 30.1, which shows how to calculate mutual inductance. See Figure 30.3 below.

$$n_1 = N_1/L.$$

$$B_1 = \mu_0 n_1 i_1 = \frac{\mu_0 N_1 i_1}{l}$$

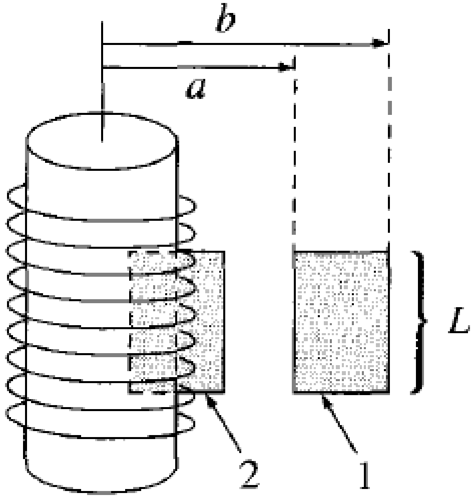
$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 B_1 A}{i_1} = \frac{N_2}{i_1}\frac{\mu_0 N_1 i_1}{l} A = \frac{\mu_0 A N_1 N_2}{l}$$

Cross-sectional area $A$



# Field of a solenoid

- A *solenoid* consists of a helical winding of wire on a cylinder.
- Follow Example 28.9 using Figures 28.22–28.24 below.




$$\oint \mathbf{B} \cdot d\mathbf{l} = [B(a) - B(b)]L = \mu_0 I_{enc} = 0,$$

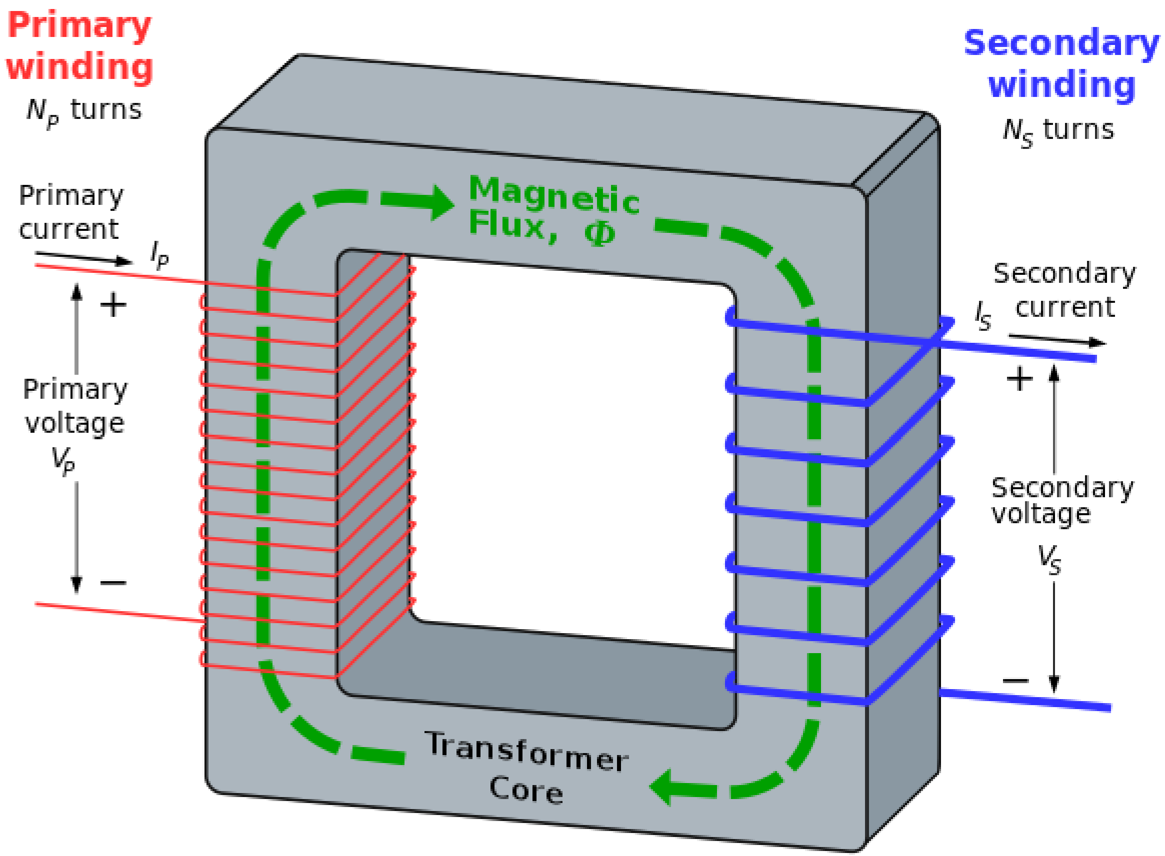
$$B(a) = B(b).$$

$$BL = \mu_0 n L I,$$

$$B = \mu_0 n I$$

Evidently the *field outside does not depend on the distance from the axis*. But we know that it goes to *zero* for large $s$. It must therefore be zero *everywhere*! (This astonishing result can also

# Mutual inductance examples




$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_2 B_1 A}{i_1} = \frac{N_2}{i_1} \frac{\mu_0 N_1 i_1}{l} A = \frac{\mu_0 A N_1 N_2}{l}$$

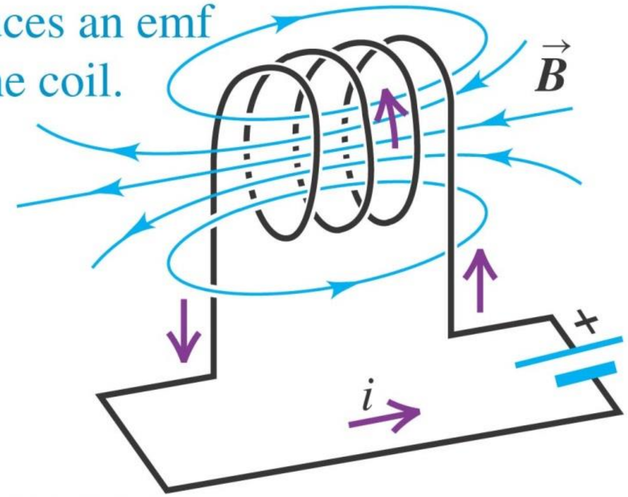
$$\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M \frac{di_2}{dt}$$

# Self-inductance

- *Self-inductance*: A varying current in a circuit induces an emf in that same circuit. See Figure 30.4 below.

*dI*

**Self-inductance:** If the current $i$ in the coil is changing, the changing flux through the coil induces an emf in the coil.




By Lenz's law, a self-induced emf always opposes the change

$$L = \frac{N\Phi_B}{i} \qquad \text{(self-inductance)}$$

$$N\frac{d\Phi_B}{dt} = L\frac{di}{dt}$$

$$\mathcal{E} = -L\frac{di}{dt} \qquad \text{(self-induced emf)}$$

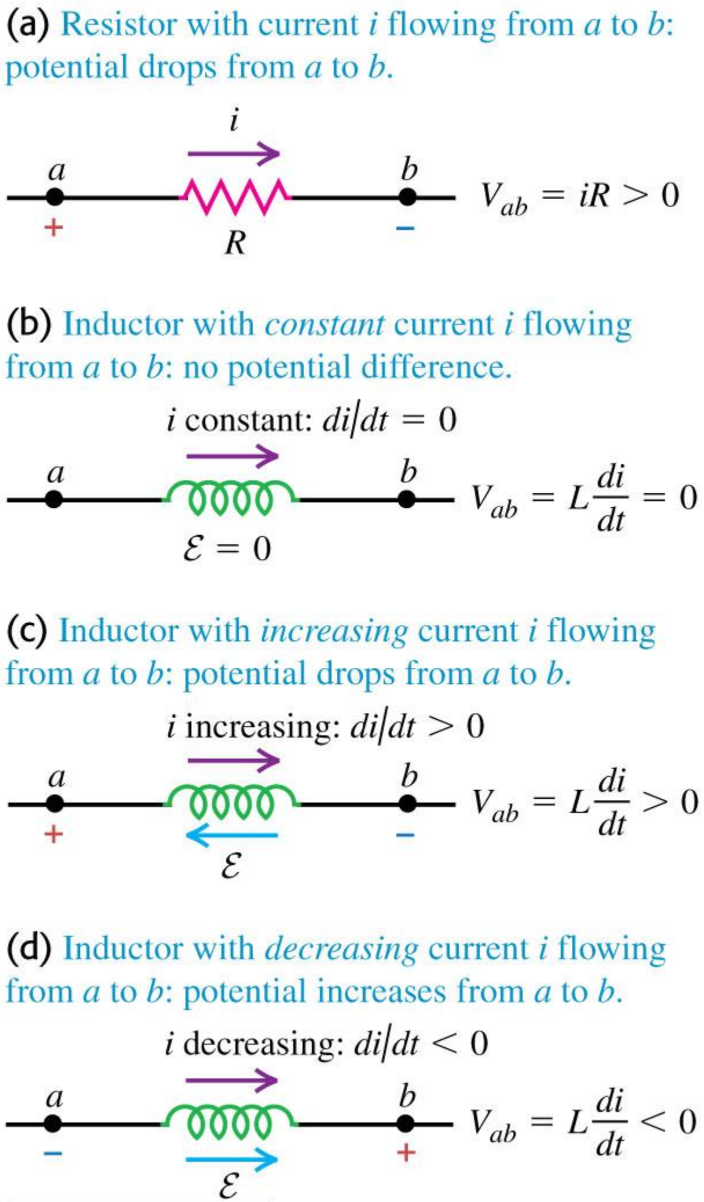
$n_1 = N_1/L.$

$$B_1 = \mu_0 n_1 i_1 = \frac{\mu_0 N_1 i_1}{l}$$

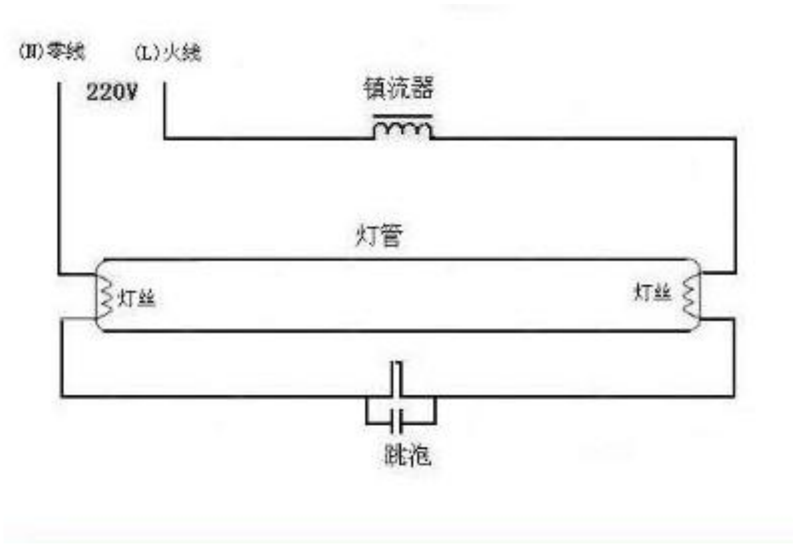
$$L = \frac{NBA}{i} = \frac{NuNiA}{i * l} = \frac{N^2 uA}{l}$$

# Potential across an inductor

- The potential across an inductor depends on the rate of change of the current through it.
- Figure 30.6 at the right compares the behavior of the potential across a resistor and an inductor.
- The self-induced emf does *not* oppose current, but opposes a *change* in the current.



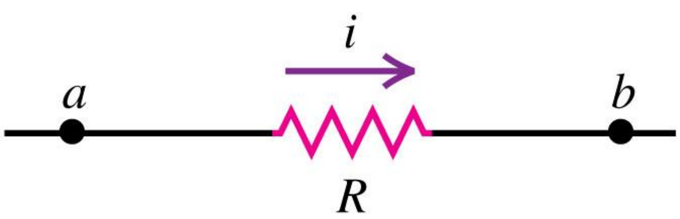
# self-induced emf




# Magnetic field energy

- The energy stored in an inductor is $U = 1/2\ LI^2$. See Figure 30.9 below.
- The energy density in a magnetic field is $u = B^2/2\mu_0$ (in vacuum) and $u = B^2/2\mu$ (in a material).
- Follow Example 30.5.

Resistor with current $i$: energy is *dissipated*.

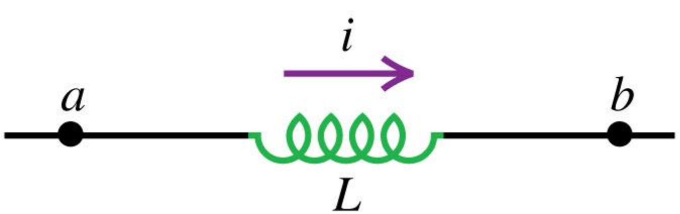



$$P = V_{ab}i = Li\frac{di}{dt}$$

$$dU = P\,dt,$$

$$dU = Li\,di$$

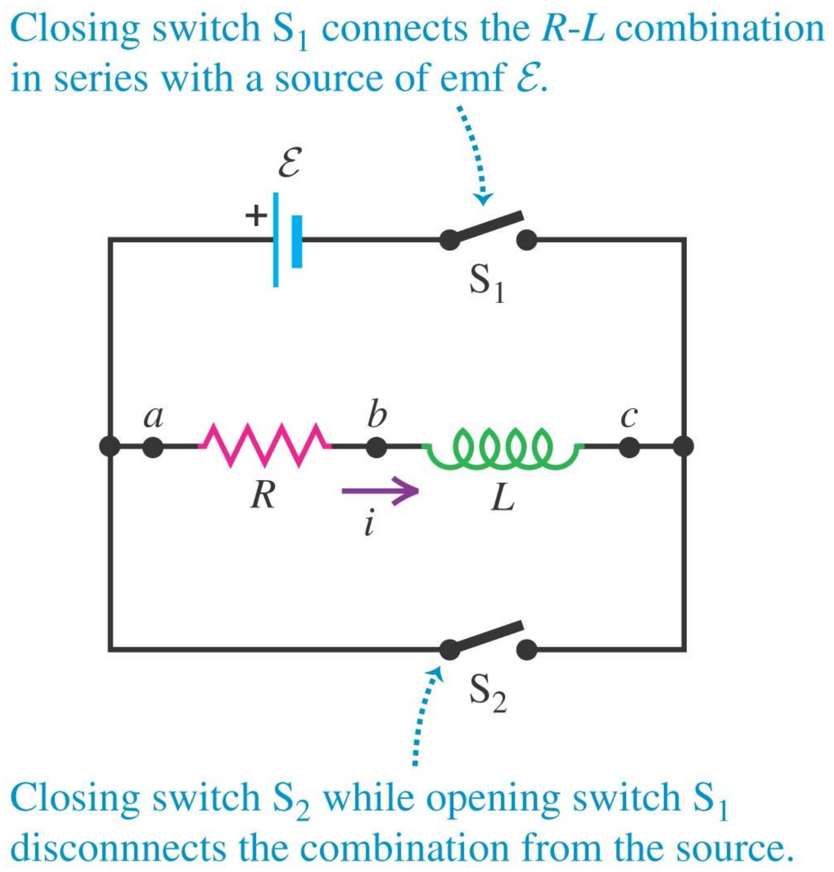
Inductor with current $i$: energy is *stored*.




$$U = L\int_0^I i\,di = \tfrac{1}{2}LI^2 \qquad \text{(energy stored in an inductor)}$$

# The *R-L* circuit

- An *R-L circuit* contains a resistor and inductor and possibly an emf source.
- Figure 30.11 at the right shows a typical *R-L* circuit.
- Follow Problem-Solving Strategy 30.1.



# Current growth in an *R*-*L* circuit

$$v_{ab} = iR$$

$$v_{bc} = L\frac{di}{dt}$$

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

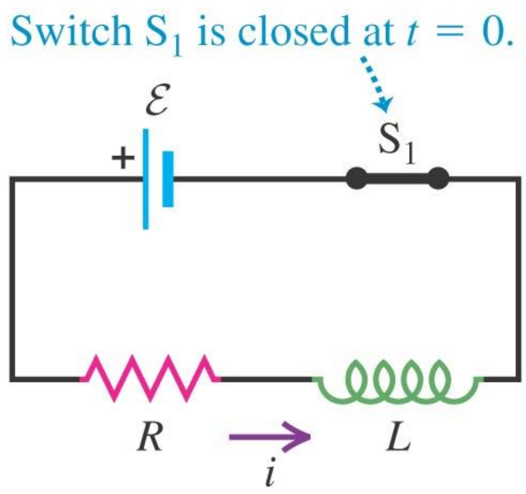
$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{\mathcal{E}}{L} - \frac{R}{L}i$$

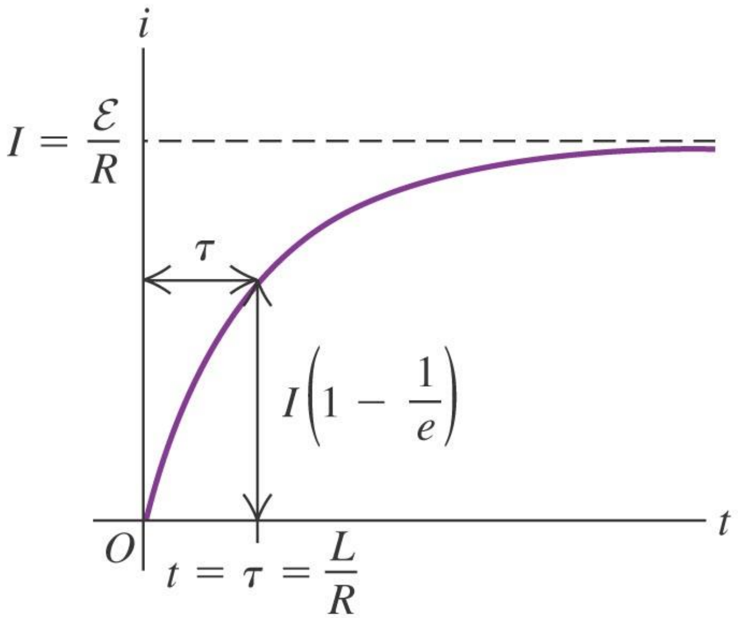
$S_1$ is first closed, $i = 0$

$$\left(\frac{di}{dt}\right)_{\text{initial}} = \frac{\mathcal{E}}{L}$$

$$\left(\frac{di}{dt}\right)_{\text{final}} = 0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \quad \text{and}$$

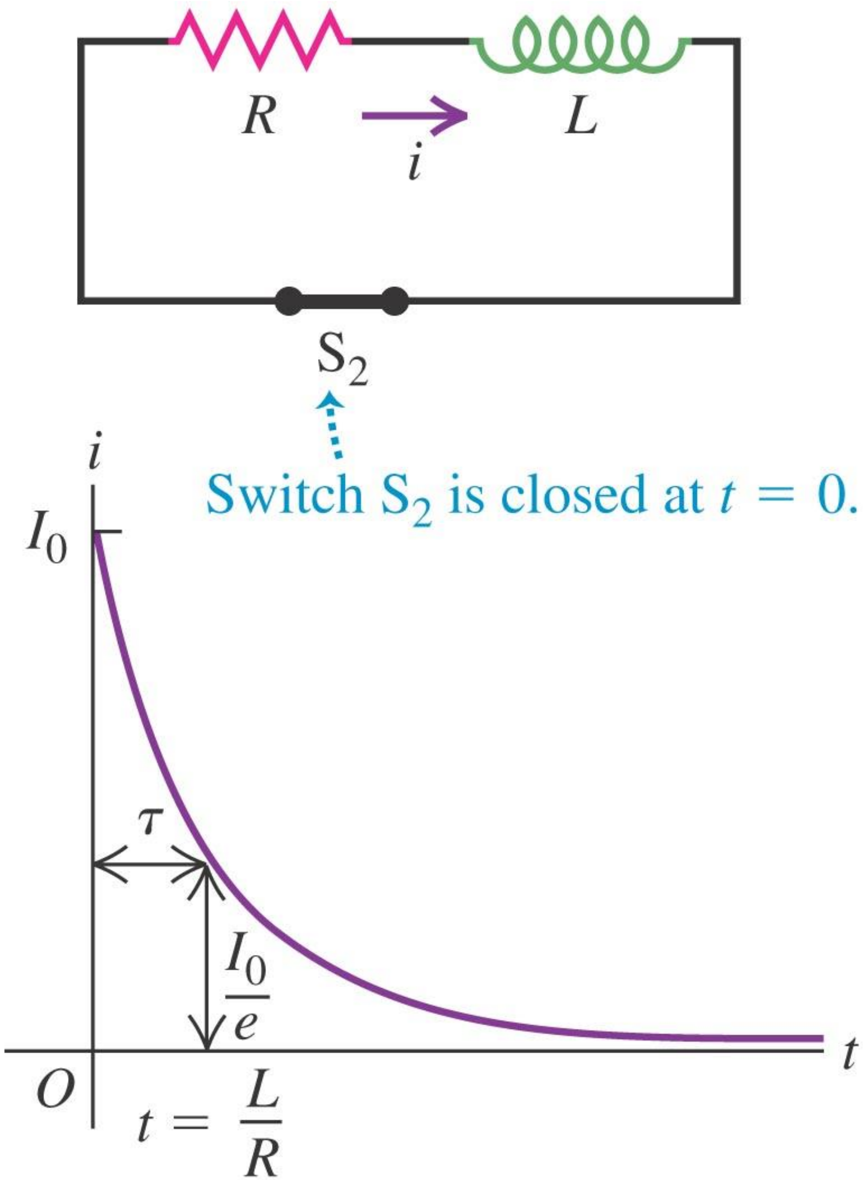
$$I = \frac{\mathcal{E}}{R}$$



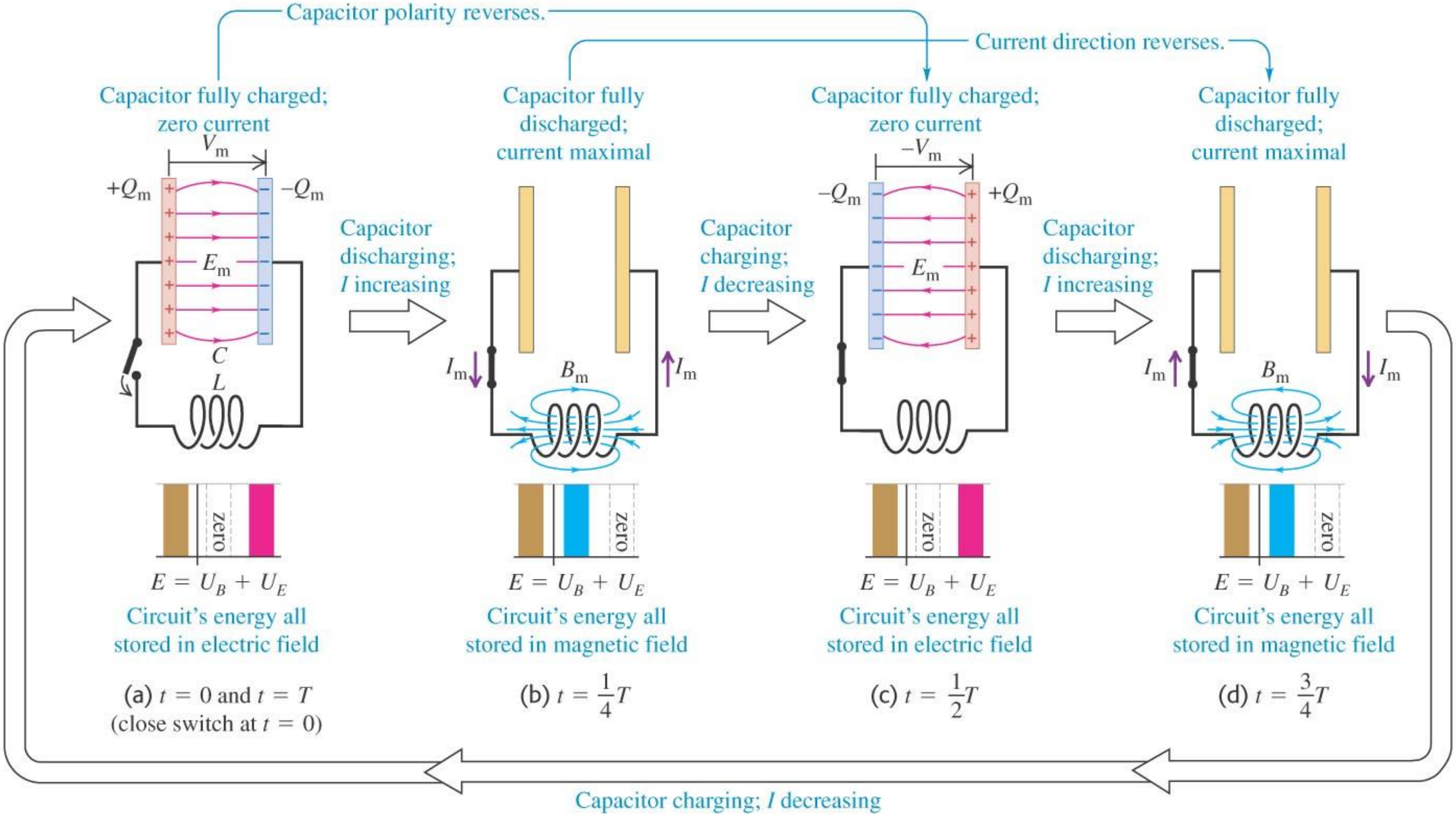
# Current decay in an *R*-*L* circuit

- Read the text discussion of current decay in an *R*-*L* circuit.
- Figure 30.13 at the right shows a graph of the current versus time.
- Follow Example 30.7.



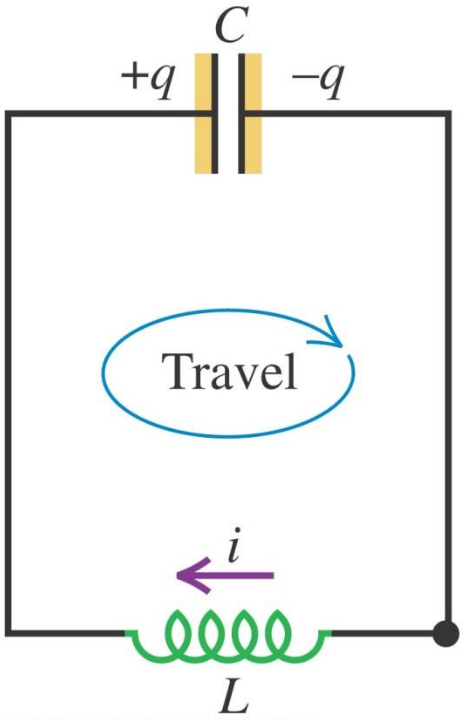
# The *L*-*C* circuit

- An *L-C circuit* contains an inductor and a capacitor and is an *oscillating* circuit. See Figure 30.14 below.



# Electrical oscillations in an *L*-*C* circuit

- Follow the text analysis of electrical oscillations and energy in an *L*-*C* circuit using Figure 30.15 at the right.



# Electrical and mechanical oscillations

- Table 30.1 summarizes the analogies between SHM and *L*-*C* circuit oscillations.
- Follow Example 30.8.
- Follow Example 30.9.

**Table 30.1** Oscillation of a Mass-Spring System Compared with Electrical Oscillation in an *L*-*C* Circuit

**Mass-Spring System**

Kinetic energy $= \frac{1}{2}mv_x^2$

Potential energy $= \frac{1}{2}kx^2$

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$v_x = dx/dt$

$\omega = \sqrt{\frac{k}{m}}$

$x = A\cos(\omega t + \phi)$

**Inductor-Capacitor Circuit**

Magnetic energy $= \frac{1}{2}Li^2$

Electric energy $= q^2/2C$

$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$

$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$

$i = dq/dt$

$\omega = \sqrt{\frac{1}{LC}}$

$q = Q\cos(\omega t + \phi)$

# The *L-R-C* series circuit

- Follow the text analysis of an *L-R-C* circuit.
- An *L-R-C* circuit exhibits *damped harmonic motion* if the resistance is not too large. (See graphs in Figure 30.16 at the right.)
- Follow Example 30.10.