vv214_SU2020_Assignment 3 **due to June 16, 2020**

Part A

Problem 1

a. Find a matrix A that does the following transformations in $\mathbb{R}^2$

$\bar{x}_1 = (2,1) \to \bar{y}_1 = (-1,1)$ and $\bar{x}_2 = (-1,\frac{3}{2}) \to \bar{y}_2 = (-2,1)$

b. Is A invertible? Find A^{-1}.

Problem 2

Find matrices of the following linear transformations:

1. T: $\mathbb{R}^2 \to \mathbb{R}^2$

a. The reflection about the line $y = x$ combined with the counterclockwise rotation through $\pi/6$.

b. The orthogonal projection onto the line $y = -x$ combined with the scaling by 2.

2. T: $\mathbb{R}^3 \to \mathbb{R}^3$

a. The orthogonal projection onto the line $x = z$ on the xz-plane.

b. The rotation about the y-axis through an angle of $\frac{\pi}{2}$, counterclockwise as viewed from the positive y-axis combined with the reflection about the xz-plane.

Problem 3

Bretscher 2.2.27, 2.2.28, 2.2.34, pp.66-67.

Problem 4

a. A matrix E obtained from I_n by one of the elementary row operations is called elementary.

Find the matrix products E_1A, E_2A, E_3A, where

$$E_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_2 = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \quad A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} \quad A =$$

and explain the relation between E_1, E_2, E_3 and A in terms of elementary row operations.

b. For the matrix $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$, represent its $rref(A)$ in the form $rref(A) = E_r \dots E_2E_1A$, where E_i are elementary matrices, write A as a product of elementary matrices, and show that A cannot be represented as a product of a lower matrix L and an upper triangular matrix U.

c. Let the coefficient matrix of a linear system $A\bar{x} = \bar{b}$ be represented the form $A = LU$. How does it help you to solve the system? (Read Bretscher p.93, exercises 2.4.90-2.4.91)

Problem 5

Bretscher 2.4.108, parts a. and b., p. 96

Problem 6

Determine whether the given vectors are linearly independent

$$\bar{x}_1 = (1,2,0,1)\ ,\bar{x}_2 = (0,1,1,2)\ ,\bar{x}_3 = (1,-1,-2,1)\ ,\bar{x}_4 = (2,2,-1,4).$$

What is the dimension of $V = span(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)$?

Problem 7

Find the reduced row-echelon form, bases of the image and the kernel for each for the following matrices

$$a.\quad \begin{pmatrix} 4 & 8 & 1 & 1 & 6 \\ 3 & 6 & 1 & 2 & 5 \\ 2 & 4 & 1 & 9 & 10 \\ 1 & 2 & 3 & 2 & 0 \end{pmatrix} \qquad b.\begin{pmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -2 & -1 \\ 2 & 4 & -4 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix}$$

Problem 8

Bretscher Exercise 3.3.84, page 136

Problem 9

Bretscher Exercise 3.1.53-3.154, page 112

Problem 10

Check if the matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 9 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & 0 & -1 \\ -1 & 4 & -1 \\ 3 & 3 & -1 \end{pmatrix}$$

are invertible. If they are, find the inverse matrices using elementary row operations.

Part B

Type the following commands in MATLAB/MATLAB Online and save the inputs and outputs/m. files/graphs as a .pdf file

Operation	Type	Comments
To enter matrices row by row	`A= [ 8 2 9; 4 9 4; 6 7 9]` `B= [1 2 3; 4 5 6; 7 8 9]`	
To find the inverses of A and B	`AI=inv(A)` `BI=inv(B)`	Explain the obtained results. What is $rref(A)$?
To get the entries in fractional format	`format` `rat AI = inv(A)`	
To form the augmented matrix $(A\|I)$	`A1= [ A eye(3) ]`	
To find $rref(A\|I)$	`A2=rref(A1)`	
To extract the inverse of A	`AI=A2(:, [4 5 6])`	
LU factorization with MATLAB		
To see LU factorization of A	`[L U] = lu(A)`	Read the exercise 2.4.90 for details
To see LU factorization of A with the permutation matrix P	`[L U P ] = lu(A)`	
To solve $L\bar{c}=\bar{a}$	`c = L\a`	Define your own vector $\bar{a}$
To solve $U\bar{x}=\bar{c}$	`x = U\c`	
To see LU factorization of B and extract L and U	`[L U P]=lu(B)`	$B\bar{x}= LU\bar{x}= L\bar{c}= \bar{a}$, $where\ \bar{c}= U\bar{x}$

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To solve $L\bar{c}= \bar{a}$	`c = L\a`	Define your own vector $\bar{a}$
To solve $U\bar{x}= \bar{c}$	`x = U\c`	Explain the obtained result.
Plotting with MATLAB		
To plot the line segment with the end-points $(1, -1)$, $(2, -5)$	`x = [1 -1]; y = [2 -5]; plot(x, y)`	Type plot(x,y,'g') for green color plot(x,y,'r--') for red dashed line
To rescale the axes	`axis ([-5 5 -3 3])`	
To add labels	`xlabel('x axis') ylabel('y axis')`	
To add a title	`title('The line segment')`	
To add another graph to the same plot	`hold on : hold off`	Enter commands for new plots after hold on and before hold off hold on

	`y= [-2 5]`
	`plot(x, y, 'k--')`
	`x=-x`
	`plot(x, y, 'b--')`
	`hold off`	

To create the .m file with the following content	`function lt2d(obj, M) plot(obj(1,:), obj(2,:), 'b') hold on y = M * obj; display(y) plot(y(1,:), y(2,:), 'r') hold off`	Save the file as ltr2d.m Add whitebg('w') for white background

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