vv214 SU2020

Assignment 6
Due: Saturday 01 August

Problem 1
For each of the following matrices, find the characteristic polynomial, eigenvalues and associated eigenvectors.

Determine algebraic and geometric multiplicities of each eigenvalue.

$$\begin{pmatrix} 2 & 7 & 6 \\ 0 & -1 & -6 \\ 0 & 2 & 7 \end{pmatrix} \quad \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad \begin{pmatrix} 4 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1001 & 3 & 5 & 7 & 8 & 11 \\ 1 & 1003 & 5 & 7 & 9 & 11 \\ 1 & 3 & 1005 & 7 & 9 & 11 \\ 1 & 3 & 5 & 1007 & 9 & 11 \\ 1 & 3 & 5 & 7 & 1009 & 11 \\ 1 & 3 & 5 & 7 & 9 & 1011 \end{pmatrix}$$

Problem 2

Determine whether the following matrices are diagonalizable and find their diagonal forms.

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \quad \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \quad \begin{pmatrix} 3 & 2 \\ -5 & 3 \end{pmatrix}$$

Problem 3
Apply the Cayley-Hamilton theorem to

1. Find A^{-1}
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{pmatrix}$$
2. Simplify $-A^3 + 4A^2 + 3A - 4I$
$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$
3. Evaluate
$$exp\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \qquad \sin\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$$

Problem 4
Find SVD of the following matrices:

$$\begin{pmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$$

Use the obtained SVD to find dim $Ker\, A$, dim $Im\, A$, dim $Ker\, A^T$, dim $Im\, A^T$.
Sketch the image, under A, of the sphere of radius 1.