

VE 320 Fall 2021

Introduction to Semiconductor Devices

Instructor: Rui Yang (杨睿)

Office: JI Building 434

rui.yang@sjtu.edu.cn

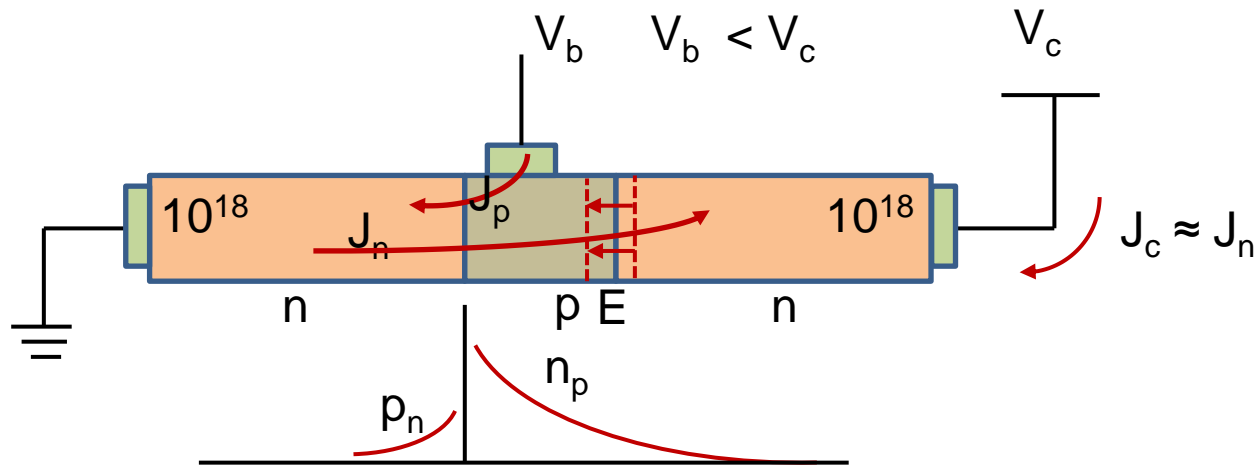


Lecture 9

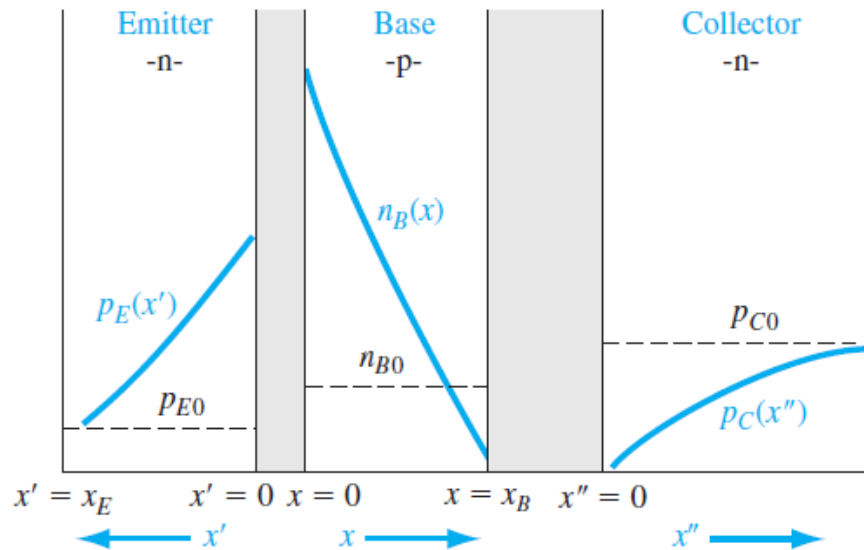
BJT

(Chapter 12)

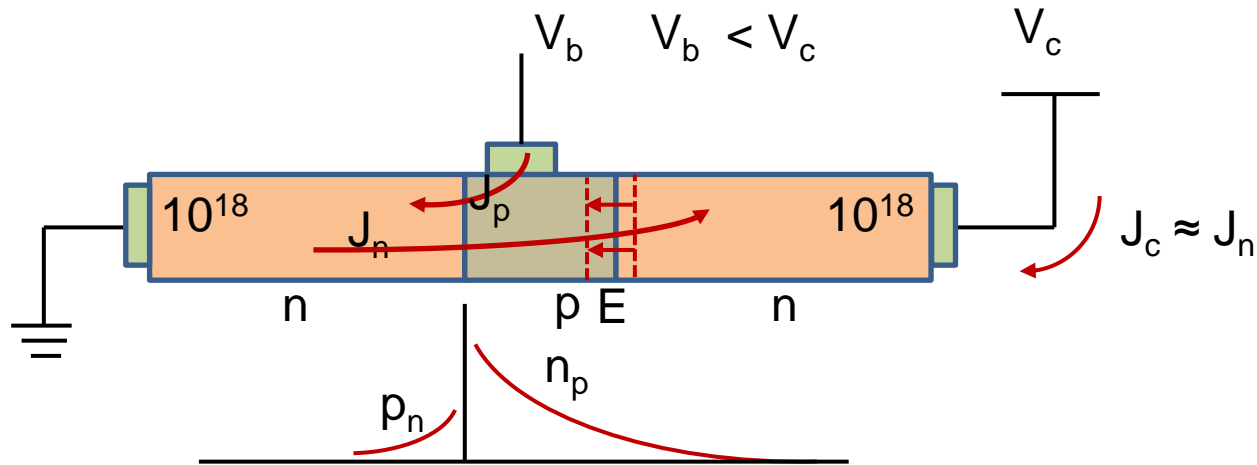
Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode



Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode
In the neutral base region

$$D_B \frac{\partial^2(\delta n_B(x))}{\partial x^2} - \frac{\delta n_B(x)}{\tau_{B0}} = 0$$

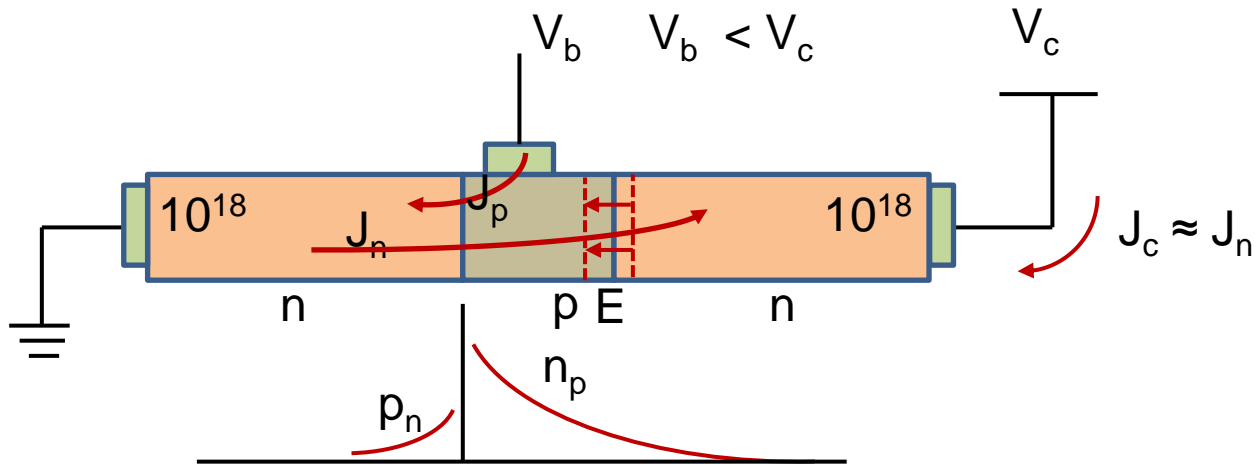
Excess electron concentration $\delta n_B(x) = n_B(x) - n_{B0}$

General solution: $\delta n_B(x) = A \exp\left(\frac{+x}{L_B}\right) + B \exp\left(\frac{-x}{L_B}\right)$ $L_B = \sqrt{D_B \tau_{B0}}$

Boundary conditions: $\delta n_B(x=0) \equiv \delta n_B(0) = A + B$

$$\delta n_B(x=x_B) \equiv \delta n_B(x_B) = A \exp\left(\frac{+x_B}{L_B}\right) + B \exp\left(\frac{-x_B}{L_B}\right)$$

Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode

In the neutral base region

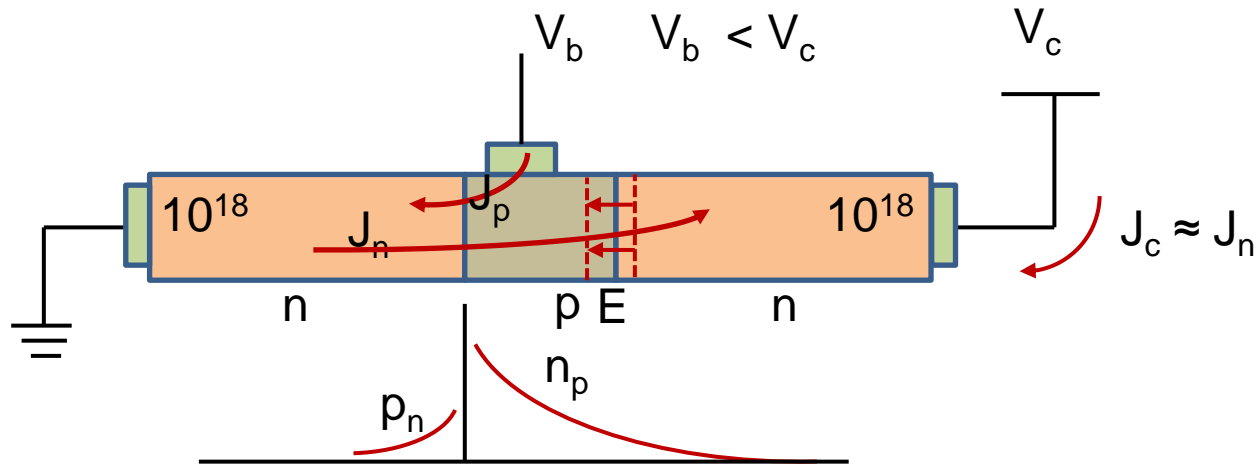
BE junction forward biased:

$$\delta n_B(0) = n_B(x=0) - n_{B0} = n_{B0} \left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right]$$

BC junction reverse biased:

$$\delta n_B(x_B) = n_B(x=x_B) - n_{B0} = 0 - n_{B0} = -n_{B0}$$

Quantitative analysis of BJT gain



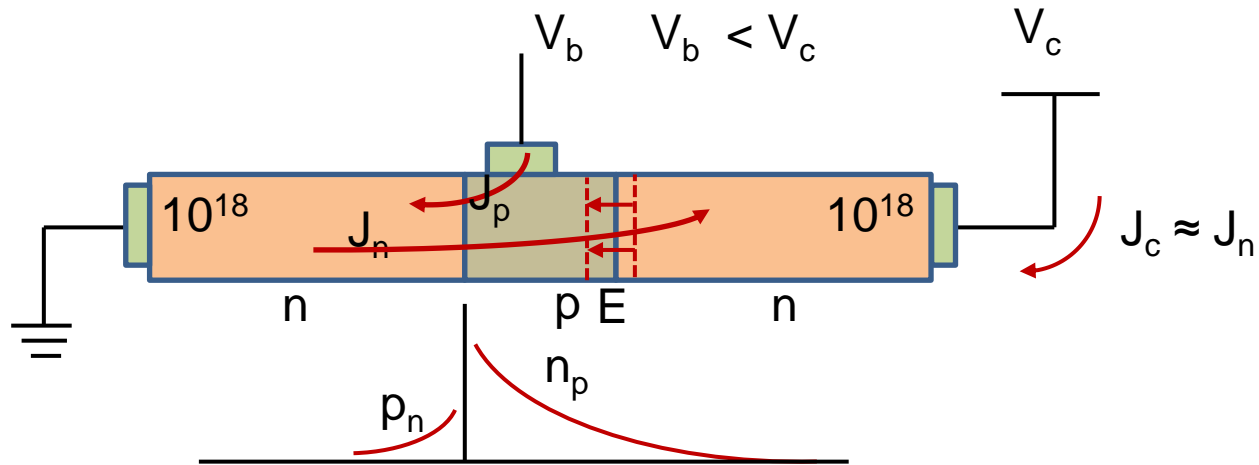
Now consider actual npn device: forward active mode
In the neutral base region

We can get

$$A = \frac{-n_{B0} - n_{B0} \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] \exp \left(\frac{-x_B}{L_B} \right)}{2 \sinh \left(\frac{x_B}{L_B} \right)}$$

$$B = \frac{n_{B0} \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] \exp \left(\frac{x_B}{L_B} \right) + n_{B0}}{2 \sinh \left(\frac{x_B}{L_B} \right)}$$

Quantitative analysis of BJT gain

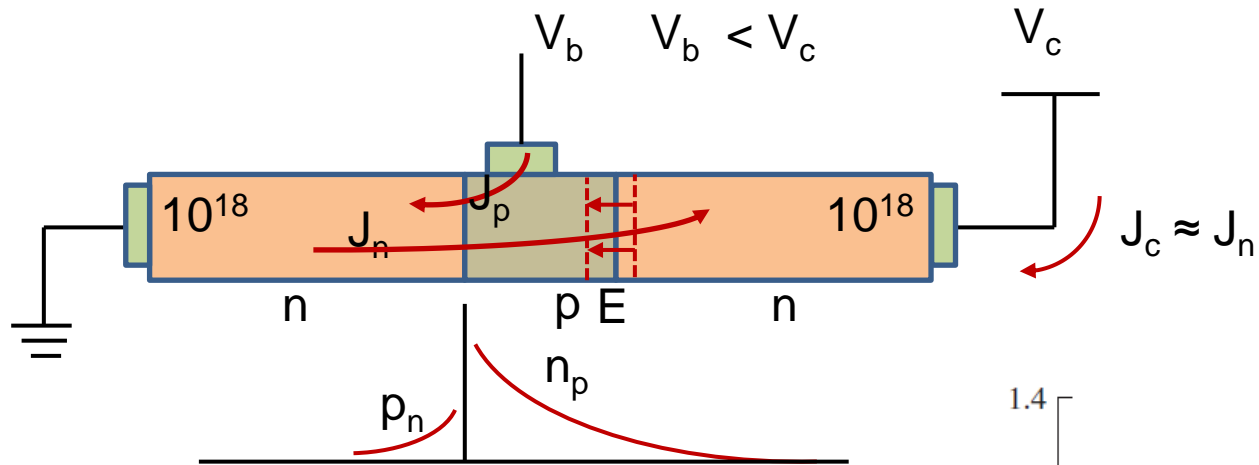


Now consider actual npn device: forward active mode
In the neutral base region

Excess minority carrier electron concentration in the base:

$$\delta n_B(x) = \frac{n_{B0} \left\{ \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] \sinh \left(\frac{x_B - x}{L_B} \right) - \sinh \left(\frac{x}{L_B} \right) \right\}}{\sinh \left(\frac{x_B}{L_B} \right)}$$

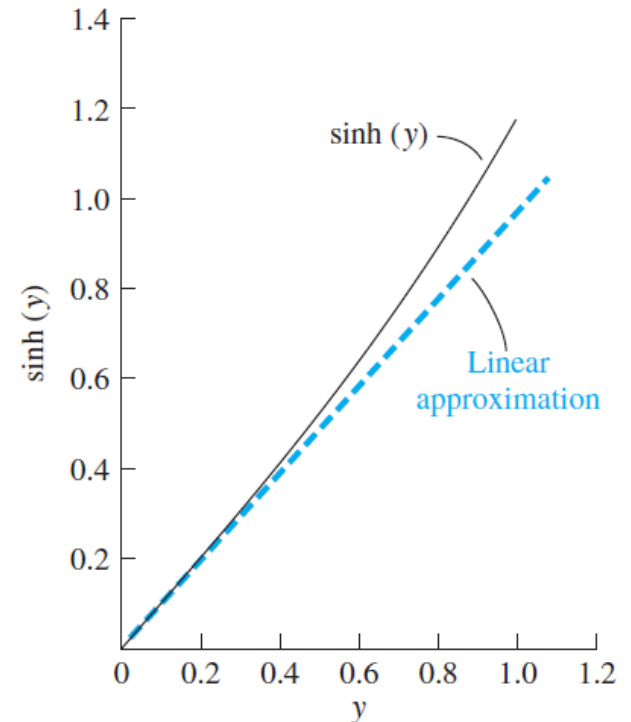
Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode
In the neutral base region

Simplify: $x_B < L_B$

Function	Taylor expansion
$\sinh(x)$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$
$\cosh(x)$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$
$\tanh(x)$	$x - \frac{x^3}{3} + \frac{2x^5}{15} + \dots$



Quantitative analysis of BJT gain

$$\sinh(x) = \frac{1}{2} [\exp(x) - \exp(-x)]$$

$$\exp(x) = 1 + x + \frac{1}{2}(x)^2 + \dots$$

$$\exp(-x) = 1 - x + \frac{1}{2}(x)^2 - \dots$$

Quantitative analysis of BJT gain

$$\sinh(x) = \frac{1}{2} [\exp(x) - \exp(-x)] = x$$

$$\begin{aligned} \exp(x) &= 1 + x + \frac{1}{2}(x)^2 + \dots \\ \exp(-x) &= 1 - x + \frac{1}{2}(x)^2 - \dots \end{aligned} \quad \begin{array}{l} \nearrow \\ \nearrow \end{array} \quad \text{if } x < 1$$

Quantitative analysis of BJT gain

$$\cosh(x) - 1 = \frac{1}{2} [\exp(x) + \exp(-x)] - 1$$

$$\exp(x) = 1 + x + \frac{1}{2}(x)^2 + \dots$$

$$\exp(-x) = 1 - x + \frac{1}{2}(x)^2 - \dots$$

Quantitative analysis of BJT gain

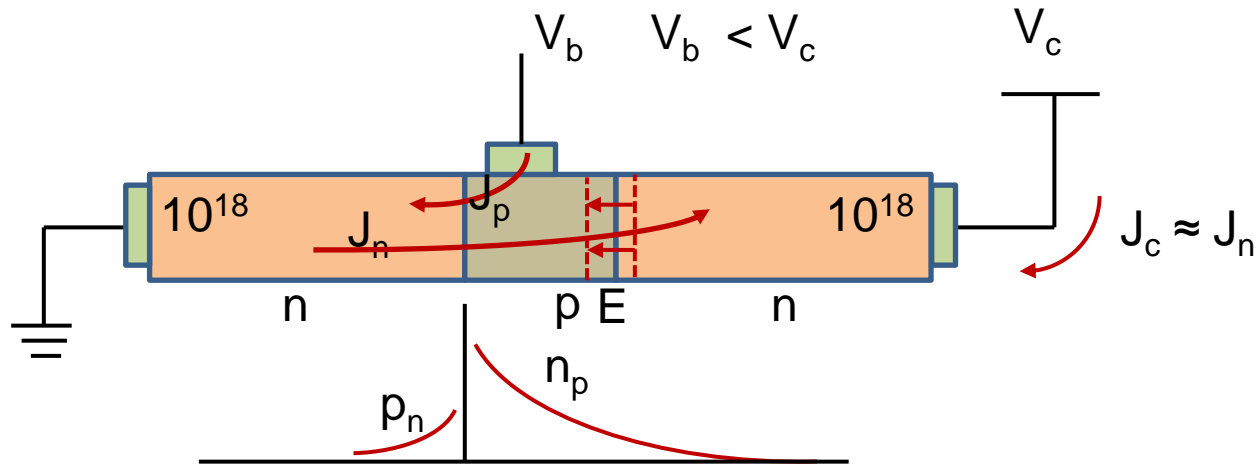
$$\cosh(x) - 1 = \frac{1}{2} [\exp(x) + \exp(-x)] - 1 = \frac{1}{2} x^2$$

$$\exp(x) = 1 + x + \frac{1}{2} (x)^2 + \dots$$

$$\exp(-x) = 1 - x + \frac{1}{2} (x)^2 - \dots$$

if $x < 1$

Quantitative analysis of BJT gain

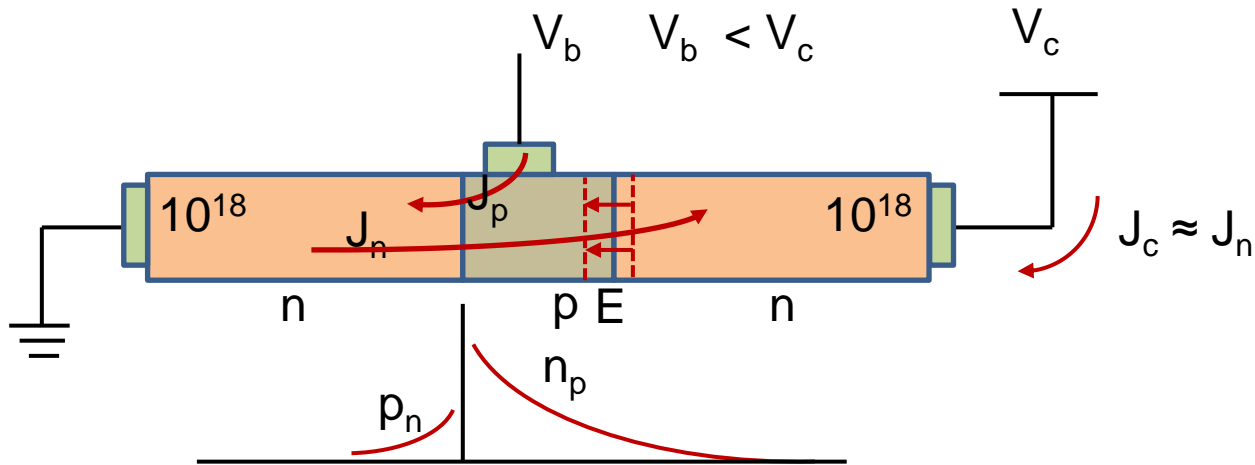


Now consider actual npn device: forward active mode
In the neutral base region

$$\delta n_B(x) = \frac{n_{B0} \left\{ \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] \sinh \left(\frac{x_B - x}{L_B} \right) - \sinh \left(\frac{x}{L_B} \right) \right\}}{\sinh \left(\frac{x_B}{L_B} \right)}$$

$$\delta n_B(x) \approx \frac{n_{B0}}{x_B} \left\{ \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] (x_B - x) - x \right\}$$

Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode
In the neutral emitter region

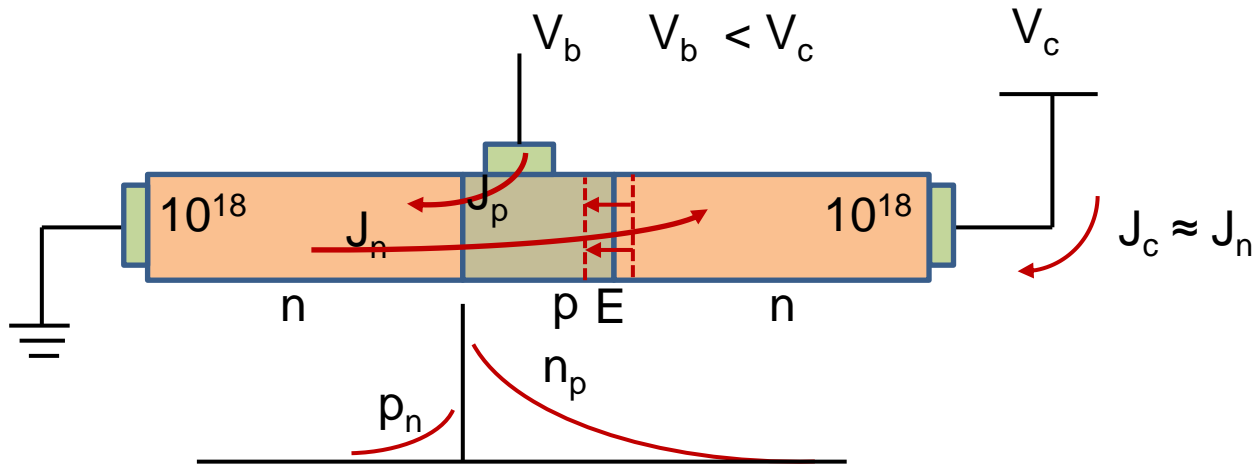
$$D_E \frac{\partial^2 [\delta p_E(x')]}{\partial x'^2} - \frac{\delta p_E(x')}{\tau_{E0}} = 0$$

$$\delta p_E(x') = p_E(x') - p_{E0}$$

General solution: $\delta p_E(x') = C \exp\left(\frac{+x'}{L_E}\right) + D \exp\left(\frac{-x'}{L_E}\right)$

$$L_E = \sqrt{D_E \tau_{E0}}$$

Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode
In the neutral emitter region

Boundary conditions: $\delta p_E(x' = 0) \equiv \delta p_E(0) = C + D$

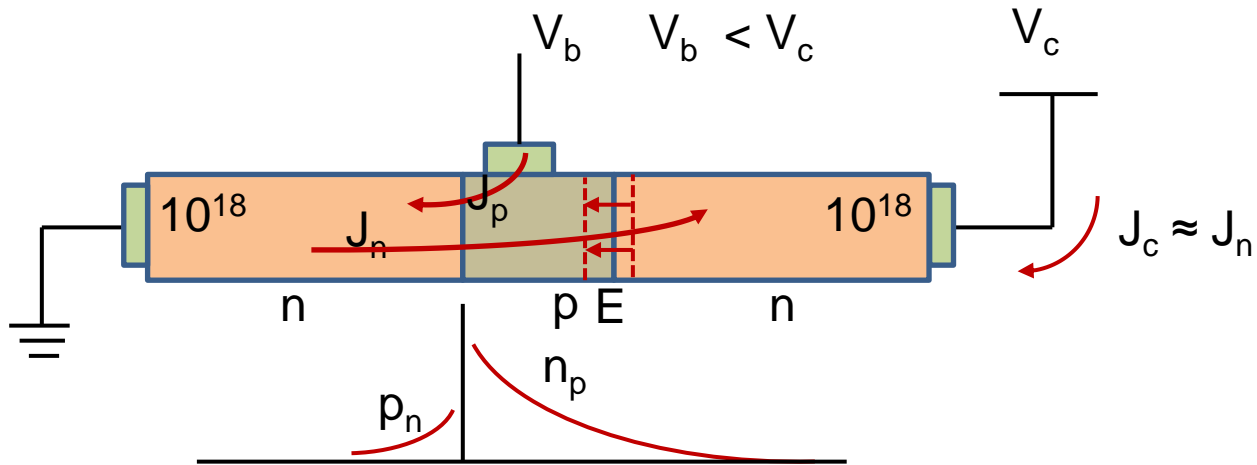
$$\delta p_E(x' = x_E) \equiv \delta p_E(x_E) = C \exp\left(\frac{x_E}{L_E}\right) + D \exp\left(\frac{-x_E}{L_E}\right)$$

BE junction forward biased: $\delta p_E(0) = p_E(x' = 0) - p_{E0} = p_{E0} \left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right]$

Assume infinite surface recombination velocity at $x' = x_E$

$$\delta p_E(x_E) = 0$$

Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode
In the neutral emitter region

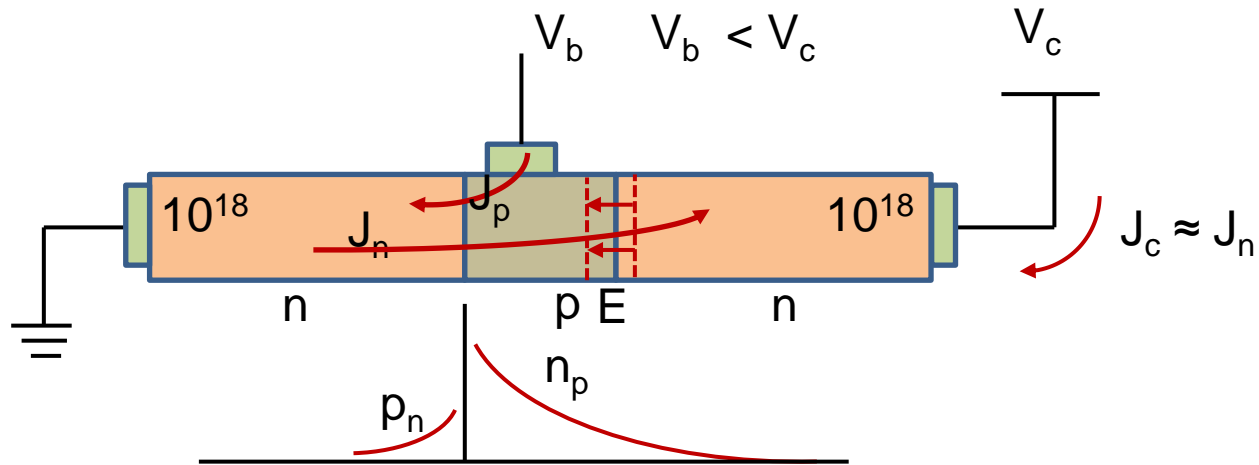
Boundary conditions:

$$\delta p_E(x') = \frac{p_{E0} \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] \sinh \left(\frac{x_E - x'}{L_E} \right)}{\sinh \left(\frac{x_E}{L_E} \right)}$$

Again, when x_E is small:

$$\delta p_E(x') \approx \frac{p_{E0}}{x_E} \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] (x_E - x')$$

Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode

In the neutral collector region

$$D_C \frac{\partial^2 [\delta p_C(x'')] }{\partial x''^2} - \frac{\delta p_C(x'')}{\tau_{C0}} = 0$$

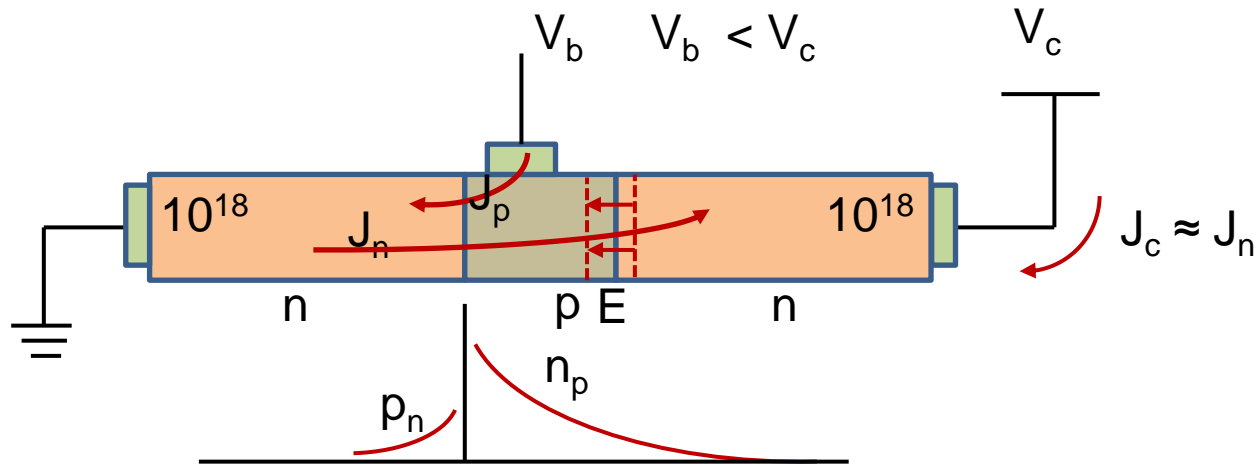
$$\delta p_C(x'') = p_C(x'') - p_{C0}$$

General solution:

$$\delta p_C(x'') = G \exp\left(\frac{x''}{L_C}\right) + H \exp\left(\frac{-x''}{L_C}\right)$$

$$L_C = \sqrt{D_C \tau_{C0}}$$

Quantitative analysis of BJT gain



Now consider actual npn device: forward active mode
In the neutral collector region

Boundary conditions:

Collector is long, and excess carrier concentration must be finite, so $G=0$

$$\delta p_c(x'' = 0) \equiv \delta p_c(0) = p_c(x'' = 0) - p_{c0} = 0 - p_{c0} = -p_{c0}$$

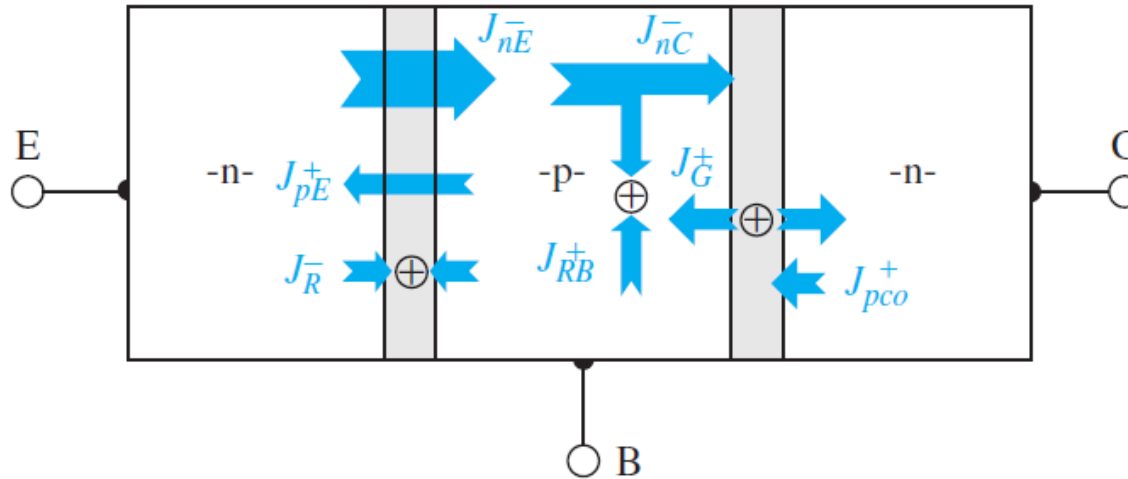
BC junction reverse biased:

$$\delta p_c(x'') = -p_{c0} \exp\left(\frac{-x''}{L_c}\right)$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

Electron and hole flux



J_{nE}^- is the electron flux injected from the emitter into the base

J_{RB}^+ is the replacement hole flux due to the recombination in the base

J_{nC}^- is the electron flux that reaches the collector

J_{pE}^+ is the holes from the base that are injected back into the emitter

J_R^- is the recombination in the space charge region of the forward-biased BE junction

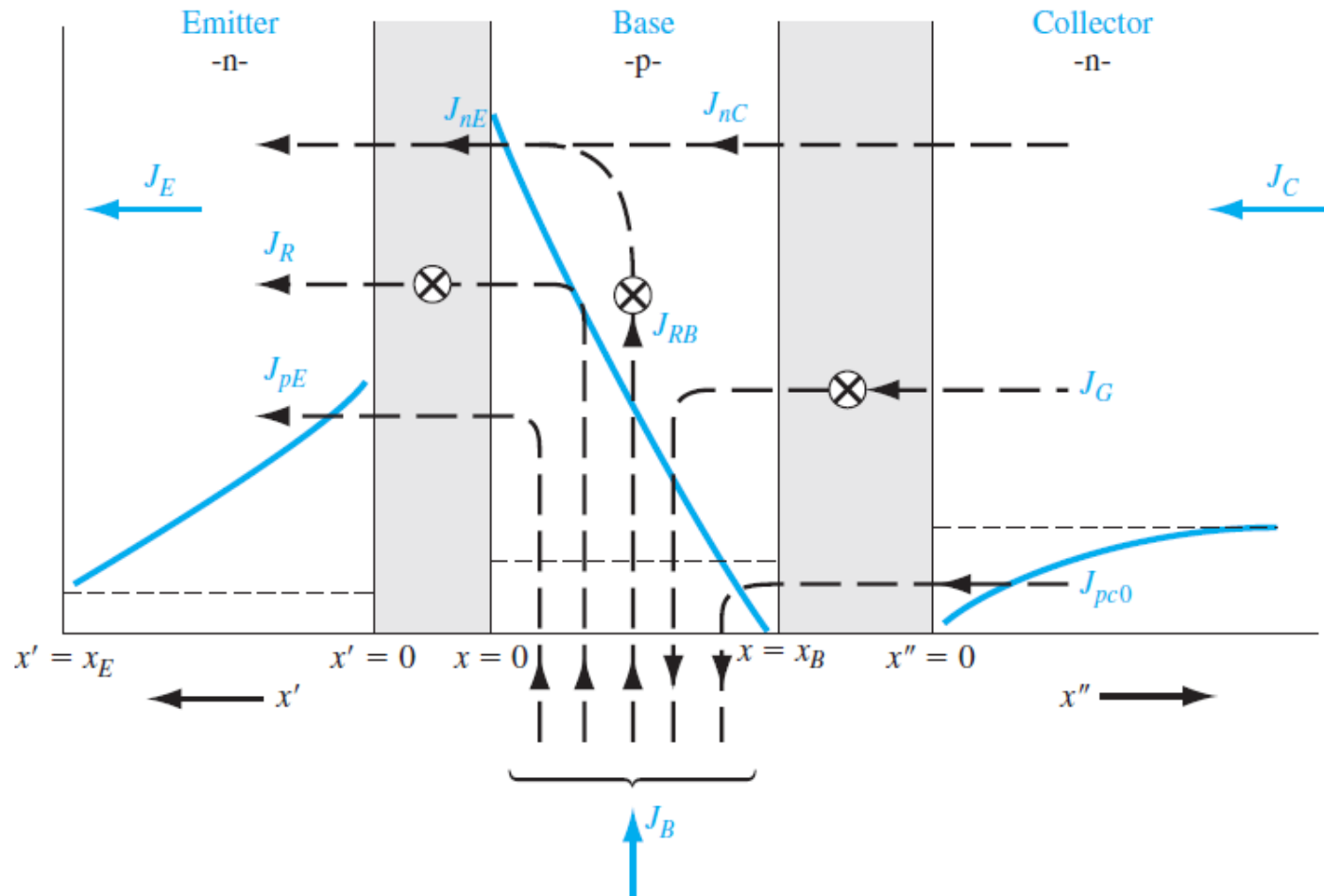
J_G^+ is the generation of electrons and holes in the reverse-biased BC junction

J_{pC0}^- is the ideal reverse-saturation current in the BC junction

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

Electron and hole current density



Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

Common-base current gain in DC $\alpha_0 = \frac{I_C}{I_E}$

$$\alpha_0 = \frac{J_C}{J_E} = \frac{J_{nC} + J_G + J_{pC0}}{J_{nE} + J_R + J_{pE}}$$

Small signal

$$\alpha = \frac{\partial J_C}{\partial J_E} = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$$

We can write

$$\alpha = \left(\frac{J_{nE}}{J_{nE} + J_{pE}} \right) \left(\frac{J_{nC}}{J_{nE}} \right) \left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \right)$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

We can write

$$\alpha = \gamma \alpha_T \delta$$

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}} \right) \quad \text{Emitter injection efficiency factor. Diffusion current } J_{pE} \text{ is not part of the collector current}$$

$$\alpha_T = \left(\frac{J_{nC}}{J_{nE}} \right) \quad \text{Base transport factor. Recombination of excess minority carrier electrons in the base}$$

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \quad \text{Recombination factor. Recombination in the forward-biased BE region}$$

We want $\alpha=1$, so every term should be close to 1

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\alpha = \gamma \alpha_T \delta$$

We can write

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}} \right) = \frac{1}{\left(1 + \frac{J_{pE}}{J_{nE}} \right)}$$

$$J_{pE} = -eD_E \left. \frac{d[\delta p_E(x')]}{dx'} \right|_{x'=0}$$

$$J_{nE} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=0}$$

$$J_{pE} = \frac{eD_E p_{E0}}{L_E} \left[\exp \left(\frac{eV_{BE}}{kT} \right) - 1 \right] \cdot \frac{1}{\tanh (x_E/L_E)}$$

$$J_{nE} = \frac{eD_B n_{B0}}{L_B} \left\{ \frac{1}{\sinh (x_B/L_B)} + \frac{[\exp (eV_{BE}/kT) - 1]}{\tanh (x_B/L_B)} \right\}$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\alpha = \gamma \alpha_T \delta$$

We can write

$$\gamma = \left(\frac{J_{nE}}{J_{nE} + J_{pE}} \right) = \frac{1}{\left(1 + \frac{J_{pE}}{J_{nE}} \right)}$$

Assume

$$\exp\left(\frac{eV_{BE}}{kT}\right) \gg 1$$

$$\frac{\exp(eV_{BE}/kT)}{\tanh(x_B/L_B)} \gg \frac{1}{\sinh(x_B/L_B)}$$

$$\gamma = \frac{1}{1 + \frac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\alpha = \gamma \alpha_T \delta$$

$$\gamma = \frac{1}{1 + \frac{p_{E0} D_E L_B}{n_{B0} D_B L_E} \cdot \frac{\tanh(x_B/L_B)}{\tanh(x_E/L_E)}}$$

For γ to be close to 1, we need $p_{E0} \ll n_{B0}$

$$p_{E0} = \frac{n_i^2}{N_E} \qquad n_{B0} = \frac{n_i^2}{N_B}$$

Then $N_E \gg N_B$

Many more electrons from the n-type emitter than holes from the p-type base will be injected across the BE space charge region

Assume $x_B \ll L_B$ and $x_E \ll L_E$

$$\gamma \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}}$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\alpha = \gamma \alpha_T \delta$$

$$\alpha_T = J_{nC} / J_{nE}$$

Since

$$J_{nC} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=x_B} \quad J_{nE} = (-)eD_B \left. \frac{d[\delta n_B(x)]}{dx} \right|_{x=0}$$

Then

$$J_{nC} = \frac{eD_B n_{B0}}{L_B} \left\{ \frac{[\exp(eV_{BE}/kT) - 1]}{\sinh(x_B/L_B)} + \frac{1}{\tanh(x_B/L_B)} \right\}$$

$$\delta n_B(x) = \frac{n_{B0} \left\{ \left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1 \right] \sinh\left(\frac{x_B - x}{L_B}\right) - \sinh\left(\frac{x}{L_B}\right) \right\}}{\sinh\left(\frac{x_B}{L_B}\right)}$$

Assume

$$V_{BE} \gg kT/e$$

$$\alpha_T = \frac{J_{nC}}{J_{nE}} \approx \frac{\exp(eV_{BE}/kT) + \cosh(x_B/L_B)}{1 + \exp(eV_{BE}/kT) \cosh(x_B/L_B)}$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\alpha = \gamma \alpha_T \delta$$

$$\alpha_T = \frac{J_{nC}}{J_{nE}} \approx \frac{\exp(eV_{BE}/kT) + \cosh(x_B/L_B)}{1 + \exp(eV_{BE}/kT) \cosh(x_B/L_B)}$$

We want α_T close to 1, so

$$x_B \ll L_B$$

$$\alpha_T \approx \frac{1}{\cosh(x_B/L_B)}$$

Taylor expansion:

$$\alpha_T \approx \frac{1}{\cosh(x_B/L_B)} \approx \frac{1}{1 + \frac{1}{2}(x_B/L_B)^2} \approx 1 - \frac{1}{2}(x_B/L_B)^2$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\alpha = \gamma \alpha_T \delta$$

Assume $J_{pE} \ll J_{nE}$

$$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}} \approx \frac{J_{nE}}{J_{nE} + J_R} = \frac{1}{1 + J_R/J_{nE}}$$

Recombination current density in a forward-biased pn junction

$$J_R = \frac{e x_{BE} n_i}{2\tau_0} \exp\left(\frac{eV_{BE}}{2kT}\right) = J_{r0} \exp\left(\frac{eV_{BE}}{2kT}\right)$$

$$J_{nE} = J_{s0} \exp\left(\frac{eV_{BE}}{kT}\right) \quad J_{s0} = \frac{eD_B n_{B0}}{L_B \tanh(x_B/L_B)}$$

$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

We want δ close to 1, so V_{BE} should be large enough

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

$$\beta_0 = I_C / I_B$$

Current $I_E = I_B + I_C$

$$\frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

Gain $\frac{1}{\alpha_0} = \frac{1}{\beta_0} + 1$

$$\beta = \frac{\alpha}{1 - \alpha}$$

$$\alpha = \frac{\beta}{1 + \beta}$$

Quantitative analysis of BJT gain

Now consider actual npn device: forward active mode

In summary

Emitter injection efficiency

$$\gamma \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}} \quad (x_B \ll L_B), (x_E \ll L_E)$$

Base transport factor

$$\alpha_T \approx \frac{1}{1 + \frac{1}{2} \left(\frac{x_B}{L_B} \right)^2} \quad (x_B \ll L_B)$$

Recombination factor

$$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Common-base current gain

$$\alpha = \gamma \alpha_T \delta \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} + \frac{1}{2} \left(\frac{x_B}{L_B} \right)^2 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Common-emitter current gain

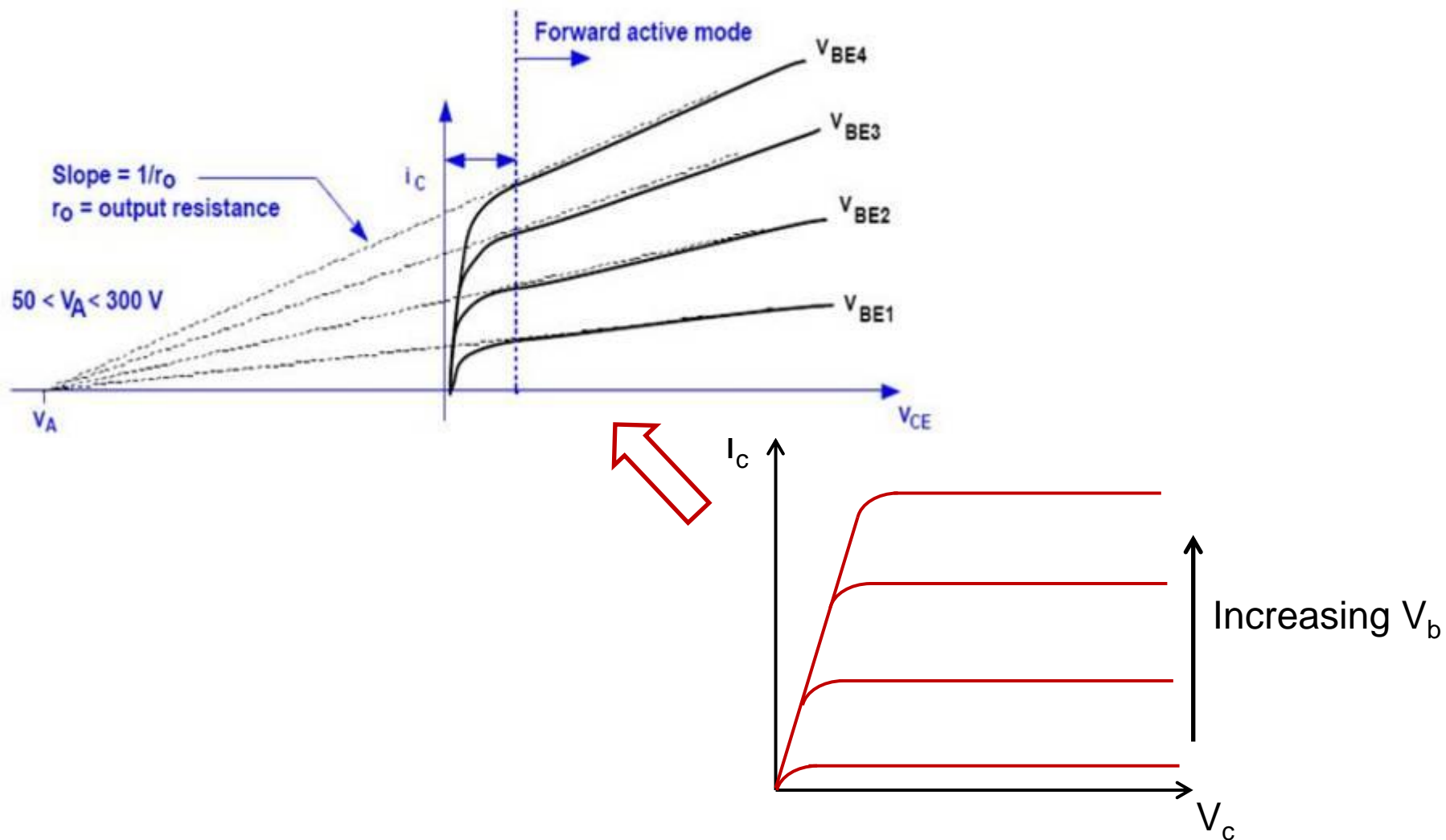
$$\beta = \frac{\alpha}{1 - \alpha} \approx \frac{1}{\frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E} + \frac{1}{2} \left(\frac{x_B}{L_B} \right)^2 + \frac{J_{r0}}{J_{s0}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}$$

Quantitative analysis of BJT gain

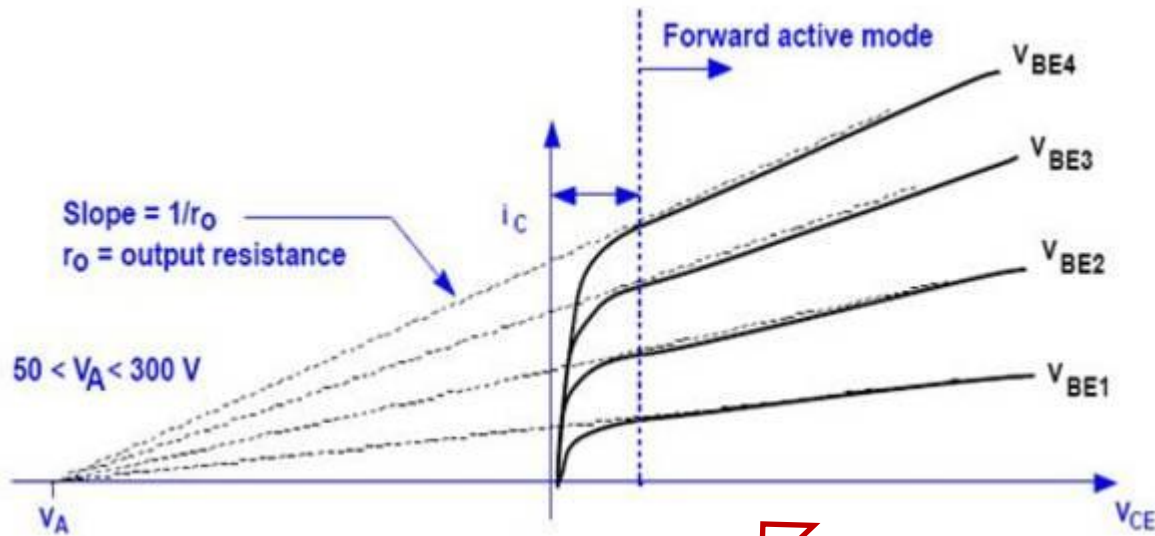
Example:

If we assume a typical value of β to be 100, then $\alpha = 0.99$. If we also assume that $\gamma = \alpha_T = \delta$, then each factor would have to be equal to 0.9967 in order that $\beta = 100$. This calculation gives an indication of how close to unity each factor must be in order to achieve a reasonable current gain.

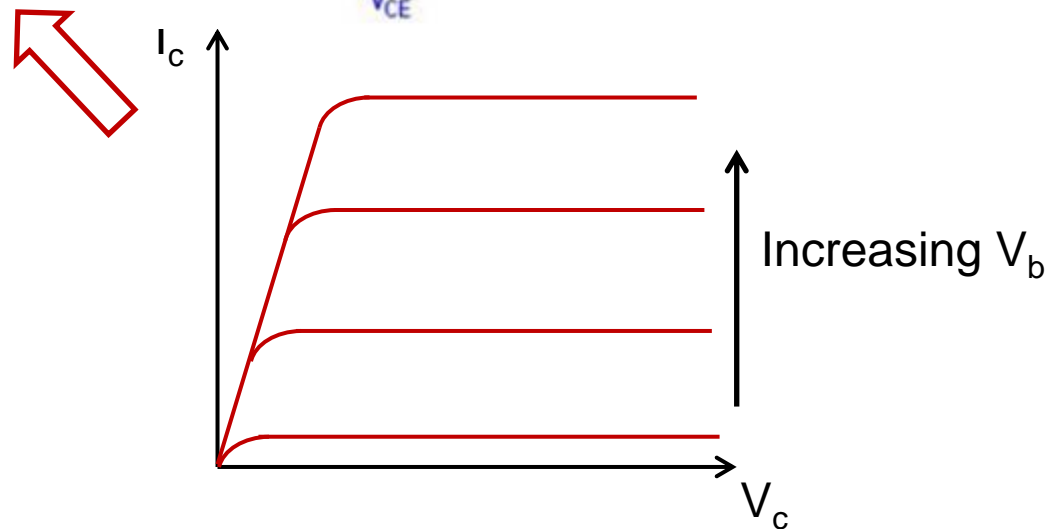
Early Effect



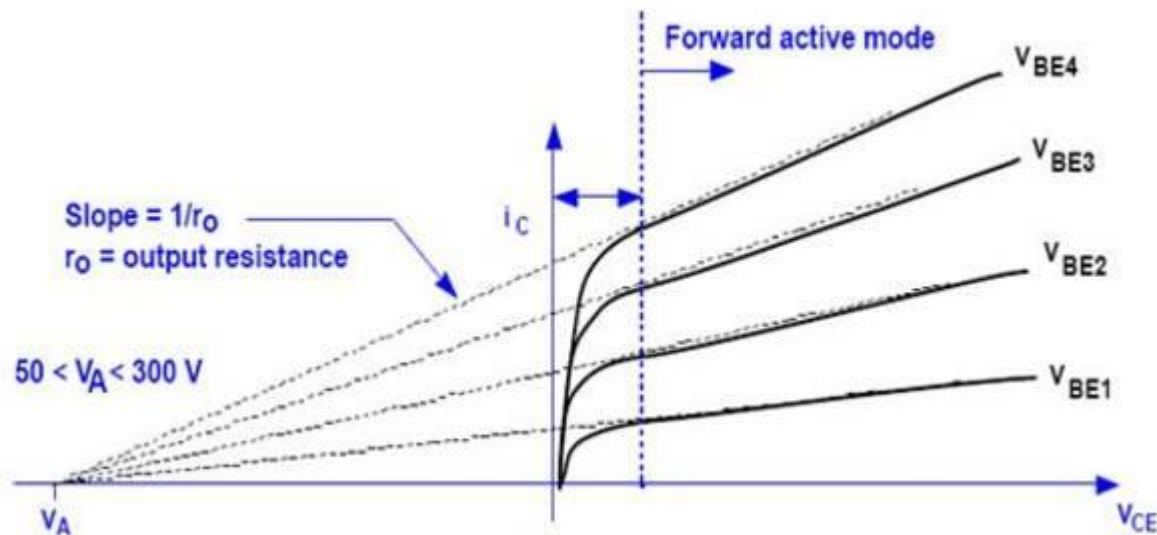
Early Effect



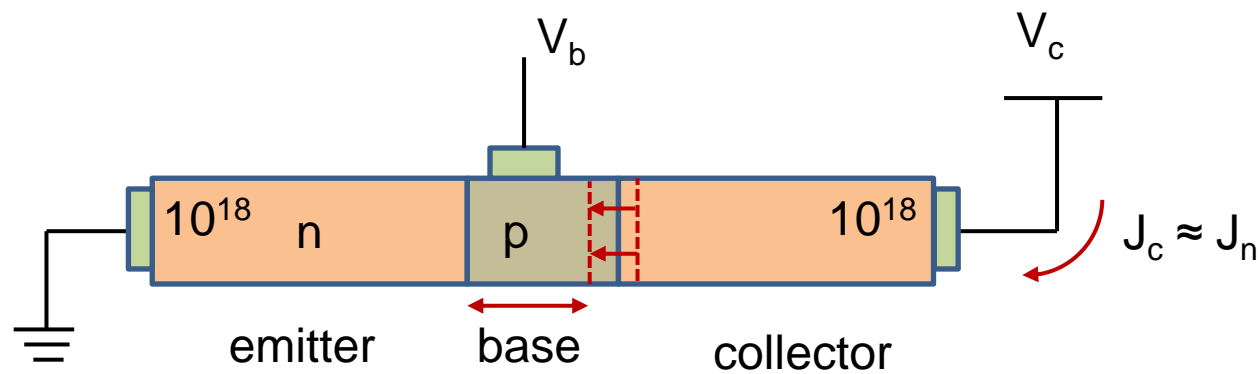
James M. Early



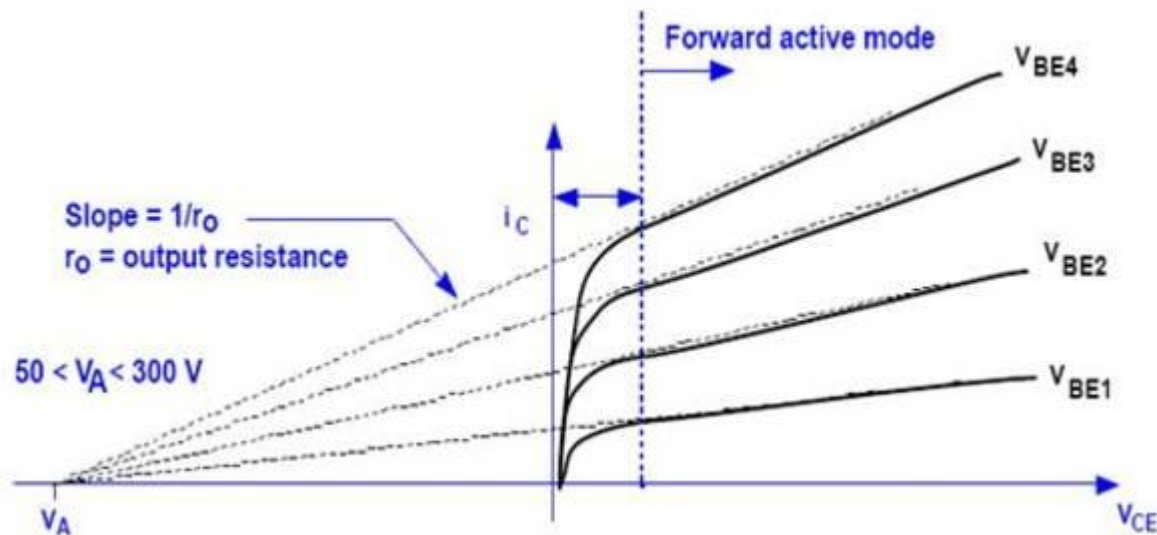
Early Effect



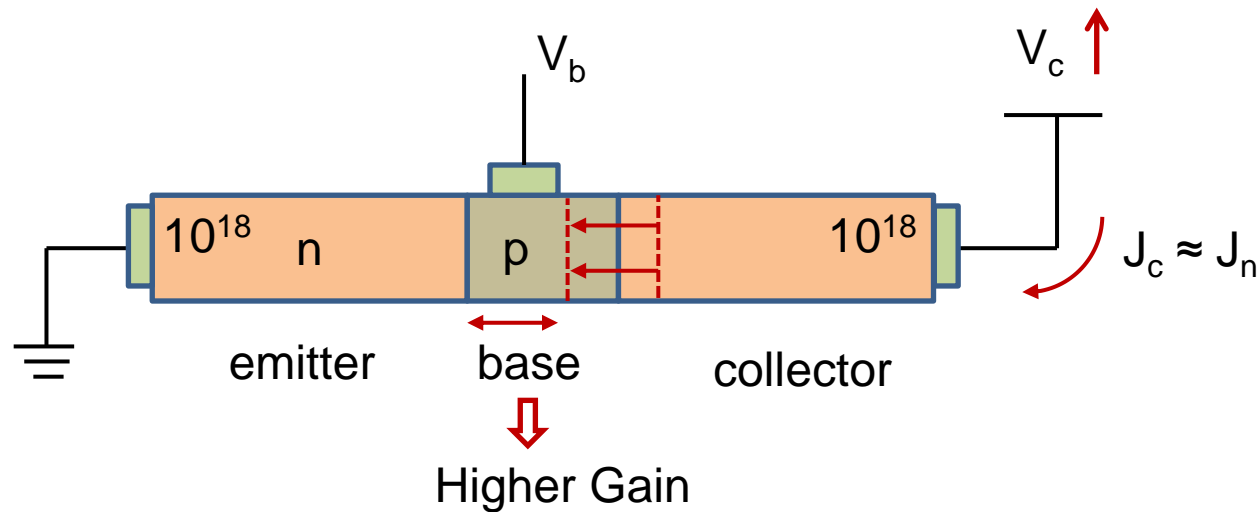
James M. Early



Early Effect



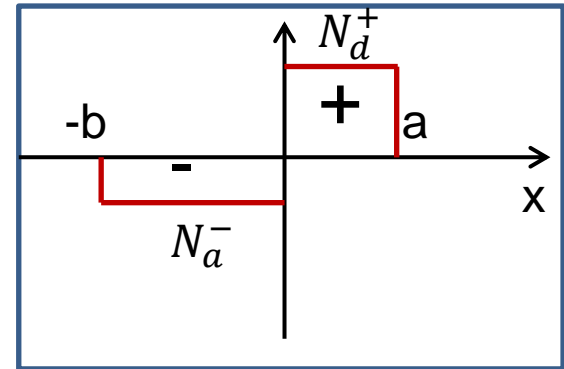
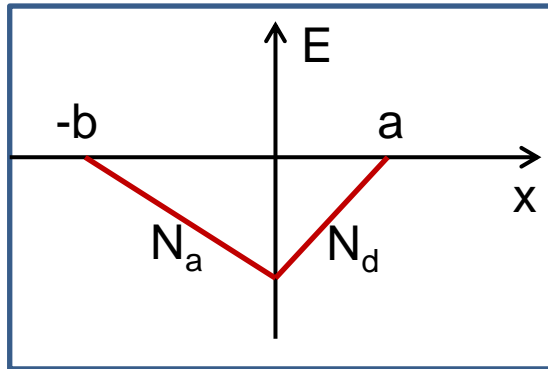
James M. Early



Previously...

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{N_a}{N_d} \frac{1}{N_a + N_d}}$$

$$N_a^- b = N_d^+ a \Rightarrow b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{N_d}{N_a} \frac{1}{N_a + N_d}}$$



Previously...

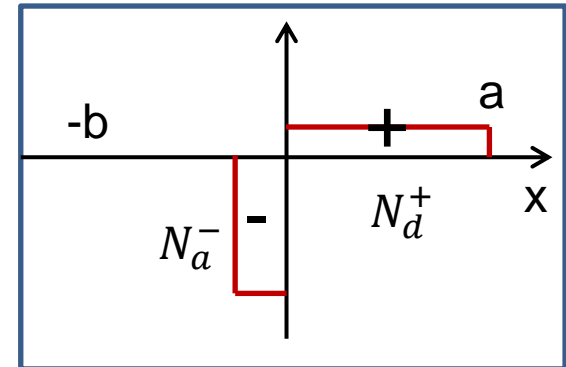
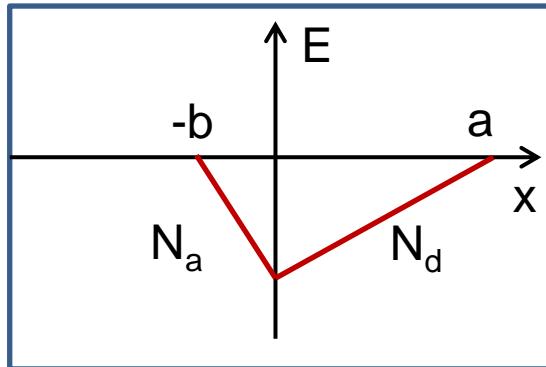
$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{N_a}{N_d} \frac{1}{N_a + N_d}}$$

$$N_a^- b = N_d^+ a \Rightarrow b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{N_d}{N_a} \frac{1}{N_a + N_d}}$$

$$N_a = 100 N_d$$

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{\cancel{100N_a}}{N_d} \frac{1}{\cancel{100N_a}}}$$

$$b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{\cancel{N_a}}{100\cancel{N_a}} \frac{1}{100N_d}}$$



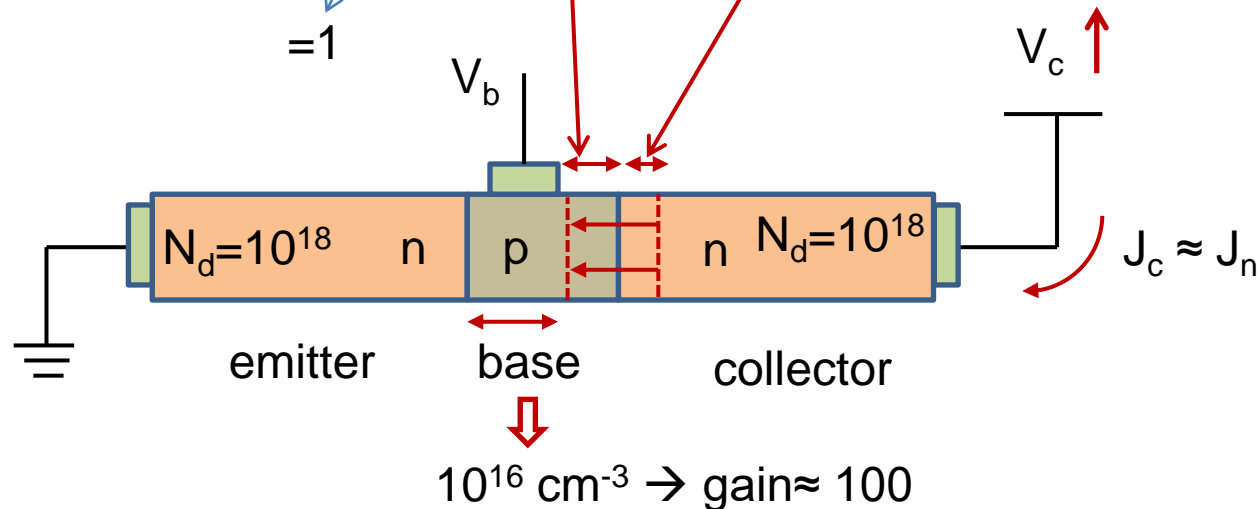
Early Effect

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{10^{16}}{10^{18}} \frac{1}{10^{16} + 10^{18}}} = 1/10000$$

$$b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{10^{18}}{10^{16}} \frac{1}{10^{16} + 10^{18}}} = 1$$



James M. Early



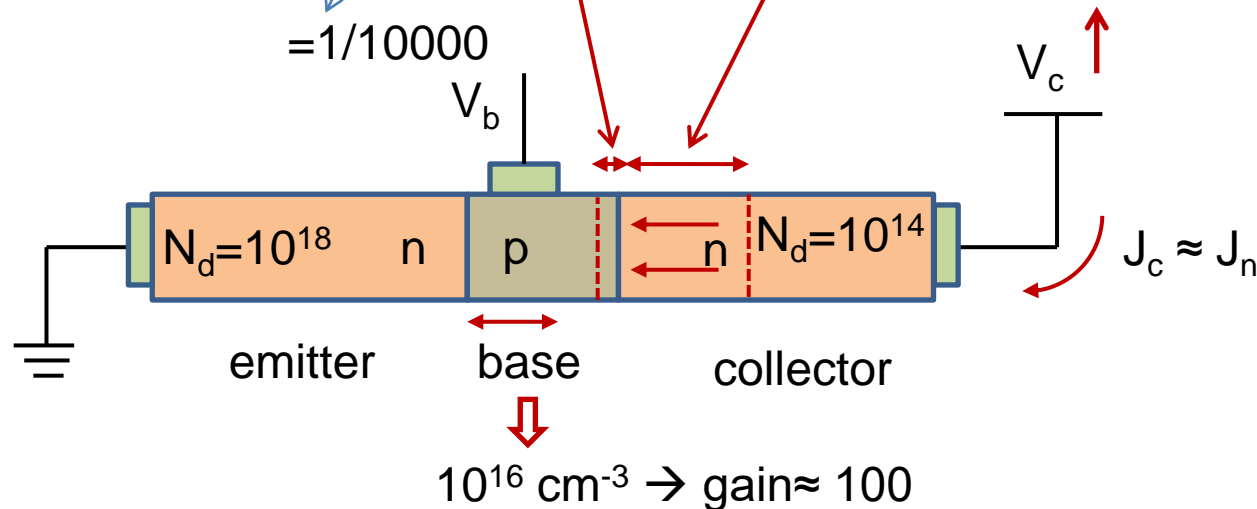
Early Effect

$$a = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{10^{16}}{10^{14}} \frac{1}{10^{16} + 10^{14}}} = 1$$

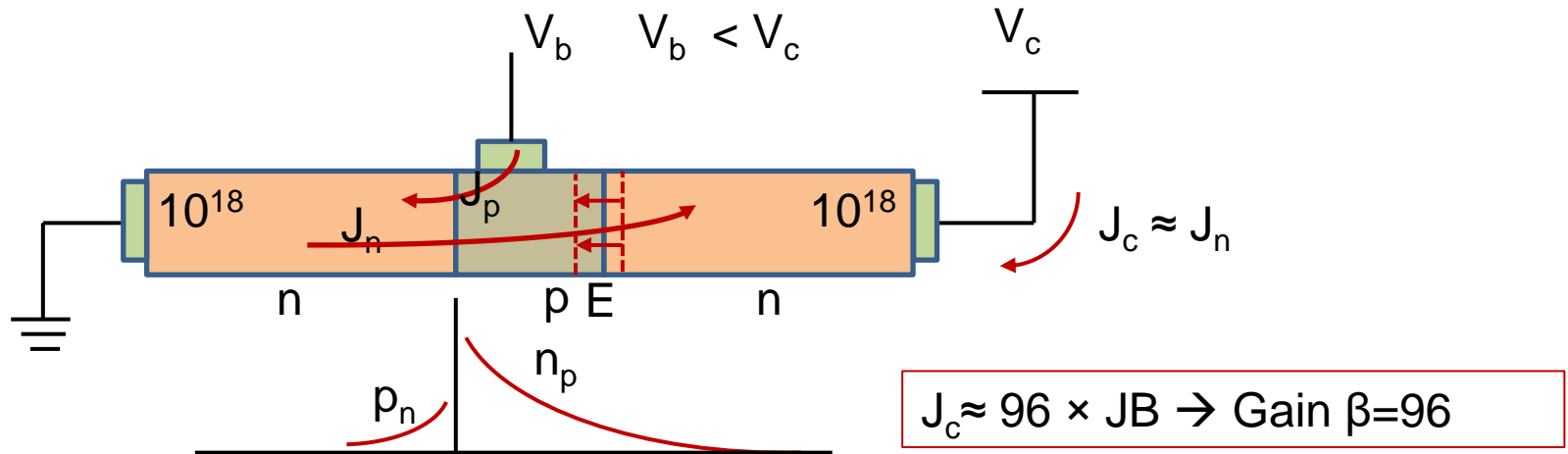
$$b = \sqrt{\frac{2\varepsilon(V_{bi}-V_R)}{e} \frac{10^{14}}{10^{16}} \frac{1}{10^{16} + 10^{14}}} = 1/10000$$



James M. Early



Summary



1. highest doping concentration is limited by solubility ($<10^{20}$)
2. Lowest doping concentration is limited by n_i and fabrication process

Basic facts:

1. Narrower base \rightarrow larger gain
2. $\beta \approx N_d/N_a$, higher emitter-to-base doping ratio \rightarrow higher gain
3. Trade-off for base doping concentration (gain and Early effect)

Breakdown

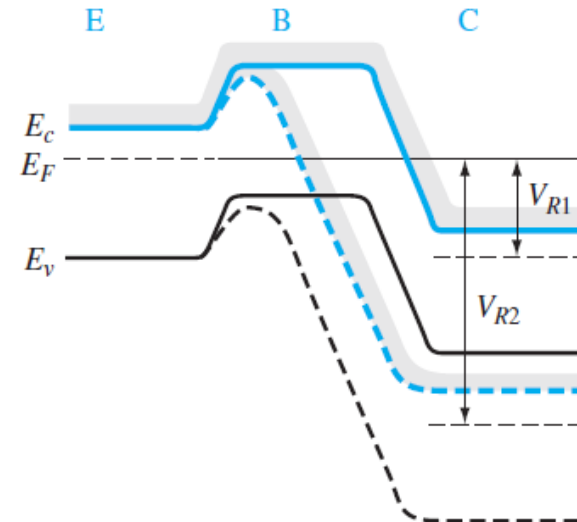
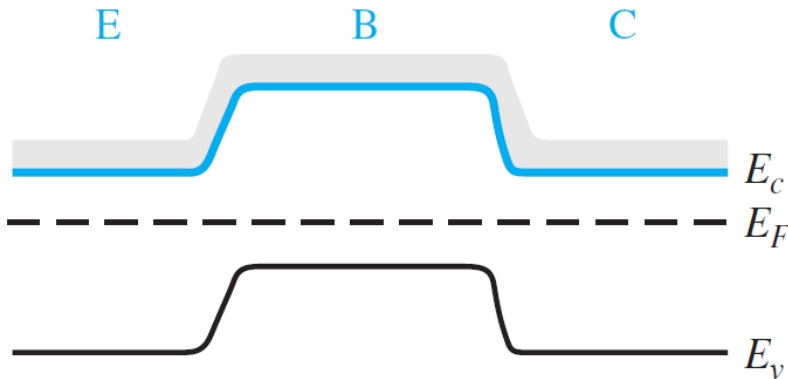
2 breakdown mechanisms:

1. Punch through
2. Avalanche breakdown: with transistor gain

Punch through: large BC junction reverse bias voltage

Depletion region extends through the base region

BE potential barrier lowered, and produce a large current

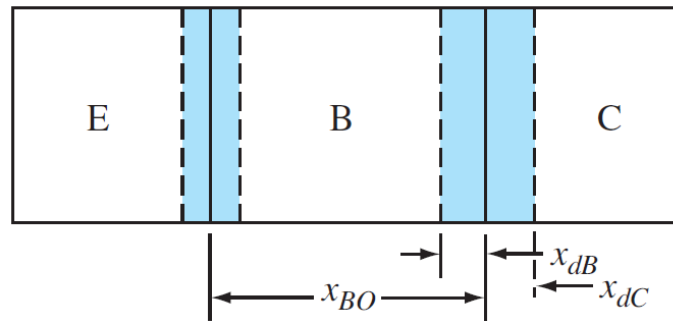


Breakdown

Punch-through: large BC junction reverse bias voltage

Depletion region extends through the base region

BE potential barrier lowered, and produce a large current



$$x_{dB} = x_{BO} = \left\{ \frac{2\epsilon_s(V_{bi} + V_{pt})}{e} \cdot \frac{N_C}{N_B} \cdot \frac{1}{N_C + N_B} \right\}^{1/2}$$

Ignore V_{bi}

$$V_{pt} = \frac{ex_{BO}^2}{2\epsilon_s} \cdot \frac{N_B(N_C + N_B)}{N_C}$$

Large breakdown voltage: low N_C