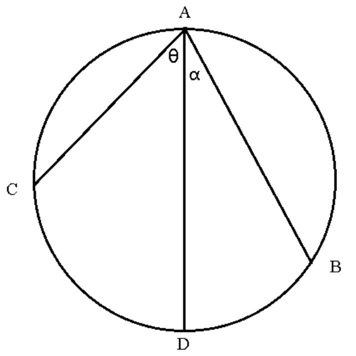
1. The pathway AB, AC, AD are all frictionless, and the diameter of the ring is d. Try to find t_{AB}, t_{AC} and t_{AD}.



AD: $x_{AD} = d = \frac{1}{2} g t_{AD}^2$

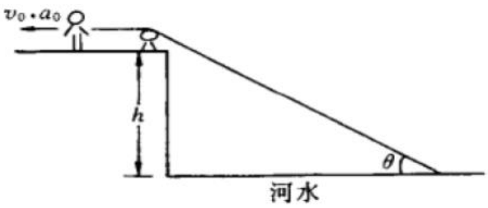
AB: $x_{AB} = d \cos\alpha = \frac{1}{2} (g \cos\alpha) t_{AB}^2$

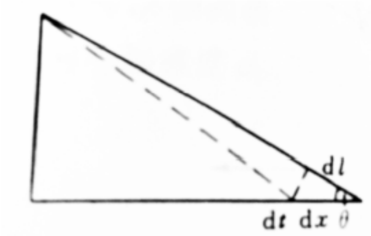
$d \cancel{\cos\alpha} = \frac{1}{2} (g \cancel{\cos\alpha}) t_{AB}^2$

$\Rightarrow t_{AD} = t_{AB}$

$\Rightarrow t_{AD} = t_{AB} = t_{AC} = \sqrt{\frac{2d}{g}}$

2. As shown in the figure, a person is dragging a boat through the string. At this instant, the person has a speed v_0 and an acceleration a_0. Find the speed v and the acceleration a of the boat at this moment. h and θ are given.





velocity:

Considering an infinitely small change as shown in the figure on the right-hand side.

$$\mathrm{d}l = \mathrm{d}x\cos\theta$$

$$v_0 = \frac{\mathrm{d}l}{\mathrm{d}t} \qquad \Rightarrow v = \frac{v_0}{\cos\theta} \qquad \text{points to left}$$

$$v = \frac{\mathrm{d}x}{\mathrm{d}t}$$

acceleration:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \dot{\theta} = \frac{v_\theta(\text{traversal component})}{\frac{h}{\sin\theta}} = \frac{v_0\tan\theta\sin\theta}{h}$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}(\frac{v_0}{\cos\theta})}{\mathrm{d}t} = v_0\frac{\mathrm{d}(\frac{1}{\cos\theta})}{\mathrm{d}t} + a_0\frac{1}{\cos\theta}$$

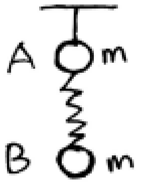
$$= v_0[-(\cos\theta)^{-2}]\cdot(\frac{\mathrm{d}\theta}{\mathrm{d}t}) + a_0\frac{1}{\cos\theta} = \frac{v_0^2\tan^3\theta}{h} + \frac{a_0}{\cos\theta} \qquad \text{points to left}$$

3. Two identical balls A and B with mass m is connected to two sides of a spring with initial length L_0, A is connected to the ceiling with a string. The spring stretches to length L because of the gravity.

(1) At the moment that the string is cut through, what is the acceleration of A and B?

(2) What motion will A and B do then?

(3) If the length of the spring return to L_0 the moment B reaches the ground, find the height of A before the string is cut.



1. $a_A = 2g$, downward

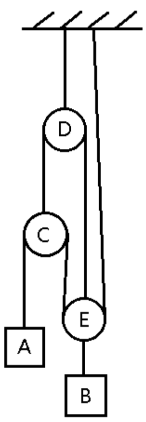
   $a_B = 0$

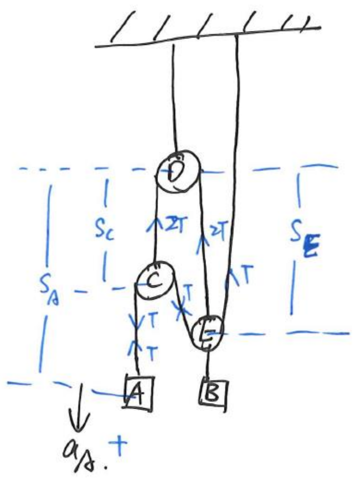
2. System A.B: free fall

   A and B: harmonic motion w.r.t the center

3. $$\left.\begin{aligned} &T = 2\pi\sqrt{\frac{m}{2k}}, \\ &mg = k(L-L_0) \\ &t = \left(\frac{N}{2}+\frac{1}{4}\right)T \\ &h = \frac{1}{2}gt^2 + \frac{L}{2} + \frac{L_0}{2} \end{aligned}\right\} \Rightarrow h$$

4. Neglect m_C, m_D, m_E, m_{string}, $m_A=m_B=m_0$, Find $\mathbf{a_A}$ and $\mathbf{a_B}$.





① $\begin{cases} S_C + S_E = const \\ S_E + (S_E - S_C) + (S_A - S_C) = const \end{cases}$

$\Rightarrow S_A + 4S_E = const$

$\Rightarrow \frac{d}{dt}(S_A + 4S_E) = \frac{d}{dt}(const) = 0.$

$\Rightarrow v_A + 4v_E = 0$

$\Rightarrow a_A + 4a_E = 0,$

② $\begin{cases} m_A g - T = mg - T = m_A a_A = m a_A. \\ m_B g - 4T = mg - 4T = m_B a_B = m a_B \end{cases}$

$\Rightarrow a_A + 4a_B = g - \frac{T}{m_0} + 4(g - \frac{4T}{m_0}) = 0.$

$\Rightarrow T = \frac{5}{17} m_0 g.$

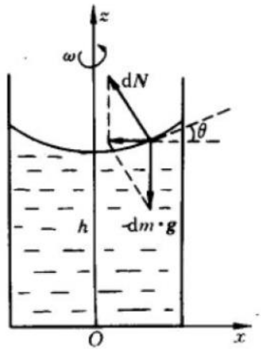
$\Rightarrow$

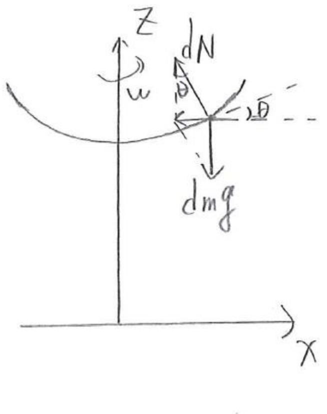
$a_A = g - \frac{T}{m_A} = \frac{12}{17} g$

$a_B = g - \frac{4T}{m_B} = -\frac{3}{17} g$ ✓

or $a_B = \frac{3}{17} g$ points upwards ✓

5. A bucket of water is rotating about its central vertical axis at constant angular velocity ω. Try to prove that when the water is static relative to the bucket, the upper surface of the water is paraboloid.





$\begin{cases} dN\cos\theta = dm\,g & -① \\ dN\sin\theta = (dm)\,\omega^2 x & -② \end{cases}$

$\frac{②}{①}$: $\tan\theta = \frac{\omega^2 x}{g}$

$\tan\theta = \frac{dz}{dx}$

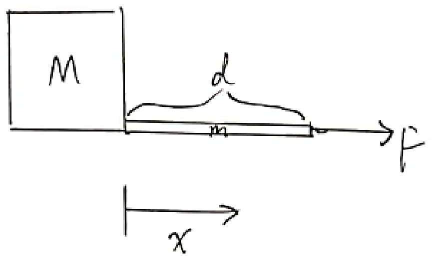
$\Rightarrow \frac{\omega^2 x}{g} = \frac{dz}{dx}$

$\int \frac{\omega^2}{g} x\,dx = \int dz \Rightarrow z = \frac{\omega^2}{g} \cdot \frac{x^2}{2} + c$

when $x = 0$, $z = h$ $\Rightarrow c = h$ $\quad z = \frac{\omega^2}{2g} x^2 + h$

6. Suppose that a uniform rope with mass m and length d, placed on a horizontal table, is attached to a block with mass M resting on the

same table. The rope is pulled from the side opposite the block with an applied horizontal force of magnitude F, and the system moves with acceleration. The coefficient of kinetic friction between the block and the surface is k, and there **does exist** friction between the rope and the surface. Find the tension in the rope as a function of the distance from the block.



Whole: $\begin{cases} F - \mu N = (m+M)a \\ N = mg + Mg \end{cases} \Rightarrow a = \dfrac{F - \mu(mg+Mg)}{m+M}$

M & $\frac{x}{d}m$: $\begin{cases} T_x - \mu N = (M + \frac{x}{d}m)a \\ N = Mg + \frac{x}{d}mg \end{cases} \Rightarrow T_x = (M + \frac{x}{d}m)\dfrac{F - \mu(mg+Mg)}{M+m}$

$+ \mu(Mg + \frac{x}{d}mg)$