<center>**Part A**</center>

Problem 1

Find the $rref$ and the rank of the following matrices:

$$\boldsymbol{a.} \quad \begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix} \quad \boldsymbol{b.} \quad \begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix}$$

Show your work.

Problem 2
Apply Gauss-Jordan elimination to solve the linear system

$$\begin{cases} 3x_1 & - 2x_2 & - 5x_3 & + x_4 = 3 \\ 2x_1 & - 3x_2 & + x_3 + 5x_4 = -3 \\ x_1 & + 2x_2 & - 4x_4 = -3 \\ x_1 & - x_2 & - 4x_3 & + 9x_4 = 22 \end{cases}$$

Show your work.

Problem 3
Show that the linear systems

$$\boldsymbol{a.} \quad \begin{cases} x_1 & + 2x_2 & - x_3 = 0 \\ 2x_1 & + 9x_2 & - 3x_3 = 0 \end{cases} \quad \boldsymbol{b.} \quad \begin{cases} 3x_1 + x_2 & - 8x_3 & + 2x_4 & + x_5 = 0 \\ 2x_1 & - 2x_2 - 3x_3 & - 7x_4 + 2x_5 = 0 \\ x_1 + 11x_2 - 12x_3 + 34x_4 - 5x_5 = 0 \\ x_1 & - 5x_2 & + 2x_3 - 16x_4 + 3x_5 = 0 \end{cases}$$

have infinitely many solutions and find those solutions.

Problem 4
For which values of the constants b and c is the vector $(3, b, c)$ a linear combination of the vectors $(1,3,2), (2,6,4), (-1,-3,-2)$?

Problem 5
Bretscher 1.2.38, p. 20

Problem 6
Show that the following sets are linear spaces:

a. the set $M_2(\mathbb{Q})$ of all square matrices over $\mathbb{Q}$
b. the set $\mathbb{Z}_p[x]$ of all polynomials with coefficients from $\mathbb{Z}_p$ where p is a prime.

Part B

Type the following commands in MATLAB/MATLAB Online and save the inputs and outputs/m.
files/graphs as a .pdf file

Operation	Type	Comments
To enter a matrix row by row	$A=$ [1 2 3 4; 5 6 7 8; 9 8 7 6; 5 4 3 2]	A space between row entries and a semicolon between rows
To create the identity matrix	$I=eye(5)$	What is the size of the obtained matrix?
To create a random square matrix	$B=rand(4)$	
To create a random matrix 7×6	$C=rand(7,6)$	
To round the entries of C to integers	$C=round(C)$	
To change the entree of the matrix	$C(3,4)=7$	To change $c_{34} \rightarrow 7$
To change the entries of the matrix	$C(1,2:3)=[5,6]$	To change the entries $c_{12} \rightarrow 5$ and $c_{13} \rightarrow 6$
To create a matrix of zeroes	$O=zeros(3)$ $O1=zeroes$ (3,5)	
To create a matrix of entries 1	$U=ones(3)$ $U1=ones(3,5)$	
To change the row of A	$A(3,:) = [0 -1 0 1]$	
To create a diagonal matrix	$D=diag([1\ 2\ 3\ 4])$	
To create the diagonal matrix from A	$D1=diag(diag(A))$	What happens if you type $diag(A)$?
To create a lower triangular matrix	$L=tril(A,k)$	Use $k=-2,-1,0,1,2$
To create an upper triangular matrix	$U=triu(A,k)$	Use $k=-2,-1,0,1,2$
To create the transpose matrix of A	A'	
To create a row-vector	$a=[1\ 2\ 3\ 4]$	A space between row entries
To create a column vector	$a1=[-4\ 3\ -2\ 1]'$	
To create a matrix with columns $a^T, a1$	$aa1=[a'\ a1]$	
To create a vector a vector with equispaced elements	$x=1:2:11$	The syntax is $x=1st\ value:step:end\ value$
To create a vector with Nv elements with the given start and end elements	$y=linspace(0,0.5,10)$	$y=linspace(start,end,Nv)$
To obtain the reduced row echelon form of the matrix A	$rref(A)$	
To find the rank of the matrix A	$Rank(A)$	
To find the sum of two matrices	$A+B$	$A4\times4, B4\times4$
To find a scalar product	$10*C$	
To find the nth power of a matrix	$A\text{^}5$	
To solve a linear system $E\bar{x}=F$	$X=E\backslash F$	Check the cases 1. $E=[1\ 4\ 1; 4\ 13\ 7; 7\ 22\ 13]$ $F=[0\ 0\ 1]$ 2. $E=[1\ 1\ -1; 4\ -1\ 5; 6\ 1\ 4]$ $F=[0\ 0\ 0]$