VE 320 Fall 2021

Introduction to Semiconductor Devices

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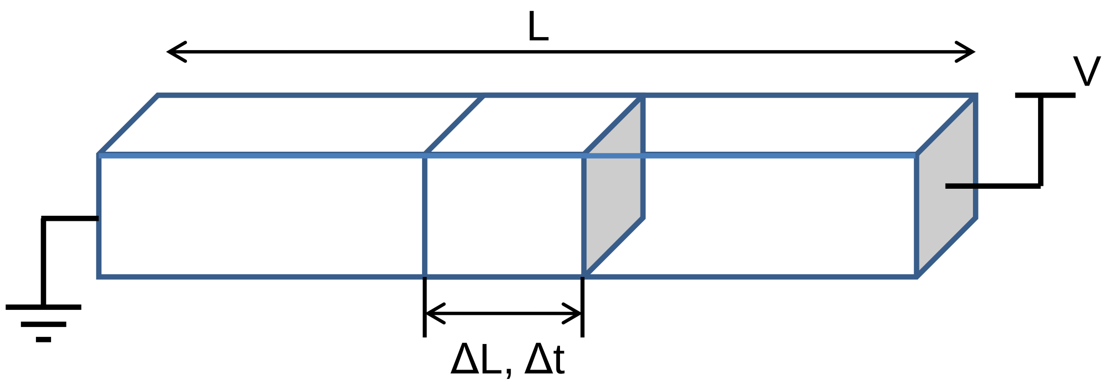
Lecture 3

Introduction to quantum theory of solids (Chapter 3)

Carrier transport



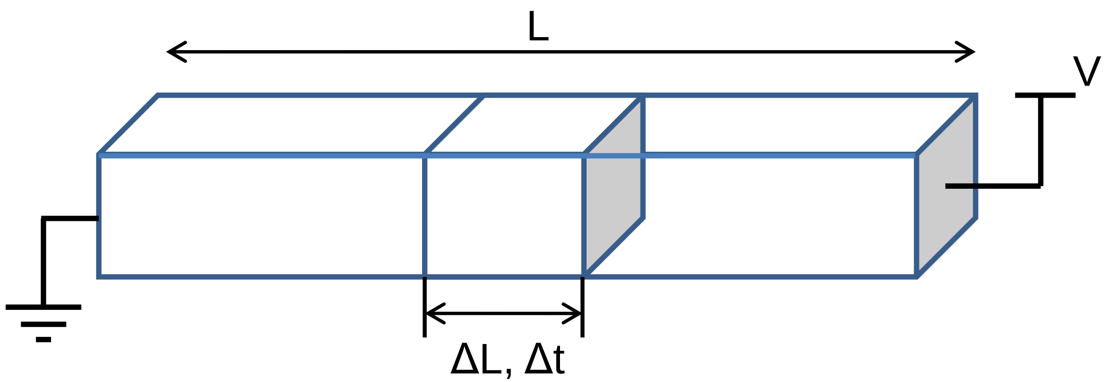
$n\ type\ semiconductor$

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c v$$

$$v = \mu E = \mu V/L$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c\mu V/L \qquad \Rightarrow \qquad \sigma = \frac{I}{V} = \frac{nqA_c\mu}{L}$$

Carrier transport



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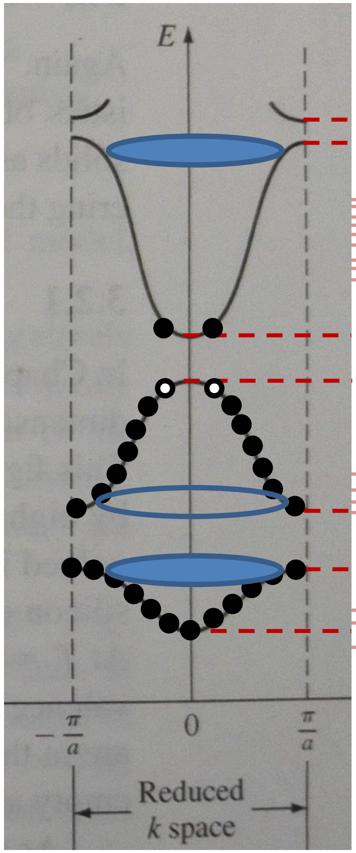
$$v = \mu E = \mu V/L$$

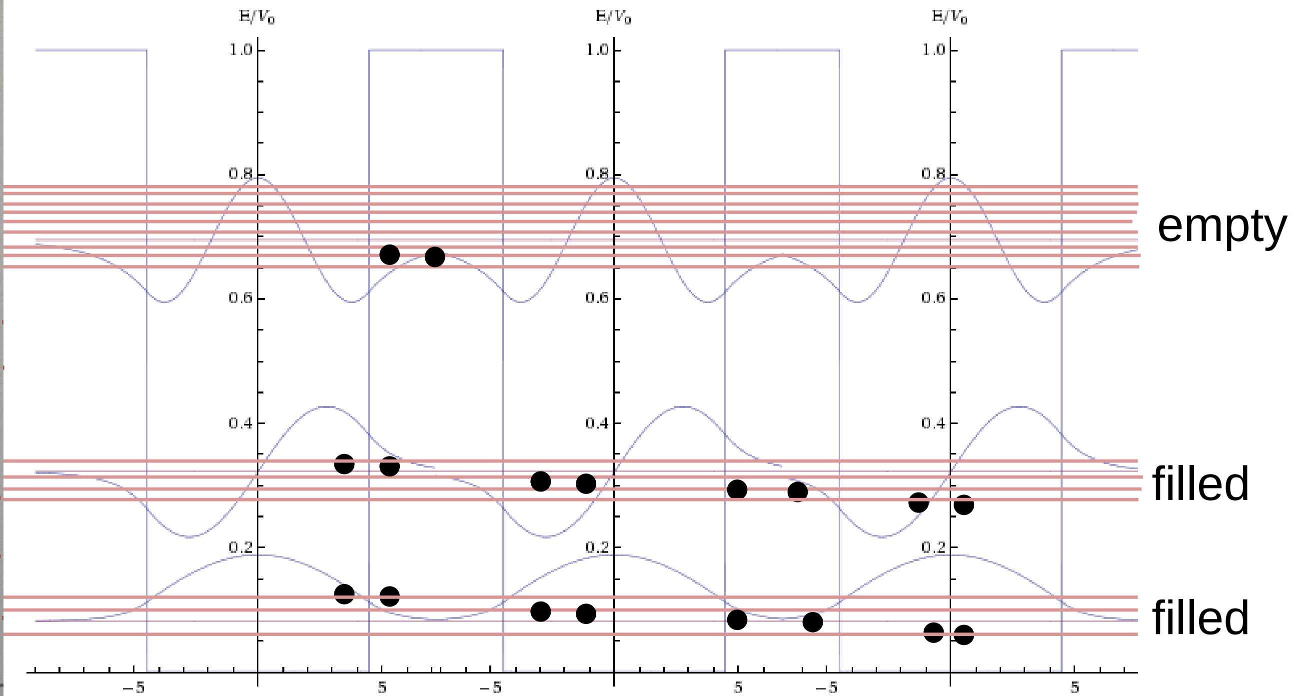
$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c\mu V/L \quad \Rightarrow \quad \sigma = \frac{I}{V} = \frac{N_D qA_c\mu}{L}$$

What if the semiconductor is intrinsic…

- Intrinsic (undoped)
- Electrons are all from the valance

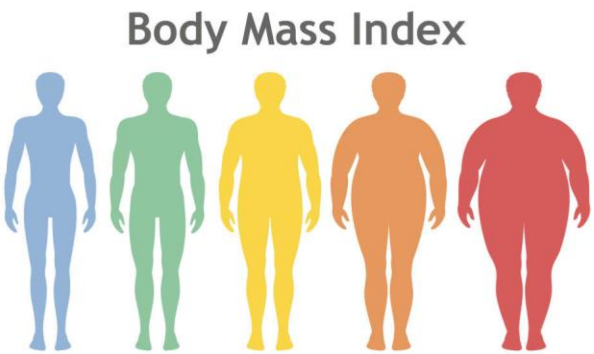


In k space

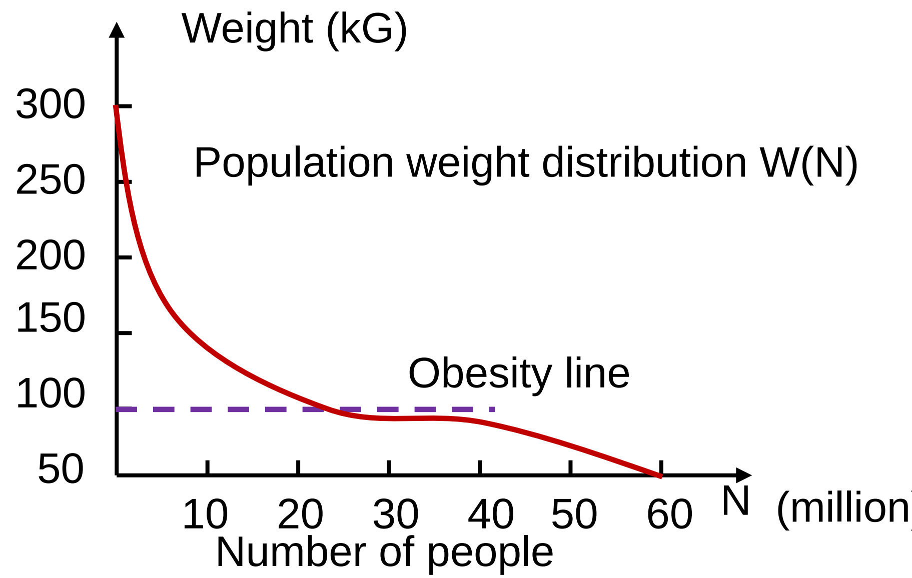


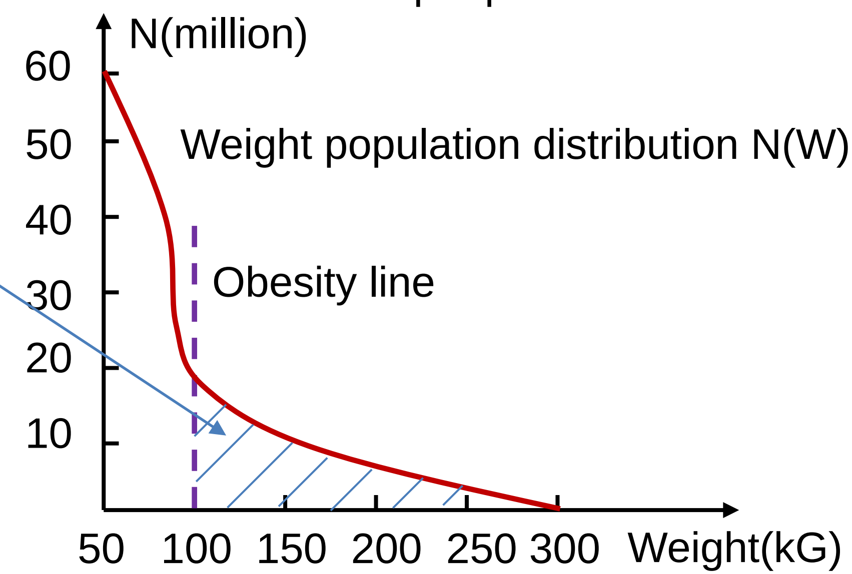
<u>How many number of electrons in the conductance band?</u>

Case study: Obesity density of population

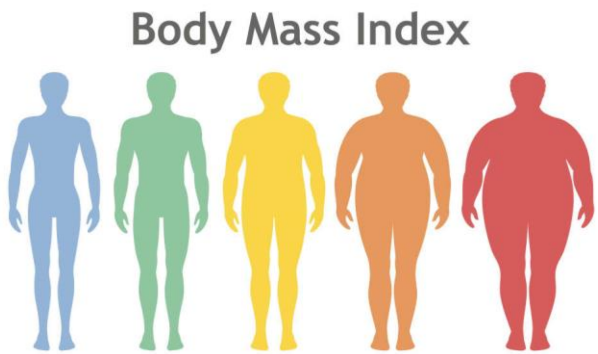


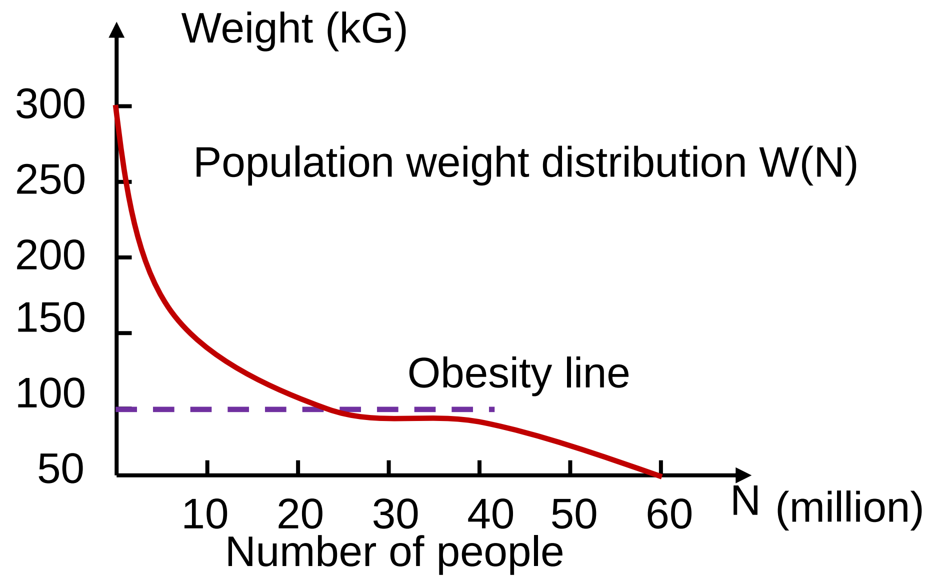
Can you find the number of people with obesity ?

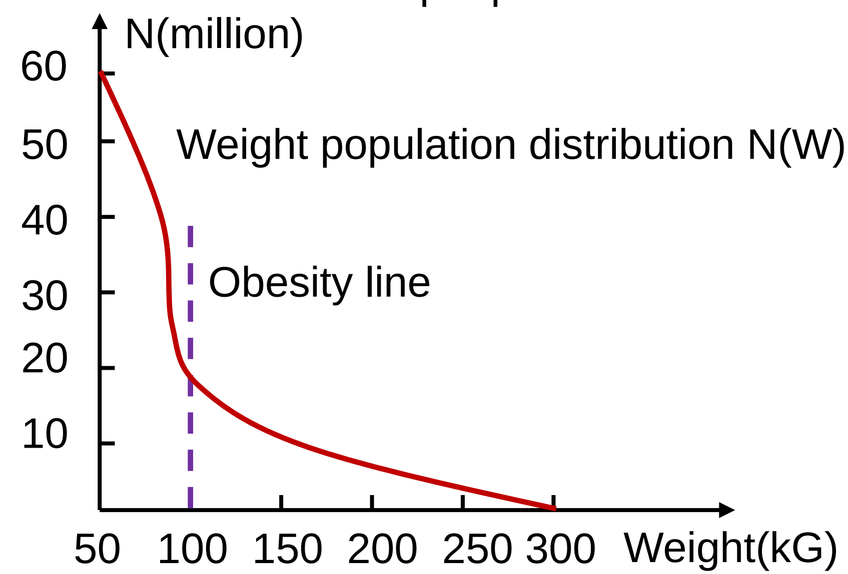




Case study: Obesity density of population







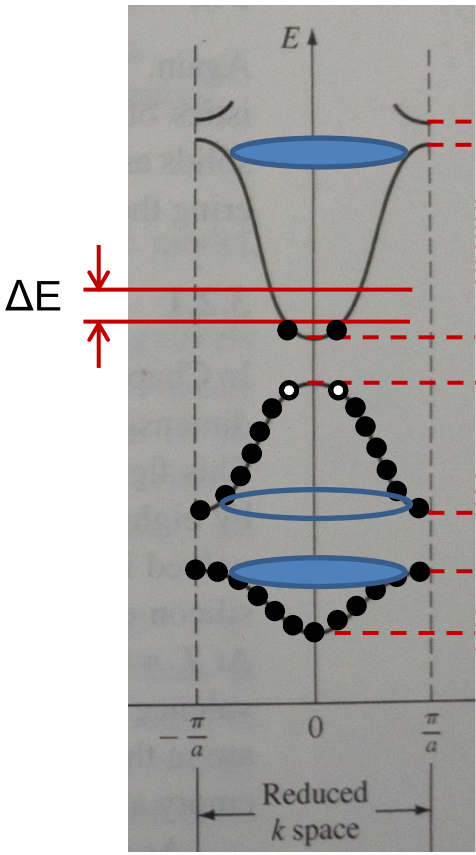
Can you find the increased number of people with obesity if the weight is increased by one unit?

$$g(W) = \frac{dN}{dW}$$

Obesity density of population

What if the semiconductor is intrinsic…

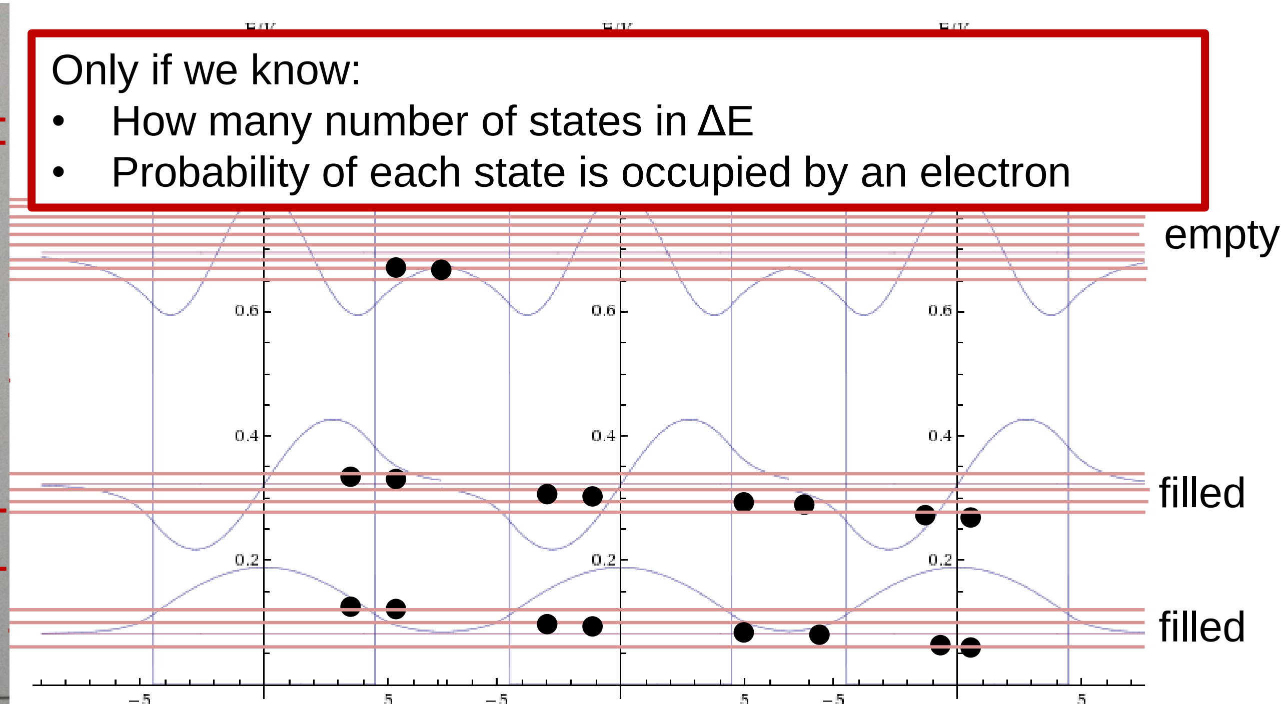
- Intrinsic (undoped)
- Electrons are all from the valance



In k space

Only if we know:

- How many number of states in ΔE
- Probability of each state is occupied by an electron



<u>How many number of electrons in the conductance band?</u>

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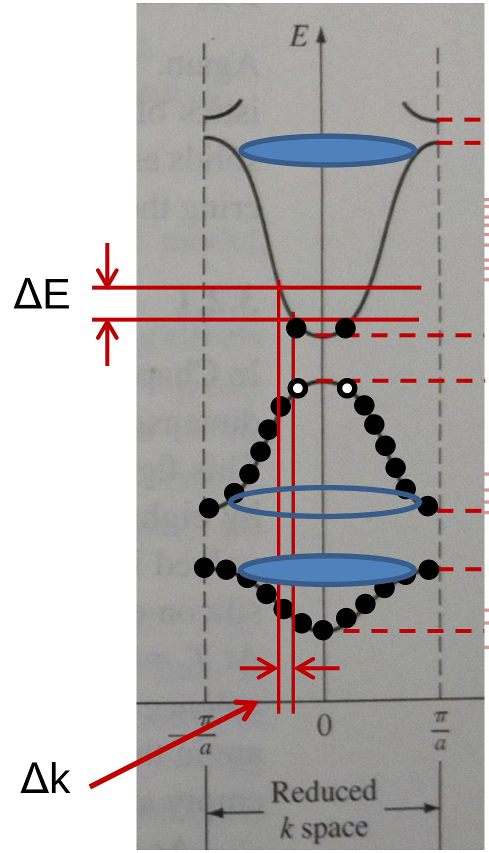

In k space

If we know:

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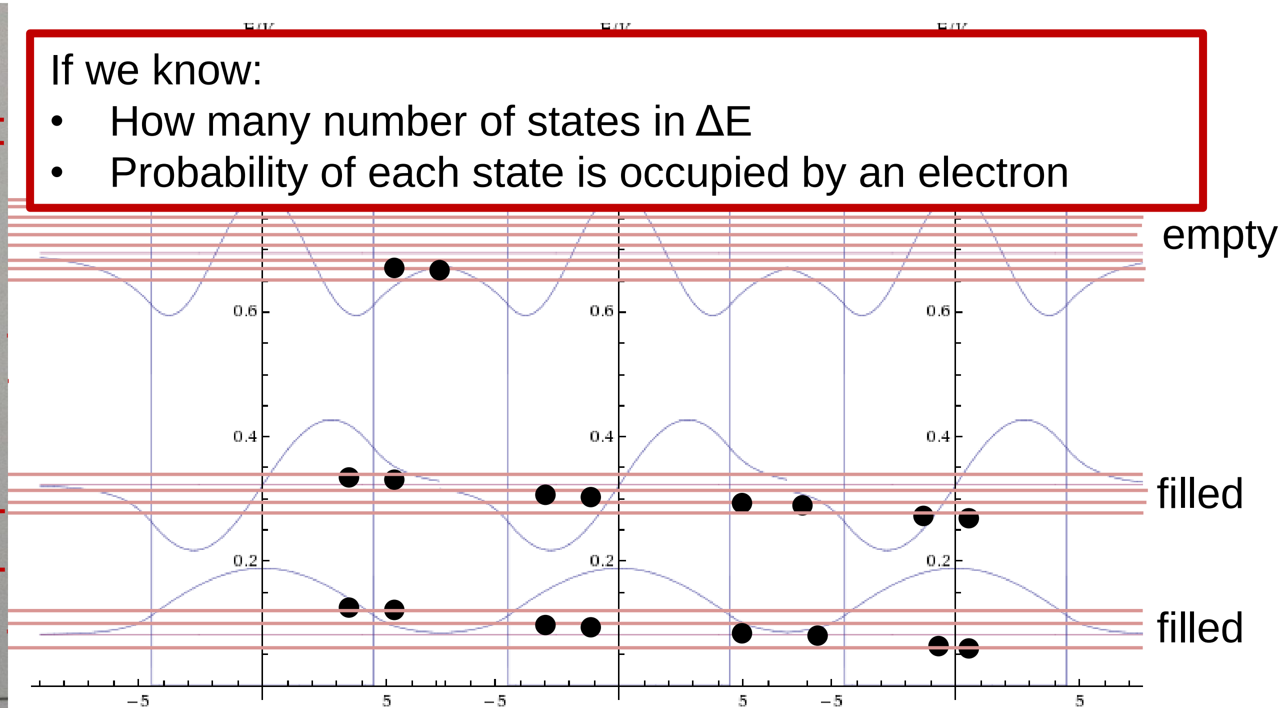


How many number of electrons in the conductance band?

- Electrons are allowed to move relatively freely in the conduction band of a semiconductor but are confined to the crystal.
- A free electron confined to a 3D infinite potential well.

$$\begin{aligned} V(x, y, z) = 0 \qquad & \text{for } 0 < x < a \\ & 0 < y < a \\ & 0 < z < a \\ V(x, y, z) = \infty \qquad & \text{elsewhere} \end{aligned}$$

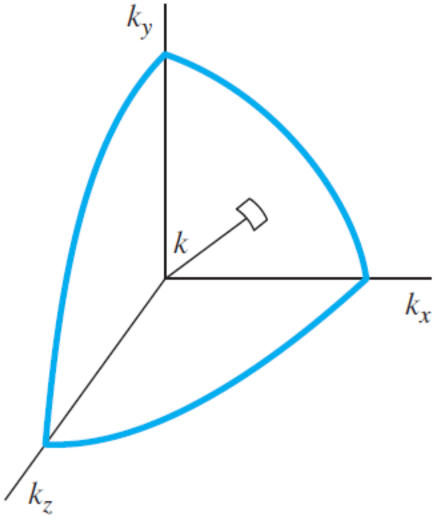
- Crystal: cube with length a
- Schrodinger's wave equation in three dimensions can be solved by using the separation of variables technique.
- Recall 1D infinite quantum well…

$$\mathrm{E} = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2} \qquad \frac{2mE}{\hbar^2} = k^2$$

- Extrapolate to 3D

$$\frac{2mE}{\hbar^2} = k^2 = k_x^2 + k_y^2 + k_z^2 = \left(n_x^2 + n_y^2 + n_z^2\right)\left(\frac{\pi^2}{a^2}\right)$$

n_x, n_y, n_z are positive integers



$$k_{x+1} - k_x = (n_x + 1)\left(\frac{\pi}{a}\right) - n_x\left(\frac{\pi}{a}\right) = \frac{\pi}{a}$$

Volume of a single quantum state is:

$$V_k = \left(\frac{\pi}{a}\right)^3$$

Density of quantum states in k space in a certain volume $4\pi k^2 dk$:

$$g_T(k)\,dk = 2\left(\frac{1}{8}\right)\frac{4\pi k^2\,dk}{\left(\frac{\pi}{a}\right)^3} = \frac{\pi k^2\,dk}{\pi^3}\cdot a^3$$

Spin Positive

- Free electron: $k^2 = \frac{2mE}{\hbar^2}$ $k = \frac{1}{\hbar}\sqrt{2mE}$

$$dk = \frac{1}{\hbar}\sqrt{\frac{m}{2E}}\,dE$$

$$g_T(k)\,dk = \frac{\pi k^2\,dk}{\pi^3}\cdot a^3 \longrightarrow g_T(E)\,dE = \frac{4\pi a^3}{h^3}\cdot(2m)^{3/2}\cdot\sqrt{E}\,dE$$

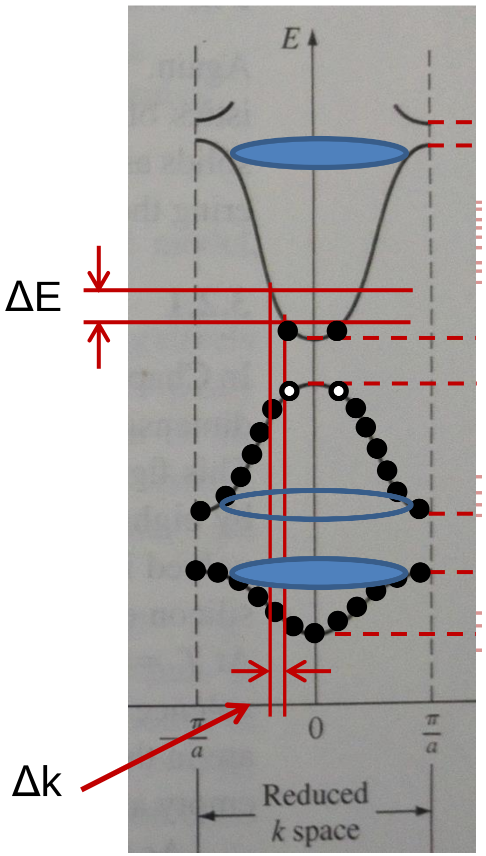
Number of energy states between E and E+dE!

- Density of quantum states per unit volume of the crystal: divide by volume a^3 and dE (per unit energy per unit volume)

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$$

- For semiconductors

- Intrinsic (undoped)
- Electrons are all from the valance



In k space

If we know:

- How many number of states in ΔE
- Probability of each state is occupied by an electron

$$\boxed{E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2} \qquad E - E_c = \frac{\hbar^2 k^2}{2m_n^*}$$

Electron in the bottom of the conduction band: a “free” electron with its own particular mass.

Density of allowed electronic energy states in the conduction band: (for $E>E_c$)

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

- Hole: approximate the *E* versus *k* curve near *k*=0 by a parabola for a "free" hole

$$E = E_v - \frac{\hbar^2 k^2}{2m_p^*}$$

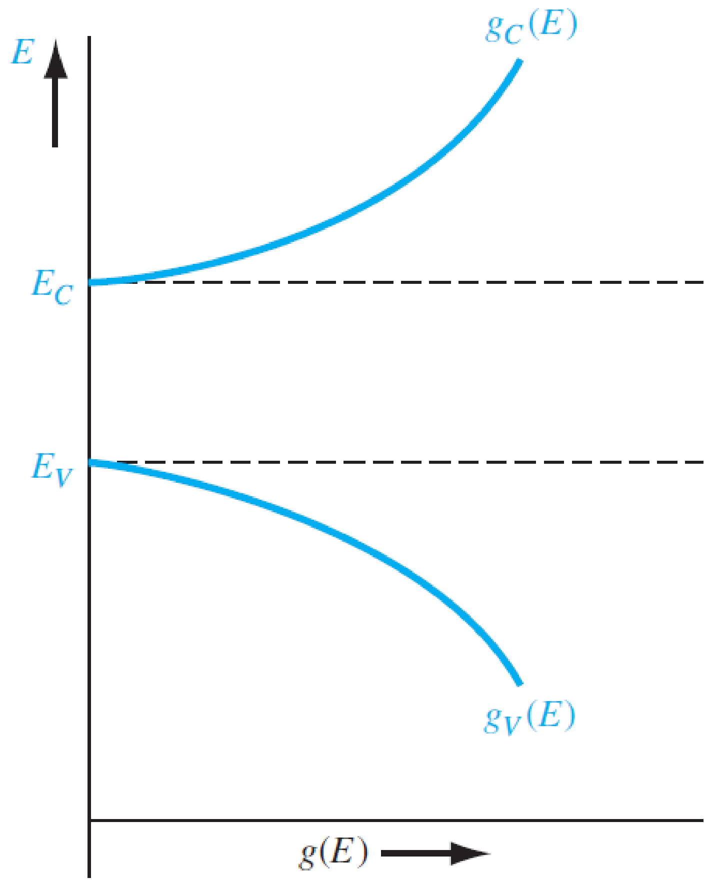
$$E_v - E = \frac{\hbar^2 k^2}{2m_p^*}$$

Density of allowed electronic energy states in the valence band: (for $E \leq E_v$)

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$$

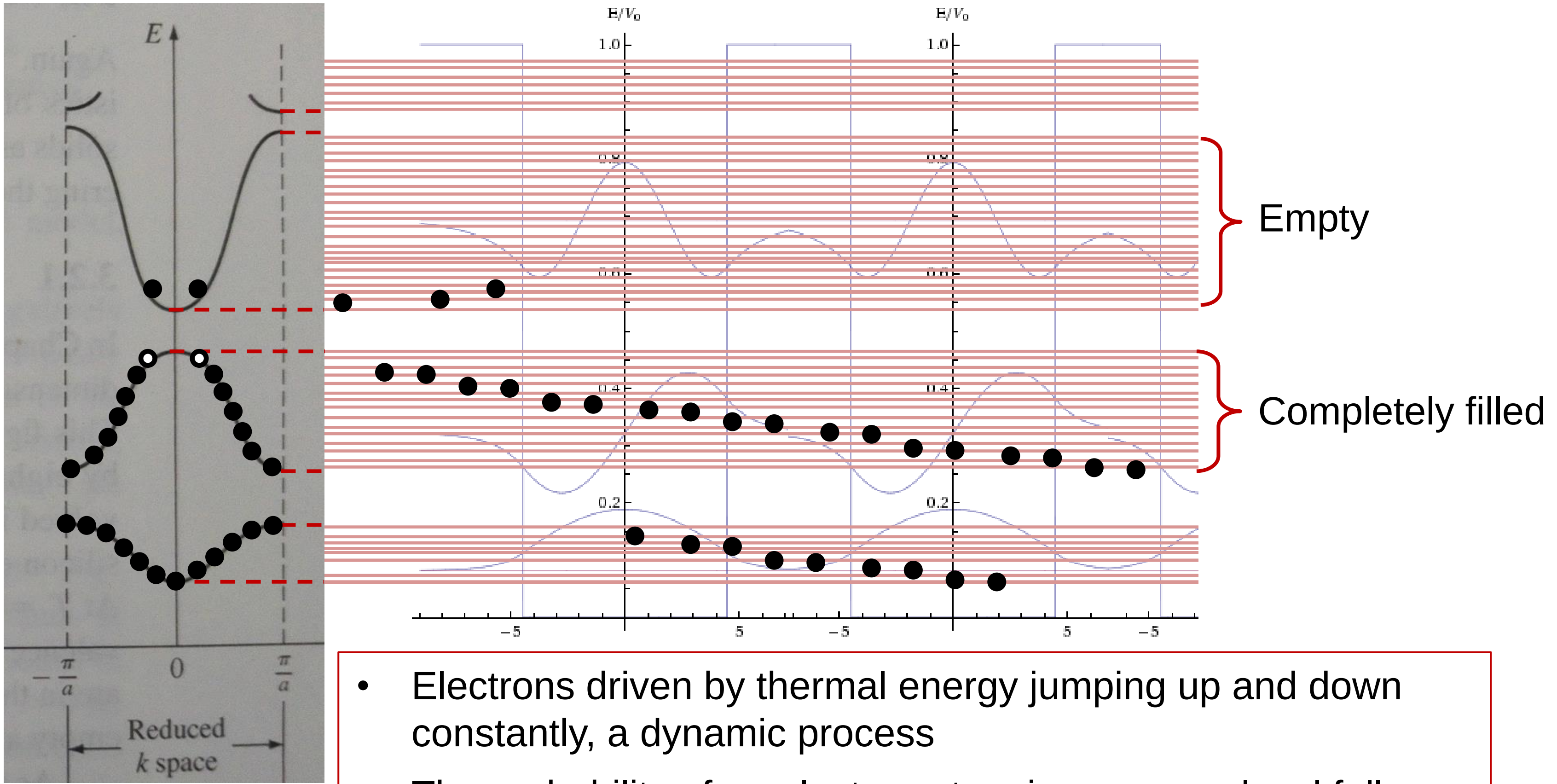
- Within the forbidden energy band: g(E)=0 for $E_v<E<E_c$

- Density of states vs. energy



- Probability of each state occupied by an electron: statistical mechanics

Semiconductors



In k space

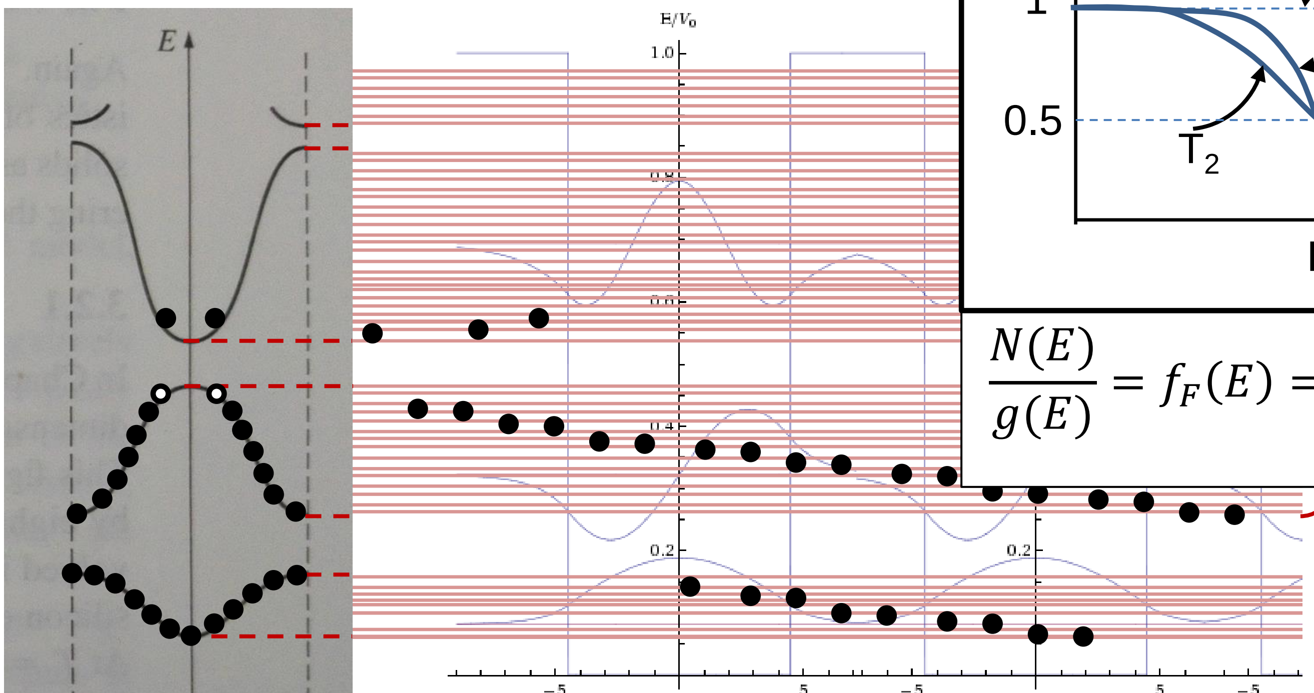
- Electrons driven by thermal energy jumping up and down constantly, a dynamic process
- The probability of an electron at a given energy level follow a statistic distribution

Statistical laws

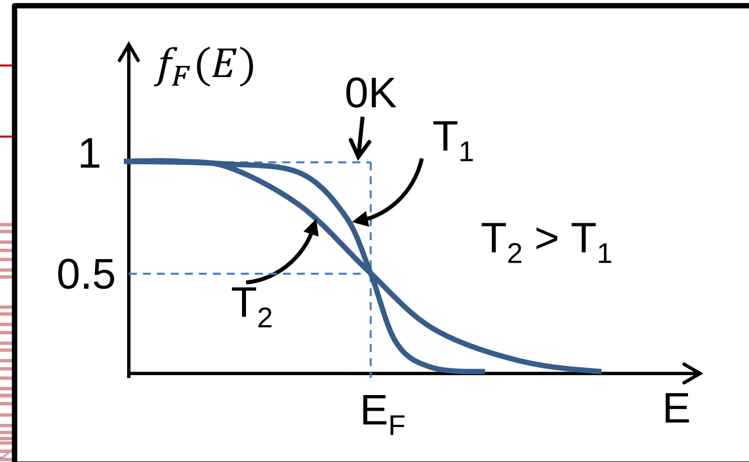
- Maxwell–Boltzmann probability function: particles are distinguishable, with no limit to the number of particles allowed in each energy state. The behavior of gas molecules in a container at fairly low pressure is an example of this distribution.
- Bose–Einstein function: particles are indistinguishable and there is no limit to the number of particles permitted in each quantum state. The behavior of photons, or black body radiation, is an example of this law. Boson
- Fermi–Dirac probability function: particles are indistinguishable, but only one particle is permitted in each quantum state. Electrons in a crystal obey this law. Fermion

Fermi-Dirac distribution and Fermi level

Semiconductors



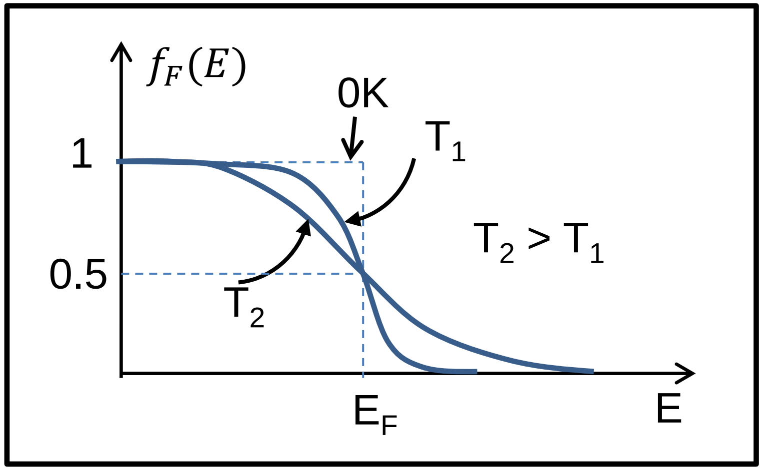
In k space



$$\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$$

- Electrons driven by thermal energy jumping up and down constantly, a dynamic process
- The probability of an electron at a given energy level follow a statistic distribution

Fermi-Dirac distribution and Fermi level



$$\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

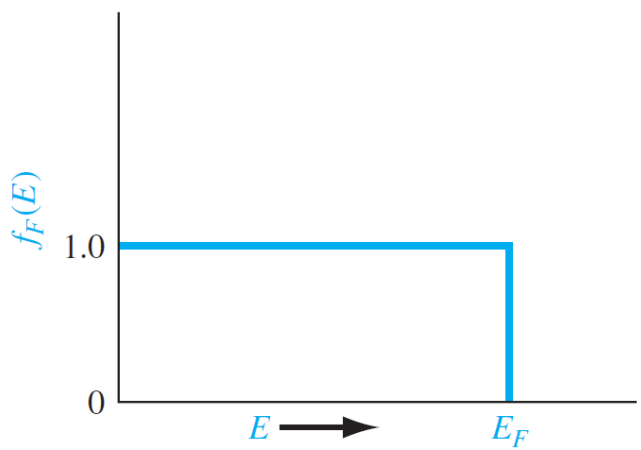
$f_F(E)$: Fermi–Dirac distribution, the probability that a quantum state at the energy *E* will be occupied by an electron. Or: the ratio of filled to total quantum states at any energy *E*.

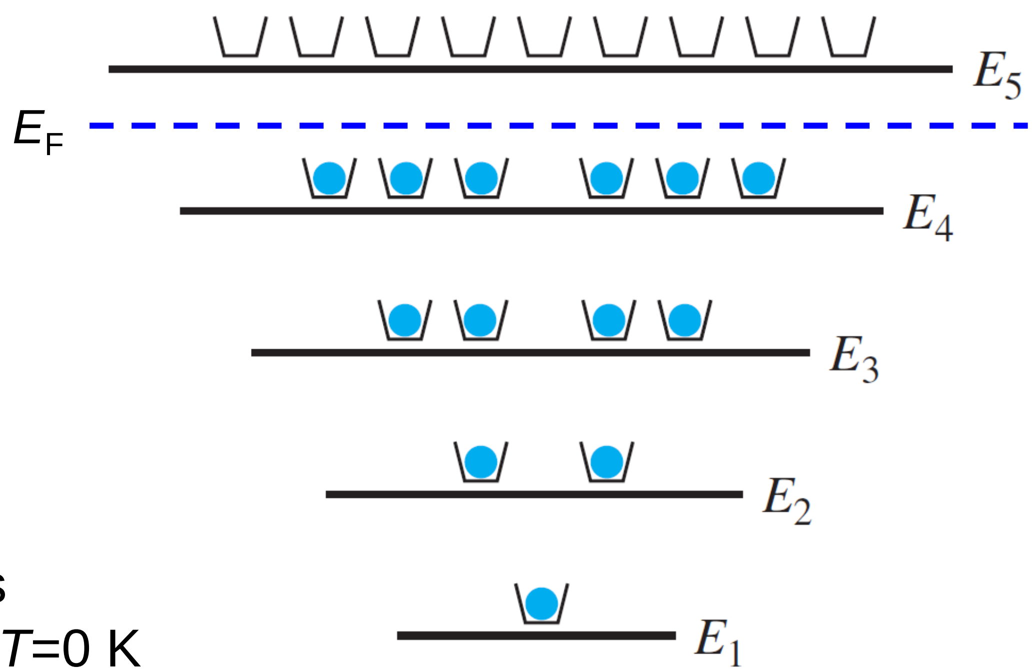
N(E): number of particles per unit volume per unit energy
g(E): the number of quantum states per unit volume per unit energy
E_F: Fermi energy

Fermi-Dirac distribution and Fermi level

T=0K:

$$\boxed{\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}}$$

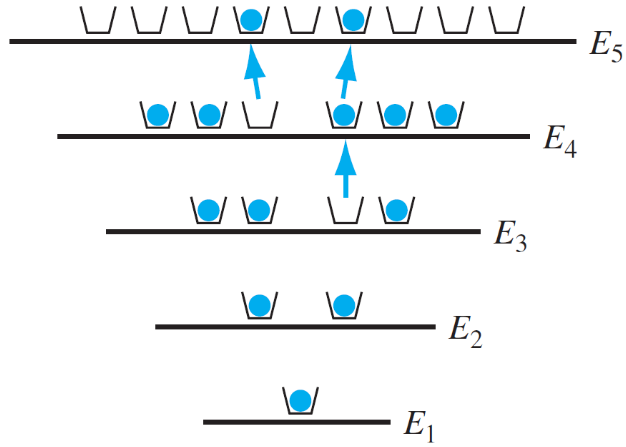




All electrons have energies below the Fermi energy at *T*=0 K

Fermi-Dirac distribution and Fermi level

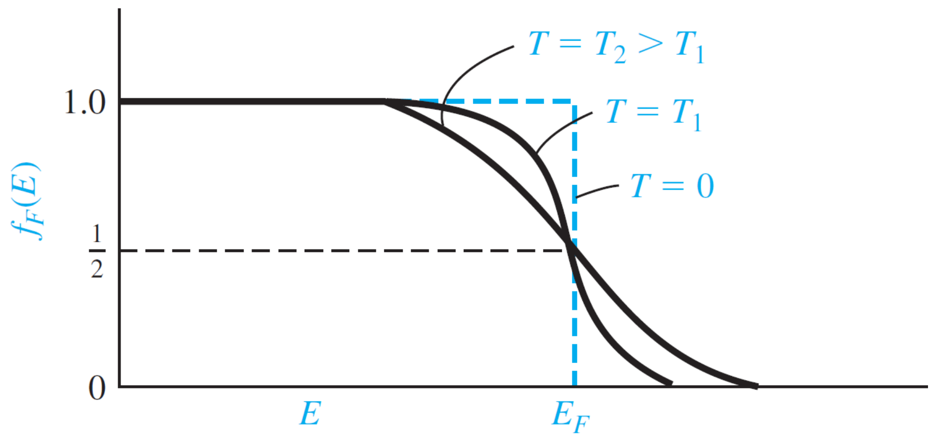
T>0K:



$$\boxed{\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}}$$

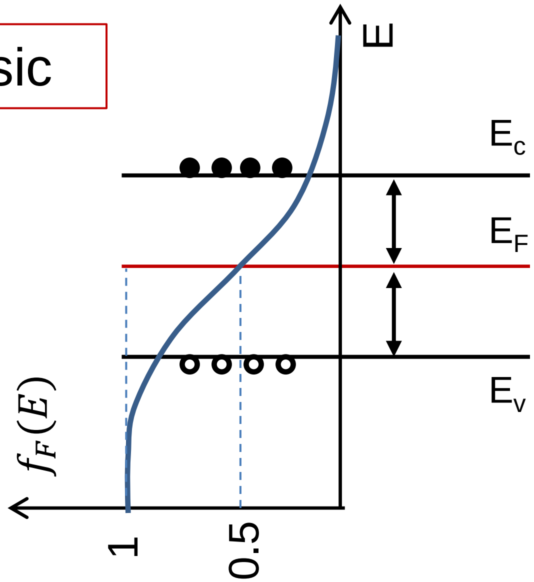
$$f_F(E=E_F) = \frac{1}{1+\exp(0)} = \frac{1}{1+1} = \frac{1}{2}$$

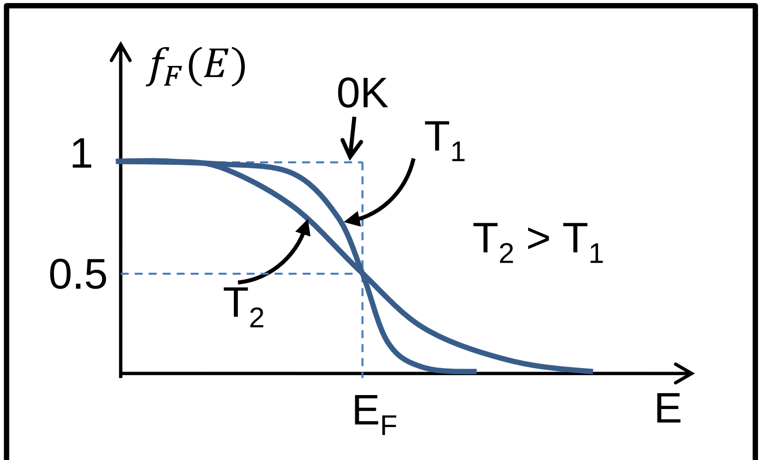
The probability of a state being occupied at E=E_F is 0.5



Fermi-Dirac distribution and Fermi level

Intrinsic





$$\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

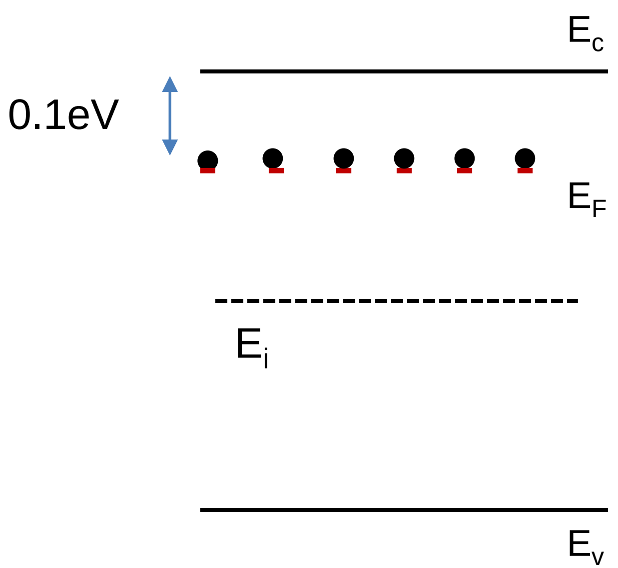
Probability of a state at E_c occupied

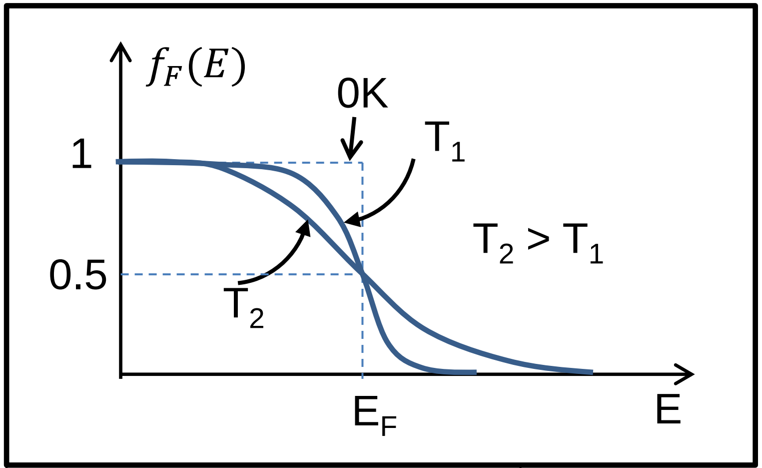
||

Probability of a state at E_v unoccupied

Fermi distribution and Fermi level

Doped





$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

① T→ 0K

② T is very high so that the majority of electrons are excited from valence band

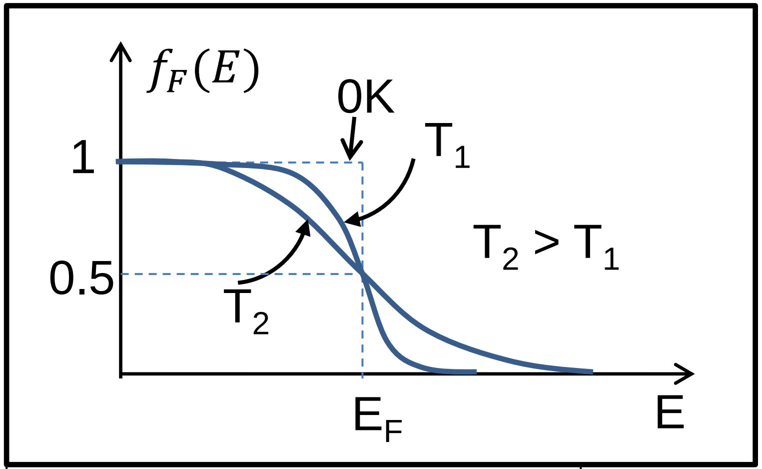
Boltzmann distribution

$$when \exp\left(\frac{E-E_F}{kT}\right) \gg 1 \Rightarrow E - E_F > 3kT$$

$$\boxed{f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}}$$

$$\boxed{f_F(E) \approx \exp(-\frac{E-E_F}{kT})}$$

Boltzmann
distribution



$$\boxed{f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}}$$

Boltzmann distribution