vv255_Assignment 5: Double Integrals.
Due to: 2019-07-08

Problem 1: Sketch the region of integration and change the order of integration

A. ***a.*** $\int_0^2 dx \int_x^{2x} f(x,y)dy$ ***b.*** $\int_{-6}^2 dx \int_{(x^2/4)-1}^{2-x} f(x,y)dy$ ***c.*** $\int_0^1 dx \int_{x^3}^{x^2} f(x,y)dy$

d. $\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)dy$ ***e.*** $\int_1^2 dx \int_{2-x}^{\sqrt{2x-x^2}} f(x,y)dy$ ***f.*** $\int_0^{2\pi} dx \int_0^{\sin x} f(x,y)dy$

B.

a. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{a\cos\varphi} f(r,\varphi)dr\,, a>0$ ***b.*** $\int_0^{\frac{\pi}{2}} d\varphi \int_0^{a\sqrt{\cos\varphi}} f(r,\varphi)dr\,, a>0$ ***c.*** $\int_a^0 d\varphi \int_0^{\varphi} f(r,\varphi)dr\,, 0<a<2\pi$

Problem 2

Let $R: 0 \le x \le t, 0 \le y \le t$ and

$$F(t) = \int_R e^{\frac{tx}{y^2}}\ dx\ dy$$

Compute $F'(t)$.

Problem 3

Recall that $S_R = \iint_R dx\ dy$

A. Find the area of the regions bounded by the following curves:

a. $xy = a^2, \quad x+y = \frac{5}{2}a \quad (a>0) Ans: \left(\frac{15}{8} - 2\ln 2\right)$ ***b.*** $(x-y)^2 + x^2 = a^2 \ (a>0) \quad Ans: \pi a^2$

B. Change variables and find the area of regions bounded by the following curves:

a. $x+y=a, \quad x+y=b, \quad y=\alpha x, \quad y=\beta x \ \ (0<a<b;\ 0<\alpha<\beta) \quad Ans: \frac{(\beta-\alpha)(b^2-a^2)}{2(\alpha+1)(\beta+1)}$

b. $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1, \quad \sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 2, \quad \frac{x}{a} = \frac{y}{b}, \quad 4\frac{x}{a} = \frac{y}{b} \quad (a>0, b>0) \quad Ans: \frac{65}{108}ab$

c. $(x^3+y^3)^2 = x^2+y^2, \quad x \ge 0, \quad y \ge 0 \quad Ans: \frac{\pi}{6} + \frac{\sqrt{2}}{3}\ln\left(1+\sqrt{2}\right)$

Problem 4

A. Sketch the solid bodies whose volumes are given by

a. $\iint\limits_{\substack{0\le x\le 1\\ x\ge 0, y\ge 0}} (x+y)dx\ dy$ *b.* $\iint\limits_{|x|+|y|\le 1} (x^2+y^2)dx\ dy$ *c.* $\iint\limits_{x^2+y^2\le x} \sqrt{x^2+y^2}\ dx\ dy$

B. Find the volume of the given solid bounded by

$$\textbf{a.}\quad z = 1 + x + y, \quad z = 0, \quad x + y = 1, \quad x = 0, \quad y = 0 \quad Ans: \frac{5}{6}$$

$$\textbf{b.}\quad z = \cos x \cos y, \quad z = 0, \quad |x + y| \le \frac{\pi}{2}, \quad |x - y| \le \frac{\pi}{2} \quad Ans: \pi$$

$$\textbf{c.}\quad z = xy, \quad z = 0, \quad x + y + z = 1 \quad Ans: \frac{17}{12} - 2\ln 2$$

C. Use polar coordinates to find the volume of the solid bodies bounded by the following surfaces:

$$\textbf{a.}\quad z^2 = xy, \quad x^2 + y^2 = a^2 \quad Ans: \frac{4}{3\sqrt{\pi}}\Gamma^2\left(\frac{3}{4}\right)a^3$$

$$\textbf{b.}\quad z = x + y, \quad (x^2 + y^2)^2 = 2xy, \quad z = 0 \quad (x > 0, y > 0) \quad Ans: \frac{\pi}{8}$$

$$\textbf{c.}\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^2} - \frac{y^2}{b^2}, \quad a, b, c > 0 \quad Ans: \frac{2}{9}abc(3\pi + 20 - 16\sqrt{2})$$

Problem 5

Find the area of the surface.

a. The part of the surface $az = xy$ that lies inside the cylinder $x^2 + y^2 = a^2$.

b. The surface is bounded by $x^2 + z^2 = a^2$, $y^2 + z^2 = a^2$ Ans: $16a^2$

c. The part of the surface $z = \sqrt{x^2 - y^2}$ that lies inside the cylinder $(x^2 + y^2)^2 = a^2(x^2 - y^2)$, $z \in \mathbb{R}$.

Ans: $\frac{\pi a^2}{2}$