Summer_2019   VV255_Assignment 3:   Functions of several variables.

Deadline: 2019-06-10

Problem 1

Find and sketch the domains of the following functions:

$$f(x,y) = \sin^{-1}(x-y), \qquad g(x,y) = \sqrt{x^2+y^2-5}$$

Problem 2

The function $z(x,y)$ is defined by the equations below. Estimate z_x and z_y at the given point.

a. $x^3 + y^3 + z^3 - 3xyz = 4,$ $\quad P(2,1,1),$ $\quad$ b. $\sqrt{x^2+y^2}+z^2-3z = 3,$ $\quad P(4,3,1)$

Problem 3

Let $u = u(x,y)$ and $x = x(r,\varphi), y = y(r,\varphi)$.

a. Let $x = r\cos\varphi, y = r\sin\varphi$. Find $\dfrac{\partial u}{\partial r}, \dfrac{\partial u}{\partial \varphi}$ and $\dfrac{\partial^2 u}{\partial r^2}$.

b. $u = \dfrac{x}{y} - \dfrac{y}{x}$, $x = r\sin 2\varphi, y = r\ \tan^2\varphi,$ $P\left(1,\dfrac{\pi}{4}\right).$ Find $\dfrac{\partial u}{\partial r}, \dfrac{\partial u}{\partial \varphi}$ at the point P.

Problem 4

a. Find the tangent plane to the surface $x^2 + z^2 - 4y^2 = -2xy$ at the point $P(-2,1,2)$.
b. Find the linear approximation at the point P.
c. What is the shortest distance between the surface and the plane $x + 6y - 2z = 15$?

Problem 5

Find the directional derivative of $x^4 - y^3 + z^2 + xyz = 0$ at the point $P(1,1,2)$ in the direction of the vector $(-2,1,2)$.

Problem 6

Consider the contour map of a function. Estimate $f(0,2)$, $f_x(0,2)$ and $f_y(0,2)$.
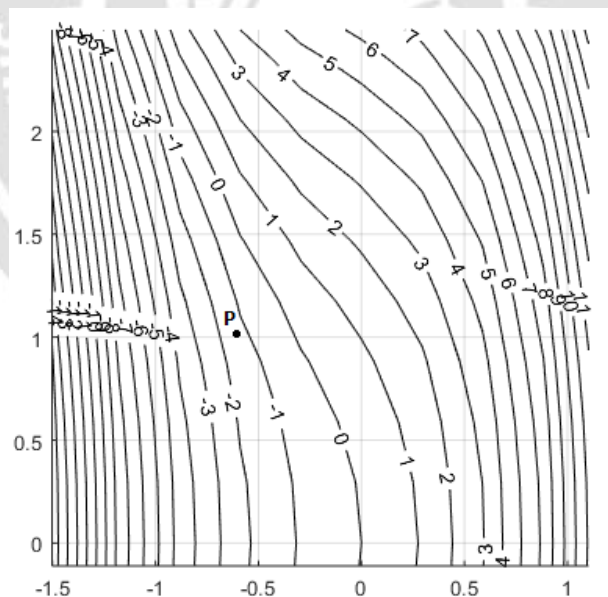
Identify the sign of $f_x(P), f_y(P), f_{xx}(P), f_{xy}(P), f_{yy}(P)$.