RC_mid1

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Before RC…

- Checklist
  - Definition on the slides
  - Theorem on the slides, especially the condition of the theorem
  - Some questions on the slides, especially those with blue words
  - Assignment, especially the exercises with wrong answers
  - Worksheet

Content

- Real Number and Sets
- The limit of a sequence

Real Number and Sets

- Upper bound: $x \leq M\ for\ every\ x \in S$
- Lower bound: $x \geq M\ for\ every\ x \in S$
- Bounded: both exist

- **NOTE**: upper bound and lower bound are **numbers**, but all of them consist of **a set**

- Supreme:$M \geq M^*\ for\ every\ upper\ bound\ M^*$
- Infimum: $m \leq m^*\ for\ every\ lower\ bound\ m^*$

Real Number and Sets

- δ-neighborhood of x: $(x - \delta, x + \delta)$
- Neighborhood: $\exists \delta, s.t. (x - \delta, x + \delta) \subseteq S$

- Open set: for every point in S, there exists δ-neighborhood
- Closed set: not the open set

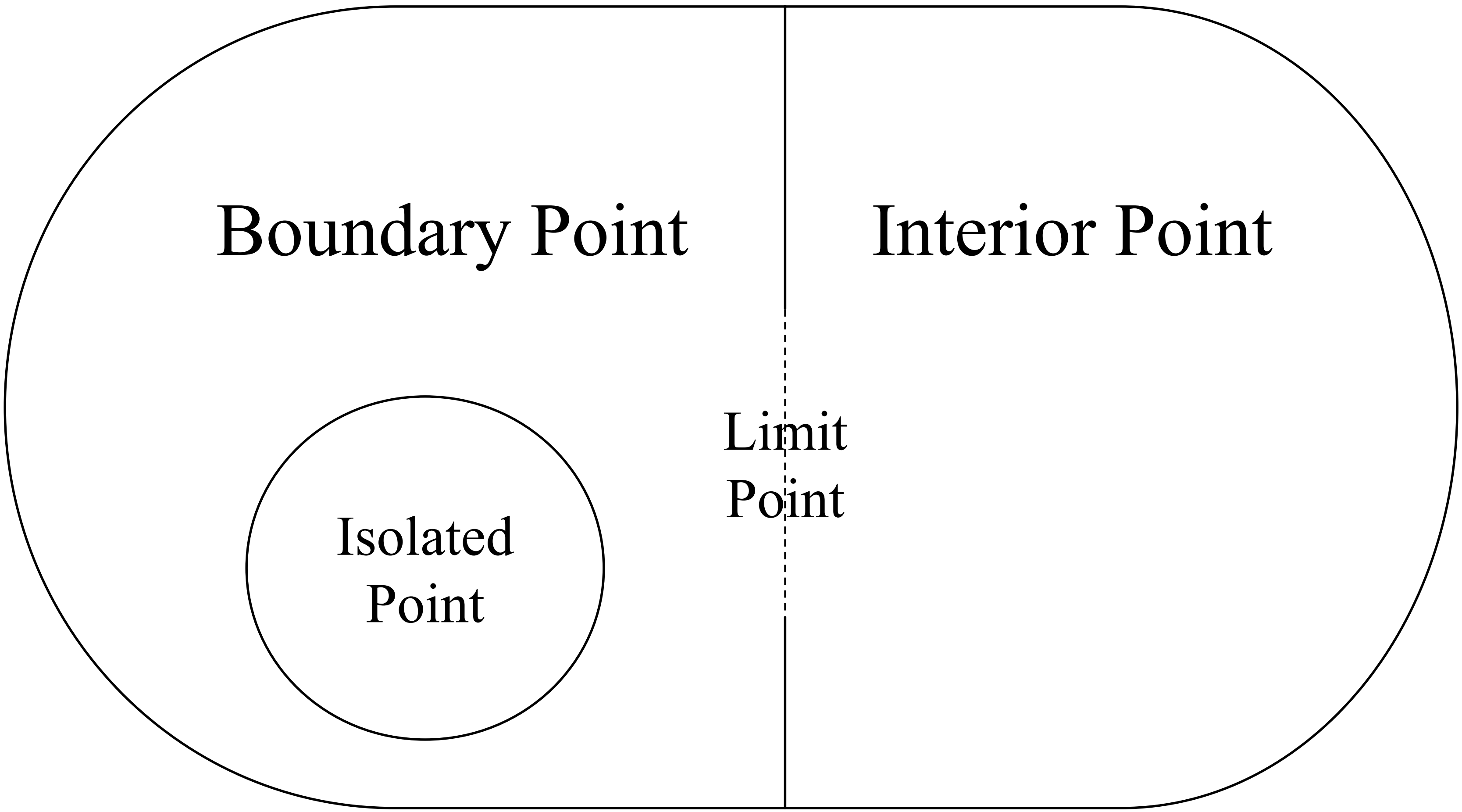
	intersection	union
Closed set	Closed set	Inconclusive
Open set	Inconclusive	Open set

Real Number and Sets

- **NOTE**:
  - some sets are **both** closed and open ($\emptyset$, $\mathbb{R}$), some sets are **neither** closed nor open ($[2,3)$)
  - **Compact**: closed and bounded

- Interior point: $\exists\delta, s.t. (x-\delta, x+\delta) \subseteq S$
- Boundary point: $\forall\delta, s.t. (x-\delta, x+\delta)$ contains a point in S and a point not in S
- Limit point: $\forall\delta, s.t. (x-\delta, x+\delta)$ contains a point in S other than itself
- Isolated point: $\exists\delta, s.t. (x-\delta, x+\delta) \cap S = x$

Real Number and Sets



The limit of a sequence

- Intuitive definition: arbitrarily close to a real value
- Rigorous definition: (if $\lim_{n \to N} a_n = L$)

$$\forall \varepsilon, \exists N, s.t.\ when\ n > N, |a_n - L| < \varepsilon$$

- Diverges to (negative) infinity: (if a_n diverges)

$$\forall M, \exists N_M, s.t. when\ n > N_M, a_n > M$$

The limit of a sequence

- Limit law: (if $\lim_{n\to\infty} a_n = L_a$ and $\lim_{n\to\infty} b_n = L_b$)
  - $\lim_{n\to\infty} a = a$
  - $\lim_{n\to\infty} (a_n \pm b_n) = L_a \pm L_b$
  - $\lim_{n\to\infty} a_n b_n = L_a L_b$
  - $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L_a}{L_b}$ if $L_b \neq 0$ and $b_n \neq 0$

The limit of a sequence

- Monotonic sequence theorem
  - Condition: monotonic
  - bounded $\Leftrightarrow$ convergent
  - For normal sequence, convergent $\Longrightarrow$ bounded
- Squeeze theorem
  - Condition: $\lim_{n\to\infty} a_n \,/\, b_n = L$; for $n > N$, $a_n \le c_n \le b_n$;
  - $\lim_{n\to\infty} c_n = L$
- Limit law → S.T. / M.S.T. → definition
- **NOTE**: we can find the relationship between function limit and sequence limit to solve problem

Exercise

- Please prove that $a_n = n^{(-1)^n}$ is unbounded, but it doesn't diverges to infinity when $n \to \infty$
- Solution:

  When $n = 2k$, $a_n = 2k$

  When $n = 2k - 1$, $a_n = \frac{1}{2k-1}$

  Because when $n = 2k$ and $n \to \infty$, a_n approaches positive infinity, it is unbounded.

  Because when $n = 2k - 1$ and $n \to \infty$, a_n approaches 0

  Note that the limit should be unique, which many of you ignored in the assignment

Exercise

- Please prove $\lim_{n \to \infty} \frac{n}{2^n} = 0$
- Solution:

  Because $2^n = (1+1)^n = 1 + n + \frac{n(n-1)}{2} + \cdots + 1 > \frac{n(n-1)}{2}$

  Then use the Squeeze Theorem

Good Luck!