Physical Quantities

Scalar quantities - defined by a single number

Examples: time, mass, length, volume, density of matter, electric charge, potential energy, pressure

Operational definitions - quantities obtained as a result of measurement operations

$$\text{density of (bulk) matter} = \text{mass} / \text{volume}$$

volume = $(\text{length})^3$

Units - metric system (or SI - Systeme Internationale)

Mathematical operations on physical quantities

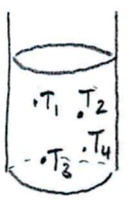
$+$ or $-$ only compatible units

$*$ or $/$ can involve quantities with different units

$\sin(\dots)$, $\ln(\dots)$, ... only dimensionless arguments

Vector quantities

Example



local temperature of a liquid

SCALAR

velocities of objects floating in a liquid

VECTOR

number (magnitude) + direction

Notation: $\vec{u}$, $\bar{u}$, $\underline{u}$

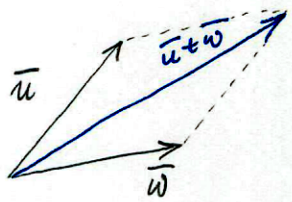
$\bar{u}$

length of $\bar{u}$: $|\bar{u}| = u$ (magnitude)

Examples: velocity, force, momentum, electric / magnetic field, angular velocity

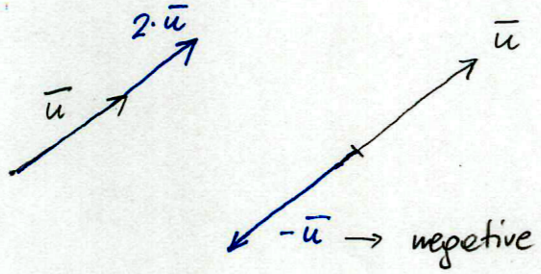
Vectors

- addition



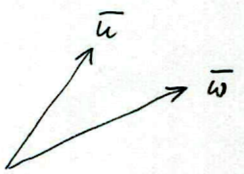
"parallelogram rule"

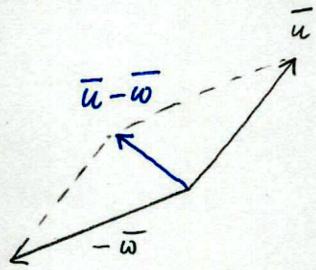
- multiplication by scalar



$-\bar{u} \to$ negative of $\bar{u}$

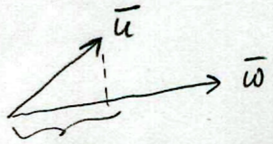
- subtraction





- multiplication

(1) scalar (dot) product



$\frac{\bar{u}\circ\bar{w}}{w} = |\bar{u}|\cos\sphericalangle(\bar{u},\bar{w})$

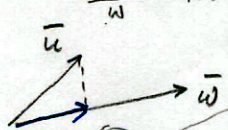
$$\bar{u}\circ\bar{w} = |\bar{u}|\cdot|\bar{w}|\cos\sphericalangle(\bar{u},\bar{w})$$

smaller angle between vectors $\bar{u}$ and $\bar{w}$

↳ number! (scalar)

Note: if $\bar{u}\perp\bar{w} \Rightarrow \bar{u}\circ\bar{w} = 0$

(and vice-versa) for non-zero vectors



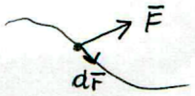
(length = 1)
unit vector in the direction of $\bar{w}$

$\left(\frac{\bar{u}\circ\bar{w}}{w}\right)\frac{\bar{w}}{w} \to$ projection of vector $\bar{u}$ on vector $\bar{w}$

dot product

multiplication by a scalar

Example elementary work



$$\delta W = \bar{F}\circ d\bar{r}$$

$\to$ infinitesimal displacement

force

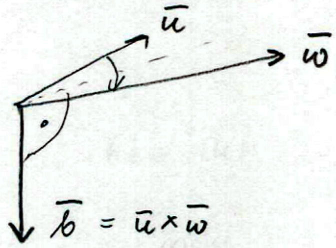
(2) vector (cross) product

smaller angle between $\bar{u}$ and $\bar{w}$

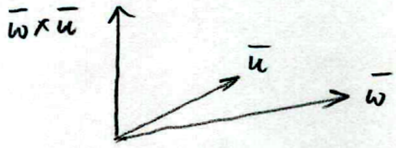
$\bar{b} = \bar{u} \times \bar{w}$ length $|\bar{b}| = |\bar{u}| \cdot |\bar{w}| \cdot \sin \angle(\bar{u}, \bar{w})$

↳ vector!
(presisely, pseudovector)

direction - right-hand rule



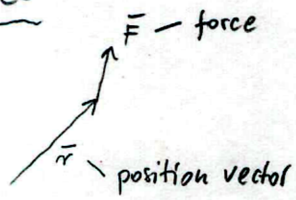
Note: (*) $\bar{u} \times \bar{w} = - \bar{w} \times \bar{u}$



(*) if $\bar{u} \parallel \bar{w} \Rightarrow \bar{u} \times \bar{w} = 0$ zero vector (and vice-versa) for non-zero vectors

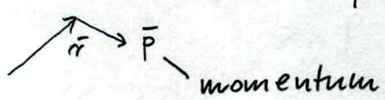
(*) $\bar{u} \times \bar{w} \perp \bar{u}$ and $\bar{u} \times \bar{w} \perp \bar{w}$

Examples



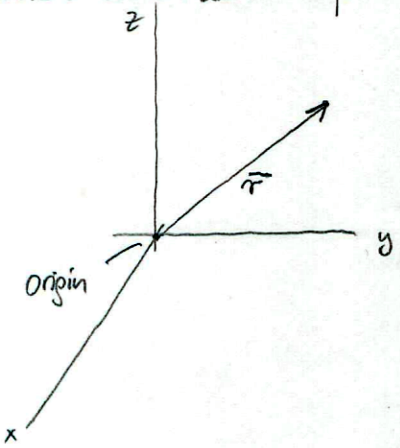
$\bar{r} \times \bar{F} = \bar{\tau}$ torque

$\bar{r} \times \bar{p} = \bar{L}$ angular momentum



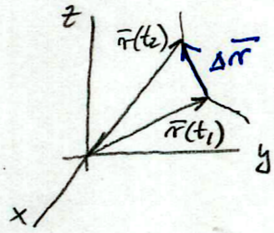
Position in space: Coordinate systems and vectors

Cartesian coordinate system



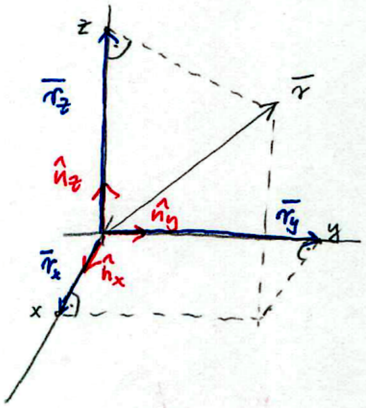
$\bar{r}$ - position vector (generally time-dependent)

$\bar{r} = \bar{r}(t)$



$\Delta\bar{r} = \bar{r}(t_2) - \bar{r}(t_1)$ displacement vector

Vector components



$\bar{r} = \bar{r}_x + \bar{r}_y + \bar{r}_z$

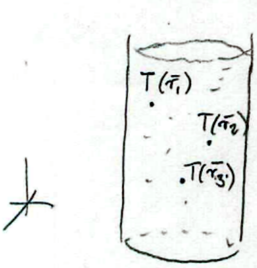
component vectors along the axes

$\bar{r} = x \cdot \hat{n}_x + y \cdot \hat{n}_y + z \hat{n}_z$

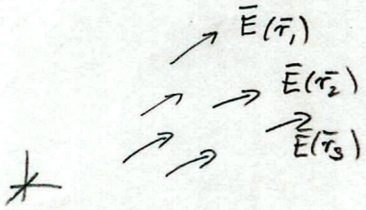
fixed unit vectors (versors) also denoted as $\hat{i}, \hat{j}, \hat{k}$ / $\hat{x}, \hat{y}, \hat{z}$

Short notation: (x, y, z) or $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Note: physical quantities may be position-dependent ⇒ fields



local temperature of a fluid



local electric field

scalar field

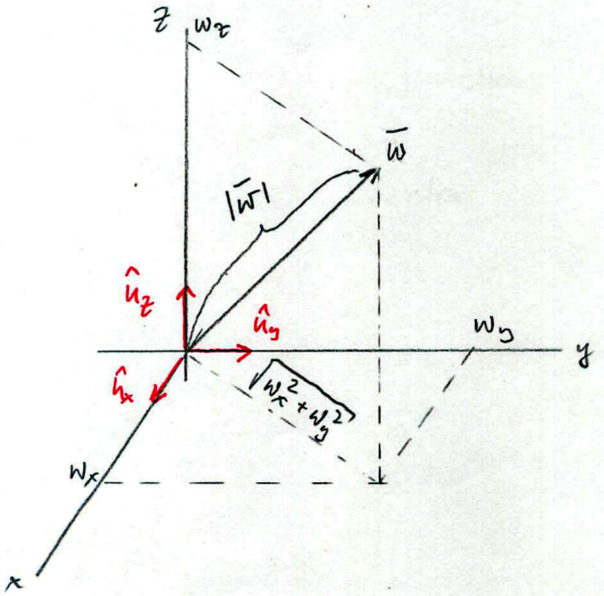
$T = T(\bar{r})$

vector field

$\bar{E} = \bar{E}(\bar{r})$

functions of the position vector

In general, for any vector



$$\bar{w} = \bar{w}_x + \bar{w}_y + \bar{w}_z =$$
$$= w_x \hat{n}_x + w_y \hat{n}_y + w_z \hat{n}_z$$

length (magnitude/modulus) of $\bar{w}$

$$|\bar{w}| = w = \sqrt{w_x^2 + w_y^2 + w_z^2}$$

Unit vectors of the Cartesien coordinate system

* mutually perpendicular (orthogonal)

$$\hat{n}_x \circ \hat{n}_y = \hat{n}_y \circ \hat{n}_z = \hat{n}_x \circ \hat{n}_z = 0$$

* unit length $|\hat{n}_x| = |\hat{n}_y| = |\hat{n}_z| = 1$; i.e. $\hat{n}_x \circ \hat{n}_x = \hat{n}_y \circ \hat{n}_y = \hat{n}_z \circ \hat{n}_z = 1$

* for right-handed system

$$\hat{n}_x \times \hat{n}_y = \hat{n}_z$$
$$\hat{n}_y \times \hat{n}_z = \hat{n}_x$$
$$\hat{n}_z \times \hat{n}_x = \hat{n}_y$$

easy to remember

cyclic permutation

Differentiation and integration of vectors in Cartesien coordinate system

$$\bar{u} = \bar{u}(t)$$
↳ usually time

$$\frac{d\bar{u}}{dt} = \dot{\bar{u}} = \frac{d}{dt}\left(u_x(t)\,\hat{n}_x + u_y(t)\,\hat{n}_y + u_z(t)\,\hat{n}_z\right) = \dot{u}_x(t)\,\hat{n}_x + \dot{u}_y(t)\,\hat{n}_y + \dot{u}_z(t)\,\hat{n}_z$$

analogously

$$\int_{t_0}^{t_1} \bar{u}\,dt = \left(\int_{t_0}^{t_1} u_x(t)\,dt\right)\hat{n}_x + \left(\int_{t_0}^{t_1} u_y(t)\,dt\right)\hat{n}_y + \left(\int_{t_0}^{t_1} u_z(t)\,dt\right)\hat{n}_z$$

($\dot{\hat{n}}_x = \dot{\hat{n}}_y = \dot{\hat{n}}_z$ because fixed unit vectors

Scalar and vector products in Cartesien coordinate system

Dot product

$$\bar{u} = u_x \hat{n}_x + u_y \hat{n}_y + u_z \hat{n}_z = (u_x, u_y, u_z)$$

$$\bar{w} = w_x \hat{n}_x + w_y \hat{n}_y + w_z \hat{n}_z = (w_x, w_y, w_z)$$

$$\bar{u} \circ \bar{w} = u_x w_x + u_y w_y + u_z w_z$$

(components with dot products of different versors vanish)
the 1st property of unit vectors
$\hat{n}_x \circ \hat{n}_y = 0, \dots$
$\hat{n}_x \circ \hat{n}_x = 1, \dots$

Note: $\bar{u} \circ \bar{u} = u_x^2 + u_y^2 + u_z^2 = u^2$

Vector product (cross product)

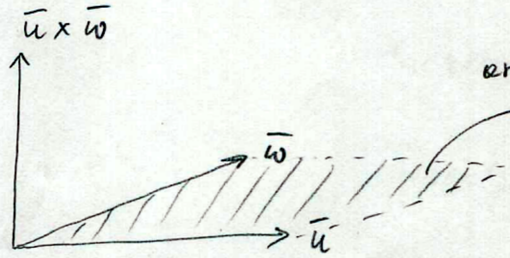
$$\bar{u} \times \bar{w} = (u_y w_z - u_z w_y)\hat{n}_x + (u_z w_x - u_x w_z)\hat{n}_y + (u_x w_y - u_y w_x)\hat{n}_z = \quad (*)$$

$$= \begin{vmatrix} \hat{n}_x & \hat{n}_y & \hat{n}_z \\ u_x & u_y & u_z \\ w_x & w_y & w_z \end{vmatrix}$$

→ "determinant" (will learn in math class)

Hint: start with $\bar{u}, \bar{w}$ written as sums of components; then use the 3rd property of unit vectors $(\hat{n}_x \times \hat{n}_y = \hat{n}_z, \dots)$ and $\hat{n}_x \times \hat{n}_x = 0, \dots$

Recall:



area

$|\bar{u} \times \bar{w}| = |\bar{u}|\,|\bar{w}| \sin\sphericalangle(\bar{u}, \bar{w})$

$\bar{u} \times \bar{w} \perp \bar{u}$

$\bar{u} \times \bar{w} \perp \bar{w}$

exercise: check that using (*)

Other operations

(*) $\bar{u} \pm \bar{w} = (u_x \hat{n}_x + u_y \hat{n}_y + u_z \hat{n}_z) \pm (w_x \hat{n}_x + w_y \hat{n}_y + w_z \hat{n}_z) =$

$= (u_x \pm w_x)\hat{n}_x + (u_y \pm w_y)\hat{n}_y + (u_z \pm w_z)\hat{n}_z = (u_x \pm w_x, u_y \pm w_y, u_z \pm w_z)$

(*) $\alpha \bar{u} = \alpha (u_x \hat{n}_x + u_y \hat{n}_y + u_z \hat{n}_z) = \dots = (\alpha u_x, \alpha u_y, \alpha u_z)$