Chapter 32

Electromagnetic Waves

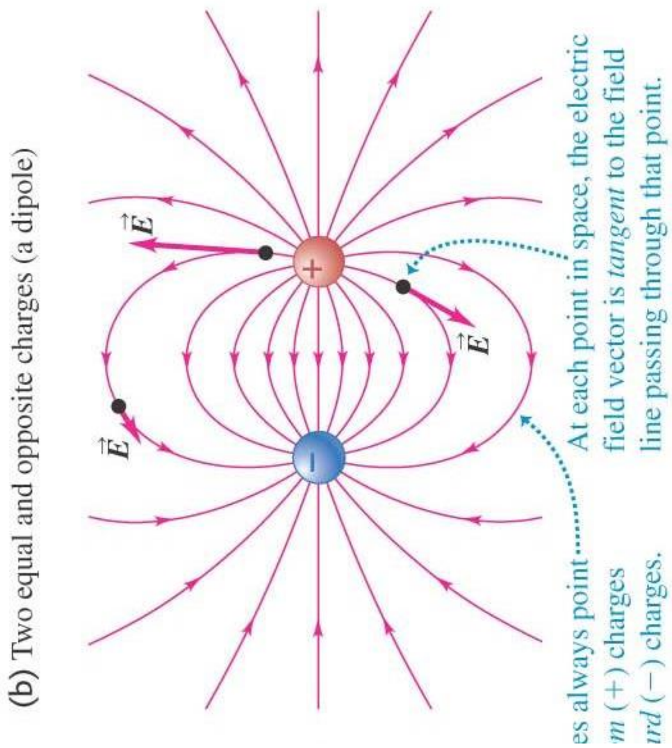
Goals for Chapter 32

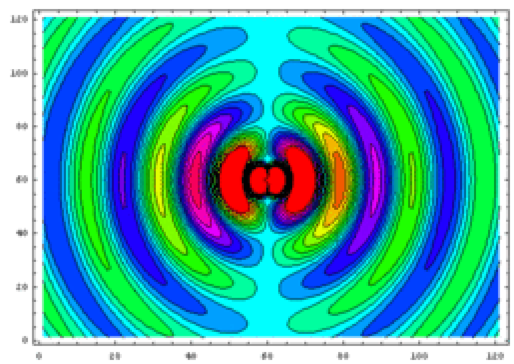
- To learn why a light wave contains both electric and magnetic fields
- To relate the speed of light to the fundamental constants of electromagnetism
- To describe electromagnetic waves
- To determine the power carried by electromagnetic waves
- To describe standing electromagnetic waves

Introduction

- Why do metals reflect light?
- We will see that light is an electromagnetic wave.
- There are many other examples of electromagnetic waves, such as radiowaves and x rays. Unlike sound or waves on a string, these waves do not require a medium to travel.

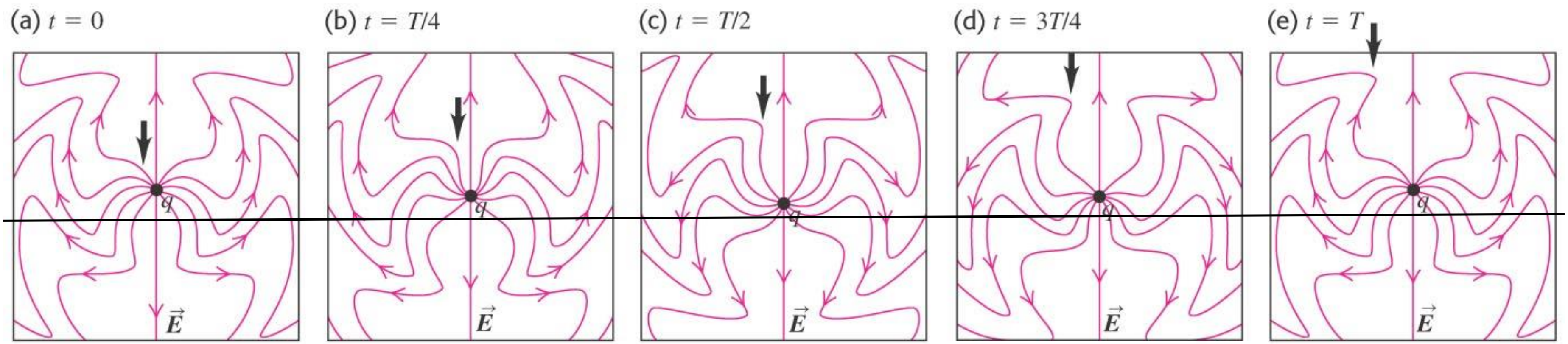
Electric field lines of point charges

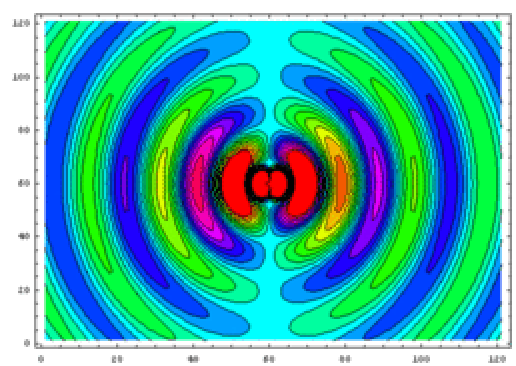




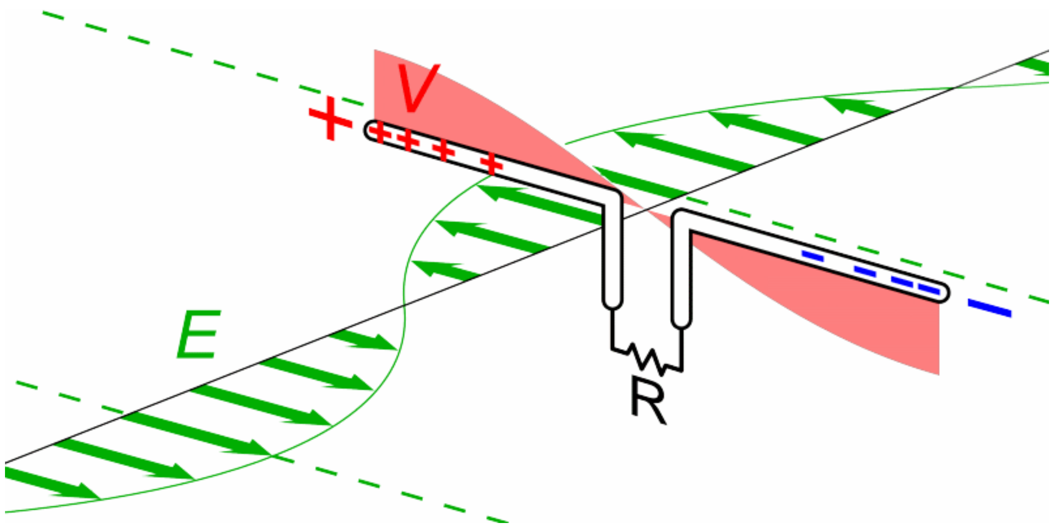
Maxwell’s equations and electromagnetic waves

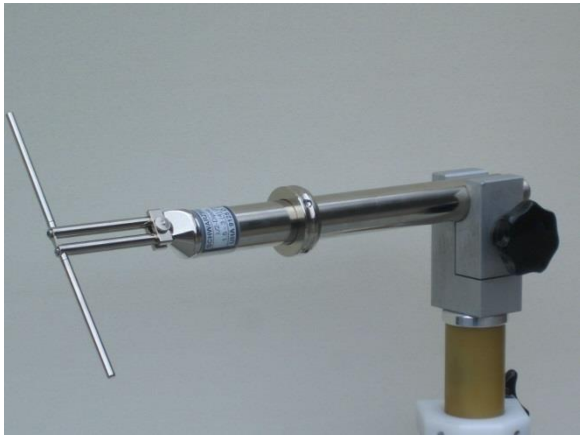
- Maxwell’s equations predicted that an oscillating charge emits *electromagnetic radiation* in the form of electromagnetic waves.

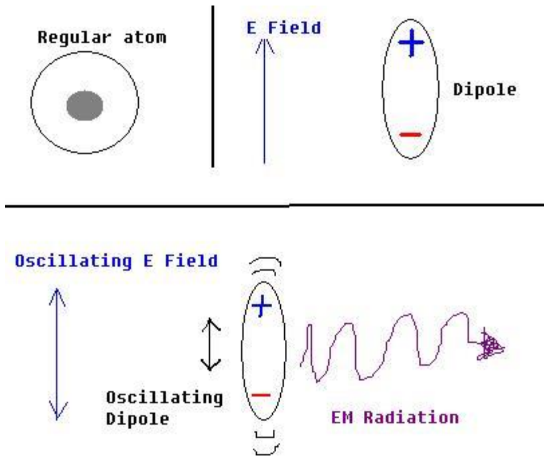


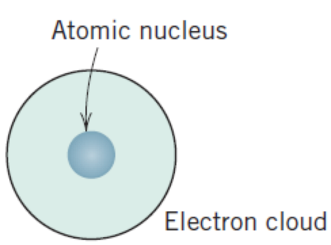


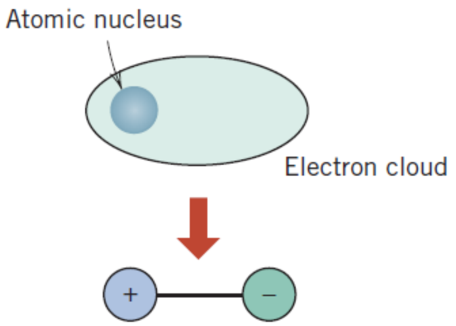
Electric field lines of point charges









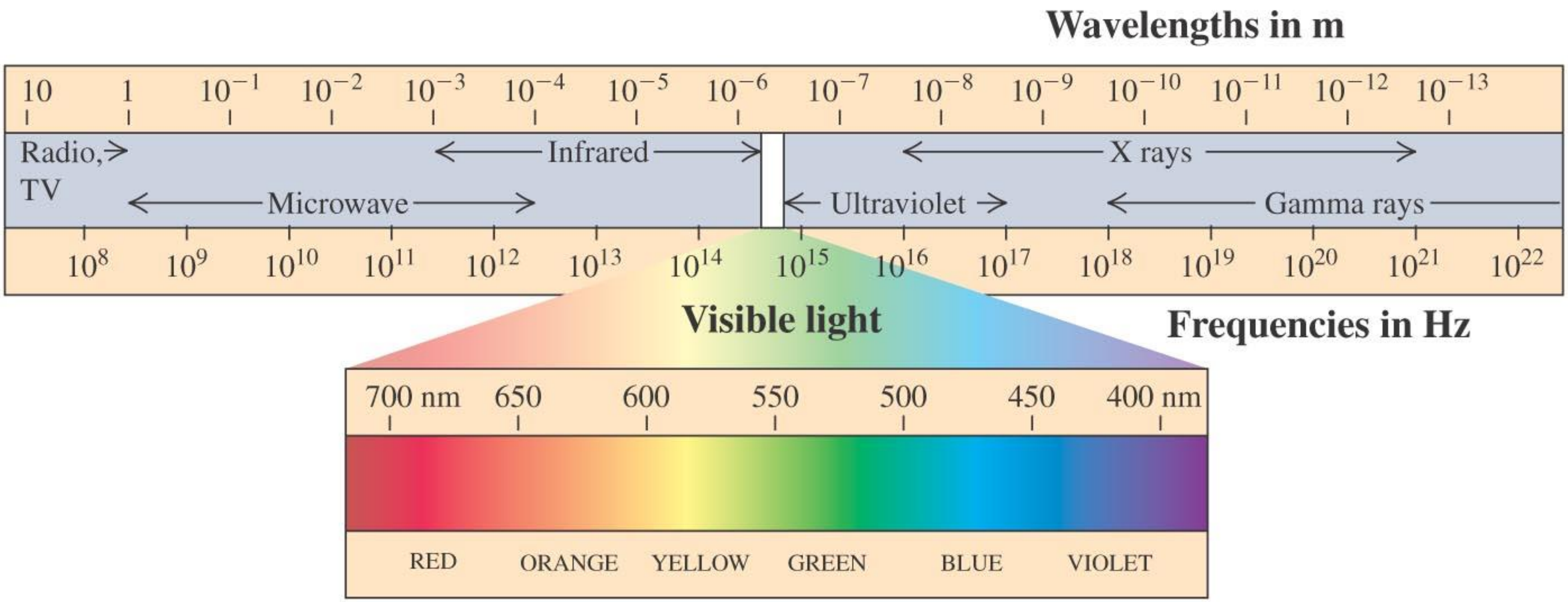


Maxwell's equations

- Gauss's law for the electric field $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$ (Gauss's law for $\vec{E}$)
- Gauss's law for the magnetic field $\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for $\vec{B}$)
- Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}}$ (Ampere's law)
- Faraday's law. $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ (Faraday's law)

The electromagnetic spectrum

- The *electromagnetic spectrum* includes electromagnetic waves of all frequencies and wavelengths. (See Figure 32.4 below.)



The electromagnetic spectrum

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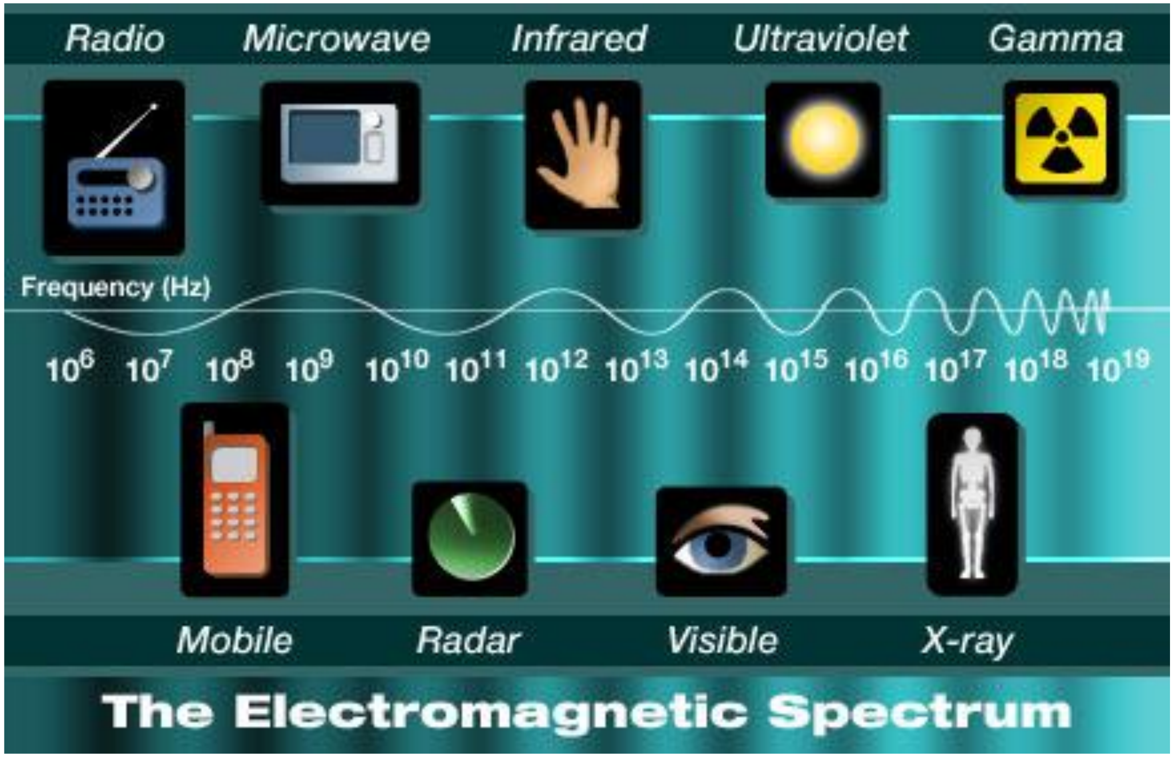
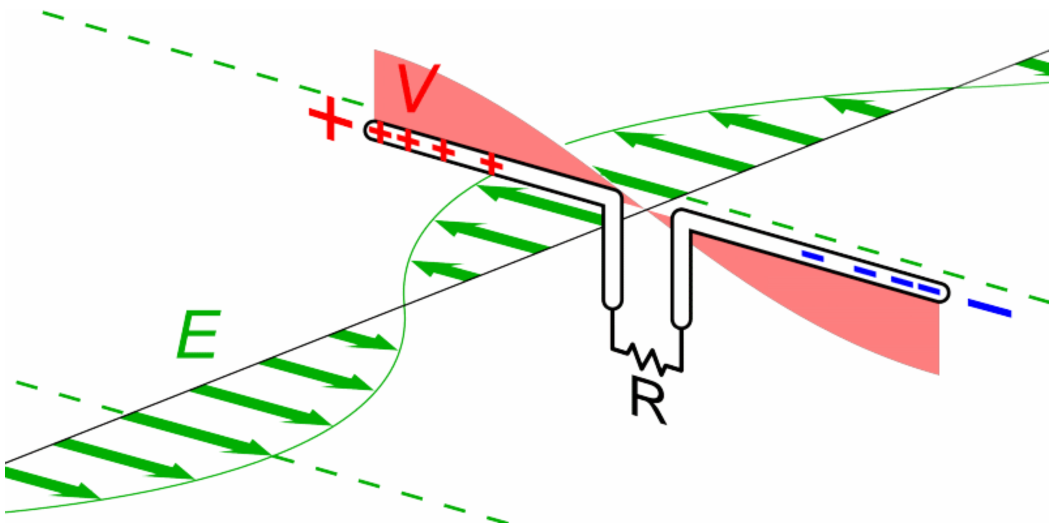
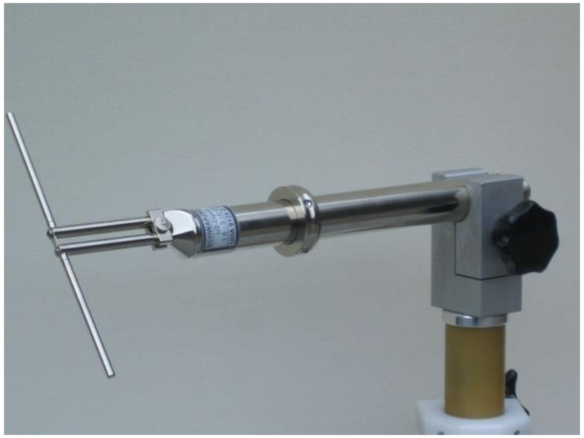
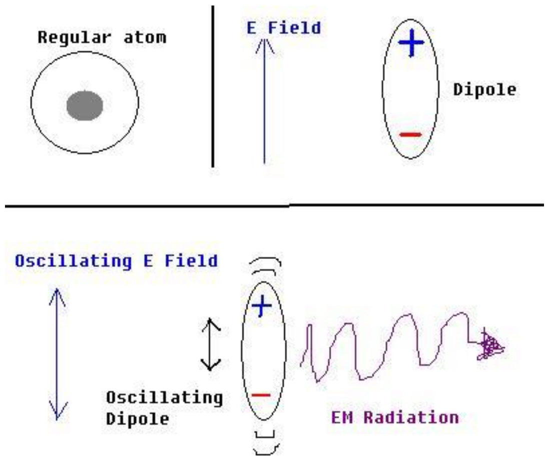
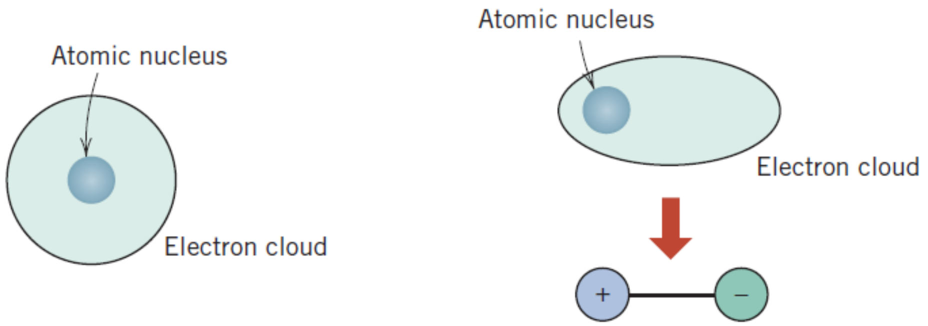


Electric field lines of point charges

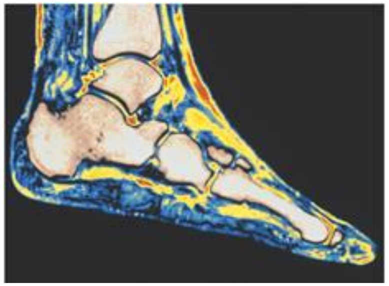


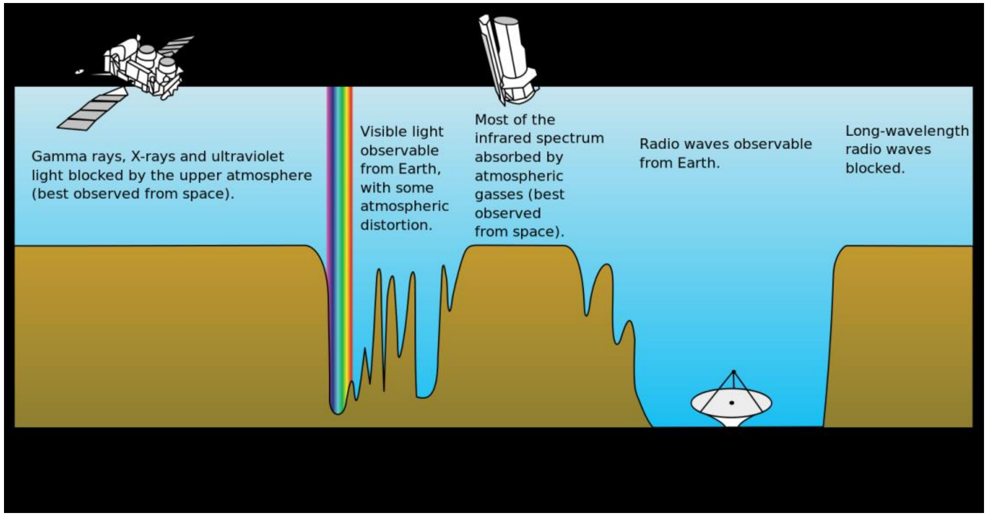




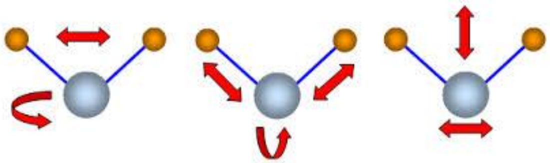


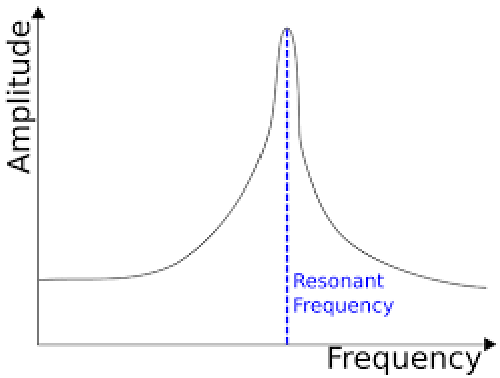
Radio Frequency

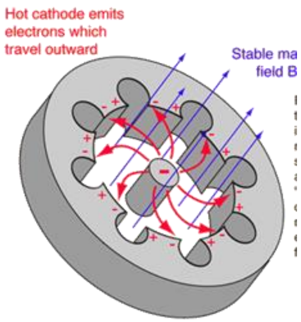




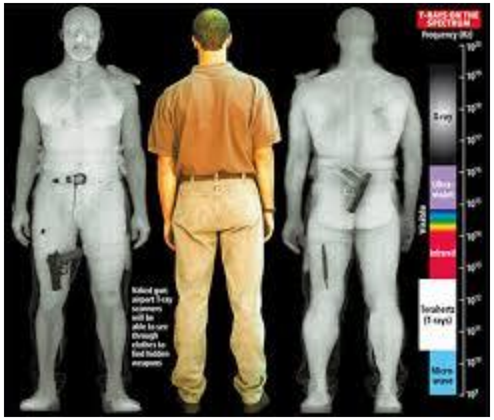
Microwave Frequency



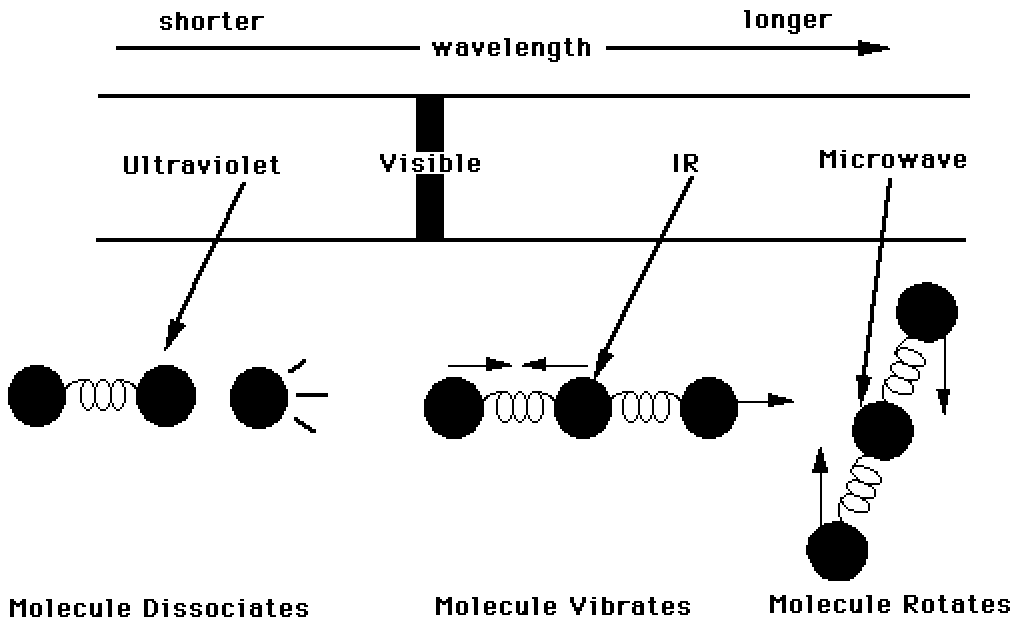


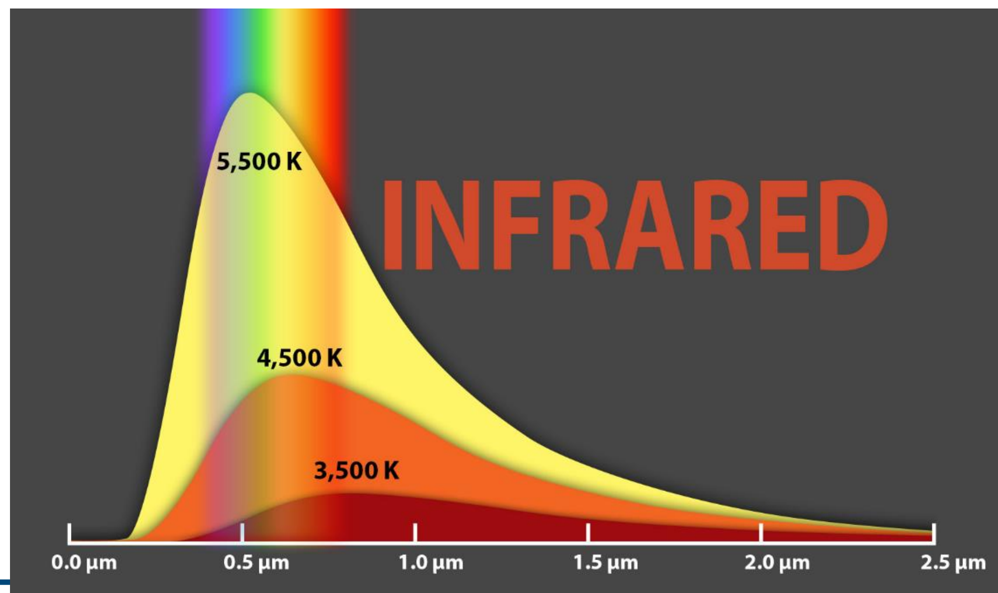


Infrared & Terahertz



Infrared & Terahertz





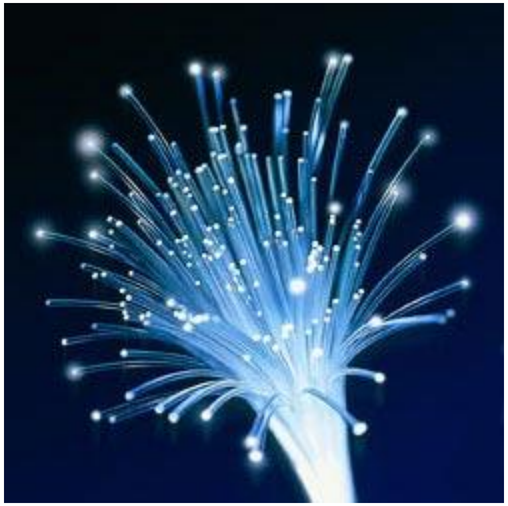
Visible light

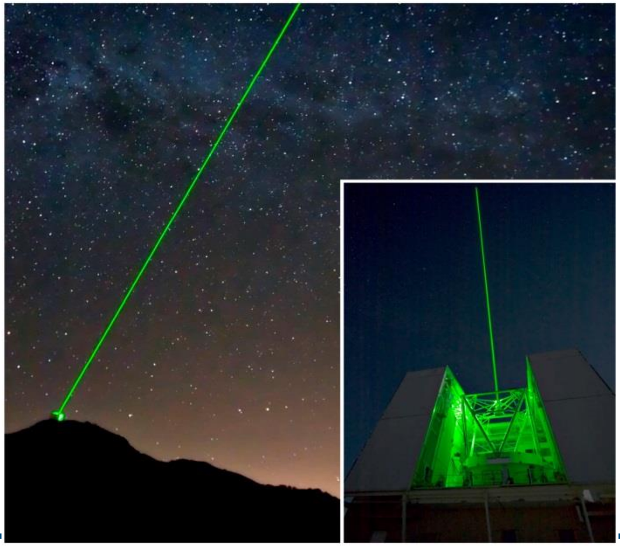
- *Visible light* is the segment of the electromagnetic spectrum that we can see.
- Visible light extends from the violet end (400 nm) to the red end (700 nm), as shown in Table 32.1.

Table 32.1 Wavelengths of Visible Light

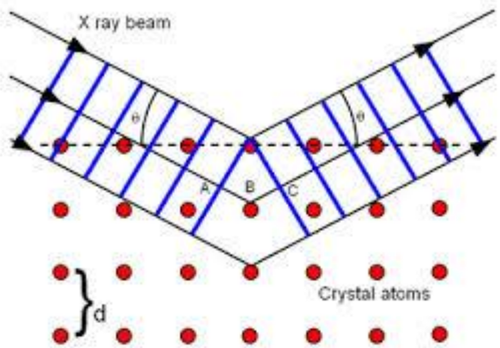
400 to 440 nm	Violet
440 to 480 nm	Blue
480 to 560 nm	Green
560 to 590 nm	Yellow
590 to 630 nm	Orange
630 to 700 nm	Red

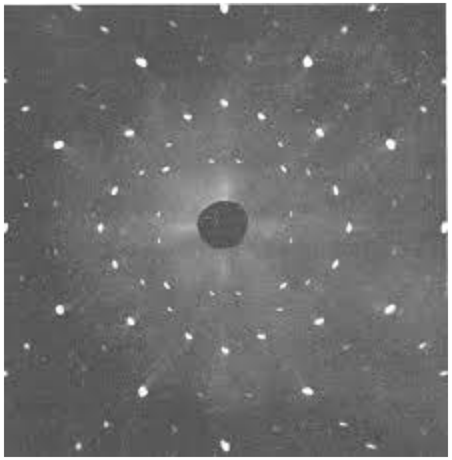
Applications



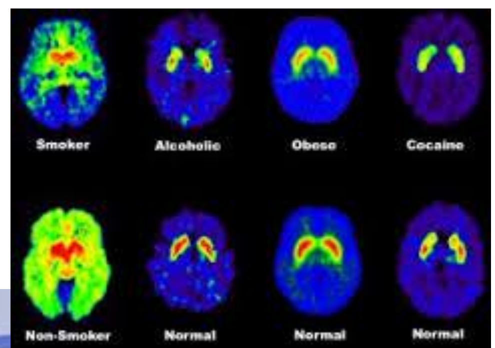


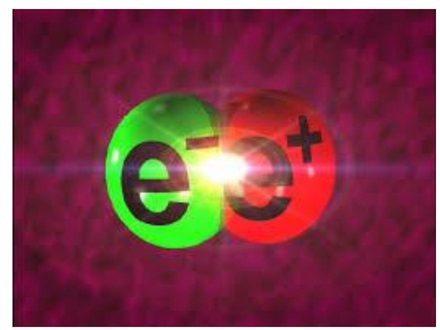
X-Ray

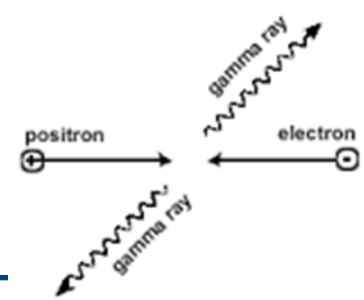




Gamma Ray

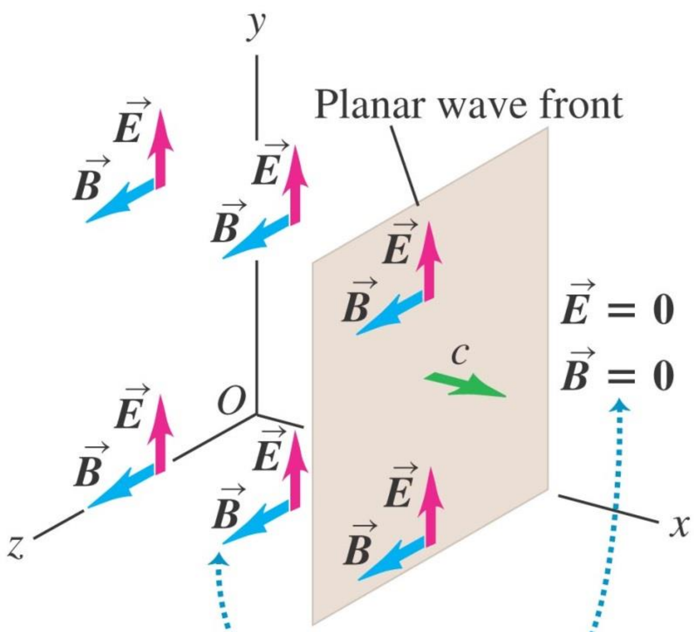

Smoker
Alcoholic
Obese
Cocaine
Non-Smoker
Normal
Normal
Normal


e-e+


gamma ray
positron
electron
gamma ray

Plane electromagnetic waves

- A *plane wave* has a planar wave front. See Figure 32.5 below.



The electric and magnetic fields are uniform behind the advancing wave front and zero in front of it.

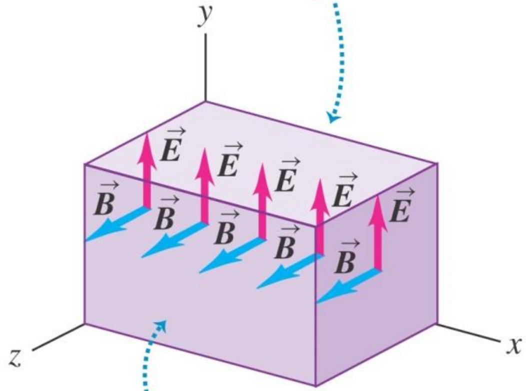
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad \text{(Gauss's law for } \vec{E}\text{)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for } \vec{B}\text{)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{encl} \qquad \text{(Ampere's law)}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

A simple plane electromagnetic wave



The electric field is the same on the top and bottom sides of the Gaussian surface, so the total electric flux through the surface is zero.

The magnetic field is the same on the left and right sides of the Gaussian surface, so the total magnetic flux through the surface is zero.

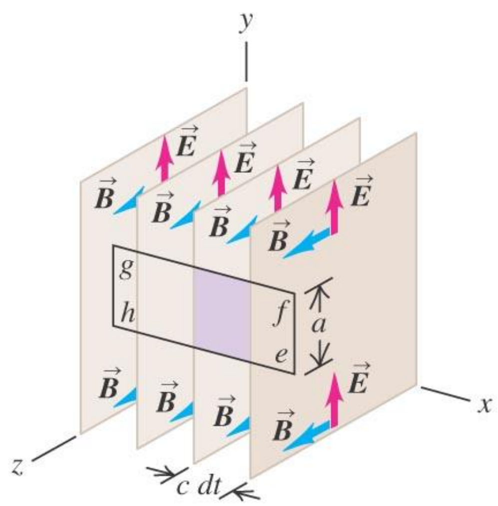
, Gauss's laws f

, the wave must be **transverse.**

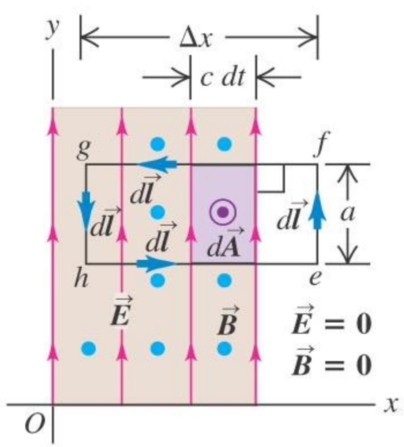
Free space only

A simple plane electromagnetic wave

(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction.



(b) Side view of situation in (a)



Faraday's law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = -Ea$$

$$d\Phi_B = B(ac\,dt),$$

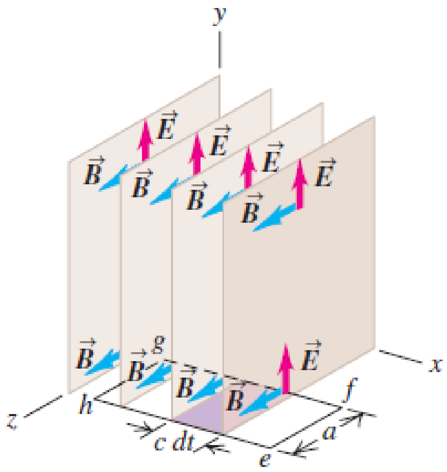
$$\frac{d\Phi_B}{dt} = Bac$$

$$-Ea = -Bac$$

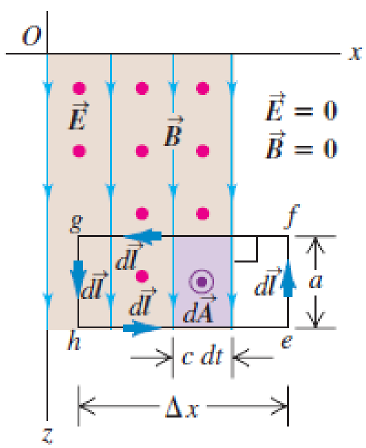
$$E = cB \qquad \text{(electromagnetic wave in vacuum)}$$

Ampere's Law

(a) In time dt, the wave front moves a distance $c\,dt$ in the $+x$-direction.



(b) Top view of situation in (a)



no conduction current $(i_C = 0)$,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = Ba$$

$$d\Phi_E = E(ac\,dt).$$

$$\frac{d\Phi_E}{dt} = Eac$$

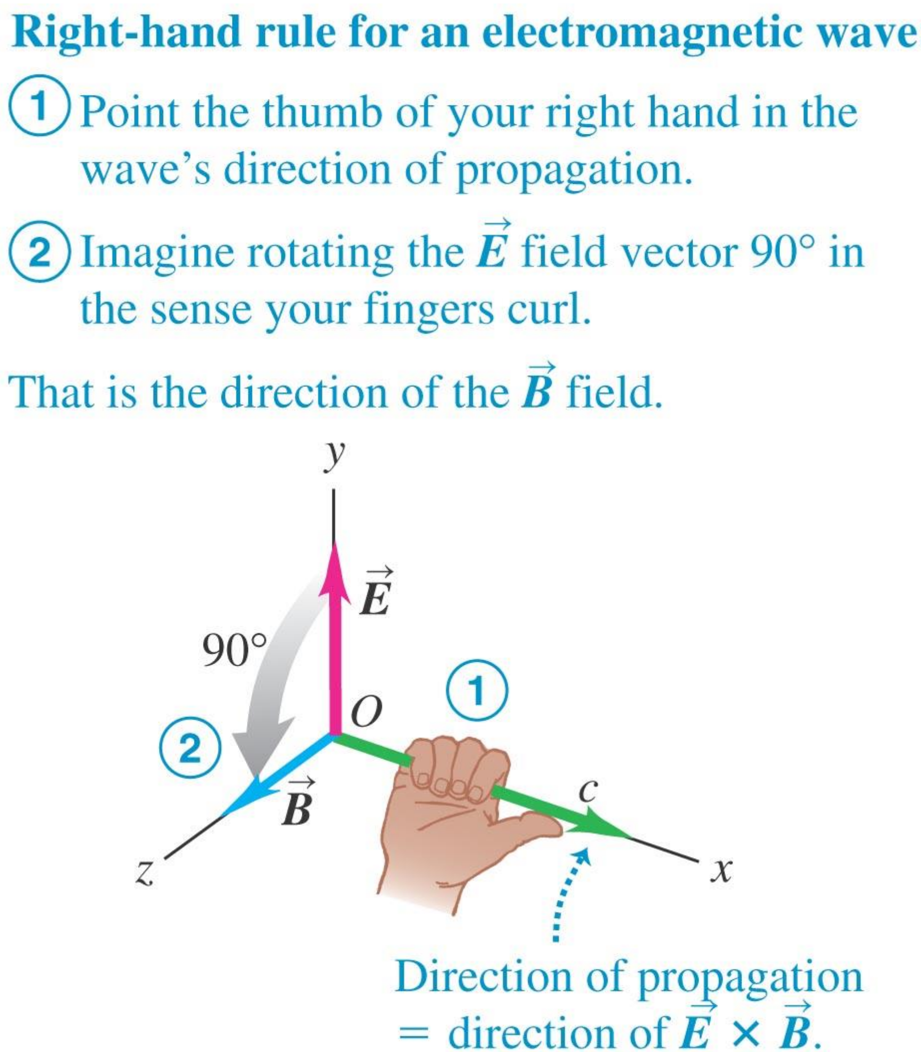
$$Ba = \epsilon_0 \mu_0 Eac$$

$$B = \epsilon_0 \mu_0 cE \qquad \text{(electromagnetic wave in vacuum)}$$

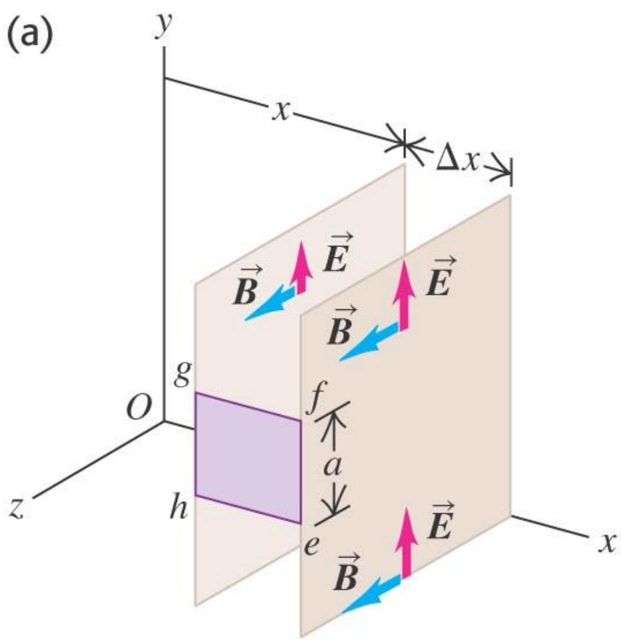
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \qquad \text{(speed of electromagnetic waves in vacuum)}$$

Key properties of electromagnetic waves

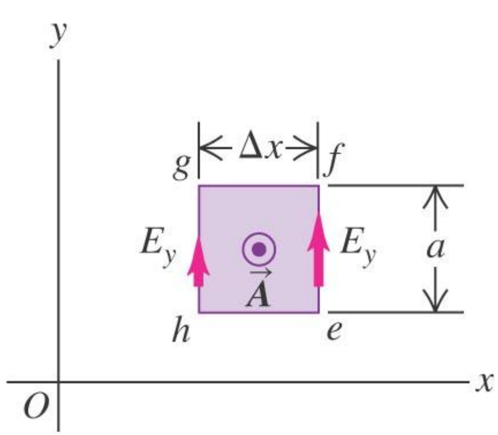
- The magnitudes of the fields in vacuum are related by $E = cB$.
- The speed of the waves is $c = 3.00 \times 10^8$ m/s in vacuum.
- The waves are *transverse.* Both fields are perpendicular to the direction of propagation and to each other. (See Figure 32.9 at the right.)



Derivation of the electromagnetic wave equation



(b) Side view of the situation in



$$\oint \vec{E} \cdot d\vec{l} = -E_y(x, t)a + E_y(x + \Delta x, t)a$$

$$= a[E_y(x + \Delta x, t) - E_y(x, t)]$$

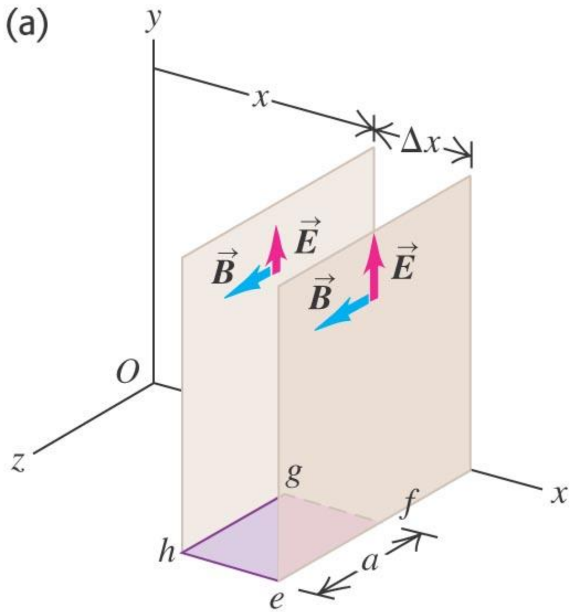
$$a[E_y(x + \Delta x, t) - E_y(x, t)] = -\frac{\partial B_z}{\partial t} a\,\Delta x$$

$$\frac{E_y(x + \Delta x, t) - E_y(x, t)}{\Delta x} = -\frac{\partial B_z}{\partial t}$$

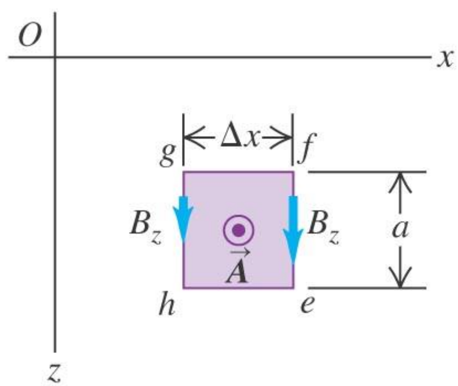
$$\frac{\partial E_y(x, t)}{\partial x} = -\frac{\partial B_z(x, t)}{\partial t}$$

Derivation of the electromagnetic wave equation

(a)



(b) Top view of the situation in (a)



$$\oint \vec{B} \cdot d\vec{l} = -B_z(x + \Delta x, t)a + B_z(x, t)a$$

$$\frac{d\Phi_E}{dt} = \frac{\partial E_y(x, t)}{\partial t} a\, \Delta x$$

$$-B_z(x + \Delta x, t)a + B_z(x, t)a = \epsilon_0 \mu_0 \frac{\partial E_y(x, t)}{\partial t} a\, \Delta x$$

$$-\frac{\partial B_z(x, t)}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y(x, t)}{\partial t}$$

$$-\frac{\partial^2 E_y(x, t)}{\partial x^2} = \frac{\partial^2 B_z(x, t)}{\partial x \partial t}$$

$$-\frac{\partial^2 B_z(x, t)}{\partial x \partial t} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x, t)}{\partial t^2}$$

$$\frac{\partial^2 E_y(x, t)}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x, t)}{\partial t^2}$$ (electromagnetic wave equation in vacuum)

$$\left.\begin{array}{llll} \text{(i)} & \nabla \cdot \mathbf{E} = 0, & \text{(iii)} & \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}, \\[2ex] \text{(ii)} & \nabla \cdot \mathbf{B} = 0, & \text{(iv)} & \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}. \end{array}\right\}$$

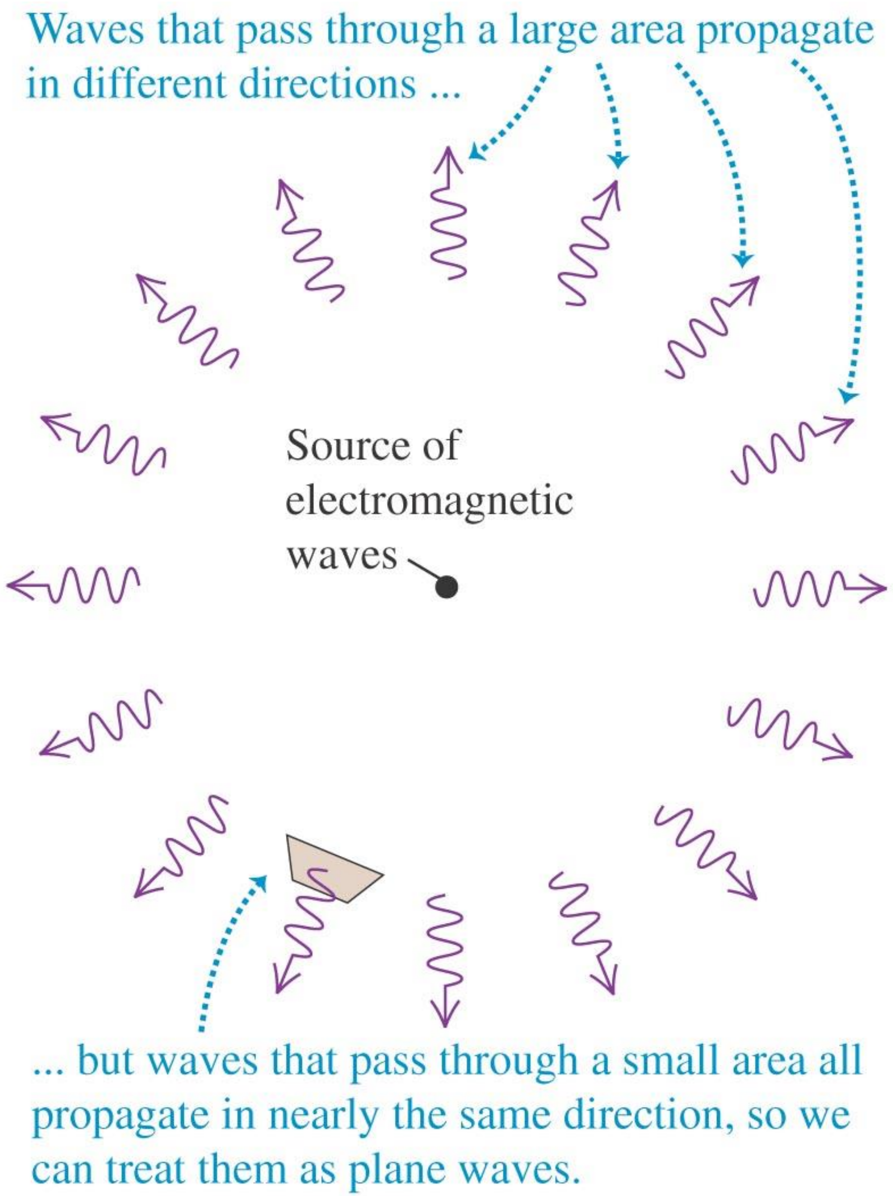
$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) \\ &= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \\ \nabla \times (\nabla \times \mathbf{B}) &= \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \nabla \times \left(\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right) \\ &= \mu_0 \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{E}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}. \end{aligned}$$

e $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$,

$$\boxed{\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.}$$

Sinusoidal electromagnetic waves

- Waves passing through a small area far from a source can be treated as plane waves. (See Figure 32.12 at the right.)



Fields of a sinusoidal wave

$$\frac{1}{v^2} = \epsilon_0\mu_0 \qquad \text{or} \qquad v = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

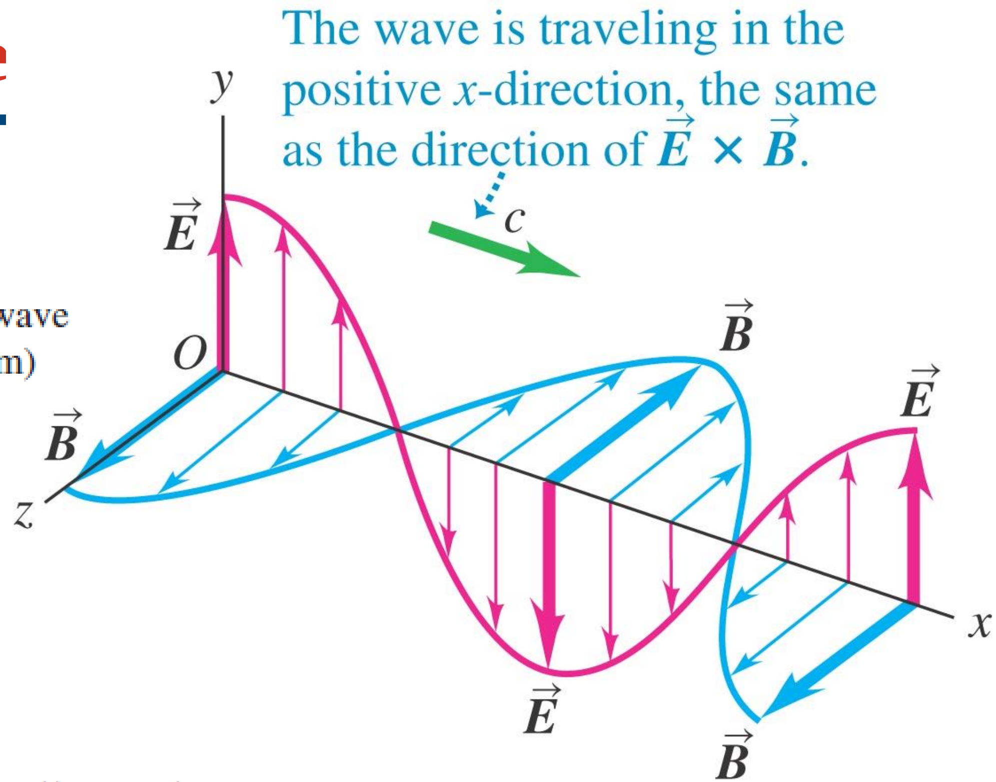
$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \epsilon_0\mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$ (electromagnetic wave equation in vacuum)

$$y(x,t) = A\cos(kx - \omega t)$$

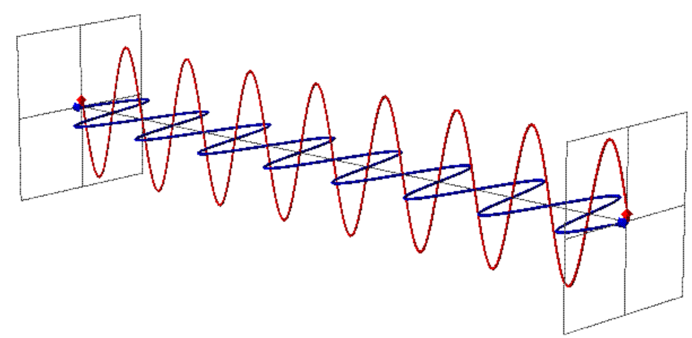
$$E_y(x,t) = E_{\max}\cos(kx - \omega t) \qquad B_z(x,t) = B_{\max}\cos(kx - \omega t)$$

$$\vec{E}(x,t) = \hat{\jmath}E_{\max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{\max}\cos(kx - \omega t)$$

$$E_{\max} = cB_{\max}$$

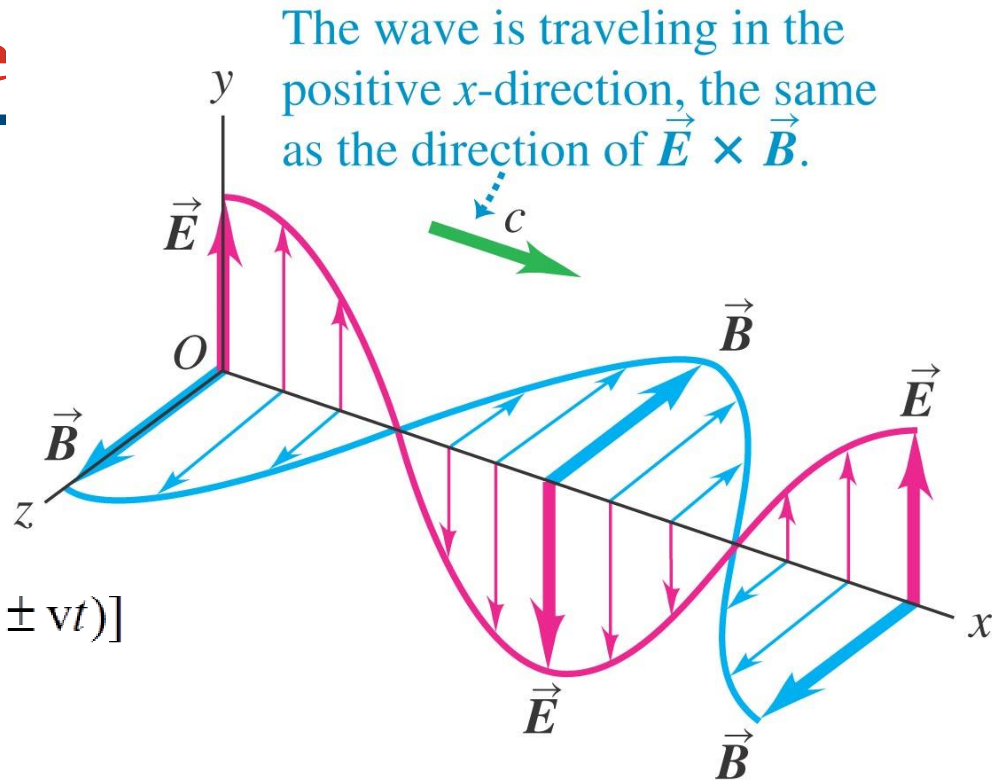


$\vec{E}$: y-component only
$\vec{B}$: z-component only



Fields of a sinusoidal wave

$$\boxed{f(x,t) = f(x \pm \mathrm{v}t)}$$



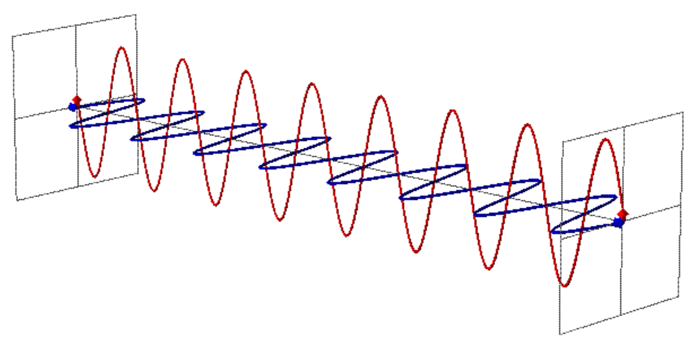
$$\mathrm{E}\;(x,t) = B\cos[k(x \pm \mathrm{v}t)] + C\sin[k(x \pm \mathrm{v}t)]$$

$$kx \pm (k\mathrm{v})t$$

$$\mathrm{E}\;(x,t) = B\cos(kx \pm \omega t) + C\sin(kx \pm \omega t)$$

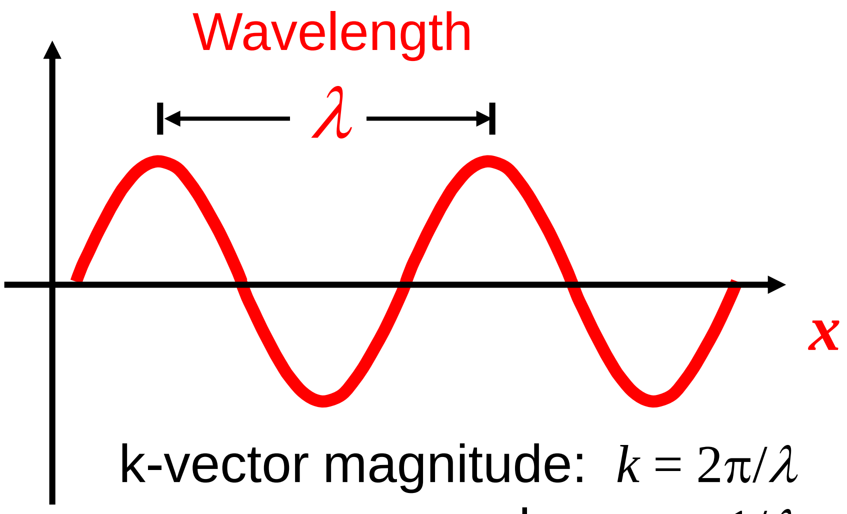
$\vec{E}$: y-component only
$\vec{B}$: z-component only

$$\boxed{\frac{\omega}{k} = \mathrm{v} = \frac{1}{\sqrt{\mu\varepsilon}}}$$



Wavelength, etc.

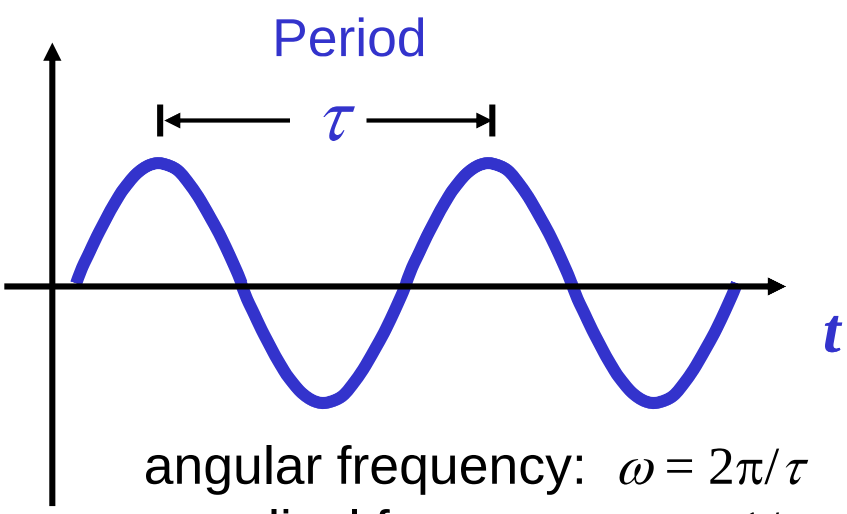
Spatial quantities:



k-vector magnitude: $k = 2\pi/\lambda$

wave number: $\kappa = 1/\lambda$

Temporal quantities:



angular frequency: $\omega = 2\pi/\tau$

cyclical frequency: $\nu = 1/\tau$

The Phase of a Wave

The phase is everything inside the cosine.

$$\boxed{E(x,t) = A\cos(\varphi),\text{ where } \varphi = kx - \omega t - \theta}$$

$\varphi = \varphi(x,t)$ and is not a constant, like θ!

In terms of the phase,

$$\omega = -\partial\varphi/\partial t$$
$$k = \partial\varphi/\partial x$$

We'll prove these results later.

And

$$\boxed{v = \frac{-\partial\varphi/\partial t}{\partial\varphi/\partial x}}$$

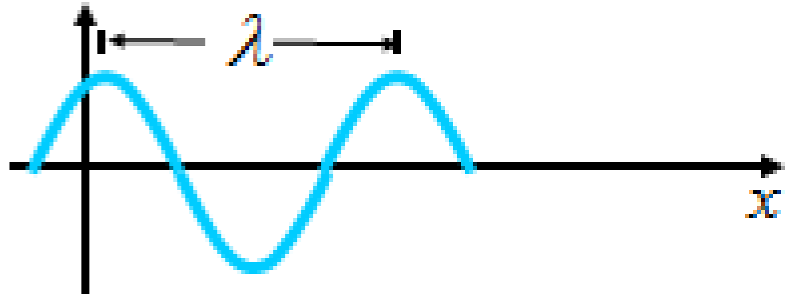
This formula is useful when the wave is really complicated.

The Phase Velocity

How fast is the wave traveling?

The **phase velocity** is the wavelength / period:

$$\boxed{v = \lambda / \tau}$$



The wave moves one wavelength, λ, in one period, τ.

Since $\nu = 1/\tau$:

$$\boxed{v = \lambda \nu}$$

In terms of the k-vector, $k = 2\pi / \lambda$, and the angular frequency, $\omega = 2\pi / \tau$, this is:

$$\boxed{v = \omega / k}$$

It's also helpful to define a phase delay, T, that a wave experiences after propagating a distance, d:

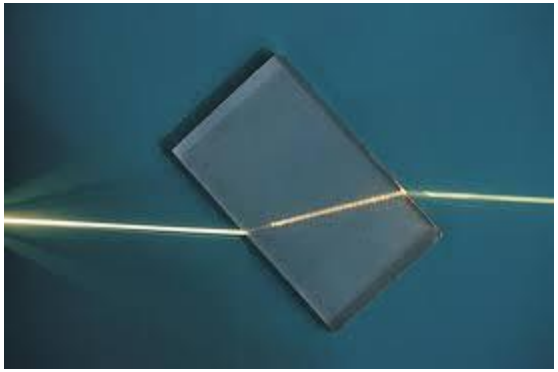
$$\boxed{T = d / v}$$

Electromagnetic waves in matter

- The *index of refraction* of a material is $n = c/v$.

Ampere's law the displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \qquad \boxed{\epsilon\, d\Phi_E/dt = K\epsilon_0\, d\Phi_E/dt,} \qquad \boxed{\mu = K_m \mu_0,}$$



$$E = vB \quad \text{and} \quad B = \epsilon\mu v E$$

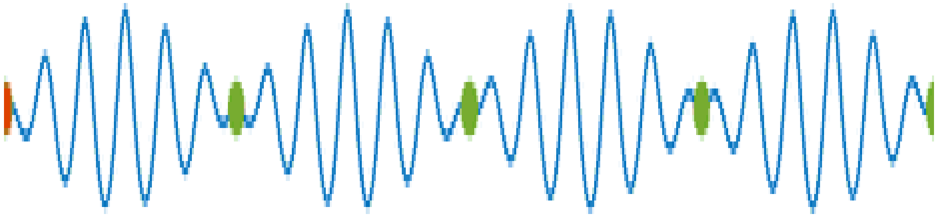
$$\frac{1}{v^2} = \epsilon_0 \mu_0 \quad \text{or} \quad v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}} \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{c}{\sqrt{KK_m}}$$ (speed of electromagnetic waves in a dielectric)

$$K_m \cong 1,$$

$$\frac{c}{v} = n = \sqrt{KK_m} \cong \sqrt{K}$$

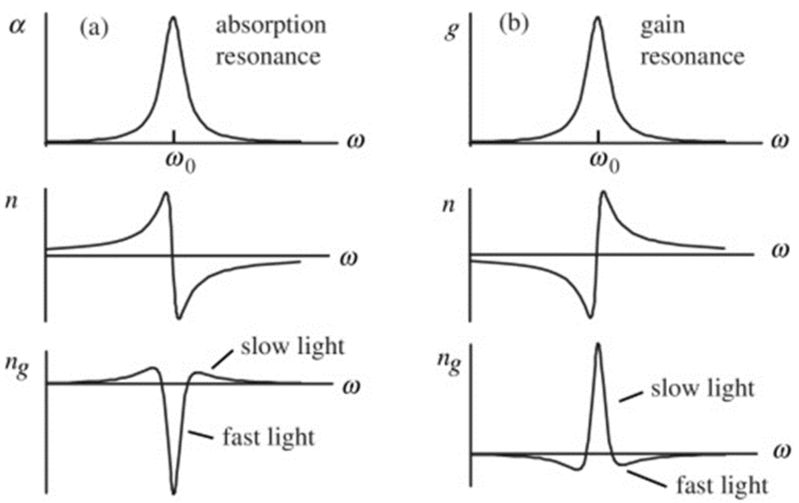
Group velocity

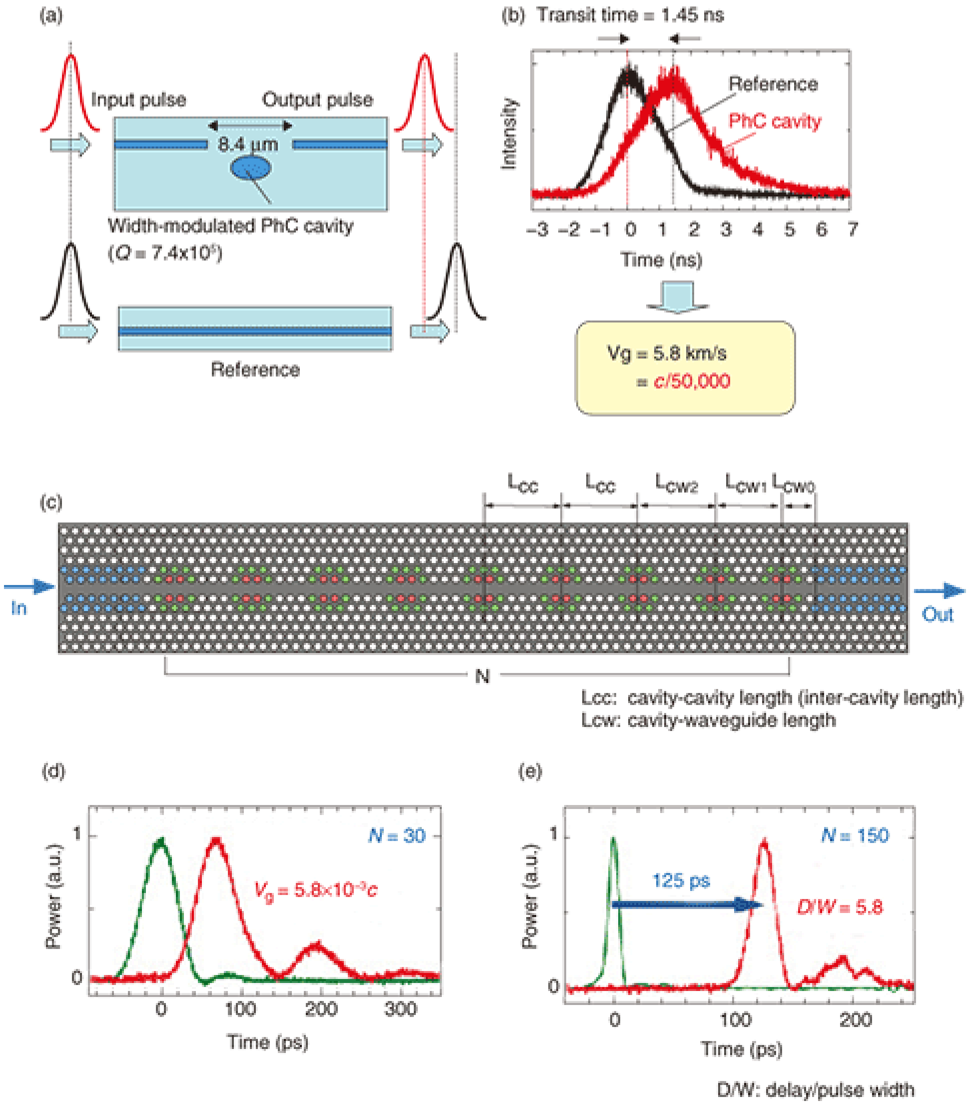


$$v_g \equiv \frac{\partial \omega}{\partial k}$$

Slow Light

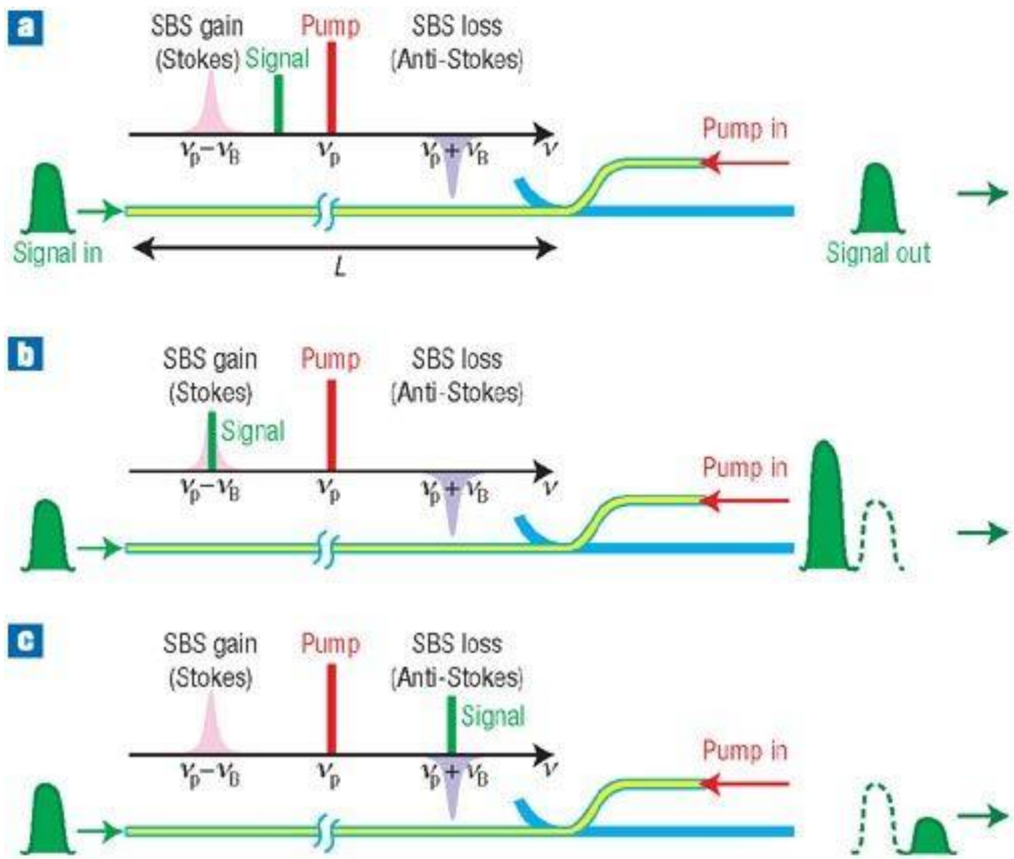
- The *index of refraction* of a material is $n = c/v$.



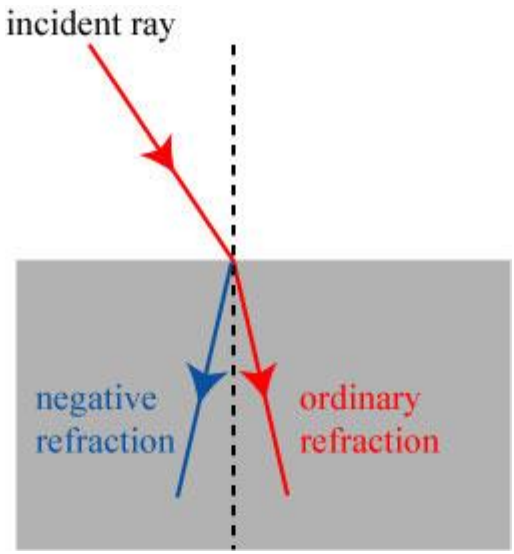


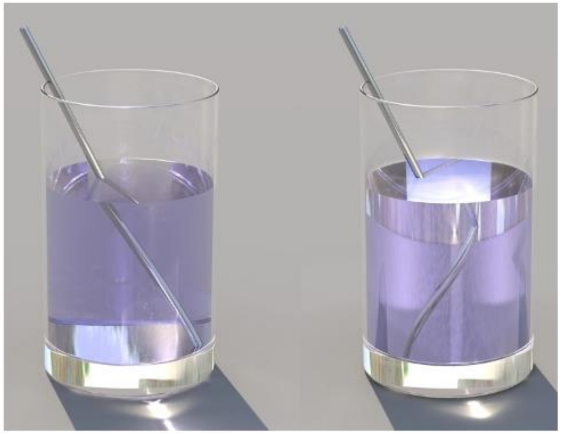
Fast Light

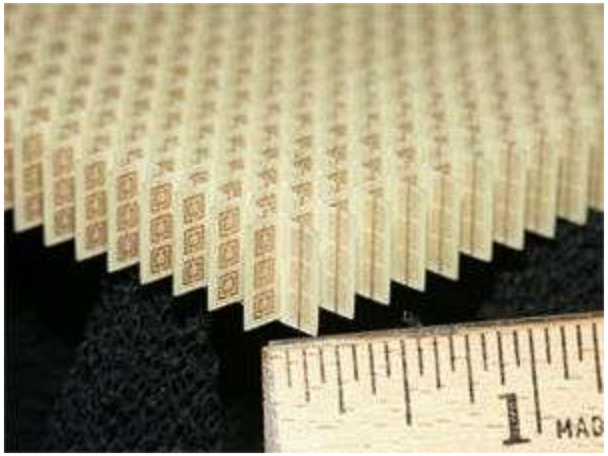
- The *index of refraction* of a material is $n = c/v$.

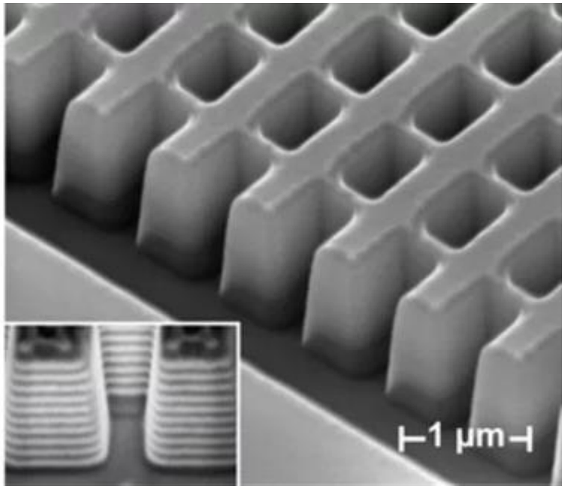


Negative Index

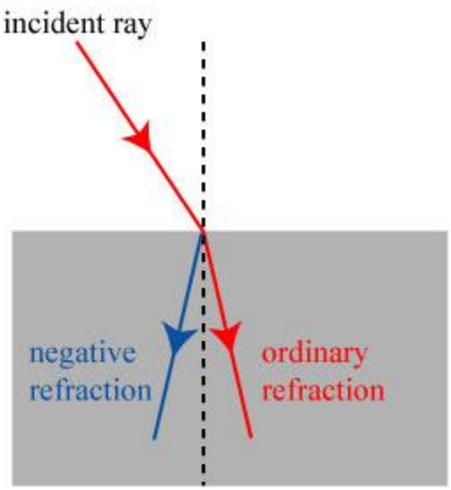


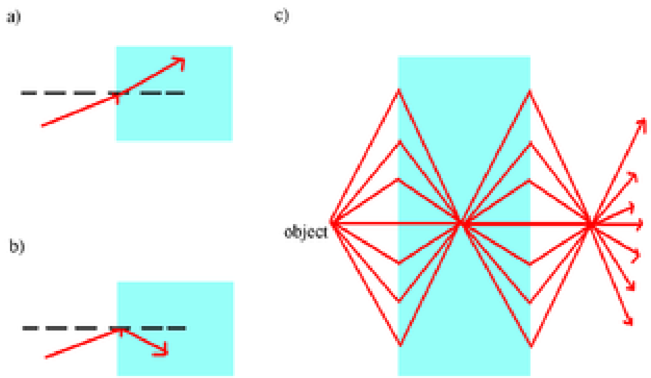


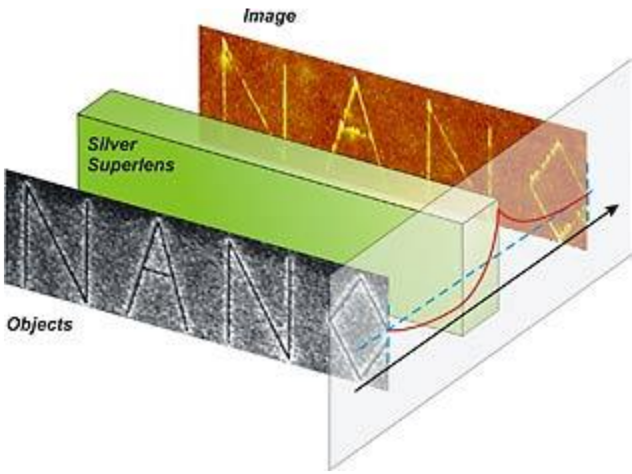


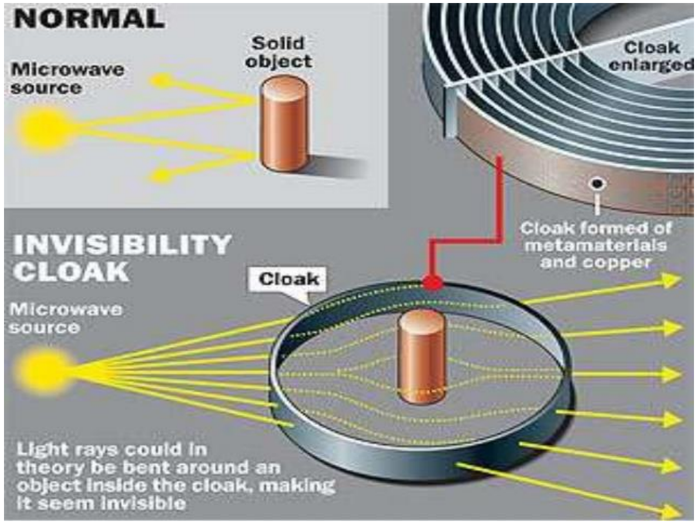


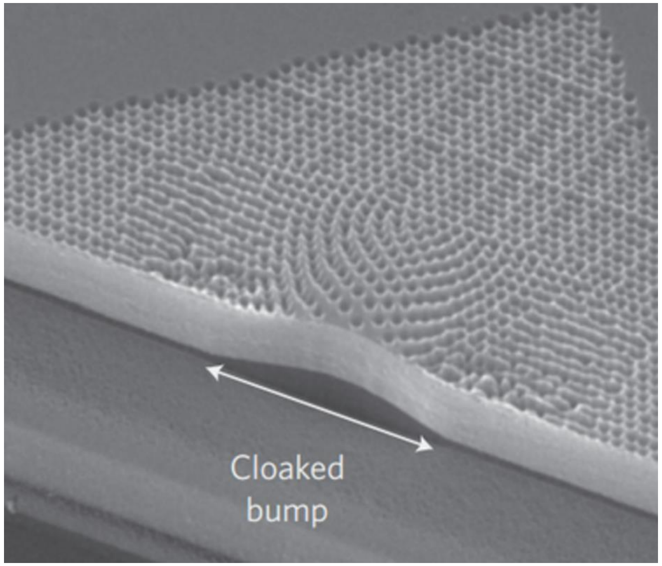
Negative Index











Energy in electromagnetic waves

$$u = \tfrac{1}{2}\epsilon_0 E^2 \qquad \text{(electric energy density in a vacuum)}$$

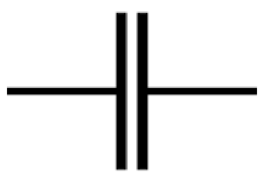
$$V = \frac{Q}{C} \qquad dW = v\,dq = \frac{q\,dq}{C}$$

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \qquad \text{(potential energy stored in a capacitor)}$$

$$u = \text{Energy density} = \frac{\tfrac{1}{2}CV^2}{Ad}$$

$C = \epsilon_0 A/d$. The de E by $V = Ed$.



$$\mathcal{E} = -L\frac{di}{dt} \qquad \text{(self-induced emf)}$$

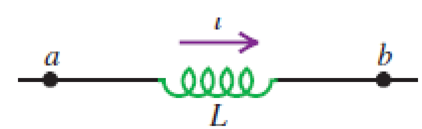
$$P = V_{ab}i = Li\frac{di}{dt}$$

$$dU = P\,dt, \qquad dU = Li\,di$$

$$U = L\int_0^I i\,di = \tfrac{1}{2}LI^2 \qquad \text{(energy stored in an inductor)}$$

$$L = \frac{\mu_0 N^2 A}{l} \qquad U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\frac{\mu_0 N^2 A}{l}I^2$$

$$u = \frac{U}{l\,A} = \tfrac{1}{2}\mu_0\frac{N^2 I^2}{(l)^2} \qquad B = \frac{\mu_0 N I}{l} \qquad u = \frac{B^2}{2\mu_0}$$



$$u = \tfrac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2 \qquad B = \frac{E}{c} = \sqrt{\epsilon_0\mu_0}\,E$$

$$\boxed{u = \tfrac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\left(\sqrt{\epsilon_0\mu_0}\,E\right)^2 = \epsilon_0 E^2}$$

Energy in electromagnetic waves

$$dU = u\,dV = (\epsilon_0 E^2)(Ac\,dt)$$

This energy passes through the area A in time dt. The energy flow per unit time per unit area, which we will call S, is

$$S = \frac{1}{A}\frac{dU}{dt} = \epsilon_0 c E^2 \qquad \text{(in vacuum)} \qquad (32.26)$$

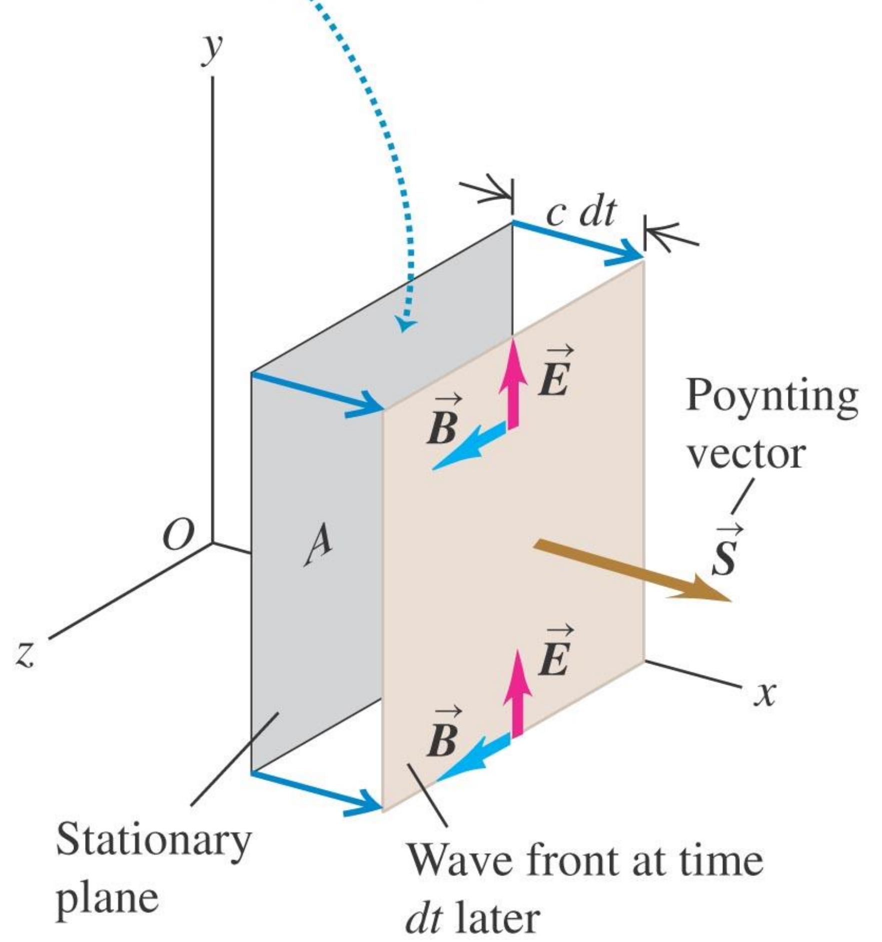
$$S = \frac{\epsilon_0}{\sqrt{\epsilon_0\mu_0}}E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}}E^2 = \frac{EB}{\mu_0}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad \text{(Poynting vector in vacuum)}$$

The total energy flow per unit time (power, P)

$$P = \oint \vec{S}\cdot d\vec{A}$$

At time dt, the volume between the stationary plane and the wave front contains an amount of electromagnetic energy $dU = uAc\,dt$.



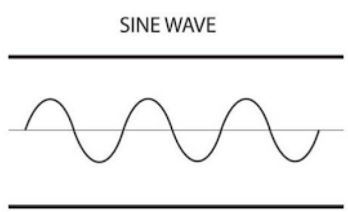
Energy in electromagnetic waves

$$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}(x,t) \times \vec{B}(x,t)$$
$$= \frac{1}{\mu_0}[\hat{\jmath}E_{\max}\cos(kx - \omega t)] \times [\hat{k}B_{\max}\cos(kx - \omega t)]$$

$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$

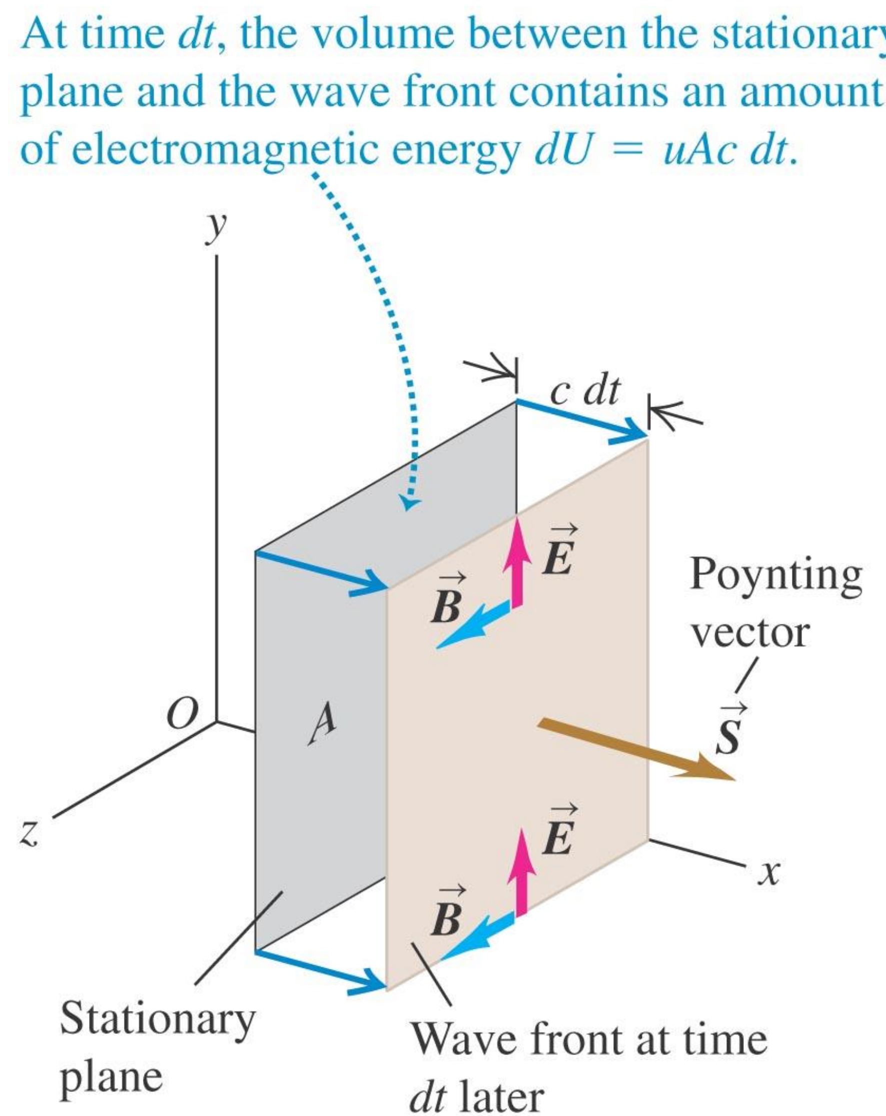
$$S_x(x,t) = \frac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx - \omega t) = \frac{E_{\max}B_{\max}}{2\mu_0}[1 + \cos 2(kx - \omega t)]$$

$$S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0}$$



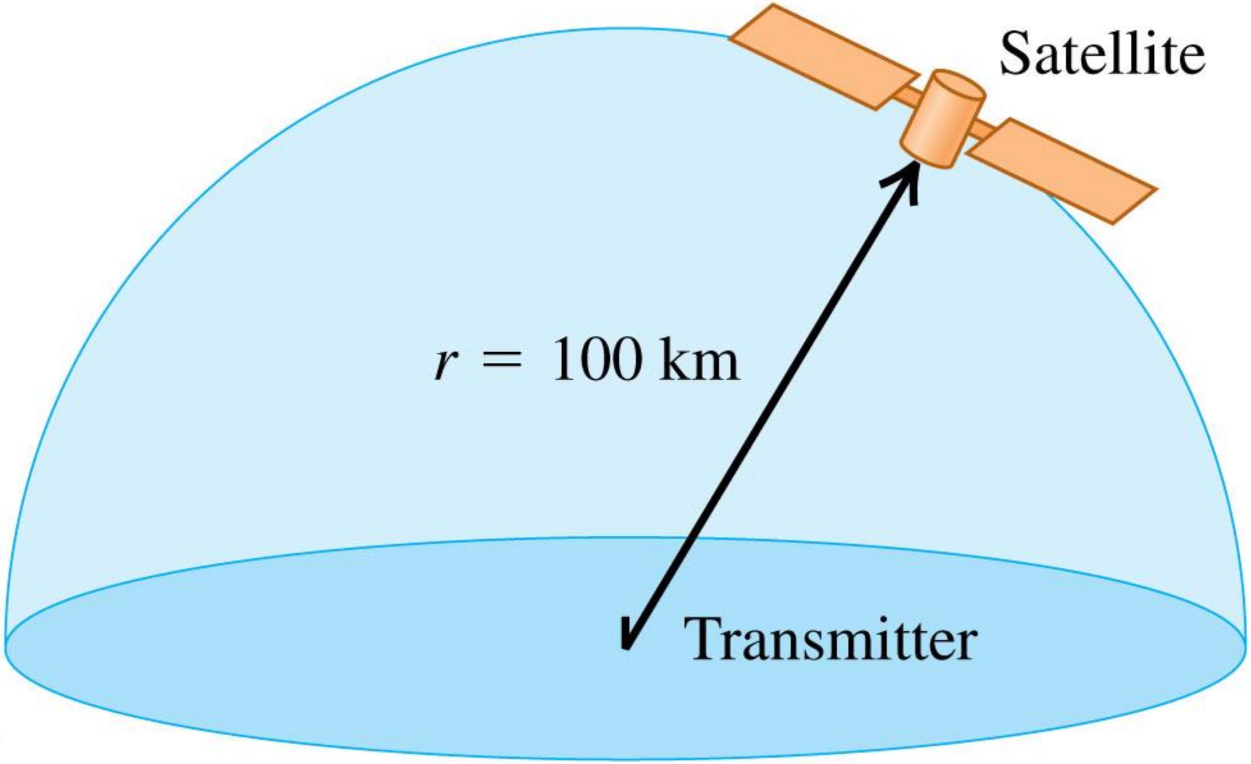
$$I = S_{\text{av}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}{}^2}{2\mu_0 c}$$
$$= \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{\max}{}^2 = \frac{1}{2}\epsilon_0 c E_{\max}{}^2$$

(intensity of a sinusoidal wave in vacuum)



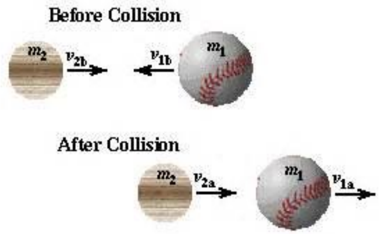
Average energy flow / time / area

Energy in sinusoidal and nonsinusoidal waves



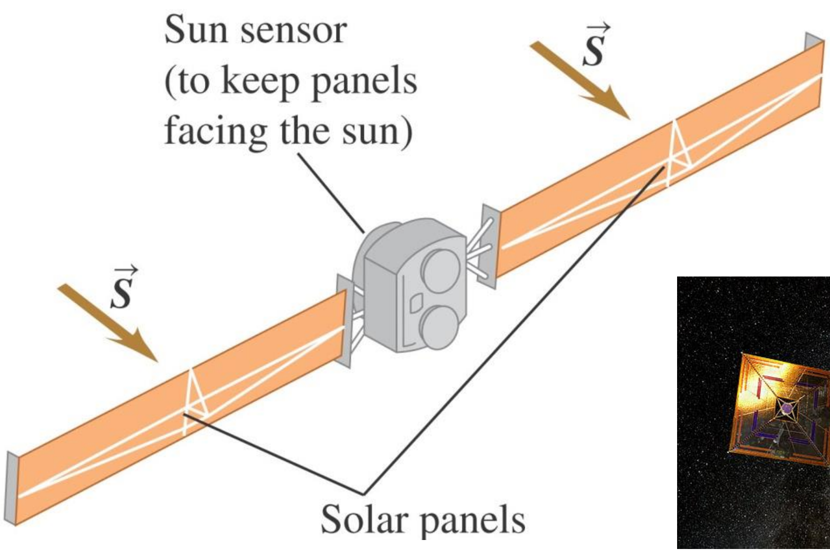
$$I = \frac{P}{A} = \frac{P}{2\pi R^2}$$

Electromagnetic momentum and radiation pressure

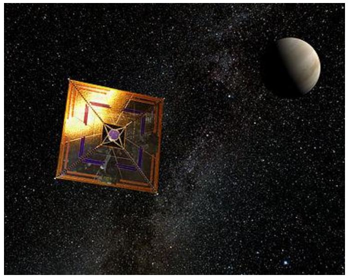


$$p_{\text{rad}} = \frac{S_{\text{av}}}{c} = \frac{I}{c} \quad \text{(radiation pressure, wave totally absorbed)}$$

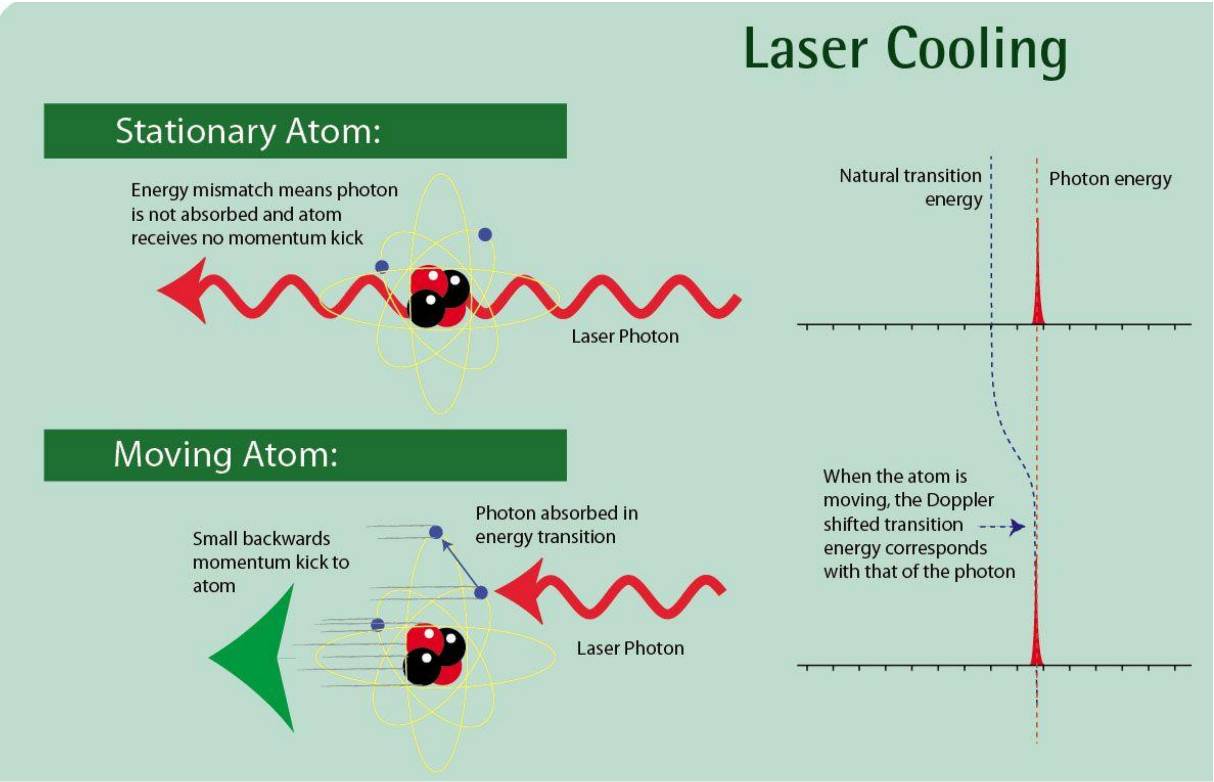
$$p_{\text{rad}} = \frac{2S_{\text{av}}}{c} = \frac{2I}{c} \quad \text{(radiation pressure, wave totally reflected)}$$

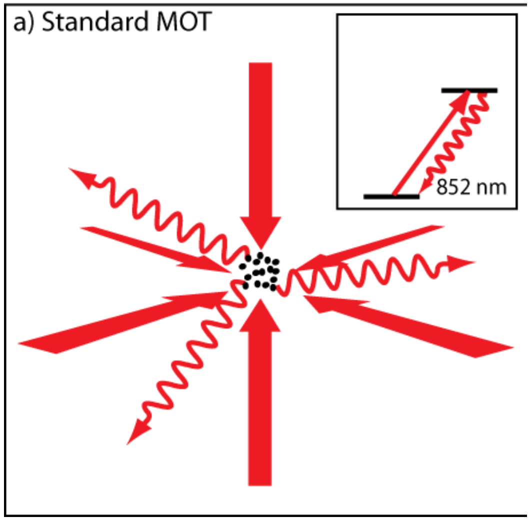


$$p_{\text{rad}} = \frac{I}{c} = \frac{1.4 \times 10^3\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = 4.7 \times 10^{-6}\ \text{Pa}$$



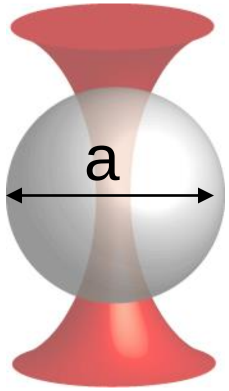
Laser Cooling



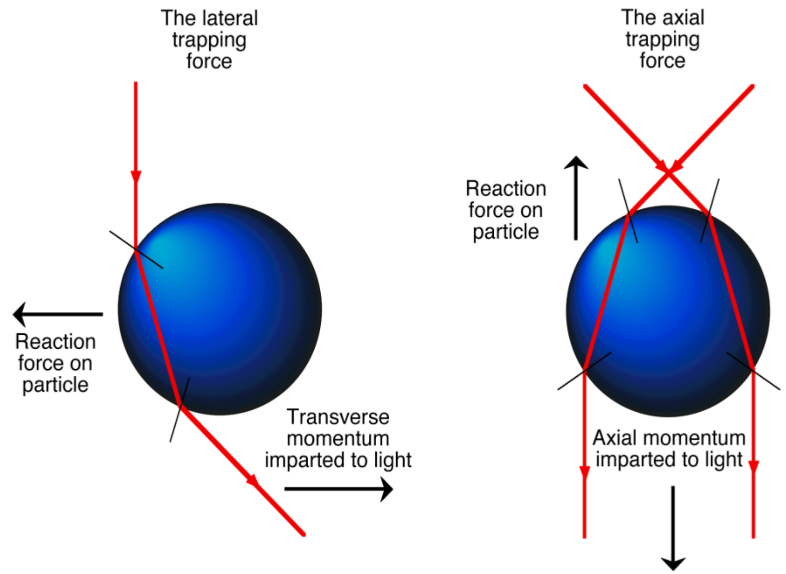


Optical Trapping - a>>λ

Conditions for Mie scattering when the particle radius *a* is larger than the wavelength of the light λ.

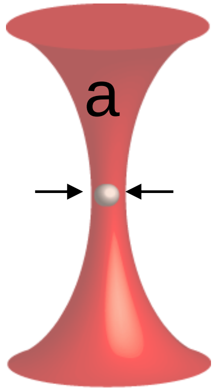


We can use a ray optics argument and look at the transfer of momentum



Optical Trapping - a<<λ

Condition for Rayleigh scattering when the particle radius *a* is smaller than the wavelength of the light λ.



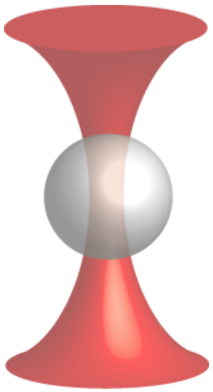
Scattering force and gradient force are separable

$$F_{scatt} = \boxed{I_0}\frac{128\pi^5 a^6 n_m}{3\lambda^4 c}\left(\frac{m^2-1}{m^2+2}\right)^2$$

$$F_{grad} = \boxed{\nabla I_0}\frac{2\pi a^3}{c}\left(\frac{m^2-1}{m^2+2}\right)$$

$F_{grad} > F_{scatt}$ requires tight focusing

The scales



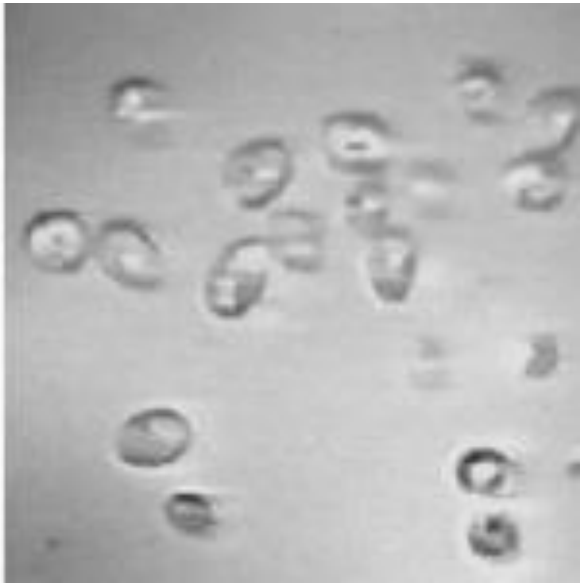
Can trap 0.1 to 10's μm

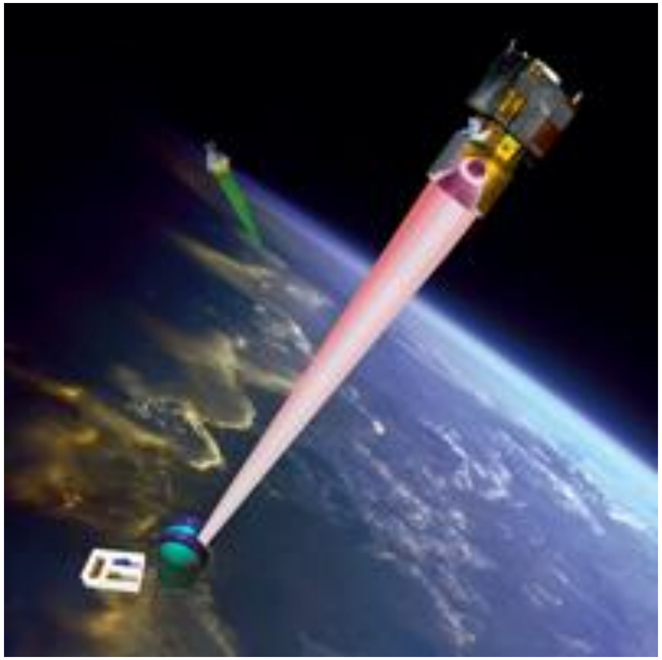
1μm is…..
…the same as 1/100th diameter of a hair.

In water, you can move a particle at about 20-30μm per sec.

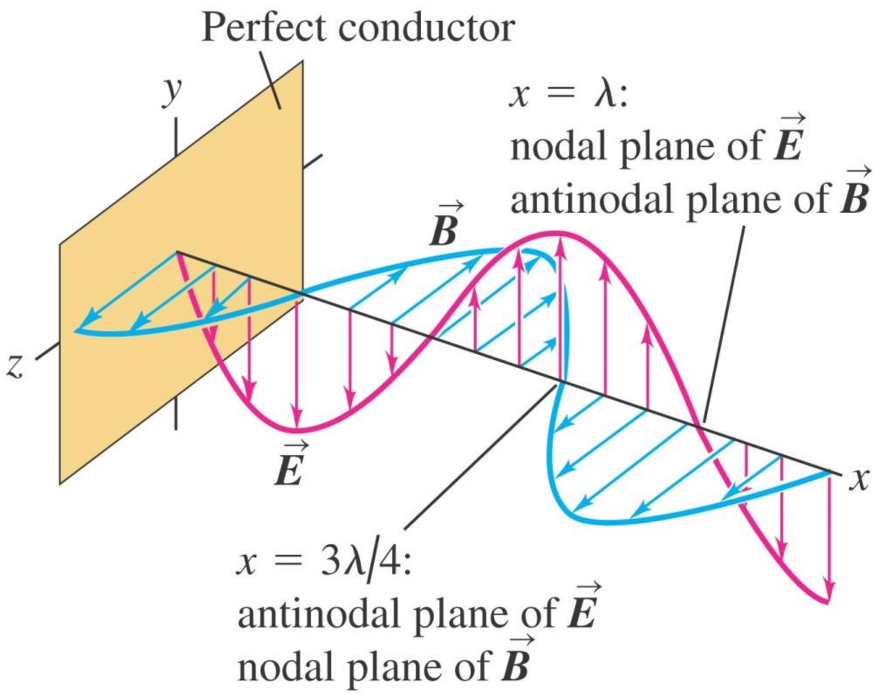
Require 10mW per trap.

Can rotate at 100's of Hz.





Standing electromagnetic waves



$$E_y(x,t) = E_{max}[\cos(kx + \omega t) - \cos(kx - \omega t)]$$

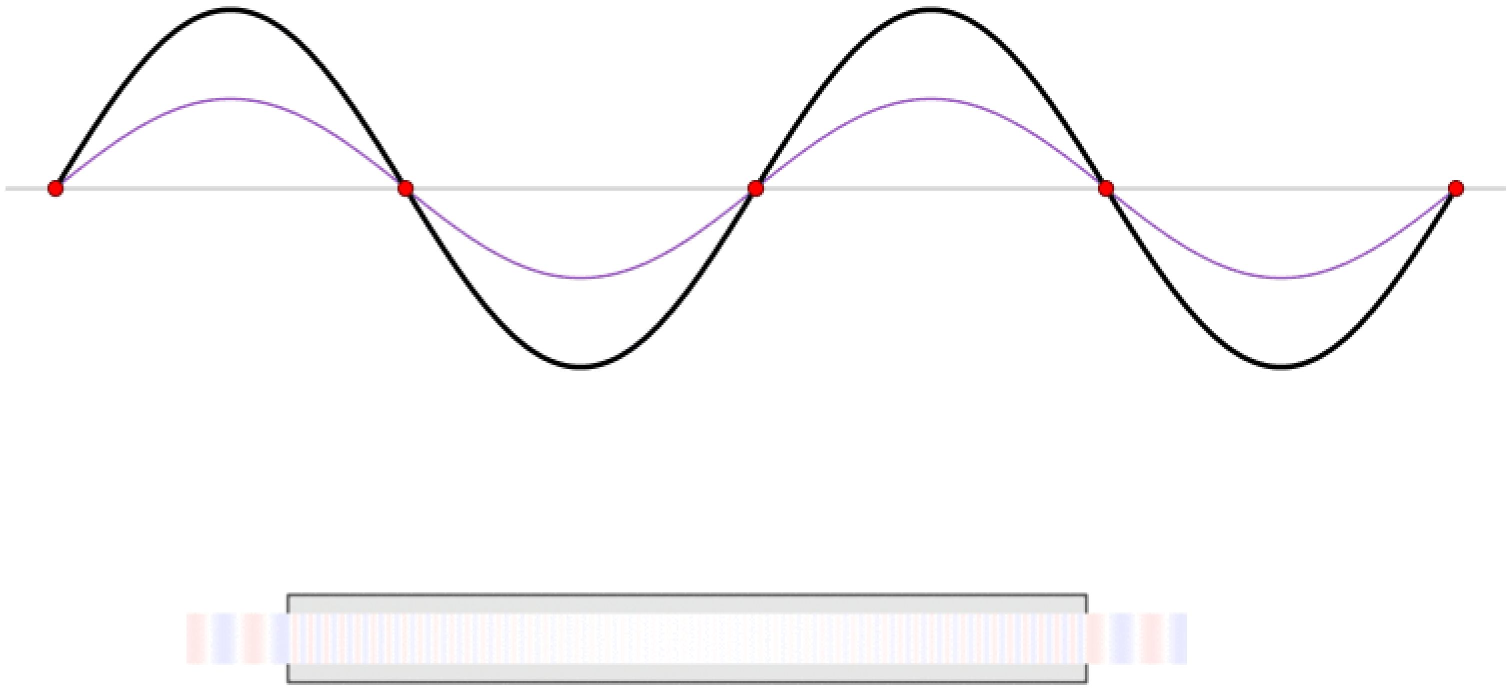
$$B_z(x,t) = B_{max}[-\cos(kx + \omega t) - \cos(kx - \omega t)]$$

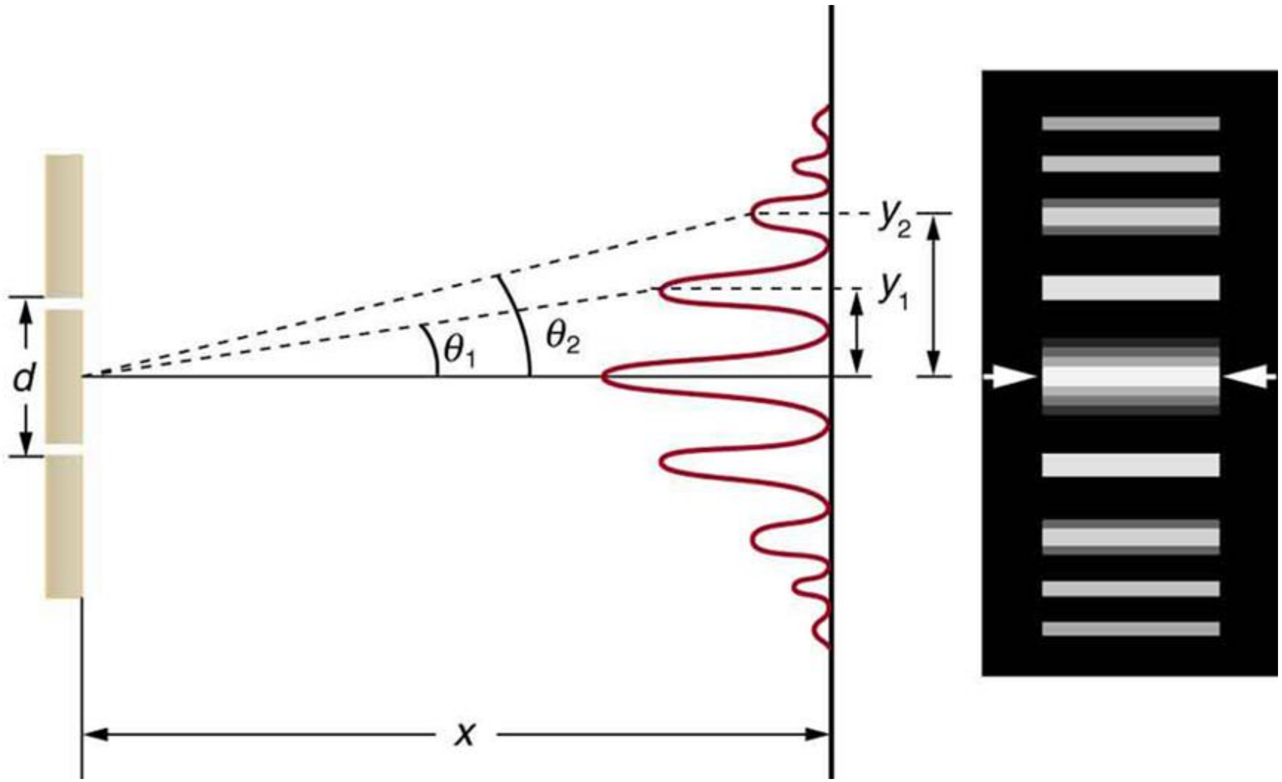
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$E_y(x,t) = -2E_{max} \sin kx \sin \omega t$$

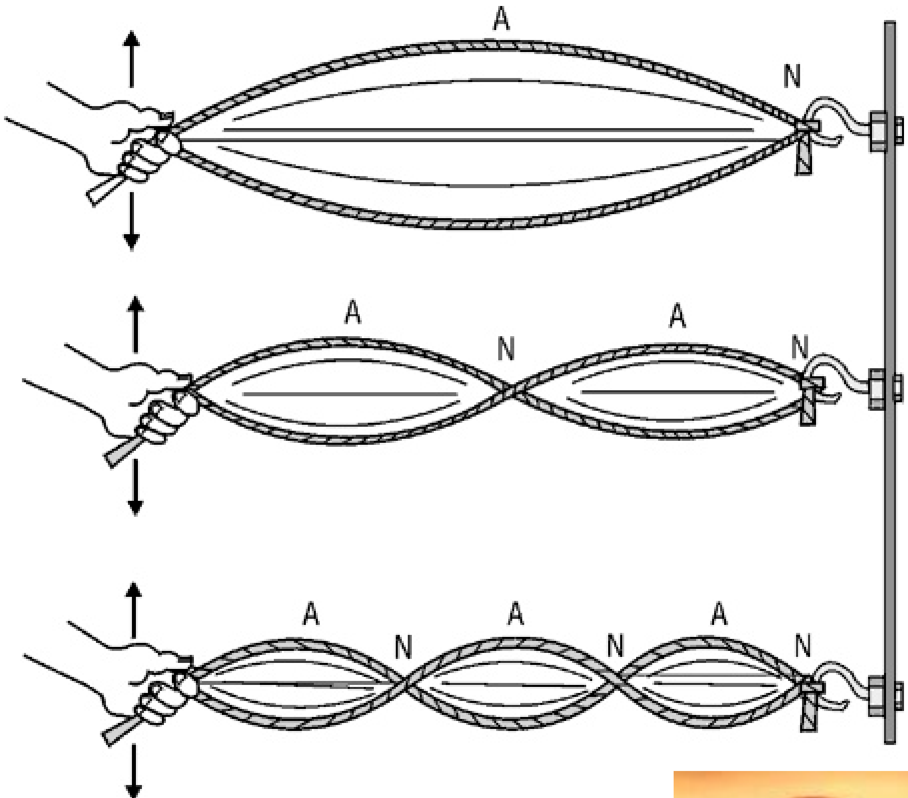
$$B_z(x,t) = -2B_{max} \cos kx \cos \omega t$$

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \qquad (\text{nodal planes of } \vec{E})$$





y_2
y_1
θ_1
θ_2
d
x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