VE 320 Fall 2021

Introduction to Semiconductor Devices

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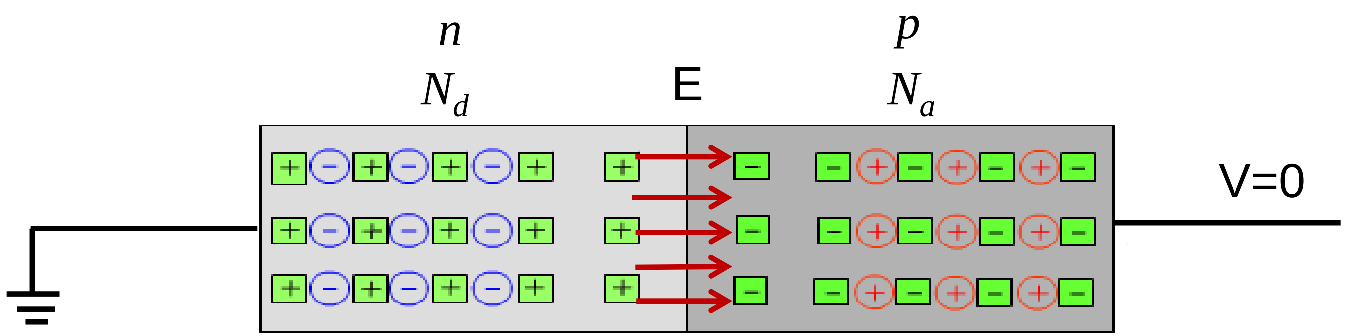
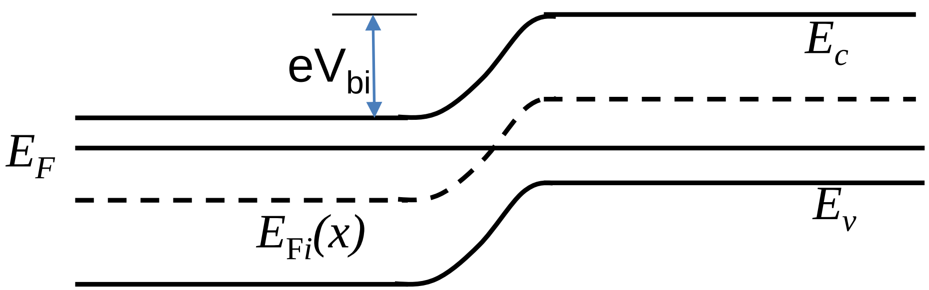
Lecture 8

pn junction diode
(Chapter 8)

pn Junction Current

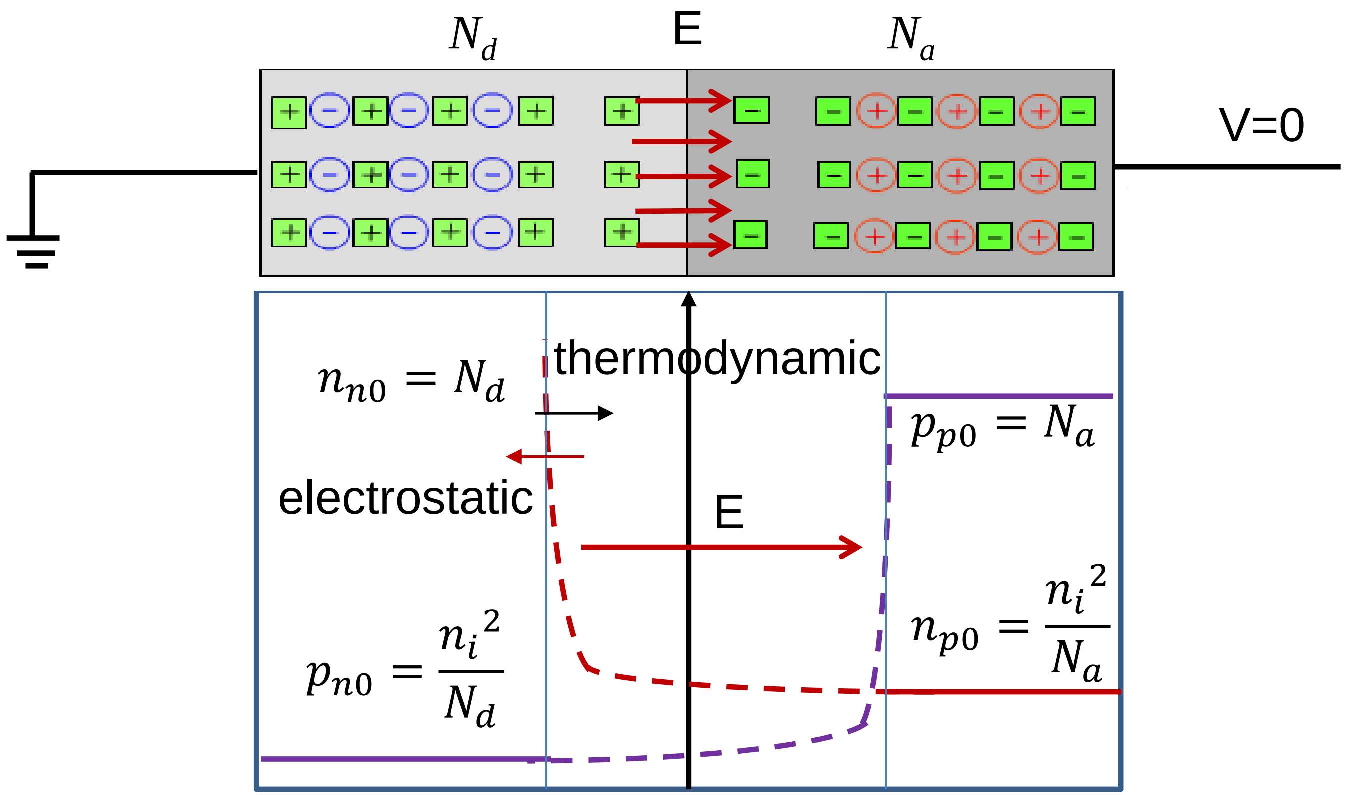
- charge carrier transport: <u>zero bias</u>





pn Junction Current

- charge carrier transport: <u>zero bias</u>

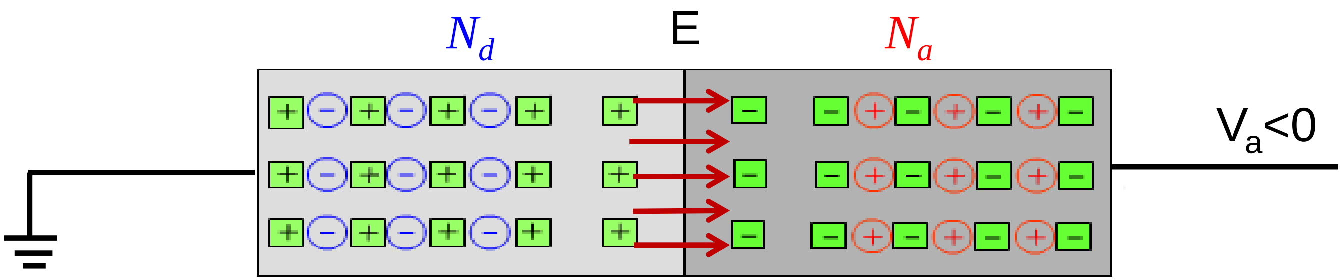


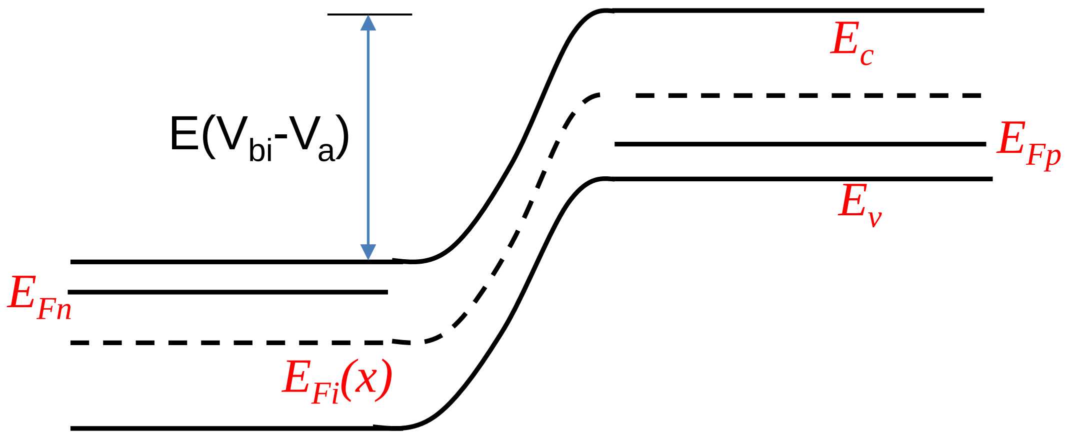
$$J_{diff,n} + J_{drift,n} = 0$$

$$J_{diff,p} + J_{drift,p} = 0$$

pn Junction Current

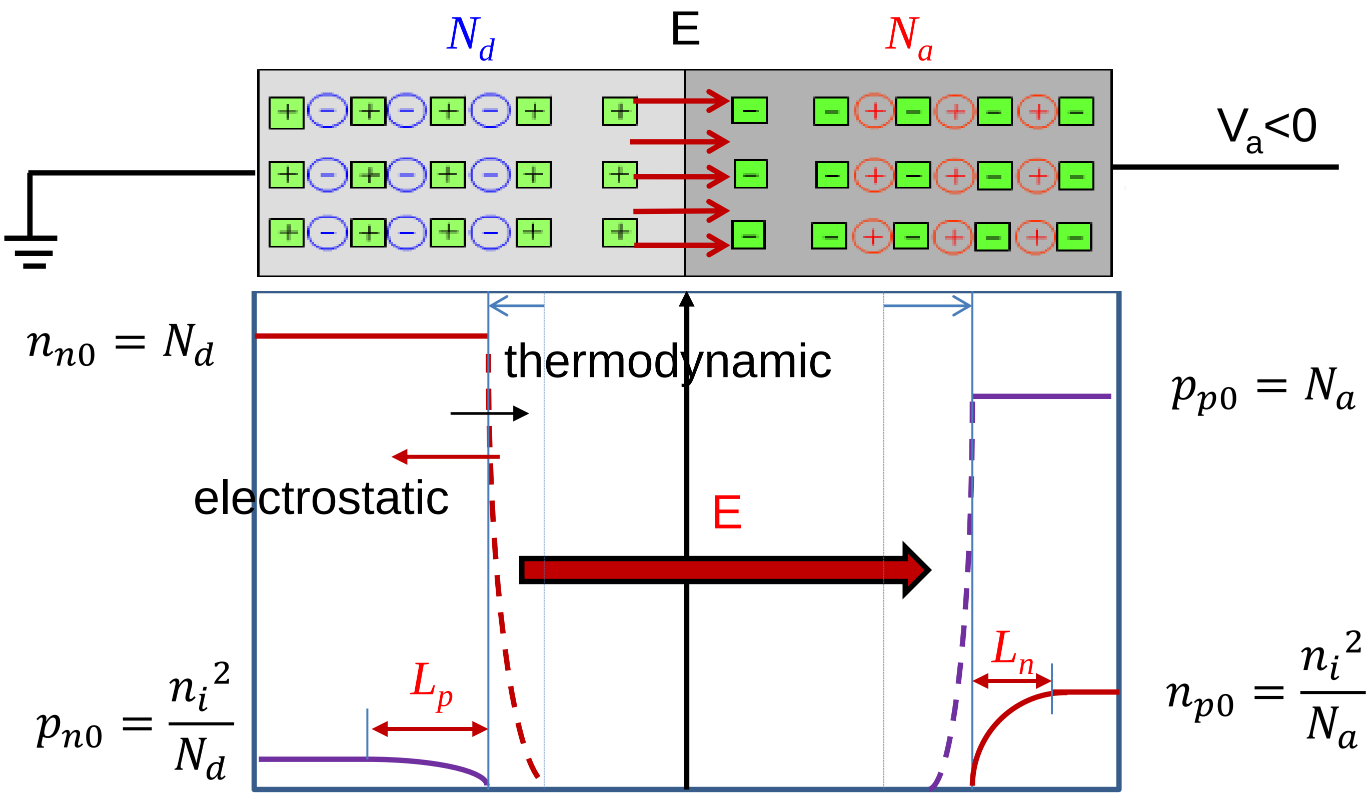
- charge carrier transport: <u>reverse bias</u>





pn Junction Current

- charge carrier transport: reverse bias

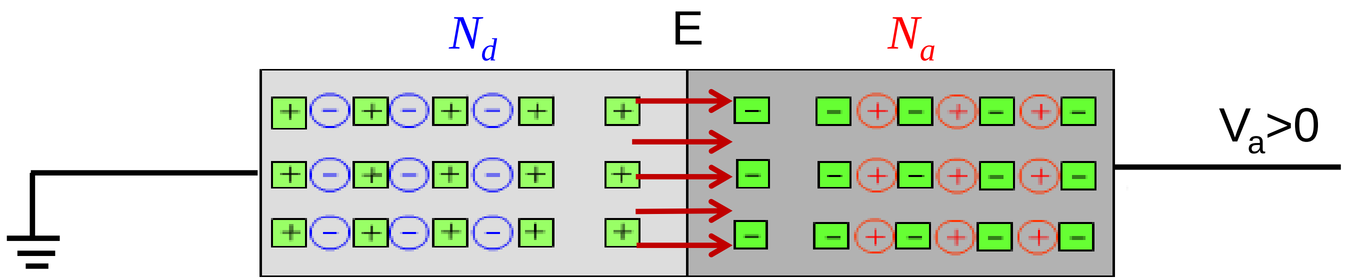


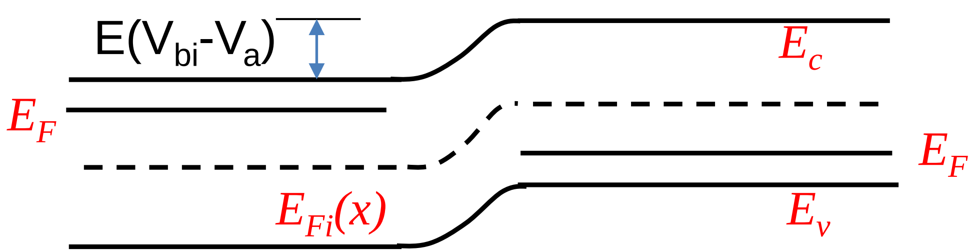
$$J_{diff,n} + J_{drift,n} < 0$$

$$J_{diff,p} + J_{drift,p} < 0$$

pn Junction Current

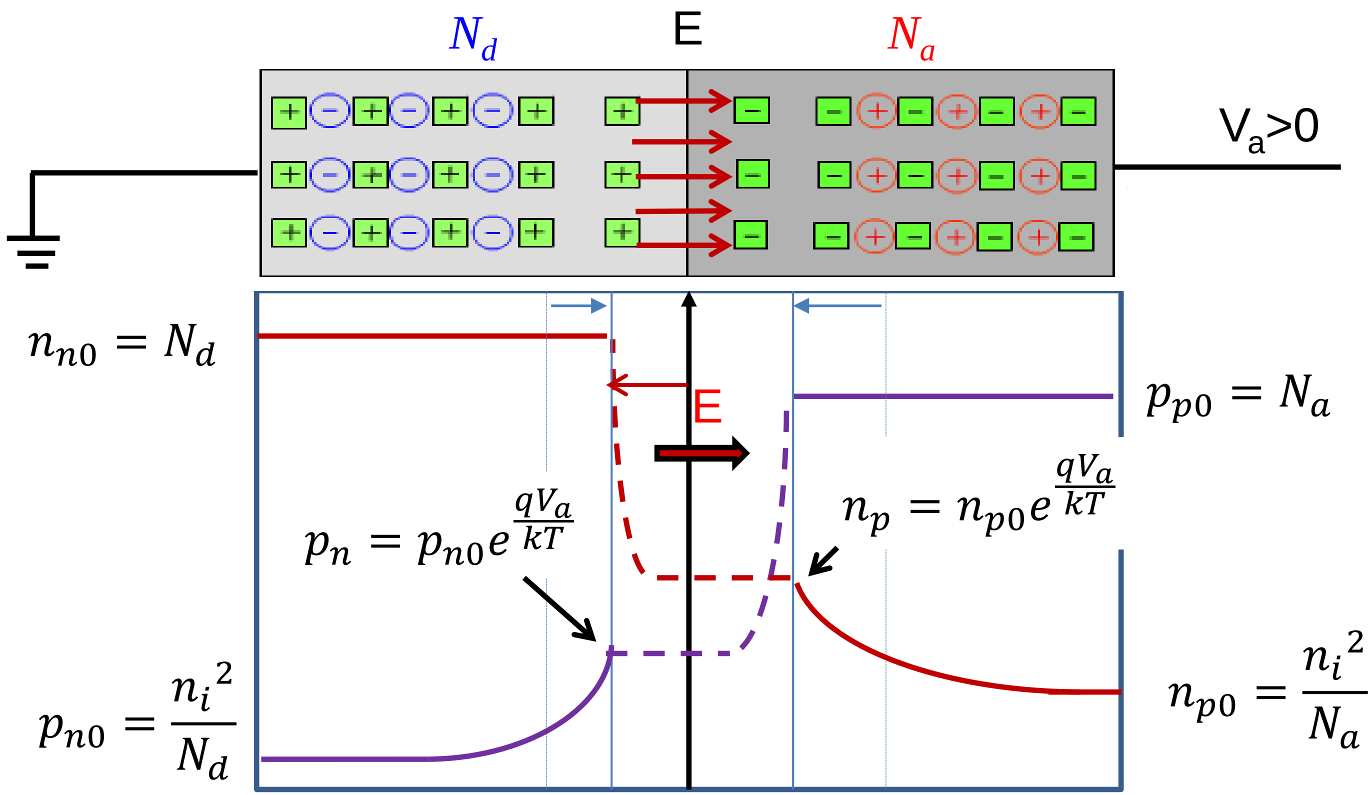
- charge carrier transport: forward bias





pn Junction Current

- charge carrier transport: forward bias



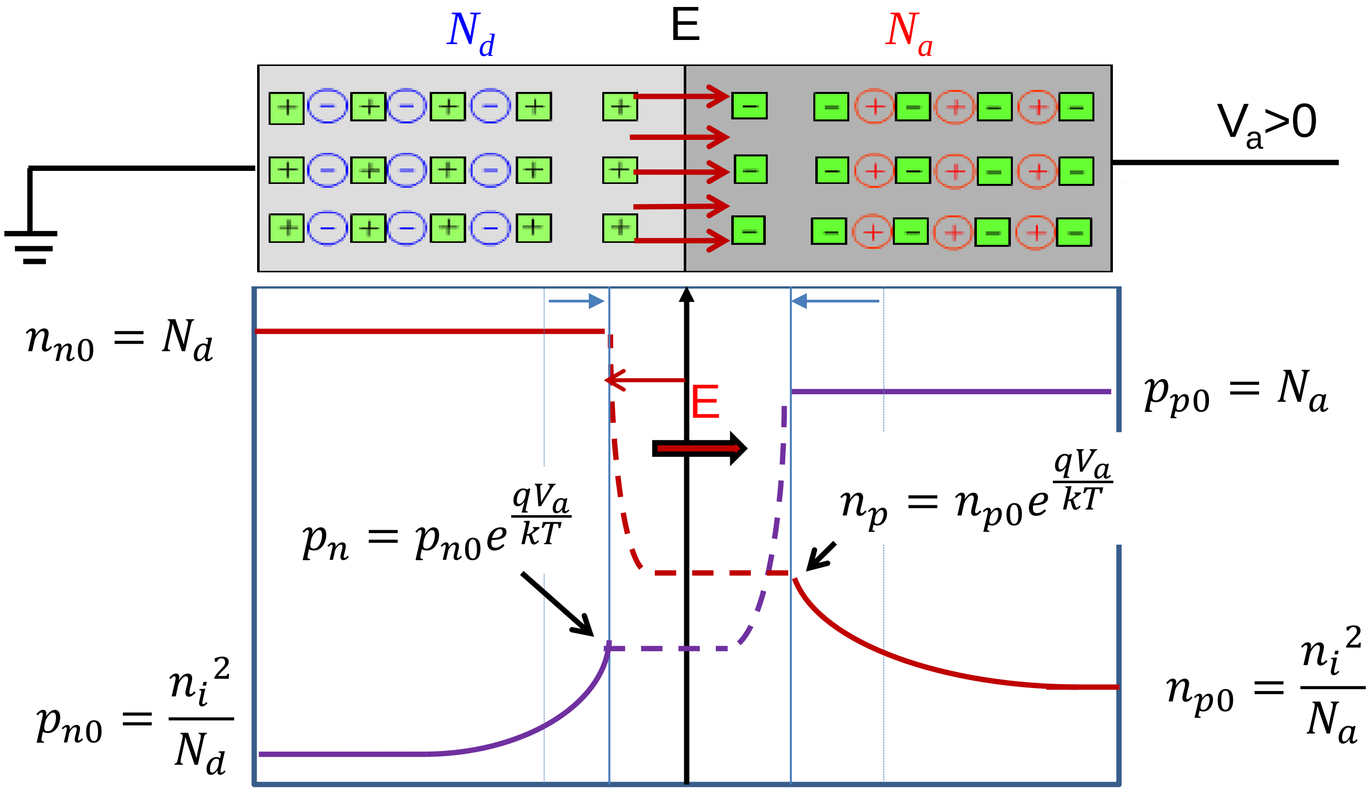
$$\boxed{J_{diff,n} + J_{drift,n} > 0}$$

$$\boxed{J_{diff,p} + J_{drift,p} > 0}$$

pn Junction Current

- charge carrier transport: <u>forward bias</u>

Electric field is reduced
Diffusion current
Minority carrier injection:
nonequilibium excess carriers!



$$J_{diff,n} + J_{drift,n} > 0$$

$$J_{diff,p} + J_{drift,p} > 0$$

pn Junction Current

Ideal *I-V* relationship assumption:

- Abrupt depletion layer approximation: The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region
- Boltzmann approximation applies to carrier statistics
- Low injection and complete ionization apply
- Current:
  - The total current is a constant throughout the entire pn structure
  - The individual electron and hole currents are continuous functions through the pn structure
  - The individual electron and hole currents are constant throughout the depletion region

pn Junction Current

Minority carrier concentration:

Previously we have $$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$ $$V_t = kT/e$$

Then $$\frac{n_i^2}{N_a N_d} = \exp\left(\frac{-eV_{bi}}{kT}\right)$$ Complete ionization: $$n_{n0} \approx N_d \qquad n_{p0} \approx \frac{n_i^2}{N_a}$$

So $$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$ relates the minority carrier on the p side to the majority carrier electron concentration on the n side in thermal equilibrium

Forward-biased pn junction: bias V_a

$$n_p = n_{n0} \exp\left(\frac{-e\left(V_{bi} - V_a\right)}{kT}\right) = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \exp\left(\frac{+eV_a}{kT}\right)$$

$$n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

Forward-bias: no longer in thermal equilibrium

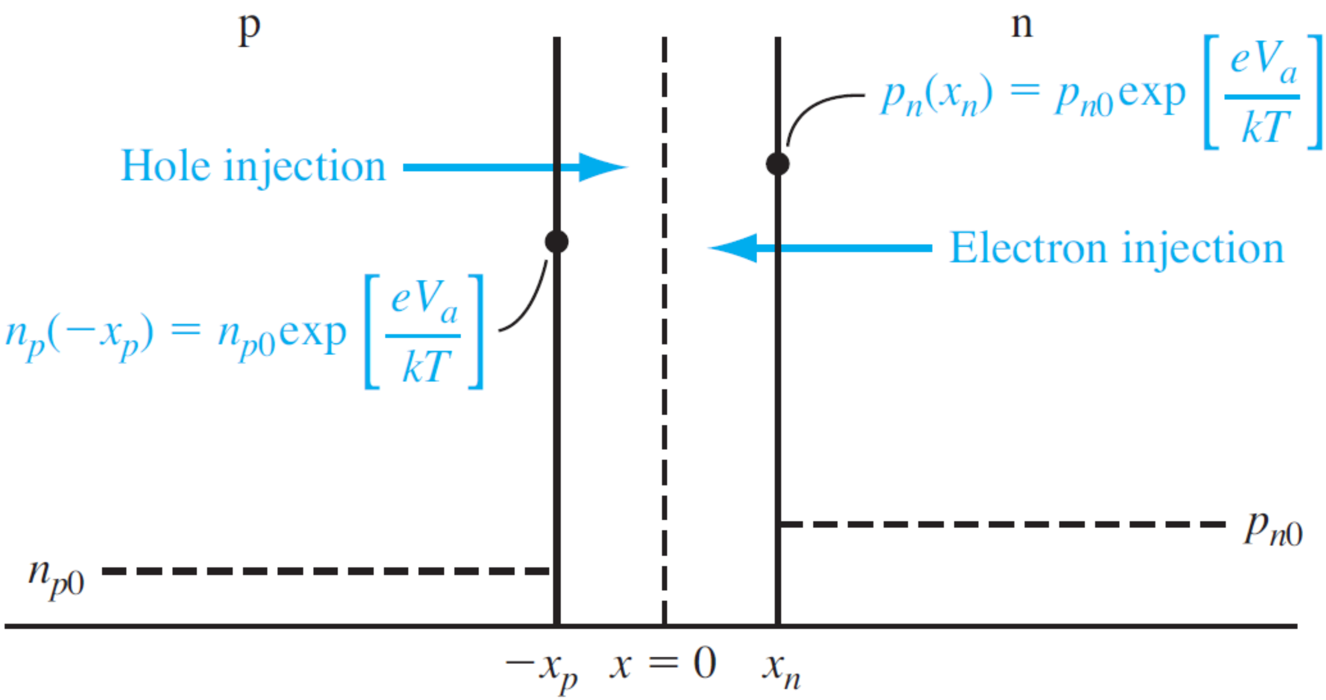
Exponential change, excess minority carriers

Concentration at the edge

Similarly, for holes in n-region: $$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

pn Junction Current

Consider Forward-biased pn junction: bias V_a



Example

Objective: Calculate the minority carrier concentrations at the edge of the space charge regions in a forward-biased pn junction.

Consider a silicon pn junction at $T = 300$ K. Assume the doping concentration in the n region is $N_d = 10^{16}$ cm^{-3} and the doping concentration in the p region is $N_a = 6 \times 10^{15}$ cm^{-3}, and assume that a forward bias of 0.60 V is applied to the pn junction.

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■ **Solution**

From Equations (8.6) and (8.7) and from Figure 8.4, we have

$$n_p(-x_p) = n_{po} \exp\left(\frac{eV_a}{kT}\right) \quad \text{and} \quad p_n(x_n) = p_{no} \exp\left(\frac{eV_a}{kT}\right)$$

The thermal-equilibrium minority carrier concentrations are

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4 \text{ cm}^{-3}$$

and

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

We then have

$$n_p(-x_p) = 3.75 \times 10^4 \exp\left(\frac{0.60}{0.0259}\right) = 4.31 \times 10^{14} \text{ cm}^{-3}$$

and

$$p_n(x_n) = 2.25 \times 10^4 \exp\left(\frac{0.60}{0.0259}\right) = 2.59 \times 10^{14} \text{ cm}^{-3}$$

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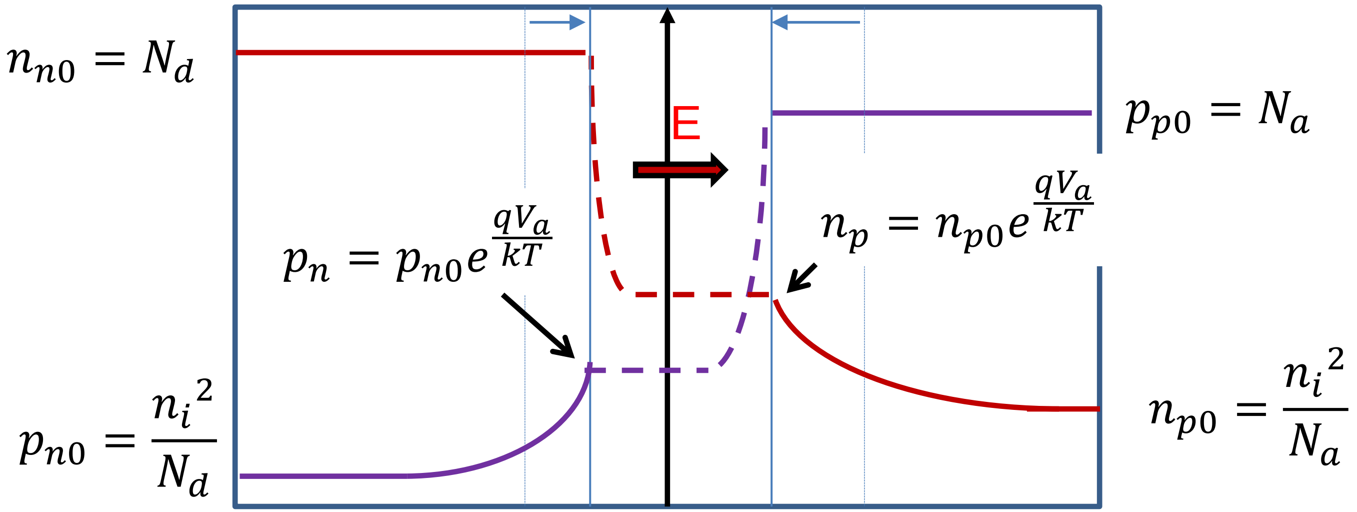
What does this tell us?

The minority carrier concentrations can increase by many orders of magnitude when a relatively small forward-bias voltage is applied. Low injection still applies, however, since the excess minority carrier concentrations at the space-charge edges are much less than the thermal-equilibrium majority carrier concentrations.

pn Junction Current

- charge carrier transport: forward bias
- In the n-region: *E*=0, and *g*'=0

$$\boxed{\underbrace{\cancel{\frac{\partial p}{\partial t}}}_{0} = D_p \underbrace{\cancel{\frac{\partial^2 p}{\partial x^2}}}_{\delta p} - \mu_p \underbrace{\cancel{E}}_{0} \frac{\partial p}{\partial x} - p\mu_p \underbrace{\cancel{\frac{\partial E}{\partial x}}}_{0} - \frac{\delta p}{\tau} + \underbrace{\cancel{g'}}_{0}}$$



pn Junction Current

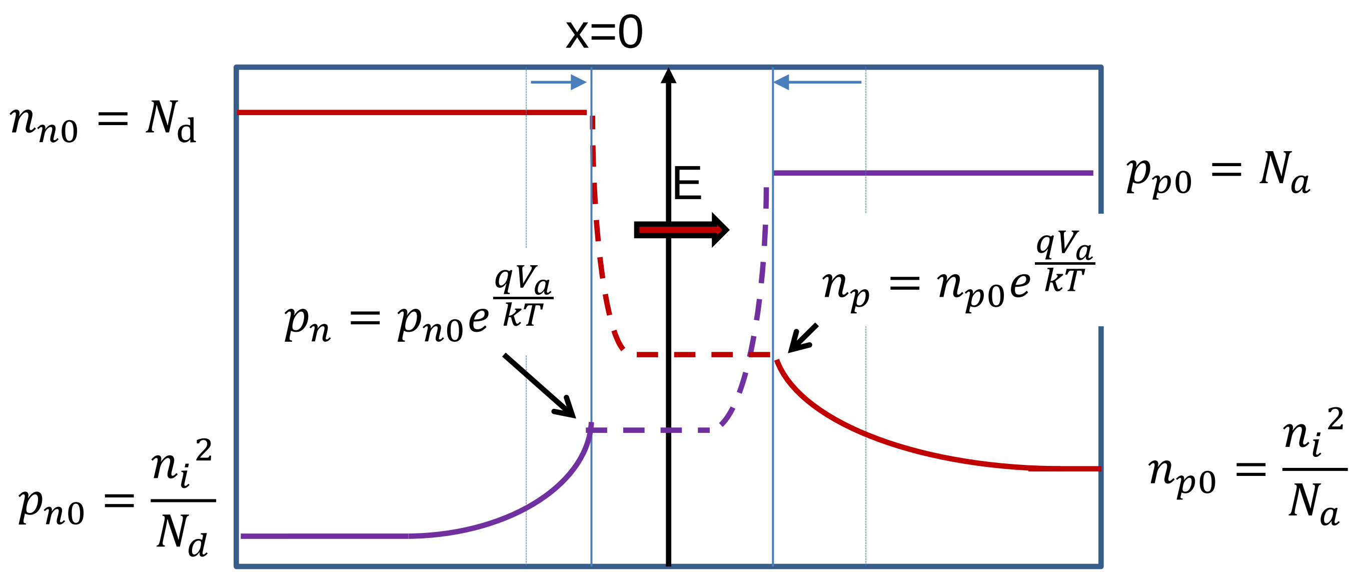
- charge carrier transport: forward bias

n-region

$$\boxed{D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau} = 0}$$

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \qquad L_p^2 = D_p \tau_{p0}$$

$$\boxed{\Rightarrow \delta p_n(x) = p_n(x) - p_{n0} = Ae^{-x/L_p} + Be^{x/L_p}}$$



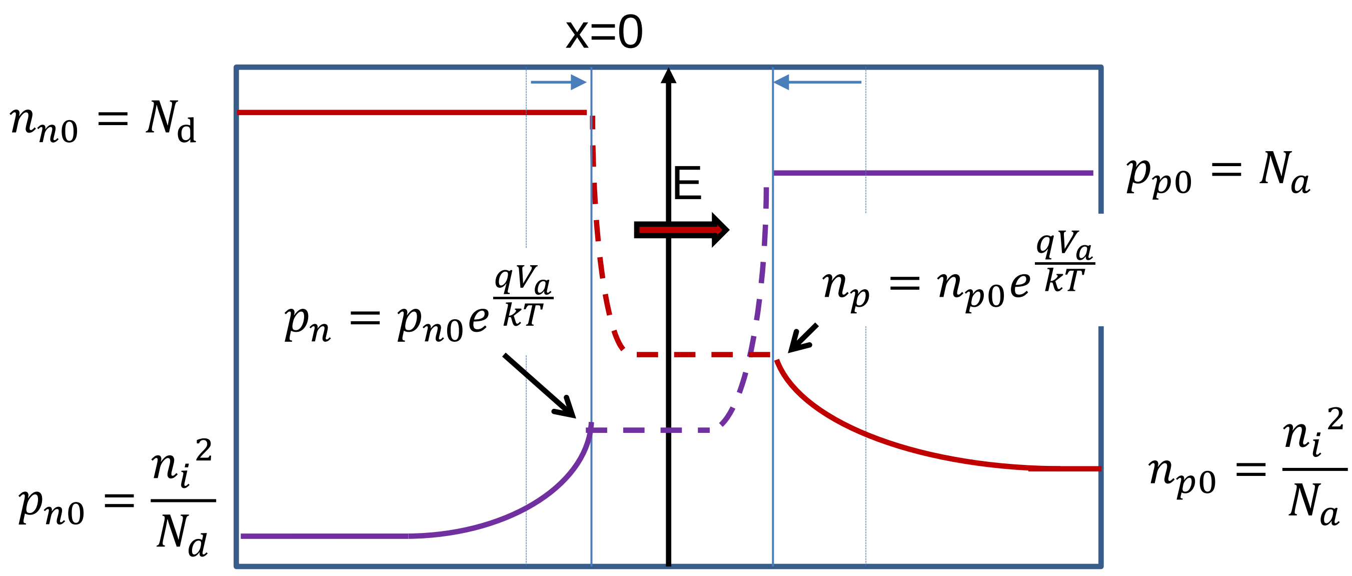
pn Junction Current

- charge carrier transport: <u>forward bias</u>

  Similarly, p-region

$$\frac{d^2(\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \qquad L_n^2 = D_n \tau_{n0}$$

$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n}$$



pn Junction Current

n on the left, p on the right

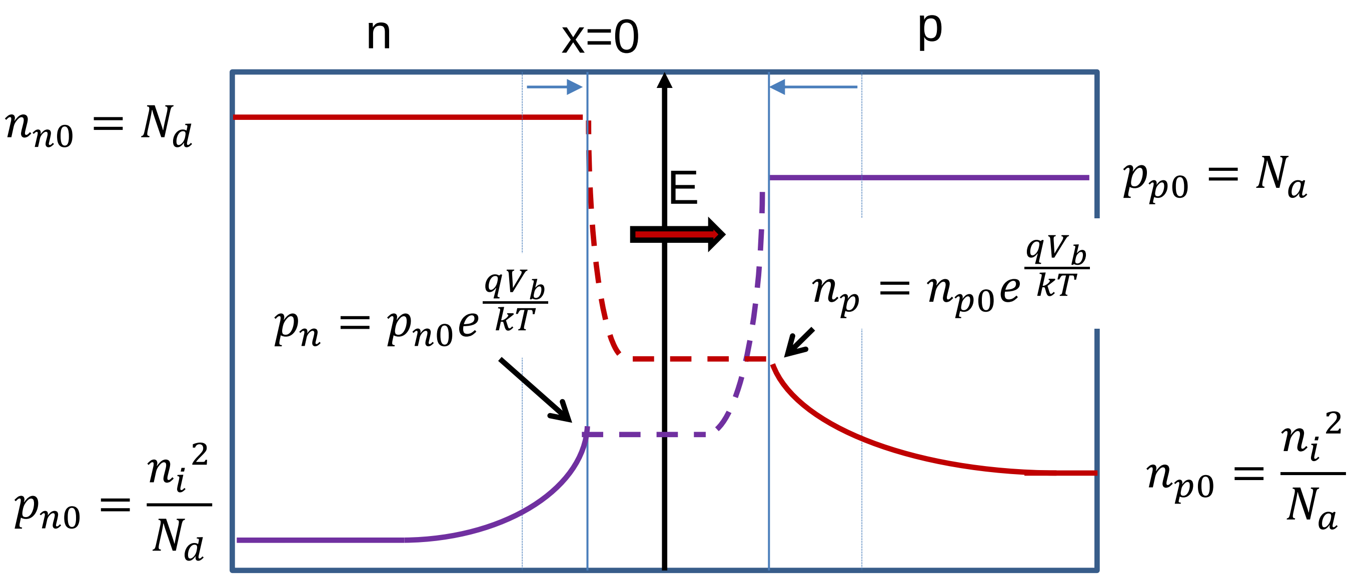
- charge carrier transport: <u>forward bias</u>

Boundary conditions:

- $p_n(-x_n) = p_{n0}\exp(\frac{eVa}{kT}),\quad n_p(x_p) = n_{p0}\exp(\frac{eVa}{kT})$
- $p_n(x \to -\infty) = p_{n0},\quad n_p\ (x \to \infty) = n_{p0}$

Or: δp must be close to 0 deep into the n region (long pn junction): recombination
δn must be close to 0 deep into the p region (long pn junction): recombination

n-region: $\Rightarrow \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}(e^{\frac{eV_a}{kT}} - 1)e^{(x+x_n)/L_p}$



pn Junction Current

n on the left, p on the right

- charge carrier transport: forward bias

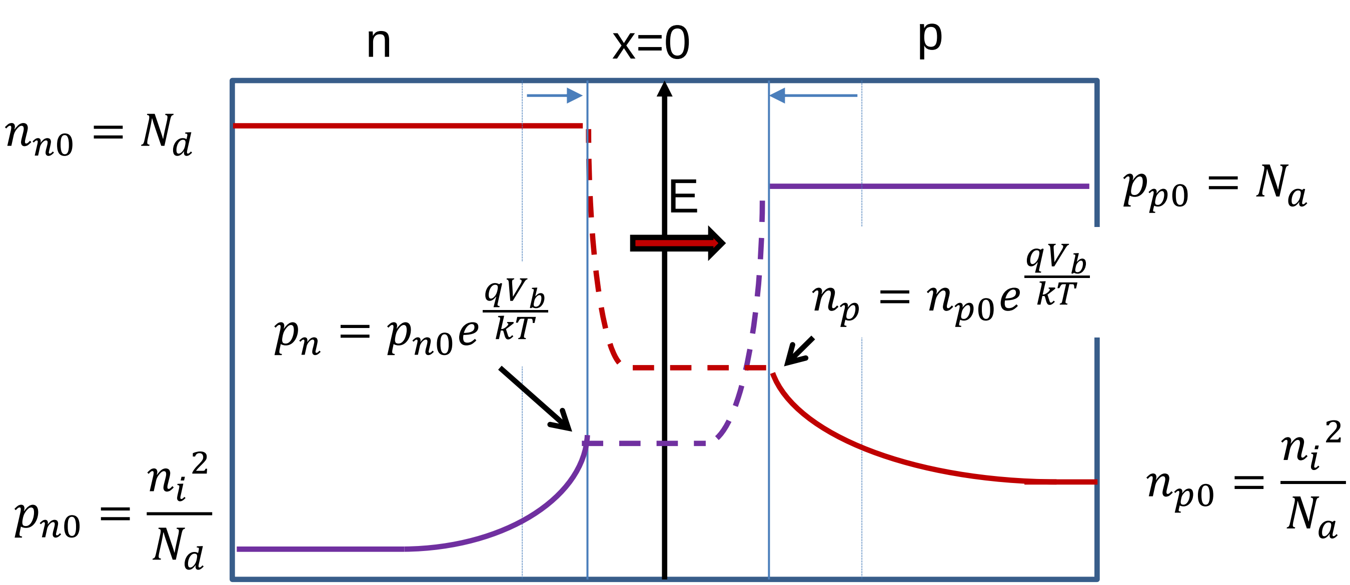
Boundary conditions:

- $p_n(-x_n) = p_{n0}\exp(\frac{eVa}{kT}),\quad n_p(x_p) = n_{p0}\exp(\frac{eVa}{kT})$

- $p_n(x \rightarrow -\infty) = p_{n0},\quad n_p\ (x \rightarrow \infty) = n_{p0}$

Or: δp must be close to 0 deep into the n region (long pn junction): recombination
δn must be close to 0 deep into the p region (long pn junction): recombination

p-region: $\Rightarrow \delta n_p(x) = n_p(x) - n_{p0} = n_{p0}(e^{\frac{eV_a}{kT}} - 1)e^{(x_p - x)/L_n}$



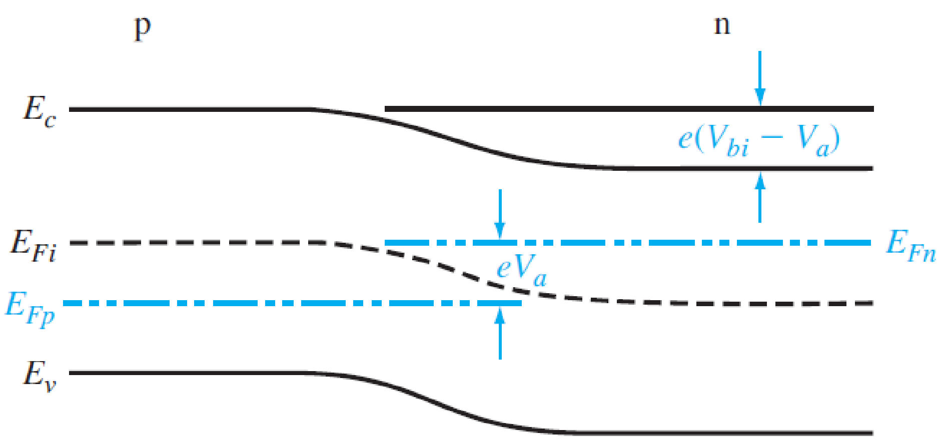
pn Junction Current

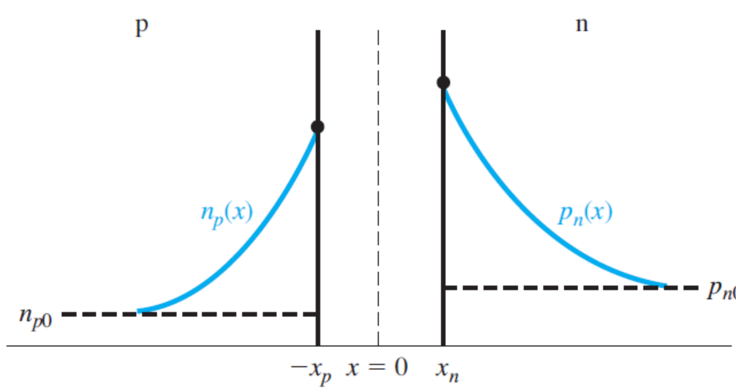
n on the right, p on the left

- charge carrier transport: forward bias

n-region ($x \geq x_n$): $$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right)$$

p-region ($x \leq -x_p$): $$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right)$$





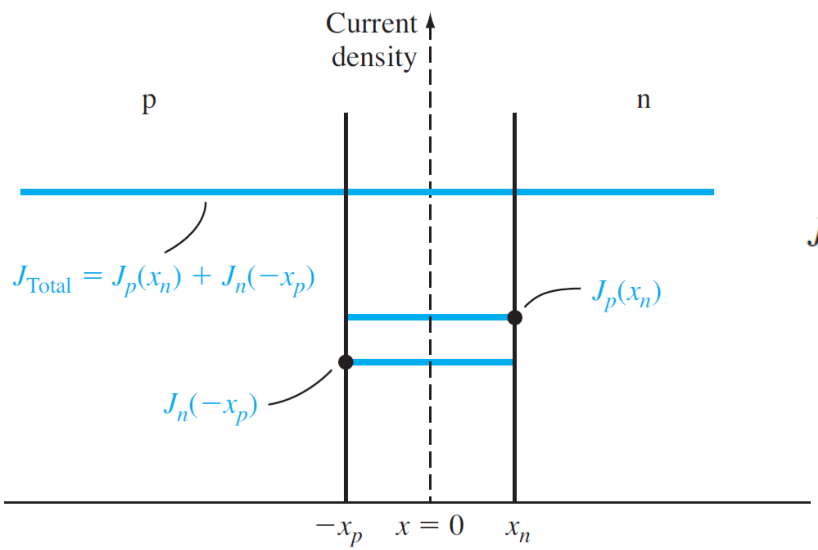
pn Junction Current

- <u>Assumption: No generation and recombination in depletion region</u>
- Ideal pn junction current
- charge carrier transport: <u>forward bias</u>

Hole diffusion: $$J_p(x_n) = -eD_p \frac{dp_n(x)}{dx}\bigg|_{x=x_n}$$

Assume uniformly doped: $$J_p(x_n) = -eD_p \frac{d(\delta p_n(x))}{dx}\bigg|_{x=x_n}$$

Remember $$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right)$$



Take the derivative of $\delta p_n(x)$

$$J_p(x) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right) \qquad (x \geq x_n)$$

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Current direction: p to n

pn Junction Current

- Assumption: No generation and recombination in depletion region
- Ideal pn junction current
- charge carrier transport: forward bias

Similarly, electron diffusion: $J_n(x) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right) \qquad (x \leq -x_p)$

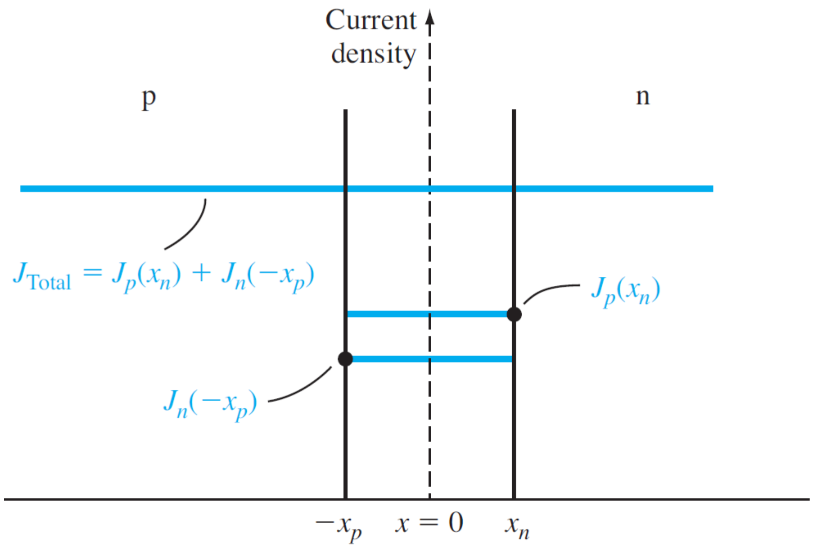
$$J_n(-x_p) = eD_n \left.\frac{d(\delta n_p(x))}{dx}\right|_{x=-x_p} = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$\boxed{\frac{J_n}{J_p} = \frac{D_n n_{p0}/L_n}{D_p p_{n0}/L_p}}$$

Current direction: also p to n, add together!

Total diffusion: ideal I-V relationship of a pn junction

$$J = J_p(x_n) + J_n(-x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$



Define $J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]$

Ideal diode equation:

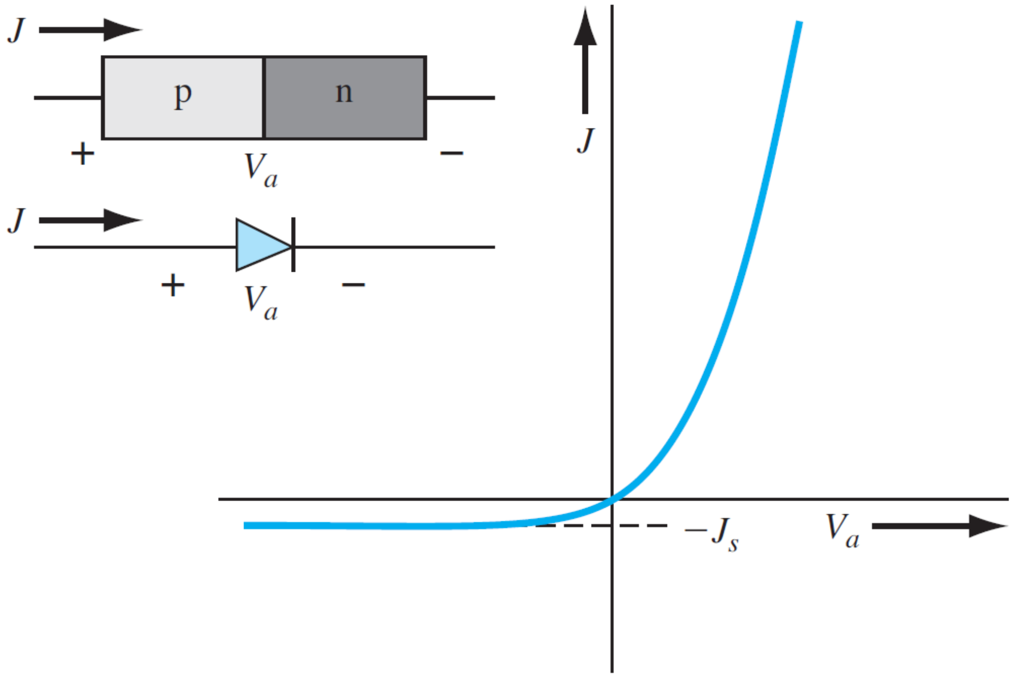
$$J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

V_a can be negative

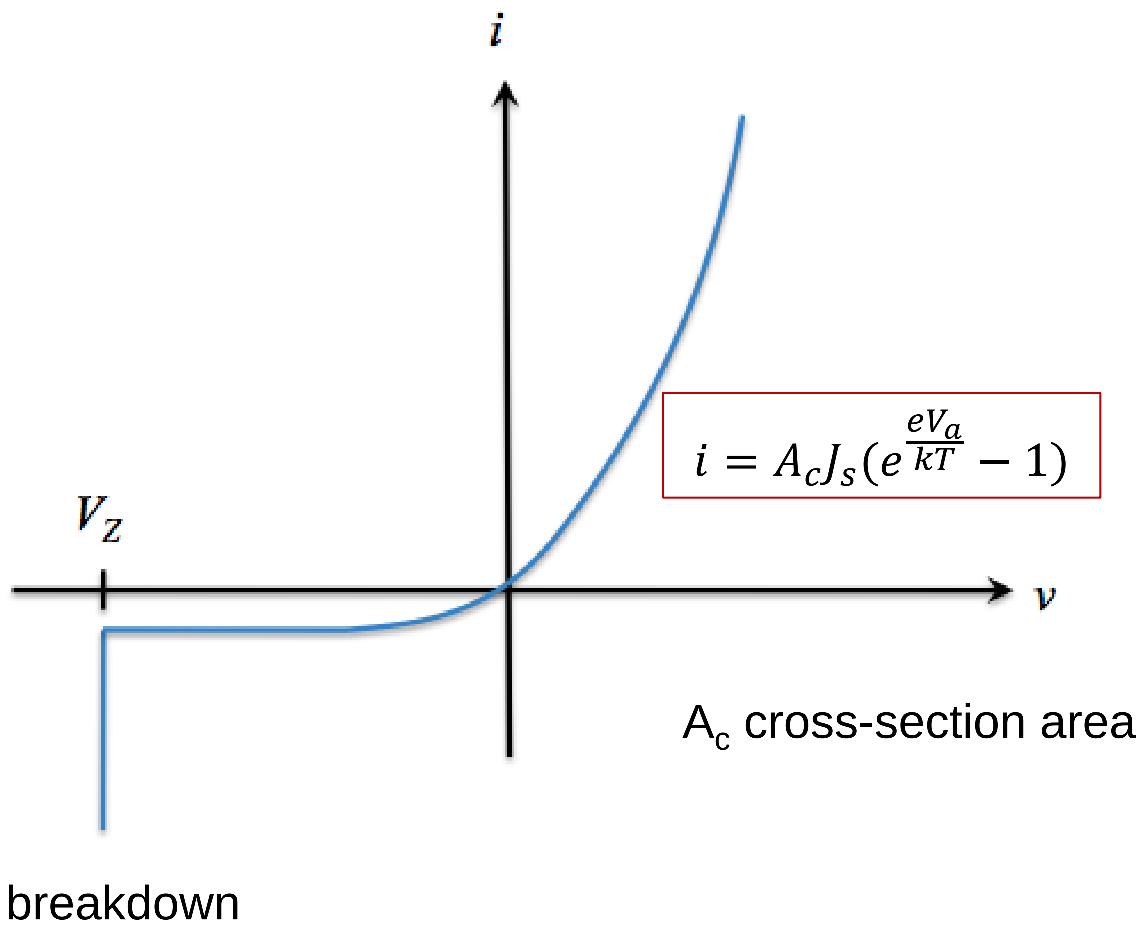
pn Junction Current

- Assumption: No generation and recombination in depletion region
- Ideal pn junction current

J_S: reverse-saturation current density



pn Junction Current



pn Junction Current

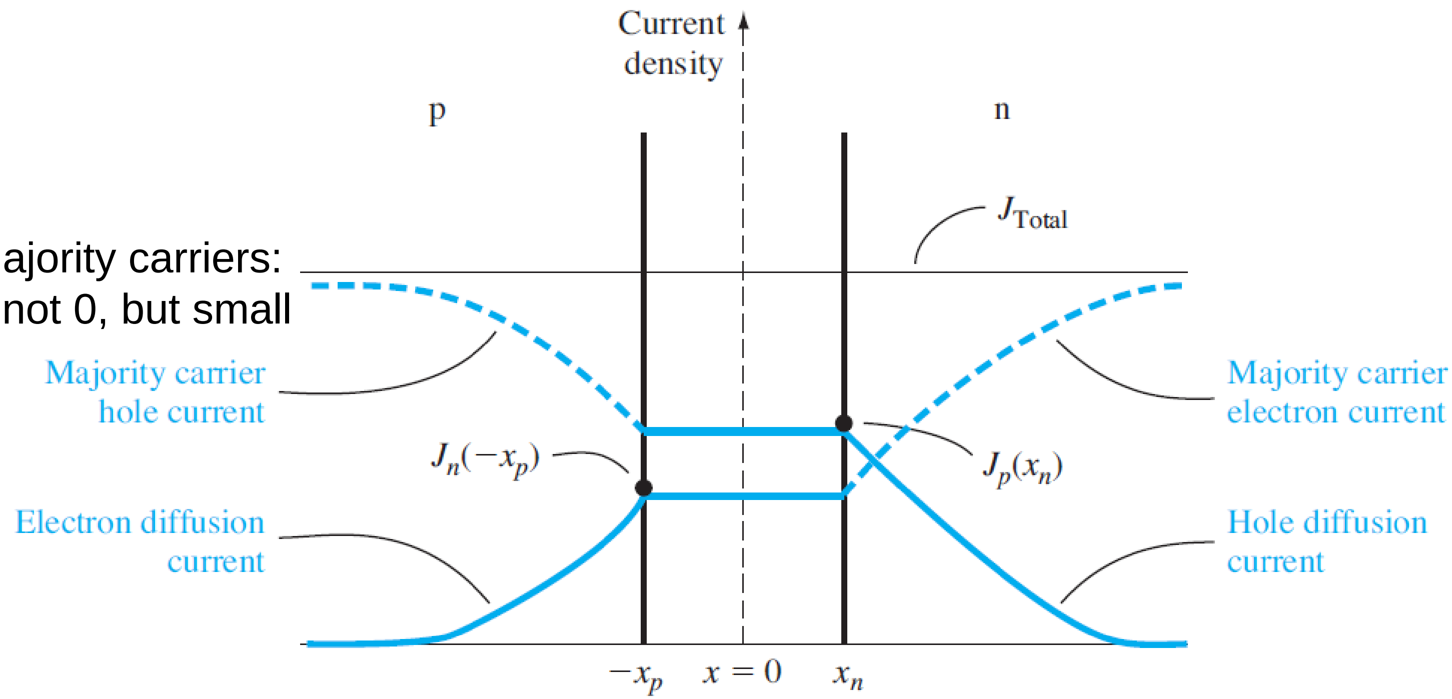
Current in p and n region:
Forward bias

$$\vec{J}_p = e\vec{F}_p = -eD_p \frac{d\delta p}{dx}\vec{x} \qquad \vec{J}_n = -e\vec{F}_n = eD_n \frac{d\delta n}{dx}\vec{x}$$

$$J_p(x) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right) \qquad (x \geq x_n)$$

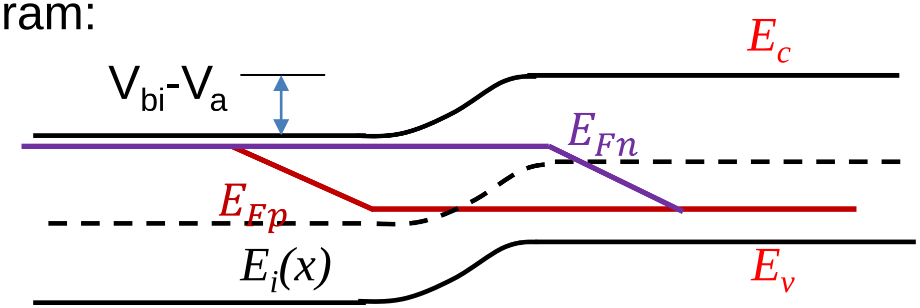
$$J_n(x) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right) \qquad (x \leq -x_p)$$

Drift of majority carriers:
E field is not 0, but small



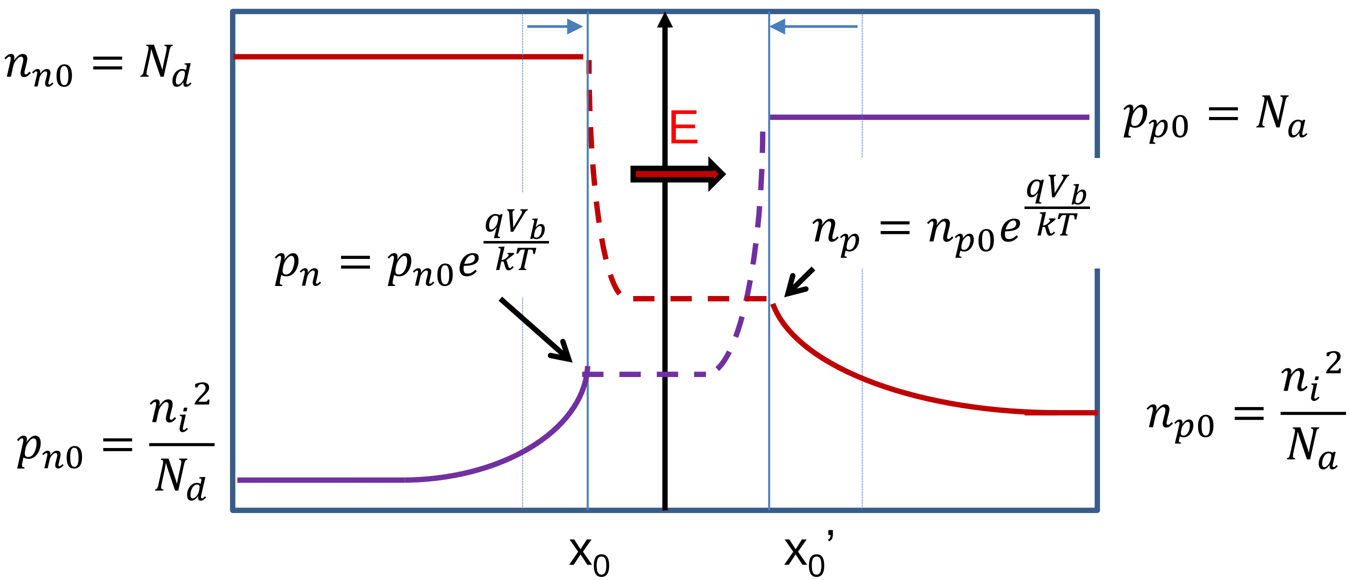
A few points about pn junction

- Energy diagram:



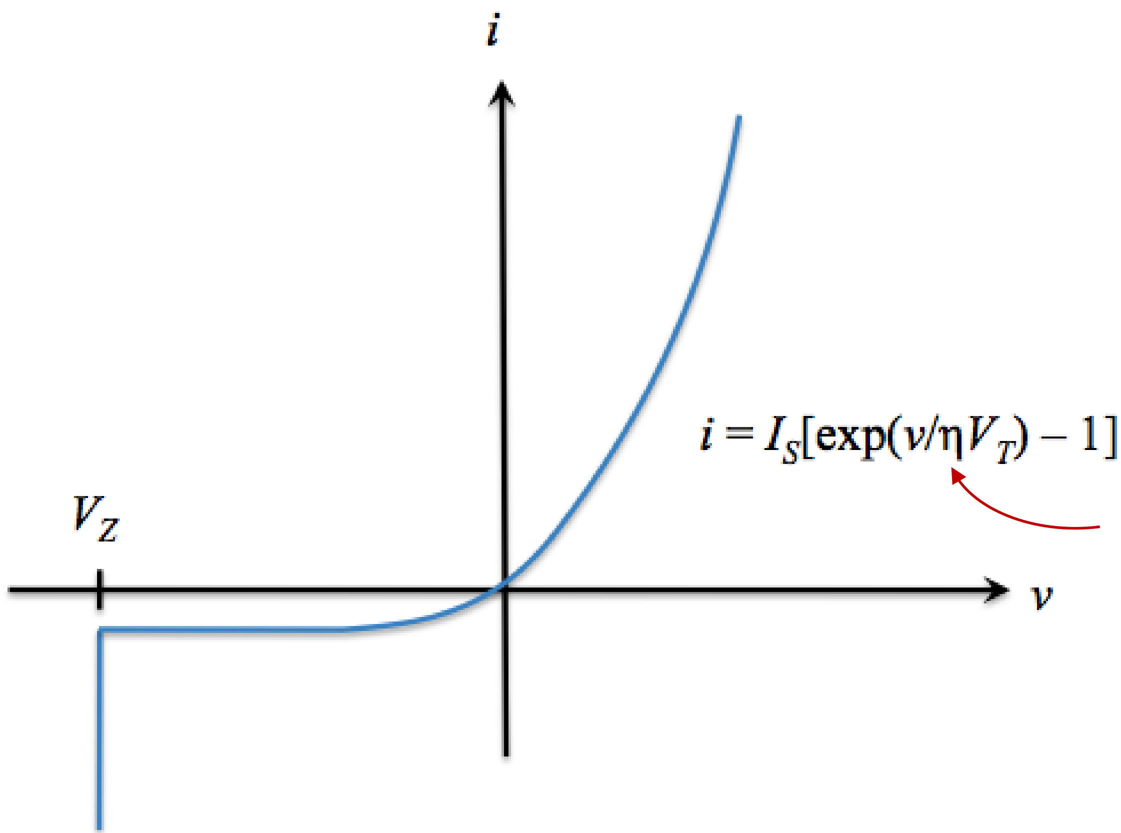
Forward bias

To understand this, see the next page.



A few points about pn junction

- Ideality factor

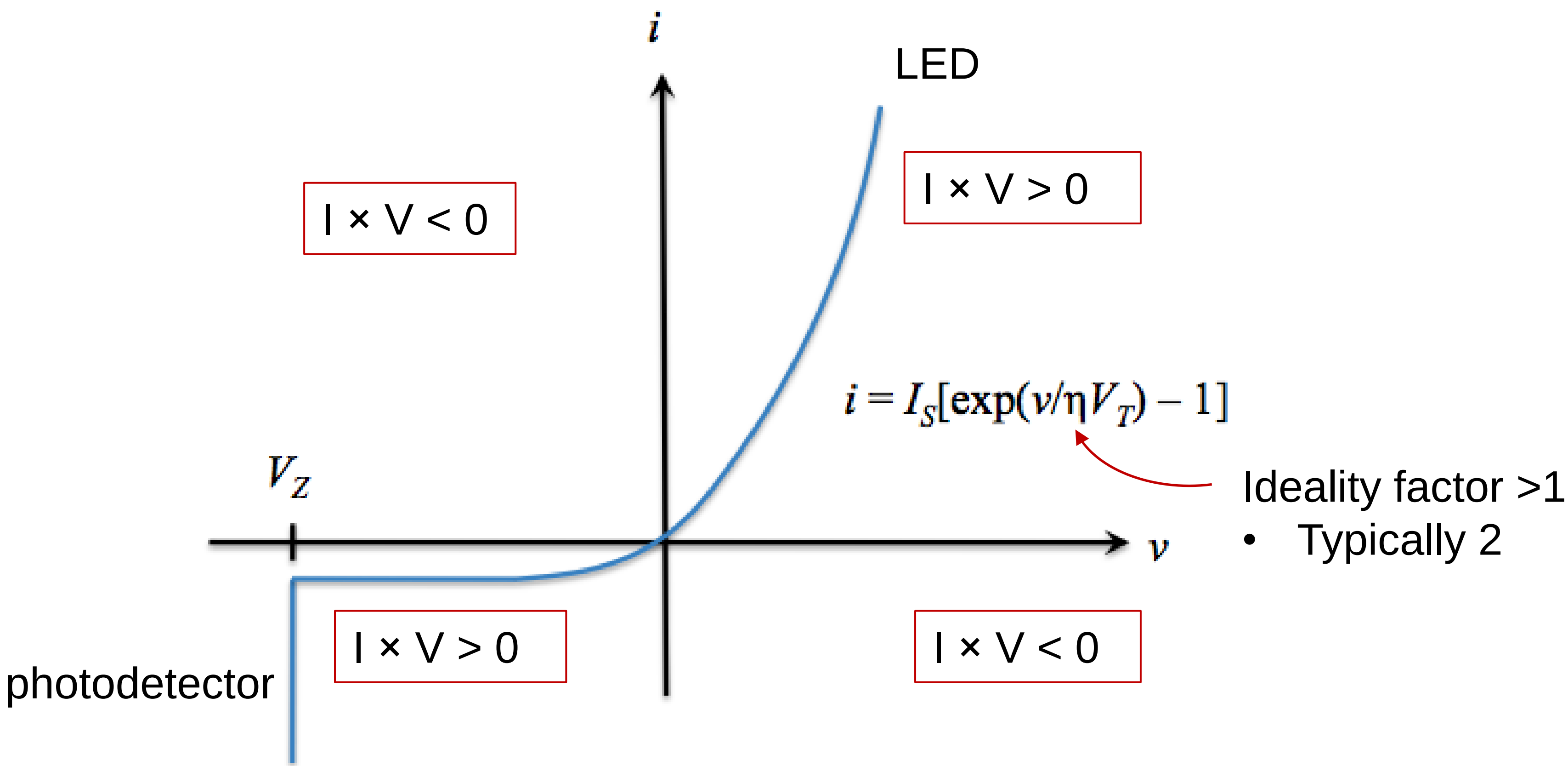


Ideality factor >1

- Typically 2

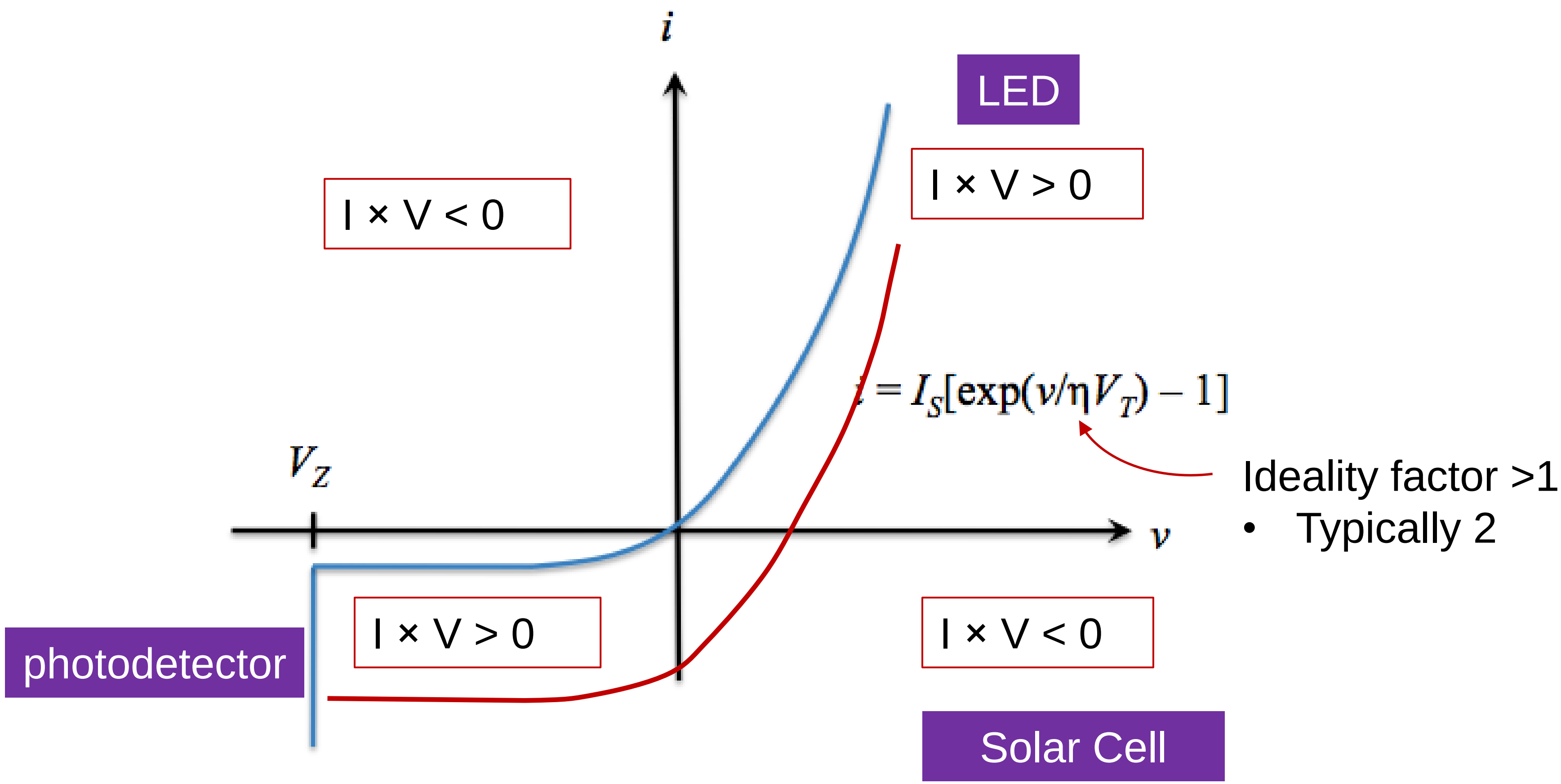
A few points about pn junction

- Energy consumption:



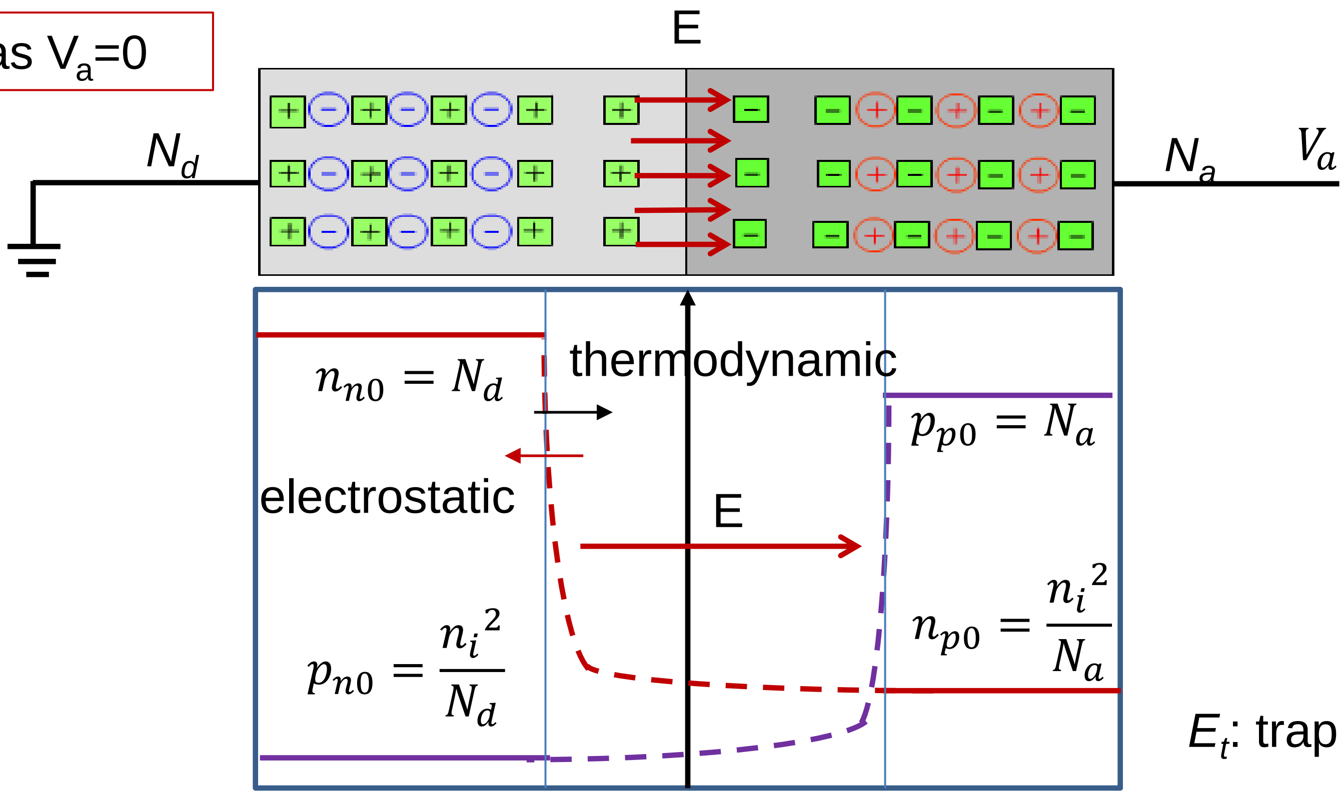
A few points about pn junction

- Energy consumption:



G-R in depletion region at zero bias

Zero Bias V_a=0



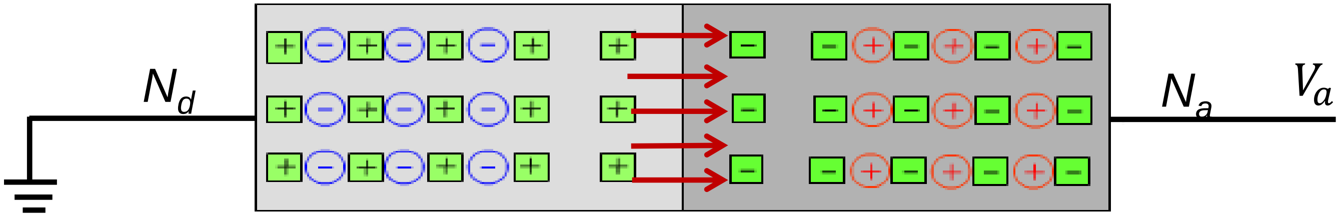
E_t: trap energy

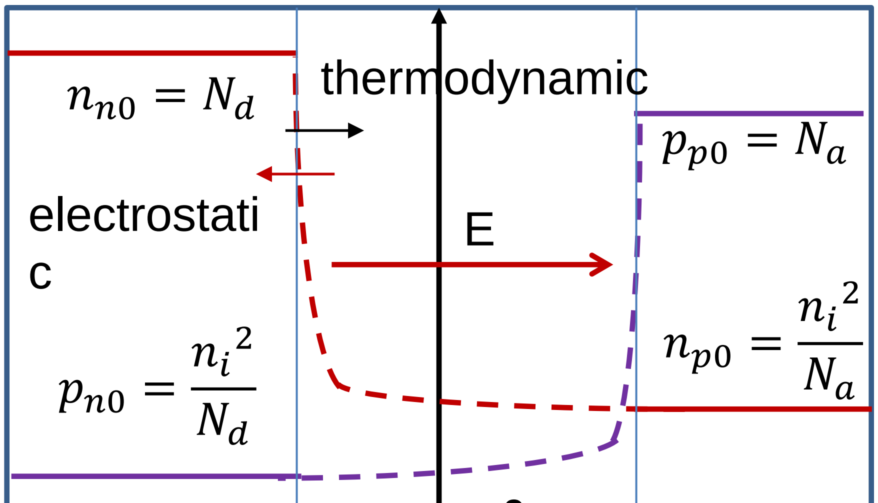
$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right)\right]}$$

$$In\ depletion\ region:\ np = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$$

G-R in depletion region at Reverse Bias

Reverse bias V_a <0





Depletion region:
n=0, p=0

$$R = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p\left[\cancelto{0}{n} + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right] + \tau_n\left[\cancelto{0}{p} + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right)\right]}$$

R<0: generation! Generated via the trap level

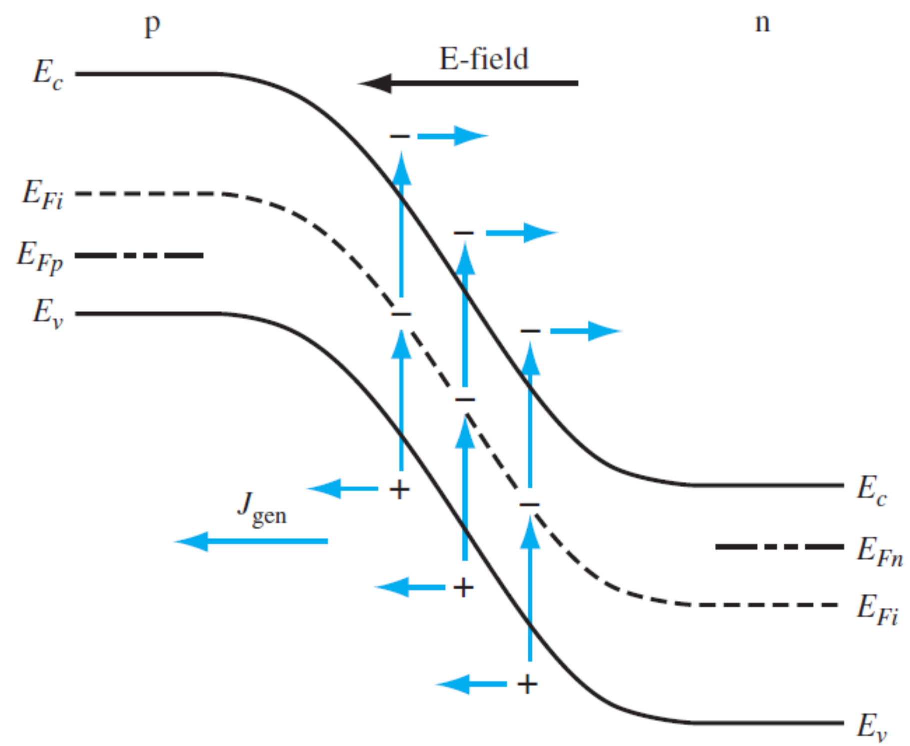
G-R in depletion region at Reverse Bias

$\boxed{\text{Reverse bias } V_a < 0}$

To simplify the calculation, we assume

$$\boxed{E_t = E_{Fi}, \tau_n = \tau_p = \tau}$$

$$\boxed{R = \frac{-n_i}{2\tau} = -G_0}$$



$$\boxed{R = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p \left[\underbrace{\cancel{n}}_{0} + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right] + \tau_n \left[\underbrace{\cancel{p}}_{0} + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right)\right]}}$$

G-R in depletion region at Reverse Bias

Reverse bias V_a <0

To simplify the calculation, we assume

$$\boxed{E_t = E_{Fi}, \tau_n = \tau_p = \tau}$$

$$\boxed{R = \frac{-n_i}{2\tau} = -G_0}$$

Reverse-biased generation current, in addition to the ideal reverse-biased saturation current

Current density from G-R in the depletion region:

$$J_{gen} = \int_0^W qG_0 dx = \frac{qWn_i}{2\tau}$$

$$\boxed{W = a + b = \sqrt{\frac{2\varepsilon(V_{bi}+V_a)}{e}\frac{N_d + N_a}{N_a N_d}}}$$

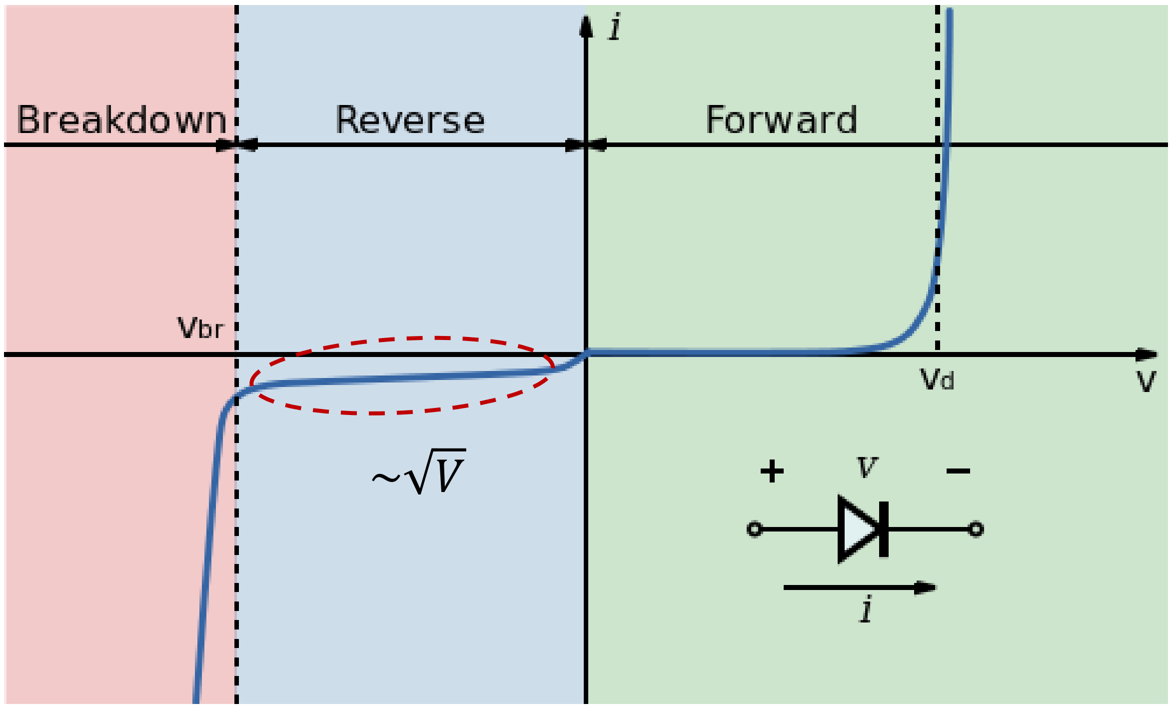
Total reverse-biased current density:

$$J_R = J_s + J_{gen}$$

Dependent on the applied voltage

G-R in depletion region at Reverse Bias

Reverse bias V_a <0



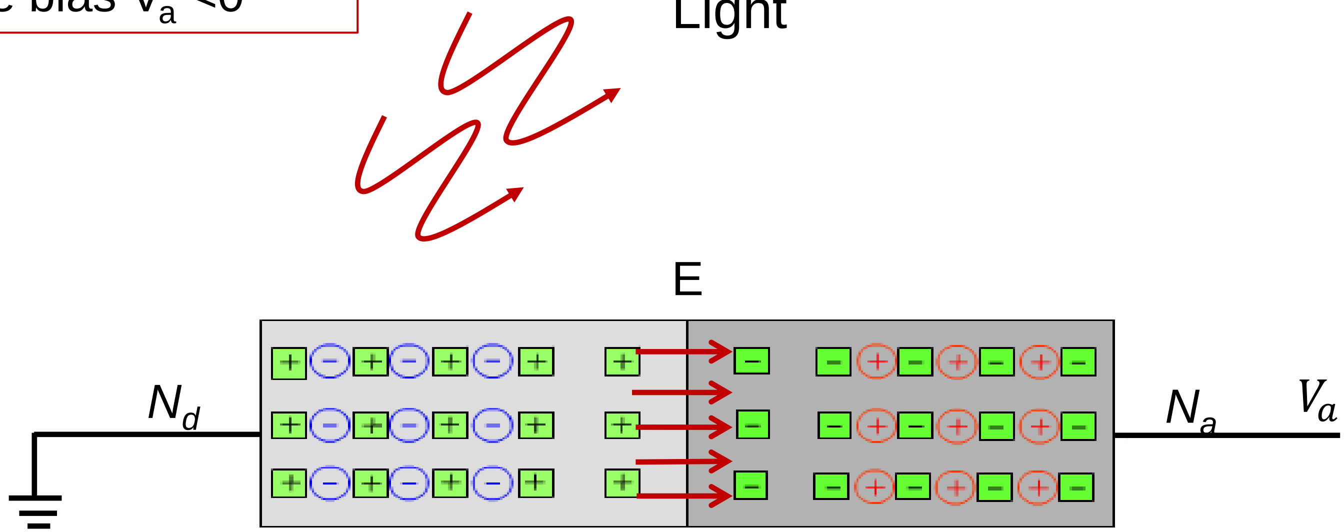
Current density from G-R in the depletion region:

$$J_r = \int_0^W eGdx = \frac{eWn_i}{2\tau}$$

$$\boxed{W = a + b = \sqrt{\frac{2\varepsilon(V_{bi}+V_a)}{e}\frac{N_d + N_a}{N_a N_d}}}$$

Photocurrent at Reverse Bias

Reverse bias $V_a < 0$



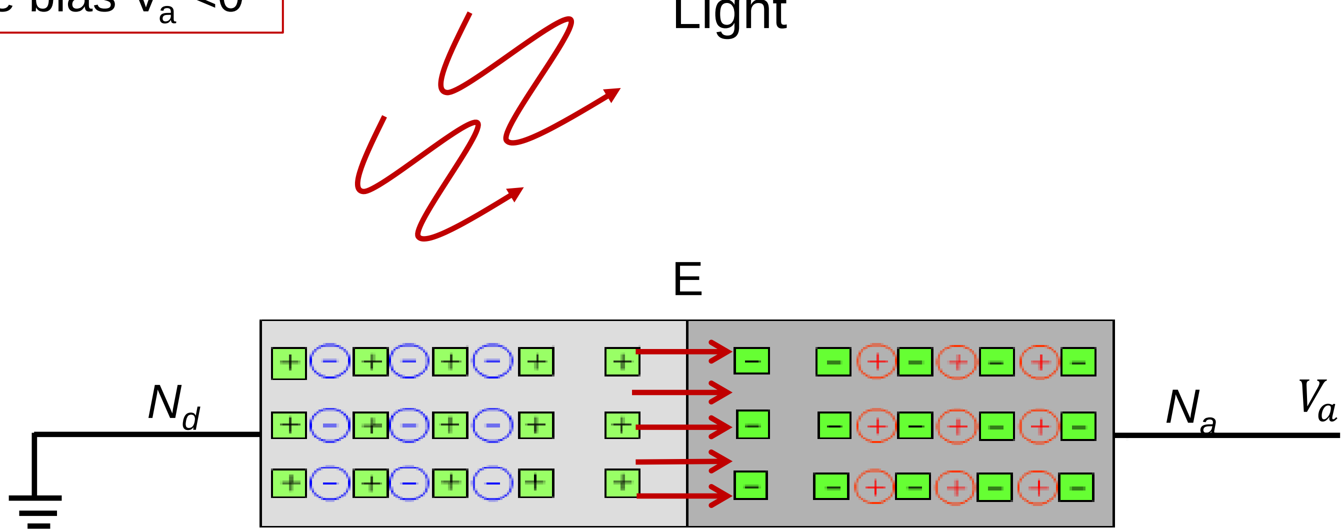
Generation: G_{ex}

$$net\ recombination: R - G = R - (G_0 + G_{ex}) = -G_0 - G_{ex}$$

n=0 and p=0 ➔Recombination in the depletion is zero

Photocurrent at Reverse Bias

Reverse bias V_a <0

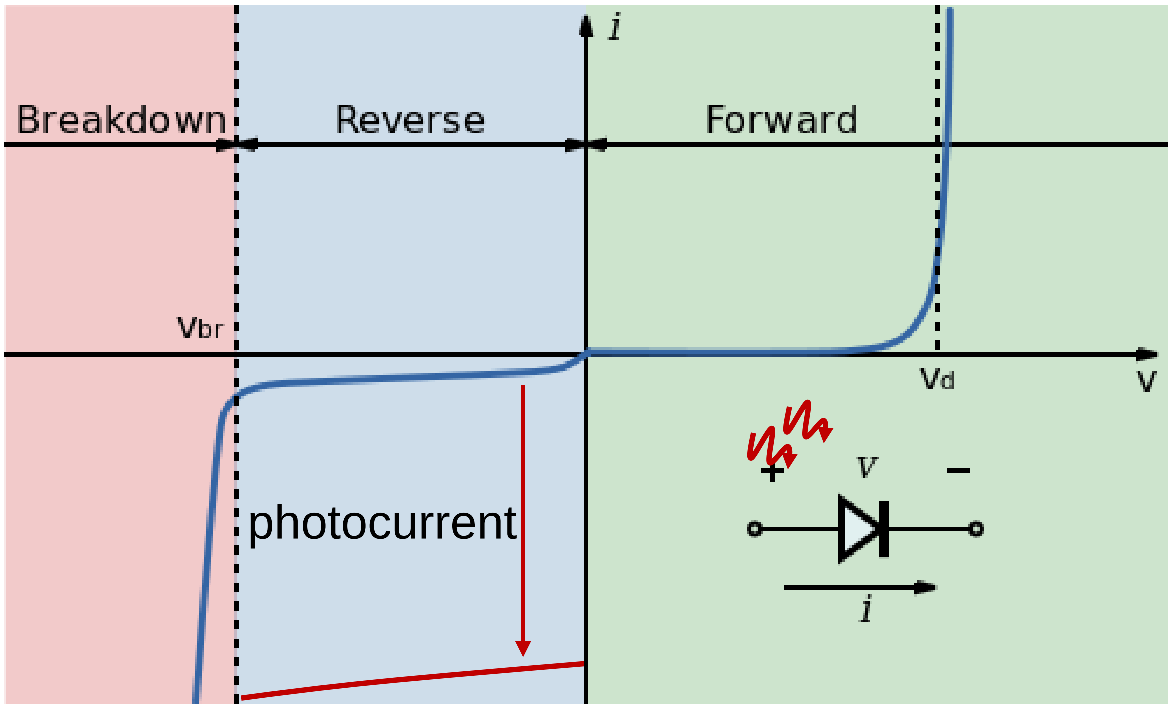


Generation: G_{ex}

$$J_{r,ph} = \int_0^W e(G_0 + G_{ex})dx = e(G_0 + G_{ex})\, W$$

Photocurrent at Reverse Bias

Reverse bias V_a <0



$$J_{r,ph} = \int_0^W e(G_0 + G_{ex})dx = e(G_0 + G_{ex})\,W$$

G-R in depletion region at Forward Bias

Forward bias $V_a > 0$

$$R = \frac{(np - n_i^2)}{\tau_p\left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right] + \tau_n\left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right)\right]}$$

$$In\ depletion\ region:\ np = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$$

At the space charge edge at $x = x_n$, we can write, for low injection

$$n_o p_n(x_n) = n_o p_{no} \exp\left(\frac{V_a}{V_t}\right) = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

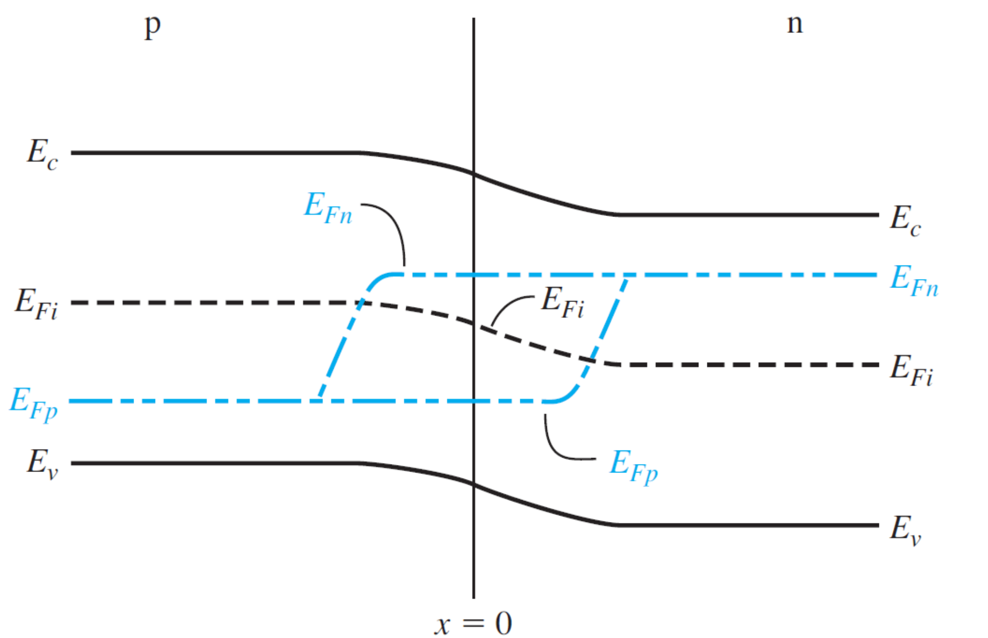
$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

To simplify the calculation, we assume

$$E_t = E_i, \tau_n = \tau_p = \tau$$

$$R = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$

Max: n=p



G-R in depletion region at Forward Bias

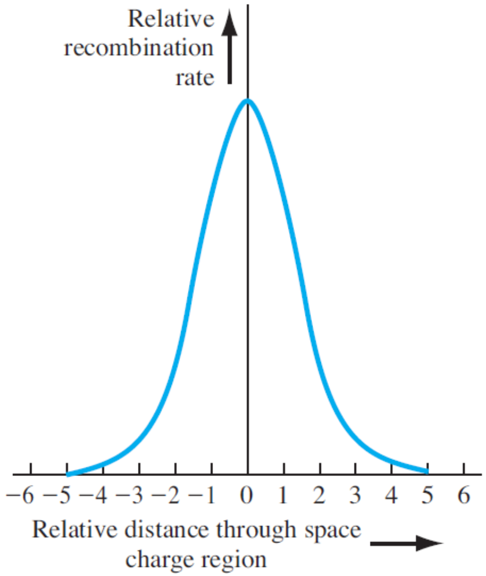
$$R = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_{Fi}}{kT}\right)\right] + \tau_n \left[p + n_i \exp\left(\frac{E_{Fi} - E_t}{kT}\right)\right]}$$

$$In\ depletion\ region:\ np = n_i^2 \exp\left(\frac{eV_a}{kT}\right)$$

To simplify the calculation, we assume

$$E_t = E_i, \tau_n = \tau_p = \tau$$

$$R = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$



Max: n=p, center of space charge region

$$R_{max} = \frac{np - n_i^2}{\tau\left[n_i \exp\left(\frac{eV_a}{2kT}\right) + n_i \exp\left(\frac{eV_a}{2kT}\right) + 2n_i\right]} = \frac{n_i\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}{2\tau\left[\exp\left(\frac{eV_a}{2kT}\right) + 1\right]}$$

G-R in depletion region at Forward Bias

$$R_{max} = \frac{np - n_i^2}{\tau\left[n_i \exp(\frac{eV_a}{2kT}) + n_i \exp(\frac{eV_a}{2kT}) + 2n_i\right]} = \frac{n_i\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}{2\tau\left[\exp\left(\frac{eV_a}{2kT}\right) + 1\right]}$$

Assume V_a>>kT/e

$$R_{max} = \frac{n_i}{2\tau}\exp\left(\frac{eV_a}{2kT}\right)$$

G-R in depletion region at Forward Bias

$$\boxed{R_{max} = \frac{n_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)}$$

Current density from G-R in the depletion region:

$$J_r = \int_0^W eR dx = \frac{eWn_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

$$J_r = J_{r0} \exp\left(\frac{eV_a}{2kT}\right)$$

Assume maximum recombination rate is effective

For a non-ideal pn junction, the total current density:

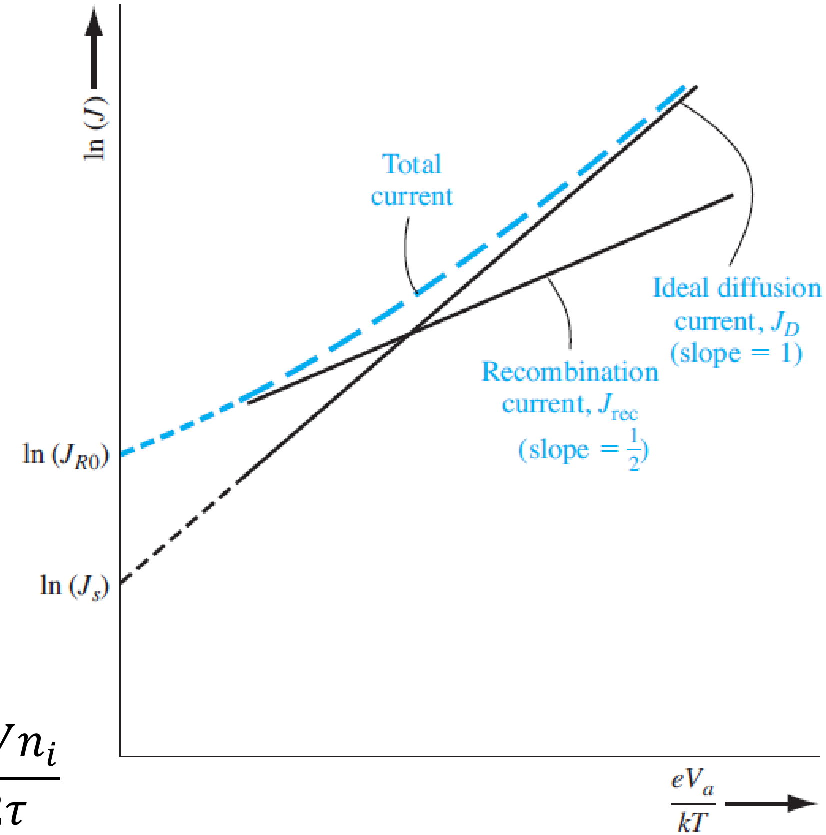
$$J = J_F + J_r = J_s \exp\left(\frac{eV_a}{kT}\right) + \frac{eWn_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

Non-ideal IV curves

Forward bias V> 3kT/e=0.078V:

$$J = J_F + J_r = J_s \exp\left(\frac{eV_a}{kT}\right) + \frac{eWn_i}{2\tau} \exp\left(\frac{eV_a}{2kT}\right)$$

At a low current density, the recombination current dominates, and at a higher current density, the ideal diffusion current dominates



Diode *I*-*V* relationship:

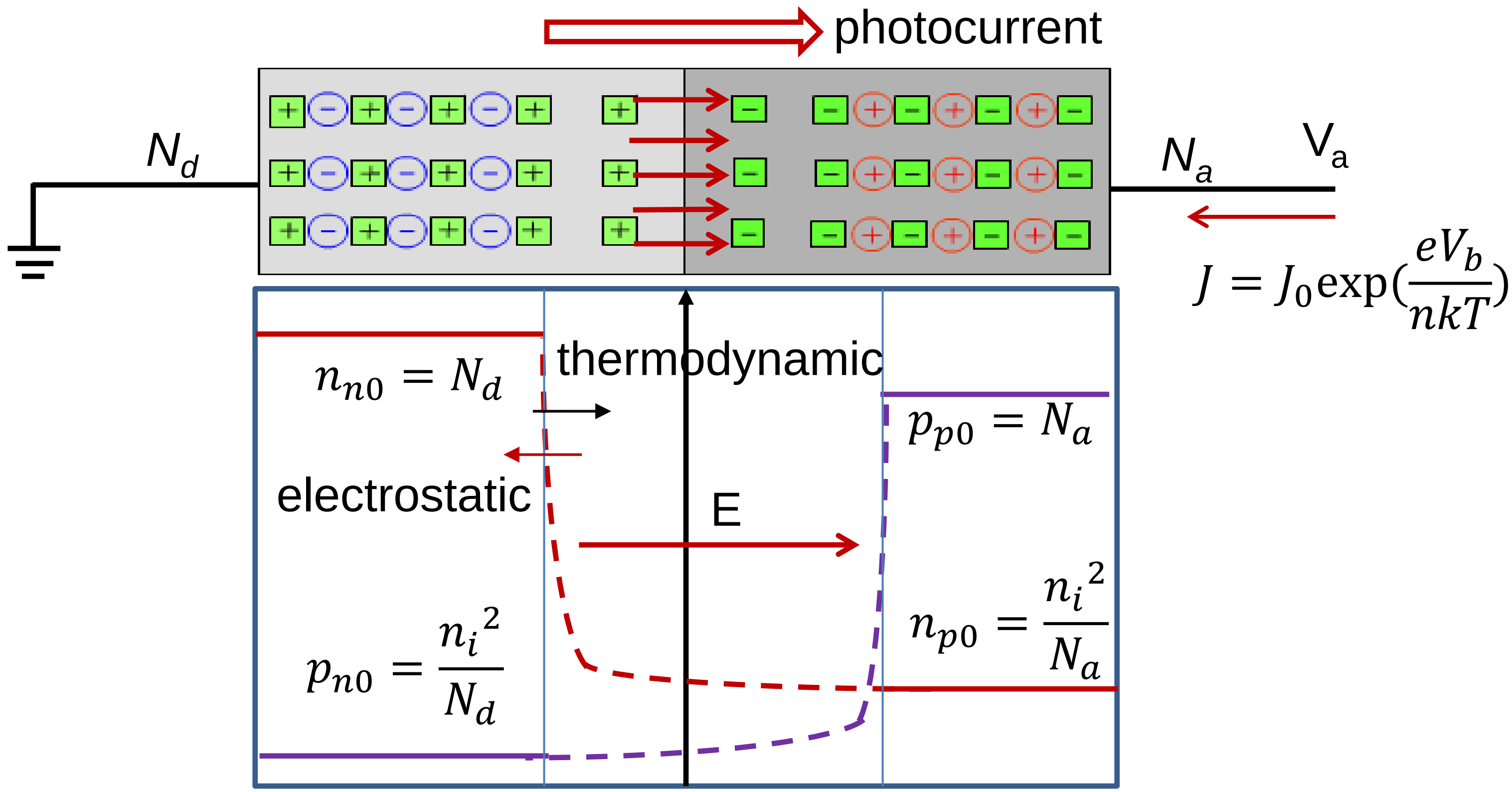
$$I = I_s\left[\exp\left(\frac{eV_a}{nkT}\right) - 1\right]$$

the ideality factor

Reverse bias:

$$J = -J_s - \frac{eWn_i}{2\tau} = -\left(\frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}\right) - \frac{eWn_i}{2\tau}$$

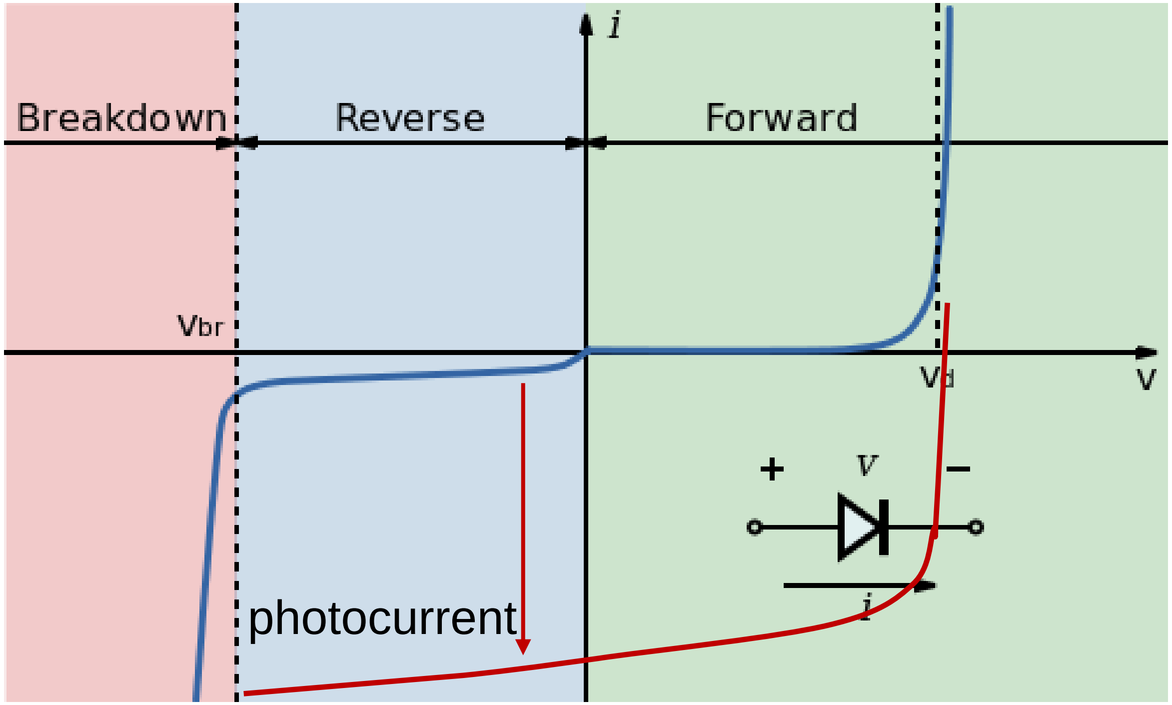
IV curve under illumination



Net current density:

$$J_{net} = J - J_{ph} = J_0 \exp\left(\frac{eV_a}{nkT}\right) - eG_{ex}W$$

IV curve under illumination

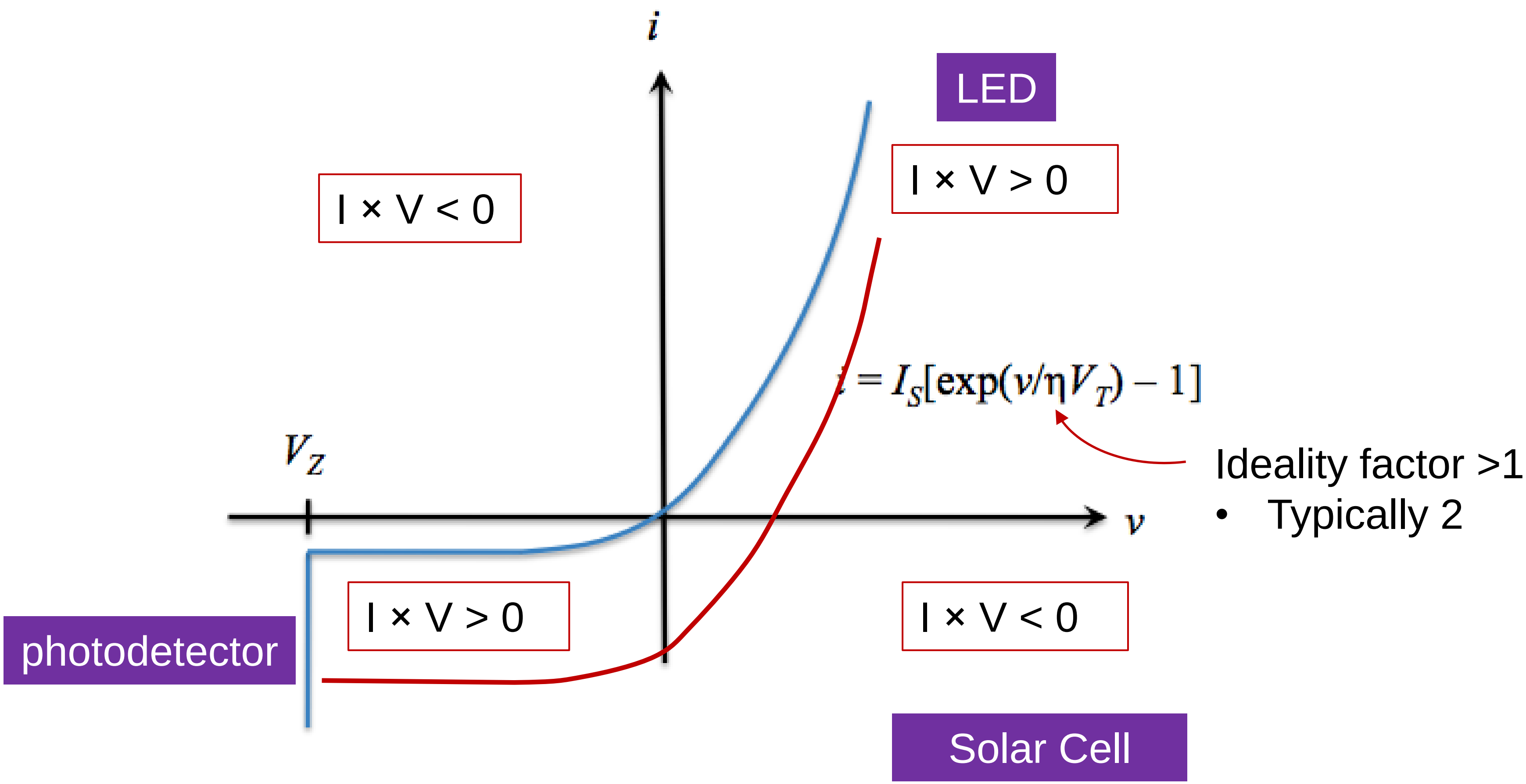


Net current density:

$$J_{net} = J - J_{ph} = J_0 \exp\left(\frac{eV_a}{nkT}\right) - eG_{ex}W$$

A few points about pn junction

- Energy consumption:



High-level injection

$$np = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

Excess carrier concentrations become comparable or even greater than the majority carrier concentration $\delta n > n_o$ and $\delta p > p_o$

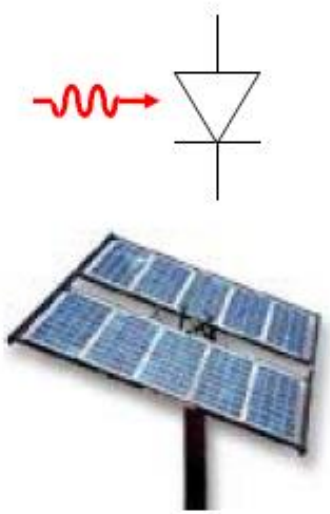
$$(\delta n)(\delta p) \cong n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$\delta n = \delta p \cong n_i \exp\left(\frac{V_a}{2V_t}\right)$$

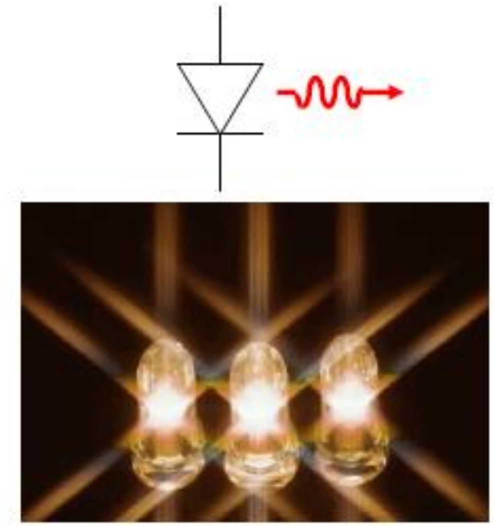
$$I \propto \exp\left(\frac{V_a}{2V_t}\right)$$

Semiconductor Optoelectronic Diodes

Detectors convert optical signals into electrical signals



Emitters are a source of optical radiation



- Photodetectors: primary purpose to detect photons
- Solar Cells: primary purpose is photo-to-electrical energy conversion

- Light-emitting diodes (LEDs)
- Lasers –may be obtained using optical cavity