JOINT INSTITUTE
交大密西根学院

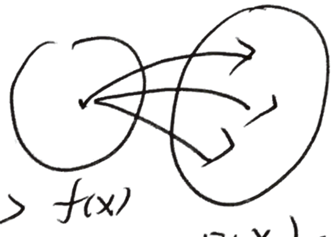
Name:
Student ID:
Course Code:
Date:

② finite ⇒ infinite
① adding large number of small things

一、§ indefinite $\boxed{\text{Integral}}$

→ Almost obvious

(*) $F(x) \xrightarrow{\frac{d}{dx}\ \langle\text{derivative}\rangle} f(x)$, $f(x) \xrightarrow{\int dx\ \langle\text{antiderivative}\rangle} F(x)$



f(x) F(x). ⟨映射理解⟩

(**) Σ vs. $\int$.

$\Sigma v(t)\Delta t$ vs. $\int v(t)dt$.

↳ exact sum ↳ dt→0时加和趋近的值

(***) Average of continuous function (finite → infinite)

$$\frac{\Sigma \text{height}}{\Sigma \text{sum of numbers} \to \infty} \Rightarrow \frac{\Sigma \text{height } \Delta x}{(\Sigma \text{sum of numbers} \cdot \Delta x)} \Rightarrow \frac{\int_a^b f(x)dx}{b-a}$$

$$\#\text{number of samples} \approx \frac{b-a}{dx}$$

1. ⟨Definition⟩

$\boxed{\text{antidifferentiation}}$ / $\boxed{\text{integration}}$

2. ⟨Definition⟩

indefinite integral → set of antiderivatives of f(x)

notation → ⟨ $\int \underbrace{\boxed{f(x)}}_{}dx$ ⟩ (integrand: f(x); dx ↳ variable of integration.)

[Note: 1. $\int f(x)dx = F(x) + \underset{\sim}{C}$ constant

2. $\int f(x)dx$ → Not a single function, but a family of functions

曲线簇 ⟩

3. ⟨properties⟩ ① $\int k f(x)dx = k\int f(x)dx$

② $\int [f(x)+g(x)]\,dx = \int f(x)dx + \int g(x)dx$

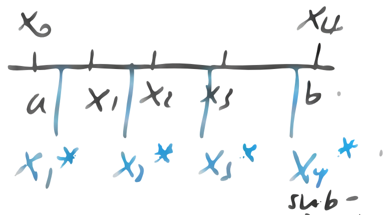
③ $\int (c_1 f_1(x) + c_2 f_2(x) + \cdots)\,dx = c_1 \int f_1(x)dx + c_2 \int f_2(x)dx + \cdots$

二. Definite integral <Riemann integral>

1. partition → a collect of distinct points → X

   <tagged> → sample points. <以点表征区间特征> → (X, X^*)

   \+ subinterval



2. norm : longest sub-interval length of the <统一表述的需要 ∵ 区间长度不一致>

$$\|P\| = \max|\Delta x_i| = \max|(x_i - x_{i-1})|$$

3. Riemann sum. $S = \sum_{i=1}^{n} f(x_i^*)\Delta x_i$

   $\langle f + P(X, X^*) \rangle$

   make use of the arbitrary.

   ↑

   The choice is important

4. <Definition> integrable.

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i \to ① \text{exists}$$

$\|P\| \to 0$

② X depends on P

∴ X (partition & / sample points)

5. <Theorem>

   $f(x) \to$ continuous / piecewise continuous bounded on $[a, b]$.

   $\to \boxed{\int_a^b f(x)\,dx}$ exists.

   ↳ 区别于不定积分 是一个 value 不是一个set

6. <properties>

   ↳ 特殊的极限 → 极限的性质的补充.

   ①②③

   ④ $\int_a^b = \int_a^c + \int_c^b$.

   ⑤ $f(x) \geq 0 \Rightarrow \int_a^b f(x)\,dx \geq 0$

   ⑥ $f(x) \geq g(x) \Rightarrow \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

☆☆ $g(x) \leq f(x) \leq h(x)$

$\Rightarrow$

$$\underbrace{\int_a^b g(x)\,dx}_{m(b-a)} \leq \int_a^b f(x)\,dx \leq \underbrace{\int_a^b h(x)\,dx}_{M(b-a)}.$$

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① 分割区间，确定norm ③ 累加取极限

<exe> ② 选择 sample points x^*

1. partition for Riemann

① $\frac{b-a}{n} = \boxed{h}$, $\boxed{a+ih} \Rightarrow \boxed{\xi_i}$ (sample points)

Δx_i

e.g.1 $\int_{-1}^{2} x^2 dx$.

1) $h = \frac{3}{n}$, check $\boxed{n\to\infty, h\to 0}$ ✓

2) $\xi_i = x^* = -1 + \frac{3}{n} i$, $0 \le i \le (n-1)$

$$\therefore \sum f(x^*)\cdot \Delta x_i = \sum_{0}^{n-1} \left(-1+\frac{3}{n}i\right)^2 \cdot \frac{3}{n}$$

$$= \sum_{0}^{n-1} (-1+ih)^2 \cdot h = nh - 2h^2 \sum_{i=0}^{n-1} i + h^3 \sum_{i=0}^{n-1} i^2$$

$$= 3 + \frac{9-9n}{2n^2} \to 3.$$

e.g.2 $\int_0^1 a^x dx$

$h = \frac{1-0}{n} = \frac{1}{n} \to \Delta x_i$ $(n\to\infty, \Delta x_i \to 0)$

$\xi_i = \frac{i}{n}$, $0 \le i \le (n-1) \to x^*$.

$= ih$.

$\therefore$

$$\sum_{i=0}^{n-1} a^{ih} h = h\frac{(a^{nh}-1)}{(a^h-1)} = \frac{(a-1)}{n(a^{\frac{1}{n}}-1)}$$

↳ 等比数列求和

$$\lim_{n\to\infty} \frac{a-1}{\frac{a^{\frac{1}{n}}-1}{\frac{1}{n}}} = \lim_{n\to\infty} \frac{a-1}{\ln a}$$

$\xi_i = \frac{k^2}{n^2}$, $\Delta x_i = \frac{(k+1)^2}{n^2} - \frac{k^2}{n^2} = \frac{2k+1}{n^2}$

$f(x) = \sqrt{x}$,

$\therefore f(x^*) \Delta x_i = \boxed{\frac{k}{n} \cdot \frac{(2k+1)}{n^2}}$

② 特殊的分割 → 等比数列.

$\boxed{\int_a^b x^m dx}$ $(0<a<b;\ m\neq -1)$

partition: $a < aq < \cdots < aq^n = b$, $q = \sqrt[n]{\frac{b}{a}}$, $\xi_i = aq^i$

$$\sum_{i=0}^{n-1} (aq^i)^m \cdot (aq^{i+1} - aq^i) = a^{m+1}(q-1) \cdot \sum_{i=0}^{n-1} q^{(m+1)i}$$

$$= a^{m+1}(q-1)\frac{q^{n(m+1)}-1}{q^{m+1}-1}$$

$$= (b^{m+1}-a^{m+1})\cdot\frac{(q-1)}{(q^{m+1}-1)} \quad \because \lim_{n\to\infty} q = 1$$

$$\Rightarrow \lim_{n\to\infty} E(\quad) = \frac{b^{m+1}-a^{m+1}}{m+1}$$

三. ① FTC <part I>

$$\int_a^b f(x)dx = F(b) - F(a) = \lim_{\|P\|\to 0} f(x_i^*)\Delta x_i.$$

<if f continuous>.

$$= F(x)\Big|_a^b = [F(x)]_a^b$$

② FTC <part II>

f continuous

$$\to F(x) = \int_a^x f(t)dt$$

$$\Leftrightarrow \frac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$$

变量变换而实现.

③ MVT for integral

$$\int_a^b f(x)dx = (b-a)f(c)$$

④ substitution.

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

⑤ By parts.

$$\int u\,dv = uv - \int v\,du.$$