9. Adaptive Ramp Metering

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Recap

- Sequencing problem
  - Hierarchical control system
  - Model formulation
- Stability analysis
  - Stochastic stability
  - Stability of Markov processes*
  - Theoretical results*
- Optimality analysis
  - Course project guidelines

Outline

- Smart highways
  - Sensing technology
  - Control technology
- Traffic flow model
  - Flow-density relation
  - Cell transmission model
- Ramp metering
  - Flow stabilization
  - Throughput maximization
  - Delay minimization

Smart highways

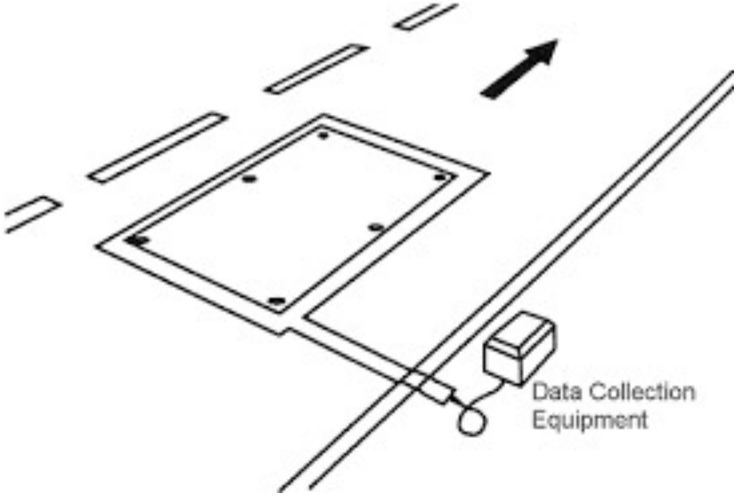
- Sensing technology
- Actuation technology

From vehicle to road

- We have been dealing with vehicles in this course.
- Now let’s switch to macroscopic level.
- We no longer consider the movement of individual vehicles.
- Instead, we look at their aggregate behavior as traffic flow
- Position/speed/acceleration -> flow/density

Induction loop

- An insulated, electrically conducting loop is installed in the pavement.
- The electronics unit applies alternating current electrical energy onto the wire loops.
- The decrease in impedance actuates the electronics unit output relay or solid-state optically isolated output.
- This sends a pulse to the traffic signal controller signifying the passage or presence of a vehicle.

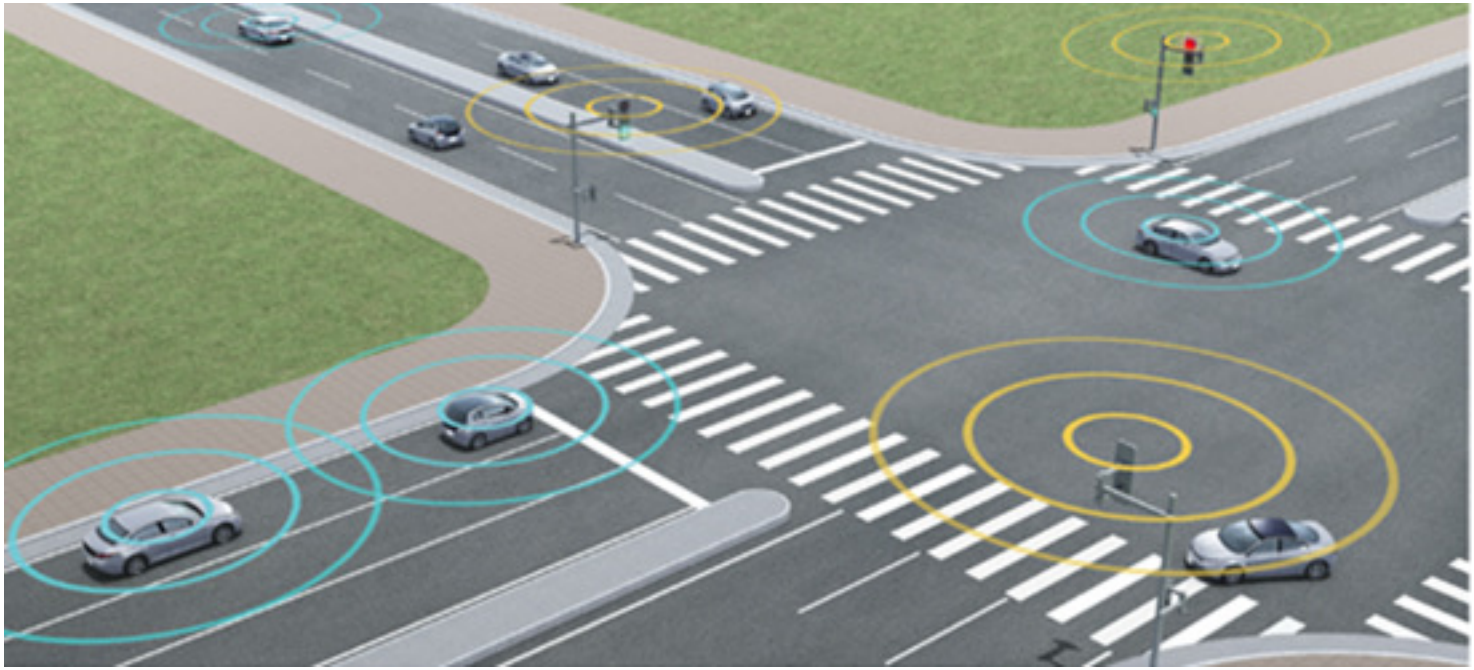


Traffic camera

- Road-side or mobile.
- Many transportation departments have linked their camera networks to the Internet on online websites, thus making them webcams which allow commuters to view current traffic conditions.
- Can count vehicles, observe flow speed, detect accidents/events…
- Questionable performance during the night

Vehicle-to-infrastructure communications

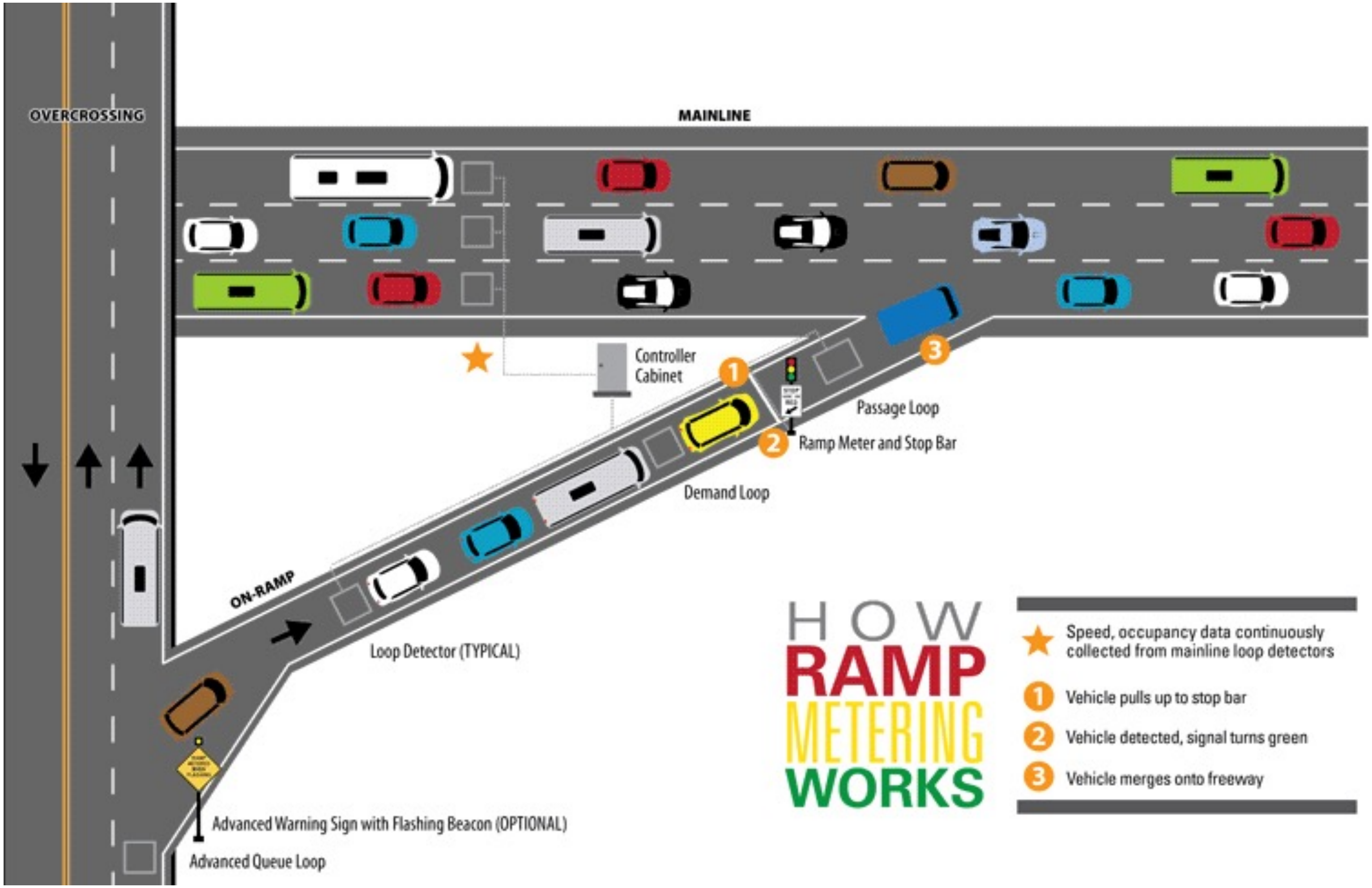
- Wireless communication between vehicles and RSU
- Can report microscopic information
- Augments macroscopic sensing



Ramp metering

- A ramp meter, ramp signal, or metering light is a device that regulates the flow of traffic entering freeways according to current traffic conditions.
- Ramp meters are used at freeway on-ramps to manage the rate of automobiles entering the freeway.
- Ramp metering systems have proved to be successful in decreasing traffic congestion and improving driver safety.

Ramp metering



Traffic flow model

- Flow-density relation
- Cell transmission model (CTM)

Flow-density relation

- Traffic flow f: # of vehicles passing a fixed cross-section during unit time [veh/hr]
- Density ρ: # of vehicles per unit road space [veh/km]
- Traffic speed v: aggregate speed of traffic [km/hr]
- Fundamental relation

$$f = \rho v$$

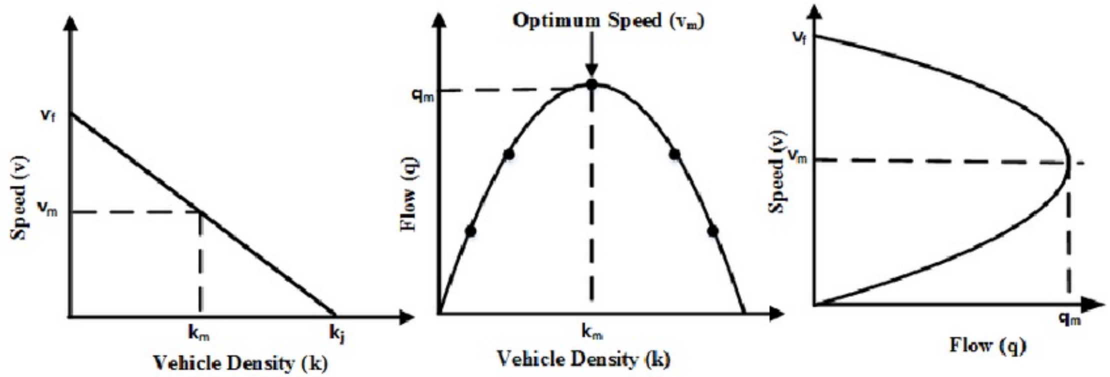
Flow-density relation

- Driving experience:
  - High traffic density -> low speed
  - Low traffic density -> high speed
- When you see something monotonic, first try a linear function -> Greeshields model
- Assume that speed linearly decreases with density…

- Greenshields, B. D., Bibbins, J. R., Channing, W. S., & Miller, H. H. (1935). A study of traffic capacity. In *Highway research board proceedings* (Vol. 1935). National Research Council (USA), Highway Research Board.

Greenshields model

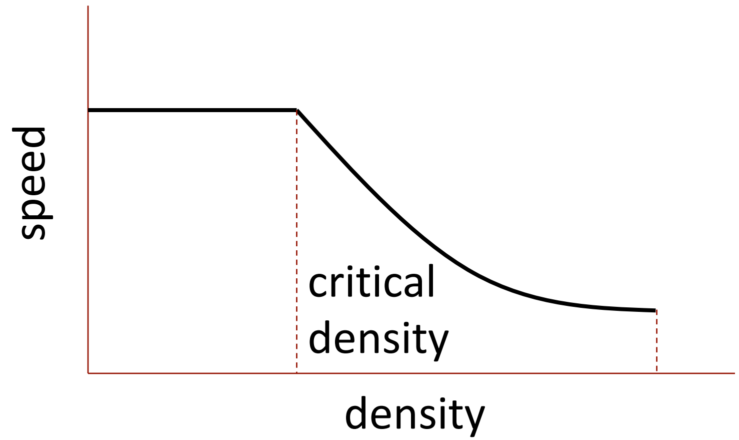
- Fundamental assumption: $v = v_0(1 - \rho/\bar{\rho})$
- v_0 = free-flow speed, $\bar{\rho}$ = jam (maximal) density

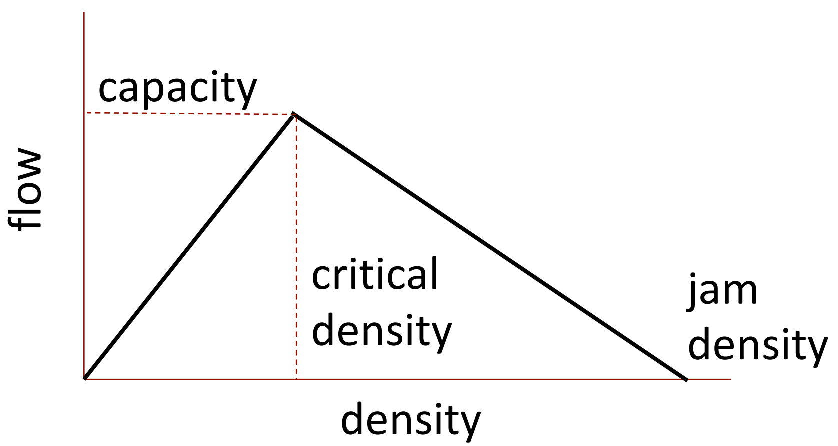


- Traffic & world in 1935…

Triangular model

- Greenshields model is problematic in the low-density regime.
- At a low density, speed is not affected by density.
- Speed is affected until the density passes a threshold, called critical density.
- Hence, we have a modification as follows:



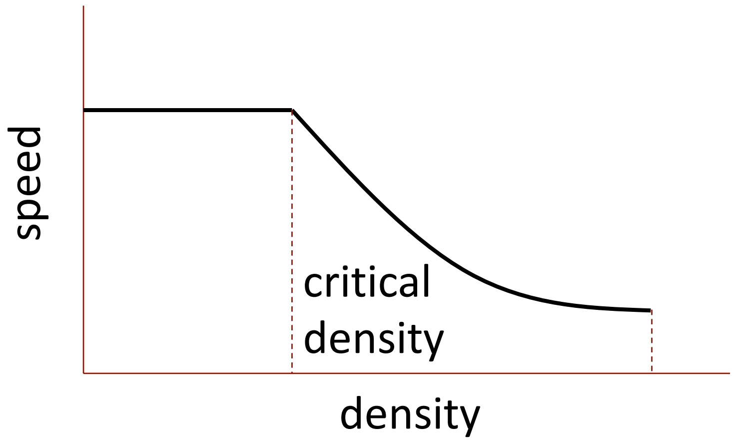


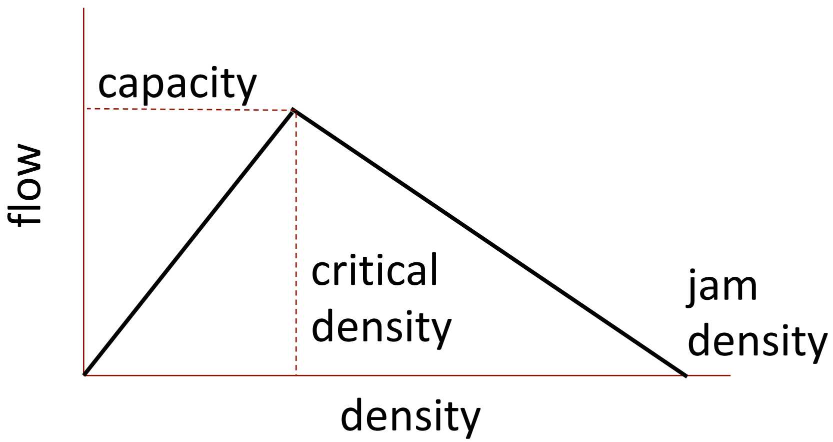
Triangular model

- Flow-density relation:

$$f = \min\{v\rho, w(\bar{\rho} - \rho)\}$$

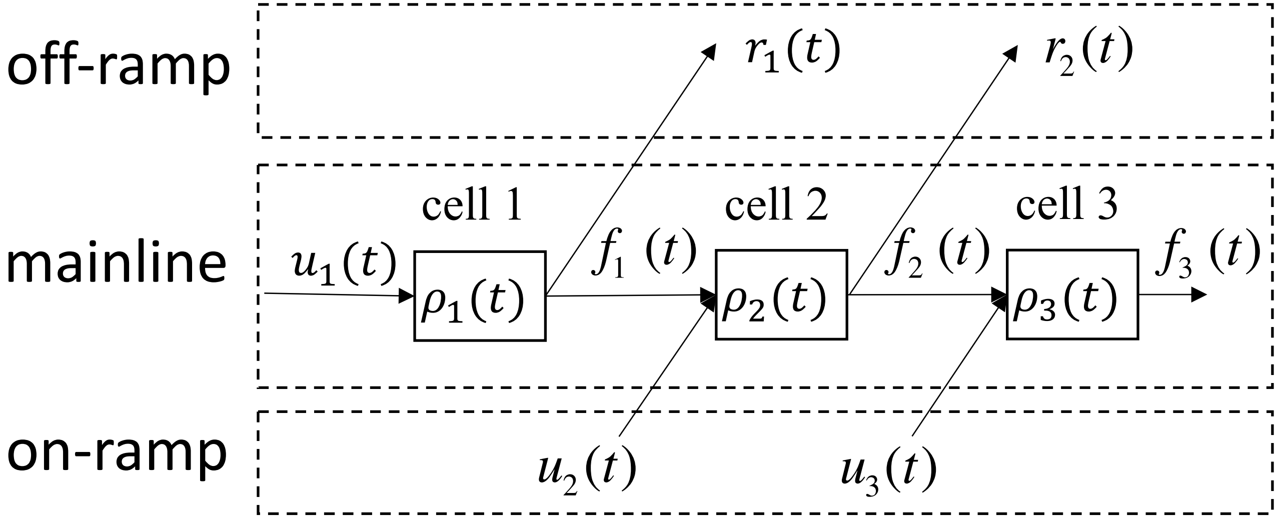
- v = free-flow speed, w = congestion wave speed
- Critical density $\rho^c = \frac{w}{v+w}\bar{\rho}$ [veh/km] ~20 veh/km/lane
- Capacity $\bar{f} = v\rho^c = \frac{vw}{v+w}\bar{\rho}$ [veh/hr] ~1600 veh/hr/lane





Cell transmission model

- Suppose that we partition a highway into many sections
- Partition: according to locations of on-/off-ramps
- State: $\rho_1(t), \rho_2(t), \ldots, \rho_n(t)$



Cell transmission model

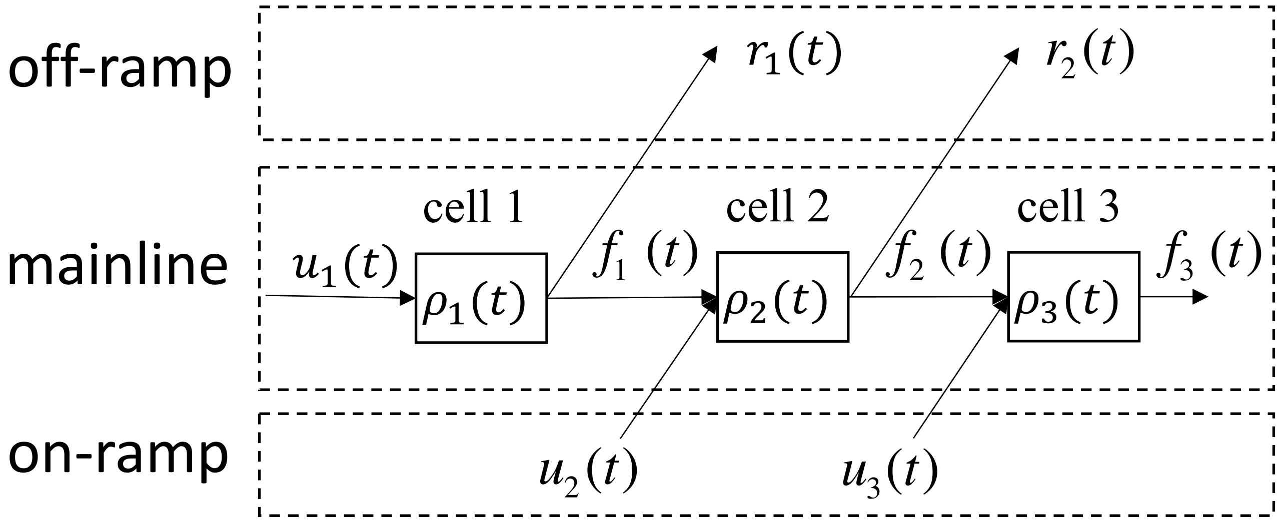
- Mainline flow

$$f_k(t) = \beta_k \min\{v\rho_k(t), \bar{f}, w(\bar{\rho} - \rho_{k+1}(t))\}$$

- Off-ramp flow

$$r_k(t) = (1 - \beta_k) \min\{v\rho_k(t), \bar{f}, w(\bar{\rho} - \rho_{k+1}(t))\}$$

- On-ramp flow $u_k(t)$: external or specified

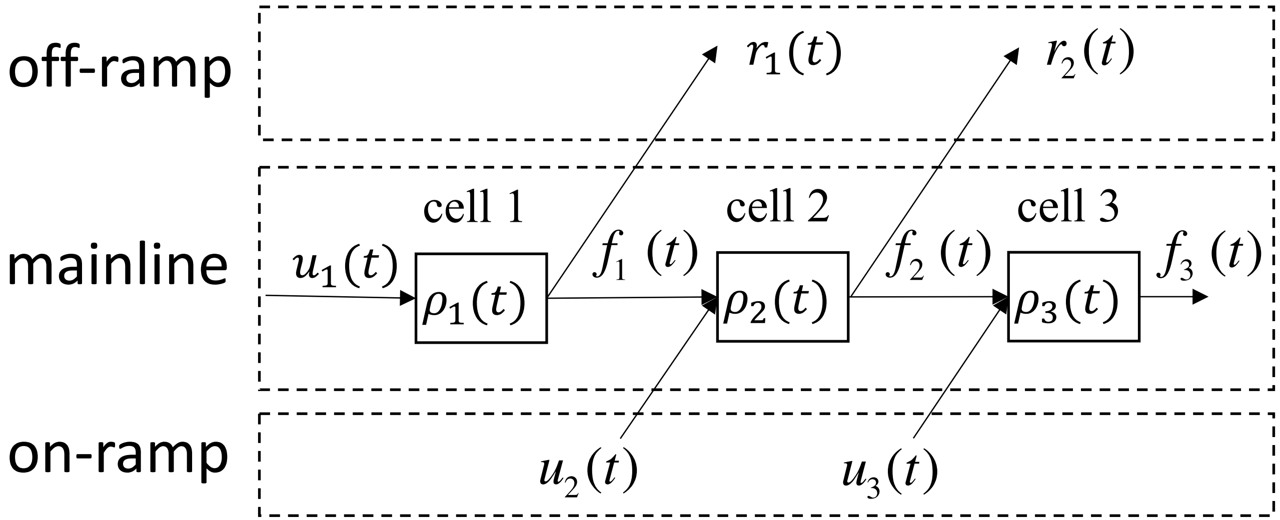


Cell transmission model

- Dynamic equation: mass conservation

$$\rho_k(t+1) = \rho_k(t) + \frac{1}{l_k}\left(f_{k-1}(t) + u_k(t) - f_k(t) - r_k(t)\right)$$

- l_k = length of cell k



Cell transmission model: properties*

- Nonlinear dynamical system
- Given constant demand $u_k(t) = a_k$, the system is convergent if and only if demand < capacity
- If demand > capacity, cells get jammed, and on-ramp queue grows.
- Steady-state densities

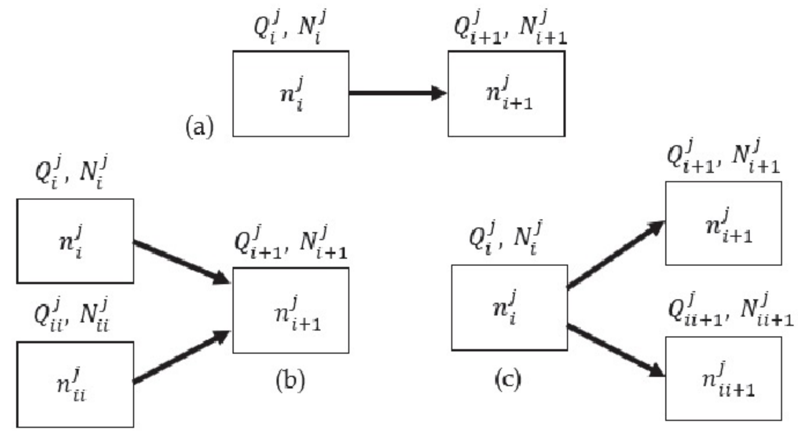
$$f_{k-1} + a_k - f_k - r_k = 0$$

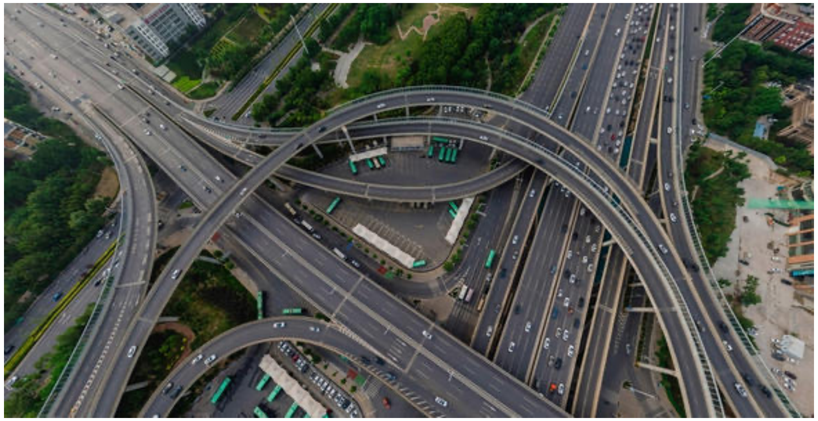
$$\rho_k^* = \frac{1}{v}(f_k + r_k) = \frac{1}{v}\left(\sum_{i=1}^{k-1} \beta_i \cdots \beta_{k-1} a_i + a_k\right)$$

- Gomes, G., Horowitz, R., Kurzhanskiy, A. A., Varaiya, P., & Kwon, J. (2008). Behavior of the cell transmission model and effectiveness of ramp metering. *Transportation Research Part C: Emerging Technologies*, *16*(4), 485-513.

Network CTM*

- Essentially same as linear CTM
- Same flow-density relation
- New: merge & diverge cells
- Merge: need to specify inflow priority
- Diverge: need to specify splitting ratios
- Less commonly used due to complexity…





Ramp metering

- Flow stabilization
- Throughput maximization
- Delay minimization

Flow stabilization

- Data:
  - Highway properties v, w, l_k
  - Demand d_k & splitting ratio β_k
- Decision variable:
  - On-ramp flow $u_k(t)$
- Constraint:
  - CTM dynamics
  - On-ramp flow $\leq$ demand
- Objective:
  - Prevent mainline congestion
  - That is, ensure $\rho_k \leq \rho_k^c$

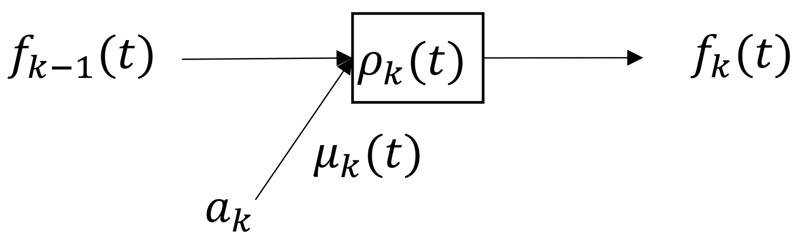
Flow stabilization

- Intuition:
  - On the one-hand, we want to let in as much on-ramp traffic as possible to prevent on-ramp queues
  - On the other hand, we do not want too much traffic on the mainline
- Insight:
  - When mainline is not congested, let in all on-ramp traffic
  - When mainline is congested, reduce the on-ramp flow
- Math

off-ramp

  - A simple "linear" controller

$$u_k(t) = \min\{a_k, v(\rho_k^c - \rho_k(t)) - f_{k-1}(t)\}$$



Flow stabilization: ALINEA

- A standard ramp metering policy

$$\mu_k(t+1) = \mu_k(t) + K(\rho_k^c - \rho_k(t))$$

- Named “asservissement linéaire d’entrée autoroutiere”
- If the measured density is found to be lower (higher) than the desired value, the second term of the right-hand side becomes positive (negative)
- The ordered on-ramp flow is increased (decreased) as compared to its last value.
- Clearly, the feedback law of above acts in the same way both for congested and for light traffic (no switchings are necessary).
- Papageorgiou, M., Hadj-Salem, H., & Blosseville, J. M. (1991). ALINEA: A local feedback control law for on-ramp metering. *Transportation Research Record, 1320*(1), 58-67.

Throughput maximization

- Suppose that each on-ramp is subject to infinite demand
- However, we only admit u_k amount of demand at each on-ramp to ensure stability
- Essentially, we are allocating “quota” for demands at each on-ramp
- How to maximize the admitted demand, i.e. the throughput?

Throughput maximization

- Data:
  - Highway parameters
- Decision variables:
  - Admitted traffic flow at each on-ramp
- Constraints:
  - Capacity constraint
- Objective:
  - Maximize steady-state throughput

Throughput maximization

- Mathematical formulation:
- max $\quad \sum_{k=1}^{n} u_k$
- s.t. $\quad \sum_{i=1}^{k} \prod_{j=i}^{k} \beta_j \, u_i \leq \bar{f}_k \quad$ (#)

  $u_k \geq 0$

- We can actually construct the optimal solution…
  - $u_1 = \min_i \frac{\bar{f}_i}{\prod_{j=1}^{i-1} \beta_j}$
  - $u_k = \min_i \frac{\bar{f}_i}{\prod_{j=k}^{i-1} \beta_j} - \sum_{i=1}^{k-1} \prod_{j=i}^{k-1} \beta_j \, u_i$
- Intuition: prioritize upstream demand #

Delay minimization

- Data:
  - Highway parameters
  - Demand $a_k(t)$
- Decision variables:
  - On-ramp flow $u_k(t)$
- Constraints:
  - On-ramp flow $\leq$ demand
  - CTM dynamics
- Objective:
  - Minimize vehicle-hours traveled
  - Minimize rejected demand

Delay minimization: VHT

- Vehicle hours traveled (VHT)
- If one vehicle spends one unit time on the road, 1 VHT is generated.
- VHT = $\sum_t$ # of vehicles at time t #
- High VHT -> congestion
- Value of time (VoT); e.g., 100 RMB/hr or 20 USD/hr
- Total cost = VoT × VHT
- Highway performance matters because our time matters.
- Time is the only non-renewable resource, and it is the largest economic cost of traveling and shipping.

Delay minimization: demand rejection

- If demand $a_k(t)$ exceeds the allowed on-ramp flow $u_k(t)$, $a_k(t) - u_k(t)$ amount of traffic is rejected
- These vehicles will leave the highway and never come back
- Rejection is costly
  - Congestion elsewhere
  - Unhappy travelers
- Hence, we assume a rejection cost

$$w\big(a_k(t) - u_k(t)\big)_+$$

- Total cost = delay + rejection cost

Delay minimization: finite horizon

- min $\quad \sum_{t=1}^{T} \sum_{k=1}^{n} \left(l_k(t)\rho_k(t) + w\big(a_k(t) - u_k(t)\big)_+ \right)$
- s.t. $\quad f_k(t) = \beta_k \min\{v\rho_k(t), \bar{f}, w\big(\bar{\rho} - \rho_{k+1}(t)\big)\},$

$$r_k(t) = (1 - \beta_k) \times \min\{v\rho_k(t), \bar{f}, w\big(\bar{\rho} - \rho_{k+1}(t)\big)\},$$

$$\rho_k(t+1) = \rho_k(t) + \frac{1}{l_k}\big(f_{k-1}(t) + u_k(t) - f_k(t) - r_k(t)\big),$$

$$0 \le u_k(t) \le \min\{a_k(t), \bar{u}_k\}.$$

- $\bar{u}_k$ = maximal on-ramp flow rate (saturation rate)

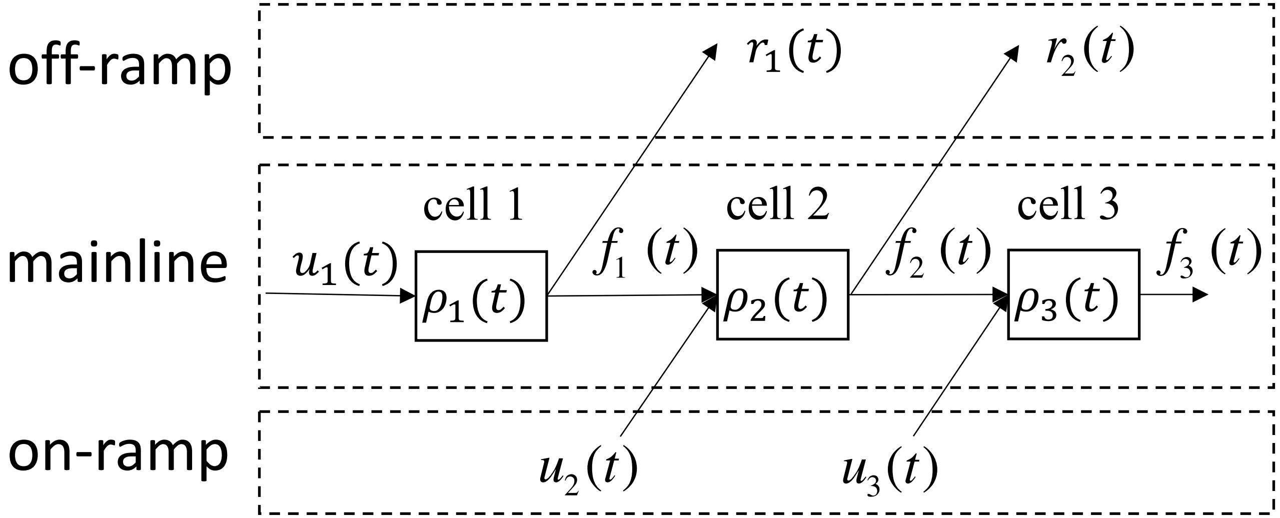
Delay minimization: infinite horizon

- Now let's extend the analysis to infinite horizon
- Two important changes!
- We can no longer specify/predict $a_k(t)$.
  - Consequently, we either assume $a_k(t)$ = constant, or assume some stochastic process (e.g. white noise), so that $a_k(t)$ can extend to infinite time.
- We can no longer directly specify $u_k(t)$.
  - If t is unbounded, $u_k(t)$ is a vector of infinite dimension.
  - You cannot handle it in optimization formulations.
  - You cannot handle it in computer programs either.
  - Consequently, we consider a ramp metering policy, or a ramp controller $\mu_k\big(\rho(t), a(t)\big)$.

Ramp controller

- Differentiate three quantities…
- Demand $a_k(t)$
- Allowed on-ramp flow $\mu_k(t)$ (control input), e.g. ALINEA
- Actual on-ramp flow

$$u_k(t) = \min\{a_k(t), \mu_k(t)\}$$



Delay minimization: infinite horizon

- Data:
  - Highway parameters
  - Demand $a_k(t)$: some model needed
- Decision variables:
  - Ramp metering policy μ_k
- Constraints:
  - On-ramp flow $\leq$ demand
  - CTM dynamics
- Objective:
  - Minimize vehicle-hours traveled
  - Minimize rejected demand

Delay minimization: infinite horizon

- Consider initial condition $\rho(0)$
- We want to design ramp metering policies μ_k that minimizes the future travel cost
- Recall from previous lectures, when we design a control policy, we need to determine two things:
  1. Form of the policy, e.g. linear/quadratic/piecewise…
  2. Parameters of the policy, e.g. slope/intercept/coefficients…
- For the ramp metering policy, typical forms include:
  1. Linear policy, i.e. $\mu = A\rho + b$
  2. ALINEA: $\dot{\mu} = A\rho + b$
  3. Neural network, i.e. $\mu = NN(\rho)$

Delay minimization: infinite horizon

- Objective:

$$\min \sum_{t=1}^{\infty} \sum_{k=1}^{n} \gamma^t \left(l_k(t)\rho_k(t) + w\big(a_k(t) - u_k(t)\big)_+ \right)$$

- $\gamma \in (0,1)$ is a discounting factor
- Discounting ensures bounded objective function
- If $a_k(t)$ is stochastic (e.g. $a_k(t) = \bar{a}_k + \epsilon$), we do a stochastic optimization

$$\min \mathrm{E} \left[\sum_{t=1}^{\infty} \sum_{k=1}^{n} \gamma^t \left(l_k(t)\rho_k(t) + w\big(a_k(t) - u_k(t)\big)_+ \right) \middle| \rho(0) \right]$$

Delay minimization: value function*

- The conditional expected cost

$$\mathrm{E}\left[\sum_{t=1}^{\infty}\sum_{k=1}^{n}\gamma^{t}\left(l_k(t)\rho_k(t)+w\big(a_k(t)-u_k(t)\big)_{+}\right)\middle|\,\rho(0)\right]$$

- Conditional expected return

$$V(\nu)=-\mathrm{E}\left[\sum_{t=1}^{\infty}\sum_{k=1}^{n}\gamma^{t}\left(l_k(t)\rho_k(t)+w\big(a_k(t)-u_k(t)\big)_{+}\right)\middle|\,\rho(0)=\nu\right]$$

- Value function

$$V(\rho)=\mathrm{E}[\text{return}|\text{initial condition }\rho]$$

Delay minimization: Monte-Carlo method

- Computing value function is extremely challenging
- We can use simulation to numerically approximate it
- Suppose some initial condition ρ and assume $a_k(t) = \bar{a}_k + \epsilon$, where ϵ is white noise
- Then, we can randomly generate $a_k(t)$ and then simulate $\rho_k(t)$
- Let $a_k^{\mathrm{sim}}(t),\ \rho_k^{\mathrm{sim}}(t)$ be the simulation results

$$V(\rho) \approx -\sum_{t=1}^{\infty}\sum_{k=1}^{n}\gamma^t\left(l_k(t)\rho_k^{\mathrm{sim}}(t) + w\left(a_k^{\mathrm{sim}}(t) - u_k(t)\right)_+\right)$$

- You can also simulate 100 runs and take average

Delay minimization: Monte Carlo method

- This is called Monte Carlo method
- Monte Carlo Casino, Monaco
- Developed by Stanisław Ulam during the US's nuclear weapon project
- Recognized by von Neumann
- Being secret, the work of von Neumann and Ulam required a code name.
- A colleague of von Neumann and Ulam, Nicholas Metropolis, suggested using the name Monte Carlo, which refers to the Monte Carlo Casino in Monaco where Ulam's uncle would borrow money from relatives to gamble.

Stanisław M. Ulam
1909-1984

Monte Carlo method

- Monte Carlo methods, or Monte Carlo experiments, are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results.
- When direct, analytical computation is hard, we can use simulation to numerically approximate.
- Only possible after the age of computers.
- Widely used in engineering, science, finance, social sciences, etc.
- In a broader scope, the idea of randomization is extensively used in control & optimization.

Delay minimization: Monte-Carlo method

- A naïve approach:
  - Guess a ramp metering policy μ
  - Select a large set of initial conditions P
  - For each initial condition $\rho \in \mathrm{P}$, simulate CTM for 100 times and compute the average return
  - Perturb μ a little, simulate CTM again, and see whether the return improves
- This would work if your computer is infinitely fast
- But unfortunately, this is not always the case
- Refinement*
  - Simulation-based optimization
  - Temporal-difference method

Summary

- Smart highways
  - Sensing technology
  - Control technology
- Traffic flow model
  - Flow-density relation
  - Cell transmission model
- Ramp metering
  - Flow stabilization
  - Throughput maximization
  - Delay minimization

Next time

- Traffic network optimization
- Quiz 1 review