# VE230 Homework 5

2021 Summer

**P1** Lightning strikes a lossy dielectric sphere - $\epsilon = 1.2\epsilon_0, \sigma = 10(\mathrm{S/m})$ - of radius $0.1(\mathrm{m})$ at time $t = 0$, depositing uniformly in the sphere a total charge $1(\mathrm{mC})$.

a) Calculate the time it takes for the charge density in the sphere to diminish to 1% of its initial value.

b) Calculate the change in the electrostatic energy stored in the sphere as the charge density diminishes from the initial value to 1% of its value. What happens to this energy?

c) Determine the electrostatic energy stored in the space outside the sphere. Does this energy change with time?

**P2** The space between two parallel conducting plates each having an area $S$ is filled with an inhomogeneous ohmic medium whose conductivity varies linearly from $\sigma_1$ at one plate ($y = 0$) to $\sigma_2$ at the other plate ($y = d$). A d-c voltage $V_0$ is applied across the plates as in Fig. 1. Determine

a) the total resistance between the plates,

b) the surface charge densities on the plates,

c) the volume charge density and the total amount of charge between the plates.

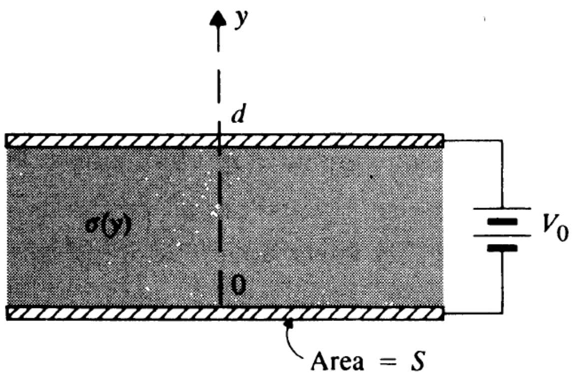


Figure 1: Inhomogeneous ohmic medium with conductivity $\sigma(y)$ (Problem 2).

**P3** Determine the resistance between two concentric spherical surfaces of radii $R_1$ and $R_2$ ($R_1 < R_2$), assuming that a material of conductivity $\sigma = \sigma_0(1 + k/R)$ fills the space between them. (Note: Laplace's equation for $V$ does not apply here.)

**P4** Assume a rectangular conducting sheet of conductivity $\sigma$, width $a$, and height $b$. A potential difference $V_0$ is applied to the side edges, as shown in Fig. 2. Find

a) the potential distribution,

b) the current density everywhere within the sheet. (Hint: Solve Laplace's equation in Cartesian coordinates subject to appropriate boundary conditions.)

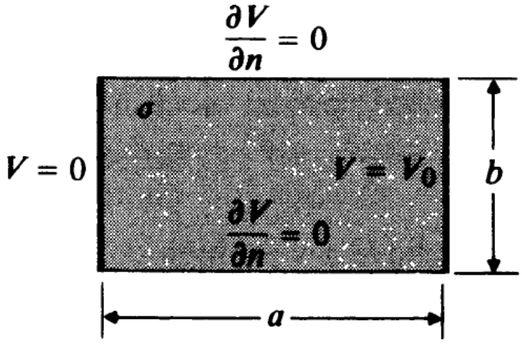


Figure 2: A conducting sheet (Problem 4). .

**P5** An electron is injected with a velocity $\mathbf{u}_0 = \mathbf{a}_y u_0$ into a region where both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ exist. Describe the motion of the electron if

a) $\mathbf{E} = \mathbf{a}_z E_0$ and $\mathbf{B} = \mathbf{a}_x \boldsymbol{B}_0$

b) $\mathbf{E} = -\mathbf{a}_z E_0$ and $\mathbf{B} = -\mathbf{a}_z B_0$

Discuss the effect of the relative magnitudes of $E_0$ and $B_0$ on the electron paths in parts (a) and (b).

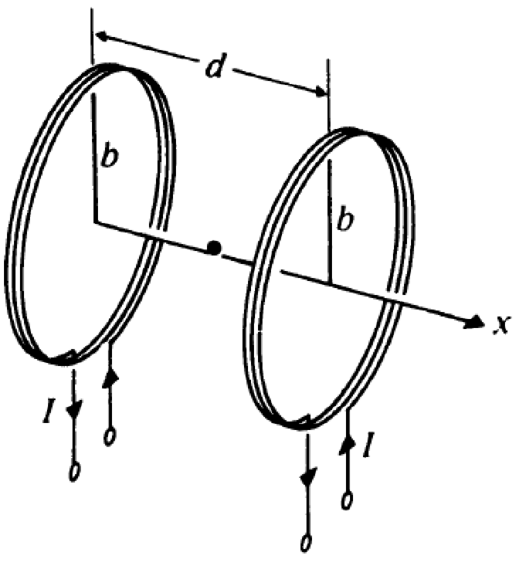


Figure 3: Helmholtz coils (Problems 6).

**P6** Two identical coaxial coils, each of $N$ turns and radius $b$, are separated by a distance $d$, as depicted in Fig. 3. A current $I$ flows in each coil in the same direction.

a) Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.

b) Show that $dB_x/dx$ vanishes at the midpoint.

c) Find the relation between $b$ and $d$ such that $d^2B_x/dx^2$ also vanishes at the midpoint. Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as Helmholtz coils.

**P7** Do the following by using $V_m = -\frac{I}{4\pi}\Omega$:

a) Determine the scalar magnetic potential at a point on the axis of a circular loop having a radius $b$ and carrying a current $I$.

b) Obtain the magnetic flux density $\mathbf{B}$ from $-\mu_0 \nabla V_m$, and compare the result with $\mathbf{B} = \mathbf{a}_z \frac{\mu_0 I b^2}{2(z^2+b^2)^{3/2}}$.