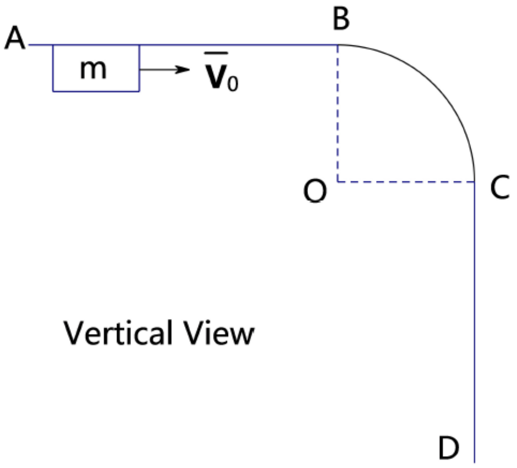
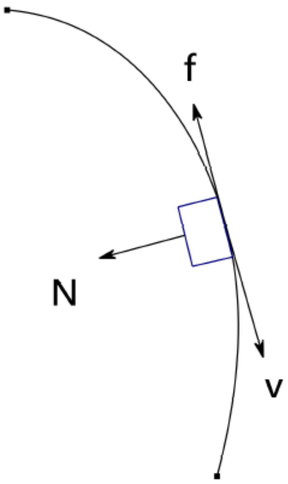
1. There is an object with mass m, is moving on a frictionless plane. Its motion is subjected to a fixed baffle on the plane. The baffle consists of two straight board and an arc board with radius R. At $t = 0$, the object is moving along AB with speed v_0. The friction coefficient is μ. Calculate the work done by firction when the object is moving from A to D.





Firstly, we try to calculate the speed at D.
when the object is moving along line segment AB, the is no interaction between the baffle and the object. When the object is moving through the arc BC, its interaction with the baffle is shown above. Accordignly, we can have the following relation:

$$-f = m\frac{dv}{dt} \tag{1}$$

$$N = m\frac{v^2}{R} \tag{2}$$

What's more,

$$f = \mu N, \tag{3}$$

By the equation above, we have

$$m\frac{dv}{dt} = -\mu m\frac{v^2}{R}$$

by substitution,

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v,$$

which implies:

$$\frac{dv}{ds} = \mu\frac{v}{R},$$

saparate variable:

$$\frac{dv}{v} = -\frac{\mu}{R}ds,$$

integral both sides:

$$\int_{v_0}^{v}\frac{dv}{v} = \int_0^{\frac{\pi R}{2}} -\frac{\mu}{R}ds,$$

Hence,

$$\ln \frac{v}{v_0} = \mu \frac{\pi}{2},$$

and v is the speed when the speed is moving to C. Since the object is moving in 1D in CD line segemnt, its sped remain constant.

$$v = v_0 e^{-\frac{\mu\pi}{2}}.$$

And only when the object is moving in BC line segment, the friction will do work, by work-energy theorem we have

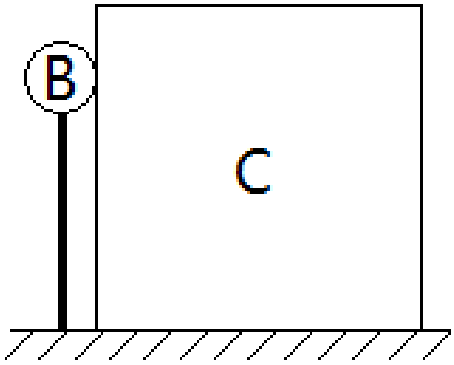
$$W_f = \int \mathbf{f} \cdot d\mathbf{r} = \Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2, \tag{4}$$

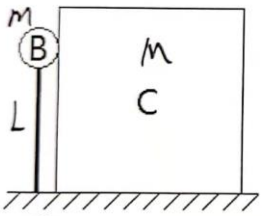
when we plug in v_f, we have

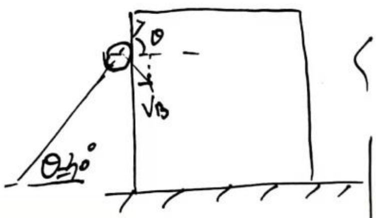
$$W_f = \frac{1}{2}mv_0^2(e^{-\mu\pi} - 1)$$

Remark: since $e^{-\mu\pi} < 1$, $W_f < 0$, which suggests the friction does negatie work.

2. The mass of B is m, the mass of C is M. The length of the light rod is L. When there is a slight turbulence, the B falls to the right. When B do not touch C, the angle between the B and C is $\pi/3$. Question: what is the magnitude of m/M? (There is no friction).





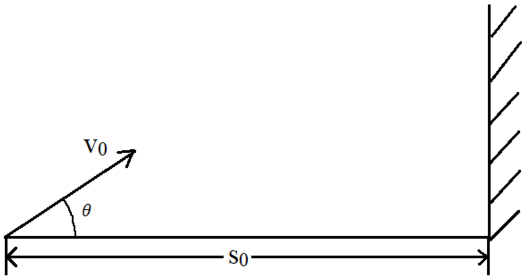


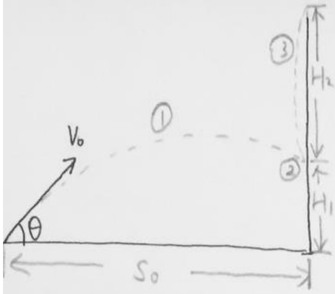
$$\begin{cases} mg(L - L\sin\theta) = \frac{1}{2}MV_C^2 + \frac{1}{2}mV_B^2 \\ V_B\sin\theta = V_C \\ m\frac{V_B^2}{L} = mg\sin\theta \end{cases}$$

Assume $\frac{m}{M} = R$

$$\begin{cases} Rg(L - L\sin\theta) = \frac{1}{2}V_C^2 + \frac{1}{2}RV_B^2 \\ V_B\sin\theta = V_C \\ V_B^2 = gL\sin\theta \end{cases} \Rightarrow \begin{cases} \frac{R}{2}gL = \frac{1}{2}V_B^2\frac{1}{4} + \frac{R}{2}V_B^2 \\ V_B^2 = \frac{1}{2}gL \end{cases} \Rightarrow R = \frac{1}{4}$$

3. A man jumps from the ground with speed v_0 to the wall, the distance between the man and the wall is s_0. When he reaches the wall, he has a vertical velocity pointing downwards. Then the wall gives the man a normal force to help him change his direction to go upward. The coefficient of kinetic friction between the man and the wall is μ. What is the value of θ when the man can jump highest?





① Projectile motion

$$\begin{cases} V_x = V_0 \cos\theta \\ V_y = V_0 \sin\theta - gt \end{cases}$$

Finally
$$\begin{cases} t_0 = \dfrac{S_0}{V_x} = \dfrac{S_0}{V_0\cos\theta} \\ V_y(t_0) = V_0 \sin\theta - \dfrac{g S_0}{V_0 \cos\theta} \end{cases} \Rightarrow H_1 = V_0\sin\theta \cdot t_0 - \frac{1}{2} g t_0^2$$
$$= V_0 \sin\theta \cdot \frac{S_0}{V_0\cos\theta} - \frac{1}{2} g \left(\frac{S_0}{V_0 \cos\theta}\right)^2$$
$$= S_0 \tan\theta - \frac{1}{2} g \frac{S_0^2}{V_0^2 \cos^2\theta}$$

(* Note that it is negative)

② Collision

→ Velocity in x direction change to 0

$$\begin{cases} \Delta P_x = N \cdot \Delta t \\ \Delta P_y = f \cdot \Delta t = \mu N \cdot \Delta t \end{cases} \Rightarrow \begin{cases} \Delta P_x = m V_0 \cos\theta \\ \Delta P_y = \mu \Delta P_x = \mu m V_0 \cos\theta \end{cases} \Rightarrow V_y(t_0^+) = V_y(t_0) + \mu V_0 \cos\theta$$
$$= V_0 \sin\theta - \frac{g S_0}{V_0 \cos\theta} + \mu V_0 \cos\theta$$

↳ V_y change direction

③ Free fall with initial velocity

$$H_2 = \frac{V_y^2(t_0^+)}{2g} = \frac{V_0^2(\sin\theta + \mu\cos\theta)^2 + \left(\frac{g S_0}{V_0 \cos\theta}\right)^2 - \frac{2 g S_0}{V_0 \cos\theta}(V_0 \sin\theta + \mu V_0 \cos\theta)}{2g}$$
$$= \frac{V_0^2(\sin\theta + \mu\cos\theta)^2}{2g} + \frac{1}{2} g \frac{S_0^2}{V_0^2 \cos^2\theta} - S_0 \tan\theta - \mu S_0$$

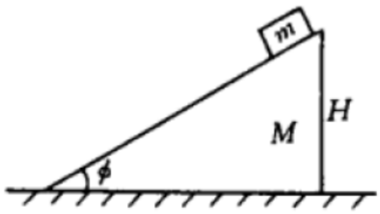
①&③ $$H = H_1 + H_2 = \frac{V_0^2(\sin\theta + \mu\cos\theta)^2}{2g} - \mu S_0 = -\mu S_0 + \frac{V_0^2(1+\mu^2)}{2g}$$

When $\sin\theta + \mu\cos\theta$ reaches maximum

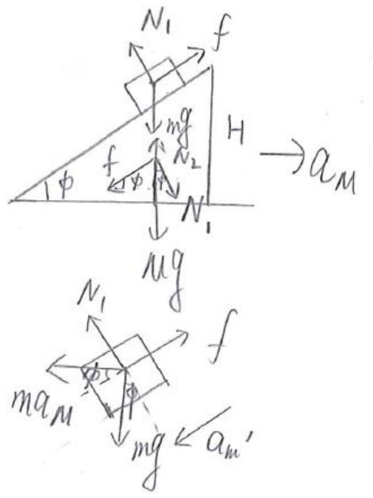
$$(\sin\theta + \mu\cos\theta)' = \cos\theta - \mu\sin\theta = 0$$

$$\Rightarrow \theta = \arctan\frac{1}{\mu} \qquad Max = \sqrt{1+\mu^2}$$

4. The system is shown in the figure. There exists friction between m and M, but the ground is smooth. Initially, the system is at rest. Then m moves along the slope to the ground, try to find the distance M travels in total. Please (a) use kinematics with the help of non-inertial FoR (the slope) (b) use the conservation of momentum to solve the problem.



(a)



$$\begin{cases} N_1\sin\phi - f\cos\phi = Ma_M \\ N_1 + ma_M\sin\phi - mg\cos\phi = 0 \\ ma_M\cos\phi + mg\sin\phi - f = ma_m' \\ f = \mu N_1 \end{cases}$$

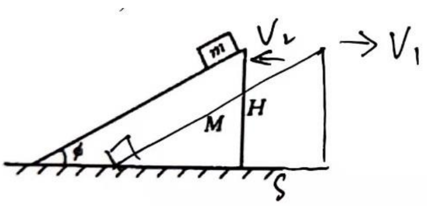
$$\Rightarrow \begin{cases} a_M = \dfrac{mg\sin\phi\cos\phi - \mu mg\cos^2\phi}{M + m\sin^2\phi - \mu m\sin\phi\cos\phi} \\ a_m' = g\sin\phi - \mu g\cos\phi + a_M(\cos\phi + \mu\sin\phi) \end{cases}$$

$$\frac{H}{\sin\phi} = \frac{1}{2}a_m' t^2 \qquad S = \frac{1}{2}a_M t^2$$

$$\Rightarrow S = \frac{a_M}{a_m'}\cdot\frac{H}{\sin\phi} = \frac{m}{M+m}H\cot\phi$$

(b)

$$\tan\varphi = \frac{H}{x}$$



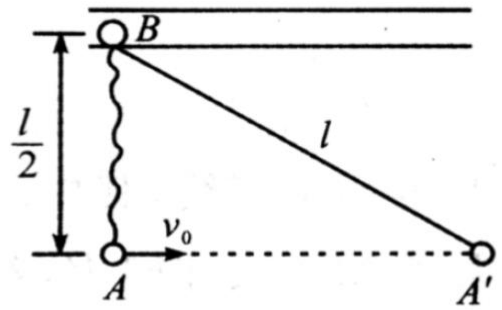
$$MV_1 = +mV_2$$

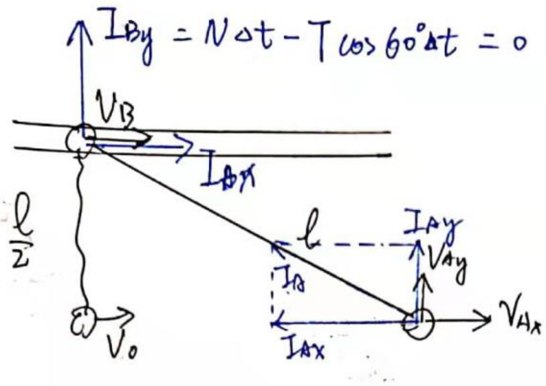
$$\Rightarrow MS = +m(-S + H\cot\varphi)$$

$$\Rightarrow (m+M)S = +mH\cot\varphi$$

$$\Rightarrow S = +\frac{m}{m+M}H\cot\varphi$$

5. Ball B with mass m is placed in a frictionless horizontal pipe. There is a narrow gap along the pipe, where ball A with mass m is connected to ball B with a inextensible massless rope with length l. Initially the distance between A and B is $\frac{l}{2}$, and ball A start moving to the right with initial speed v_0. When the rope is tightened, determine the speed of ball B v_B.





t from 0^- to 0^+

$$\begin{cases} I_{Bx} = mV_B - 0 = mV_B & (1) \\ -I_{Ax} = mV_{Ax} - mV_0 & (2) \\ I_{Ay} = mV_{Ay} & (3) \\ I_{Ax} = I_{Bx} & (4) \end{cases}$$

at $t = 0$

$$V_B \cos 30^\circ = V_{Ax}\cos 30^\circ - V_{Ay}\cos 60^\circ \qquad (5)$$

$$\begin{cases} I_{Ax} = I_A \cos 30^\circ & (6) \\ I_{Ay} = I_A \sin 30^\circ & (7) \end{cases}$$

solve (1) to (7) we can get the answer.