---

**Question1** (1 points)

Consider the following set

$$\mathcal{A} = \left\{ \frac{m}{n} \mid m, n \in \mathbb{N}_1, m < n \right\}$$

Show $\mathcal{A}$ has no minimum or maximum element in spite of having $\sup(\mathcal{A}) = 1$ and $\inf(\mathcal{A}) = 0$.

**Question2** (1 points)

Let $\mathcal{F}$ be the collection of sets

$$\mathcal{S}_r = \{x \mid r < x \leq 1 + r\}, \qquad 0 < r \leq \frac{1}{2}$$

Find the union $\bigcup \{\mathcal{S}_r \mid \mathcal{S}_r \in \mathcal{F}\}$ and the intersection $\bigcap \{\mathcal{S}_r \mid \mathcal{S}_r \in \mathcal{F}\}$.

**Question3** (3 points)

Suppose $\mathcal{A}$ is a nonempty proper subset of $\mathbb{R}$. Determine whether each of the following statements is true. If not, briefly explain why it is false.

(a) (1 point) The set $\mathcal{A}$ is either open or closed.

(b) (1 point) The interior points and boundary points of $\mathcal{A}$ are limit points.

(c) (1 point) If every element of $\mathcal{A}$ is an isolated point, then $\mathcal{A}$ is closed.

**Question4** (5 points)

Let $\mathcal{D} = (-\infty, -5] \cup (3, 4) \cup \{7\}$.

(a) (1 point) Find the interior of $\mathcal{D}$, that is, the set of all interior points of $\mathcal{D}$.

(b) (1 point) Find the boundary of $\mathcal{D}$, that is, the set of boundary points of $\mathcal{D}$, which is often denoted by $\partial\mathcal{D}$. The closure of $\mathcal{D}$, often denoted by $\overline{\mathcal{D}}$, is the union of $\mathcal{D}$ and $\partial\mathcal{D}$.

(c) (1 point) Find the exterior of $\mathcal{D}$.

(d) (1 point) Find all limit points of $\mathcal{D}$.

(e) (1 point) Find all isolated points of $\mathcal{D}$.

**Question5** (0 points)

(a) (1 point (bonus)) Let $\mathcal{A}$ and $\mathcal{B}$ be two subsets of $\mathbb{R}$. Find the difference of $\mathcal{A} - \mathcal{B}$, where $\mathcal{A}$ is the closed interval between 0 and 1, and $\mathcal{B}$ is the singleton $\{0\}$. The set $\mathcal{A} - \mathcal{B}$ is also known as the complement of $\mathcal{B}$ relative to $\mathcal{A}$.

(b) A set of real number $\mathcal{S} \subset \mathbb{R}$ is disconnected if there are open sets $\mathcal{U}$, $\mathcal{V} \subset \mathbb{R}$ such that $\mathcal{U} \cap \mathcal{V}$ are empty, and $\mathcal{S} \cap \mathcal{U}$ and $\mathcal{S} \cap \mathcal{V}$ are nonempty and

$$\mathcal{S} = (\mathcal{S} \cap \mathcal{U}) \cup (\mathcal{S} \cap \mathcal{V})$$

A set is connected if it is not disconnected.

i. (1 point (bonus)) Show the set $\{0, 1\}$ is disconnected.

ii. (1 point (bonus)) Let $\mathcal{S} \subset \mathbb{R}$, show $\mathcal{S}$ is connected if and only if it is an interval.

(c) (1 point (bonus)) Show a point $x^*$ is a limit point of a set $\mathcal{S}$ if and only if there is a sequence $\{x_n\}$ of points in $\mathcal{S}$ such that $x_n \neq x^*$ for $n \geq 1$, and $x_n \to x^*$ as $n \to \infty$.