Vv557 Methods of Applied Mathematics II

Green Functions and Boundary Value Problems

Assignment 6

Date Due: 12:55 PM, Wednesday, the 7th of April 2021

Exercises (14 Marks)

Exercise 6.1

Find the adjoint boundary value problem to

$$L = -\frac{d^2}{dx^2}, \qquad -1 < x < 1, \qquad B_1 u = u(-1) - u(1), \qquad B_2 u = u'(-1) - u'(1).$$

State the solvability condition for

$$Lu = f, \qquad B_1 u = \gamma_1, \qquad B_2 u = \gamma_2$$

and find the modified Green's function.
(9 Marks)

Exercise 6.2

Find the solvability condition for the forced harmonic oscillator

$$-u'' - u = f, \quad -\pi < x < \pi, \qquad u(\pi) - u(-\pi) = \gamma_1, \qquad u'(\pi) - u'(-\pi) = \gamma_2.$$

Suppose that $\gamma_1 = \gamma_2 = 0$. Interpret the result in terms of the type of forcing function f that can give a periodic solution.
(5 Marks)