Chapter 28

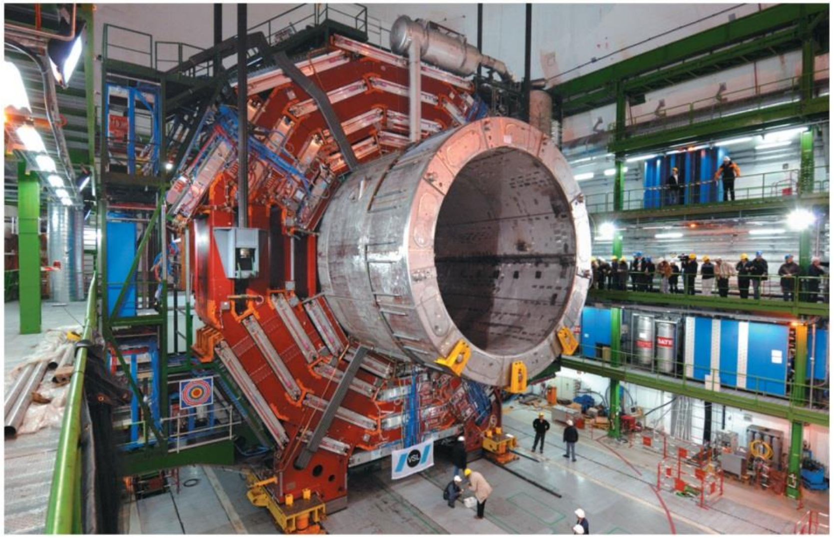
Sources of Magnetic Field

Goals for Chapter 28

- To determine the magnetic field produced by a moving charge
- To study the magnetic field of an element of a current-carrying conductor
- To calculate the magnetic field of a long, straight, current-carrying conductor
- To study the magnetic force between current-carrying wires
- To determine the magnetic field of a circular loop
- To use Ampere's Law to calculate magnetic fields

Introduction

- What can we say about the magnetic field due to a solenoid?
- What actually creates magnetic fields?
- We will introduce Ampere's law to calculate magnetic fields.



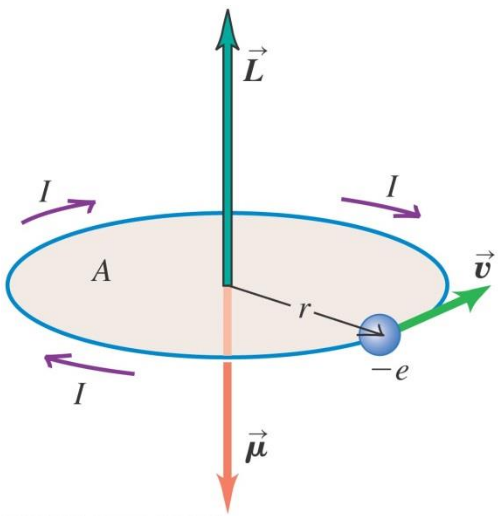
The Bohr magneton and paramagnetism

- Follow the text discussions of the *Bohr magneton*

$$\mu = IA; \qquad A = \pi r^2.$$

$$T = 2\pi r/v. \quad I = \frac{e}{T} = \frac{ev}{2\pi r}$$

$$\mu = \frac{ev}{2\pi r}(\pi r^2) = \frac{evr}{2} \qquad L = mvr \qquad h/2\pi,$$



$$\mu = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi m}$$

e **Bohr magneton,** denoted b

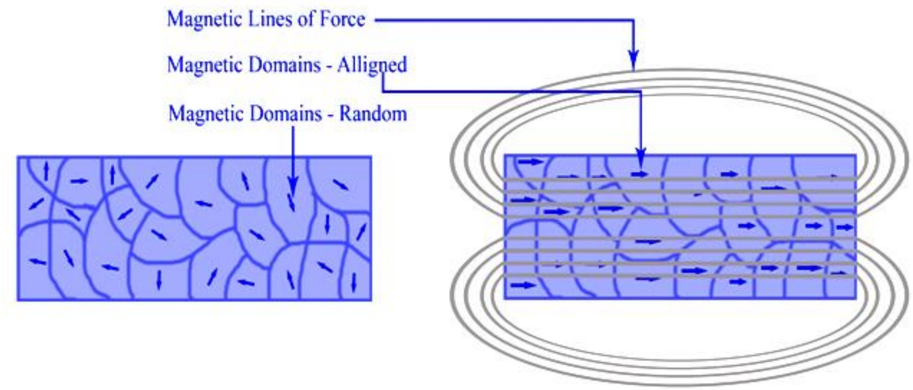
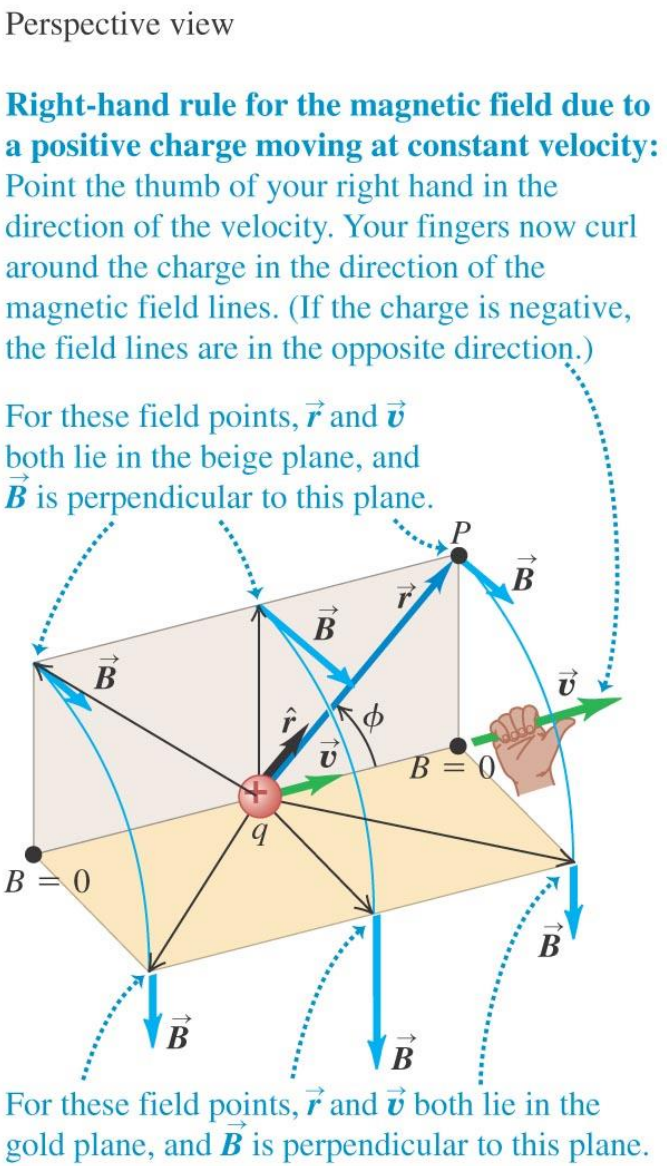
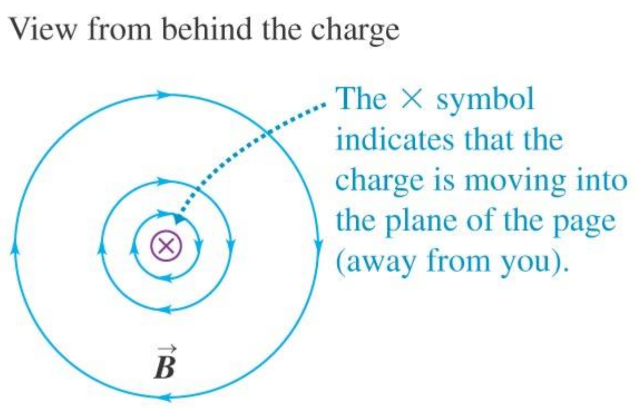


Fig. Ferromagnetism (a) Unmagnetized Material (b) Magnetized Material

The magnetic field of a moving charge

- **A moving charge generates a magnetic field** that depends on the velocity of the charge.
- Figure 28.1 shows the direction of the field.





The magnetic field of a moving charge

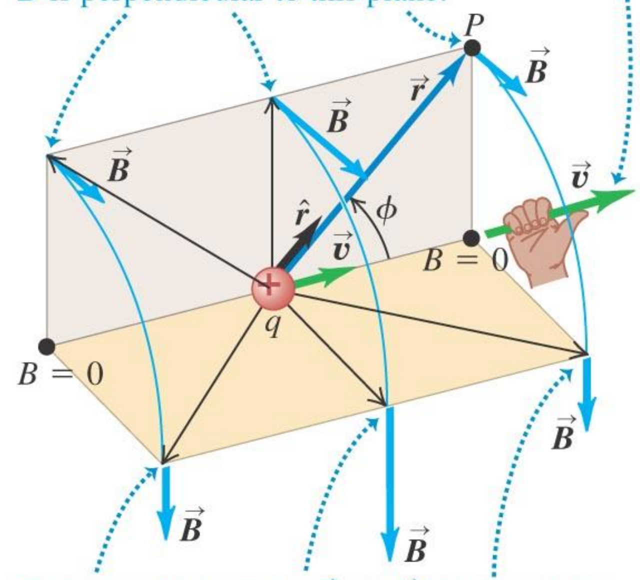
$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2} \quad \text{(magnetic field of a point charge with constant velocity)}$$

Perspective view

Right-hand rule for the magnetic field due to a positive charge moving at constant velocity: Point the thumb of your right hand in the direction of the velocity. Your fingers now curl around the charge in the direction of the magnetic field lines. (If the charge is negative, the field lines are in the opposite direction.)

For these field points, $\vec{r}$ and $\vec{v}$ both lie in the beige plane, and $\vec{B}$ is perpendicular to this plane.



For these field points, $\vec{r}$ and $\vec{v}$ both lie in the gold plane, and $\vec{B}$ is perpendicular to this plane.

Magnetic force between moving protons

$$F_E = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

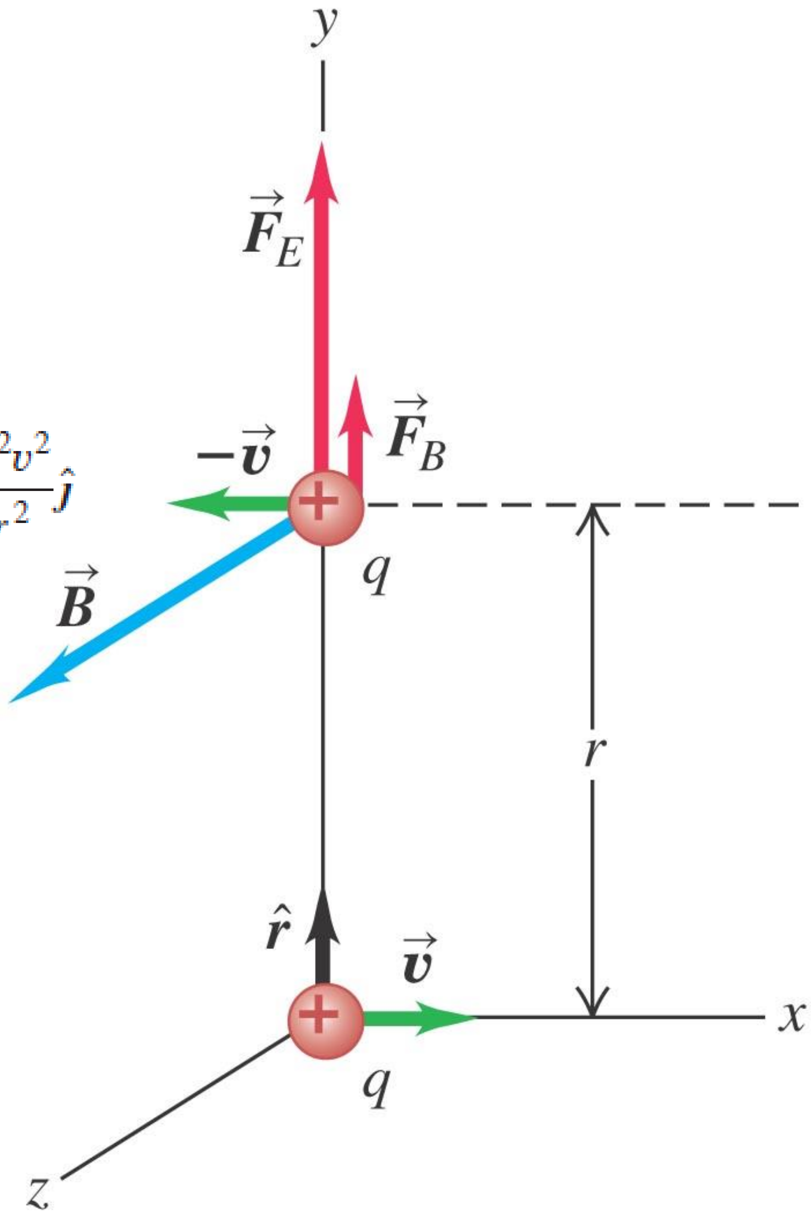
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(v\hat{\imath})\times\hat{\jmath}}{r^2} = \frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k}$$

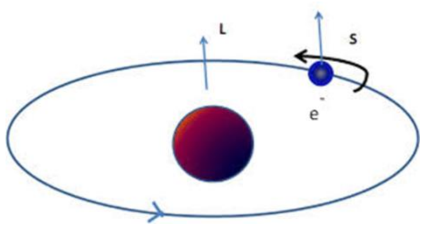
$$\vec{F}_B = q(-\vec{v})\times\vec{B} = q(-v\hat{\imath})\times\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}\hat{\jmath}$$

$$\frac{F_B}{F_E} = \frac{\mu_0 q^2v^2/4\pi r^2}{q^2/4\pi\epsilon_0 r^2} = \frac{\mu_0 v^2}{1/\epsilon_0} = \epsilon_0\mu_0 v^2$$

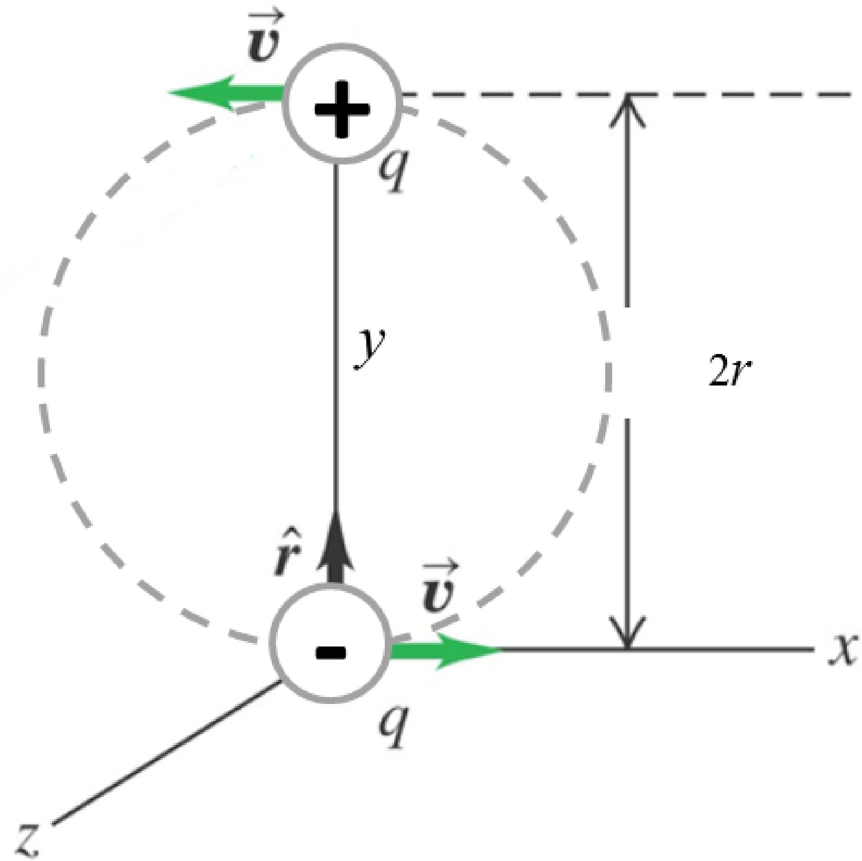
$$\epsilon_0\mu_0 = 1/c^2,$$

$$\frac{F_B}{F_E} = \frac{v^2}{c^2}$$





10^6 m



$\vec{v}$
+
q
y
$2r$
$\hat{r}$
$\vec{v}$
-
x
q
z

Magnetic field of a current element

- *law of Biot and Savart.*

$$dQ = nqA\,dl$$

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|v_{\mathrm{d}}\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{n|q|v_{\mathrm{d}}A\,dl\sin\phi}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2}$$

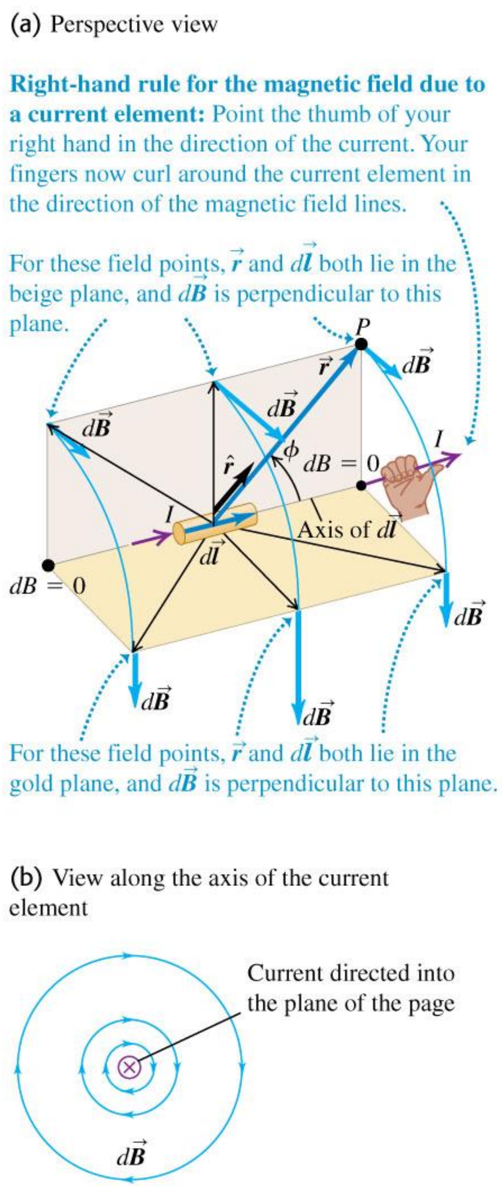
$n|q|v_{\mathrm{d}}A$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{(magnetic field of a current element)}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

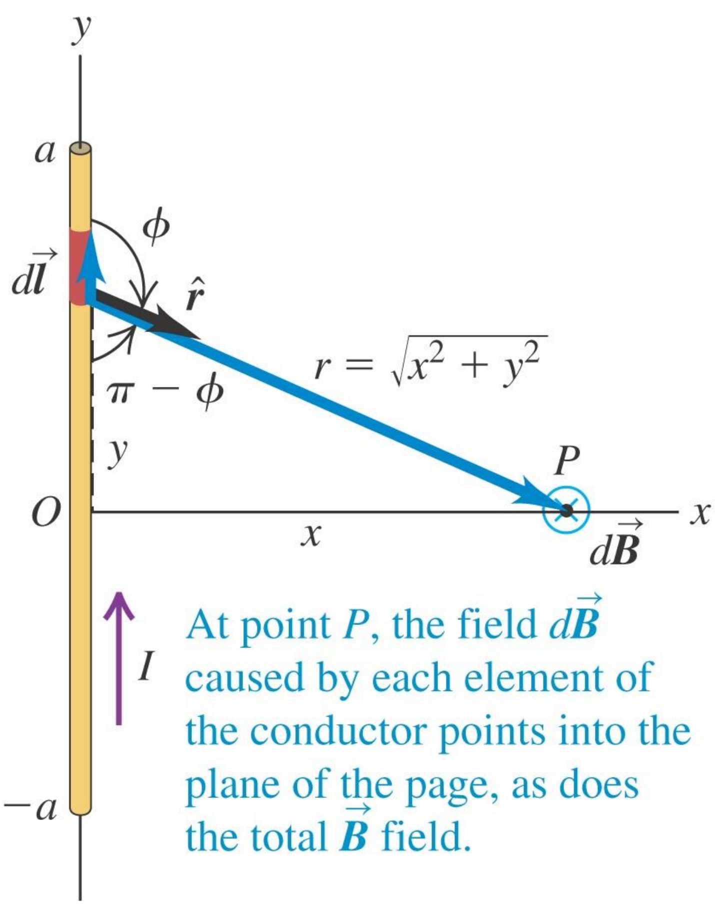
For surface and volume currents the Biot-Savart law becomes

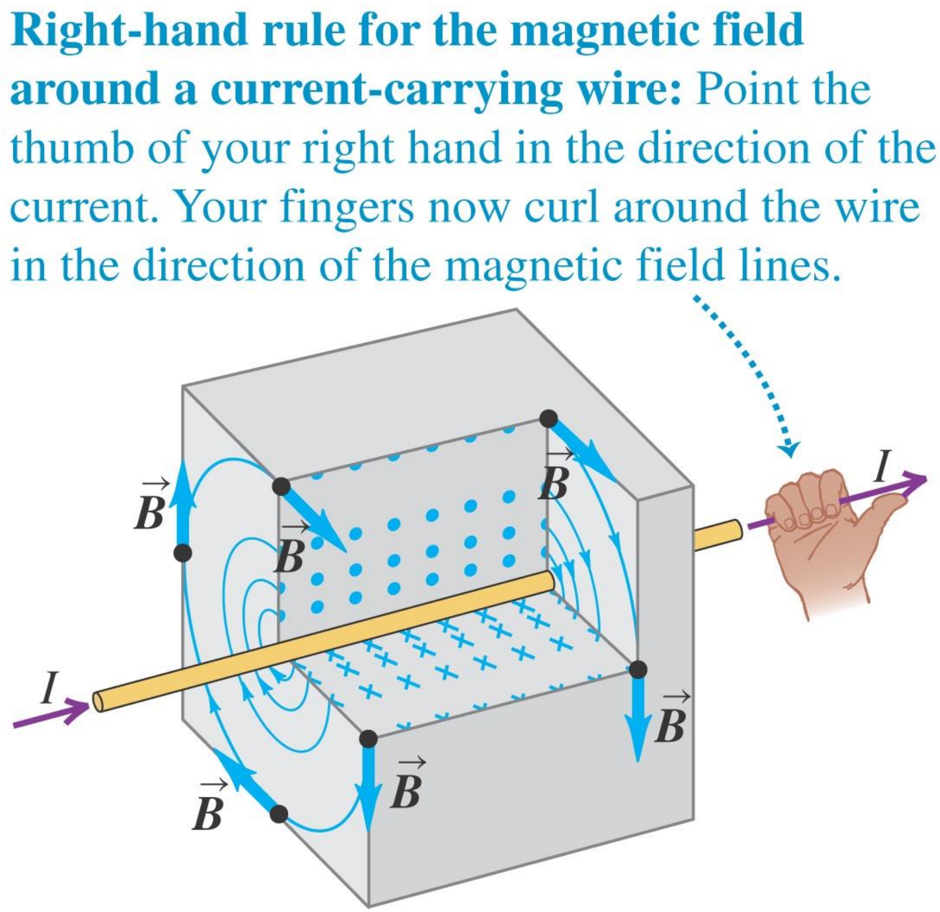
$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{K}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,da' \quad \text{and} \quad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')\times\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,d\tau',$$



Magnetic field of a straight current-carrying conductor

- If we apply the law of Biot and Savart to a long straight conductor, the result is $B = \mu_0 I/2\pi x$. See Figure 28.5 below left. Figure 28.6 below right shows the right-hand rule for the direction of the force.





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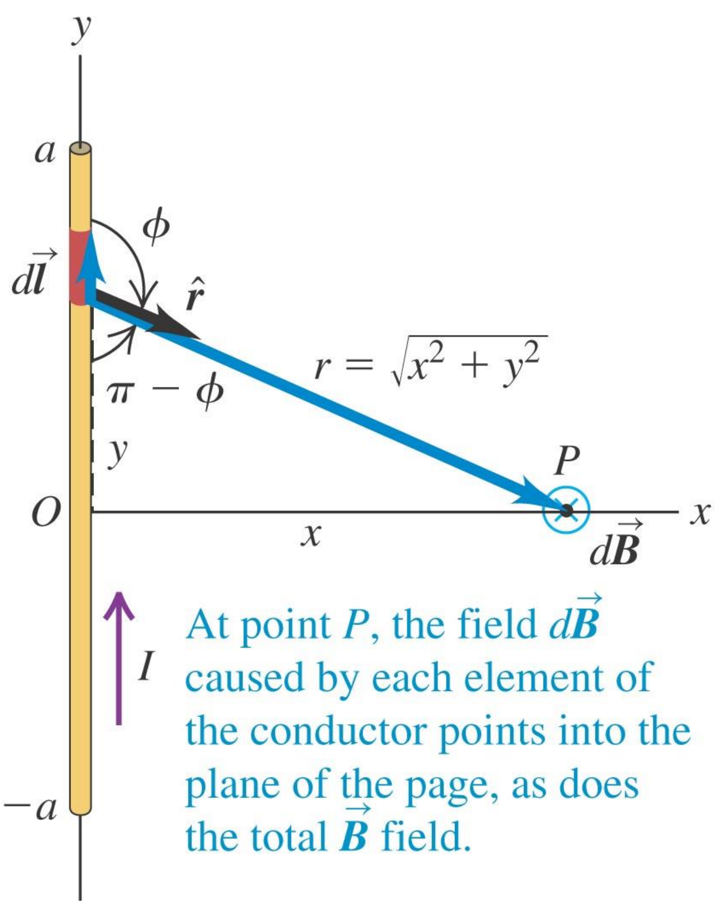


$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2} \qquad \text{(magnetic field of a current element)}$$

$$\sin\phi = \sin(\pi - \phi) = x/\sqrt{x^2 + y^2}.$$

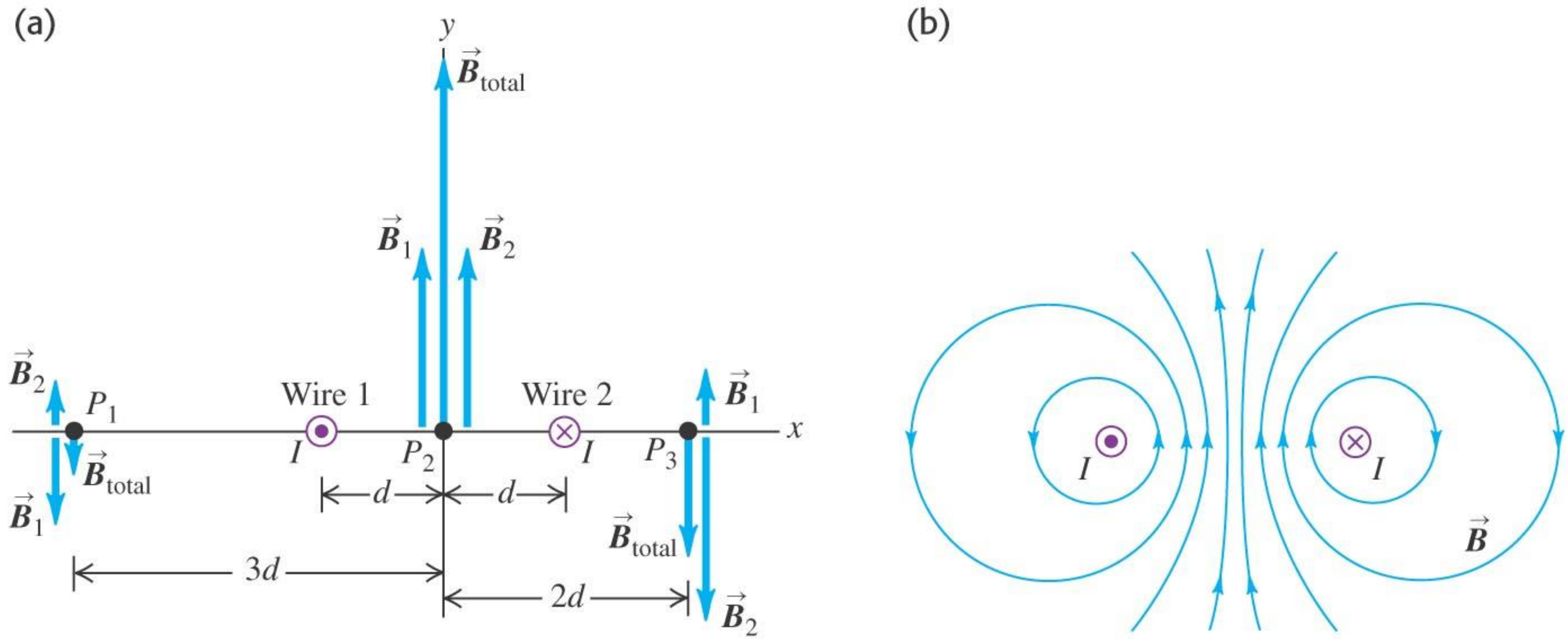
$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{x\,dy}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2 + a^2}}$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{(near a long, straight, current-carrying conductor)}$$

Magnetic fields of long wires

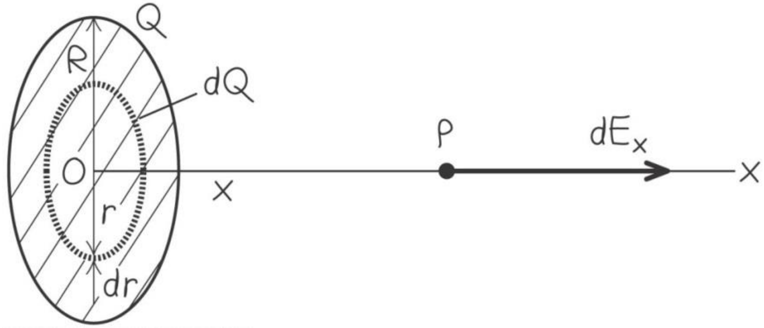
- Follow Example 28.3 for one wire.
- Follow Example 28.4 for two wires. Use Figure 28.7 below.

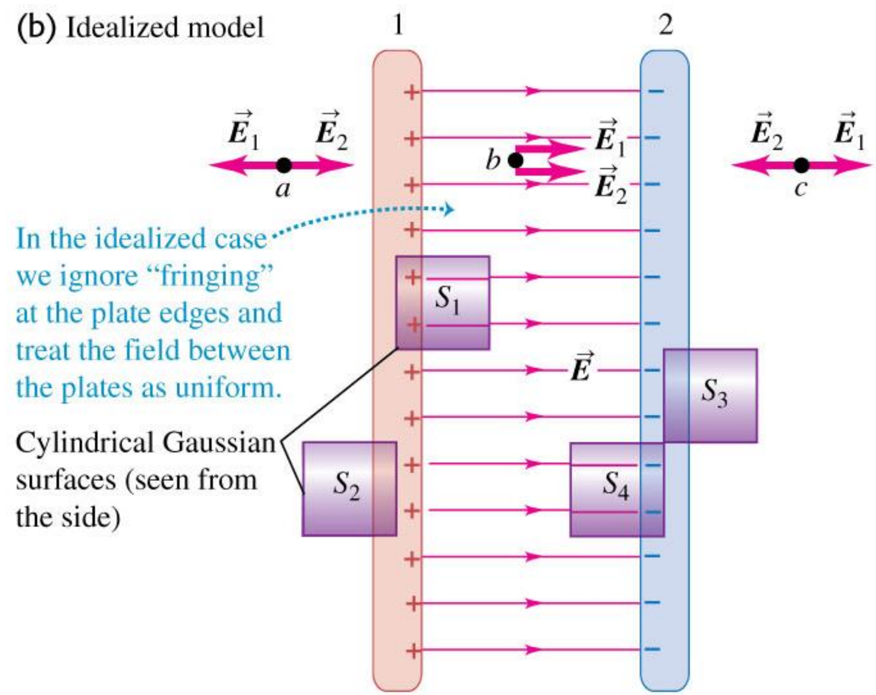


$$B = \frac{\mu_0 I}{2\pi r}$$

Field between two parallel conducting plates

- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.





$$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right] \qquad (21.11)$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad (21.12)$$

$$EA = \sigma A/\epsilon_0$$

$$E = \frac{\sigma}{\epsilon_0} \text{ (field between oppositely charged conducting plates)}$$

Force between parallel conductors

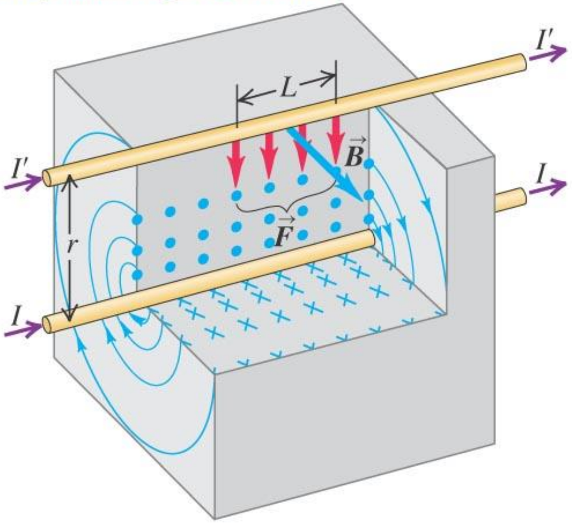
The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

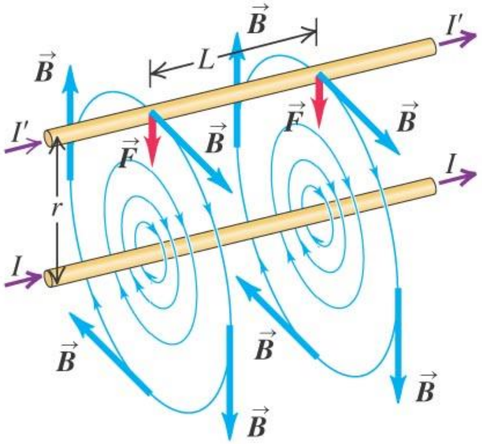
If the wires had currents in *opposite* directions, they would *repel* each other.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \qquad \text{(two long, parallel, current-carrying conductors)}$$





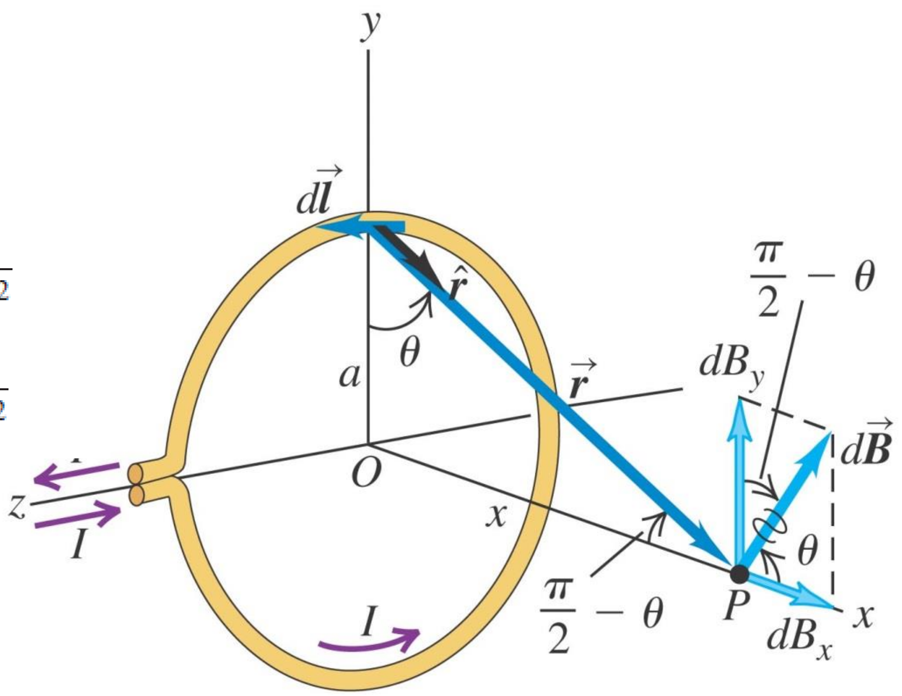
Magnetic field of a circular current loop

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)}$$

The components of the vector $d\vec{B}$ are

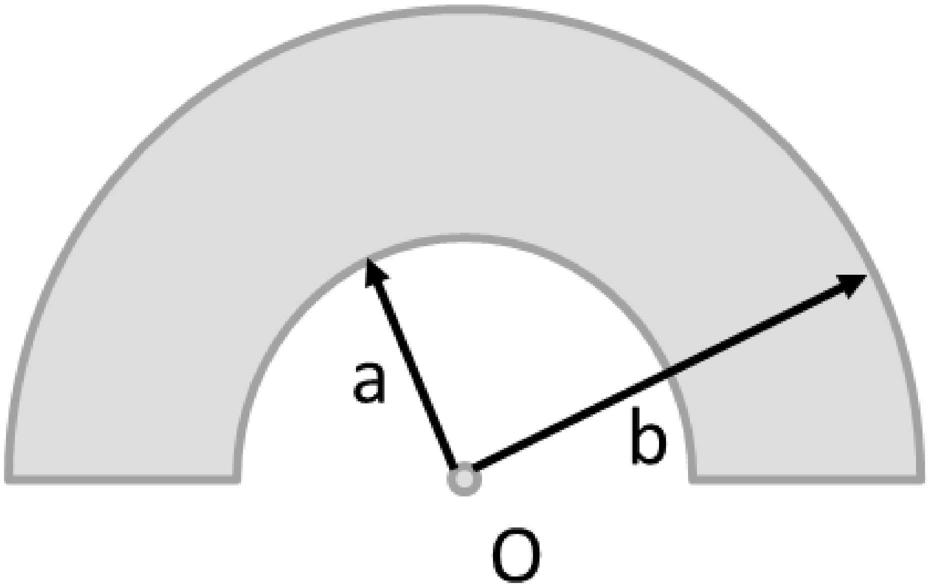
$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{a}{(x^2 + a^2)^{1/2}}$$

$$dB_y = dB\sin\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}}$$



$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a\,dl}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi(x^2 + a^2)^{3/2}} \int dl$$

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \qquad \text{(on the axis of a circular loop)}$$


a
b
O

Magnetic field of a coil

- Figure 28.13 (top) shows the direction of the field using the right-hand rule.
- Figure 28.14 (below) shows a graph of the field along the *x-a*xis.
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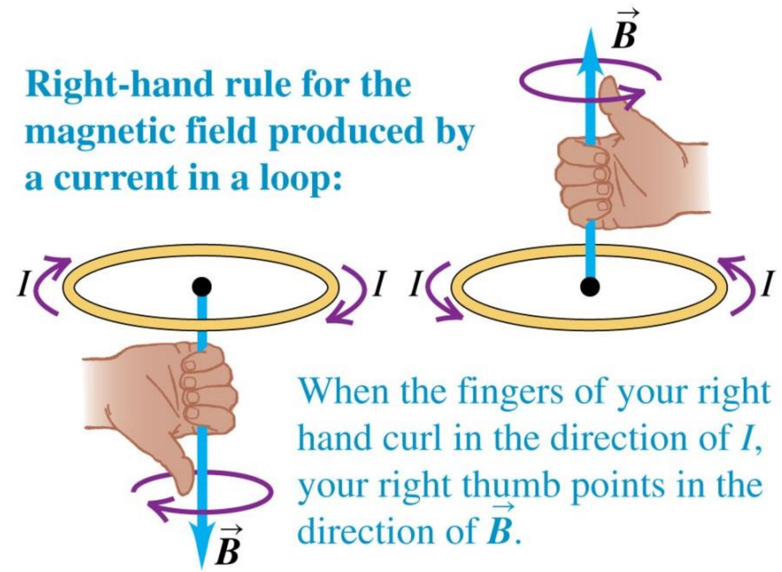
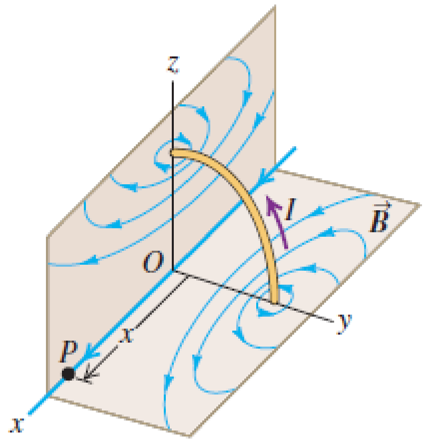
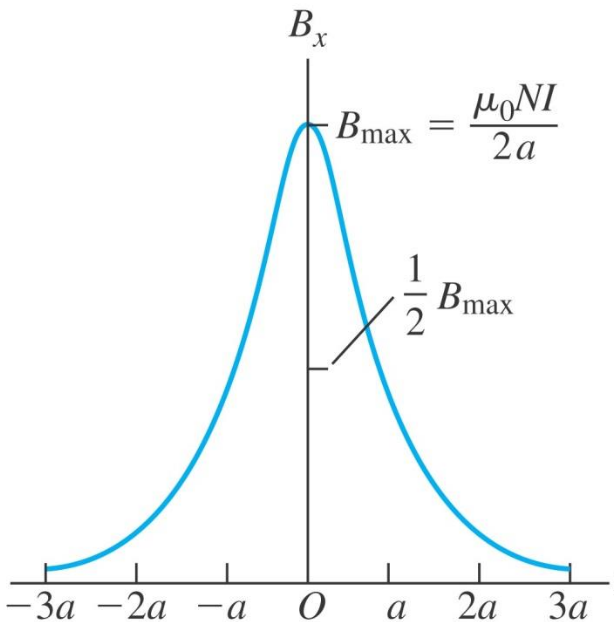
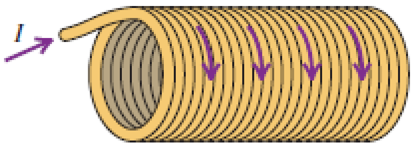




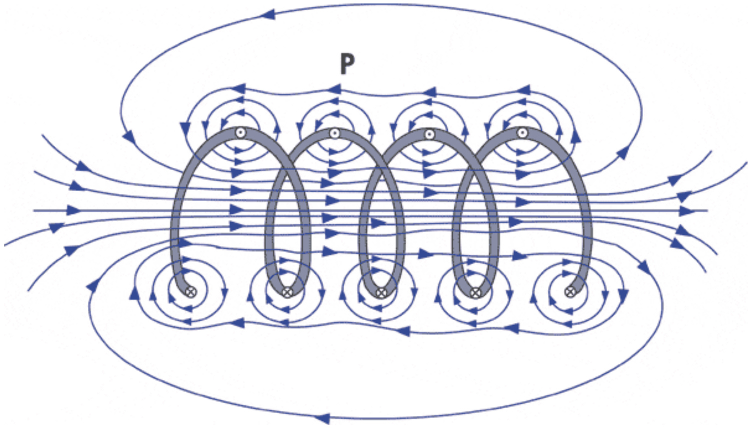


Magnetic field of a coil

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} \quad \text{(on the axis of } N \text{ circular loops)}$$

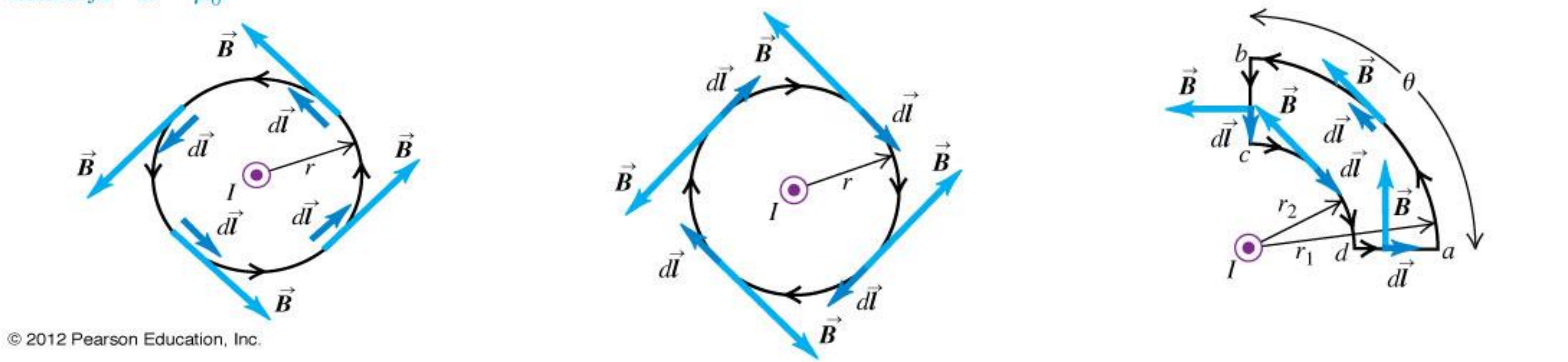


$$B_x = \frac{\mu_0 N I}{2a} \quad \text{(at the center of } N \text{ circular loops)}$$



Ampere's law (special case)

- Follow the text discussion of Ampere's law for a circular path around a long straight conductor, using Figure 28.16 below.



(a) Integration path is a circle centered on the conductor; integration goes around the circle counterclockwise. Result: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

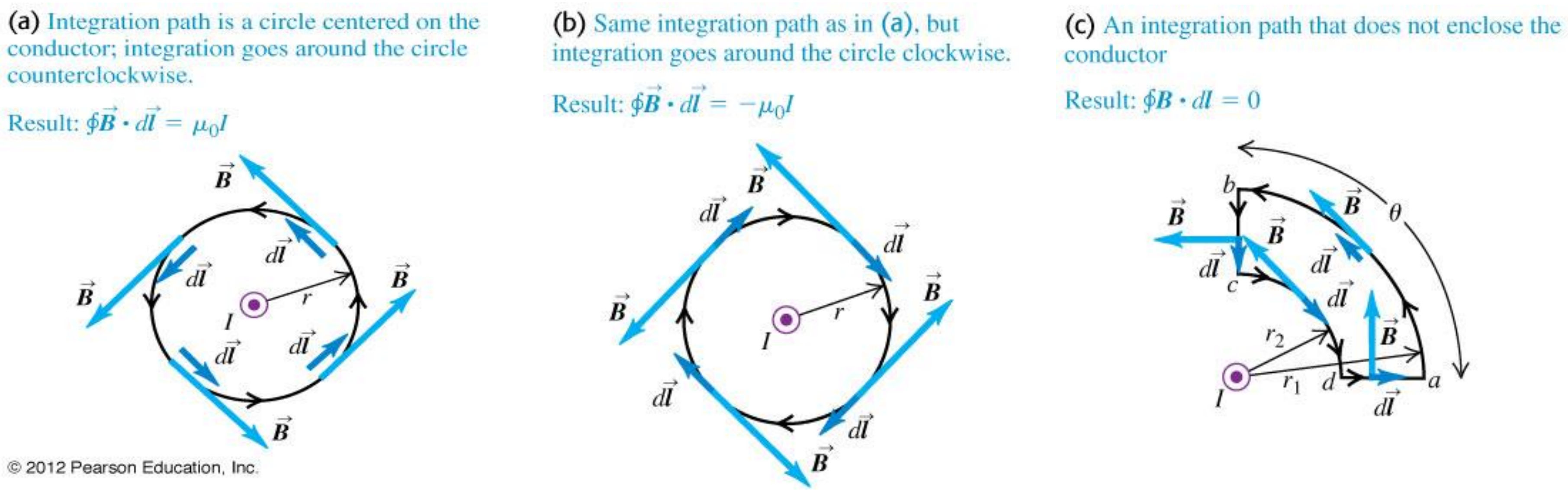
(b) Same integration path as in (a), but integration goes around the circle clockwise. Result: $\oint \vec{B} \cdot d\vec{l} = -\mu_0 I$

(c) An integration path that does not enclose the conductor. Result: $\oint \vec{B} \cdot d\vec{l} = 0$

$$\oint \vec{B} \cdot d\vec{l} = \oint B_\parallel \, dl = B_1 \int_a^b dl + (0) \int_b^c dl + (-B_2) \int_c^d dl + (0) \int_d^a dl$$
$$= \frac{\mu_0 I}{2\pi r_1}(r_1 \theta) + 0 - \frac{\mu_0 I}{2\pi r_2}(r_2 \theta) + 0 = 0$$

Ampere's law (special case)

- Follow the text discussion of Ampere's law for a circular path around a long straight conductor, using Figure 28.16 below.



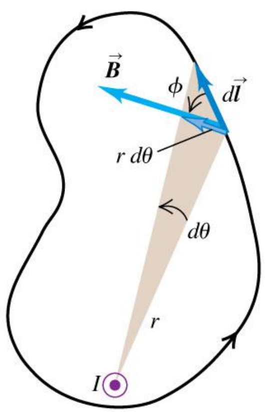
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$\oint \vec{B} \cdot d\vec{l} = \oint B_{\parallel}\, dl = B \oint dl = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

Ampere's law (general statement)

- Follow the text discussion of the general statement of Ampere's law, using Figures 28.17 and 28.18 below.

(a)

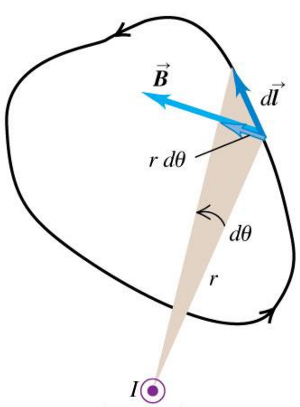


$$\vec{B}\cdot d\vec{l} = B\,dl\cos\phi$$

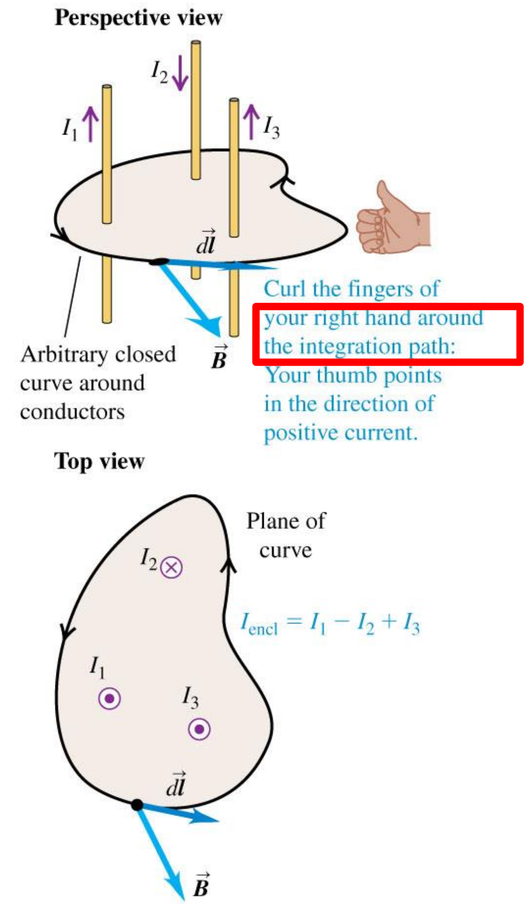
$$dl\cos\phi = r\,d\theta,$$

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi r}(r\,d\theta) = \frac{\mu_0 I}{2\pi}\oint d\theta$$

(b)



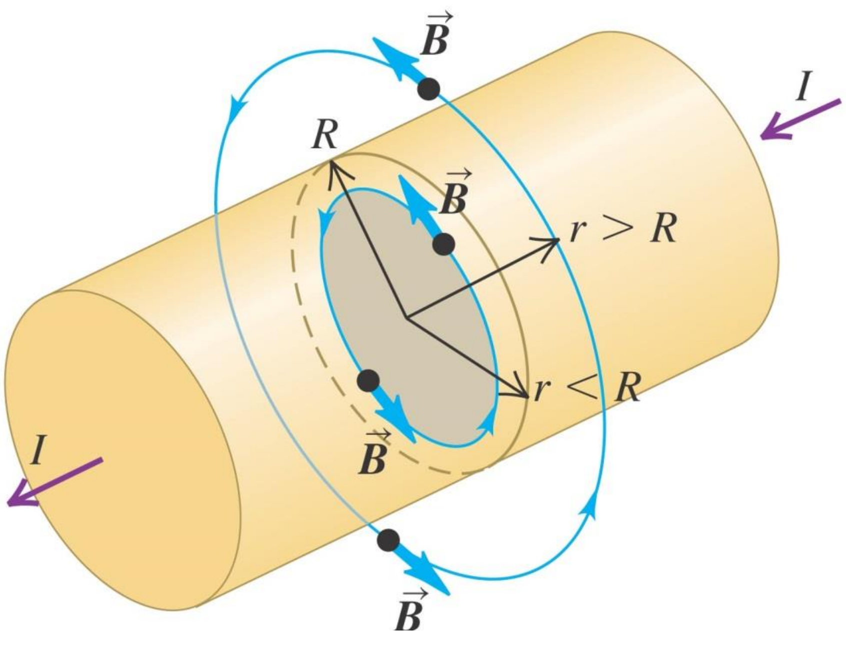
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \qquad \text{(Ampere's law)}$$

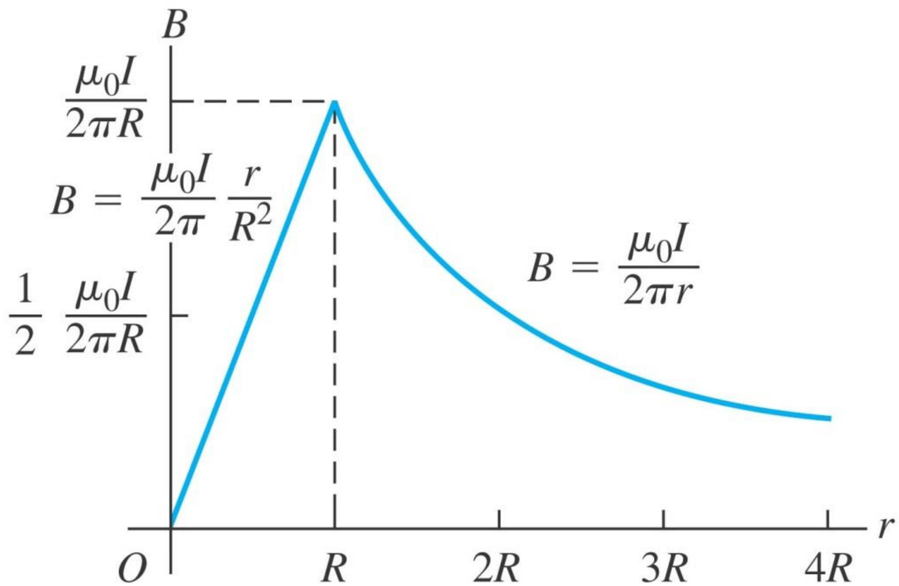


Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

Magnetic fields of long conductors

- Read Problem-Solving Strategy 28.2.
- Follow Example 28.7 for a long straight conductor.
- Follow Example 28.8 for a long cylinder, using Figures 28.20 and 28.21 below.





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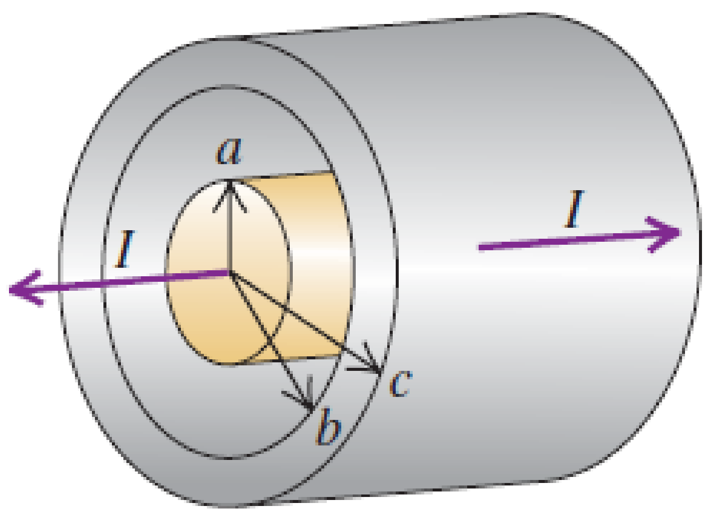

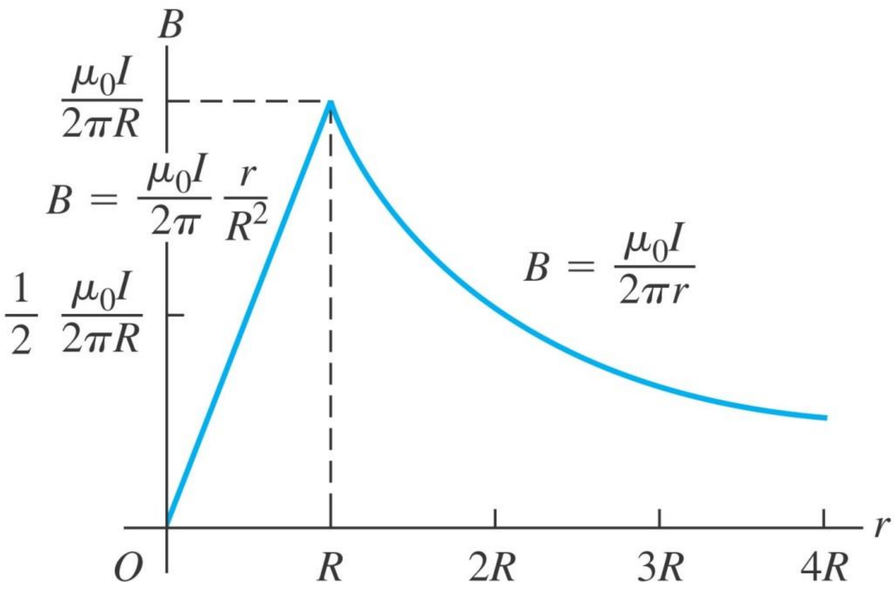

Magnetic field of a coil

- Figure 28.13 (top) shows the direction of the field using the right-hand rule.
- Figure 28.14 (below) shows a graph of the field along the *x*-axis.
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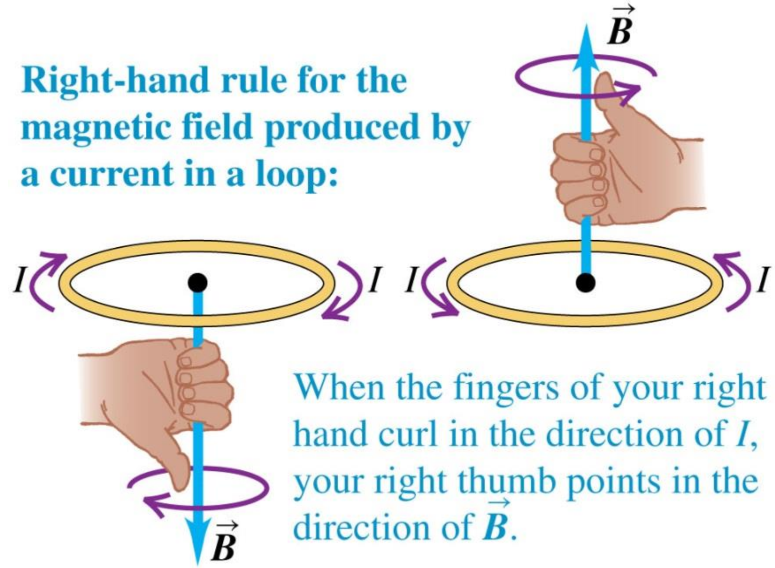
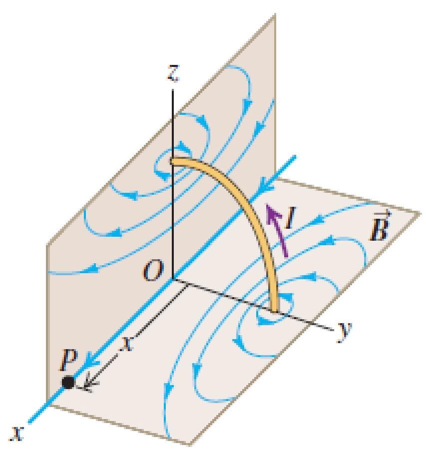
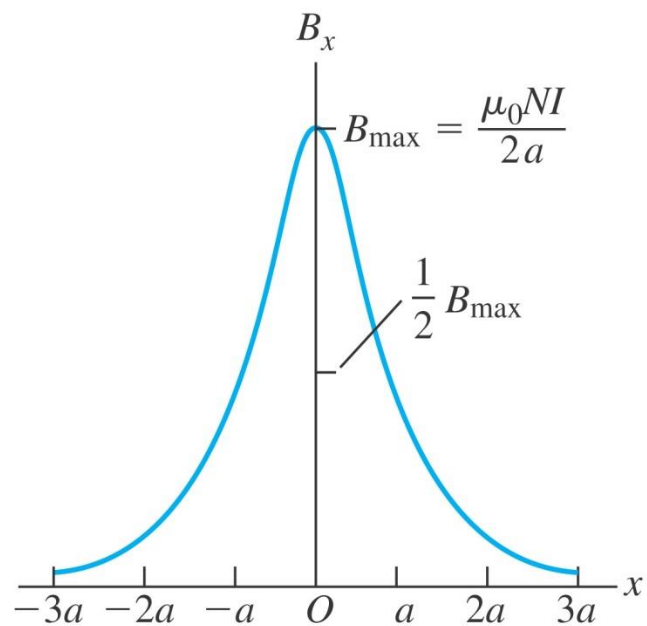
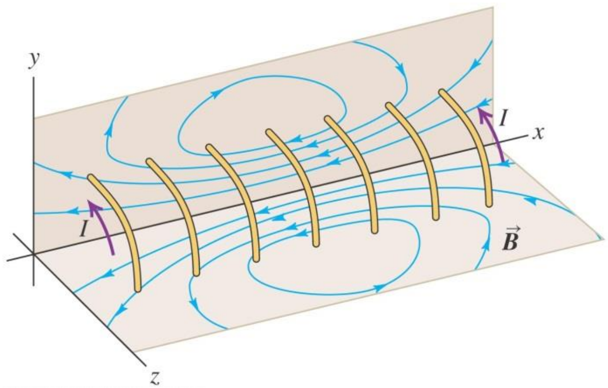


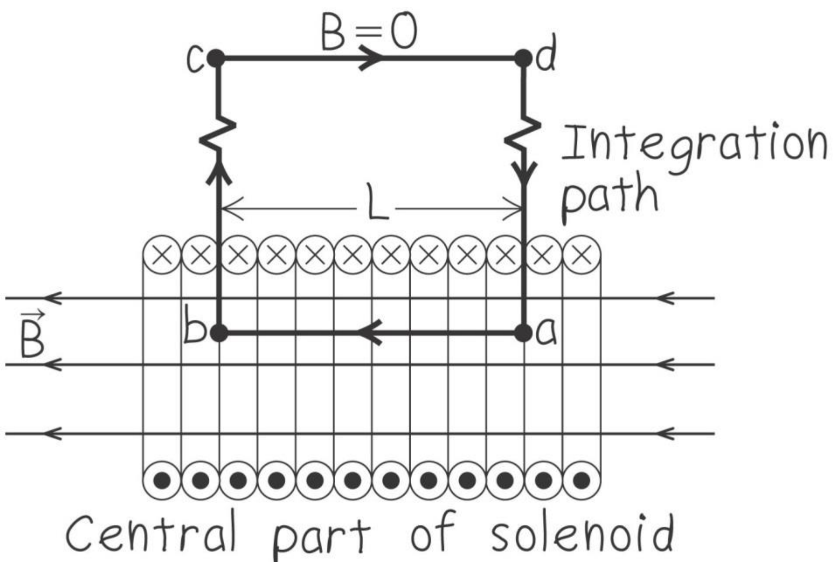


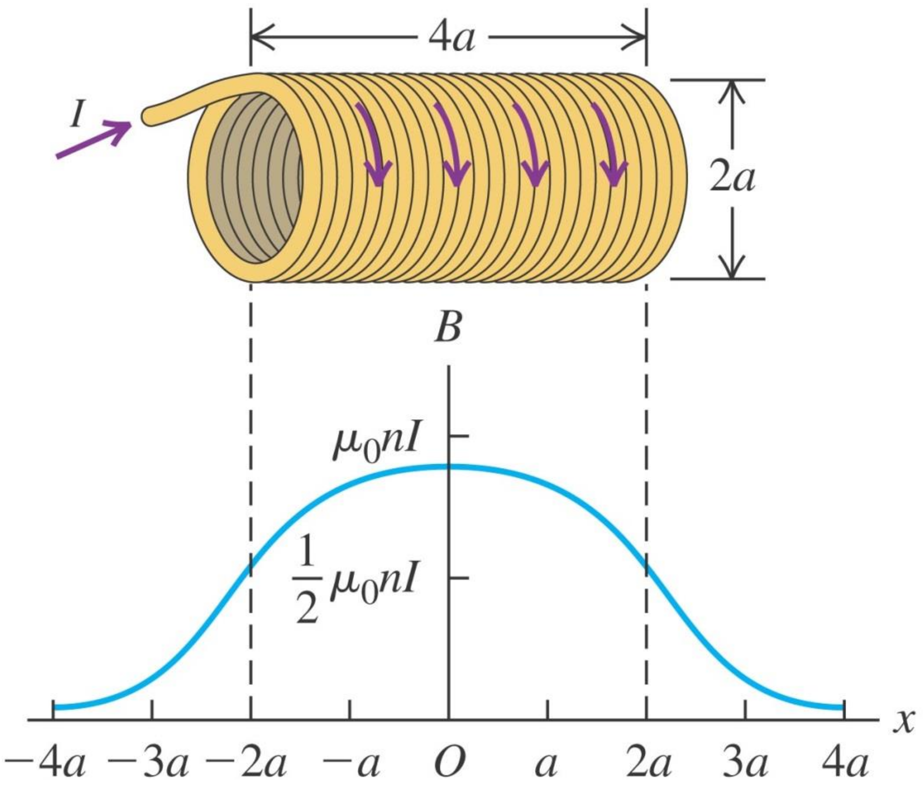


Field of a solenoid

- A *solenoid* consists of a helical winding of wire on a cylinder.
- Follow Example 28.9 using Figures 28.22–28.24 below.







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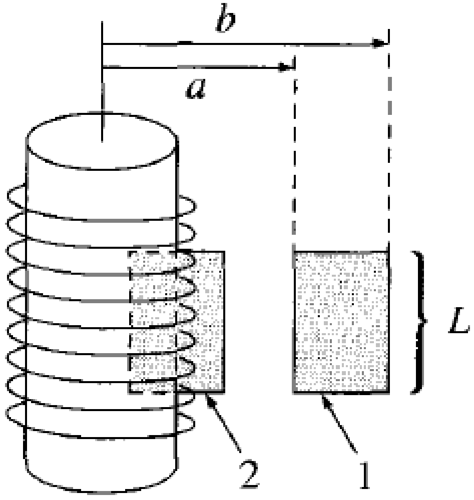


$$\oint \mathbf{B} \cdot d\mathbf{l} = [B(a) - B(b)]L = \mu_0 I_{enc} = 0,$$

$$B(a) = B(b).$$

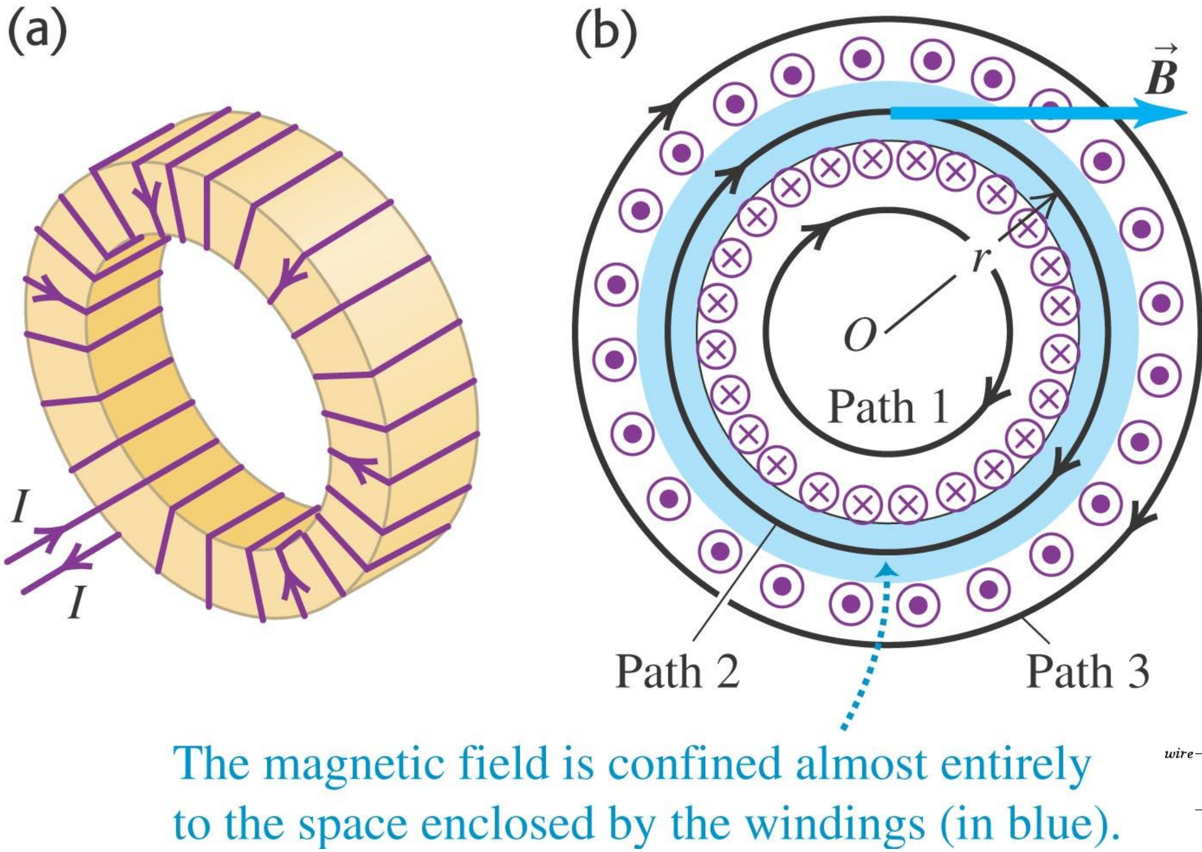
$$BL = \mu_0 nLI,$$

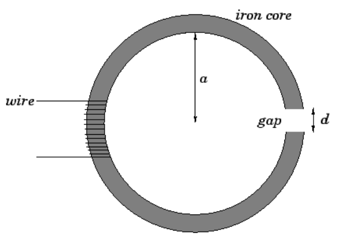
$$B = \mu_0 nI$$

Evidently the *field outside does not depend on the distance from the axis*. But we know that it goes to *zero* for large s. It must therefore be zero *everywhere*! (This astonishing result can also

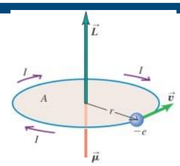
Field of a toroidal solenoid

- A *toroidal solenoid* is a doughnut-shaped solenoid.
- Follow Example 28.10 using Figure 28.25 below.



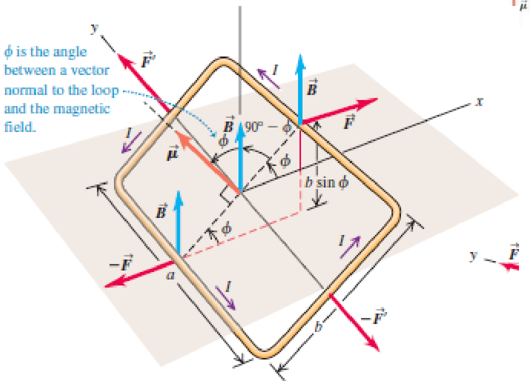


The Bohr magneton and paramagnetism



$$\mu = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi m}$$

e **Bohr magneton,** denoted b



$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

Table 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20°C$

Material	$\chi_m = K_m - 1\ (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

e **magnetization** of the material, denoted by $\vec{M}$:

$$\vec{M} = \frac{\vec{\mu}_{\text{total}}}{V}$$

, the total magnetic field $\vec{B}$ in the material is

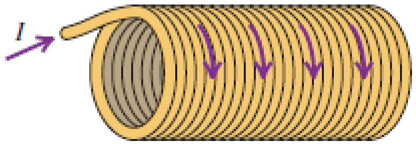
$$\vec{B} = \vec{B}_0 + \mu_0\vec{M}$$

as μ and is called the **permeability**

$$\mu = K_m\mu_0 \qquad (2$$

Magnetic field of a coil

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$$ (on the axis of N circular loops)

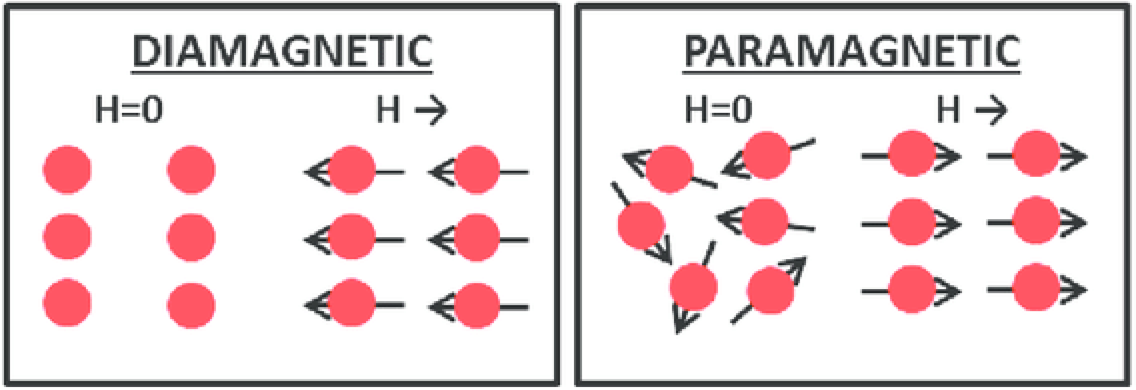


$$B_x = \frac{\mu_0 N I}{2a}$$ (at the center of N circular loops)

$$A = \pi a^2, \qquad \mu = N I \pi a^2.$$

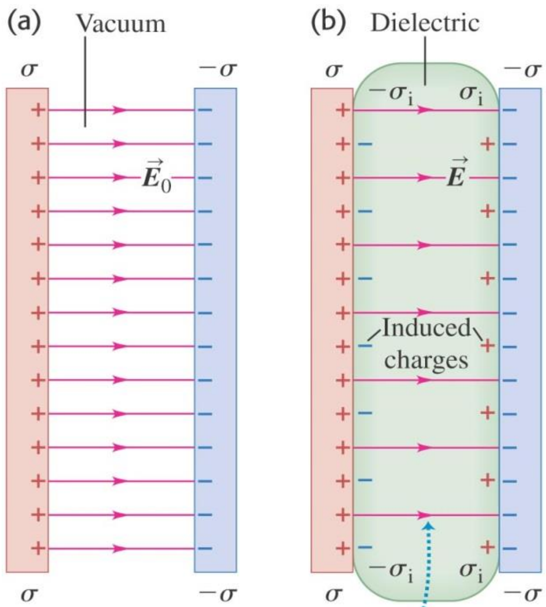
$$B_x = \frac{\mu_0 \mu}{2\pi(x^2 + a^2)^{3/2}}$$ (on the axis of any number of circular loops)

Diamagnetism and Paramagnetism



Molecular model of induced charge - II

- Figure 24.20 below shows *polarization* of the dielectric and how the induced charges reduce the magnitude of the resultant electric field.



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A}$$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

$$E = \frac{E_0}{K} \qquad \text{(when } Q \text{ is constant)}$$

$$E_0 = \frac{\sigma}{\epsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \qquad \text{(induced surface charge density)}$$

The product $K\epsilon_0$ is called the **permittivity** of the dielectric,

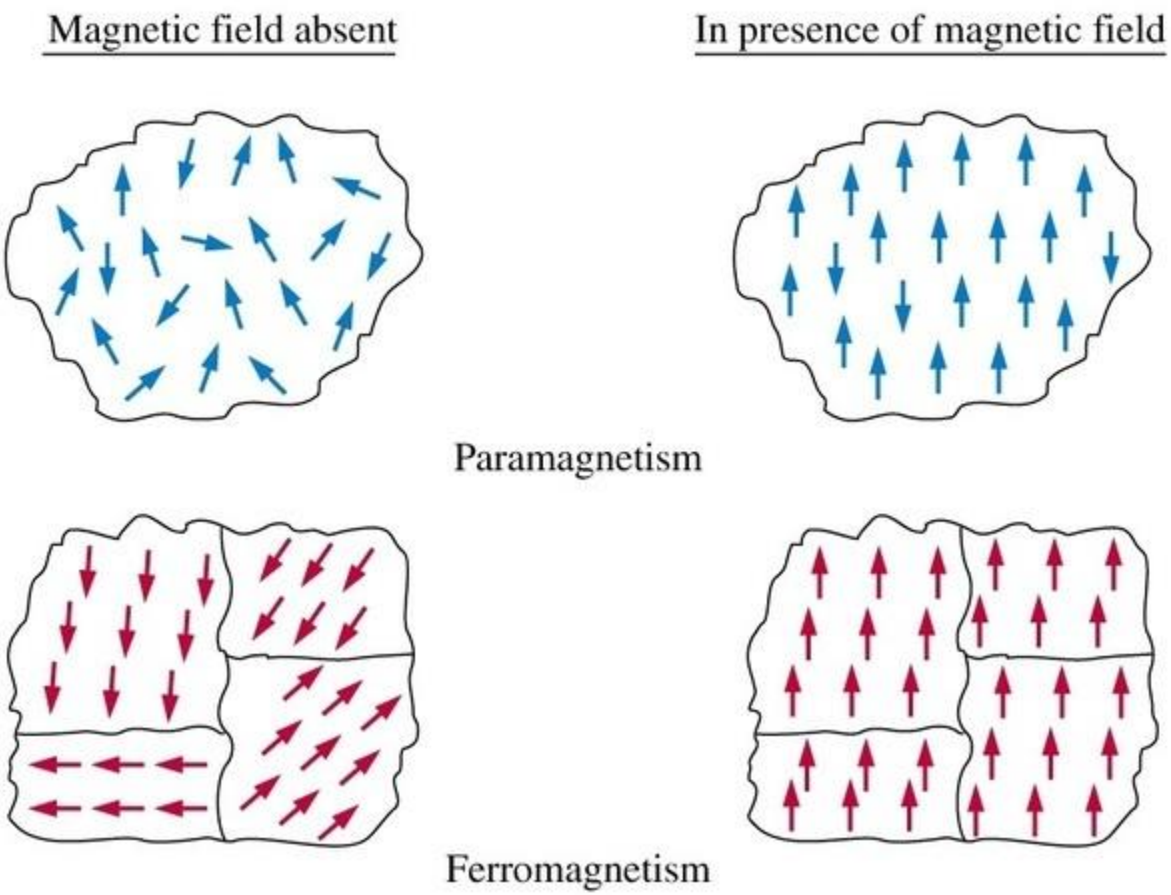
$$\epsilon = K\epsilon_0 \qquad \text{(definition of permittivity)}$$

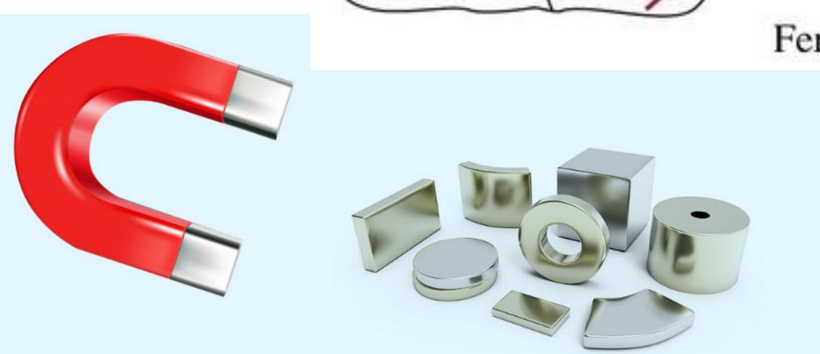
$$\boxed{E = \frac{\sigma}{\epsilon}}$$

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \qquad \text{(parallel-plate capacitor, dielectric between plates)} \qquad (24.19)$$

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \qquad \text{(electric energy density in a dielectric)} \qquad (24.20)$$

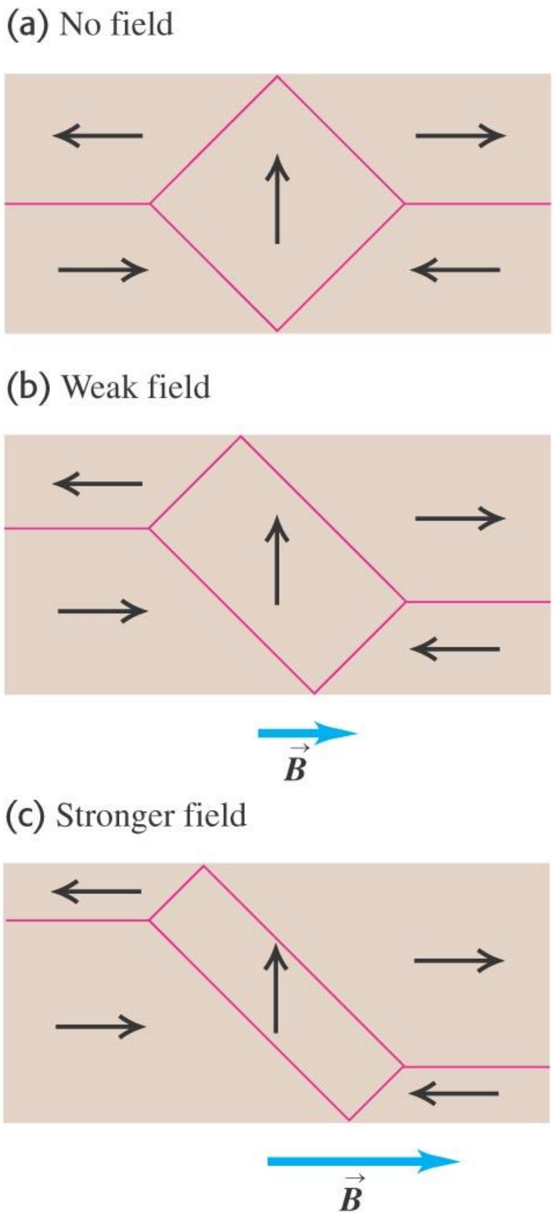
ferromagnetism

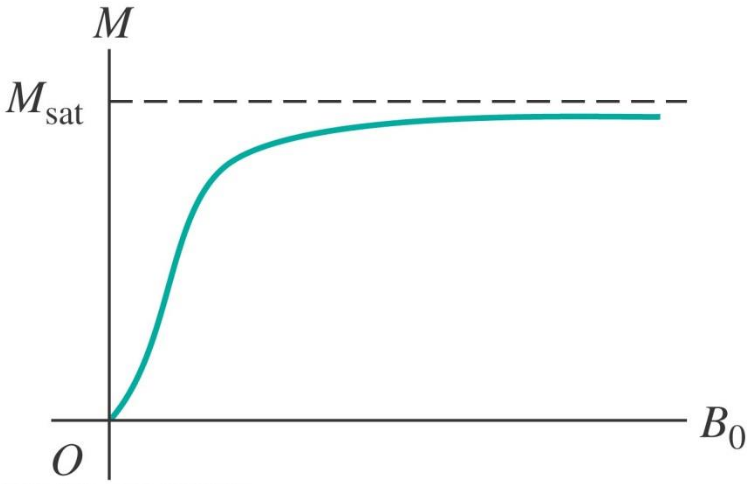




Diamagnetism and ferromagnetism

- Figure 28.27 at the right shows how *magnetic domains* react to an applied magnetic field.
- Figure 28.28 below shows a magnetization curve for a ferromagnetic material.





Hysteresis

- Read the text discussion of *hysteresis* using Figure 28.29 below.
- Follow Example 28.12.