

# 7. Vehicle Platooning II

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- Background
  - Signalized & unsignalized intersections
  - Connected & autonomous vehicles
  - Vehicle-to-infrastructure connectivity
- Control in nominal setting
  - Centralized approach
  - Decentralized approach
  - Hierarchical control
  - **HW2**
- Control in face of disruptions
  - How to address latency
  - How to address packet loss
  - How to address malicious attacks

- Macroscopic decisions for platooning
- Link level: headway regulation
  - Hybrid fluid model
  - Analysis
  - Design
- Junction level: cooperative scheduling
  - Static approach
  - Dynamic approach
- Network level: cooperative routing
  - Problem formulation
  - Fundamental tradeoff

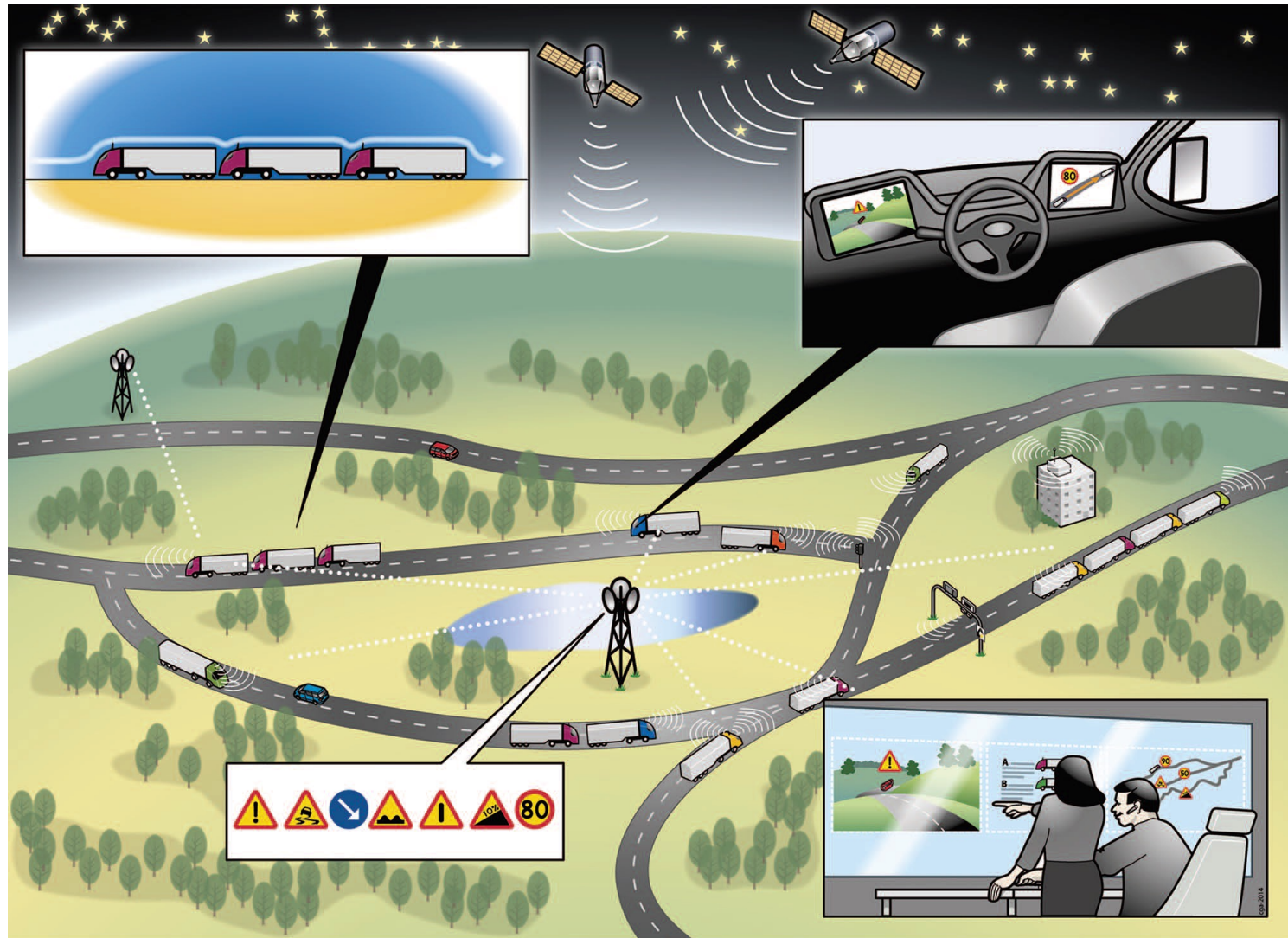
# Macroscopic decisions for platooning



# Platooning is more than CACC

- For two vehicles to form a platoon, they have to
  1. travel on fully or partially overlapping routes -> cooperative route planning
  2. arrive at the overlapping section at the same time -> cooperative scheduling
- Furthermore, platoons are not traveling on empty roads
  - Other vehicles on the road may influence the formation and movement of platoons.
  - Platoons may induce a non-trivial impact on neighboring traffic (“moving bottleneck”).
- All the above requires macroscopic (i.e. system-level) analysis and design!

# A hierarchical decision-making framework



# A hierarchical decision-making framework

- High level: global (network) planning
  - Schedule trips
  - Plan routes
  - Done by transportation authority or fleet manager
- Middle level: local (link & junction) coordination
  - Headway regulation
  - Coordinated time of arrival
  - Done by road-side unit (control tower)
- Low level: CACC (vehicle)
  - Joining & leaving maneuvers
  - Vehicle following
  - Done by onboard computer driver

# Link level: headway regulation

- Hybrid fluid model
- Analysis
- Design



# Main questions

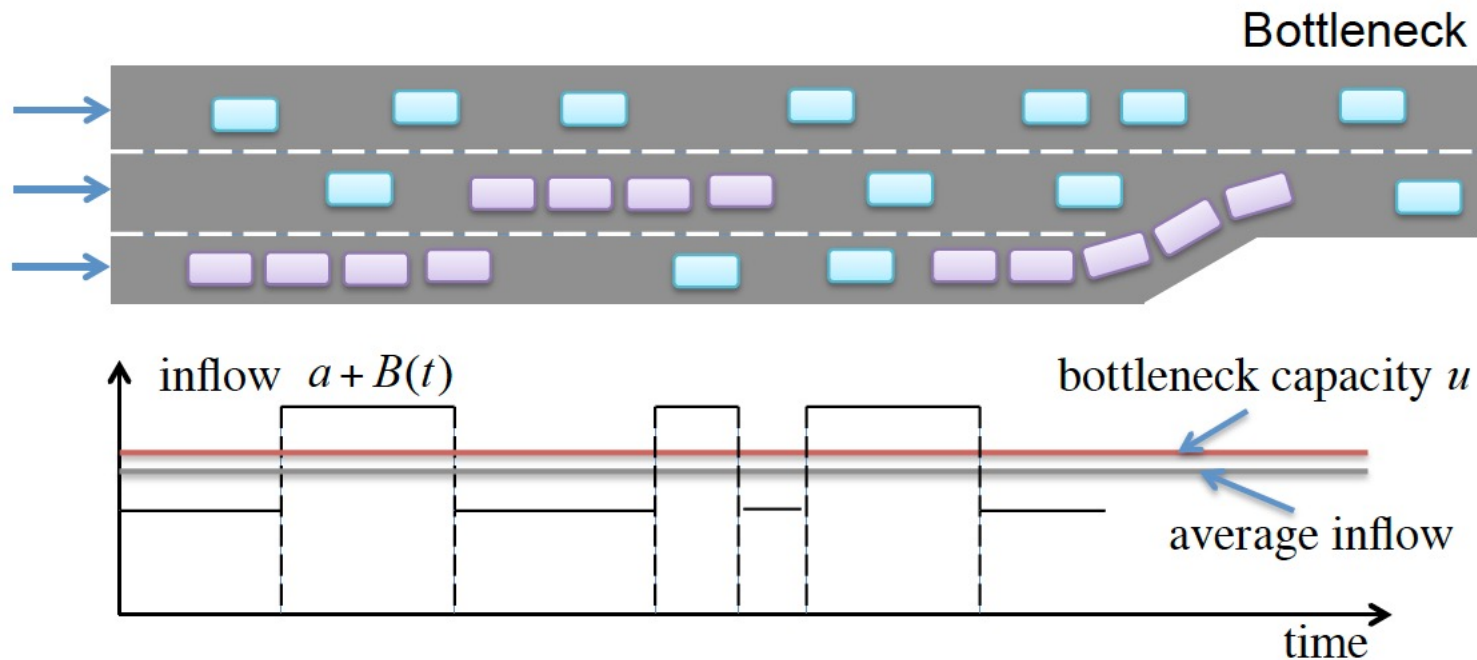
Main questions:

- How to model macroscopic interaction between CAV platoons and background traffic?
- How to design platooning operations to improve highway performance?



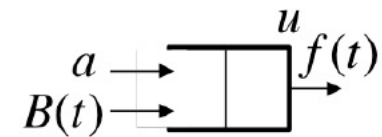
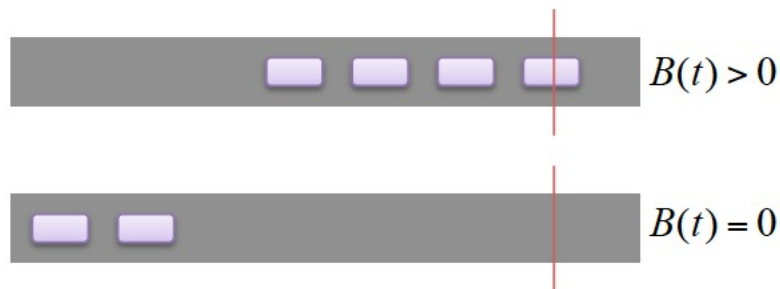
# Platoons at bottlenecks

- CAV Platoons are less flexible than non-CAVs
- Platoons may lead to local congestion and disrupt local traffic flow

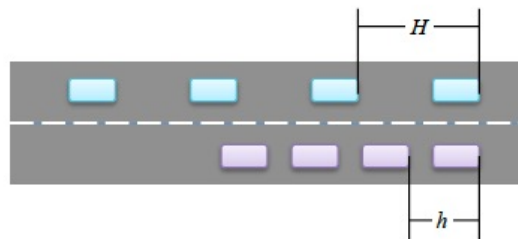


# Hybrid fluid model

- Two classes of traffic
  - Background traffic: non-CAVs, constant inflow rate  $a \in \mathbb{R}_{\geq 0}$
  - Platooned traffic: CAV platoons, switching inflow rate  $B(t) \in \{0, b\}$

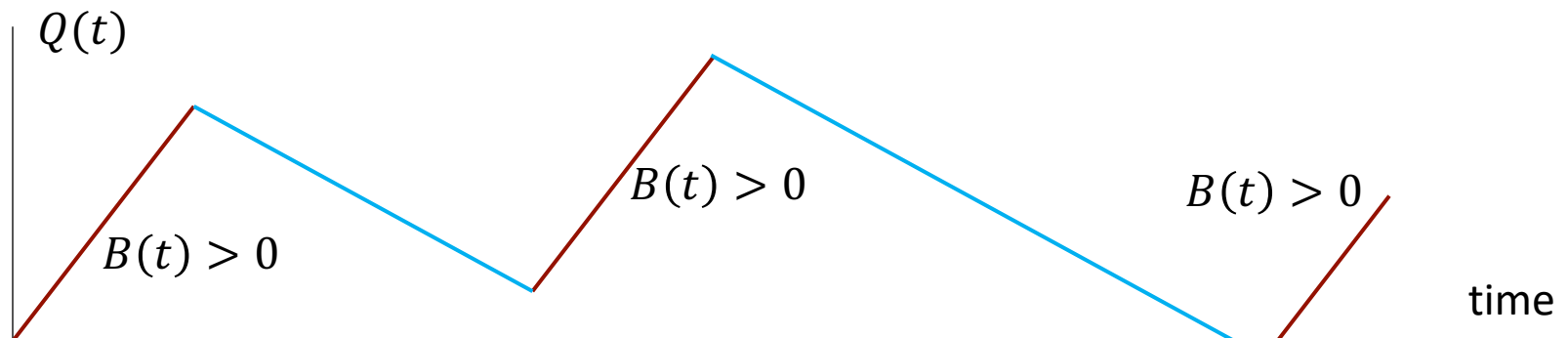


- Platoons are “condensed” traffic



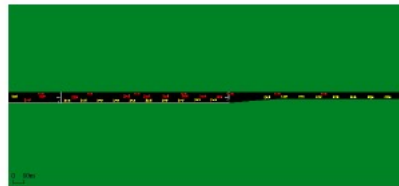
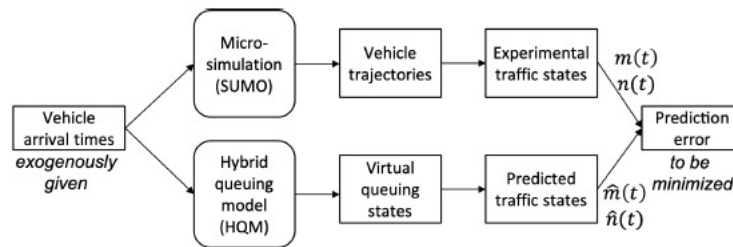
# Hybrid fluid model (HFM)

- Actual queue:  $Q_a(t) + Q_b(t)$ .
- Actual inflow:  $a + B(t)$ .
- A platoon of  $n$  CAVs  $\frac{h}{H}n$  non-CAVs
- Effective queue:  $Q(t) = Q_a(t) + \frac{h}{H}Q_b(t)$ .
- Effective inflow:  $R(t) = a + \frac{h}{H}B(t)$ .
- Queuing dynamics:  
 $\dot{Q}(t) = a + \frac{h}{H}B(t) - f(t)$  (a stochastic process)

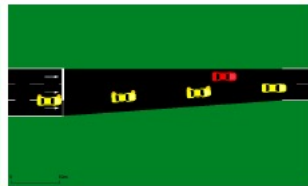


# Model validation

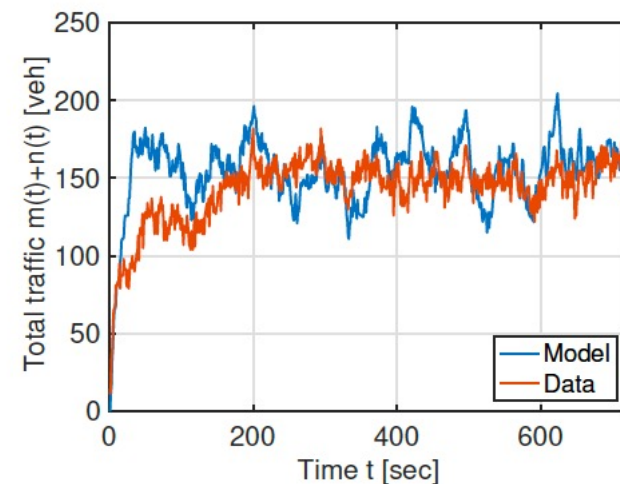
- **Very important! Wrong model + good math = nothing.**
- Essential task: show consistency between model and reality
  - Reality = intuition, actual data, simulation, experiments...
  - Consistency needs quantification: prediction error



(a) Part of highway section.



(b) Bottleneck.



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Mix

# Model validation

- Something philosophical

*All models are wrong, but some are useful*

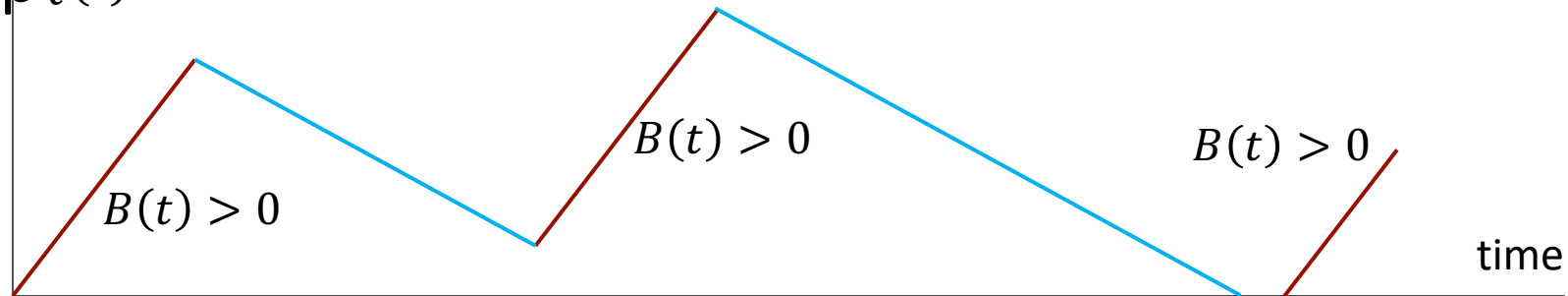
- Statistical or scientific models always fall short of the complexities of reality but can still be of use.
  - Vehicle dynamic model
  - Vehicle kinematic model
  - Intersection model
  - Hybrid queuing model
- Modeling is both science and art
  - We need models for analysis & design
  - But we cannot include every complexity
  - A tradeoff is needed



George E. P. Box

# Stability of HFM

- Intuitively, the HFM is stable if  $Q(t)$  does not “blow up”



- Since  $Q(t)$  is stochastic, we can only specify stability in a probabilistic sense:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \int_{s=0}^t \mathbb{E}[Q^p(s)] ds \leq Z$$

- $\mathbb{E}[Q^p(s)]$  is called the  $p$ th moment.

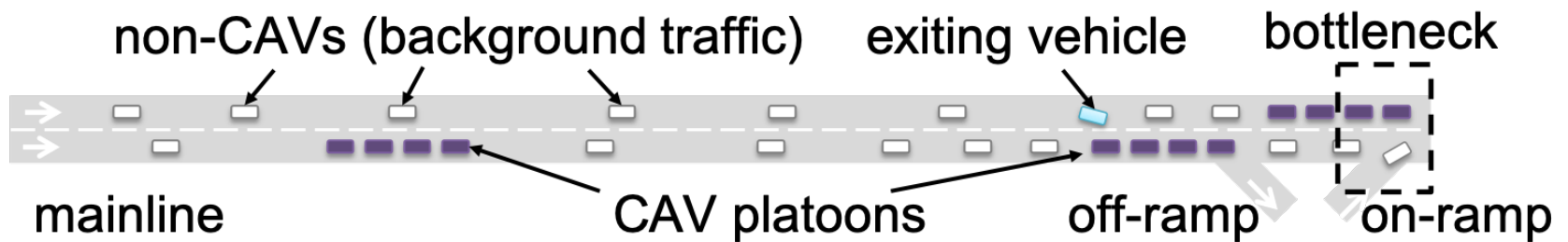
## Theorem

- $Q(t)$  is stable if and only if average inflow  $<$  capacity.
- $Q(t)$  converges to a unique steady-state distribution that can be analytically computed  $\rightarrow q$  &  $Var(q)$
- Proof idea: construct a switched Lyapunov function for the HFM, and apply a drift condition.
- Jin, L., Čičić, M., Amin, S. and Johansson, K.H., 2018, April. Modeling the impact of vehicle platooning on highway congestion: A fluid queuing approach. In *Proceedings of the 21st International Conference on Hybrid Systems: Computation and Control (HSCC, part of CPS Week)* (pp. 237-246). ACM.

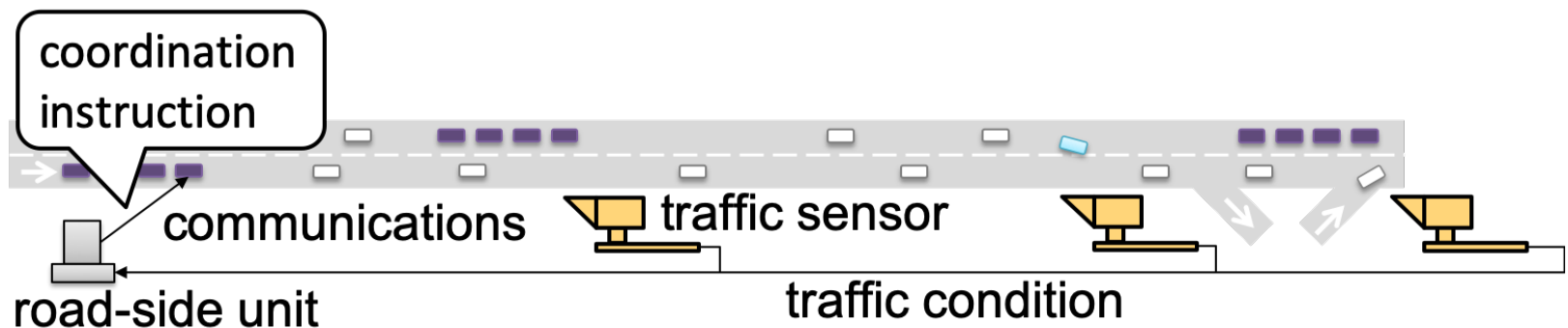


# Local congestion

- We should avoid clustering platoons!
- Uncontrolled:



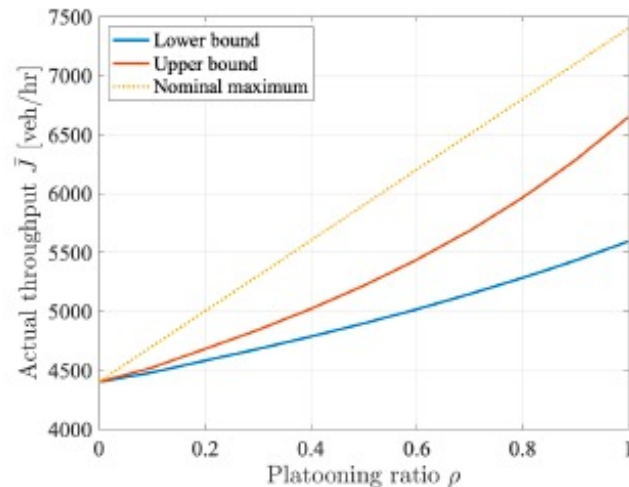
- Controlled:



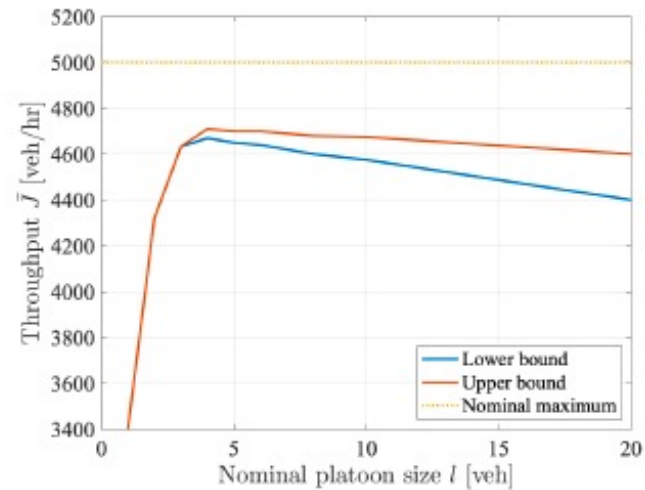
# Theoretical results\*

## Theorem

- The throughput of uncontrolled system can be analytically computed



(a) Throughput vs. fraction.



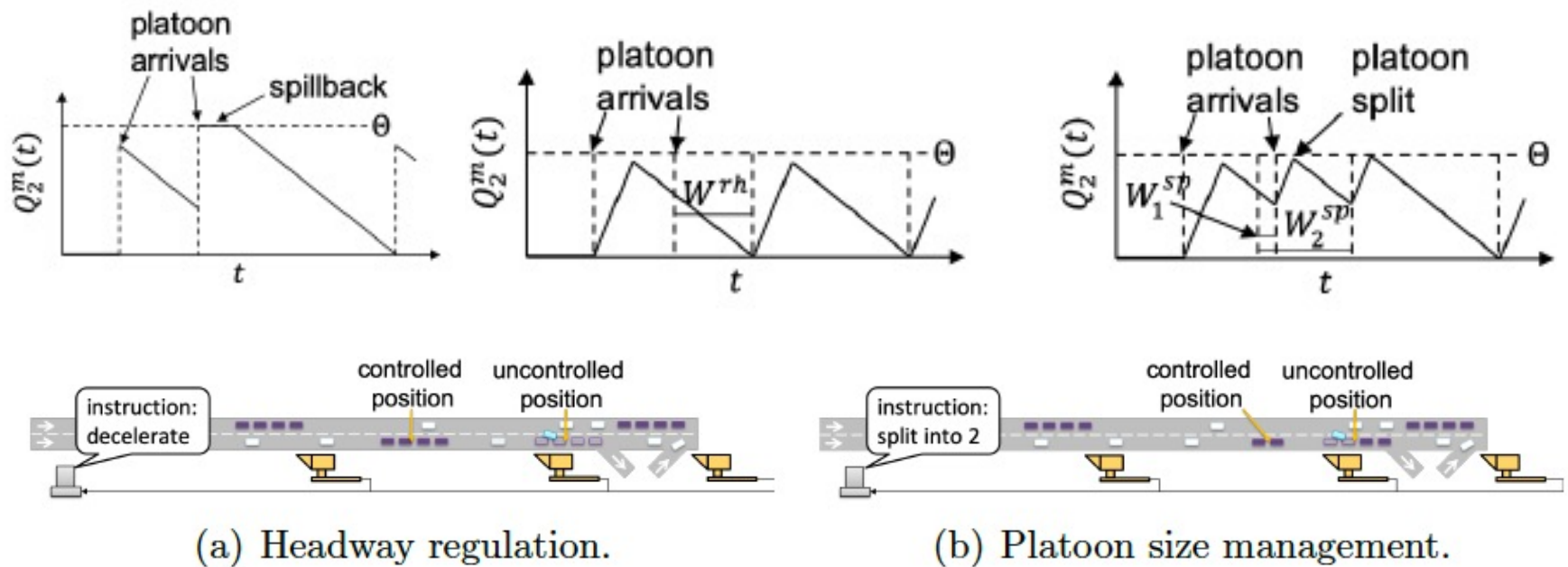
(b) Throughput vs. platoon size.

- Jin, L. \*, Čičić, M., Johansson, K.H. and Amin, S. Analysis and design of vehicle platooning operations on mixed-traffic highways. *IEEE Transactions on Automatic Control*, accepted.

# Theoretical results\*

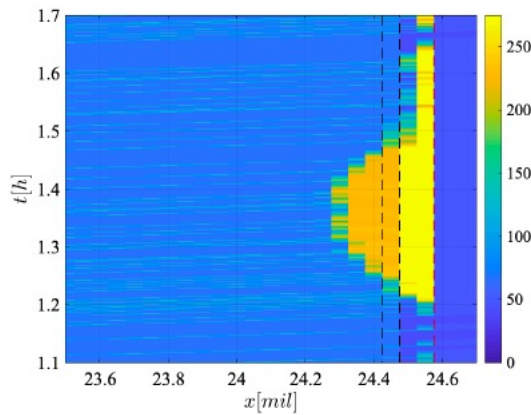
## Theorem

We can attain maximal throughput and minimal delay by either (i) imposing a minimal headway between platoons or (ii) decomposing long platoons.

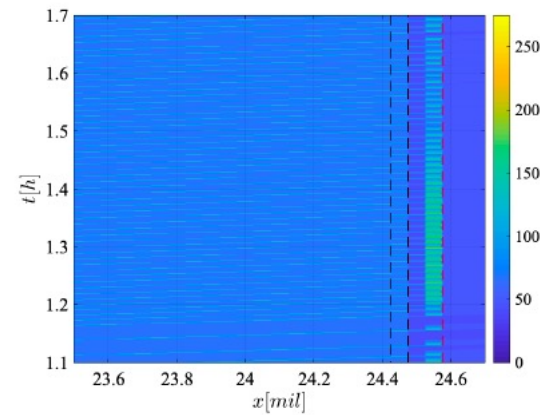


# Validation

- Macroscopic simulation



(a) Without coordination.



(b) With coordination.

- Microscopic simulation



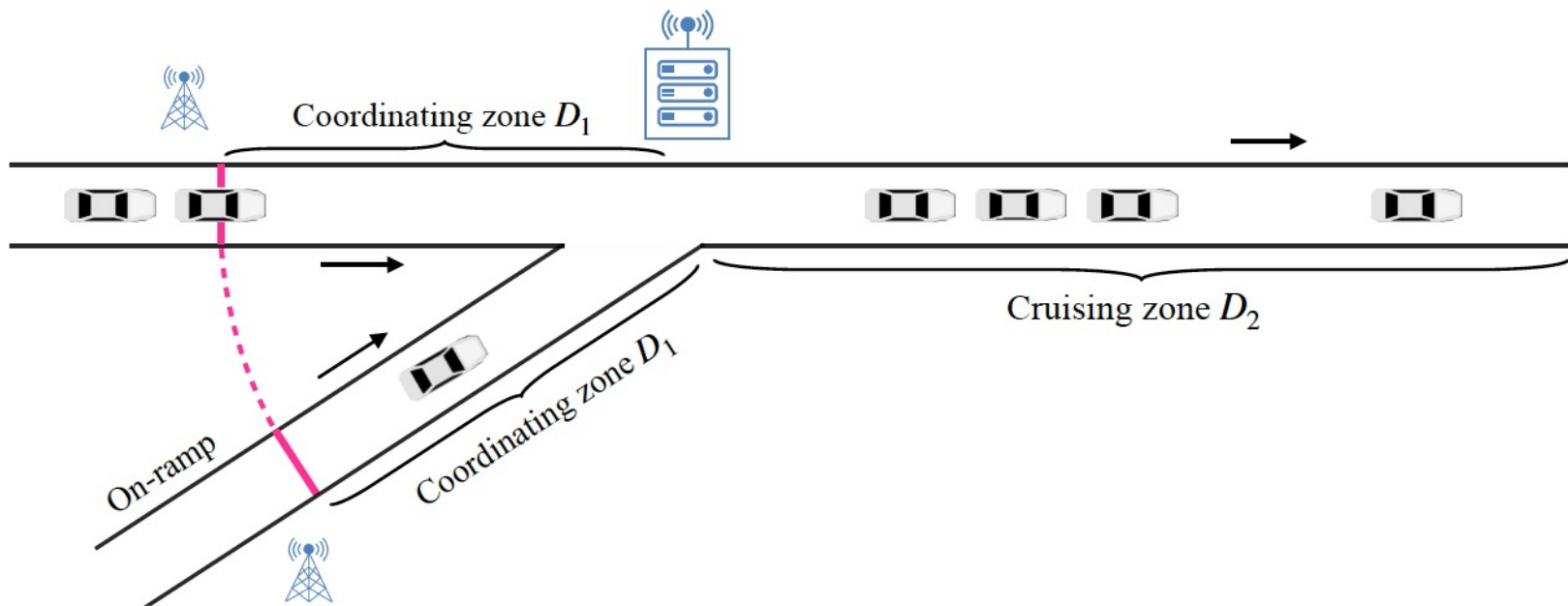
Strategy	VHT, CTM	Improve-ment (VHT)	Traverse time, SUMO [min]	Improve-ment [min]
No coordination	3471	0	36.26	0
Theoretically optimal	3466	5.3	33.27	2.99
Simulation optimal	3465	6.2	32.58	3.68

# Junction level: cooperative scheduling

- Cooperative scheduling
- Static approach
- Dynamic approach

# Cooperative scheduling

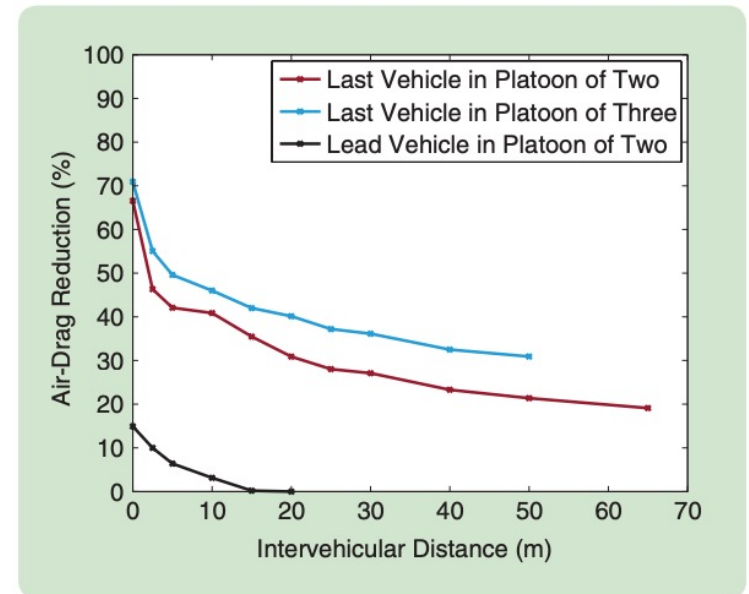
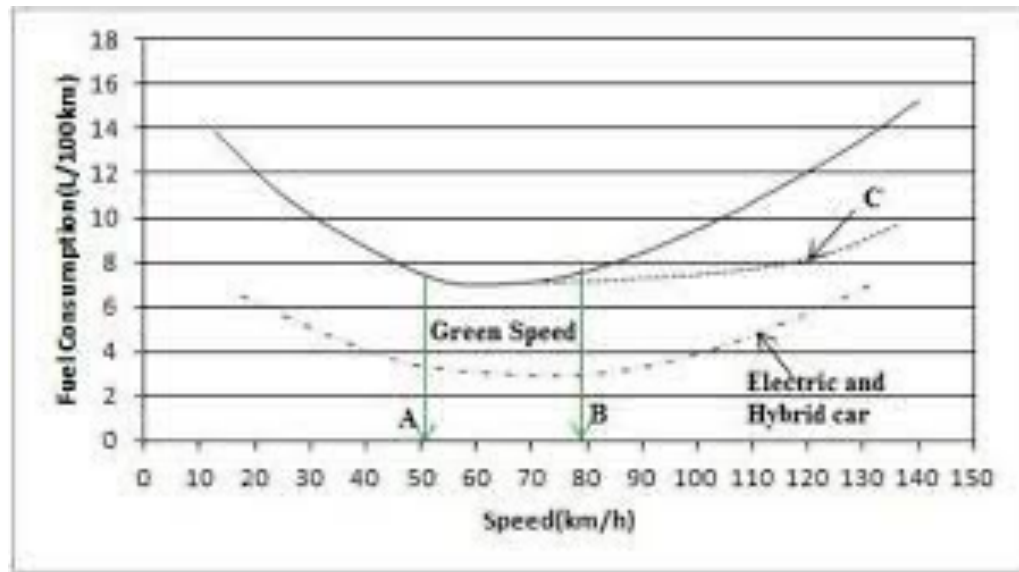
When a CAV enters coordinating zone, system operator instructs the CAV to merge with the leading CAV or CAV platoon or not



# The Trade-off to Study

- Benefits of merging:
  - Fuel savings over cruising zone
  - Reduced travel time
- Cost of merging:
  - Increased fuel consumption due to acceleration
  - Lower chance of platooning with following CAVs
- State of the art: very limited methods for link/network level coordination
- Our approach:
- Two threshold-based coordination policies are proposed to minimize travel cost (time + fuel) in the link layer
  - **Static approach**: threshold for coordinating zone entering time, static optimization
  - **Dynamic approach**: threshold for actual headway upon CAV arrival, dynamic programming

# To platoon or not to platoon?

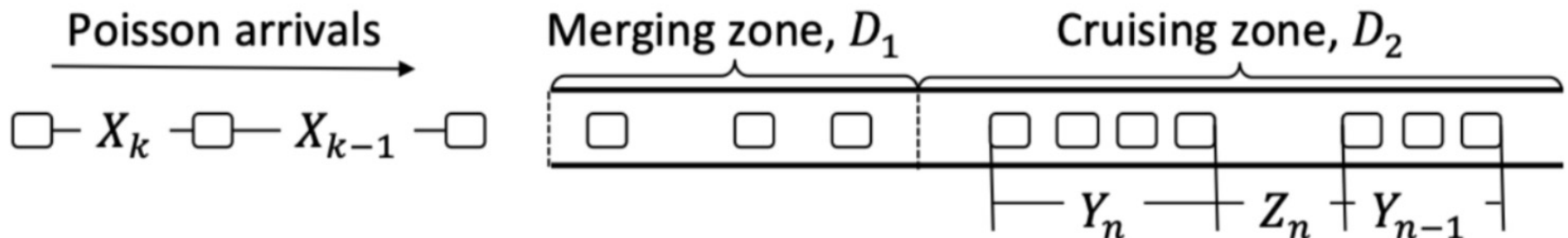




# Static approach

## Assumptions

- We assume the vehicle arrival is a Poisson process. The inter-arrival time  $X_k$  is independent and identically distributed, which follows the exponential distribution with parameter  $\lambda$ .
- Let  $r$  be the threshold for platooning. After the merging process, we assume the headway between platoon is  $h_0 \approx 0$ .
- Three ways to coordinate merging process: acceleration only, deceleration only and cooperation.



# Static approach

- When operating platoons on the highway, we need to identify some essential characteristics, including
  - platoon size;
  - headway between platoons;
  - time reduction due to platooning.
- Note that all the results consider the process in the steady state.
- Main question: how does the decision variable  $r$  affect platoon characteristics?

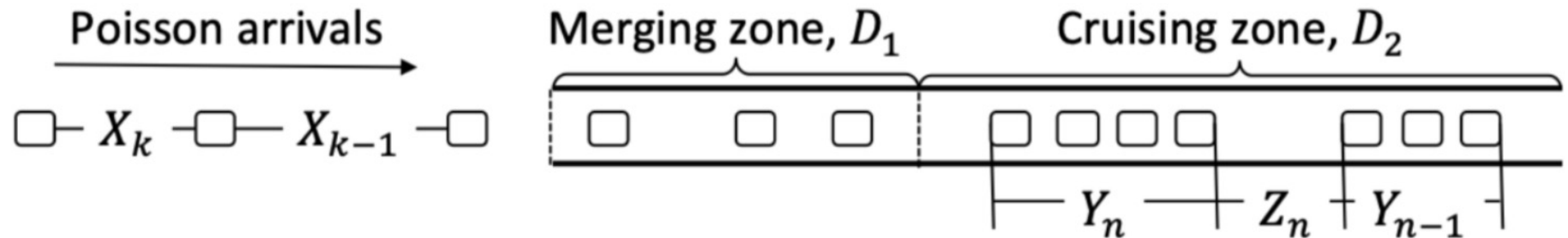
Reference: Xiong, X., Xiao, E. and Jin, L., 2019, December. Analysis of a stochastic model for coordinated platooning of heavy-duty vehicles. In 2019 IEEE 58th Conference on Decision and Control (CDC) (pp. 3170-3175). IEEE.

# Static approach\*

## Proposition

In the steady state, the number of vehicles in a platoon  $Y$  follows the probability mass function (PMF):

$$P_Y(y) = e^{-\lambda r} (1 - e^{-\lambda r})^{y-1}, \quad y = 1, 2, 3, \dots$$



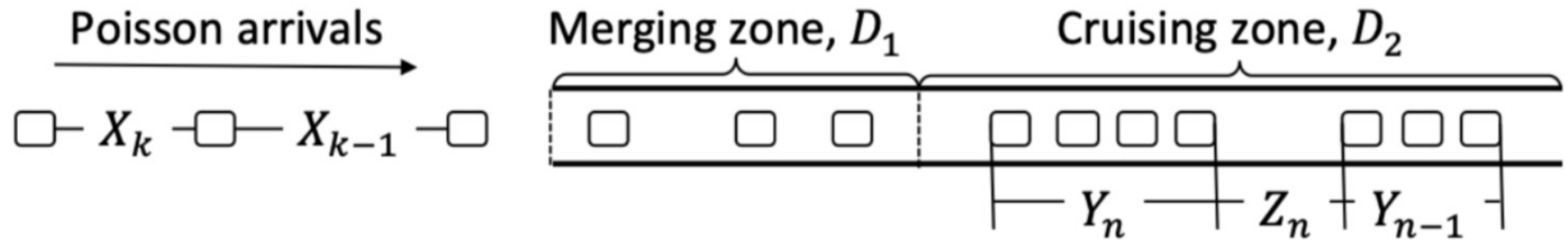
Interpretation: platoon size distribution decreases exponentially.

# Static approach\*

## Proposition

- In the steady state, the headways  $Z$  between a platoon has the expected value

$$E[Z] = \frac{e^{\lambda r}}{\lambda}$$

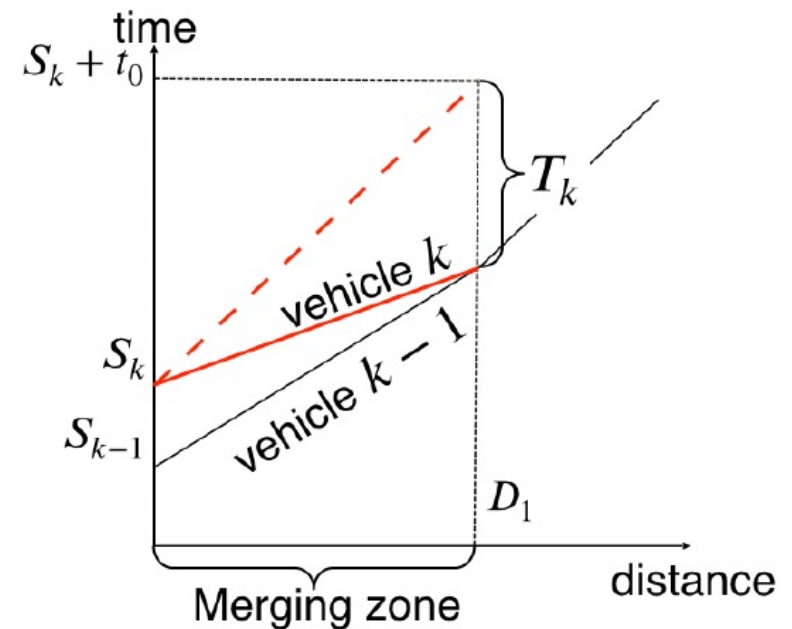


- Interpretation: headway exponentially increases with threshold.

# Static approach\*

- Time reduction due to platooning
  - the nominal traverse time  $t_0$ , which is identical for all vehicles,
  - the increment  $-T_k \leq 0$  due to acceleration for merging.
- The time at which the  $k$ th vehicle leaves the merging zone is  $S_k + t_0 - T_k$ .
- **Proposition**

$$E[T_k] = \frac{1}{\lambda} e^{\lambda r} - r - \frac{1}{\lambda}$$



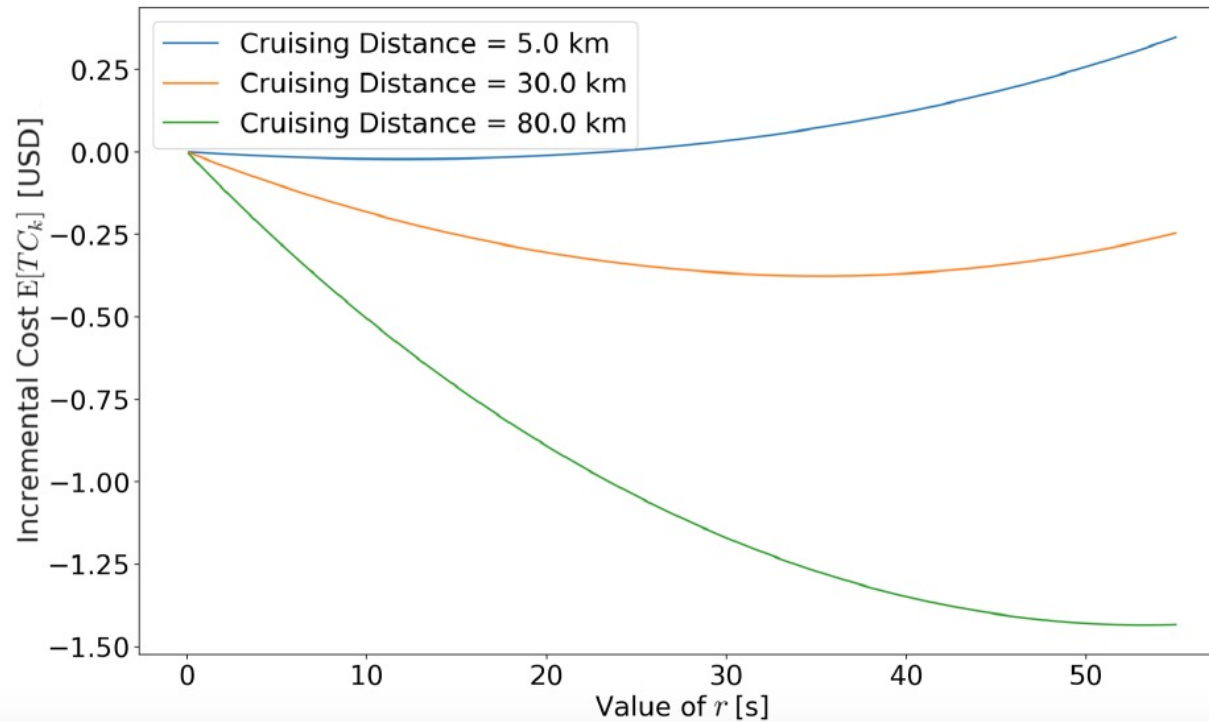
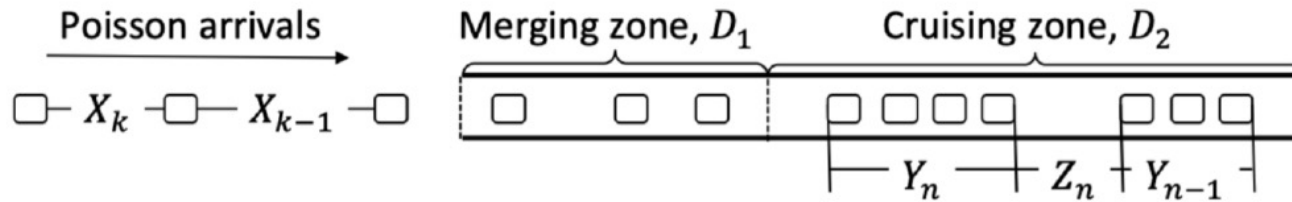
# Static approach

- Optimizing Platooning Threshold
- The benefits of platooning include two parts: reducing travel time and fuel consumption.
- The total cost  $TC_k$  is the incremental total cost instead of absolute value.
- We divide the cost of platooning into three parts: time reduction  $T_k$ , increased fuel  $F_1$  during merging and fuel reduction  $F_2$  after merging. The total cost  $TC_k$  can be expressed as:

$$TC_k = -w_1 T_k + w_2 \Delta F_1 - w_2 \Delta F_2,$$

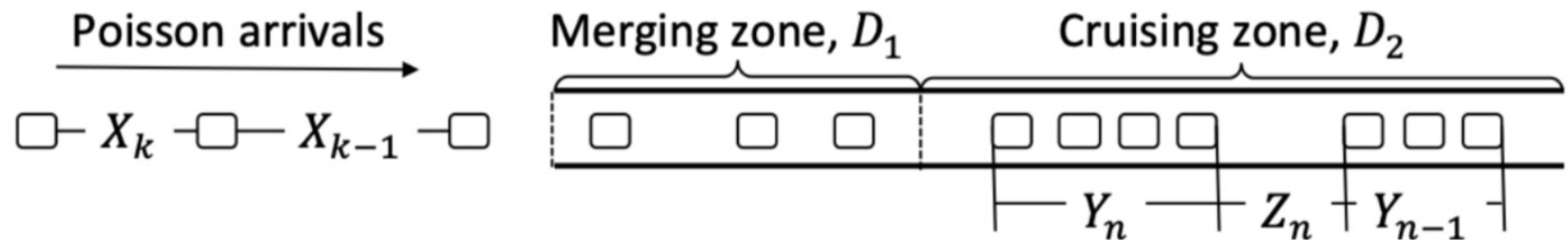
- $w_1$  is value of time (VOT) and  $w_2$  is fuel price.

# Static approach



# Dynamic approach

- Previously we use inter-arrival time  $X_k$  as the decision variable, which would lead to consequent acceleration and high catch-up speed.
- We now use the time reduction of platooning  $T_k$  as the decision variable (platoon-size sensitive).
- The decision is made for an incoming CAV based on the position and speed of the CAV ahead when the incoming CAV enters the merging zone.





## Dynamic programming formulation

- State  $S_k$  = predicted headway between CAVs  $k$  and  $k - 1$  at the junction.
- Action  $A_k = T_k$  , i.e. time reduction of CAV  $k$ .
- Reward  $R_k$  = - travel time cost - fuel cost for CAV  $k$ ;
- State update equation:  $S_{k+1} = S_k + 1 + A_k$ .
- Objective: find the optimal policy to maximize accumulative rewards

$$\max E \left[ \sum_{k=0}^{\infty} \gamma^k R_k \right]$$

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## Theorem

The optimal policy to the DP of coordinated platooning is such that

$$\mu^*(s) = \begin{cases} s & s \leq \theta \\ c & s > \theta \end{cases}$$

where  $\theta > 0$  and  $c < 0$  are constants.

- Holds for general arrival processes.
- Interpretation:
  - If the predicted headway between two CAVs is less than a threshold, then merge;
  - Otherwise, the following CAV should slightly decelerate in anticipation of further subsequent CAVs to merge.
- Idea of proof: Bellman optimality equation\*

# Dynamic approach\*

- The above theorem only states the structure of the optimal policy.
  - This significantly reduces computation time!

Headway Distribution		Bounded Value Iteration	Recursive Approximation	Refinement
Exponential	$\lambda e^{-\lambda x}$	9.6 h	0.9 h	< 1 s
Discrete random variable	$p_1 h_{c1} + p_2 h_{c2}$	37 s	10 s	--
Constant	$h_c$	29 s	8 s	--

Reference: Xiong, X., Sha, J. and Jin, L. Optimizing coordinated vehicle platooning: An analytical approach based on stochastic dynamic programming. *Transportation Research Part B: Methodological*, conditionally accepted.

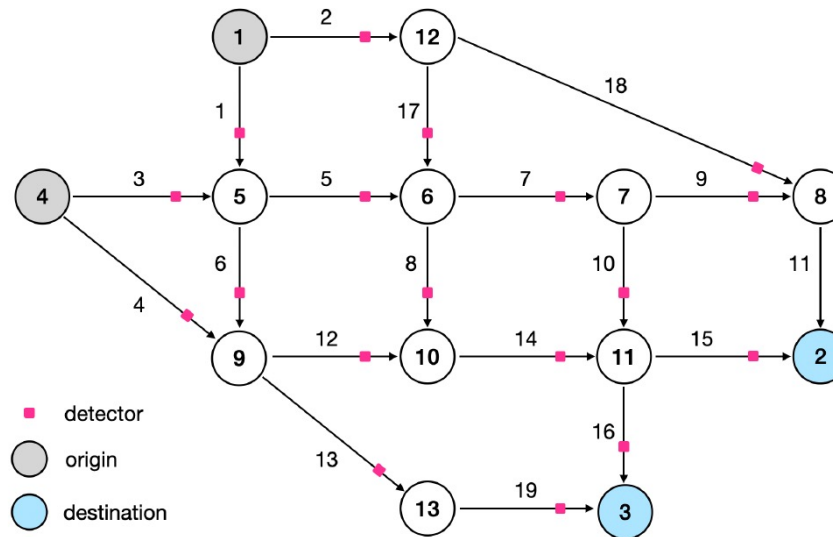
# Network level: cooperative routing

- Problem formulation
- Fundamental tradeoff

# Platooning Coordination in Networks

## Coordinated platooning at junctions

- State: predicted headway and vehicle destinations;
- Action: time reduction and routing decisions.
- System-level coordination and cooperation
- Challenge: spatial-temporal correlations in networks



# Fundamental tradeoff

- On the one hand, we want CAVs' routes to have as many overlapping legs as possible to maximize the chance of platooning.
- On the other hand, clustering all CAVs on a small number of legs may lead to congestion, which compromises non-CAV traffic.
- This tradeoff is very complex, and learning-based methods will be needed to approximately compute the optimal decisions.

# Summary

- Macroscopic decisions for platooning
- Link level: headway regulation
  - Hybrid fluid model
  - Analysis
  - Design
- Junction level: trip coordination
  - Markov decision process
  - Optimal coordination policy
- Network level: cooperative routing
  - Problem formulation
  - Fundamental tradeoff



## Analysis & design of intersection sequencing

- Modeling
- Analysis
- Design