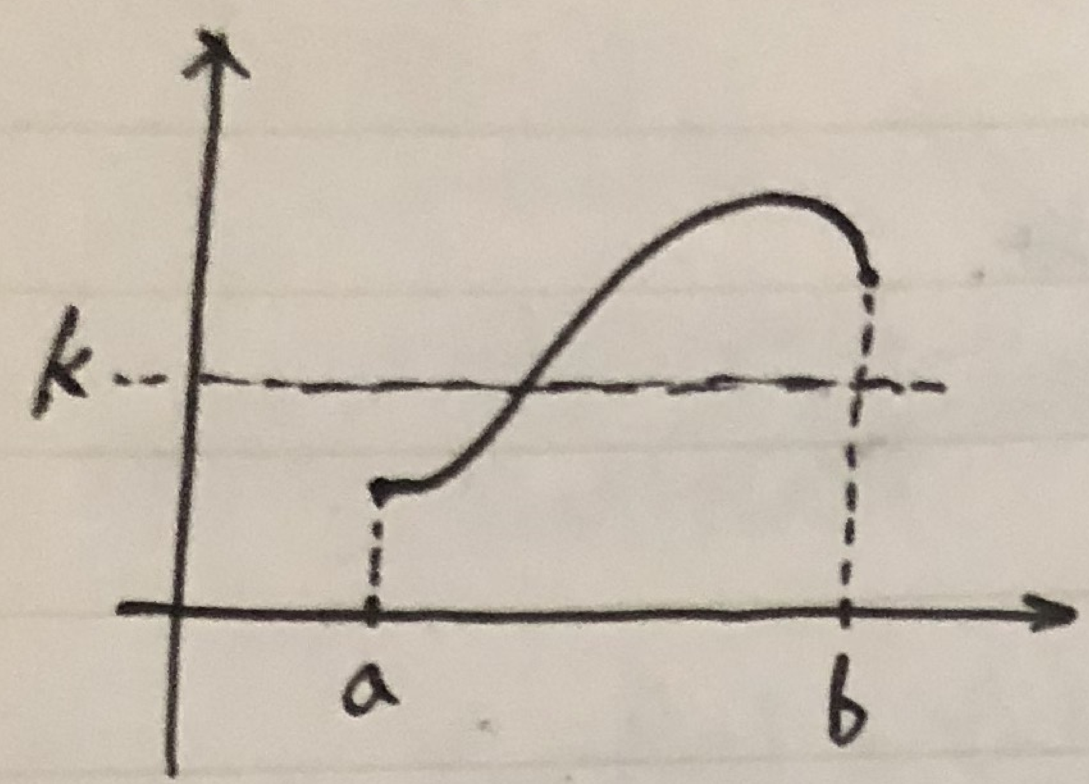
IVT

continuous , closed interval $[a,b]$



$k \in (f(a), f(b))$, $f(a) \neq f(b)$

$\Rightarrow \exists c$, $f(c) = k$

"介质"定理.

Application: 证实有解. $f(x) = e^x, \sin x, \cos x$, etc.

show $f(x) = c$

then: plug in value , say , 0, $\frac{k}{2}\pi$, etc.

with derivative: 证实仅有若干解.

Differentiability

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

Differentiable $\Rightarrow$ Continuous

Why? $f'(x)$ exist. $h \to 0$.

$$\lim_{h \to 0} [f(x+h) - f(x)] = 0$$

$$\therefore \lim_{h \to 0} f(x+h) = f(x).$$

Notation:

$y'(x)$. $y''(x)$ ...

$\dot{y}(x)$, $\ddot{y}(x)$ ...

$\frac{dy}{dx}$. $\frac{d^2y}{dx^2}$ ...

Non-differentiable

① Verticle. ~~tan x at x = π/2~~

$\sqrt{2-x}$ at $x = 2$

② corner $|x|$ left ≠ right.

③ Discontinuous

3 kinds of discontinuity

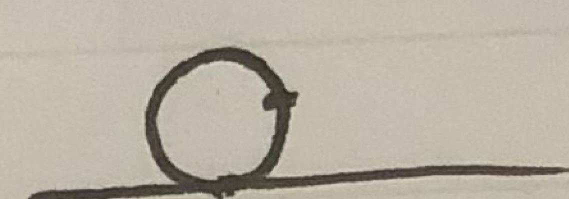
① essential : one single-sided limit doesn't exist

② jump left ≠ right

③ removable limit ✓ but doesn't equal

Vertical cusp. 左右极限均为∞. 一正一负.

点的轨迹



旋轮线 尖点

$$\begin{cases} x = r(\theta - \sin\theta) \\ y = r(1-\cos\theta) \end{cases}$$

过点 Q5 给大家

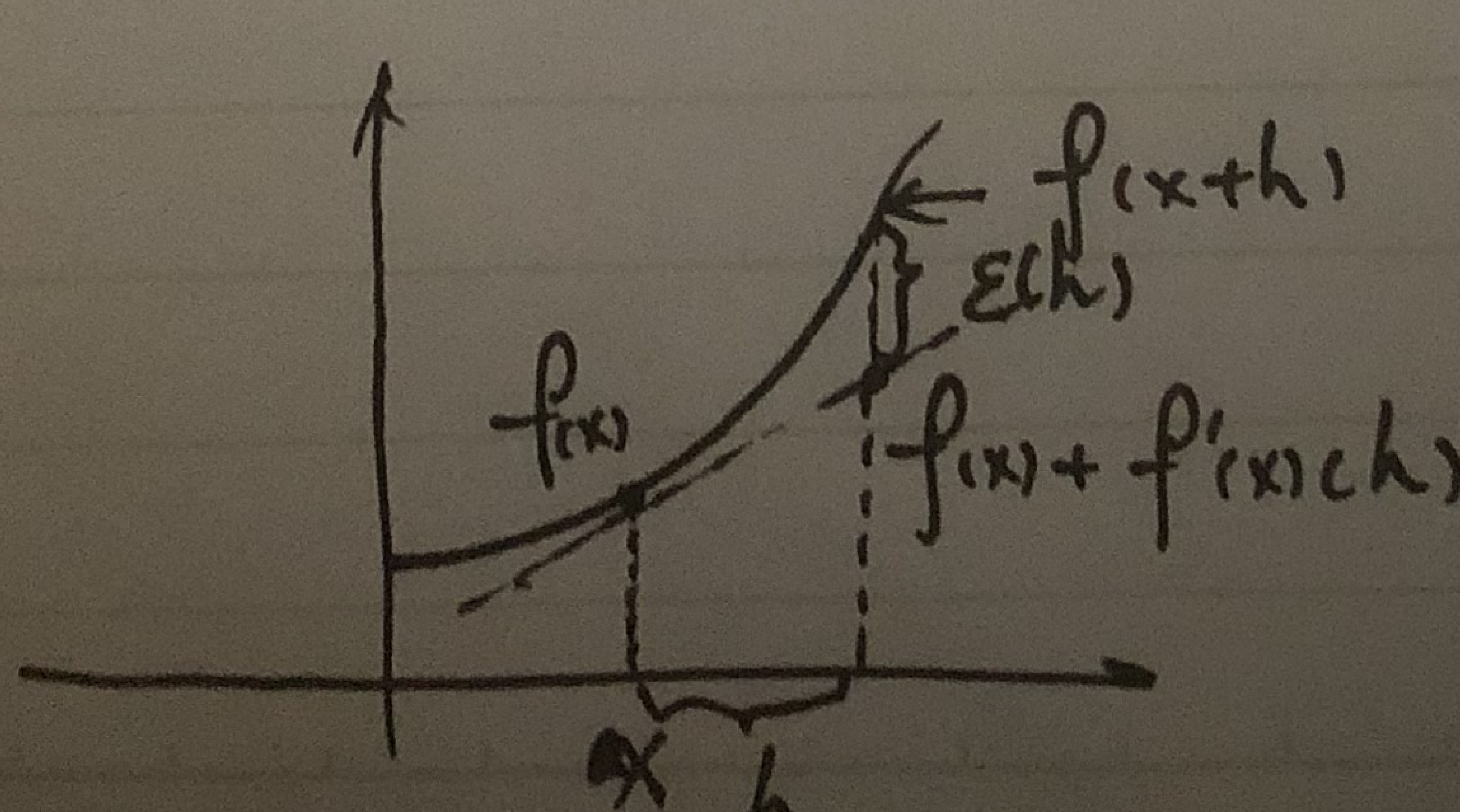
推导参数方程如何求导!

Linear approximation

$$f(x) \approx f(a) + f'(a)(x-a)$$

what

$$f(x+h) = f(x) + f'(x)(h) + \varepsilon(h)$$



当h减小时, ε(h)减小更快.

$$\lim_{h \to 0} \frac{\varepsilon(h)}{h} = 0$$

Is the linear approximation unique ?

$$f(x+h) = f(x) + Ah + \varepsilon(h).$$
$$\varepsilon(h) + Ah = f(x+h) - f(x)$$
$$\frac{\varepsilon(h)}{h} + A = \frac{f(x+h) - f(x)}{h} \quad \text{as } h \to 0$$
$$\therefore \quad A = f'(x).$$

⑤ law

① $+ . -$ ② $\times$ $(uv)' = u'v + uv'$ ③ $\div$ $(u/v)' = \dfrac{u'v - uv'}{v^2}$

④ fundamental

see Q_1 row 1, 2.

⑤ Inverse.

How to remember ? $y = f^{-1}(x) \Rightarrow x = f(y)$ $/dx$

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

we are looking for $\frac{dy}{dx}$ $\quad 1 = f'(y)\frac{dy}{dx}$

$$\therefore \frac{dy}{dx} = \frac{1}{f'(y)} = \frac{1}{f'(f^{-1}(x))}$$

Easier to remember using implicit equation's differential

see Q_1 row 3

⑥ Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

See Q_2 (1) (5) (7) (3)

$$2^{3^{x^2}} \qquad \ln 2 \cdot 2^{3^{x^2}} (\ln 3\, 3^{x^2})(2x)$$

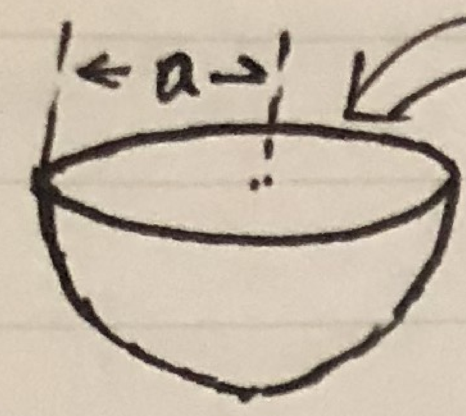
$$\ln \ln \ln y \qquad \frac{1}{\ln \ln y} \frac{1}{\ln y} \frac{1}{y}$$

$$(1-x^2)^{10} \qquad 10(1-x^2)^9 (-2x)$$

$$\arctan(e^{\sqrt{x}}) = \frac{1}{1+e^{2\sqrt{x}}} e^{\sqrt{x}} \frac{1}{2} \frac{1}{\sqrt{x}}$$

what is special, we can add variable into it.

$\dot{V} = 5\pi a^3$



find $\frac{dh}{dt}$. what we have: $\frac{dV}{dt}$.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = y''(x)$$

$$\frac{dh}{dt} = \frac{dh}{dV}\frac{dV}{dt} = \frac{1}{\frac{dV}{dh}}\frac{dV}{dt} = \frac{1}{\pi(a^2-(a-h)^2)}5\pi a^3$$

cross-section area at h

or you can just do an integral in the future

$$\left(\frac{dy}{dx}\right)^2 = (y'(x))^2$$

⑦ Implicit.

* $y = \frac{f(x)g(x)}{h(x)w(x)}$ du dx dy.

$\ln y = \ln f(x) + \ln g(x) - \ln h(x) - \ln w(x)$ will the domain ~~affect~~ the answer

we do a little change here.

$\cancel{\ln|y| = \ln\left|\frac{f(x)g(x)}{h(x)w(x)}\right|}$

case ① when both sides are negative

$\ln -y = \ln -\frac{f(x)g(x)}{h(x)w(x)}$ same function and can take the ln.

$$\frac{-1}{-y}\frac{dy}{dx} = \frac{d}{dx}\left(\ln -\frac{f(x)g(x)}{h(x)w(x)}\right)$$

$\frac{1}{y}\frac{dy}{dx}$ just the same as we treat y as positive.

case ② $f(x)$, $g(x)$ might both be negative.
But y is positive

$$y = \frac{f(x)\,g(x)}{h(x)\,w(x)}$$

$$\cancel{|y| = \frac{|f(x)||g(x)|}{|h(x)||w(x)|}} \Rightarrow y = \frac{|f(x)||g(x)|}{h(x)\,w(x)}$$

$$\ln y = \ln|f(x)| + \ln|g(x)| - \ln h(x) - \ln w(x).$$

$$\ln y = \ln(-f(x)) + \ln(-g(x)) - \ln h(x) - \ln w(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{-f'(x)}{-f(x)} + \frac{-g'(x)}{-g(x)} - \frac{h'(x)}{h(x)} - \frac{w'(x)}{w(x)}$$

$$= \frac{f'(x)}{f(x)} + \frac{g'(x)}{g(x)} - \frac{h'(x)}{h(x)} - \frac{w'(x)}{w(x)}.$$

chain rule has taken care of the sign problem!

this might give you confidence when you use the method!

See Q3 (1) (3) (4) * (6).

Before.
Other Application

Comment:

① differential rules

② definition $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

分段函数, like Q8 and Q14.

then we can solve questions one by one.