[1].

(a) $\{x[n]\} = \{\underset{\uparrow}{1}, -1\} = \delta[n] - \delta[n-1]$

$X(w) = 1 - e^{-jw}$

(b) $x[n] = u[n] - u[n-5] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$

$X(w) = 1 + e^{-jw} + e^{-2jw} + \cdots + e^{-4jw}$

$= \frac{1-e^{-5jw}}{1-e^{-jw}}$

(c) $x[n] = \sin(\frac{\pi}{4}n)$, $w_0 = \frac{\pi}{4}$

$X(w) = -j\pi[\delta(w-\frac{\pi}{4}) - \delta(w+\frac{\pi}{4})]$

(d) $x[n] = \alpha^n \cos(w_0 n) u[n]$, $|\alpha| < 1$

① $\alpha = 0$: $x[n] = 0$, $X(w) = 0$

② $\alpha \neq 0$: $x[n] = \cos(w_0 n) \cdot x_1[n]$, $x_1[n] = \alpha^n \cdot u[n]$.

$X_1(w) = \frac{1}{1-\alpha e^{-jw}}$,

$X(w) = \frac{1}{2} X_1(w-w_0) + \frac{1}{2} X_1(w+w_0) = \frac{\frac{1}{2}}{1-\alpha e^{-j(w-w_0)}} + \frac{\frac{1}{2}}{1-\alpha e^{-j(w+w_0)}}$

[2]. $X(w) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$

(a) for $X_d(0)$: $w=0 \Rightarrow e^{-jwn} = 1$

so $X_d(0) = \sum_{n=-\infty}^{\infty} x[n] = 9$

(b) for $X_d(\pi)$: $w = \pi \Rightarrow e^{-jwn} = \begin{cases} 1 & n \text{ is even} \\ -1 & n \text{ is odd} \end{cases}$

So $X_d(\pi) = x[-2] - x[-1] + x[0] - x[1] + x[2]$

$= 1$

(c) since $x[n]$ is real-valued: $X_d(w) = \sum_{n=-\infty}^{\infty} Re\{x[n]\}\cos(wn) - jRe\{x[n]\}\sin wn$

. we can see that $Im\{X_d(w)\} = \sum_{n=-\infty}^{\infty} Re\{x[n]\}\sin wn$

$= \sin(-2w) + \sin(2w) + 2[\sin(-w) + \sin w] + 3 \cdot \sin(0)$

$= 0$

so $\angle X_d(w) = 0$

(d) from inverse DTFT: $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w) \cdot e^{jwn} dw$

consider $n=0$: $x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w) dw$

$\therefore \int_{-\pi}^{\pi} X_d(w) = 2\pi x[0] = 6\pi$

(e) from Parseval's relation: $\sum_{n=-\infty}^{\infty} |x[n]|^2 \overset{F}{\longleftrightarrow} \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(w)|^2 dw$

$\therefore \int_{-\pi}^{\pi} |X_d(w)|^2 dw = 2\pi \cdot \sum_{n=-\infty}^{\infty} |x[n]|^2 = 38\pi$