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section:

PROBLEM SET#3

[1]

(a)

	B	B^c
A	0.25	0.25
A^c	0.25	0.25

$P(AB) = P(A)\cdot P(B) = 0.25$

$P(AB^c) = P(A)\cdot P(B^c) = 0.25$

$P(B) + P(B^c) = 1 \Rightarrow P(A) = P(AB) + P(AB^c) = 0.5$

$\therefore P(B) = 0.5 \Rightarrow P(A) = P(A^c) = P(B) = P(B^c) = 0.5$

$\therefore P(AB) = P(AB^c) = P(A^cB) = P(A^cB^c) = 0.25$

(b) A^c and C is exclusive $\Rightarrow P(A^c\cdot C) = 0 \Rightarrow P(A^c\cdot BC) = P(A^c\cdot B^c\cdot C) = 0$

	AB	AB^c	A^cB	A^cB^c
C	0.1	0.1	0	0
C^c	0.15	0.15	0.25	0.25

$P(ABC) = 0.1 \quad P(C) = 0.2$

$P(C) = P(ABC) + P(AB^cC) + P(A^cBC) + P(A^cB^cC)$

$\therefore P(AB^cC) = P(C) - P(ABC) = 0.1$

$P(AB) = P(ABC) + P(ABC^c)$

$\Rightarrow P(ABC^c) = P(AB) - P(ABC) = 0.15$

the same for the second row of K-map.

(c) $P(A\cup B\cup C) = 1 - P(A^cB^cC^c) = 0.75$

[2]

(a) the possible value of X: 1, 4, 16, 64, 256, 1024

(b) $P_X(1024) = \frac{1}{2^5} = \frac{1}{32}$

$P_X(256) = \frac{C_5^1}{2^5} = \frac{5}{32}$

$P_X(64) = \frac{C_5^2}{2^5} = \frac{10}{32} = \frac{5}{16}$

$P_X(16) = \frac{C_5^3}{2^5} = \frac{5}{16}$

$P_X(4) = \frac{C_5^4}{2^5} = \frac{5}{32}$

$P_X(1) = \frac{1}{2^5} = \frac{1}{32}$

	1	4	16	64	256	1024
P	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$	$\frac{1}{32}$

1045

1+20+160+640+1280+1024 1205

1+45

(c): $E(X) = \sum p\cdot x = 1\cdot\frac{1}{32} + 4\cdot\frac{5}{32} + 16\cdot\frac{5}{16} + 64\cdot\frac{5}{16} + 256\cdot\frac{5}{32} + 1024\cdot\frac{1}{32}$

$= \frac{3125}{32}$

the brochure is quite accurate

(d): the probability of losing money: $P = \frac{1}{32} + \frac{5}{32} + \frac{5}{16} = \frac{1}{2}$

[3]: A: $p = 0.85$ C: $p = 0.15$

(a): the pmf of Y: $P_Y(n) = p\cdot(1-p)^{n-1}$, $n = 1, 2\cdots$

(b): $E[Y] = \frac{1}{p}$, for the A student:

$E[Y] = \frac{100}{85} = \frac{20}{17}$

for the C student: $E[Y] = \frac{100}{15} = \frac{20}{3}$

(c): for the A student:

$P(Y>5) = (1-p)^5 = (0.05)^5$

for the C student:

$P(Y>5) = (1-p)^5 = (0.85)^5 \quad P(Y\le 5) = 1-(0.85)^5 \approx 0.5563$

(d): the A student. Each time getting invitation from both is independant.

[4]:

(a): let L_i become the number of times one roll until outcome 4 occurred.

obviously, $X = L_1 + L_2 + L_3$, and L_1, L_2, L_3 are independant.

Thus, the pmf of X: $P_X(n) = (\frac{1}{6})^2\cdot(\frac{5}{6})^{n-3}\cdot C_{n-1}^2\cdot\frac{1}{6}$

the expected value of X: $E(X) = E(L_1) + E(L_2) + E(L_3) = \frac{3}{\frac{1}{6}} = 18$

the variance of X: $Var(X) = Var(L_1) + Var(L_2) + Var(L_3) = 3\cdot\frac{1-\frac{1}{6}}{(\frac{1}{6})^2} = 90$

(b) let L: the number of times one roll until outcome 1 or 2 occurred

obviously, $Y = L_1 + L_2 + L_3 + L_4$, and $L_1 \sim L_4$ are independent

Thus, the pmf of Y: $P_Y(n) = (\frac{1}{3})^3 \cdot (\frac{2}{3})^{n-4} \cdot C^3_{n-1} \cdot \frac{1}{3}$

the expected value of Y: $E(Y) = E(L_1) + E(L_2) + E(L_3) + E(L_4) = 4 \cdot \frac{1}{\frac{1}{3}} = 12$

the variance of Y: $Var(Y) = 4 \cdot \frac{1-\frac{1}{3}}{(\frac{1}{3})^2} = 24$

[5]. consider minute as unit time

(a) let Y be the call per minute.

$E(Y) = \lambda = 4$

$p_Y(k) = \frac{e^{-4} \cdot 4^k}{k!}$, $k \geq 0$

let X be the call in an interval of 3 minutes.

obviously, $E(X) = 3\lambda = 12$

$p_X(2) = \frac{e^{-12} \cdot 12^2}{2!} = 0.000442$

(b) $P(Y \geq 3) = 1 - P(Y=0) - P(Y=1) - P(Y=2)$

$= 1 - e^{-4} - 4 \cdot e^{-4} - 8e^{-4}$

$= 0.762$

[6].

(a) all the passengers showing up get the seat means:

at most 100 people show up

so the probability should be:

$P = 1 - (0.8)^{105} - C^1_{105} \cdot (0.8)^{104} \cdot 0.2 - C^2_{105} \cdot (0.8)^{103} \cdot (0.2)^2 - C^3_{105} \cdot (0.8)^{102} \cdot (0.2)^3 - C^4_{105} \cdot (0.8)^{101} \cdot (0.2)^4$

$= 0.9999985335$

(b) consider X passengers show up at the gate

$E(X) = 105 \times 0.8 = 84$

(c) each person's no-shows is independent of any other passenger.

and for each person, they all have probability $p = 0.2$ for no-shows.

There are 105 person with $p = 0.8$ for show-up (the "0" event) and

$p = 0.2$ for no-shows (the "1" event), thus the number of no-shows

can be modeled as a binomial random variable Y with

parameters ($n = 105$, $p = 0.2$)

(d) for the poisson approximation.

$\lambda = np = 105 \cdot 0.2 = 21$

$P(Y \geq 5) = 1 - P(Y=0) - P(Y=1) - P(Y=2) - P(Y=3) - P(Y=4)$

$= 1 - e^{-21} - 21 \cdot e^{-21} - \frac{21^2}{2} \cdot e^{-21} - \frac{21^3}{6} \cdot e^{-21} - \frac{21^4}{24} \cdot e^{-21}$

$= 0.999925013$

we can see that the answer from Poisson approximation is

really close to the answer in part a, with error less than 0.00001.