University of Illinois **Fall 2023**

ECE 313: Problem Set 9

Due: Friday, October 27 at 7:00:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.7–3.10

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON Canvas
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings. **5 points will be deducted for a submission with incorrectly-assigned page numbers.**

1. **[Function of a RV]**
   Suppose that X is a uniform random variable with mean 2 and variance 1/3, and $Y = X^2 + 2X + 3$.
   (a) What is the set of possible values of Y?
   (b) Find the pdf of Y.
   (c) Find $E[Y]$.

2. **[Cauchy Distribution]**
   A random variable X is said to have the Cauchy distribution if its pdf is given by $f_X(u) = \frac{1}{\pi(1+u^2)}$ for $-\infty < u < \infty$ (see Example 3.8.6 of the notes). Let $Y = 1/X$. Find the pdf of Y.

3. **[Generating a RV]**
   Let U be uniformly distributed in $[0, 1]$. Find a function g such that $X = g(U)$ is Gaussian with $N(2, 2)$.

4. **[Binary hypothesis testing]**
   Consider the following binary hypothesis testing problem. Under H_0, the random variable X has the pdf f_0, while under H_1, the random variable X has the pdf f_1, where

$$f_0(u) = \begin{cases} \frac{1}{4} & u \in \left[\frac{-1}{2}, \frac{3}{2}\right] \cup \left[\frac{5}{2}, \frac{9}{2}\right], \\ 0 & \text{else} \end{cases}$$

   and

$$f_1(u) = \begin{cases} \frac{1}{4}u & u \in [0, 2], \\ \frac{-1}{4}u + 1 & u \in (2, 4], \\ 0 & \text{else} \end{cases}$$

   Assume that $4\pi_0 = \pi_1$.

(a) Find the ML rule.

(b) Find $p_{false\,alarm}$, p_{miss}, and p_e for the ML rule.

(c) Find the MAP rule.

(d) Find $p_{false\,alarm}$, p_{miss}, and p_e for the MAP rule.

5. **[ML Parameter Estimation]**
The random variable X is assumed to have a Gaussian distribution. It is observed that $X = 1$.

(a) Suppose that variance of X is known to be 1. Find the ML estimate of the mean of X given the observation.

(b) Suppose the mean of X is known to be 0. Find the ML estimate of the variance of X given the observation.