UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Fall 2023

Homework 7

Profs. Do, Snyder, Moustakides **Due: October 13, 2023 on Gradescope**

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1. For each of the following LTI systems with impulse response $h[n]$, determine the (i) frequency response $H(\omega)$, (ii) magnitude response $|H(\omega)|$, (iii) phase response $\angle H(\omega)$, and (iv) plot the magnitude and phase responses for $\pi \leq \omega \leq \pi$.

   (a) $h[n] = \{\underset{\uparrow}{2}, 0 - 1, 0, 2\}$

   (b) $h[n] = \{\underset{\uparrow}{1}, -1, 1, -1\}$

2. The frequency response of a real-valued LTI system is given by

$$H(\omega) = j, \ \pi \leq \omega \leq 2\pi.$$

   for the interval of $\pi \leq \omega \leq 2\pi$. Considering the symmetries of the DTFT:

   (a) Plot $|H(\omega)|$ for $-\pi \leq \omega \leq \pi$.

   (b) Plot $\angle H(\omega)$ for $-\pi \leq \omega \leq \pi$.

   (c) Determine the system's impulse response $h[n]$.

3. Consider an LTI system with the following frequency response:

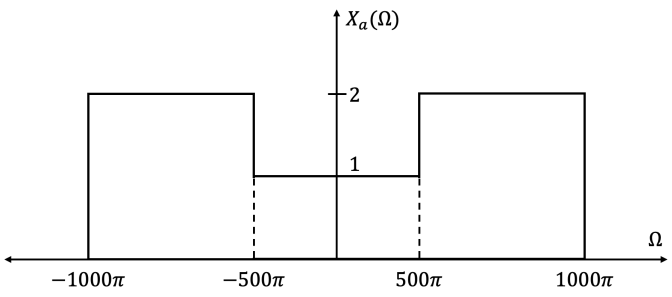
$$H(\omega) = \omega^2 e^{j \sin(\omega)}.$$

   (a) Is this system real-valued?

   (b) Compute the system response $y[n]$ to input $x[n] = e^{j\frac{\pi}{2}n} - 2\cos\left(\frac{5\pi}{6}n\right)$.

   (c) Compute the system response $y[n]$ to input $x[n] = 5 - e^{j\frac{\pi}{4}n} + 3\sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$

4. The continuous-time Fourier transform $X_a(\Omega)$ of an analog signal $x_a(t)$ is given below. We sample $x_a(t)$ at some sampling rate $f_s = \frac{1}{T}$ to obtain discrete-time signal $x[n]$.



   (a) What is the minimum sampling rate such that we avoid aliasing when obtaining $x[n]$?

(b) Sketch the DTFT of $x[n]$, $X_d(\omega)$, for $-3\pi \leq \omega \leq 3\pi$ when $T = \frac{1}{2000}$ s. Please carefully label your axes.

(c) Sketch $X_d(\omega)$ for $-3\pi \leq \omega \leq 3\pi$ when $T = \frac{1}{1000}$ s.

(d) Sketch $X_d(\omega)$ for $-3\pi \leq \omega \leq 3\pi$ when $T = \frac{3}{2000}$ s.