UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Fall 2023

Homework 12

Profs. Do, Snyder, Moustakides **Due: Friday, Nov 17, 2023 on Gradescope**

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1. The frequency response of a generalized linear phase (GLP) filter can be expressed as $H_d(\omega) = A(\omega)e^{j(-\alpha\omega+\beta)}$ where $A(\omega)$ is a real function, and α, β are real constants. For each of the following filters, determine whether it is a GLP filter. If it is, find $A(\omega)$, α, and β, and indicate whether it is also a linear phase filter.
   (a) $\{h_n\}_{n=0}^{2} = \{2, 1, 2\}$
   (b) $\{h_n\}_{n=0}^{2} = \{-1, 3, 1\}$
   (c) $\{h_n\}_{n=0}^{2} = \{1, 0, -1\}$
   (d) $\{h_n\}_{n=0}^{3} = \{1, 1, -1, -1\}$
   (e) $\{h_n\}_{n=0}^{3} = \{2, 1, 1, 2\}$

   In each case, the remaining samples of the impulse response of the filter are zero.

2. Use the windowing method to design a length-$(M + 1)$ low-pass, generalized linear phase FIR filter with cut-off frequency $\pi/3$.
   (a) Find an expression for the filter coefficients $\{h[n]\}_{n=0}^{M}$ if the rectangular window is used for the design.
   (b) Find an expression for the filter coefficients $\{h[n]\}_{n=0}^{M}$ if the Hamming window is used for the design.
   (c) Comment on the pros and cons of the frequency responses of the designed filters obtained by the above two different windows.

3. We are interested in designing a lowpass filter with passband $[-\omega_p, \omega_p]$ and stopband $[-\pi, -\omega_s] \cup [\omega_s, \pi]$ where $\omega_s > \omega_p$. We would like to approximate the desired characteristics with a Type-I filter:
   (a) Specify which ideal characteristic you are going to approximate with the windowing method.
   (b) If we are interested in a stopband attenuation or a passband ripple of 10^{-4} will this be possible to achieve using any of the known windows for a sufficiently large filter size?

4. Let $x_c(t)$ be a *bandlimited* continuous time signal and $\dot{x}_c(t)$ its continuous time derivative.
   (a) If $X_c(\Omega)$ denotes the continuous time Fourier transform (CTFT) of $x_c(t)$, using the properties of the Fourier transform specify the CTFT of the derivative.
   (b) Suppose now that $x_c(t)$ and $\dot{x}_c(t)$ are being sampled with sampling period T_s corresponding to a sampling frequency that exceeds the Nyquist limit, generating the discrete time signals $x[n]$ and $y[n]$ respectively. Express the two discrete time Fourier transforms $X(\omega), Y(\omega)$ in terms of the CTFT of the two original continuous time signals in the interval $[-\pi, \pi]$. Compare the two expression and propose an ideal *discrete time* filter capable of computing $y[n]$ directly from $x[n]$ by specifying its frequency response $\mathcal{D}(\omega)$ and showing that $Y(\omega) = \mathcal{D}(\omega)X(\omega)$. This system is called *Digital Differentiator*.

(c) Next assume that for the discrete time signal $x[n]$ generated by sampling $x_c(t)$ as in (b) you would like to compute the derivative of the part of the signal that has frequencies in the interval $[-\omega_c, \omega_c]$. You can achieve this with two methods: 1) Apply first a lowpass filter and then the differentiator introduced in (b). 2) Combine the lowpass filter and the differentiator into a single ideal response since the two systems are used in series. Assuming that you are approximating the ideal filter responses with *generalized linear phase and causal FIR filters*, specify the filter types you are going to use in each method and also the final delay in the output you are going to experience due to the linear phase. Plot the ideal frequency response of each system you are going to use.