[1]:

(a): $h[n] = \delta[n] + 2\delta[n+2] - 3\delta[n-4]$

$H(z) = 1 + 2z^{2} - 3z^{-4}$, ROC: all z except $z=\infty$ and $z=0$

(b) $h[n] = (\frac{1}{3})^{n} u[n+1] + (\frac{1}{2})^{n} u[-(n+1)]$

$h_1[n] = (\frac{1}{3})^{n} u[n+1] = 3(\frac{1}{3})^{n+1} \cdot u[n+1]$

$H_1(z) = 3 \cdot \frac{z}{1-\frac{1}{3z}}$, ROC: $|z| > \frac{1}{3}$ and $z \neq \infty$

$H_2(z) = -\frac{1}{1-\frac{1}{2z}}$, ROC: $|z| < \frac{1}{2}$

$\therefore H(z) = 3 \cdot \frac{z}{1-\frac{1}{3z}} - \frac{1}{1-\frac{1}{2z}}$

$= \frac{9z^{2}}{3z-1} - \frac{2z}{2z-1}$ ROC: $\frac{1}{3} < |z| < \frac{1}{2}$

(c): $h[n] = n(\frac{1}{3})^{n} u[n-2] + (\frac{1}{2})^{n} u[n+3]$

$= (n-2) \cdot (\frac{1}{3})^{n-2} \cdot \frac{1}{9} u[n-2] + 2 \cdot \frac{1}{9} \cdot (\frac{1}{3})^{n-2} u[n-2] + (\frac{1}{2})^{n+3} \cdot 8\, u[n+3]$

$H(z) = z^{-2} \cdot \frac{1}{9} \cdot \frac{\frac{1}{3z}}{(1-\frac{1}{3z})^{2}} + \frac{2}{9} \cdot \frac{z^{-2}}{1-\frac{1}{3z}} + 8 \cdot \frac{z^{3}}{1-\frac{1}{2z}}$

$= \frac{1}{3z(3z-1)^{2}} + \frac{2}{3z(3z-1)} + \frac{16z^{4}}{2z-1}$, ROC: $|z| > \frac{1}{2}$, $z \neq \infty$

(d): $h[n] = \{A_1(z_1)^{n} + A_2(z_2)^{n}\} u[n]$

$= A_1(z_1)^{n} u[n] + A_2(z_2)^{n} u[n]$

$H(z) = A_1 \cdot \frac{1}{1-\frac{z_1}{z}} + A_2 \cdot \frac{1}{1-\frac{z_2}{z}}$

$= \frac{A_1 z}{z-z_1} + \frac{A_2 z}{z-z_2}$ ROC: $|z| > \max\{|z_1|, |z_2|\}$

[2] $x[n] \xleftrightarrow{z} X(z) = \frac{1}{1-\frac{1}{2z}}$, ROC: $|z| > |\frac{1}{2}|$, $\alpha \neq 0$

(a) $y[n] = x[n-n_0]$, $Y(z) = z^{-n_0} X(z) = \frac{z^{-n_0}}{1-\frac{1}{2z}}$

ROC: $\begin{cases} n_0 < 0: & |z| > |\frac{1}{2}|,\ z \neq \infty \\ n_0 \geq 0: & |z| > |\frac{1}{2}| \end{cases}$

(b): $y[n] = n(n-1)x[n]$

$= n^{2} x[n] - n x[n]$

let $a[n] = n x[n]$, $A(z) = -z \cdot \frac{dX(z)}{dz} = -z \cdot \frac{2(2z-1) - 2z \cdot 2}{(2z-1)^{2}} = \frac{2z}{(2z-1)^{2}}$

$b[n] = n^{2} x[n]$, $B(z) = -z \cdot \frac{dA(z)}{dz} = -z \cdot \frac{2(2z-1)^{2} - 2z \cdot 2 \cdot 2(2z-1)}{(2z-1)^{4}} = \frac{4z^{2}+2z}{(2z-1)^{3}}$

$Y(z) = B(z) - A(z) = \frac{4z^{2}+2z}{(2z-1)^{3}} - \frac{2z}{(2z-1)^{2}} = \frac{4z^{2}+2z-2z(2z-1)}{(2z-1)^{3}} = \frac{4z}{(2z-1)^{3}}$, ROC: $|z| > \frac{1}{2}$

(c): $y[n] = \beta^{n} x[n]$

$Y(z) = X(\frac{z}{\beta}) = \frac{1}{1-\frac{\beta}{2z}} = \frac{2z}{2z-\beta}$ ROC: $|z| > \frac{1}{2} \cdot |\beta|$

(d): $y[n] = x[n] * x[-n]$

$Y(z) = X_1(z) \cdot X_2(z)$

$= X(z) \cdot X(z^{-1})$

$= \frac{2z}{2z-1} \cdot \frac{1}{1-\frac{1}{2}z}$

$= \frac{2z}{2z-1} \cdot \frac{2}{2-z}$

$= \frac{4z}{-2z^{2}+5z-2}$ ROC: $\frac{1}{2} < |z| < 2$

[3]

(a): $x[-n] = \rho^{n} u[n]$, both z-transform

$X(z^{-1}) = \frac{1}{1-\rho z^{-1}}$

$X(z) = \frac{1}{1-\rho z}$

$\therefore x[n] = \rho^{-n} u[-n]$

(b) $n x[n] = 3x[n]$, $X(z)|_{z=1} = 1$, both z-transform

$\Rightarrow 3X(z) = -z \cdot \frac{dX(z)}{dz}$

$\Rightarrow X(z) = \frac{1}{z^{3}}$ ROC: $|z| > 0$

$\therefore x[n] = \delta[n-3]$

(c) $x[n]=1.3x[n-1]-0.4x[n-2]+\delta[n]+\delta[n-1]$, both z-transform

$$X(z)=1.3\cdot z^{-1}\cdot X(z)-0.4\cdot z^{-2}X(z)+1+z^{-1}$$

$$(z^2-1.3z+0.4)\,X(z)=z^2+z$$

$$X(z)=\frac{z^2+z}{z^2-1.3z+0.4}\ ,\ \text{ROC: } |z|>\frac{4}{5}$$

$$=1+\frac{2.3z-0.4}{(z-\frac{4}{5})(z-\frac{1}{2})}$$

$$=1+\frac{4.8}{z-0.8}-\frac{2.5}{z-0.5}$$

$$\therefore x[n]=\delta[n]+6\cdot\left(\tfrac{4}{5}\right)^n\cdot u[n-1]-5\cdot\left(\tfrac{1}{2}\right)^n u[n-1]$$

(d): $\left(\frac{1}{2}\right)^n x[n]=\left[1-\left(\frac{1}{3}\right)^n\right]u[n]$, both z-transform

$$X\left(\frac{z}{2}\right)=\frac{z}{z-1}-\frac{3z}{3z-1}\qquad \left(\text{ROC: }\left|\frac{z}{2}\right|>1\right)$$

$$X(z)=\frac{z}{z-2}-\frac{3z}{3z-2}\quad \text{ROC: } |z|>2$$

$$\therefore x[n]=2^n u[n]-\left(\tfrac{2}{3}\right)^n u[n]$$