[1]

(a) $X[0] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi\cdot 0}{N}\cdot n} = \sum_{n=0}^{N-1} x[n]$

since $\{x[0] \cdots x[N-1]\}$ is real-valued, $X[0]$ is real-valued

$X[L] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi L}{N}\cdot n} = \sum_{n=0}^{N-1} x[n] e^{-j\cdot\pi n} = \sum_{n=0}^{N-1} (-1)^n x[n]$

since $\{x[0] \cdots x[N-1]\}$ is real-valued, $X[0]$ is real-valued

(b) i. $x_1[n] = x[n] + x[\langle n-L\rangle_N]$

$X_1[k] = X[k] + e^{-j\cdot\frac{2\pi L}{N}n} X[k]$

$= X[k] + e^{-j\cdot\pi k} X[k]$

$= \begin{cases} 0 & k \text{ is odd} \\ 2X[k] & k \text{ is even} \end{cases}$

ii. $x_2[n] = x[n] - x[\langle n-L\rangle_N]$

$X_2[k] = X[k] - e^{-j\frac{2\pi L}{N}k} X[k]$

$= X[k] - e^{-j\cdot\pi k} X[k]$

$= \begin{cases} 0 & k \text{ is even} \\ 2X[k] & k \text{ is odd} \end{cases}$

iii. $x_3[n] = (-1)^n x[n] = e^{j\cdot\pi n} x[n] = e^{j\cdot\frac{2\pi L}{N}n} x[n]$

$\therefore X_3[k] = X[\langle k-L\rangle_N]$

(c) from (a) we know $X[L] = \sum_{n=0}^{N-1} (-1)^n x[n]$

if $x[n] = x[N-1-n]$ for $n \in \{0, 1 \cdots N-1\}$ is right

$X[L] = x[0] - x[1] + x[2] \cdots + x[N-2] - x[N-1]$. (since $N=2L$, we know the $(-1)^n$)

$= (x[0] - x[N-1]) + (-x[1] + x[N-2]) + \cdots$

$= 0$

(d) if $x[n] = -x[N-1-n]$ for $n \in \{0, 1, \cdots N-1\}$ is right

$X[0] = x[0] + x[1] + \cdots + x[N-2] + x[N-1]$

$= (x[0] + x[N-1]) + (x[1] + x[N-2]) + \cdots + (x[L-1] + x[L])$

$= 0$

[2]. $x_c(t) = \cos(25\pi t)$ 60Hz for 5s $\{x[n]\}_{n=0}^{N-1}$ $N=300$

(a) $x[n] = \cos\left(\frac{25\pi\cdot 5}{300}\cdot n\right) = \cos\left(\frac{5}{12}\pi n\right)$

$X(w) = \frac{1}{2}C\left(w - \frac{5}{12}\pi\right) + \frac{1}{2}C\left(w + \frac{5}{12}\pi\right)$

$X[k] = X(w)\big|_{w=\frac{2\pi k}{N}} = \frac{1}{2}C\left(\frac{2\pi k}{N} - \frac{5}{12}\pi\right) + \frac{1}{2}C\left(\frac{2\pi k}{N} + \frac{5}{12}\pi\right)$

$\frac{2\pi k}{N} - \frac{5}{12}\pi = 0 \Rightarrow k = \frac{5}{12}\pi \cdot \frac{N}{2\pi} = \frac{5N}{24}$; $\frac{2\pi k}{N} + \frac{5}{12}\pi = 0 \Rightarrow k = -\frac{5}{24}N \Rightarrow k = \pm\frac{125}{2}$

so $k = 62, 63, 237, 238$, $X[k]$ have the greatest magnitude.

(b) $X[k] = X(w)\big|_{w=\frac{2\pi k}{M}} = \frac{1}{2}C\left(\frac{2\pi k}{M} - \frac{5}{12}\pi\right) + \frac{1}{2}C\left(\frac{2\pi k}{M} + \frac{5}{12}\pi\right)$

$\frac{2\pi k}{M} - \frac{5}{12}\pi = 0 \Rightarrow k = \frac{5}{24}M$; $\frac{2\pi k}{M} + \frac{5}{12}\pi = 0 \Rightarrow k = -\frac{5}{24}M$

So $k = 107, 405$, $X[k]$ have the greatest magnitude

[3]. $x[n] = e^{jw_0 n}$ w_0 $X(w) = 2\pi\delta(w - w_0)$ $s[n] = \{x[0], \cdots x[N-1]\}$

$S(w) = e^{-j\cdot\frac{N-1}{2}(w-w_0)} \frac{\sin(\frac{N}{2}(w-w_0))}{\sin(\frac{1}{2}(w-w_0))}$ $w_0 = \frac{2\pi l}{m}$

(a) when $N = rm$, $k = rl$, $w = \frac{2\pi}{N}\cdot k = \frac{2\pi l}{m} = w_0$

$S\left(\frac{2\pi l}{m}\right) = e^{-j\cdot\frac{N-1}{2}\cdot 0} \cdot \underbrace{\frac{\sin(\frac{N}{2}\cdot 0)}{\sin(\frac{1}{2}\cdot 0)}} = N$ ← L' Hopital rule

and we know for $S(w)$, $w = \frac{2\pi k}{N} + w_0$ and $w \neq w_0$

we have $S(w) = 0$.

for N: if we want w_0 to appear in $w_k = \frac{2\pi k}{N}$,

obviously $N = rm$, then consider k: $w = \frac{2\pi k}{rm}$

$S(w) = e^{-j\frac{N-1}{2}(w-w_0)} \frac{\sin(\frac{N}{2}(w-w_0))}{\sin(\frac{1}{2}(w-w_0))}$

$= e^{-j\cdot\frac{N-1}{2}(w-w_0)} \frac{\sin[\frac{N}{2}(\frac{2\pi k}{N} - \frac{2\pi rl}{N})]}{\sin[\frac{1}{2}\cdot(\frac{2\pi k}{N} - \frac{2\pi rl}{N})]}$

$= e^{-j\cdot\frac{N-1}{2}(w-w_0)} \cdot \frac{\sin(\pi k - \pi rl)}{\sin(\frac{\pi k - \pi rl}{N})}$

$\sin(\pi k - \pi rl) = 0 \Rightarrow S(w) = 0$

so in order for the frequency to appear as $w_k = \frac{2\pi}{N}\cdot k$,

we need $N = rm$, $k = rl$

relationship: $w_0 = \frac{2\pi k}{N}$, $k \in \mathbb{Z}^+$

(b) see in part (a)

[4]. $x_c(t) = A_0 \cos(\Omega_0 t) + A_1 \cos(\Omega_1 t)$, $f = 32 \times 10^{12}$ Hz

(a) from the figure, we know $N = 4 + 28 = 32$

$w_0 = \frac{2\pi \cdot 4}{32} = \frac{\pi}{4}$, $A_0 = 2$, $\Omega_0 = w_0 f = 8\pi \cdot 10^{12}$

$w_1 = \frac{2\pi \cdot 12}{32} = \frac{3\pi}{4}$, $A_1 = 1$, $\Omega_1 = w_1 f = 24\pi \cdot 10^{12}$

(b) $f = 64 \times 10^{12}$ Hz , $t = 10^{-12}$ s, $N = 64$, $w_0 = \frac{\pi}{8}$, $w_1 = \frac{3\pi}{8}$

$\frac{2\pi k}{64} = \frac{\pi}{8}$, $k = 4$, $\frac{2\pi k}{64} = \frac{3\pi}{8}$, $k = 12$