NAME: Junsheng Huang

netID: jh103

section:

PROBLEM SET #2

[1] (a) $P(X=12) = \frac{1}{6}$

$P(X=6) = \frac{1}{3}$

$P(X=-m) = \frac{1}{2}$

(b) $E(X) = \frac{1}{6}\cdot 12 + \frac{1}{3}\cdot 6 + \frac{1}{2}\cdot(-m) = -1$

$\Rightarrow -\frac{1}{2}m + 4 = -1,\ m = 10$

(c) $Var(X) = E[(X+E(X))^2]$

$= E[(X+1)^2]$

$= 85$

[2] (a) X can be: 0, 1, 2

$P(X=0) = \frac{C_4^2}{C_{10}^2} = \frac{4\times 3}{10\times 9} = \frac{2}{15}$

$P(X=1) = \frac{4\times 6}{C_{10}^2} = \frac{4\times 6\times 2}{10\times 9} = \frac{24}{45} = \frac{8}{15}$

$P(X=2) = \frac{C_6^2}{C_{10}^2} = \frac{6\times 5}{10\times 9} = \frac{1}{3}$

(b) $E[(X+1)(X+2)]$

$= \frac{2}{15}\cdot 2 + \frac{24}{45}\cdot 2\cdot 3 + \frac{1}{3}\cdot 3\cdot 4$

$= \frac{4}{15} + \frac{48}{15} + 4$

$= \frac{112}{15}$

[3] (a) we need to consider the order:

$\Omega = \{(1,2,3)\ (1,3,2)\ (2,1,3)\ (2,3,1)\ (3,1,2)\ (3,2,1)\}$

$|\Omega| = 6$

(b) X can only take 0, 1, 3 (no 2 because if 2 cards fit the box, the left one should also fit the box)

$P(X=0) = \frac{1}{3}$ $P(X=1) = \frac{1}{2}$ $P(X=3) = \frac{1}{6}$

(c) $E(X) = \frac{1}{3}\cdot 0 + \frac{1}{2}\cdot 1 + \frac{1}{6}\cdot 3 = 1$

(d) $Var(X) = E[(X-E(X))^2]$

$= \frac{1}{3}\cdot(0-1)^2 + \frac{1}{2}\cdot(1-1)^2 + \frac{1}{6}\cdot(1-3)^2$

$= 1$

[4] (a) event A: Cookie Monster eats the cookie

$P(A) = \frac{1}{2} + \frac{1}{2}\cdot(\frac{1}{2})^3 = \frac{9}{16}$

(b) event B: first toss is head

$P(B) = \frac{1}{2},\ P(AB) = \frac{1}{2}$

$\Rightarrow P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{1}{2}}{\frac{9}{16}} = \frac{8}{9}$

[5] (a) there are n cells O O ··· O

$|\Omega| = C_n^2 = \frac{n(n-1)}{2}$

event A: two cells fail next to each other

$|A| = n-1$ (obviously, if you choose the cell from index 1 to $n-1$)

$P(A) = \frac{2}{n}$

(b) event B: both two cells fail among first four cells.

$P(B) = \frac{C_4^2}{C_n^2}$

event C: at least one cells fail among first four cells.

$P(C) = \frac{C_4^2 + C_4^1\cdot C_{n-4}^1}{C_n^2}$

$\therefore P(B|C) = \frac{C_4^2}{C_4^2 + C_4^1\cdot C_{n-4}^1} = \frac{6}{6+4(n-4)} = \frac{3}{2n-5}$