[1]. (a)

$$H = \begin{array}{c} \begin{matrix} 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \end{matrix} \\ \begin{bmatrix} 1 & & & & & & & \\ -1 & 1 & & & & & & \\ & -1 & 1 & & & & & \\ & & -1 & 1 & & & & \\ & & & -1 & 1 & & & \\ & & & & -1 & 1 & & \\ & & & & & -1 & 1 & \\ & & & & & & -1 & 1 \\ & & & & & & & -1 \end{bmatrix} \end{array} \qquad x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$y = H \cdot x = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ -2 \end{bmatrix} = \{\underset{\uparrow}{1}, 0, 0, 0, 1, 0, 0, 0, -2\}$$

(b) $x[n] = (\frac{1}{2})^n u[n]$, $h = \{\underset{\uparrow}{1}, 1\}$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$
$$= x[n] + x[n-1]$$
$$= (\tfrac{1}{2})^n u[n] + (\tfrac{1}{2})^{n-1} u[n-1]$$

$n=0$: $y[0] = 1$, $n \geq 1$: $y[n] = (\frac{1}{2})^n + (\frac{1}{2})^{n-1} = 3 \cdot (\frac{1}{2})^n$

(c) $x = \{\underset{\uparrow}{0}, 0, 2\}$ $h[n] = (\frac{1}{2})^n u[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[n-k] \cdot x[k]$$
$$= 2h[n-2]$$
$$= 2 \cdot (\tfrac{1}{2})^{n-2} \cdot u[n-2]$$
$$= (\tfrac{1}{2})^{n-3} u[n-2]$$

(d) $x[n] = \delta[n-4]$, $h[n] = u[n] - u[n-4]$

$y[n] = \sum_{-\infty}^{\infty} x[k] \cdot h[n-k]$, $x[k] = \delta[k-4]$ only when $k=4$ has value 1.

$\Rightarrow y[n] = h[n-4] = u[n-4] - u[n-8]$

$$\therefore y[n] = \begin{cases} 0 & n < 4,\ n \geq 8 \\ 1 & 4 \leq n < 8 \end{cases}$$

[2] $y = g$ for $x = \{\underset{\uparrow}{0}, 2\} = 2\delta[n-1]$, $x \xrightarrow{T} y = g$

$g = T(2\delta[n-1])$, $h = T(\delta[n]) = \frac{1}{2} g[n+1]$

[3] (a) impulse response: $h = \delta[n] + 2\delta[n-1] = \begin{cases} 1 & n=0 \\ 2 & n=1 \\ 0 & \text{other} \end{cases}$

(b)
$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$
$$= x[n] + 2x[n-1]$$
$$= u[n] - u[n-4] + 2(u[n-1] - u[n-5])$$
$$= \begin{cases} 1 & 0 \leq n < 1 \\ 3 & 1 \leq n < 4 \\ 2 & 4 \leq n < 5 \\ 0 & n < 0,\ n \geq 5 \end{cases} = \{\underset{\uparrow}{1}, 3, 3, 3, 2, 0\}$$

[4]. $y[n] = \frac{1}{2} y[n-1] + x[n] + 2x[n-1]$. $\underbrace{y[n] = 0 \text{ for } n < 0}_{\text{zero initial condition.}}$

(a) impulse response: $x = \delta[n]$, $y = \{\underset{\uparrow}{1}, \frac{5}{2}, \frac{5}{4} \cdots\}$

$\therefore h[n] = u[n] - u[n-1] + 5(\frac{1}{2})^n u[n-1]$

$= u[n] + [5 \cdot (\frac{1}{2})^n - 1] u[n-1]$

(b)
$$y[n] = \sum_{-\infty}^{\infty} h[n-k]x[k]$$
$$= \sum_{k=0}^{3} h[n-k]$$
$$= h[n] + h[n-1] + h[n-2] + h[n-3]$$
$$= u[n] + 5 \cdot (\tfrac{1}{2})^n u[n-1] + 5(\tfrac{1}{2})^{n-1} u[n-2] + 5 \cdot (\tfrac{1}{2})^{n-2} u[n-3] + [5(\tfrac{1}{2})^{n-3} - 1] u[n-4]$$
$$= \begin{cases} 0 & n < 0 \\ \frac{7}{2} & n = 1 \\ \frac{19}{4} & n = 2 \\ \frac{43}{8} & n = 3 \\ \frac{75}{2^n} & n \geq 4 \end{cases}$$