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section:

PROBLEM SET #5

[1].

	$x=1$	$x=2$	$x=3$	$x=4$
H_0	0.25	0.25	0.25	0.25
H_1	0.1	0.2	0.3	0.4

(a) see the figure above

(b)

	$x=1$	$x=2$	$x=3$	$x=4$
H_0	0.25	0.25	0.25	0.25
H_1	0.1	0.2	0.3	0.4
Λ	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{8}{5}$

$\Lambda<1$: choose H_0

$\Lambda>1$: choose H_1

so declare H_0 when $x=1, x=2$,

otherwise declare H_1 $\Rightarrow$ the same as (a)

(c) $P_{\text{false alarm}} = 0.25+0.25 = 0.5$

$P_{\text{miss}} = 0.1+0.2 = 0.3$

(d) $P_e = \pi_0 P_{\text{false alarm}} + \pi_1 \cdot P_{\text{miss}} = \frac{1}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{3}{10} = \frac{11}{30}$

[2].

	$x=1$	$x=2$	$x=3$	$x=4$
H_0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
H_1	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$

$\pi_0=\frac{1}{3}$ $\pi_1=\frac{2}{3}$

(a) see figure above

(b)

	$x=1$	$x=2$	$x=3$	$x=4$
H_0	0.25	0.25	0.25	0.25
H_1	0.1	0.2	0.3	0.4
Λ	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{8}{5}$

threshold $\tau=\frac{\pi_0}{\pi_1}=\frac{1}{2}$

$\Lambda<\frac{1}{2}$: H_0

thus when $x=1$, declare H_0 is true,

otherwise declare H_1 is true $\Rightarrow$ the same as (a)

(c) $P_e = \frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{15} = \frac{19}{60}$

(d) the p_e for the MAP is smaller for the p_e for the ML rule.

[3].

(a) the process is geometric distribution

H_1: $p_1(x=k) = (1-p_1)^{k-1}\cdot p_1$

H_0: $p_0(x=k) = (1-p_0)^{k-1}\cdot p_0$

likelihood ratio test:

$\Lambda = \frac{p_1(k)}{p_0(k)} = \left(\frac{1-p_1}{1-p_0}\right)^{k-1}\cdot\frac{p_1}{p_0}$

$\Lambda \geq 1$: $\left(\frac{1-p_1}{1-p_0}\right)^{k-1}\cdot\frac{p_1}{p_0} \geq 1$

$\left(\frac{1-p_1}{1-p_0}\right)^{k-1} \geq \frac{p_0}{p_1}$

$(k-1)\ln\left(\frac{1-p_1}{1-p_0}\right) \geq \ln\left(\frac{p_0}{p_1}\right)$

since $0<p_0<p_1<1$, $\frac{1-p_1}{1-p_0}<1$, $\ln\left(\frac{1-p_1}{1-p_0}\right)<0$

$k-1 \leq \frac{\ln\left(\frac{p_0}{p_1}\right)}{\ln\left(\frac{1-p_1}{1-p_0}\right)}$

$k \leq \frac{\ln\left(\frac{p_0}{p_1}\right)}{\ln\left(\frac{1-p_1}{1-p_0}\right)}+1$

so when $k \leq \frac{\ln\left(\frac{p_0}{p_1}\right)}{\ln\left(\frac{1-p_1}{1-p_0}\right)}+1$, we declare H_1 is true.

otherwise, we declare H_0 is true.

(b) $\Lambda(1) = \frac{0.75}{0.25} = 3$ $\Lambda(2) = \frac{0.75}{0.25}\cdot\frac{1-0.75}{1-0.25} = 1$

so for $k=1$ and 2, we declare H_1 is true.

otherwise, we declare H_0 is true.

so the $P_{\text{false alarm}} = P_0(X=1)+P_0(X=2) = \frac{1}{4}+\frac{1}{4}\times\frac{3}{4} = \frac{7}{16}$

$P_{\text{miss}} = P_1(X>2) = (1-p_1)^2 = 0.25^2 = \frac{1}{16}$

[4]. $\pi_0 = 3\pi_1 \Rightarrow \pi_0 = \frac{3}{4}$, $\pi_1 = \frac{1}{4}$

(a) for H_1: $P_1(X=k) = \frac{1}{4}\cdot(1-p_1)^{k-1}\cdot p_1$

for H_0: $P_0(X=k) = \frac{3}{4}\cdot(1-p_0)^{k-1}\cdot p_0$

$\Delta = \frac{P_1(k)}{P_0(k)} = \frac{1}{3}\cdot\left(\frac{1-p_1}{1-p_0}\right)^{k-1}\cdot\frac{p_1}{p_0}$

$\Delta \geq 1 \Rightarrow \frac{1}{3}\cdot\left(\frac{1-p_1}{1-p_0}\right)^{k-1}\cdot\frac{p_1}{p_0} \geq 1$

$\Rightarrow \left(\frac{1-p_1}{1-p_0}\right)^{k-1} \geq \frac{3p_0}{p_1}$

again like [3], we can get $k \leq \frac{\ln\left(\frac{3p_0}{p_1}\right)}{\ln\left(\frac{1-p_1}{1-p_0}\right)} + 1$

so when $k \leq \frac{\ln\left(\frac{3p_0}{p_1}\right)}{\ln\left(\frac{1-p_1}{1-p_0}\right)} + 1$, we declare H_1 is true.

otherwise, we declare H_0 is true.

(b) $\Delta(1) = \frac{1}{3}\cdot 3 = 1$ $\Delta(2) = \frac{1}{3}\cdot\frac{1}{3}\cdot 3 = \frac{1}{3}$

so when $k=1$, we declare H_1 is true,

otherwise we declare H_0 is true.

$p_e = P(H_0, X=1) + P(H_1, X>1)$

$= \frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot(1-\frac{3}{4})^1$

$= \frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4}$

$= \frac{1}{4}$