UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Fall 2023

Homework 3

Profs. Do, Snyder, Moustakides **Due: Sep 15 on Gradescope**

---

1. We are given the following impulse responses of LTI systems:

   (a) $h[n] = \delta[n] + 2\delta[n+2] - 3\delta[n-4]$

   (b) $h[n] = \left(\frac{1}{3}\right)^n u[n+1] + \left(\frac{1}{2}\right)^n u[-(n+1)]$

   (c) $h[n] = n\left(\frac{1}{3}\right)^n u[n-2] + \left(\frac{1}{2}\right)^n u[n+3]$

   (d) $h[n] = \left\{A_1(z_1)^n + A_2(z_2)^n\right\}u[n]$, where A_1, A_2, z_1, z_2 are constant numbers not depending on n and A_1, A_2 are not zero.

   Using the definition and the properties of the $\mathcal{Z}$-transform, determine for each case the $\mathcal{Z}$-transform of the impulse response (transfer function) and the corresponding ROC.

2. We are given the $\mathcal{Z}$-transform pair

$$x[n] \overset{\mathcal{Z}}{\leftrightarrow} X(z) = \frac{1}{1 - 0.5z^{-1}}, \quad \text{ROC} : |z| > 0.5, \quad \text{with } \alpha \neq 0.$$

   Using the $\mathcal{Z}$-transform properties determine the $\mathcal{Z}$-transform and the corresponding ROC of the following sequences **without computing $x[n]$**:

   (a) $y[n] = x[n - n_0]$, where n_0 is a constant integer.

   (b) $y[n] = n(n-1)x[n]$

   (c) $y[n] = \beta^n x[n]$, where $\beta \neq 0$ is a constant.

   (d) $y[n] = x[n] * x[-n]$

3. Using the properties of the $\mathcal{Z}$-transform determine the signal $x[n]$ that satisfies the following equations:

   (a) $x[-n] = \rho^n u[n]$

   (b) $nx[n] = 3x[n]$, with the $\mathcal{Z}$-transform of $x[n]$ satisfying $X(z)|_{z=1} = 1$

   (c) $x[n] = 1.3x[n-1] - 0.4x[n-2] + \delta[n] + \delta[n-1]$, where $x[n]$ is rightsided

   (d) $\left(\frac{1}{2}\right)^n x[n] = \left(1 - \left(\frac{1}{3}\right)^n\right)u[n]$