ECE 313: Problem Set 10

Due: Friday, November 3 at 7:00:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 4.1–4.3
  Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

  Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON Canvas
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.
5 points will be deducted for a submission with incorrectly-assigned page numbers.

1. [**Joint cumulative distribution functions**]
   Consider the following function:

   $$F(u, v) = \begin{cases} 0, & u + v \leq 1, \\ 1, & u + v > 1. \end{cases}$$

   Is this a valid joint CDF? Why or why not? Prove your answer and show your work.

2. [**Joint pmf**]
   The joint pmf $p_{X,Y}(u, v)$ of X and Y is shown in the table below:

	$u = 0$	$u = 1$	$u = 2$	$u = 3$
$v = 4$	0	0.1	0.1	0.2
$v = 5$	0.2	0	0	0
$v = 6$	0	0.2	0.1	0.1

   (a) Find the marginal pmfs $p_X(u)$ and $p_Y(v)$.

   (b) Let $Z = X + Y$. Find p_Z, the pmf of Z.

   (c) Find $p_{Y|X}(v|3)$ for all v and find $E[Y|X = 3]$.

3. [**Joint pmf**]
   Two fair six-sided dice are rolled. One of the dice shows Z_1 pips (pips are small dots on each face of a six-sided die), the other shows Z_2 pips. The random variables X and Y are defined as follows:

   $$X = \min(Z_1, Z_2)$$
   $$Y = \max(Z_1, Z_2)$$

(a) Sketch the support, in the (u, v) plane, of the joint pmf $p_{X,Y}(u, v)$.

(b) Find the joint pmf $p_{X,Y}(u, v)$.

(c) Find the marginal pmf of Y.

(d) Find $E[Y - X]$.

4. **[Joint pdf]**
   The jointly continuous random variables X and Y have joint pdf

$$f_{X,Y}(u, v) = \begin{cases} 1.5 & 0 \leq u < 1, \ 0 \leq v < 1, \ 0 \leq u + v < 1, \\ 0.5 & 0 \leq u < 1, \ 0 \leq v < 1, \ 1 \leq u + v < 2, \end{cases}$$

and zero elsewhere.

(a) Find the marginal pdf of Y.

(b) Find $P(X + Y \geq \frac{3}{2})$.

(c) Find $P(X^2 + Y^2 \leq 1)$.

5. **[Joint pdf]**
   X and Y are two random variables with the following joint pdf:

$$f_{X,Y}(u, v) = \begin{cases} A(1 - |u - v|), & 0 < u < 1, 0 < v < 1, \\ 0, & \text{else.} \end{cases}$$

(a) Find A.

(b) Find marginal pdfs for X and Y.

(c) Find $P\{X > Y\}$.

(d) Find $P\{X + Y < 1 | X > 1/2\}$.