[1].

(a) $\{h_n\}_{n=0}^{2} = \{2,1,2\} = 2\delta[n]+\delta[n-1]+2\delta[n-2]$

$H(w) = 2 + e^{-jw} + 2e^{-2jw}$

$= e^{-jw}(2e^{jw}+1+2e^{-jw})$

$= (4\cos w+1)\cdot e^{-jw}$

it is a GLP filter, $A(w)=4\cos w+1$, $\alpha=1, \beta=0$

it is a linear phase filter

(b) $\{h_n\}_{n=0}^{2} = \{-1,3,1\} = -\delta[n]+3\delta[n-1]+\delta[n-2]$

$H(w) = -1+3e^{-jw}+e^{-j\cdot 2w}$

$=e^{-jw}(-e^{jw}+e^{-jw}+3)$

$= (3+2j\sin w)\, e^{-jw}$

it is not a GLP filter

(c) $\{h_n\}_{n=0}^{2} = \{1,0,-1\} = \delta[n]-\delta[n-2]$

$H(w) = 1-e^{-2jw} = e^{-jw}(e^{jw}-e^{-jw})$

$= 2j\sin w\cdot e^{-jw}$

$= 2\sin w\cdot e^{-jw+\frac{\pi}{2}}$

it is a GLP filter, $A(w)=2\sin w$, $\alpha=1$, $\beta=\frac{\pi}{2}$

it is not a linear phase filter.

(d) $\{h_n\}_{n=0}^{3} = \{1,1,-1,-1\} = \delta[n]+\delta[n-1]-\delta[n-2]-\delta[n-3]$

$H(w) = 1+e^{-jw}-e^{-j2w}-e^{-j3w} = e^{-j\cdot\frac{3}{2}w}[e^{j\cdot\frac{3}{2}w}+e^{j\cdot\frac{1}{2}w}-e^{-j\cdot\frac{1}{2}w}-e^{-j\frac{3}{2}w}]$

$= 2j\,[\sin(\frac{1}{2}w)+\sin(\frac{3}{2}w)]\cdot e^{-j\cdot\frac{3}{2}w}$

$=2[\sin(\frac{1}{2}w)+\sin(\frac{3}{2}w)]\,e^{-j\cdot\frac{3}{2}w+\frac{\pi}{2}}$

it is a GLP filter, $A(w)=2[\sin(\frac{1}{2}w)+\sin(\frac{3}{2}w)]$, $\alpha=\frac{3}{2}$, $\beta=\frac{\pi}{2}$

it is not a linear phase filter.

(e) $\{h_n\}_{n=0}^{3} = \{2,1,1,2\} = 2\delta[n]+\delta[n-1]+\delta[n-2]+2\delta[n-3]$

$H(w) = 2+e^{-jw}+e^{-j2w}+2e^{-j3w}$

$= e^{-j\cdot\frac{3}{2}w}[2(e^{j\cdot\frac{3}{2}w}+e^{-j\cdot\frac{3}{2}w})+e^{j\cdot\frac{1}{2}w}+e^{-j\cdot\frac{1}{2}w}]$

$= [4\cos(\frac{3}{2}w)+2\cos(\frac{1}{2}w)]\,e^{-j\frac{3}{2}w}$

it is a GLP filter, $A(w) = 4\cos(\frac{3}{2}w)+2\cos(\frac{1}{2}w)$, $\alpha=\frac{3}{2}$, $\beta=0$

it is a linear phase filter

[2] length: M+1, lowpass, GLP FIR filter, cut-off freq: $\frac{\pi}{3}$

$h[n] = g[n]\cdot w[n]$, $g[n]=d[n-\alpha] = \frac{\sin(w_c(n-\alpha))}{\pi(n-\alpha)}$, $\alpha=\frac{M+1-1}{2}=\frac{M}{2}$, $w_c=\frac{\pi}{3}$

$\Rightarrow h[n] = \frac{\sin(\frac{\pi}{3}(n-\frac{M}{2}))}{\pi(n-\frac{M}{2})}\cdot w[n]$

(a) rectangular window: $\{h[n]\}_{n=0}^{M} = \frac{\sin(\frac{\pi}{3}(n-\frac{M}{2}))}{\pi(n-\frac{M}{2})}$

(b) hamming window, $\{h[n]\}_{n=0}^{M} = \frac{\sin(\frac{\pi}{3}(n-\frac{M}{2}))}{\pi(n-\frac{M}{2})}\cdot[0.54-0.46\cos(\frac{2\pi n}{M-1})]$

(c) rectangular window: narrowest transition band, weakest stopband attenuation

hamming window, widest transition band, strongest stopband attenuation.

[3]

(a) the ideal characteristic should have a cutoff frequency at W_p, with passband ripple as small as possible (like 0.1dB) and the stopband should contain $(-\pi,-W_s]\cup[W_s,\pi)$, with stopband attenuation about -100dB (10^{-5})

(b) no: for a stopband attenuation 10^{-4}, that is -80dB, thus from the figure of window method we taught in class, the best is hamming window with stopband attenuation around -60dB.

[4].

(a) the CTFT of the derivative: $Y_c(\Omega) = j\Omega X_c(\Omega)$

(b) $x_c(t)$, $x_c'(t)$ $T_s \Rightarrow x[n]$ $y[n]$

$$X_d(w) = \frac{1}{T_s}\sum_{-\infty}^{\infty} X_a\left(\frac{w-2n\pi}{T_s}\right)$$

$$Y_d(w) = \frac{jw}{T_s^2}\sum_{-\infty}^{\infty} X_a\left(\frac{w-2n\pi}{T_s}\right)$$

$D(w) = \frac{jw}{T_s}$, $-w_c \le w \le w_c$, obviously, $Y_d(w) = D(w)\cdot X_d(w)$

(c) for method (1): first using type I filter, then using type III filter.

for method (2): using type III filter

lowpass filter: $\frac{\sin(w_c n)}{\pi n} \xrightarrow{\text{shift}} \frac{\sin(w_c(n-\alpha))}{\pi(n-\alpha)}$, so $X(w) \longrightarrow X(w)\cdot e^{-j\cdot\alpha w}$, $\alpha = \frac{N-1}{2}$, delay: $-\frac{d\angle H(w)}{dw} = \alpha$

$\Rightarrow$ final delay: $\frac{N-1}{2}$

ideal frequency response $H(w)$: lowpass $H_1(w) = e^{-j\alpha w}$ differentiator $H_2(w) = \frac{jw}{T_s}$

method (1):

$H(w) = H_1(w)\cdot H_2(w) = \frac{jw}{T_s}\cdot e^{-j\alpha w}$

method (2):