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section:

PROBLEM SET #6

[1].

(a) for a particular subsystem fails, at least two server failures

$p_0 = 1-(1-p)^9 - 9\cdot(1-p)^8\cdot p$, when $p=0.01$, $p_0 = 0.0000358$

(b) for the overall system fails, at least two subsystem fails

$p_1 = 1-(1-p_0)^9 - 9\cdot(1-p_0)^8\cdot p_0$, when $p=0.01$, $p_1 = 0.00000004621$

(c) consider each server as $S_1 \sim S_9$, the subsystem is A

$p_0 = P(A) = P\left(\bigcup_{1\le i<j\le 9} S_iS_j\right) \le \sum_{1\le i<j\le 9} P(S_iS_j)$

$= 36p^2$

consider each subsystem as $A_1 \sim A_9$, the whole system is E

$p_1 = P(E) = P\left(\bigcup_{1\le i<j\le 9} A_iA_j\right) \le \sum_{1\le i<j\le 9} P(A_iA_j)$

$= 36\cdot p_0^2$

when $p=0.001$, $p_0 = 3.6\times10^{-5}$ $p_1 = 4.6656\times10^{-8}$

[2].

(a) $\int_0^A \sin(u)du = 1$

$\int_0^A \sin(u)du = [-\cos u]_0^A = -\cos A + 1$

$\Rightarrow -\cos A = 0$, $A = \frac{\pi}{2}+2k\pi, k\in\mathbb{Z}$, notice that $\sin(u)\ge0$ should always right $\Rightarrow A=\frac{\pi}{2}$

(b) if $c\le0$: $F_X(c)=0$ if $c\ge A$, $F_X(c)=1$

if $0<c<A$: $F_X(c) = \int_0^c \sin(u)du = 1-\cos(c)$

so $F_X(c) = \begin{cases} 0 & c\le 0 \\ 1-\cos(c) & 0<c<\frac{\pi}{2} \\ 1 & c\ge\frac{\pi}{2} \end{cases}$

(c) $E[X] = \int_0^A u\sin u\,du$

$= \int_0^A u\,d(-\cos u)$

$= [-u\cos u]_0^A + \int_0^A \cos u\,du$

$= 0 + [\sin u]_0^{\frac{\pi}{2}}$

$= 1$

(d) $E[X^2] = \int_0^A u^2\sin u\,du$

$= [-u^2\cos u]_0^{\frac{\pi}{2}} + \int_0^A \cos u\,du^2$

$= 0 + 2\int_0^A u\cos u\,du$

$= [2u\sin(u)]_0^{\frac{\pi}{2}} - 2\int_0^A \sin u\,du$

$= \pi + 2[\cos u]_0^A$

$= \pi - 2$

$Var(X) = E(X^2) - [E(X)]^2$

$= \pi - 3$

(e) $Y = 2X$:

pdf $f_Y(u) = \begin{cases} \frac{1}{2}\sin(\frac{u}{2}) & 0\le u\le\pi \\ 0 & \text{otherwise} \end{cases}$

CDF $F_Y(c)$: when $c\le0$, $F_Y(c)=0$, when $c\ge\pi$, $F_Y(c)=1$

when $0<c<\pi$:

$F_Y(c) = \int_0^c \frac{1}{2}\sin(\frac{u}{2})du$

$= -[\cos(\frac{u}{2})]_0^c$

$= 1-\cos(\frac{c}{2})$ $\Rightarrow$ so $F_Y(c) = \begin{cases} 0 & c\le0 \\ 1-\cos(\frac{c}{2}) & 0<c<\pi \\ 1 & c\ge\pi \end{cases}$

[3].

$$F_X(c) = \begin{cases} 0 & c < -10 \\ \frac{1}{2} & -10 \le c < -5 \\ \frac{2}{3} & -5 \le c < 0 \\ \frac{4}{5} & 0 \le c < 5 \\ 1 & c \ge 5 \end{cases} \Rightarrow$$

X should be discrete random variable

$p_X(-10) = \frac{1}{2}$ $\quad p_X(-5) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$p_X(0) = \frac{4}{5} - \frac{2}{3} = \frac{2}{15}$ $\quad p_X(5) = 1 - \frac{4}{5} = \frac{1}{5}$

(a) $P(X \le 3.5) = p_X(-10) + p_X(-5) + p_X(0) = \frac{4}{5}$

(b) $P(X \ge -4.37) = p_X(0) + p_X(5) = \frac{1}{3}$

(c) $P(|X| \le 2) = p_X(0) = \frac{2}{15}$

(d) $P(X^2 \le 9) = P(|X| \le 3) = p_X(0) = \frac{2}{15}$

(e) $E[X] = -10 \cdot \frac{1}{2} - 5 \cdot \frac{1}{6} + 0 \cdot \frac{2}{15} + 5 \cdot \frac{1}{5} = -\frac{29}{6}$

(f) $E[X^2] = 100 \cdot \frac{1}{2} + 25 \cdot \frac{1}{6} + 25 \cdot \frac{1}{5} = \frac{355}{6}$

$Var(X) = E[X^2] - E[X]^2 = \frac{355}{6} - \frac{841}{36} = \frac{1289}{36}$

[4] consider the cut off point as X

$$p_X(u) = \begin{cases} 1 & 0 \le u \le 1 \\ 0 & \text{otherwise} \end{cases}$$

p

consider the expected length as Y

$$E[Y] = \int_0^p (1-u)\,du + \int_p^1 u\,du$$
$$= [u - \tfrac{1}{2}u^2]_0^p + [\tfrac{1}{2}u^2]_p^1$$
$$= p - \tfrac{1}{2}p^2 + \tfrac{1}{2} - \tfrac{1}{2}p^2$$
$$= p + \tfrac{1}{2} - p^2$$

[5].

(a) consider $p_X(u) = \begin{cases} \frac{1}{a-b} & b \le u \le a \\ 0 & \text{else} \end{cases}$

$E[X] = 5 = \frac{a+b}{2}$ $\quad Var(X) = 12 = \frac{(a-b)^2}{12}$

$\therefore a - b = 12,\ a + b = 10$

$\Rightarrow a = 11,\ b = -1$

pdf $p_X(u) = \begin{cases} \frac{1}{12} & -1 \le u \le 11 \\ 0 & \text{else} \end{cases}$

(b) $p_X(u) = \begin{cases} \frac{1}{4} & -2 \le u \le 2 \\ 0 & \text{else} \end{cases}$

$E[X^4] = \int_{-2}^{2} \frac{1}{4} u^4\,du = [\frac{1}{20}u^5]_{-2}^{2} = \frac{2 \cdot 2^5}{20} = \frac{16}{5}$

$E[X^5] = \int_{-2}^{2} \frac{1}{4} u^5\,du = [\frac{1}{24}u^6]_{-2}^{2} = 0$

$E[e^X] = \int_{-2}^{2} \frac{1}{4} e^u\,du = [\frac{1}{4}e^u]_{-2}^{2} = \frac{1}{4}(e^2 - e^{-2})$