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section:

PROBLEM SET # 4

[1]. $\hat{p}_a = 1 - \frac{E}{n}$

(a) the probability distribution of E is: $P_E(k) = C_n^k \cdot (1-p_a)^k \cdot p_a^{n-k}$, $k = 0, 1, 2 \cdots n$

(b) take $a = 2$, $\frac{a}{2\sqrt{n}} = 0.1$

The true accuracy of her design p_a lies in the interval 0.95 ± 0.1 with a confidence level greater than 75%. I think it is not ok to say the accuracy of $p_a = 0.95$ because the confidence interval is really big.

(c) $1 - \frac{1}{a^2} = 95\% \Rightarrow a = \sqrt{20}$, $\frac{a}{2\sqrt{n}} = 0.01$, $\frac{\sqrt{5}}{\sqrt{n}} = \frac{1}{100}$ $n = 50000$

(d) $1 - \frac{1}{a^2} = 80\% \Rightarrow a = \sqrt{5}$, $\frac{a}{2\sqrt{n}} = 0.01$ $\frac{\sqrt{5}}{2\sqrt{n}} = \frac{1}{100}$ $\sqrt{n} = 50\sqrt{5}$, $n = 2500 \cdot 5 = 12500$

thus, for each chip: $\frac{12500}{50} = 250$ images are needed

[2]. the process is a binomial distribution, conside it as random variable X: X heads are observed during 10 times tossing coin.

(a) $P_X(6) = C_{10}^6 \cdot p^6 \cdot (1-p)^4$

$\frac{dP_X(6)}{dp} = C_{10}^6 \cdot [6p^5(1-p)^4 - p^6 \cdot 4(1-p)^3]$

$= C_{10}^6 \cdot 2p^5(1-p)^3 \cdot [3(1-p) - 2p]$

$= 2 \cdot C_{10}^6 \cdot p^5 \cdot (1-p)^3 \cdot [3 - 5p]$

when $p = \frac{3}{5}$, $P_X(6)$ is largest, thus the maximum likelihood estimate $\hat{p}_{ML} = 0.6$

(b). consider the random variable Y: Y time tossed, 6 time heads are observed.

$P_Y(n) = C_n^6 \cdot 0.05^6 \cdot 0.95^{n-6}$, $n \geq 6$

$\frac{P_Y(n-1)}{P_Y(n)} \leq 1 \Rightarrow \begin{cases} \frac{1}{0.95} \cdot \frac{n-6}{n} \leq 1 \\ 0.95 \cdot \frac{n+1}{n-5} \leq 1 \end{cases} \Rightarrow 6 \leq 0.05n \leq 5.95$, $119 \leq n \leq 120$

$\frac{P_Y(n+1)}{P_Y(n)} \leq 1$ so $\hat{n}_{ML} = 119$ or 120.

(c) this process is a geometric distribution. let it be random variable Z

$P_Z(10) = (1-p)^9 \cdot p$, $\frac{dP_Z(10)}{dp} = -9(1-p)^8 \cdot p + (1-p)^9 = (1-p)^8 \cdot (1-p-9p) = (1-p)^8 \cdot (1-10p)$

$\Rightarrow p = \frac{1}{10}$, $P_Z(10)$ max, $\hat{p}_{ML} = \frac{1}{10}$

[3].

(a) for each bit of the word, should be $\{0,1\}$ from $\{0,1,2,3\}$, thus $p = \frac{1}{2}$,

each bit is independent, so probability is: $(\frac{1}{2})^8 = \frac{1}{256}$

(b) the probability is $\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{256} = \frac{257}{512}$

(c) the probability is $\frac{\frac{1}{2}}{\frac{257}{512}} = \frac{256}{257}$

[4]. bug 1: $\frac{1}{5}$ message 1 (M1), $\frac{4}{5}$ message 2 (M2)

bug 2: $\frac{2}{3}$ M1 , $\frac{1}{3}$ M2

bug 1: $\frac{5}{6}$ bug 2: $\frac{1}{6}$

(a) the probability is $\frac{5}{6} \cdot \frac{1}{5} + \frac{1}{6} \cdot \frac{2}{3} = \frac{5}{18}$

(b) the probability is $\frac{\frac{5}{6} \cdot \frac{1}{5}}{\frac{5}{18}} = \frac{3}{5}$

[5]

(a) $\frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot (1-p) = \frac{7}{12}$

$\therefore \frac{1}{3}(1-p) = \frac{1}{4}$, $1-p = \frac{3}{4}$,

$p = \frac{1}{4}$

(b) a head and a tail's probability is $\frac{1}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot [\frac{1}{2} p + \frac{1}{2} \cdot (1-p)] = \frac{1}{2}$

the conditional probability is $\frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$