UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Fall 2023

Homework 13

Profs. Do, Snyder, Moustakides **Due: Wednesday, December 6, 11:59PM on Gradescope**

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1. Suppose we have the digital rate conversion system given below with input signal $x[n]$ and output signal $y[n]$. The following two parts are unrelated.

$$x[n] \longrightarrow \boxed{\downarrow 3} \longrightarrow y[n]$$

(a) Let $x[n] = 5\sin\left(\frac{\pi}{6}n\right)$. Determine and sketch $y[n]$ for $0 \leq n < 12$.

(b) Let $X_d(\omega)$ be the DTFT of $x[n]$ where

$$X_d(\omega) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}.$$

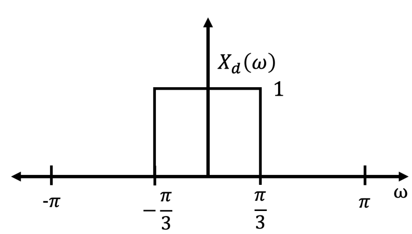
Show that the output DTFT $Y_d(\omega)$ is given by

$$Y_d(\omega) = \frac{1}{1 - \frac{1}{8}e^{-j\omega}}.$$

2. Consider the following digital rate conversion system.

$$x[n] \longrightarrow \boxed{\downarrow 4} \longrightarrow y[n] \longrightarrow \boxed{\uparrow 2} \longrightarrow z[n]$$

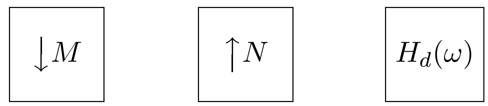
Let $X_d(\omega)$ be the DTFT of input $x[n]$ given below.



(a) Sketch the DTFT of $y[n]$, $Y_d(\omega)$, for $-\pi \leq \omega \leq \pi$. **Please carefully label your axes!**

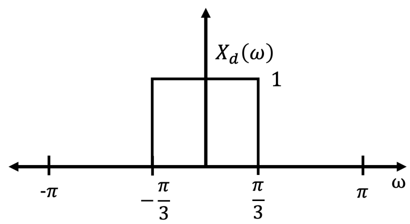
(b) Sketch the DTFT of $z[n]$, $Z_d(\omega)$, for $-\pi \leq \omega \leq \pi$.

3. We would like to implement a digital rate conversion system that **increases** the implicit sampling rate of a digital signal $x[n]$ by a factor of $\frac{5}{3}$. You are given the below building blocks for your system.



(a) Sketch the implementation of your rate conversion system, i.e. the full block diagram of your system. Specify your choices of M and N, and sketch the necessary digital filter $H_d(\omega)$.

(b) Verify that your system works properly by sketching the output of each stage with the following input signal. *Hint:* you should have three plots in total.

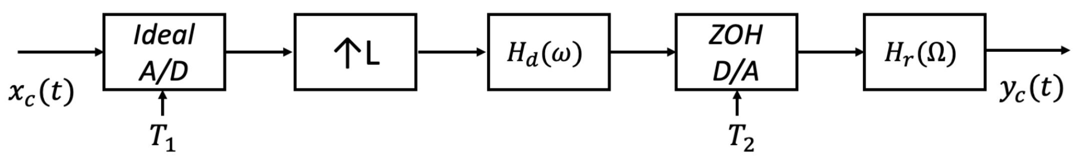


4. Let $x[n] = \delta[n-1] + 3\delta[n-4]$ be a discrete-time signal. For each of the following parts, we perform digital-to-analog conversion with a sampling period of $T = 1/10$ seconds to recover $x_a(t)$.

(a) Sketch the resulting analog $x_a(t)$ from an ideal D/A converter.

(b) Sketch the resulting $x_a(t)$ from a ZOH, i.e. with no compensation filter.

5. Consider the following practical D/A system.



Suppose $x_c(t)$ is a bandlimited continuous-time signal with maximum radial frequency $\Omega_0 = 4\pi \times 10^3$ and we sample $x_c(t)$ with sampling period $T_1 = \frac{1}{5{,}000}$ seconds. Above, $H_r(\Omega)$ is the analog compensation filter for a practical D/A.

(a) Let $L = 1$ and thus $H_d(\omega)$ is simply an all-pass system that does not modify the input signal.

   i. What is the proper choice of T_2 in this scheme?

   ii. What is the resulting transition bandwidth for the compensation filter $H_c(\Omega)$?

(b) Let $L = 6$ where we now have an oversampled practical D/A system.

   i. Sketch the corresponding $H_d(\omega)$.

   ii. What is the proper choice of T_2 in this scheme?

   iii. What is the new transition bandwidth for the compensation filter $H_c(\Omega)$?