[1].

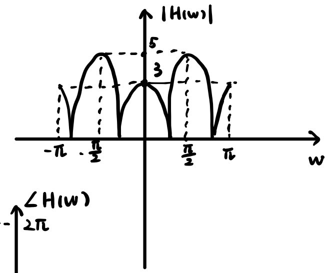
(a) $h[n]=\{\underset{\uparrow}{2},0,-1,0,2\} = 2\delta[n]-\delta[n-2]+2\delta[n-4]$

frequency response, $H(w) = 2 - e^{-j\cdot 2w} + 2e^{-j\cdot 4w}$.

magnitude response: $|H(w)| = \sqrt{(2-e^{-2jw}+2e^{-4jw})(2-e^{2jw}+2e^{4jw})}$

$= \sqrt{9+4(e^{-4jw}+e^{4jw})-4(e^{-2jw}+e^{2jw})}$

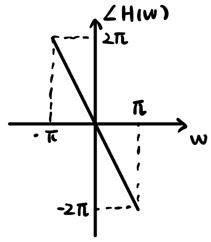
$= \sqrt{9+8\cos 4w-8\cos 2w} = |4\cos 2w - 1|$



phase response : $\angle H(w) = \tan^{-1}\left(\frac{\sin(-2w)[4\cos(-2w)-1]}{\cos(-2w)[4\cos(-2w)-1]}\right)$

$= \tan^{-1}\left(\frac{\sin(-2w)}{\cos(-2w)}\right)$

$= -2w$



(b) $h[n]=\{\underset{\uparrow}{1},-1,1,-1\} = \delta[n]-\delta[n-1]+\delta[n-2]-\delta[n-3]$

frequency response: $H(w) = 1-e^{-jw}+e^{-2jw}-e^{-3jw}$

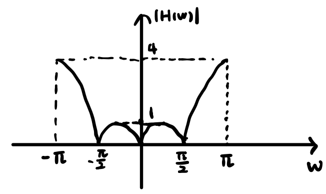
magnitude response: $|H(w)| = \sqrt{H(w)\cdot H^*(w)}$

$= \sqrt{(1-e^{-jw}+e^{-2jw}-e^{-3jw})(1-e^{jw}+e^{2jw}-e^{3jw})}$

$= \sqrt{4-e^{jw}+e^{2jw}-e^{3jw}-e^{-jw}-e^{jw}+e^{2jw}+e^{-2jw}-e^{-jw}-e^{jw}-e^{-3jw}+e^{-2jw}-e^{-jw}}$

$= \sqrt{4-(e^{3jw}+e^{-3jw})-3(e^{jw}+e^{-jw})+2(e^{-2jw}+e^{2jw})}$

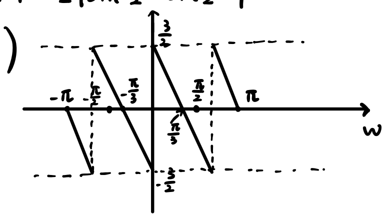
$= \sqrt{-2\cos(3w)+4\cos(2w)-6\cos w+4} = 2\left|\sin\frac{w}{2}-\sin\frac{3}{2}w\right|$



phase response $\angle H(w) = \tan^{-1}\left(\frac{-\sin(-3w)+\sin(-2w)-\sin(w)}{-\cos(-3w)+\cos(-2w)-\cos(w)+1}\right)$

$= \arctan\left(\frac{\cos\frac{3}{2}w}{\sin\frac{3}{2}w}\right)$

$= \arctan\left(\frac{1}{\tan\frac{3}{2}w}\right)$

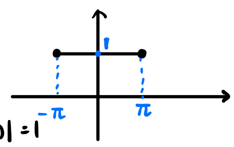


[2]. $H(w) = j$, $\pi \le w \le 2\pi$

(a) due to the 2π periodicity, $H(w-2\pi) = H(w) = j$,

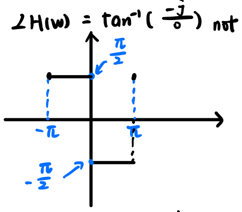
so when $-\pi \le w < 0$, $H(w) = j$, $|H(w)| = 1$

real-valued LTI $\Rightarrow H^*(w) = H(-w) \Rightarrow$ when $0 < w \le \pi$, $H(w) = H^*(-w) = -j$, $|H(w)| = 1$



(b) $\angle H(w) = \tan^{-1}\left(\frac{Im(H(w))}{Re(H(w))}\right)$ not exist for $w \in [-\pi, 0] \Rightarrow \angle H(w) = \frac{\pi}{2}$

$\angle H(w) = \tan^{-1}\left(\frac{-j}{0}\right)$ not exist for $w \in [0,\pi] \Rightarrow \angle H(w) = -\frac{\pi}{2}$



(c) $h[n] = \frac{1}{2\pi}\int_{-\pi}^{0} j\cdot e^{jwn}\,dw + \frac{1}{2\pi}\int_{0}^{\pi}(-j)e^{jwn}\,dw$

$= \frac{1}{2\pi}\cdot\frac{1}{n}e^{jwn}\Big|_{-\pi}^{0} - \frac{1}{2\pi}\cdot\frac{1}{n}e^{jwn}\Big|_{0}^{\pi}$

$= \frac{1}{2\pi n}\cdot(e^0+e^0-e^{-jn\pi}-e^{jn\pi})$

$= \frac{2}{2\pi n}[1-\cos(n\pi)]$

$= \frac{1-\cos(n\pi)}{n\pi}$

[3]. $H(w) = w^2 e^{j\sin w}$

(a) $H^*(w) = w^2 e^{-j\sin w}$

$= w^2[\cos(-\sin w) + j\cdot\sin(-\sin w)]$

$= w^2[\cos(\sin w) - j\sin(\sin w)]$

$= H(-w)$

so this system is real-valued.

(b) $x[n] = e^{j\frac{\pi}{2}n} - 2\cos(\frac{5\pi}{6}n)$

$X(w) = 2\pi\delta(w-\frac{\pi}{2}) - 2\pi[\delta(w-\frac{5\pi}{6}) + \delta(w+\frac{5}{6}\pi)]$

$Y(w) = X(w)\cdot H(w) = 2\pi[\delta(w-\frac{\pi}{2}) - \delta(w-\frac{5}{6}\pi) - \delta(w+\frac{5}{6}\pi)]\cdot w^2 e^{j\sin(w)}$

$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(w)\cdot e^{jwn}dw$

$= \int_{-\pi}^{\pi}[\delta(w-\frac{\pi}{2}) - \delta(w-\frac{5}{6}\pi) - \delta(w+\frac{5}{6}\pi)]\cdot w^2 e^{j\sin w}\cdot e^{jwn}dw$

consider $a(w) = w^2 e^{j\sin(w)}\cdot e^{jwn}$, from sifting property:

$y[n] = a(\frac{\pi}{2}) - a(\frac{5}{6}\pi) - a(-\frac{5}{6}\pi)$

$= (\frac{\pi}{2})^2 e^{j\sin(\frac{\pi}{2})}\cdot e^{j\cdot\frac{\pi}{2}n} - (\frac{5}{6}\pi)^2[e^{j\sin(\frac{5}{6}\pi)}\cdot e^{j\cdot\frac{5}{6}\pi n} + e^{j\sin(-\frac{5}{6}\pi)}\cdot e^{j\cdot(-\frac{5}{6}\pi)n}]$

$= (\frac{\pi}{2})^2 e^{j}\cdot e^{j\cdot\frac{\pi}{2}n} - (\frac{5}{6}\pi)^2\cdot 2\cos(\frac{5}{6}\pi n + \frac{1}{2})$

$= \frac{\pi^2}{4}e^{j(\frac{\pi}{2}n+1)} - \frac{25}{18}\pi^2\cdot\cos(\frac{5}{6}\pi n + \frac{1}{2})$

(c) $x[n] = 5 - e^{j\cdot\frac{\pi}{4}n} + 3\sin(\frac{\pi}{3}n + \frac{\pi}{4})$

$= 5 - e^{j\cdot\frac{\pi}{4}n} + \frac{3\sqrt{2}}{2}[\sin(\frac{\pi}{3}n) + \cos(\frac{\pi}{3}n)]$

$X(w) = 10\pi\delta(w) - 2\pi\delta(w-\frac{\pi}{4}) + \frac{3\sqrt{2}}{2}\pi[\delta(w-\frac{\pi}{3}) + \delta(w+\frac{\pi}{3})] - j\pi[\delta(w-\frac{\pi}{3}) - \delta(w+\frac{\pi}{3})]$

$Y(w) = X(w)\cdot H(w)$,

$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(w)e^{jwn}dw$

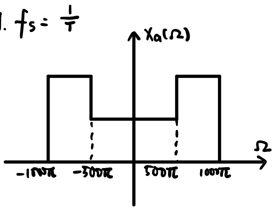
$= 5\cdot e^0\cdot 0^2\cdot e^{j\cdot\sin 0} - (\frac{\pi}{4})^2 e^{j\sin(\frac{\pi}{4})}e^{j\cdot\frac{\pi}{4}n} + \frac{3\sqrt{2}}{4}\cdot[(\frac{\pi}{3})^2 e^{j\cdot\frac{\sqrt{3}}{2}}\cdot e^{j(\frac{\pi}{3}n)} + (\frac{\pi}{3})^2 e^{-j\cdot\frac{\sqrt{3}}{2}}e^{-j(\frac{\pi}{3}n)} - j\cdot(\frac{\pi}{3})^2 e^{j\frac{\sqrt{3}}{2}}\cdot e^{j(\frac{\pi}{3}n)} + j(\frac{\pi}{3})^2 e^{-j\cdot\frac{\sqrt{3}}{2}}\cdot e^{-j(\frac{\pi}{3}n)}]$

$= -(\frac{\pi}{4})^2\cdot e^{j\cdot\frac{\sqrt{2}}{2}}\cdot e^{j\cdot\frac{\pi}{4}n} + \frac{3\sqrt{2}}{4}\cdot\frac{\pi^2}{9}\cdot 2\cos(\frac{\pi}{3}n + \frac{\sqrt{3}}{2}) + \frac{3\sqrt{2}}{4}\cdot\frac{\pi^2}{9}\cdot(-j)\cdot(2j)\cdot\sin(\frac{\pi}{3}n + \frac{\sqrt{3}}{2})$

$= -\frac{\pi^2}{16}e^{j(\frac{\pi}{4}n + \frac{\sqrt{2}}{2})} + \frac{\sqrt{2}\pi^2}{6}[\cos(\frac{\pi}{3}n + \frac{\sqrt{3}}{2}) + \sin(\frac{\pi}{3}n + \frac{\sqrt{3}}{2})]$

[4]. $f_s = \frac{1}{T}$

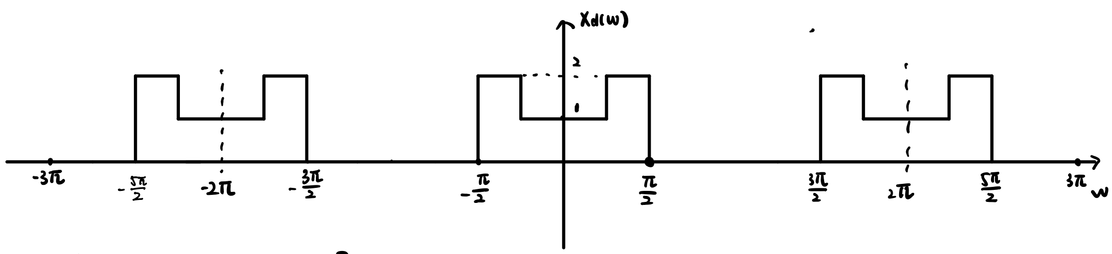
(a)



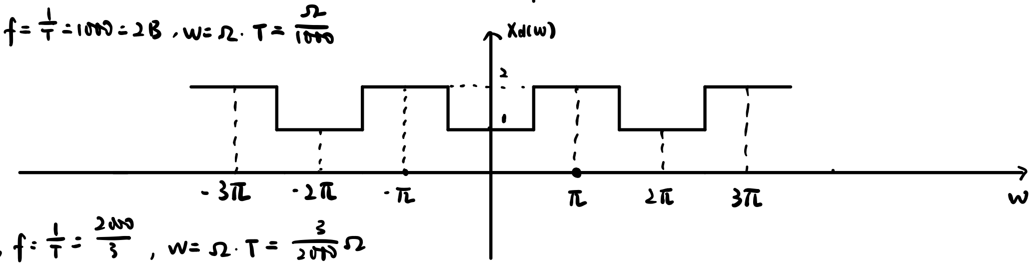
$B = \frac{1000\pi}{2\pi} = 500$

minimum sampling rate: $f = 2B = 1000$

(b) $T = \frac{1}{2000}$, $f = \frac{1}{T} = 2000 = 4B$, $w = \Omega\cdot T = \frac{\Omega}{2000}$



(c) $T = \frac{1}{1000}$, $f = \frac{1}{T} = 1000 = 2B$, $w = \Omega\cdot T = \frac{\Omega}{1000}$



(d) $T = \frac{3}{2000}$, $f = \frac{1}{T} = \frac{2000}{3}$, $w = \Omega\cdot T = \frac{3}{2000}\Omega$