ECE310 Recitation Fall 2023

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1. The frequency response of a generalized linear phase (GLP) filter can be expressed as $H(w) = A(w)e^{j(\alpha w+\beta)}$ where $A(w)$ is a real function, and α, β are real constants. For each of the following filters, determine whether it is a GLP filter. If it is, find $A(w)$, α, and β, and indicate whether it is also a linear phase filter.

   a)
$$\{h_n\}_{n=0}^{2} = \{-1, 4, 5\}$$

   b)
$$\{h_n\}_{n=0}^{2} = \{2, 0, -2\}$$

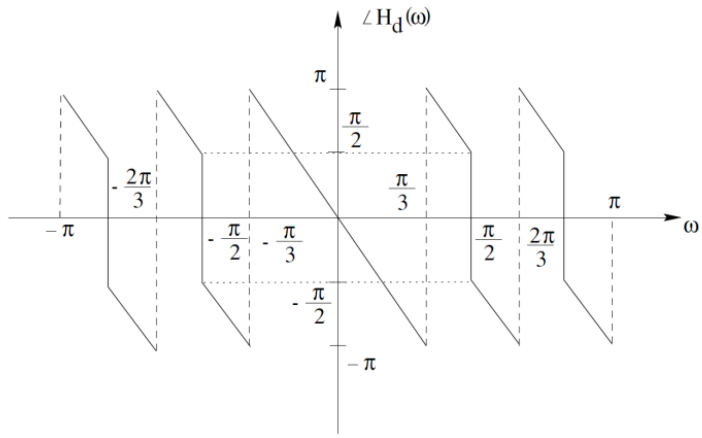
   c)
$$\{h_n\}_{n=0}^{3} = \{0, 1, 1, 0\}$$

a) **Solution:** The filter doesn't display any form of symmetry. Hence, this is not a GLP filter.

b) **Solution:** The filter displays an odd symmetry about the midpoint as $h[0] = -h[2]$ and the midpoint $h[1]$ is zero. Therefore, this is a GLP filter.
$H_d(\omega) = 2 - 2e^{-j2\omega} = 2e^{-j\omega}(e^{j\omega} - e^{-j\omega}) = 2e^{-j\omega}(2jsin(\omega)) = 4sin(\omega)e^{j(-\omega+\frac{\pi}{2})}$. $A(\omega) = 4sin(\omega)$, $\alpha = 1$ and $\beta = \frac{\pi}{2}$. $A(\omega)$ has a zero crossing at $\omega = 0$ and thus there will be a π jump in the phase plot. Hence, this is not a strict linear phase filter.

c) **Solution:** The filter displays an even symmetry and an odd order. Therefore, this filter is a Type II GLP filter.
$H_d(\omega) = e^{-j\omega} + e^{-j2\omega} = e^{-j1.5\omega}(e^{-j0.5\omega} + e^{j0.5\omega}) = 2cos(0.5\omega)e^{-j1.5\omega}$. $A(\omega) = 2cos(0.5\omega)$, $\alpha = 1.5$ and $\beta = 0$.
For $-\pi < \omega < \pi$, $A(\omega) > 0$. There won't be π jumps in the phase plot. Hence, this is a strict linear phase filter.

2. . Given the following phase response $\angle H_d(w)$ of a generalized linear-phase FIR filter, answer the following questions. Explain you answers.



Determine the system output y[n] for the following input:
a) Is the filter (i) type-1 GLP, (ii) type-2 GLP, or (iii) neither type-1 GLP nor type-2 GLP?
b) Determine the filter length from the phase plot.
c) Can you characterize the filter as (i) possibly low-pass, (ii) possibly high-pass, (iii) neither highpass nor low-pass, or (iv) the given information is insufficient to make any of the preceding statements? (Specify all correct answers).
d) Determine $H_d(\frac{\pi}{2})$

a) Is the filter (i) type-1 GLP, (ii) type-2 GLP, or (iii) neither type-1 GLP nor type-2 GLP?

Solution: For any GLP filter, $H_d(\omega) = A(\omega)e^{j(\beta-\alpha\omega)}$, where $A(\omega)$ is real. Since $\angle H_d(\omega)$ has only π or 2π jumps and constant slopes, this is a GLP filter. Since the phase is zero when $\omega = 0$, this is a type-1 GLP.

b) Determine the filter length from the phase plot.

Solution: The slope of the phase plot is -3. Therefore, the filter length is $2 \times 3 + 1 = 7$.

c) Can you characterize the filter as (i) possibly low-pass, (ii) possibly high-pass, (iii) neither high-pass nor low-pass, or (iv) the given information is insufficient to make any of the preceding statements? (Specify **all** correct answers).

Solution: Since the GLP filter is type-1 and it has an odd length, both $H_d(0)$ and $H_d(\pi)$ are not necessarily zero. This means there are no restrictions on this filter. (i), (ii) and (iii) are all correct. (Selecting iv with sufficient reasoning and explanations is also acceptable.)

d) Determine $H_d(\frac{\pi}{2})$.

Solution: From the phase plot, we can see that there is a π jump at $\omega = \frac{\pi}{2}$. From this, we can infer that $A(\omega)$ has a zero crossing at $\omega = \frac{\pi}{2}$ and thus $A(\frac{\pi}{2}) = 0$. Therefore, $H_d(\frac{\pi}{2}) = 0$.

3. Design a length-24 FIR lowpass filter with cutoff frequency $w_c = \frac{\pi}{5}$ radians using the window design method.
a) Find an expression for the filter coefficients $\{h_n\}_{n=0}^{23}$ if the rectangular window is used for the design.
b) Find an expression for the filter coefficients $\{h_n\}_{n=0}^{23}$ if the Hanning window is used for the design.

a) **Solution:** Find an expression for the filter coefficients $\{h_n\}_{n=0}^{23}$ if the rectangular window is used for the design. The impulse response of an ideal LPF is given by:

$$h_{lp}[n] = \frac{\sin(\omega_c n)}{\pi n}$$

Before applying the window, we need to shift this filter by a factor of $\frac{M}{2}$ so that our filter will be causal and exhibit generalized linear phase. This lines up with the definition of a LPF given in the textbook, where we set $\omega_c = \frac{\pi}{5}$ and $\alpha = \frac{23}{2}$. Our shifted ideal LPF is then:

$$\begin{aligned} h_{lp}[n] &= \frac{\sin(\frac{\pi}{5}(n - \frac{23}{2}))}{n - \frac{23}{2}} \\ &= \frac{1}{5} sinc(\frac{\pi}{5}(n - \frac{23}{2})) \end{aligned}$$

Here, and in following problems, we define the sinc function as $sinc(x) = \frac{\sin(x)}{x}$. Applying the rectangular window gives the solution:

$$\{h[n]\}_{n=0}^{23} = \begin{cases} \frac{1}{5} sinc(\frac{\pi}{5}(n - \frac{23}{2})) & 0 \leq n \leq 23 \\ 0 & else \end{cases}$$

b) **Solution:** Find an expression for the filter coefficients $\{h_n\}_{n=0}^{23}$ if the Hann window is used for the design. The definition of the Hann window has been given in the texbook. We simply apply that to $h_{lp}[n]$ in place of the rectangular window to get the answer:

$$\{h[n]\}_{n=0}^{23} = \begin{cases} (0.5 - 0.5\cos(\frac{2\pi n}{23}))\frac{1}{5} sinc(\frac{\pi}{5}(n - \frac{23}{2})) & 0 \leq n \leq 23 \\ 0 & else \end{cases}$$