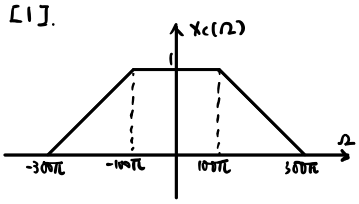
[1].

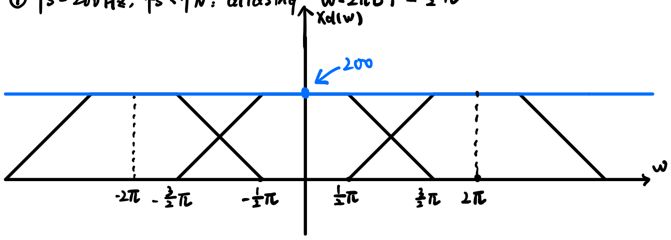


$f_s = \frac{1}{T}$ $\quad x[n] = x_c(nT)$

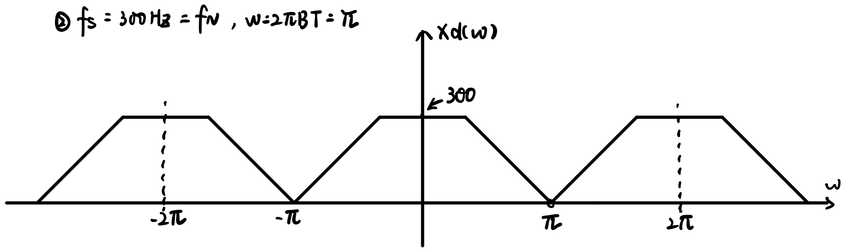
$\Omega = 300\pi$, $B = \frac{\Omega}{2\pi} = 150$

$f_N = 2B = 300\,Hz$

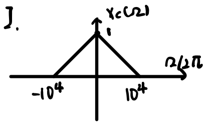
① $f_s = 200\,Hz$, $f_s < f_N$: aliasing, $w = 2\pi B T = \frac{3}{2}\pi$



② $f_s = 300\,Hz = f_N$, $w = 2\pi B T = \pi$

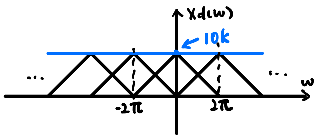


[2].

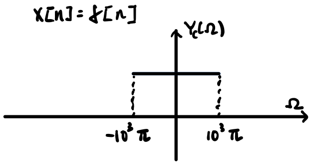


$B = 10^4$, $f_N = 2 \cdot B = 2 \cdot 10^4$

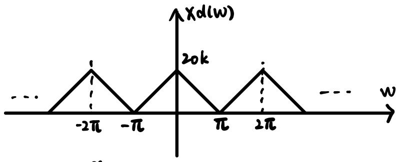
① $f_s = 10kHz$, $w = 2\pi B T = \frac{2\pi B}{f_s} = 2\pi$



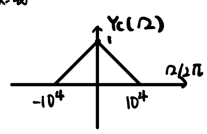
$x[n] = \delta[n]$



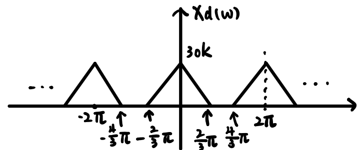
② $f_s = 20kHz = f_N$, $w = \frac{2\pi B}{f_s} = \pi$



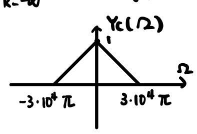
$x[n] = \sum_{k=-\infty}^{\infty} e^{j2\pi k n} \cdot \frac{1}{2\pi} \mathrm{Sinc}^2\left(\frac{n}{4\pi}\right)$



③ $f_s = 30kHz$, $w = \frac{2\pi B}{f_s} = \frac{2}{3}\pi$



$x[n] = \sum_{k=-\infty}^{\infty} e^{j2\pi k n} \cdot \frac{3}{8\pi^2} \mathrm{Sinc}^2\left(\frac{3n}{8\pi}\right)$



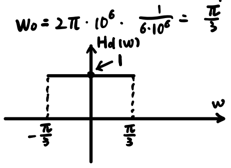
[3].

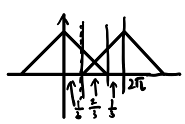
(a) bandlimited to 3MHz $\Rightarrow B = 3\cdot10^6$

Nyquist sampling rate: $f_N = 2B = 6\cdot10^6$

(b) only pass the frequency up to 1MHz:

$w_0 = 2\pi \cdot 10^6 \cdot \frac{1}{6\cdot10^6} = \frac{\pi}{3}$

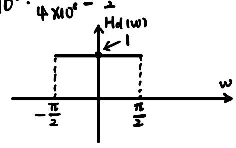




(c) since we only need the frequencies within 1MHz

$\frac{4}{3}w = 2\pi$, $w = \frac{3}{2}\pi$, $f = 4\times10^6\,Hz$, $T = 2.5\times10^{-7}s$

(d) $w_0 = 2\pi \cdot 10^6 \cdot \frac{1}{4\times10^6} = \frac{\pi}{2}$



[4]. $\{X[k]\}_{k=0}^{20}$ 21-point DFT $\{x[n]\}_{n=0}^{7}$ zero padding to 21. $\Rightarrow \{\hat{x}[n]\}_{n=0}^{21}$

(a) $X[19] = \sum_{n=0}^{20} \hat{x}[n] e^{-j\cdot\frac{2\pi\cdot19}{21}n} = \sum_{n=0}^{20} \hat{x}[n] \cdot e^{j\cdot(2\pi - \frac{2\pi\cdot19}{21}n)} = \sum_{n=0}^{20} \hat{x}[n] \cdot e^{j\frac{4\pi}{21}n}$

$X_d(-\frac{4\pi}{21}) = \sum_{n=0}^{20} \hat{x}[n] \cdot e^{j\cdot\frac{4\pi}{21}n}$

so $X[19] = X_d(-\frac{4\pi}{21})$

(b) $X[2] = \sum_{n=0}^{20} \hat{x}[n] e^{-j\cdot\frac{2\pi\cdot2}{21}n} = \sum_{n=0}^{20} \hat{x}[n] e^{-j\cdot\frac{4\pi}{21}n}$

since $x[n]$ is real-valued: $X_d^*(w) = X(-w)$

$\therefore X_d^*(-\frac{4\pi}{21}) = X_d(\frac{4\pi}{21}) = \sum_{n=0}^{20} \hat{x}[n] \cdot e^{-j\cdot\frac{4\pi}{21}n}$ $\quad 42-24=18$

so $X[2] = X_d^*(-\frac{4\pi}{21})$

(c) $X[12] = \sum_{n=0}^{20} \hat{x}[n] e^{-j\cdot\frac{2\pi\cdot12}{21}n} = \sum_{n=0}^{20} \hat{x}[n] e^{j\cdot(2\pi - \frac{2\pi\cdot12}{21}n)} = \sum_{n=0}^{20} \hat{x}[n] \cdot e^{j\cdot\frac{18\pi}{21}n}$

$X_d(-\frac{4\pi}{21}) = \sum_{n=0}^{20} \hat{x}[n] e^{j\frac{4\pi}{21}n}$

they shouldn't be the same.

(d) $X[4] = \sum_{n=0}^{20} \hat{x}[n] e^{-j \cdot \frac{2\pi \cdot 4}{21} n}$

$X_d^*(-\frac{4\pi}{21}) = X_d(\frac{4\pi}{21}) = \sum_{n=0}^{20} \hat{x}[n] \cdot e^{-j \cdot \frac{4\pi}{21} n}$

they shouldn't be the same.

[5]. $\{x[n]\}_{n=0}^{39}$ length 40 zero padding to 64: $\{y[n]\}_{n=0}^{63}$

$X(k) = \sum_{n=0}^{39} x[n] \cdot e^{-j \frac{2\pi \cdot k}{40} n}$

$Y(k') = \sum_{n=0}^{63} y[n] \cdot e^{-j \cdot \frac{2\pi k'}{64} n}$

$X(k) = Y(k')$ if and only if $\frac{2\pi k}{40} = \frac{2\pi k'}{64} \Rightarrow \frac{k}{40} = \frac{k'}{64}$

(a) $k=0, k'=0 \Rightarrow \frac{k}{40} = \frac{k'}{64}$ correct, so $X[0] = Y[0]$

(b) $k=5, k'=8 \Rightarrow \frac{k}{40} = \frac{k'}{64}$ correct, so $X[5] = Y[8]$

(c) $k=10, k'=16 \Rightarrow \frac{k}{40} = \frac{k'}{64}$ correct, so $X[10] = Y[16]$

(d) $k=12, k'=18 \Rightarrow \frac{k}{40} = \frac{k'}{64}$ wrong. $X[12] \neq Y[18]$

(e) $k=39, k'=63 \Rightarrow \frac{k}{40} = \frac{k'}{64}$ wrong $X[39] \neq Y[63]$