UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering
ECE 310 Digital Signal Processing – Fall 2023

Homework 10

Profs. Do, Snyder, Moustakides **Due: Sunday, November 5, 11:59PM on Gradescope**

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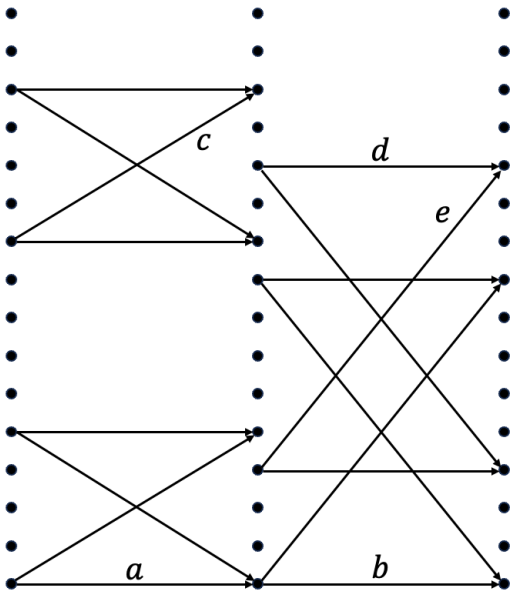
1. The diagram below depicts part of the FFT butterfly diagram for a 16-point, radix-2, decimation-in-time FFT. Determine the branch weights for the requested values $a, b, c, d,$ and e. Hint: consider the width of each butterfly to determine the length of the DFTs being merged in each stage.



2. Consider the following finite-length signals:

$$x[n] = \{\underset{\uparrow}{1}, 4, -2, 0, 3, -1\},\ h[n] = \{\underset{\uparrow}{1}, 0, -1\}$$

   (a) Compute the linear convolution $x[n] * h[n]$.
   (b) Compute the circular convolution $x[n] \circledast_6 h[n]$, where $\circledast_N$ denotes N-point circular convolution.
   (c) What is the minimum N such that $x[n] \circledast_N h[n] = x[n] * h[n]$?

3. We can perform the linear convolution of two sequences $x[n]$ and $h[n]$ via the DFT method as follows:

$$x[n] * h[n] = \text{DFT}^{-1}\{\text{DFT}\{x[n]\} \cdot \text{DFT}\{h[n]\}\}.$$

   Suppose we are given that $\{x[n]\}_{n=0}^{56}$ and $\{h[n]\}_{n=0}^{14}$ are lengths 57 and 15, respectively.

   (a) What is the minimum number of zeros we must pad to $x[n]$ and $h[n]$, respectively, to ensure that linear convolution via the DFT method will compute $x[n] * h[n]$ correctly?
   (b) Suppose we will use a radix-2 FFT to compute each of the DFTs for the DFT method. What is the minimum number of zeros that we must pad to $x[n]$ and $h[n]$, respectively?
   (c) In part (a), can zeros be padded at the beginning of each signal (instead of the end)? If so, how can we obtain $x[n] * h[n]$ from $\text{DFT}\{x[n]\} \cdot \text{DFT}\{h[n]\}$?