

ECE 313: Problem Set 6

Due: Friday, October 6 at 7:00:00 p.m.

Reading: *ECE 313 Course Notes*, Sections 2.12 - 3.4

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Mondays, and is due by 11:59 p.m. on the following Monday. You must upload handwritten homework to Gradescope. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:

NAME AS IT APPEARS ON Canvas

NETID

SECTION

PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[The reliability of a hierarchical backup system]**

Consider a parallel storage system composed of nine subsystems, each of which contains nine servers. Each subsystem can tolerate a single server failure, and the overall system can tolerate a single sub-system failure. Thus, in order for the overall system to fail, there has to be at least two subsystems that each have at least two server failures. Suppose servers fail independently with probability p .

- (a) Find an expression for the exact probability, p_0 , that a particular subsystem fails, in terms of p . Also, compute the numerical value of p_0 assuming that $p = 0.001$.
- (b) Find an expression for the exact probability, p_1 , that the overall system fails, in terms of p_0 . Also, compute the numerical value of p_1 assuming that $p = 0.001$.
- (c) Give an upper bound on p_0 and an upper bound on p_1 using the union bound, and compute their numerical values assuming that $p = 0.001$.

2. **[Continuous Random Variables]**

Random variable X is distributed with the following pdf:

$$f_X(u) = \begin{cases} \sin(u) & 0 \leq u \leq A \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is the value of the constant A ?
- (b) What is the corresponding CDF, $F_X(c)$?
- (c) What is $E[X]$?
- (d) What is $\text{Var}(X)$?
- (e) Let $Y = 2X$. Find the CDF $F_Y(c)$ and the pdf $f_Y(u)$.

3. **[Using a CDF]**

Let X be a random variable with a CDF defined as follows:

$$F_X(c) = \begin{cases} 0 & c < -10 \\ \frac{1}{2} & -10 \leq c < -5 \\ \frac{2}{3} & -5 \leq c < 0 \\ \frac{4}{5} & 0 \leq c < 5 \\ 1 & c \geq 5 \end{cases}$$

Sketch the CDF and compute the following values:

- (a) $P(X \leq 3.5)$
- (b) $P(X \geq -4.37)$
- (c) $P(|X| < 2)$
- (d) $P(X^2 \leq 9)$
- (e) $\mathbb{E}[X]$
- (f) $\text{Var}(X)$

4. **[LOTUS for continuous random variables]**

Consider a stick of length 1 containing some fixed point p , where $0 \leq p \leq 1$. Suppose we cut the stick at some other point that is uniformly distributed over $(0, 1)$. This separates the stick into two pieces. Determine the expected length of the piece that contains the point p .

5. **[Uniform random variables]**

- (a) Suppose X is a uniform random variable with $\mathbb{E}[X] = 5$ and $\text{Var}(X) = 12$, find the pdf $f_X(u)$ of X .
- (b) Let $X \sim \text{Uniform}(-2, 2)$. Find $\mathbb{E}[X^4]$, $\mathbb{E}[X^5]$, and $\mathbb{E}[e^X]$.