UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering
ECE 310 Digital Signal Processing – Fall 2023

Homework 4

Profs. Do, Snyder, Moustakides **Due: September 22, 2023 on Gradescope**

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1. Consider a causal discrete-time system given by the following LCCDE:

$$y[n] = \frac{7}{4}y[n-1] - \frac{3}{4}y[n-2] + \frac{1}{2}x[n] - x[n-1].$$

Assuming zero initial conditions, compute system response $y[n]$ for each of the following input signals.

(a) $\delta[n] - \delta[n-1]$
(b) $\left(\frac{1}{3}\right)^n u[n]$
(c) $2^n u[n-2]$

2. Two LTI systems with impulse responses

$$h_1[n] = \frac{12}{13}(3)^n u[n] + \frac{1}{13}\left(-\frac{1}{4}\right)^n u[n], \qquad h_2[n] = \left(\frac{1}{2}\right)^{n+1} u[n] - \frac{3}{2}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

are in **series** connection.

(a) Determine the transfer function of each individual system and state whether each system is BIBO stable. Explain your reasoning.
(b) Find the transfer function and impulse response of the overall system. Is the overall system BIBO stable? Explain your reasoning.
(c) Give the LCCDE that expresses the overall system.

3. Determine all possible ROCs for the following z-transform and the corresponding $x[n]$ for each case.

$$X(z) = \frac{1}{\left(1 - \frac{2}{3}z^{-1}\right)\left(1 - \frac{3}{2}z^{-1}\right)}$$

4. For each of the following transfer functions, determine if they represent a BIBO stable system. If a given system is not BIBO stable, give an example **real-valued** bounded input that would yield an unbounded output. You may assume all systems are causal.

(a) $3 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} - 7z^{-3}$
(b) $\frac{1-(1/6)z^{-1}}{1-(1/3)z^{-1}+(1/9)z^{-2}}$
(c) $\frac{1}{1-2z^{-1}+(8/9)z^{-2}}$
(d) $\frac{1}{1+z^{-2}}$

5. Consider the following LCCDE where α is a real-valued constant:

$$y[n] = \alpha y[n-2] + 3x[n] - x[n-1].$$

Assuming the system is causal, find the condition on α such that the system is BIBO stable.