UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Fall 2023

# Homework 6

**Profs. Do, Snyder, Moustakides** **Due: Oct 6 on Gradescope**

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1. We are given the transfer function $H(z) = \frac{1-z^{-1}}{1-rz^{-1}}$ of a causal system where $0 < r < 1$ and $r$ is **close to 1**.

   (a) Determine the frequency response $H(\omega)$ of this system.

   (b) Plot the magnitude response $|H(\omega)|$ when $r = 0.99$ for $-\pi \leq \omega \leq \pi$. Your plot does not need to be precise, but you can evaluate a few values of $\omega \in [-\pi, \pi]$ to get the general shape of the magnitude response. In particular, make sure to note the values at $\omega = 0, \pm\frac{\pi}{2}, \pm\pi$.

   (c) What is the practical role of an LTI system of this form when it is applied to a signal $x[n]$?

   (d) Repeat parts (a–c) for $H(z) = \frac{1+z^{-1}}{1+rz^{-1}}$.

2. Consider an FIR system of length $N$ with the following impulse response:

$$\{h[n]\}_{n=0}^{N-1} = \frac{1}{N}\sum_{k=0}^{N-1}\delta[n-k] = \left\{\underset{\uparrow}{\frac{1}{N}}, \frac{1}{N}, \frac{1}{N} \cdots\right\}$$

   (a) Compute the frequency response $H(\omega)$ of this system.

   (b) Plot the magnitude response $|H(\omega)|$ when $N = 5$.

   (c) What will happen to an input signal $x[n]$ if the length of the system $N$ becomes very large? **Hint**: you may want to consider what the magnitude response will look like as $N$ becomes very large.

3. The DTFT $X(\omega)$ of a sequence $x[n]$ is given by

$$X(\omega) = \begin{cases} -1 & \text{for } -\pi \leq \omega < 0 \\ 1 & \text{for } 0 \leq \omega < \pi. \end{cases}$$

   (a) Apply the inverse DTFT to show that for $k = \ldots, -2, -1, 0, 1, 2, \ldots$ we have

$$x[n] = \begin{cases} 0 & \text{for } n = 2k \\ \frac{2j}{\pi(2k+1)} & \text{for } n = 2k+1. \end{cases}$$

   (b) Use the result in part (a) and Parseval's Theorem to demonstrate that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$