[1] $y[n] = \frac{7}{4}y[n-1] - \frac{3}{4}y[n-2] + \frac{1}{2}x[n] - x[n-1]$

both z-transform.

$Y(z) = \frac{7}{4}z^{-1}Y(z) - \frac{3}{4}z^{-2}Y(z) + \frac{1}{2}X(z) - \frac{1}{2}\cdot z^{-1}X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{1}{2} - z^{-1}}{1-\frac{7}{4}z^{-1}+\frac{3}{4}z^{-2}} = \frac{2-4z^{-1}}{(4-3z^{-1})(1-z^{-1})}$$

(a) $x[n] = \delta[n] - \delta[n-1] \Rightarrow X(z) = 1 - z^{-1}$

$\therefore Y(z) = H(z)\cdot X(z) = \frac{2-4z^{-1}}{4-3z^{-1}} = \frac{4}{3} - \frac{5}{6}\cdot\frac{1}{1-\frac{3}{4}z^{-1}}$

$\therefore y[n] = \frac{4}{3}\delta[n] - \frac{5}{6}\cdot(\frac{3}{4})^n u[n]$

(b) $x[n] = (\frac{1}{3})^n u[n] \Rightarrow X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} = \frac{-3}{z^{-1}-3}$

$\therefore Y(z) = H(z)\cdot X(z) = \frac{3(2-4z^{-1})}{(4-3z^{-1})(1-z^{-1})(3-z^{-1})} = \frac{3}{z^{-1}-1} + \frac{9}{2}\cdot\frac{1}{1-\frac{3}{4}z^{-1}} - \frac{1}{1-\frac{1}{3}z^{-1}}$

$y[n] = -3u[n] + \frac{9}{2}(\frac{3}{4})^n u[n] - (\frac{1}{3})^n u[n]$

(c) $x[n] = 2^n u[n-2] = 4\cdot 2^{n-2}\cdot u[n-2] \Rightarrow X(z) = 4\cdot z^{-2}\cdot\frac{1}{1-2z^{-1}}$

$Y(z) = H(z)\cdot X(z) = \frac{8z^{-2}}{(z^{-1}-1)(3z^{-1}-4)} = \frac{8}{3} + \frac{8}{1-z^{-1}} - \frac{32}{3}\cdot\frac{1}{1-\frac{3}{4}z^{-1}}$

$y[n] = \frac{8}{3}\delta[n] + 8u[n] - \frac{32}{3}\cdot(\frac{3}{4})^n u[n]$

[2].

(a) $h_1[n] = \frac{12}{13}(3)^n u[n] + \frac{1}{13}\cdot(-\frac{1}{4})^n u[n]$

$H_1(z) = \underbrace{\frac{12}{13}\cdot\frac{1}{1-3z^{-1}}}_{ROC: |z|>3} + \underbrace{\frac{1}{13}\cdot\frac{1}{1+\frac{1}{4}z^{-1}}}_{ROC: |z|>\frac{1}{4}}$ ROC: $|z|>3$

thus. $H_1(z)$ don't contain the unit circle $\Rightarrow$ not BIBO stable.

$h_2[n] = (\frac{1}{2})^{n+1}u[n] - \frac{3}{2}\cdot(\frac{1}{2})^{n-1}u[n-1]$

$H_2(z) = \underbrace{\frac{1}{2}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}}_{ROC: |z|>\frac{1}{2}} - \underbrace{\frac{3}{2}\cdot z^{-1}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}}_{ROC: |z|>\frac{1}{2}}$, ROC: $|z|>\frac{1}{2}$

thus. $H_2(z)$ contain the unit circle $\Rightarrow$ BIBO stable.

(b) since $h_1[n]$, $h_2[n]$ are in series. the transfer function is:

$H(z) = H_1(z)\cdot H_2(z) = \frac{12}{13}\cdot(\frac{1}{2}-\frac{3}{2}z^{-1})\cdot\frac{1}{1-3z^{-1}}\cdot\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{13}\cdot(\frac{1}{2}-\frac{3}{2}z^{-1})\cdot\frac{1}{1+\frac{1}{4}z^{-1}}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}$

$= \frac{4}{8-2z^{-1}-z^{-2}} = \frac{2}{3}\frac{1}{4+z^{-1}} + \frac{2}{3}\cdot\frac{1}{2-z^{-1}} = \frac{1}{6}\frac{1}{1+\frac{1}{4}z^{-1}} + \frac{1}{3}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}$, ROC: $|z|>\frac{1}{2}$

impulse response is: $h[n] = \frac{1}{6}\cdot(-\frac{1}{4})^n u[n] + \frac{1}{3}\cdot(\frac{1}{2})^n u[n]$

the overall system is BIBO stable because $H(z)$'s ROC contain unit circle.

(c) for the LCCDE, consider $H(z)$ again:

$H(z) = \frac{12}{13}\cdot(\frac{1}{2}-\frac{3}{2}z^{-1})\cdot\frac{1}{1-3z^{-1}}\cdot\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{13}\cdot(\frac{1}{2}-\frac{3}{2}z^{-1})\cdot\frac{1}{1+\frac{1}{4}z^{-1}}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}$

$= \frac{4}{-z^{-2}-2z^{-1}+8} = \frac{\frac{1}{2}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$

so the LCCDE should be:

$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = \frac{1}{2}x[n]$

[3]: $X(z)=\frac{1}{(1-\frac{2}{3}z^{-1})(1-\frac{3}{2}z^{-1})} = \underbrace{-\frac{4}{5}\frac{1}{1-\frac{2}{3}z^{-1}}}_{X_1(z)} + \underbrace{\frac{9}{5}\frac{1}{1-\frac{3}{2}z^{-1}}}_{X_2(z)}$

basically, the different case comes from whether is generated by $u[n]$ or $u[-(n+1)]$

so there are four cases to analyze:

right-sided

① $X_1(z), X_2(z)$ are both from $u[n]$: the ROC should be $|z|>\frac{3}{2}$

$x[n]=-\frac{4}{5}\cdot(\frac{2}{3})^n u[n]+\frac{9}{5}\cdot(\frac{3}{2})^n u[n]$

one right-sided and one left-sided

② $X_1(z)$ from $u[n]$, $X_2(z)$ from $u[-(n+1)]$: the ROC should be $\frac{2}{3}<|z|<\frac{3}{2}$

$x[n]=-\frac{4}{5}\cdot(\frac{2}{3})^n u[n]-\frac{9}{5}\cdot(\frac{3}{2})^n u[-(n+1)]$

③ $X_1(z)$ from $u[-(n+1)]$, $X_2(z)$ from $u[n]$, the ROC should be $\frac{3}{2}<|z|<\frac{2}{3}$ not exist!

left-sided

④ $X_1(z), X_2(z)$ both from $u[-(n+1)]$: the ROC should be $|z|<\frac{2}{3}$

$x[n]=\frac{4}{5}(\frac{2}{3})^n u[-(n+1)]-\frac{9}{5}\cdot(\frac{3}{2})^n u[-(n+1)]$

[4].

(a) $H(z)=3-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}-7z^{-3}$, ROC contains unit circle $|z|=1$ $\Rightarrow$ BIBO stable

(b) $H(z)=\frac{1-\frac{1}{6}z^{-1}}{1-\frac{1}{3}z^{-1}+\frac{1}{9}z^{-2}} = \frac{1-\frac{1}{6}z^{-1}}{(\frac{1}{3}z^{-1}-\frac{1}{2})^2+\frac{3}{4}}$, ROC contains unit circle $|z|=1$ $\Rightarrow$ BIBO stable

(c) $H(z)=\frac{1}{\frac{8}{9}z^{-2}-2z^{-1}+1} = \frac{9}{(4z^{-1}-3)(2z^{-1}-3)} = \frac{6}{4z^{-1}-3}+\frac{-3}{2z^{-1}-3} = \frac{2}{1-\frac{4}{3}z^{-1}}-\frac{1}{1-\frac{2}{3}z^{-1}}$, ROC: $|z|>\frac{4}{3}$

since ROC doesn't contain unit circle $|z|=1$ $\Rightarrow$ not BIBO stable

$h[n]=2\cdot(\frac{4}{3})^n u[n]-(\frac{2}{3})^n u[n]$

bounded input $x[n]=\delta[n]$, output $y[n]$,

we have $Y(z)=H(z)\cdot X(z)=H(z)$, thus $y(z)=h(z)=2\cdot(\frac{4}{3})^n u[n]-(\frac{2}{3})^n u[n]$ is unbounded for $n\to\infty, y[n]\to\infty$

(d) $H(z)=\frac{1}{1+z^{-2}}=\frac{1}{(z^{-1}-j)(z^{-1}+j)}$ ROC not contain $z=\pm i$, thus not contain whole unit circle $\Rightarrow$ not BIBO stable

$H(z)=\frac{j}{2}\cdot\frac{1}{z^{-1}+j}-\frac{j}{2}\cdot\frac{1}{z^{-1}-j}$

$=\frac{1}{2}\cdot\frac{1}{1-j\cdot z^{-1}}+\frac{1}{2}\cdot\frac{1}{1+jz^{-1}}$

$h[n]=\frac{1}{2}\cdot(j)^n u[n]+\frac{1}{2}(-j)^n u[n]$

$=\frac{1}{2}\cdot e^{\frac{\pi}{2}jn}u[n]+\frac{1}{2}\cdot e^{\frac{3}{2}\pi jn}u[n]$

Consider anothe way:

$H(z)=\frac{1}{1+z^{-2}} \Rightarrow y[n]=x[n]-y[n-2]$

let $x[n]=\begin{cases}(-1)^{\frac{n}{2}} & n \text{ even}\\ (-1)^{\frac{n-1}{2}} & n \text{ odd}\end{cases}$ (bounded input)

thus, $y[n]=\begin{cases}(-1)^{\frac{n}{2}}\cdot(1+\frac{n}{2}) & n \text{ even}\\ (-1)^{\frac{n-1}{2}}\cdot(\frac{n+1}{2}) & n \text{ odd}\end{cases}$ which is unbounded output.

[5]. $y[n]=\alpha y[n-2]+3x[n]-x[n-1]$

both z-transform: $Y(z)-\alpha z^{-2}Y(z)=3X(z)-z^{-1}X(z)$

transform function $H(z)=\frac{Y(z)}{X(z)}=\frac{3-z^{-1}}{1-\alpha z^{-2}}$

$1-\alpha z^{-2}=0 \Rightarrow |z|=|\sqrt{\alpha}|$, since the system is casual,

the ROC should be $|z|>|\sqrt{\alpha}|$, thus if $|\alpha|<1$, the system is BIBO stable.

condition: $-1<\alpha<1$