UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 310 Digital Signal Processing – Fall 2023

Homework 8

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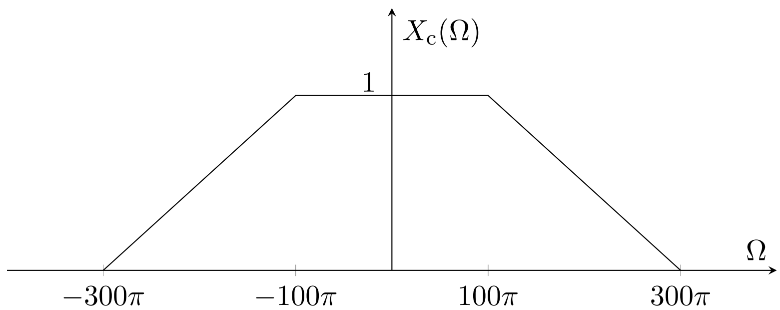
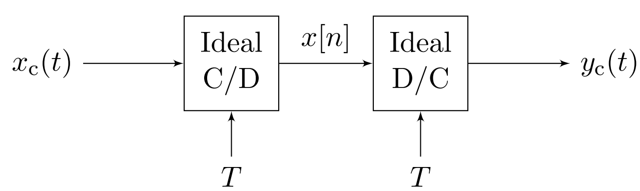
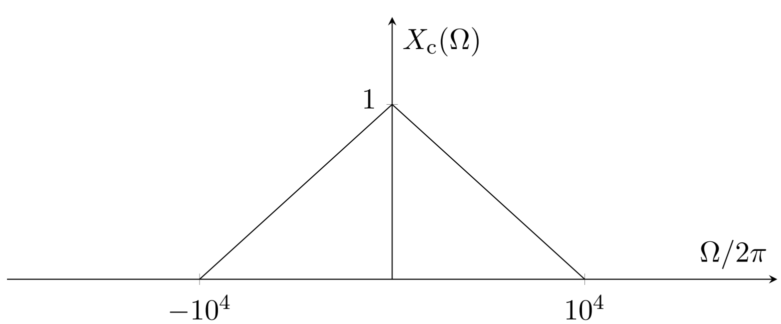
1. The continuous-time signal $x_c(t)$ has the continuous-time Fourier transform (CTFT) shown in the figure below. The signal $x_c(t)$ is sampled with sampling interval T, or sampling frequency $f_s = 1/T$, to get the discrete-time signal $x[n] = x_c(nT)$. Sketch $X_d(\omega)$ (the DTFT of $\{x[n]\}$) for each of the following sampling frequencies: $f_s = 200$ Hz and $f_s = 300$ Hz. Clearly label all of the axes in your sketches.



2. Consider the following signal processing chain with an ideal continuous-to-discrete (C/D) or analog-to-digital converter (ADC), followed by an ideal discrete-to-continuous (D/C) or digital-to-analog converter (DAC), both for sampling interval T, or sampling frequency $f_s = 1/T$.

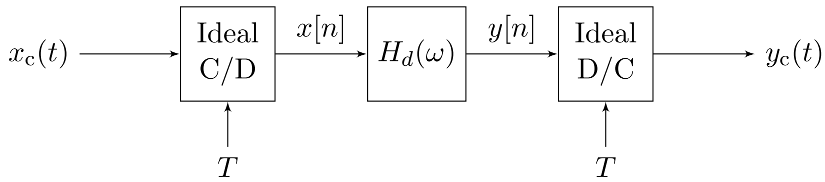


Most of human speech can be considered to be bandlimited to 10 kHz. To be definite, consider a speech signal $x_c(t)$ that has the following continuous-time Fourier transform (CTFT):



Sketch the DTFT $X_d(\omega)$ of the output signal $x[n]$ after the C/D or ADC, and the CTFT $Y_c(\Omega)$ of the output signal $y_c(t)$ after the D/C or DAC, for each of the following sampling frequencies $f_s = 10$ kHz, $f_s = 20$ kHz, and $f_s = 30$ kHz. Clearly label all of the axes in your sketches.

3. A continuous-time signal $x_c(t)$ is assumed to be bandlimited to 3 MHz. We would like to filter $x_c(t)$ with a lowpass filter that will pass only the frequencies up to 1 MHz in $x_c(t)$ by using a digital filter $H_d(\omega)$ sandwiched between an ideal ADC and an ideal DAC as follows.



   (a) Determine the Nyquist sampling rate for the input signal $x_c(t)$.
   (b) Sketch the frequency response $H_d(\omega)$ for the necessary discrete-time filter, when sampling at the Nyquist rate.
   (c) Find the largest sampling period T for which the ADC, digital filter response $H_d(\omega)$, and DAC can perform the desired filtering function. (Hint: some amount of aliasing may be permissible during ADC for this part.)
   (d) For the system using T from part (c), sketch the necessary $H_d(\omega)$.

4. Let $\{X[k]\}_{k=0}^{20}$ and $X_d(\omega)$ respectively be the 21-point DFT and DTFT of a *real-valued* sequence $\{x[n]\}_{n=0}^{7}$ that is zero-padded to length 21. Determine all the correct relationships in the following and justify your answers.

   (a) $X[19] = X_d(-\frac{4\pi}{21})$.
   (b) $X[2] = X_d^*(-\frac{4\pi}{21})$
   (c) $X[12] = X_d(-\frac{4\pi}{21})$
   (d) $X[4] = X_d^*(-\frac{4\pi}{21})$

5. Assume $\{x[n]\}_{n=0}^{39}$ is a finite-duration sequence of length 40, and $\{y[n]\}_{n=0}^{63}$ is obtained by zero-padding $\{x[n]\}_{n=0}^{39}$ to length 64. That is, $y[n] = x[n]$, for $n = 0, 1, \ldots, 39$, and $y[n] = 0$, for $n = 40, 41, \ldots, 63$.

   Let $\{X[k]\}_{k=0}^{39}$ and $\{Y[k]\}_{k=0}^{63}$ be the DFT of $\{x[n]\}_{n=0}^{39}$ and $\{y[n]\}_{n=0}^{63}$, respectively. Determine all the correct relationships in the following and justify your answers.

   (a) $X[0] = Y[0]$
   (b) $X[5] = Y[8]$
   (c) $X[10] = Y[16]$
   (d) $X[12] = Y[18]$
   (e) $X[39] = Y[63]$