University of Illinois
ECE 310

Profs. Dokmanic, Liang, Radhakrishnan

Midterm Exam II

7:00-8:30pm, Thursday, April 6, 2016

Problem	Pts.	Score
1	10	
2	6	
3	4	
4	6	
5	3	
6	3	
7	3	
8	15	
9	25	
10	25	
Total	100	

Name Key

Section: 9:00 AM 12:00 PM 3:00 PM

Score ____________

Instructions

- You may not use any books, calculators, or notes other than two handwritten two-sided sheets of 8.5" x 11" paper.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.

(10 Pts.)

1. Answer **True** or **False** to each of the following statements:

   (a) The causal LSI system $H(z) = \frac{1+z^{-1}}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$ is stable. **True/False.**

   (b) An FIR filter is always BIBO stable. **True/False**

   (c) Output $y[n]$ of a discrete BIBO stable system always satisfies $|y[n]| \leq M < \infty$ for every n and some positive constant M. **True/False**

   (d) Knowing the ROC of the transfer function of a causal system is sufficient to determine whether the system is BIBO stable. **True/False**

   (e) If the impulse response of a system is $h[n] = \delta[n] + \delta[n-1] + (-1-j)^n$ then the system is unstable. **True/False**

(6 Pts.)

2. For each of the filters shown in the table, indicate by "**yes**" or "**no**" whether the properties indicated apply to the filter. (In each case, the remaining terms of the unit pulse response $h[n]$ of the filter are zero.)

$h[n]$	Type-1 GLP	Type-2 GLP	Strict LP	
$\{h_n\}_{n=0}^{3} = \{1, 5, 5, -1\}$	NO	NO	NO	0.5 pt each
$\{h_n\}_{n=0}^{4} = \{2, 1, 6, -1, -2\}$	NO	NO	NO	
$\{h_n\}_{n=0}^{41} = (-1)^n \text{sinc}\left(\frac{\pi}{3}(n - \frac{41}{2})\right)$	NO	YES	NO	1 pt each

(4 Pts.)

3. A system's transfer function is $H(z)$ and its input is $\cos(\omega_0 n)$. Under what conditions does it hold that $y[n] = |H(\omega)| \cos(\omega_0 n + \angle H_d(\omega))$, where $H_d(\omega) = H(z)\big|_{z=e^{j\omega}}$? Check all that apply.
   (-1 pt for each wrong answer)

   Note the typo
   (Otherwise, (e) is the right answer)

   (a) The system must be causal.
   (b) The system must be BIBO stable.
   (c) The impulse response of the system must be real.
   (d) The impulse response of the system must be of finite length.
   (e) It never holds.

(6 Pts.)

4. Consider an LSI system with transfer function $H(z) = \frac{z^2}{z^2+1}$. Determine which of the following inputs will produce an **unbounded** output. (**-1pt for wrong answer**).

   (a) $x[n] = \delta[n]$
   (b) $x[n] = \sin\left(\frac{\pi n}{2}\right)$
   (c) $x[n] = u[n]$
   (d) $x[n] = \cos(\pi n)$
   (e) $x[n] = 5^n u[n]$

(3 Pts.)

5. If the response of a discrete LTI system to input $x_1[n] = \sin\left(\frac{\pi}{4}n\right)$ is $y_1[n] = e^{j\left(\frac{\pi}{4}n+\pi\right)}$, then its response to $x_2[n] = \cos\left(\frac{\pi}{4}n+1\right)$ is

   (a) $y_2[n] = j\cos\left(\frac{\pi}{4}n+1+\pi\right)$
   (b) $y_2[n] = -je^{j\left(\frac{\pi}{4}n+1\right)}$
   (c) $y_2[n] = \sin\left(\frac{\pi}{4}n+1-\pi\right)$
   (d) There is not enough information to determine the correct answer.

(3 Pts.)

6. Given that the z-transform of $x[n] = 5^n u[n]$ is $X(z) = \frac{z}{z-5}$, the DTFT of $x[n]$ is given by

   (a) $\frac{e^{j\omega}}{e^{j\omega}-5}$
   (b) $\frac{e^{-j\omega}}{e^{-j\omega}-5}$
   (c) $\frac{1}{1-5e^{-j\omega}}$
   (d) Does not exist

(3 Pts.)

7. The transfer function of a causal LSI system is: $H(z) = \frac{z-1}{z+j}$. Find a **bounded, real-valued** input to the system which will produce an **unbounded** output $y[n]$. Give an expression for the input in the z-domain, $X(z)$.

1pt a) To given unbounded output, $X(z)$ needs to have a pole at $z=-j$

1pt b) To make the input real-valued, $X(z)$ needs to have a pole at $z=(-j)^* = j$

So: $X(z)$ can be

1pt $$\frac{1}{(z+j)(z-j)} = \frac{1}{z^2+1}$$

or $\frac{z}{z^2+1}$, etc.

PROBLEM 8

a) $$H(z) = H_2(z)H_1(z)H_2(z)$$

$$= \frac{\left[\frac{z^{-1}}{2}\cancel{(1-2z^{-1})}\right]^2}{\left(1-\cos(1)z^{-1}+\frac{1}{4}z^{-2}\right)\cancel{(1-2z^{-1})^2}}$$

Pole-zero cancelation!

$$= \frac{1}{4}\,\frac{z^{-2}}{\left(1-\cos(1)z^{-1}+\frac{1}{4}z^{-2}\right)}$$

$\cos(1) = 2r\cos\gamma$

$r^2 = \frac{1}{4} \Rightarrow r = \frac{1}{2},\ \gamma = 1$

$z_{1,2} = \frac{1}{2}e^{\pm j}$

$$= \frac{1}{4}\,\frac{z^{-2}}{\left(1-\frac{e^{j}}{2}z^{-1}\right)\left(1-\frac{e^{-j}}{2}z^{-1}\right)}$$

$$|z_{1,2}| = \frac{1}{2} < 1 \Rightarrow \text{BIBO stable}$$

(causal so ROC includes the unit circle)

$$H_d(\omega) = H(e^{j\omega}) = \frac{1}{4}\,\frac{e^{-2j\omega}}{\left(1-\frac{1}{2}e^{j(1-\omega)}\right)\left(1-\frac{1}{2}e^{-j(1+\omega)}\right)}$$

b) At least 3 ways to solve:

i) $y[n] = |H(1)|\cos(n+\sphericalangle H(1)) - H(-1)e^{-jn}$

ii) $x[n] = \frac{1}{2}(e^{jn}+e^{-jn}) - e^{-jn} = j\sin(n)$

$$\Rightarrow y[n] = j|H(1)|\sin(n+\sphericalangle H(1))$$

iii) $x[n] = \frac{1}{2}e^{jn} - \frac{1}{2}e^{-jn}$

$$\Rightarrow y[n] = \frac{1}{2}H(1)e^{jn} + \frac{1}{2}H(-1)e^{-jn}$$

with: $$H(1) = \frac{1}{4}\,\frac{e^{-2j}}{\left(1-\frac{1}{2}\right)\left(1-e^{-2j}/2\right)} = \frac{e^{-2j}}{2-e^{-2j}}$$

$$H(-1) = \frac{1}{4}\,\frac{e^{+2j}}{\left(1-e^{2j}/2\right)\left(1-\frac{1}{2}\right)} = \frac{e^{2j}}{2-e^{2j}} = H(1)^*$$

(of course ...)

(25 Pts.)

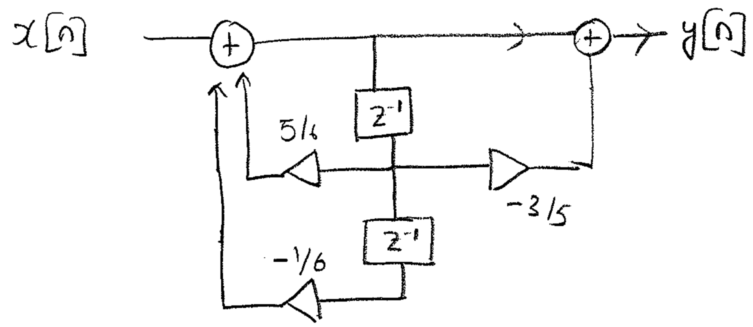
9. An LSI system has the transfer function, $H(z) = \frac{z(z-\frac{3}{5})}{(z-\frac{1}{2})(z-\frac{1}{3})}$

   (a) Determine a difference equation, in causal form $y[n]$ =?, relating the input $x[n]$ and output $y[n]$ of the system.

$$Y(z)\left[z^2 - \frac{5}{6}z + \frac{1}{6}\right] = X(z)\left[z^2 - \frac{3}{5}z\right]$$

$$\Rightarrow y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n] - \frac{3}{5}x[n-1]$$

   (b) Draw the direct form II structure for the difference equation determined in part (a).



   (c) Determine the output, $y[n]$, of the system to the input $x[n] = \left(\frac{3}{5}\right)^n u[n]$.

$$X(z) = \frac{z}{z - \frac{3}{5}}$$

$$Y(z) = H(z)X(z) = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}}$$

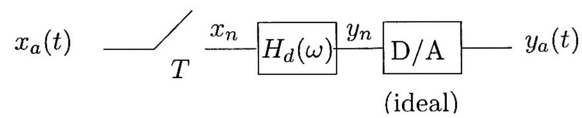
$$\Rightarrow z = A(z - 1/3) + B(z - 1/2) \Rightarrow \begin{array}{l} A = 3 \\ B = -2 \end{array}$$

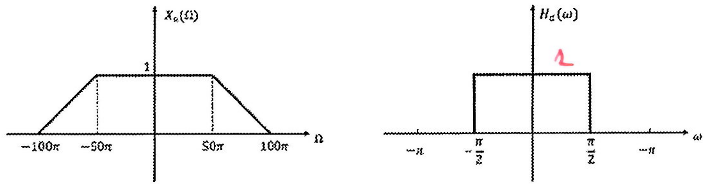
$$\therefore \frac{Y(z)}{z} = \frac{3}{z-\frac{1}{2}} \quad 5 \quad - \frac{2}{z - 1/3}$$

$$\Rightarrow y[n] = 3\left(\tfrac{1}{2}\right)^n u[n] - 2\left(\tfrac{1}{3}\right)^n u[n]$$

(25 Pts.)

10. Consider the following system with the given input and digital filter:





(a) Find the maximum value of T allowed without aliasing errors in $x[n]$.

$$T = \frac{\pi}{100\pi} = 10^{-2} \text{ sec}$$

5pt

(b) Find the maximum value of T allowed without aliasing errors in $y[n]$.

$$2\pi - 100\pi T \geq \frac{\pi}{2}$$

$$T \leq 1.5 \times 10^{-2} \text{ sec}$$

$$T_{max} = 1.5 \times 10^{-2} \text{ sec}$$

5pt

(c) Determine and sketch $H_a(\Omega) = Y_a(\Omega)/X_a(\Omega)$ for the value of T in a) and b), respectively.

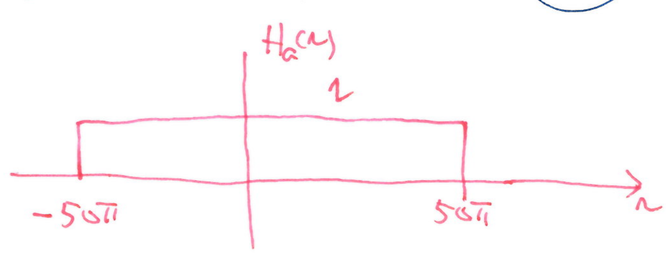
15pt

For both cases, there is no aliasing in $y[n]$ and $y_a(t)$.

therefore: $H_a(\Omega) = H_d(\Omega T)$

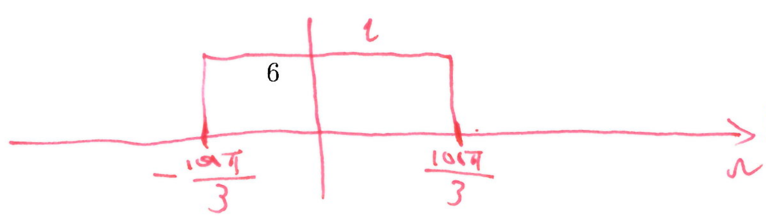
5pt

Case 1:



5pt

Case 2:



5pt