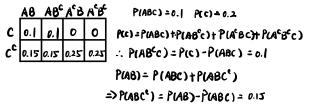
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PROBLEM SET#3

 $L_{1}J$

(a)
$$\frac{B}{A} = \frac{B^{C}}{A \cdot 25}$$
 $\frac{A^{C}}{A \cdot 25} = \frac{A^{C}}{A \cdot 25}$

(b) A and C is exclusive => P(ACC) =0 => P(ACBC)= P(ACBC)=0



the same for the second row of K-map.

[2]

(a) the possible value of X: 1.4. 16.64,256,1024

(b)
$$P_{X}(1874) = \frac{1}{2^{x}} = \frac{1}{32}$$
 $P_{X}(276) = \frac{C_{1}^{2}}{2^{x}} = \frac{5}{32}$
 $P_{X}(64) = \frac{C_{1}^{2}}{2^{x}} = \frac{10}{32} = \frac{5}{16}$
 $P_{X}(16) = \frac{C_{1}^{2}}{2^{x}} = \frac{5}{16}$
 $P_{X}(4) = \frac{C_{1}^{2}}{2^{x}} = \frac{5}{32}$
 $P_{X}(4) = \frac{C_{1}^{2}}{2^{x}} = \frac{5}{32}$

(c):
$$E(X) = \sum_{i} P \cdot X = 1 \cdot \frac{1}{32} + 4 \cdot \frac{5}{32} + 16 \cdot \frac{5}{16} + 64 \cdot \frac{5}{16} + 256 \cdot \frac{5}{52} + 1024 \cdot \frac{1}{12}$$

= $\frac{3125}{32}$

the brochuse is quite accurate

(d): the probability of losing money:
$$P = \frac{1}{32} + \frac{5}{32} + \frac{5}{16} = \frac{1}{2}$$

for the C student: ECY] = 150 = 20

(a): let Li become the number of times one roll vertil outcome 4 occussed.

Thus, the pmf of x :
$$P_x(n) = (\frac{1}{6})^2 \cdot (\frac{5}{6})^{n-3} \cdot C_{n-1} \cdot \frac{1}{6}$$

the expected value of X:
$$E(X) = E(L_1) + E(L_2) + E(L_3) = \frac{3}{6} = 18$$

the variance of X: Var(X) = Var(L1) + Var(L2) + Var(L3) = 3 ·
$$\frac{1-\frac{1}{6}}{(\frac{1}{2})^2}$$
 = 90

- (b) let L: the number of times one roll until outcome 1 or 2 occurred obviously, Y=L+tL2+L3+L4, and L1~L4 are independent. Thus, the pmf of Y: $P_Y(n) = (\frac{1}{5})^3 \cdot (\frac{2}{3})^{n-4} \cdot C_{n-1}^3 \cdot \frac{1}{3}$ the expected value of Y: $E(Y) = E(L_1) + E(L_2) + E(L_3) + E(L_4) = 4 \cdot \frac{1}{3} = 12$ the variance of Y: $Var(Y) = 24 \cdot \frac{1-3}{(\frac{1}{3})^2} = 24$
- [5]. consider minute as unit time
- (a) let Y be the call per minute.

$$E(Y) = \lambda = 4$$

 $P_Y(k) = \frac{e^{-4} \cdot 4^k}{k!}, k \ge 0$

let X be the call in an interval of 3 minutes.

$$p_{x}(2) = \frac{e^{-12} \cdot 12^{2}}{2!} : 0.000442$$

[6].

- (a) all the passengers showing up get the seat means:
 - at most 100 people show up

so the probability should be:

$$P = 1 - (0.8)^{105} - C_{105}^{1} \cdot (0.8)^{104} \cdot 0.2 - C_{105}^{2} \cdot (0.8)^{103} \cdot (0.2)^{2} - C_{105}^{3} \cdot (0.8)^{102} \cdot (0.2)^{3} - C_{105}^{4} \cdot (0.8)^{101} \cdot (0.2)^{4}$$

$$= 0.9999885335$$

(b) consider X passengers show up at the gate

E(X) = 105 x 0.8 = 84

(C) each person's no-shows is independent of any other passenger.

and for each parson, they all have probability p=0.2 for no-shows.

There are 105 person with p=0.8 for show-up (the "O" event) and p=0.2 for no-shows (the "I" event), thus the number of no-shows

can be modeled as a binomial random variable Y with

parameters (n=105, p=0.2)

(d) for the poisson approximation.

$$\begin{split} P(Y25) &= |-P(Y=0) - P(Y=1) - P(Y=2) - P(Y=3) - P(Y=4) \\ &= |-e^{-21} - 2| \cdot e^{-21} - \frac{21^2}{2} \cdot e^{-21} - \frac{21^4}{6} \cdot e^{-21} - \frac{21^4}{14} \cdot e^{-21} \\ &= 0.111112503 \end{split}$$

we can see that the answer from Poisston approximation is really close to the answer in part a, with error less than a coocal.