[1]. $H(z) = \frac{1-z^{-1}}{1-rz^{-1}}$

(a) $H(z) = \frac{1}{1-rz^{-1}} - \frac{z^{-1}}{1-rz^{-1}}$

$h[n] = (r)^n u[n] - (r)^{n-1} u[n-1]$

$H(w) = \frac{1}{1-re^{-jw}} - \frac{e^{-jw}}{1-re^{-jw}} = \frac{1-e^{-jw}}{1-re^{-jw}}$

(b) $|H(w)| = \sqrt{H(w)\cdot H^*(w)}$

$= \sqrt{\frac{(1-e^{-jw})(1-e^{jw})}{(1-re^{-jw})(1-re^{jw})}}$

$= \sqrt{\frac{1+1-(e^{-jw}+e^{jw})}{1+r^2-r(e^{-jw}+e^{jw})}}$

$= \sqrt{\frac{2-2\cos(w)}{1+r^2-2r\cdot\cos(w)}}$

$|H(0)| = \sqrt{\frac{2-2}{1+r^2-2r}} = 0$

$|H(\pm\frac{\pi}{2})| = \sqrt{\frac{2}{1+r^2}} = \sqrt{\frac{2}{1+0.99^2}} = 1.00501$

$|H(\pm\pi)| = \sqrt{\frac{4}{1+r^2+2r}} = \sqrt{\frac{4}{(1+r)^2}} = \frac{2}{1+r} = \frac{2}{1.99} = 1.00502$

(c) $H(w) = \frac{(1-e^{-jw})(1-re^{jw})}{(1-re^{-jw})(1-re^{jw})}$

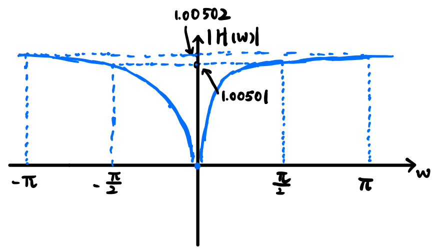
$= \frac{1+r-(e^{-jw}+re^{jw})}{1+r^2-2r\cos(w)}$

$= \frac{1+r-2r\cos(w)-(1-r)e^{-jw}}{1+r^2-2r\cos(w)}$

$\angle H(w) = \tan^{-1}\left(\frac{(1-r)\sin w}{1+r-2r\cos w-(1-r)\cos(w)}\right)$

$= \tan^{-1}\left(\frac{(1-r)\sin w}{1+r-(1+r)\cos w}\right)$

practical role: high pass filter



(d) $H(z) = \frac{1+z^{-1}}{1+rz^{-1}}$

[a]. $h[n] = (-r)^n u[n] + (-r)^{n-1} u[n-1]$

$H(w) = \frac{1}{1+re^{-jw}} + \frac{e^{-jw}}{1+re^{-jw}} = \frac{1+e^{-jw}}{1+re^{-jw}}$

[b].

$|H(w)| = \sqrt{H(w)\cdot H^*(w)}$

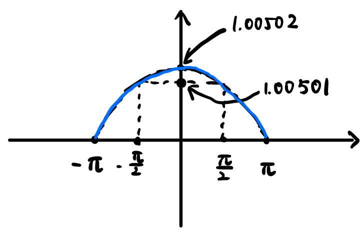
$= \sqrt{\frac{(1+e^{-jw})(1+e^{jw})}{(1+re^{-jw})(1+re^{jw})}}$

$= \sqrt{\frac{2+2\cos w}{1+r^2+2r\cos w}}$

$|H(0)| = \sqrt{\frac{4}{(r+1)^2}} = \frac{2}{r+1} = \frac{2}{1.99}$

$|H(\pm\frac{\pi}{2})| = \sqrt{\frac{2}{1+r^2}} = \sqrt{\frac{2}{1+0.99^2}}$

$|H(\pm\pi)| = 0$



[c].

$H(w) = \frac{1+e^{-jw}}{1+re^{-jw}} = \frac{(1+e^{-jw})(1+re^{jw})}{(1+re^{-jw})(1+re^{jw})}$

$= \frac{1+r+e^{-jw}+re^{jw}}{1+r^2+2r\cos w}$

$= \frac{[r+1+\cos(-w)+r\cos w]+j\cdot[\sin(-w)+\sin(w)]}{1+r^2+2r\cos w}$

$\tan^{-1}(H(w)) = \tan^{-1}\left(\frac{\sin(-w)+\sin(w)}{r+1+(1+r)\cos w}\right) = 0$

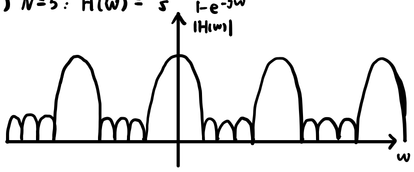
practical role: low pass filter

[2] FIR. $\{h[n]\}_{n=0}^{N-1} = \frac{1}{N}\sum_{k=0}^{N-1}\delta[n-k] = \{\frac{1}{N}, \frac{1}{N}, \frac{1}{N} \cdots\}$

(a) $H(w) = \frac{1}{N}\cdot[1+e^{-jw}+\cdots+e^{-j(N-1)w}]$

$= \frac{1}{N}\cdot\frac{1-e^{-jNw}}{1-e^{-jw}}$

(b) $N=5$: $H(w) = \frac{1}{5}\cdot\frac{1-e^{-5jw}}{1-e^{-jw}}$



(c) $H(w) = \frac{1}{N}\cdot\frac{1-e^{-jNw}}{1-e^{-jw}}$

if N is very large, then $|H(w)|$ is very close to 0.

so for the input signal $x[n]$, it will

$y(n) = h[n] * x[n]$

$= \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \frac{1}{N}\sum_{k=-\infty}^{\infty} x[k]$,

if $x[n]$ is FIR, then $y[n]$ is very close to 0.

if $x[n]$ is IIR, then $y[n]$ equals $\frac{\sum_{n=-\infty}^{\infty} x[n]}{N}$

[3].

$$X(w) = \begin{cases} -1 & -\pi \le w < 0 \\ 1 & 0 \le w < \pi \end{cases}$$

(a):

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w) e^{jwn} dw$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} -e^{jwn} dw + \int_{0}^{\pi} e^{jwn} dw\right]$$

for $n=2k$: $e^{jwn} = 0$, so $x[n]=0$

for $n=2k+1$: $e^{jwn} = e^{jw}$

$$x[n] = \frac{1}{2\pi}\left\{\left[-\frac{1}{jn}e^{jwn}\right]_{-\pi}^{0} + \left[\frac{1}{jn}e^{jwn}\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left(-\frac{1}{jn} + \frac{1}{jn}\cdot e^{-jn\pi} - \frac{1}{jn} + \frac{1}{jn}\cdot e^{jn\pi}\right)$$

$$= -\frac{4}{2\pi jn}$$

$$= \frac{2j}{\pi(2k+1)}$$

so: $x[n] = \begin{cases} 0 & n=2k \\ \dfrac{2j}{\pi(2k+1)} & n=2k+1 \end{cases}$

(b) from parseval's relation:

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(w)|^2 dw$$

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{4}{\pi^2}\cdot\sum_{k=-\infty}^{\infty}\frac{1}{(2k+1)^2} = \frac{8}{\pi^2}\left(\frac{1}{1^2}+\frac{1}{3^2}+\cdots\right)$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(w)|^2 dw = \frac{1}{2\pi}\cdot\int_{-\pi}^{\pi} 1\, dw = 1$$

$$\therefore \frac{8}{\pi^2}\cdot\left(\frac{1}{1^2}+\frac{1}{3^2}+\cdots\right) = 1 \Rightarrow \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots = \frac{\pi^2}{8}$$