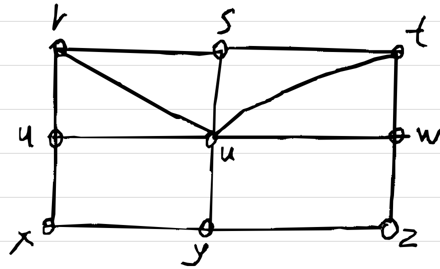
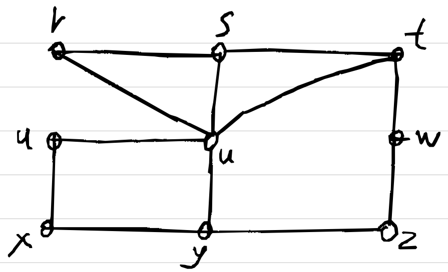
4

1.11. Let G be the graph of figure 1.20. Let $X = \{e, f\}$ were $e = ru$ and $f = uw$, and let $U = \{u, w\}$. Draw the subgraphs $G - X$ and $G - U$ of G

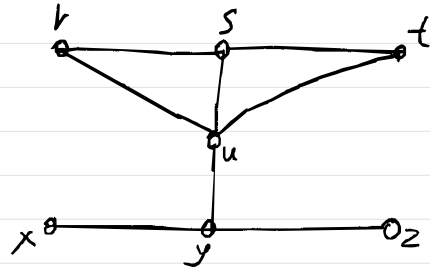


Solution 1.11

$G - X$

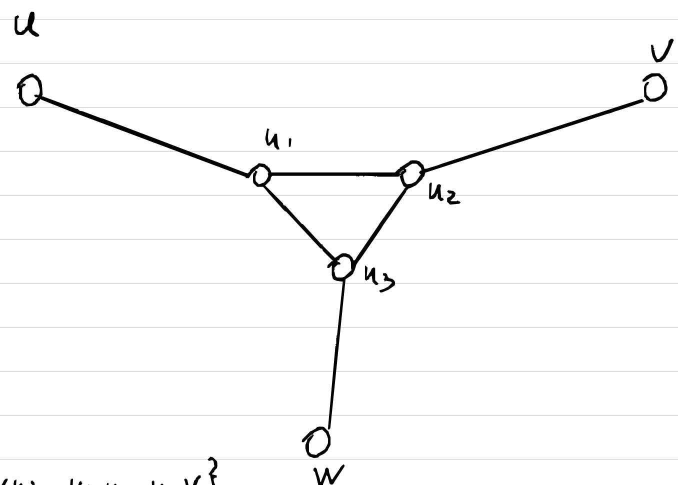


$G - U$



1.13

a) Give an example of an connected graph G containing three vertixes u, v, w such that $d(u,v) = d(u,w) = d(v,w) = diam(G) = 3$



$d(w,v) = \{wu_3, u_3u_2, u_2v\}$
$d(w,u) = \{wu_3, u_3u_1, u_1u\}$
$d(u,v) = \{uu_1, u_1u_2, u_2v\}$
$diag(G) = 3$

b) Is there some relation between the number of vertixes and the diameter of a connected graph?

1.15

Draw all connected graphs of order 5 in which the distance between every two distinct vertices is odd. Explain why you know that you have drawn all such graphs.

Solution

(0) $dist(u,v) \geq 1$ as G connected.

(1) $dist(u,v)$ is the length (no of edges) of the geodesic path

(2) for any path W, 2.1 $|W| = |E(W)|$

2.2 $|E(W)| + 1 = |V(W)|$

2.3 $|W| = |V(W)| - 1$

(3) $\forall$ Path P, $|P| \leq n-1$, where $n = |V(G)|$

Paths may not have more verexes than the total sum of verexes in a graph

(4) Therefore from 2.3 and 3 we conclude for order 5...

$\forall u,v \in V(G)$

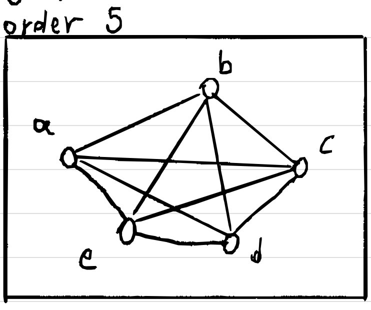
- $dist(v,v) \geq 1$ Connected Graph
- $dist(u,v) \leq |V(G)| - 1 \Rightarrow dist(u,v) \leq 4$ 2.3 & 3
- $dist(u,v)$ is odd Given

(5) So from 4. we can see

$\forall u,v \in V(G)$...

$dist(u,v) = 1 \vee dist(u,v) = 3$

(6) The graph where $dist(u,v) = 1$ is trivial. Is the complete graph of order 5



(7) There is no graph of order 5 s.t $\forall u,v \in V(G)$, $dist(u,v) = 3$

- let a G of order 5 where $dist(u,v) = 3$ $\forall u,v$.
- let the Path $P = \{v_0 = u, x, v\}$
- There is a path $u-x$ of length $= 1$
- Impossible

(8) Is there any that has $dist(u,v) = 1$ OR $dist(u,v) = 3$?

- The complete graph of order 5 honors this criterion
- Therefore, we searching for a graph G where
  - $dist(u,v) = 1$ OR $dist(u,v) = 3$
  - $\exists u,v \in V(G)$ st $dist(u,v) = 3$ at least one

(9) We will show that such a graph does not exist.

Let G with $|G| = 5$ where

- $dist(u,v) = 1$ OR $dist(u,v) = 3$ $\forall u,v \in V(G)$
- $\exists u,v \in V(G)$ st $dist(u,v) = 3$

let Geodesic Path P $u-v$ of size 3

$P = (V_0 = v, x, y, u)$ $x, y \in V(G)$

Path P $(V_0 = V, x, y, u)$ reveals path $v-y$

path P has size 2

Impossible

(10) The only graph G of order 5 that honors this criterion is the Complete graph of order 5

1.17 : Το λύσαμε στο lecture 15.

1.19 : Theorem 1.10 states that a graph G of order 3 or more is connected if and only if G contains two distinct vertices u, v such that $G-u$, $G-v$ are connected.

Based on this one, one might suspect that the following statement is true ...

"Every connected graph G of order 4 or more contains three distinct vertexes u, v and w such that $G-u$, $G-v$, $G-w$ connected"

Is it true?

Solution

let $x \in V(G)$, $x \neq u \neq v \neq w$

1. $\exists$ $u-x$ on $G-v$ AND $G-w$
2. $\exists$ $v-x$ on $G-u$ AND $G-w$
3. $\exists$ $w-x$ on $G-u$ AND $G-v$

Therefore

1. From 1 & 2, $G-v$, $G-u$, $G-w$ are connected between each other
2. $\exists$ path $u-v$, $u-w$, $v-w$ via x (1+2+3)
3. G is connected.