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TURTLE AND GOOD STRINGS

Problem: *Link*

For the solution, check the next page.

Let s be the input string with size n. We will show that $s[0] \neq s[n-1]$ is necessary and sufficient condition to answer YES. In other words, we can represent the string as $s = t_1 + t_2 + ... + t_k$ for $k \geqslant 2$ s.t for all $1 \leqslant i < j \leqslant k$, the first character of t_i isn't equal to the last character of t_j **if and only if** $s[0] \neq s[n-1]$.

Proof. (=>) Suppose we can represent the string as we described. Now suppose that $s[0] == s[n-1]$. This means that the first character of t_1 and the last character of t_k is the same. Hence, we found two strings (for $i = 1$ and $j = k$) that the first character of the first one is equal to the last character of the second one. But this is impossible.

(<=) Now suppose that $s[0] \neq s[n-1]$. If $n == 2$ then obviously we can take $t_1 = s[0]$ and $t_2 = s[1]$ and answer YES. Now let $n \geqslant 3$. Then we can choose an arbitrary d such that $0 < d < n-1$. Now consider $t_1 := s[0]...s[d], t_2 := s[d+1]...s[n-1]$. Then $s = t_1 + t_2$ and the first character of t_1 ($s[0]$) isn't equal to the last character of t_2 ($s[n-1]$) by assumption. This completes the proof. $\square$