Parametric Analysis of Variance

The same principles used in the two-sample means test can be applied to more than two sample means.

In these cases we call the test *analysis of variance*.

The null hypothesis for analysis of variance (AOV or ANOVA) is:

$H_0 : \mu_1 = \mu_2 \ldots = \mu_k$

And this may be applied over both time and space.

Assumptions of analysis of variance testing:

1. Observations between and within samples are random and independent (methodological issue).

2. The observations in each category are normally distributed (run normality tests on each group).

3. The population variances are assumed to be equal (homogeneity of variances test).

Generic Data Set

	j ↓ Group1	Group2	Group3
i → Obs 1	$X_{1,1}$	$X_{2,1}$	$X_{3,1}$
Obs 2	$X_{1,2}$	$X_{2,2}$	$X_{3,2}$
Obs 3	$X_{1,3}$	$X_{2,3}$	$X_{3,3}$
Obs 4	$X_{1,4}$	$X_{2,4}$	$X_{3,4}$
Obs 5	$X_{1,5}$	$X_{2,5}$	$X_{3,5}$

Note that i denotes rows and j denotes columns.

AOV:

- Compares the variation *within* the groups (columns) to the variation *among* the group means.
  - If the variation *among* the group means is *much greater* than within the groups, reject H_0.
  - If the variation *among* the group means *is not* much greater than within the groups, accept H_0.

There are 3 pieces of information that need to be determined to perform AOV:

TSS (total sum of squares) – the sum of the squared deviations of the observation from the overall mean.

BSS (between sum of squares) – the sum of the squared deviations between the groups.

WSS (within sum of squares) – the sum of the squared deviations with the groups.

We will use this set of equations because they are easier to calculate.

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^{2} - C \qquad where\ \ C = \frac{\left(\sum \sum X_{ij}\right)^2}{N}$$

$$BSS = \sum_{i=1}^{n_i} \frac{\left(\sum_{j=1}^{n_i} X_{ij}\right)^2}{n_i} - C$$

$$WSS = TSS - BSS$$

We test for differences among the groups using the F test.

- The standard F test is simply a ratio of variances.

- Here the F test is slightly modified to take into account:

    1. The number of groups.
    2. Differences in the number of group members.

The *F* statistic is calculated as:

$$F = \frac{BSS/_{k-1}}{WSS/_{N-k}}$$

where k is the number of groups and N is the total sample size. This is basically the sample size weighted ratio of the within group variation to the between group variation.

There are 2 *types* of degrees of freedom for the *F* statistic:

- k – 1 (where k is the number of columns) for the numerator.

- N – k (where N is the total sample size) for the denominator.

Note that the table from the book is different than the web table.

- Book p values are < or > a set alpha level.

- Web table p values can be a range.

A few words on Sum of Squares…

- In all cases, the sum of squares is a measure of total variation *from the mean* in a data set.

- If the sum of squares is *large*, then the data set has a lot of variation. This makes it more difficult to reject H_o (find a difference).

- If the sum of squares is *small*, then the data set has little variation. This makes it easier to reject H_o (find a difference).

- Once the sum of squares has been determined there are many ways of using the information.

In AOV the sum of squares is partitioned into 3 distinct types.

- This is because we are working with subsets of the total data set (3+ groups).

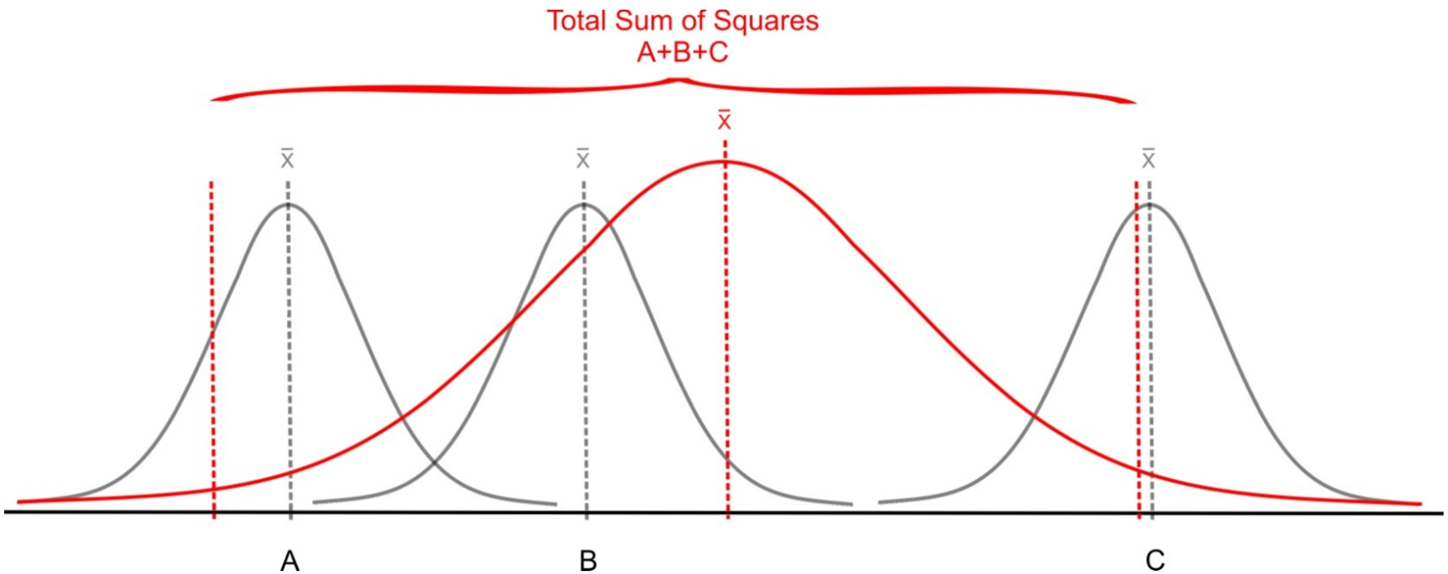
It can be thought of as performing multiple and simultaneous difference in means tests (t-tests).

Understanding the role of the total sum of squares (TSS) is straightforward: it is the total amount of variation in the data set.

$$\boldsymbol{TSS = \sum_{i}\sum_{j}(X_{ij} - \overline{X}_{ij})^2}$$

$\Big\downarrow$ **is equivalent to**

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^2 - C \qquad where\ C = \frac{\left(\sum\sum X_{ij}\right)^2}{N}$$



For TSS the data are pooled together.

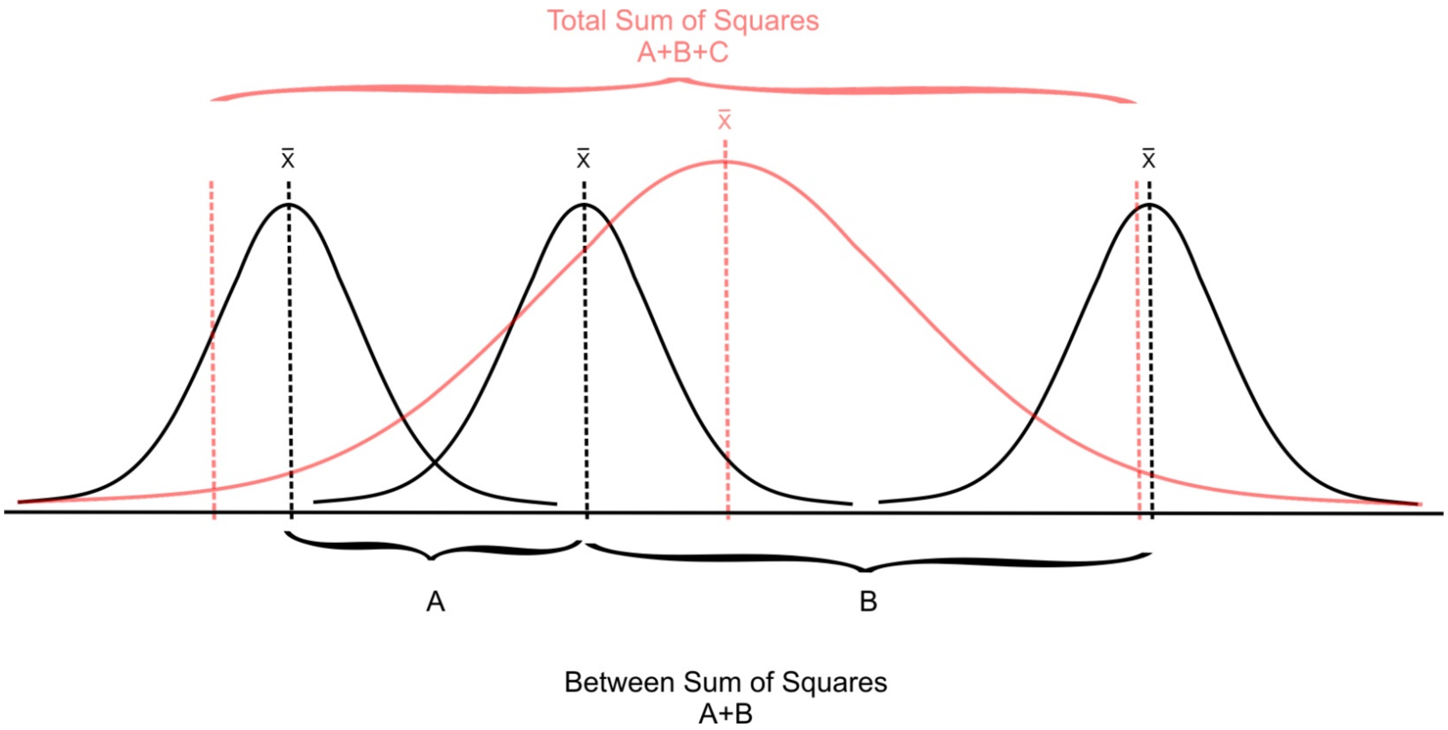
The between sum of squares (BSS) is the “treatment” variance.

- We want to compare these to each other.

$$\boldsymbol{BSS = \sum_{j} n_j(\overline{X}_j - \overline{X}_{ij})^2}$$

$$\Big\downarrow \ \textbf{is equivalent to}$$

$$BSS = \sum_{i=1}^{n_i} \frac{\left(\sum_{j=1}^{n_i} X_{ij}\right)^2}{n_i} - C$$



We are examining the distance between the means of each group (A and B).

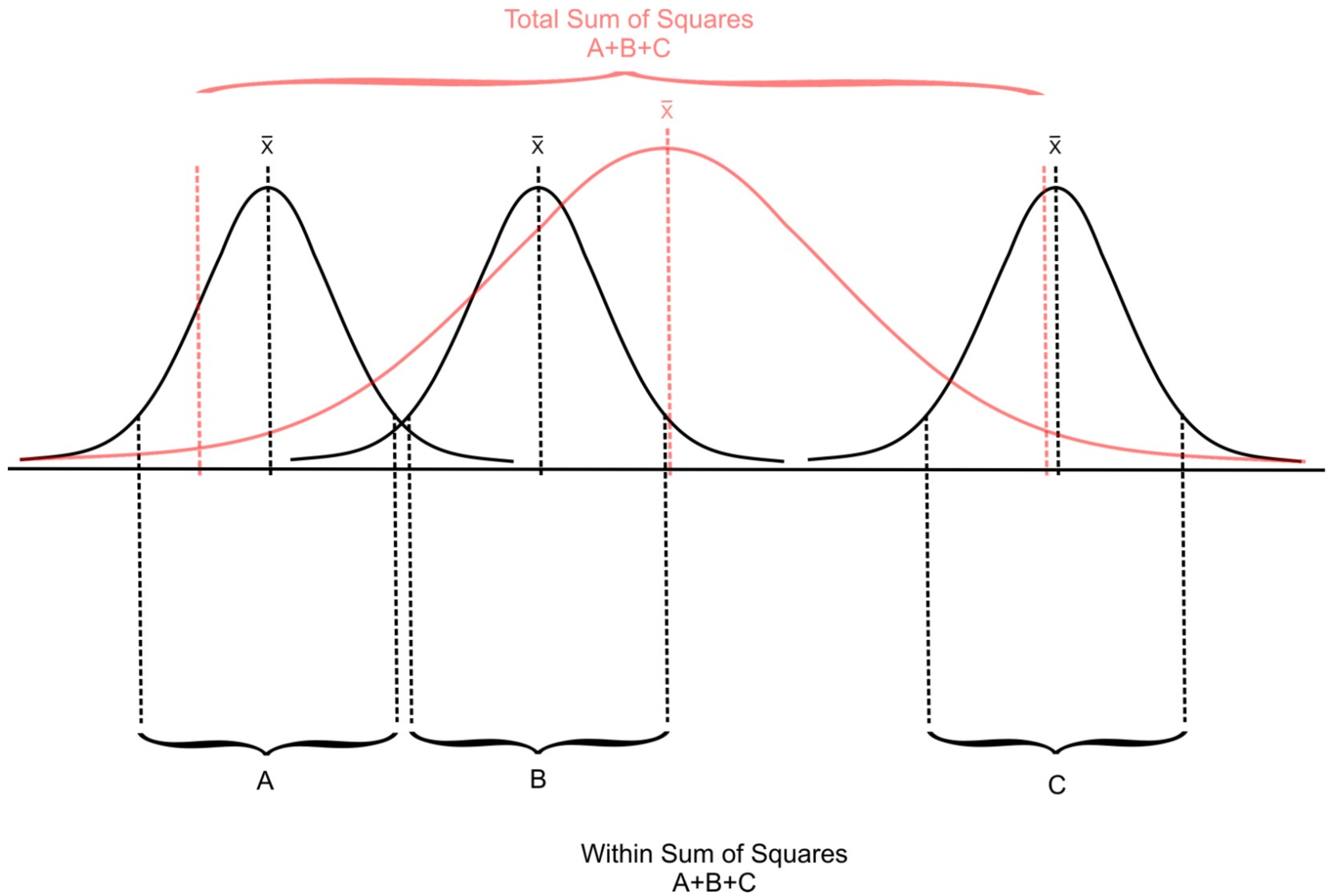
The within sum of squares (WSS) can be thought of as the "error" variance.

- We want to factor out variation within the groups.

$$\boldsymbol{WSS = \sum_{i}\sum_{j}(X_{ij} - \overline{X}_{j})^2}$$

$$\Big\downarrow \text{ **is equivalent to**}$$

$$WSS = TSS - BSS$$

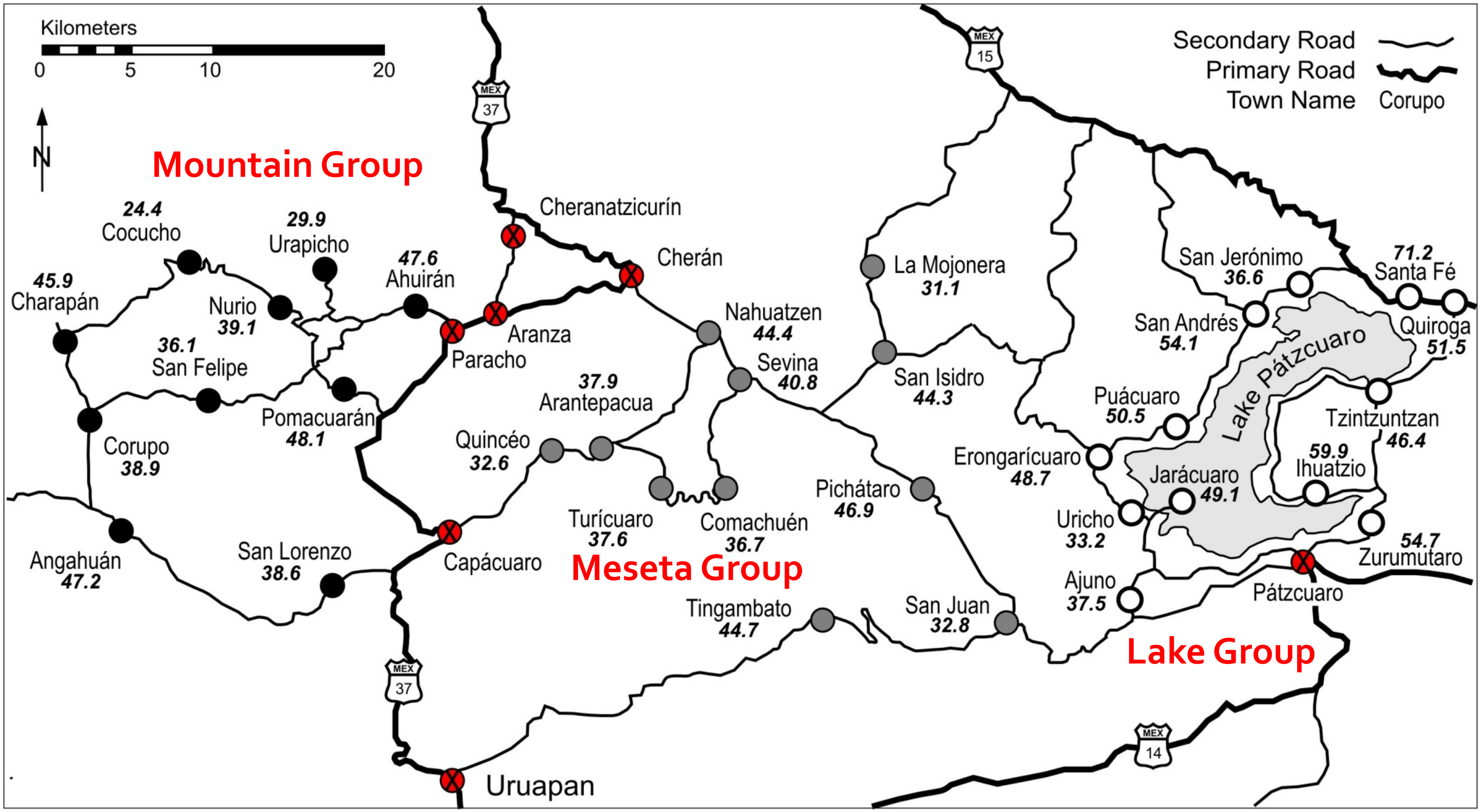

Total Sum of Squares
A+B+C
x̄
x̄
x̄
x̄
A
B
C
Within Sum of Squares
A+B+C

The F test is simply the ratio between the error variance (within groups) and the treatment variance (between groups).

$$F = \frac{BSS / (k-1)}{WSS / (N-k)}$$

IF the ratio is small then the difference is also small (accept H_o).
IF the ratio is large then the difference is also large (reject H_o).

Tarascan Village Percent Population Employed



Mountain **Meseta** **Lake**

We will drop the towns along the highways since we are interested only in rural towns.

Lake	
Quiroga	51.55
San Andres	54.12
San Jeronimo	36.61
Erongaricuaro	48.73
Ihuatzio	59.91
Jaracuaro	49.09
Puacuaro	50.52
Santa Fe	71.19
Uricho	33.22
Zurumutaro	54.68
Tzintzuntzan	46.38
Ajuno	37.52

Meseta	
Arantepacua	37.88
Comachuen	36.72
La Mojonera	31.08
Nahuatzen	44.39
Pichataro	46.90
Quinceo	32.64
San Isidro	44.32
San Juan	32.76
Sevina	40.76
Tingambato	44.72
Turicuaro	37.63

Mountain	
Ahuiran	47.56
Angahuan	47.21
Charapan	45.92
Cocucho	24.41
Corupo	38.92
Nurio	39.14
Pomacuaran	48.09
San Felipe	36.11
San Lorenzo	38.64
Urapicho	29.90

Normality Tests by Group

Ho: Village economic activity measurements are not significantly different than normal.

$$q_{Lake} = \frac{37.97}{10.52} = 3.61 \qquad q_{Meseta} = \frac{15.82}{5.53} = 2.86 \qquad q_{Mountain} = \frac{23.68}{7.97} = 2.97$$

$n_{Lake} = 12$
$q_{critical} = 2.80 – 3.91$
Accept Ho.

$n_{Meseta} = 11$
$q_{critical} = 2.74 – 3.80$
Accept Ho.

$n_{Mountain} = 10$
$q_{critical} = 2.67 – 3.69$
Accept Ho.

All village groups are not significantly different than normal (w/s $q_{Lake\ 3.61}$, $p > 0.05$, $q_{Meseta\ 2.86}$, $p > 0.05$, $q_{Mountain\ 2.97}$, $p > 0.05$)

Equality of Variances Test (Hartley Test[1])

Ho: The group variances are not significantly different.

$$F_{Max} = \frac{Max\ s^2}{Min\ s^2}$$

Group variances: Lake = 110.67
Meseta = 30.60
Mountain = 63.54

$$df = n_{\max} - 1, k$$

$$df = 11, 3$$

$$n_{lake} = 12$$

$$F_{\max} = \frac{110.67}{30.60} = 3.62$$

$$F_{critical} = 4.85$$

The group variances are not significantly different ($F_{3.62}$, $p > 0.05$).

[1] SPSS uses the less conservative Levene's Test.

Critical Values of F_{max} for Hartley's Homogeneity of Variance Test

The upper value in each box is for α = 0.05. The lower value is for α=0.01. The test assumes that there are equal sample sizes in each group (n). For unequal sample sizes, use the smaller of the df for the two variances being compared.

DF (n-1)	Number of treatments (k)										
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	39.0 199	87.5 448	142 729	202 1036	266 1362	333 1705	403 2063	475 2432	550 2813	626 3204	714 3605
3	15.4 47.5	27.8 85.0	39.2 120	50.7 151	62.0 184	72.9 21	83.5 24	93.9 28	104 31	114 33	124 36
4	9.6 23.2	15.5 37.0	20.6 49.0	25.2 59	29.5 69	33.6 79	37.5 89	41.1 97	44.6 106	48.0 113	51.4 120
5	7.2 14.9	10.8 22.0	13.7 28.0	16.3 33	18.7 38	20.8 42	22.9 46	24.7 50	26.5 54	28.2 57	29.9 60
6	5.82 11.1	8.38 15.5	10.4 19.1	12.1 22	13.7 25	15.0 27	16.3 30	17.5 32	18.6 34	19.7 36	20.7 37
7	0.99 8.89	6.94 12.1	8.44 14.5	9.70 16.5	10.8 18.4	11.8 20	12.7 22	13.5 23	14.3 24	15.1 26	15.8 27
8	4.43 7.50	6.00 9.90	7.18 11.7	8.12 13.2	9.03 14.5	9.78 15.8	10.5 16.9	11.1 17.9	11.7 18.9	12.2 19.8	12.7 21
9	4.03 6.54	5.34 8.50	6.31 9.9	7.11 11.1	7.80 12.1	8.41 13.1	8.95 13.9	9.45 14.7	9.91 15.3	10.3 16.0	10.7 16.6
Use next lower value. → **10**	3.72 5.85	$\boxed{4.85}$ 7.40	5.67 8.6	6.34 9.6	6.92 10.4	7.42 11.1	7.87 11.8	8.28 12.4	8.66 12.9	9.01 13.4	9.34 13.9
12	3.28 4.91	4.16 6.1	4.75 6.9	5.30 7.6	5.72 8.2	6.09 8.7	6.42 9.1	6.72 9.5	7.00 9.9	7.25 10.2	7.43 10.6
15	2.86 4.07	3.54 4.9	4.01 5.5	4.37 6.0	4.68 6.4	4.95 6.7	5.19 7.1	5.40 7.3	5.59 7.5	5.77 7.8	5.95 8.0
20	2.46 3.32	2.95 3.8	3.29 4.3	3.54 4.6	3.76 4.9	3.94 5.1	4.10 5.3	4.24 5.5	4.37 5.6	4.49 5.8	4.59 5.9
30	2.07 2.63	2.40 3.0	2.61 3.3	2.78 3.4	2.91 3.6	3.02 3.7	3.12 3.8	3.21 3.9	3.29 4.0	3.36 4.1	3.39 4.2
60	1.67 1.96	1.85 2.2	1.96 2.3	2.04 2.4	2.11 2.4	2.17 2.5	2.22 2.5	2.26 2.6	2.30 2.6	2.33 2.7	2.36 2.7
∞	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00

- The Hartley test is VERY sensitive to the normality assumption. Even small deviations from normality can influence the results.

- The test works best when the group sample sizes are equal or roughly equal. If very unequal use the group with the largest sample size (rather than largest variance) for determining n.

- If in doubt, double check using the SPSS Levene's test.

Hypotheses

H_o: There is no significant difference in the percent working population *among* the three Tarascan village groups.

H_a: There is a significant difference in the percent working population *among* the three Tarascan village groups.

Use *between* for two-group testing and *among* for 3+ group testing.

Lake		Meseta		Mountain	
Quiroga	51.55	Arantepacua	37.88	Ahuiran	47.56
San Andres	54.12	Comachuen	36.72	Angahuan	47.21
San Jeronimo	36.61	La Mojonera	31.08	Charapan	45.92
Erongaricuaro	48.73	Nahuatzen	44.39	Cocucho	24.41
Ihuatzio	59.91	Pichataro	46.90	Corupo	38.92
Jaracuaro	49.09	Quinceo	32.64	Nurio	39.14
Puacuaro	50.52	San Isidro	44.32	Pomacuaran	48.09
Santa Fe	71.19	San Juan	32.76	San Felipe	36.11
Uricho	33.22	Sevina	40.76	San Lorenzo	38.64
Zurumutaro	54.68	Tingambato	44.72	Urapicho	29.90
Tzintzuntzan	46.38	Turicuaro	37.63		
Ajuno	37.52				
Column Sums	*593.5*		*429.8*		*395.9*

$n_1 = 12$

$n_2 = 11$

$n_3 = 10$

$n_{total} = 33$

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^2 - C \qquad where\ C = \frac{\left(\sum \sum X_{ij}\right)^2}{N}$$

Lake		x^2	Meseta		x^2	Mountain		x^2
Quiroga	51.55	2657.4	Arantepacua	37.88	1434.9	Ahuiran	47.56	2262.2
San Andres	54.12	2928.9	Comachuen	36.72	1348.7	Angahuan	47.21	2228.4
San Jeronimo	36.61	1340.4	La Mojonera	31.08	965.7	Charapan	45.92	2108.9
Erongaricuaro	48.73	2374.9	Nahuatzen	44.39	1970.4	Cocucho	24.41	596.0
Ihuatzio	59.91	3589.6	Pichataro	46.90	2199.6	Corupo	38.92	1515.1
Jaracuaro	49.09	2409.9	Quinceo	32.64	1065.7	Nurio	39.14	1532.3
Puacuaro	50.52	2552.2	San Isidro	44.32	1964.5	Pomacuaran	48.09	2313.0
Santa Fe	71.19	5068.2	San Juan	32.76	1072.9	San Felipe	36.11	1303.9
Uricho	33.22	1103.8	Sevina	40.76	1661.3	San Lorenzo	38.64	1493.3
Zurumutaro	54.68	2990.1	Tingambato	44.72	1999.8	Urapicho	29.90	893.9
Tzintzuntzan	46.38	2151.5	Turicuaro	37.63	1416.0			
Ajuno	37.52	1407.8						
Column Sums	***593.5***	***30574.6***		***429.8***	***17099.5***		***395.9***	***16246.9***

Sum of the observations **Sum of the squared observations**

Total observation sum = 593.5 + 429.8 + 395.9 = 1419.2 $\longleftarrow \left(\sum \sum X_{ij}\right)$

Total squared sum = 30574.6 + 17099.5 + 16246.9 = 63921 $\longleftarrow \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^2$

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{n_i} X_{ij}^{2} - C \qquad where \quad C = \frac{\left(\sum \sum X_{ij}\right)^2}{N}$$

$$C = \frac{(1419.2)^2}{33} = 61034.2$$

$$TSS = 63921 - 61034.2 = 2886.8$$

$$BSS = \sum_{i=1}^{n_i} \frac{\left(\sum_{j=1}^{n_i} X_{ij} \right)^2}{n_i} - C$$

Lake *n=12*		x^2	Meseta *n=11*		x^2	Mountain *n=10*		x^2
Quiroga	51.55	2657.4	Arantepacua	37.88	1434.9	Ahuiran	47.56	2262.2
San Andres	54.12	2928.9	Comachuen	36.72	1348.7	Angahuan	47.21	2228.4
San Jeronimo	36.61	1340.4	La Mojonera	31.08	965.7	Charapan	45.92	2108.9
Erongaricuaro	48.73	2374.9	Nahuatzen	44.39	1970.4	Cocucho	24.41	596.0
Ihuatzio	59.91	3589.6	Pichataro	46.90	2199.6	Corupo	38.92	1515.1
Jaracuaro	49.09	2409.9	Quinceo	32.64	1065.7	Nurio	39.14	1532.3
Puacuaro	50.52	2552.2	San Isidro	44.32	1964.5	Pomacuaran	48.09	2313.0
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Uricho	33.22	1103.8	Sevina	40.76	1661.3	San Lorenzo	38.64	1493.3
Zurumutaro	54.68	2990.1	Tingambato	44.72	1999.8	Urapicho	29.90	893.9
Tzintzuntzan	46.38	2151.5	Turicuaro	37.63	1416.0			
Ajuno	37.52	1407.8						
Column Sums	*593.5*	*30574.6*		*429.8*	*17099.5*		*395.9*	*16246.9*

$$BSS = \frac{(593.5)^2}{12} + \frac{(429.8)^2}{11} + \frac{(395.9)^2}{10} - 61034.2 = 786.5$$

$$TSS = 2886.8$$ Only used to calculate the WSS

$$BSS = 786.5$$

$$WSS = 2886.8 - 786.5 = 2100.3$$

$$F = \frac{BSS / (k-1)}{WSS / (N-k)}$$

$$F = \frac{786.5 / (3-1)}{2100.3 / (33-3)} = \frac{393.25}{70.01} = 5.62$$

df = (k-1) = (3-1) = 2,
(n-k) = (33-3) = 30

To get the probability range on this table, read DOWN the column.

Critical Values of the *F*-Distribution (cont.)
Taken from Rohlf and Sokal, 1981 Table 16

Numerator Degrees of Freedom (V1)

Denominator Degrees of Freedom (V2)	α	1	2	3	4	5	6	7	8	9	10	11	12	α	
	.75	.104	.291	.406	.480	.532	.571	.600	.624	.644	.660	.674	.686	.75	
	.50	.468	.712	.810	.861	.893	.915	.930	.942	.951	.959	.965	.970	.50	
	.25	1.38	1.46	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.37	1.36	1.35	.25	
	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.83	1.81	.10	
26	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	.05	26
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.54	2.49	.025	
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	.01	
	.005	9.41	6.54	5.41	4.79	4.38	4.10	3.89	3.73	3.60	3.49	3.40	3.33	.005	
	.001	13.7	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.64	4.48	4.35	4.24	.001	
	.75	.104	.291	.406	.480	.532	.571	.601	.624	.644	.660	.674	.686	.75	
	.50	.467	.711	.809	.861	.892	.914	.930	.941	.950	.958	.964	.969	.50	
	.25	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.35	.25	
	.10	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.82	1.80	.10	
27	.05	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	.05	27
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.51	2.47	.025	
	.01	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.45	3.06	2.99	2.93	.01	
	.005	9.34	6.49	5.36	4.74	4.34	4.06	3.85	3.69	3.56	3.45	3.36	3.28	.005	
	.001	13.6	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.57	4.41	4.28	4.17	.001	
	.75	.103	.290	.406	.480	.532	.571	.601	.625	.644	.661	.675	.687	.75	
	.50	.467	.711	.808	.860	.892	.913	.929	.940	.950	.957	.963	.968	.50	
	.25	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	.25	
	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.81	1.79	.10	
28	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	.05	28
	.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.49	2.45	.025	
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	.01	
	.005	9.28	6.44	5.32	4.70	4.30	4.02	3.81	3.65	3.52	3.41	3.32	3.25	.005	
	.001	13.5	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.50	4.35	4.22	4.11	.001	
	.75	.103	.290	.406	.480	.532	.571	.601	.625	.645	.661	.675	.687	.75	
	.50	.467	.710	.808	.859	.891	.912	.928	.940	.949	.956	.962	.967	.50	
	.25	1.38	1.45	1.45	1.43	1.41	1.40	1.38	1.37	1.36	1.35	1.35	1.34	.25	
	.10	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	2.86	1.83	1.80	1.78	.10	
29	.05	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	.05	29
	.025	5.59	4.20	3.60	3.27	3.04	2.88	2.76	2.67	2.59	2.51	2.47	2.43	.025	
	.01	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93	2.87	.01	
	.005	9.23	6.40	5.28	4.66	4.26	3.98	3.77	3.61	3.48	3.38	3.29	3.21	.005	
	.001	13.4	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.45	4.29	4.16	4.05	.001	
	.75	.103	.290	.406	.480	.532	.571	.601	.625	.645	.661	.676	.688	.75	
	.50	.466	.709	.807	.858	.890	.912	.927	.939	.948	.955	.961	.966	.50	
	.25	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.34	.25	
	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77	.10	
30	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	.05	30
	.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.46	2.41	.025	
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.90	2.84	.01	
	.005	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.25	3.18	.005	
	.001	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	4.24	4.11	4.00	.001	

F = 5.62

Numerator Degrees of Freedom (V1)

Denominator Degrees of Freedom (V2)	α	1	2	3	4	5	6	7	8	9	10	11	12	α	
	.75	.103	.289	.404	.480	.533	.572	.603	.627	.647	.662	.679	.689	.75	
	.50	.463	.705	.802	.854	.885	.907	.922	.934	.943	.950	.956	.961	.50	
	.25	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	.25	
	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.74	.10	
40	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.04	.05	40
	.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.33	2.29	.025	
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	.01	
	.005	8.83	6.07	4.98	4.37	3.99	3.71	3.51	3.35	3.22	3.12	3.03	2.95	.005	
	.001	12.6	8.25	6.60	5.70	5.13	4.73	4.44	4.21	4.02	3.87	3.74	3.64	.001	
	.75	.102	.289	.405	.480	.534	.573	.604	.629	.650	.667	.682	.695	.75	
	.50	.461	.701	.798	.849	.880	.901	.917	.928	.937	.945	.951	.956	.50	
	.25	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.29	.25	
	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66	.10	
60	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	.05	60
	.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.22	2.17	.025	
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	.01	
	.005	8.49	5.79	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.81	2.74	.005	
	.001	12.0	7.76	6.17	5.31	4.76	4.37	4.09	3.87	3.69	3.54	3.41	3.31	.001	
	.75	.102	.288	.405	.481	.534	.574	.606	.631	.652	.670	.686	.699	.75	
	.50	.458	.697	.793	.844	.875	.896	.912	.923	.932	.939	.945	.950	.50	
	.25	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.27	1.26	.25	
	.10	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60	.10	
120	.05	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.87	1.83	.05	120
	.025	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.10	2.05	.025	
	.01	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.40	2.34	.01	
	.005	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.62	2.54	.005	
	.001	11.4	7.32	5.79	4.95	4.42	4.04	3.77	3.55	3.38	3.24	3.11	3.02	.001	
	.75	.102	.288	.404	.481	.535	.576	.608	.634	.655	.674	.690	.703	.75	
	.50	.455	.693	.789	.839	.870	.891	.907	.918	.927	.934	.939	.945	.50	
	.25	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.24	.25	
	.10	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.57	1.55	.10	
∞	.05	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.79	1.75	.05	∞
	.025	5.02	3.69	3.11	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.99	1.94	.025	
	.01	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18	.01	
	.005	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.43	2.36	.005	
	.001	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	2.96	2.84	2.74	.001	

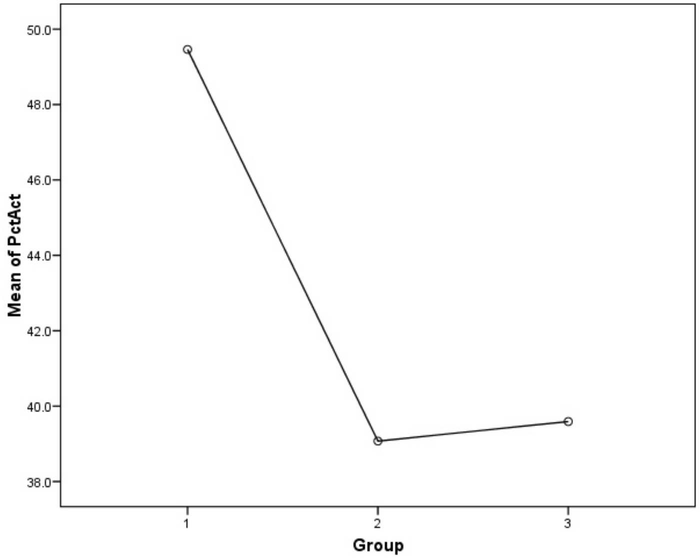
Since 5.62 > 3.32 reject H_o

There is a significant difference in the percent employed population among the three groups of Tarascan villages ($F_{5.62,}$ $0.01 > p > 0.005$).

Test of Homogeneity of Variances

PctAct

Levene Statistic	df1	df2	Sig.
.876	2	30	.427



ANOVA

PctAct

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	786.816	2	393.408	5.633	.008
Within Groups	2095.240	30	69.841		
Total	2882.056	32			

Multiple Comparisons

Dependent Variable: PctAct

LSD

(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1	2	10.3886*	3.4885	.006	3.264	17.513
	3	9.8698*	3.5783	.010	2.562	17.178
2	1	-10.3886*	3.4885	.006	-17.513	-3.264
	3	-.5188	3.6515	.888	-7.976	6.938
3	1	-9.8698*	3.5783	.010	-17.178	-2.562
	2	.5188	3.6515	.888	-6.938	7.976

*. The mean difference is significant at the .05 level.

Comparing our results to those of SPSS:

	SPSS	Us
TSS	2882.1	2886.6
BSS	786.8	786.8
WSS	2095.2	2100.3
F	5.63	5.62
p	0.008	0.01 - 0.005

Multiple Comparison (AOV Post Hoc) Tests

- Available in SPSS to compare all group pairs.
- Used to determine which means are significantly different.
- Similar to (but not exactly like) performing two group means tests while holding the other group(s) constant.
- The post hoc test listed below are useful for non-paired comparisons and non treatment-control comparisons.

For treatment-control (e.g. before- after) comparisons, use the Dunnet test.

LSD (Fisher test)

- The Least Significant Difference test assumes that if the null hypothesis is incorrect, as indicated by a significant F-test, Type I errors are not really possible.

- LSD test has been criticized for not sufficiently controlling for Type I errors.

- Although this is the first test on the list in SPSS, it is probably not the best.

Bonferroni

- Probably the most commonly used post hoc test because it is flexible, simple to compute, and can be used with any type of statistical test.

- The traditional Bonferroni tends to lack power.

- The Bonferroni overcorrects for Type I error.

Sidak

- A relatively simple modification of the Bonferroni formula that would have less of an impact on statistical power but retain much of the flexibility of the Bonferroni method.
- This approach is convenient but has not received any systematic study.
- It is unlikely that a single, simple correction will result in the most efficient balance of Type I and Type II errors.

Tukey (Tukey's HSD)

- The HSD (*honest significant difference*) has greater power than the other tests under most circumstances.
- Requires equal sample sizes.
- If sample sizes are unequal, the Tukey-Kramer test is used by SPSS which computes the harmonic mean of the sample sizes for each group, slightly lowering the power of the test.

Games-Howell

- This test can be used when variances are unequal and also takes into account unequal group sizes.

- Severely unequal variances can lead to increased Type I error, and with smaller sample sizes, more moderate differences in group variance can also lead to increases in Type I error.

- This test appears to do better than the Tukey HSD if variances are very unequal (or moderately so in combination with small sample size) or can be used if the sample size per cell is very small (e.g., <6).

- This is probably the most reliable post hoc test, but also the most conservative.

Probabilities from 5 Post Hoc Tests

Group Comparisons	Games-Howell	Tukey HSD	LSD	Bonferroni	Sidak
1 – 2	0.021	0.015	0.006	0.017	0.017
1 – 3	$\boxed{0.053}$	0.026	0.010	0.029	0.029
2 – 3	0.984	0.989	0.888	1.000	0.999

Note that the differences are slight, except for the Games-Howell test.

Texas Death Row Inmates (2016)

Race/ Ethnicity	Age	Race/ Ethnicity	Age	Race/ Ethnicity	Age
White	44	Black	29	Hispanic	26
White	37	Black	36	Hispanic	23
White	27	Black	19	Hispanic	20
White	28	Black	23	Hispanic	25
White	40	Black	25	Hispanic	29
White	18	Black	19	Hispanic	36
White	19	Black	28		
White	41	Black	22		
		Black	23		

Are the ages (at time of offense) of Texas death row inmates by race/ethnicity different?