CS5800: Algorithms — Virgil Pavlu

Homework 8

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Collaborators:

Instructions:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- Please review the grading policy outlined in the course information page.
- You must also write down with whom you worked on the assignment. If this changes from problem to problem, then you should write down this information separately with each problem.
- Problem numbers (like Exercise 3.1-1) are corresponding to CLRS 3^{rd} edition. While the 2^{nd} edition has similar problems with similar numbers, the actual exercises and their solutions are different, so make sure you are using the 3^{rd} edition.

1. **(50 points)** Answers to problem are 1-4 in code files

2. **(50 points)**

Consider an ordinary binary min-heap data structure supporting the instructions INSERT and EXTRACT-MIN that, when there are n items in the heap, implements each operation in $O(\lg n)$ worst-case time. Give a potential function Φ such that the amortized cost of INSERT is $O(\lg n)$ and the amortized cost of EXTRACT-MIN is $O(1)$, and show that your potential function yields these amortized time bounds. Note that in the analysis, n is the number of items currently in the heap, and you do not know a bound on the maximum number of items that can ever be stored in the heap.

Solution::

Consider the potential function $\Phi = \sum_{i=1}^{n} \lg i$ for a binary min-heap with n elements. For INSERT, when adding a new element, the potential increases by $\lg(n+1)$. Combined with the actual cost of $O(\lg n)$, this yields an amortized cost of $O(\lg n)$.

For EXTRACT-MIN, while the actual cost is $O(\lg n)$, removing an element decreases the potential by $\lg n$. This decrease in potential compensates for the actual cost, resulting in an $O(1)$ amortized cost. The potential function effectively banks cost during insertions and spends it during extractions, achieving our desired bounds of $O(\lg n)$ for INSERT and $O(1)$ for EXTRACT-MIN.