

# CSP Ex09

## Task1:

$$\begin{aligned}
 \mathcal{H} &= T + V_{\text{int}} + \lambda \mathcal{K} & \mathcal{K} &= \frac{1}{2} (\dot{a}^2 - \dot{r}^2) \stackrel{!}{=} 0 \\
 &= \frac{1}{2} 4\epsilon \left( \left( \frac{6}{r} \right)^{12} - \left( \frac{6}{r} \right)^6 \right) + \lambda \frac{1}{2} (\dot{a}^2 - \dot{r}^2) \\
 \frac{d}{dt} \mathcal{H} &= \left( 24\epsilon \left( 2 \left( \frac{6}{r} \right)^{12} - \left( \frac{6}{r} \right)^6 \right) \frac{1}{r^2} - \lambda \dot{r} \right) \dot{a} - \lambda (\dot{a}^2 - \dot{r}^2) \\
 \Rightarrow \quad 24\epsilon \left( 2 \left( \frac{6}{r} \right)^{12} - \left( \frac{6}{r} \right)^6 \right) \frac{1}{r^2} &= \lambda \dot{r} & \Rightarrow \quad \lambda &= 24\epsilon \left( 2 \left( \frac{6}{r} \right)^{12} - \left( \frac{6}{r} \right)^6 \right) \frac{1}{\dot{r} r^2} \\
 \frac{1}{2} (\dot{a}^2 - \dot{r}^2) &= 0 & \Rightarrow \quad \dot{a}^2 &= \dot{r}^2 \Rightarrow \dot{a} = \dot{r} \\
 \lambda &= 24\epsilon \left( 2 \left( \frac{6}{a} \right)^{12} - \left( \frac{6}{a} \right)^6 \right) \frac{1}{\dot{a}^2}
 \end{aligned}$$

## Task2:

Note that I created a video. It looks pretty funny.

Parameters I used:

$N = 432$

$L = 10$

$rc = 2.5$

$T = 1000$

$dt = 10^{-3}$

$m = 1.0$

$\epsilon = 1.0$

$\sigma = 1.0$

$d = \epsilon / 10$

The simulation looks pretty good. It behaves like expected. If you watch the video you see the system will explode at a certain point. It is very unstable. The Energy plot looks this time more normal than last time. It only has a big spike at the beginning due to the high initial velocity. After that, it goes stable downwards. I was very careful this time with the initialization of the particles. I set them in a grid spaced equally (Could use noise to make it a little random but I am ok with it).

