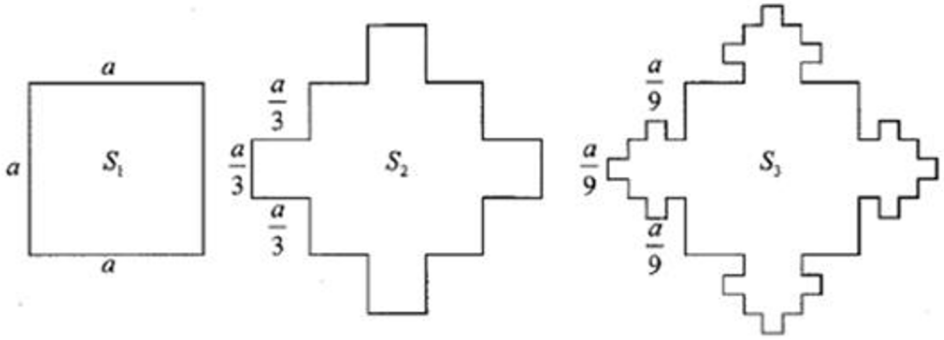
Engaging Complexity
Quiz 3

THE LONDON INTERDISCIPLINARY SCHOOL

1. (4 points) The 1-dimensional cellular automata is a cellular automata where the current state is only one dimension and where each cell is either black or white. The next state is determined by the state of the cell above and its neighbours as dictated by the rules. The neighbourhood includes the adjacent cells.

   Program a simulation in Python that runs 20 iterations of the 1-dimensional cellular automata with the rule shown below. Plot your results.

2. (6 points) The figure below shows part of a sequence S_1, S_2, S_3, ... of model snowflakes, based on squares (in the prep you saw one based on triangles). The first term S_1 consists of a single square of side a. To obtain S_2, the middle third of each edge is replaced with a new square, of side $\frac{a}{3}$, as shown in the figure. Subsequent terms are obtained by replacing the middle third of each external edge of a new square formed in the previous snowflake, by a square $\frac{1}{3}$ of the size, as illustrated by S_3 in the figure.



   (a) Deduce that to form S_4, 36 new squares of side $\frac{a}{27}$ must be added to S_3.

   (b) Demonstrate that the perimeters of S_2 and S_3 are $\frac{20a}{3}$ and $\frac{28a}{3}$ respectively.

   (c) Find the perimeter of S_n.

   (d) Describe what happens to the perimeter of S_n as n increases.

   (e) Find the areas of S_1, S_2 and S_3.

   (f) Find the smallest value of the constant S such that the area of S_n is strictly less than S, for all values of n.