Engaging Complexity
Quiz 1

THE LONDON INTERDISCIPLINARY SCHOOL

In the problems below, where it says 'plot' this means generating the plot in the computer, you can do this by using Python, Excel, or analogous software.. Remember to label your figures and include units (where required).

1. (2 points) As blood moves through a vein or an artery, its velocity v is greatest along the central axis and decreases as the distance r from the central axis increases. The formula that gives v as a function of r is called the law of laminar flow. For an artery with radius 0.5 cm, the relationship between v (in cm/s) and r (in cm) is given by the function
$$v(r) = 18500(0.25 - r^2), \qquad 0 \leq r \leq 0.5.$$
   (a) Find $v(0.1)$ and $v(0.4)$.
   (b) What do your answers to part (a) tell you about the flow of blood in this artery?
   (c) Make a table of values of $v(r)$ for $r = 0$, 0.1, 0.2, 0.3, 0.4, 0.5.
   (d) Sketch by hand the graph of v.

2. (2 points)
   (a) As a weather balloon is inflated, the thickness T of its rubber skin is related to the radius of the balloon by
$$T(r) = \frac{0.5}{r^2},$$
   where T and r are measured in centimetres. Plot the function T for values of r between 10 and 100.
   (b) The power produced by a wind turbine depends on the speed of the wind. If a windmill has blades 3 metres long, then the power P produced by the turbine is modeled by
$$P(v) = 14.1v^3,$$
   where P is measured in watts (W) and v is measured in metres per second (m/s). Plot the function P for wind speeds between 1 m/s and 10 m/s.

3. (2 points) A sequence is defined recursively by the given formulas. Find the first five terms of each sequence.
   (a) $a_n = 2(a_{n-1} + 3)$ and $a_1 = 4$.
   (b) $a_n = \frac{1}{1+a_{n-1}}$ and $a_1 = 1$.
   (c) $a_n = a_{n-1} - a_{n-2}$ and $a_1 = 1$, $a_2 = 3$.

4. (2 points) Plot the first 100 terms of the following sequences:

   (a) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ and $a_1 = a_2 = a_3 = 1$.

   (b) $a_n = a_{n-1} - 2a_{n-2}$ and $a_1 = a_2 = 1$.

   (c) $a_n = 4a_{n-1}(1 - a_{n-1})$ and $a_1 = 0.2$.

5. (2 points) Plot the first 50 terms of the sequence $a_n = a_{n-1} + a_{n-2}$ with initial conditions $a_1 = 0$, $a_2 = 1$. In a different figure, plot the ratio $\frac{a_n}{a_{n-1}}$. Comment on the long-time behaviour (as n becomes large) of the plots.