Engaging Complexity
Quiz 2

Notation: log denotes logarithm base 10, ln denotes natural logarithm.

1. (3 points) The relative growth rate of world population has been decreasing steadily in recent years. On the basis of this, some population models predict that world population will eventually stabilise at a level that the planet can support. One such logistic model is

$$P(t) = \frac{73.2}{6.1 + 5.9e^{-0.02t}},$$

where $t = 0$ is the year 2000 and population P is measured in billions.

(a) What world population does this model predict for the year 2200? For 2300?

(b) Sketch a graph of the function P for the years 2000 to 2500.

(c) What size does the world population approach as time goes on?

2. (2 points)

(a) A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function

$$m(t) = 13e^{-0.015t},$$

where $m(t)$ is measured in kilograms. Find the mass at time $t = 0$. How much of the mass remains after 45 days?

(b) When a certain medical drug is administered to a patient, the number of milligrams remaining in the patient's bloodstream after t hours is modeled by

$$D(t) = 50e^{-0.2t}.$$

What is the half life of the drug (i.e. the time required for the quantity of the drug to reduce to half of its initial value)?

3. (3 points)

(a) A certain strain of bacteria divides every 3 hours. If a colony is started with 50 bacteria, then the time t (in hours) required for the colony to grow to N bacteria is given by

$$t = \frac{\log(N/50)}{\log(2)}.$$

Find the time required for the colony to grow to a million bacteria.

(b) The time required to double the amount of an investment at an interest rate r compounded continuously is given by

$$t = \frac{\ln(2)}{r}.$$

Find the time required to double an investment at 6%, 7%, and 8%.

(c) The rate at which a battery charges is slower the closer the battery is to its maximum charge C_0. The time (in hours) required to charge a fully discharged battery to a charge C is given by

$$t = -k \ln \left(1 - \frac{C}{C_0} \right),$$

where k is a positive constant that depends on the battery. For a certain battery, $k = 0.25$. If this battery is fully discharged, how long will it take to charge to 90% of its maximum charge C_0?

4. (2 points) Vilfredo Pareto (1848-1923) observed that most of the wealth of a country is owned by a few members of the population. Pareto's Principle is

$$\log(P) = \log(c) - k \log(W),$$

where W is the wealth level (how much money a person has) and P is the number of people in the population having that much money.

(a) Solve the equation for P.

(b) Assume that $k = 2.1$ and $c = 8000$, and that W is measured in millions of dollars. Use part (a) to find the number of people who have $2 million or more. How many people have $10 million or more?