### MIXED DX: A SAS® Macro for Two-Level Linear Model Diagnostics

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### **ABSTRACT**

Multilevel modeling has become a common analytic technique across a variety of disciplines including medicine and the social and behavioral sciences. However, because many researchers who use multilevel modeling in their research do not report if the data were screened for potential violations of distributional assumptions and outliers, it is unclear if these diagnostic procedures are being conducted. Thus, in an effort to make the process of checking the assumptions for two-level linear models easier for the applied researcher, this paper provides a SAS macro for conducting two-level linear model diagnostics, including examinations of residual normality, linearity, homogeneity of variance, and influential outliers. By utilizing data from PROC MIXED ODS tables, the macro produces box-and-whisker plots, summary tables of statistical output, histograms, and plots of residuals by predicted values. Macro outputs are produced for both level-1 and level-2 data. This paper provides the macro programming language, as well as results from an executed example of the macro.

### INTRODUCTION

As multilevel modeling techniques (also called hierarchical linear modeling or mixed modeling) continue to be used with increasing frequency across a variety of disciplines, it is important for researchers to conduct appropriate model diagnostics when using these statistical methods. Like most parametric statistical procedures, there are certain distributional assumptions underlying the validity of the Type I error control when conducting multilevel analyses. Thus, just as researchers were trained to evaluate the underlying assumptions for multiple regression and ANOVA, they, too, need to assess and report on the assumptions associated with multilevel models. More specifically, although most of the assumptions associated with two-level linear models are similar to OLS model assumptions (i.e., residual normality, linearity, and homogeneity of variance), few articles in which multilevel modeling techniques are used contain this information, despite recommendations to be transparent when reporting multilevel modeling research processes and findings (Ferron, Hogarty, Dedrick, Hess, Niles, & Kromrey, 2008).

One likely reason for the omission of this information could be related to the cumbersome nature of conducting model diagnostics of two-level linear models. First, it is not easy to locate all of the data necessary to conduct diagnostics for both level-1 and level-2 residuals. For example, the level-1 and level-2 residuals are stored in two different ODS tables, which not only have to be requested separately, but the commands necessary to request each ODS table occurs at two different places within the PROC MIXED code (i.e., the ODS table for level-1 residuals is an option that can be added to the model statement whereas the ODS table for level-2 residuals is requested through an ODS OUTPUT command). Second, the ODS table that contains the level-2 residuals does not contain predicted values for level-2 units. Thus, the data must be manipulated to generate these values. Third, although PROC MIXED does allow the INFLUENCE option on the model statement, the oodles of data produced by this option can be overwhelming, especially to a novice PROC MIXED user. Thus, in an effort to reduce the burden of conducting these important, yet unwieldy, analyses this paper provides a SAS macro to conduct diagnostic evaluations of two-level linear models, including examinations of residual normality, linearity, homogeneity of variance, and influential outliers. Diagnostics are conducted for both level-1 and level-2 data and the macro includes the data screening techniques recommended by authors such as Hox (2002), Longford (2001), and Raudenbush and Bryk (2002).

### STATISTICAL ASSUMPTIONS AND THEIR EVALUATION

Raudenbush and Bryk (2002) correctly assert that the validity of inferences based on models depend on the degree to which assumptions are upheld about the structural and random parts of the model. They go on to suggest that "skillful analysts pay close attention to the assumptions required by their models" (p. 253). Unfortunately, based on reporting practices, it seems that the assumptions frequently go unexamined or under-examined by analysts conducting multilevel modeling (Ferron et al., 2008). However, it is not clear if this common omission is a function of a lack of understanding or the effort and time needed to assess model adequacy. This work seeks to address the latter issue by facilitating the generation of information necessary to assess to the tenability of the model assumptions.

Under Ordinary Least Squares (OLS) regression, the error terms are assumed to be normally distributed, independent, and homoscedastic. These same assumptions apply to linear multilevel models. In particular, the assumption of normally distributed errors must be made for both level-1 and level-2 variables, with violations adversely affecting level-2 estimated standard errors and inferential statistics (Raudenbush & Bryk, 2002). The effects of the violation at level-1 may include distorted random effect coefficients and variance-covariance components. To examine the distribution of the errors, data analysts should consider the following options. First, one might plot the standardized residuals against their normal scores to observe how closely the plot follows a diagonal line (Hox, 2002). Second, to assess the distribution of the overall

residuals at levels one and two, one might also use histograms or box-and-whisker plots. Similarly, box-and-whisker plots of the level-1 residuals for each level-2 unit and of the level-2 residuals for each level-2 effect will help assess deviations from a normal distribution and identify extreme values. Third, to assess assumptions of normality, linearity, and homogeneity of variance simultaneously, analysts should examine the plot of predicted values against the residuals. This should be done for level-1 residuals, as well as the residuals for each level-2 effect. Scatterplots that contain roughly equivalent frequencies of points above and below their mean value, with no particular structure, provide evidence that the assumptions have not been violated (Hox, 2002).

To supplement the visual level-1 assumption diagnostics, summary statistics (i.e., skewness, kurtosis, variance and standard deviation, and a statistical test for normality such as the Shapiro-Wilk or Kolmogorov-Smifnov test) for level-1 residuals within each level-2 unit should also be reviewed for information regarding level-1 normality and variance homogeneity. In addition, Levene's test can be used to assess homogeneity of variance of the level-1 residuals across each level-2 unit. Next, when evaluating the tenability of assumptions for the level-2 residuals, multivariate summary statistics such as skewness, kurtosis, and Mahalanobis distances should be generated and reviewed. Multivariate normality can also be evaluated visually through a histogram of Mahalanobis distances. And, as with the level-1 residuals, Levene's test for assessing homogeneity of variance of the level-2 residuals can also be conducted for each level-2 effect.

The MIXED\_DX macro provides analysts with a comprehensive approach to assessing the degree to which one or more of the linearity, normality, or homogeneity of variance assumptions has been violated. The output includes both visual (e.g., box-and-whisker plots, histograms, scatter plots) and statistical information for both level-1 and level-2 residuals. A unique feature of our macro is the calculation of level-2 predicted values for each level-2 effect. Unlike the level-1 ODS output table in SAS (generated through the outp option) that contains both residuals and predicted values for each level-1 observation, the level-2 ODS output table in SAS (SolutionR) does not contain predicted values. By calculating predicted values for each level-2 effect, our macro allows analysts to create and examine residual by predicted scatterplots for level-2 effects (something that cannot be done using the default SAS output alone). Furthermore, the MIXED\_DX macro generates output for all level-2 units, with an option for a user-defined minimum of cases per level-2 unit to be included in these analyses.

#### **INFLUENCE**

Nearly all statistical analyses involve post-hoc diagnostic investigations to determine how well models fit the data. One of the more well-established diagnostics for ordinary least-squares linear regression is the detection of influential observations in determining parameter estimates. Previous work by Cook (1977), Belsley, Kuh, and Welsch (1980), and Andrews & Pregibon (1978) provided useful statistics for determining how much influence a particular observation had over estimated parameters or the performance of the model. With the development of more complex multilevel models, the detection of influential observations becomes more complex as well. For example, mixed model analyses generate more results than may be of potential interest to the researcher. A researcher may be interested in predicting a particular outcome variable, in which case fixed effect results are of primary interest, whereas another researcher might be interested in explaining the variance in a particular outcome, thus variance component results become the primary concern. Further, with multilevel models, the concept of influence is often applied to higher-level units (i.e., group or cluster influence) vs. with OLS diagnostics where influence diagnostics are concerned with the influence of individual observations. This interest in the influence of higher-level clusters or groups calls for set-deletion analyses where an entire cluster or group is removed to determine its influence vs. the removal of individual observations as is done in OLS diagnostic algorithms.

Littell, Milliken, Stroup, and Wolfinger (2006) recommend a "drill-down" approach to mixed model influence diagnostics, beginning with a global assessment of the influence on the overall model, followed by a more detailed exploration of the case-sets should they be warranted. The likelihood distance (Cook & Weisberg, 1982) provides an assessment of a unit's influence on the overall model. A group or unit's influence on parameter estimates can be determined using Cook's *D* or the multivariate DFFITS statistics. The larger the value for these statistics, the greater the influence a unit has on parameter estimates (i.e., the change in the parameter estimate is large relative to the variability of the estimate; Schabenberger, 2004). Researchers can also examine a unit's effect of the precision of an estimate through the covariance trace (COVTRACE) and covariance ratio (COVRATIO) statistics. For these statistics, benchmark values used to determine a unit's level of influence are zero and one, respectively (Schabenberger, 2004). Each of these influence diagnostics can be generated for both fixed effects and covariance parameters, however calculation for the latter requires an iterative process due to the potential impact of observations on covariance matrices. The INFLUENCE option within PROC MIXED allows the researcher to utilize non-iterative or iterative diagnostics, and provides the option to control the number of iterations when re-calculating estimates and covariance matrices (Littell et. al., 2006).

In addition to examining a unit's influence on the change in parameter estimates and the change in the precision of estimates, influence on fitted and predicted values can also be examined through the Predicted Residual Sum of Squares (PRESS) statistic. The PRESS provides a comparison of the predicted marginal mean and the observed mean when the predicted value is calculated without the deleted observation in question (Schabenberger, 2004). Again, larger values suggest more influential units. Other measures of overall influence provided through the INFLUENCE option in PROC MIXED include RMSE and Restricted Likelihood Distance (RLD). RMSE values represent the model RMSE with a specific level-2 unit deleted. On the other hand, RLD functions more as an 'indicator' statistic in that it does not convey what part of the model is likely to change given the removal of a specific level-2 unit. Instead, it suggests that the overall influence of

a particular level-2 unit stands out comparatively to other level-2 units (Littell et al., 2006). To determine what model components are influenced by a given level-2 unit, analysts need to examine the individual influence statistics such as MDFFITS or COVRATIO.

To facilitate detection of influential observations in a two-level linear model, the MIXED\_DX macro creates a ranked summary table of the influence statistics automatically created in the SAS ODS influence table (Influence). More specifically, for each statistic provided in the summary table, the macro employs the RANK procedure to determine the percentile rank of each statistic's value for each level-2 unit. MIXED\_DX allows the user to choose a threshold percentile rank of interest for the detection of influential observations via the 'PR = ' argument to the macro. The default, 'PR = 90', will flag any level-2 unit with a percentile rank greater than 90 for each statistic included in the ODS output. To further facilitate inspection of the summary table, particularly for data sets with a large number of level-2 units, we summed the number of flagged statistics and sorted the table in descending order, thus, placing level-2 units most frequently identified as influential at the top of the summary table. In addition, the MIXED\_DX influence output is calculated at the level-2 unit (controlled by the 'EFFECT =' option in Influence command in PROC MIXED) and based on five iterations (controlled by the 'ITER =' option in the Influence command in PROC MIXED).

### MACRO MIXED\_DX

The MIXED\_DX macro inputs consist of ODS table names generated from the MIXED procedure including (a) ModelInfo,, (b) Dimsensions, (c) SolutionF, (d) SolutionR, (e) Level1, and (f) Influence, as well as two user specified arguments, MIN\_NJ and PR. MIN\_NJ defines the smallest level-2 unit to be included in the within-unit box-and-whisker plots and normality assessments (allowing the exclusion of very small units from these analyses), whereas PR defines the threshold percentile rank of influence diagnostics that trigger the flagging of level-2 units. If the MIN\_NJ subanalysis is not wanted, the user will need to override the default value of five by specifying some value for MIN\_NJ such as MIN\_NJ = 1.

The MIXED\_DX macro produces numeric (i.e., output located in the output window) and visual (i.e., output located in the graph window) output for both level-1 and level-2 data.

Level-1 numeric output includes: (a) normality summary statistics table for the overall level-1 residual and for each level-2 unit, (b) normality summary statistics table for the overall level-1 residual and for each level-2 unit where  $n_j > \min_n j$ , (c) PROC UNIVARIATE plot output for the variance of all level-1 residuals, (d) PROC UNIVARIATE plot output for the variance of level-1 residuals where  $n_j > \min_n j$ , and (e) PROC GLM output using Levene's Test for homogeneity of variance of level-1 residuals.

Level-1 visual output includes: (a) side-by-side box-and-whisker plots for the overall level-1 residual and for each level-2 unit, (b) side-by-side box-and-whisker plots for the overall level-1 residual and for each level-2 unit where  $n_j > \min_n j$ , (c) histogram of the variance of all level-1 residuals, (d) histogram of the variance of all level-1 residuals where  $n_j > \min_n j$ , and (e) plot of level-1 residuals by predicted values.

Level-2 numeric output includes: (a) PROC UNIVARIATE plot output of level-2 residuals for each level-2 effect, (b) multivariate normality and outlier summary table, (c) Mahalanobis distance values for level-2 units, (d) PROC UNIVARIATE plot output output of Mahalanobis distance values for level-2 units, and (e) table of ranked influence diagnostics.

Level-2 visual output includes: (a) plot of level-2 residuals by predicted values for each level-2 effect, (b) histogram of level-2 residuals for each level-2 effect, and (c) histogram of Mahalanobis distance values.

#### **EXAMPLE OF MACRO MIXED DX**

Below is an example of PROC MIXED code used in conjunction with the macro MIXED\_DX, followed by select examples of the numeric and visual output.

```
ods exclude influence SolutionR;
title;
proc mixed data = temp covtest noclprint NAMELEN=32;
class schoolid;
model mathach = SIZE FEMALE SES MEANSES SES*MEANSES SES*SIZE FEMALE*SIZE/
solution outp=L1Resid influence(effect=schoolid iter=5);
random intercept FEMALE SES / sub=schoolid solution type=un;
ods output SolutionR=L2Resid SolutionF=Fixed ModelInfo=ModelStuff
Dimensions=DatStuff Influence=influence; run;
%mixed_dx (ModelI = modelstuff, Dims = datstuff, solnF = Fixed, solnR =
L2Resid, Level1 = L1Resid, Influence = influence, min_nj = 5, pr = 90);
run;
```

Figure 1 contains one 'window' of the level-1 residual box-and-whisker plot output for the overall level-1 residual (the first plot in the series) and for each level-2 unit. These plots can be used to evaluate the normality and heterogeneity of variance assumption for the overall level-1 residual, as well as for the level-1 residuals within each level-2 unit. Figure 2 contains similar output as Figure 1, however, instead of including box-and-whisker plots for all level-2 units, the output in Figure 2 displays the overall level-1 residual and the residuals for level-2 units in which  $n_j \ge 5$ , where the sample size of five was specified through the MIN\_NJ = 5 option in the call to the macro. As with Figure 1, the box-and-whisker plots included in Figure 2 allow an analyst to evaluate the tenability of the normality and heterogeneity of variance assumptions for level-1 residuals. However, instead of examining level-1 residuals within each level-2 unit, this part of the MIXED\_DX output s limited to level-2 units with a user specified minimum number of observations. This approach allows users to focus on level-2 units with enough observations that normality and equal variances might be plausible (i.e., when level-2 units do not have many observations in them, normally distributed errors and equal variances with other, more dense level-2 units is highly improbable given the tenets of Central Limit Theorem).

The histogram displaying the distribution of the variance of level-1 residuals for all level-2 units shown in Figure 3 provides users with summary information regarding the magnitude of level-1 residual variances across level-2 units. Although this part of the output is not directly related to evaluating the heterogeneity of variance assumption for level-1 residuals, it offers supplemental information regarding the nature of the variability in level-1 residuals. Figure 4 displays the plot of level-1 residuals by predicted values for all observations included in the PROC MIXED analysis. Similarly, Figures 5 and 6 contain plots of level-2 residuals by predicted values for the intercept and the random effect for the level-1 variable 'female' (i.e., level-2 effect = female), respectively. Each scatterplot provides users a way to evaluate simultaneously the normality, linearity, and homoscedasticity of level-1 (Figure 4) and level-2 residuals (Figures 5 and 6).

Figure 7 contains a histogram displaying the distribution of Mahalanobis distance values for each level-2 unit which allows researchers to easily examine the range of values to determine if potentially problematic outliers are evident. Moreover, when evaluating data for possible outliers or violations of statistical assumptions, nicely summarized distributions such as the histogram in Figure 7 are particularly helpful and efficient when there are oodles of level-2 units.

Figures 8, 9, 10, and 11 contain different elements of the numerical output generated by MIXED\_DX. Specifically, Figure 8 is one 'window' of the normality summary table for level-1 residuals, Figure 9 contains the PROC GLM out for Levene's test of homogeneity of variance of level-1 residuals, Figure 10 includes the level-2 multivariate normality and outlier assessment table, and Figure 11 includes one 'window' of the ranked influence diagnostic summary table for level-2 units. The statistical output provided in the normality summary table for level-1 residuals (Figure 8) can be used to supplement the visual evaluation of the normality and homogeneity of variance assumption of level-1 residuals. Similarly, the Levene's test of homogeneity of variance output provided in Figure 9 can also be used to supplement any visual assessment of equal variances of level-1 residuals across level-2 units. The statistical output in Figure 10 allows users a method of evaluating the multivariate normality of level-2 residuals. Also, results from the inferential examination of the largest Mahalanobis distance can be used to supplement the visual evaluation of potentially problematic outliers. By providing a nice summary of the influential nature of each level-2 unit, the ranked influence diagnostics output in Figure 11 provides users a way to easily examine which, if any, level-2 units might be influencing the magnitude or precision of fixed and/or random effects in their model.

#### **CONCLUSIONS**

The macro MIXED\_DX was created to facilitate the evaluation of statistical assumptions of linear two-level multilevel models, including the examination of residual normality, linearity, and homogeneity of variance, as well as influential outliers. By utilizing data provided in various ODS tables in PROC MIXED, MIXED\_DX makes it much easier for the applied data analyst to evaluate assumptions for both level-1 and level-2 residuals, through both numeric and visual assessments such as statistical summary tables, box-and-whisker plots, histograms, and plots of residuals by predicted values. Furthermore, MIXED\_DX is easily modifiable. For example, users can add to the macro to invoke procedures that are not currently included, such as examining the distribution of level-2 sample sizes. Conversely, users can also easily comment out any unwanted elements of the macro. Thus, given the ease in calling the macro when estimating two-level linear models via PROC MIXED, we hope that more researchers will not only evaluate the distributional assumptions and outliers, but also report said evaluation in their published research.

# Overall level-1 residual and level-1 residual for each level-2 unit

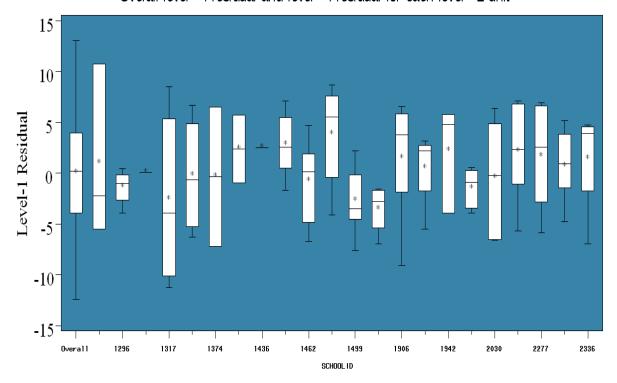


Figure 1. Distribution of level-1 residuals: Overall residual and for each level-2 unit



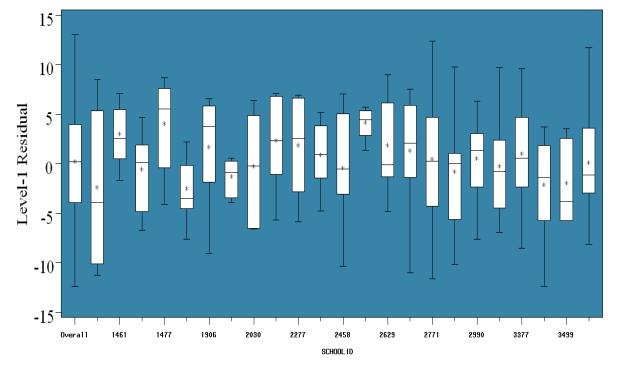


Figure 2. Distribution of level-1 residuals: Overall residual and for each level-2 unit where  $n_i \ge 5$ 

### Variance of level-1 residuals for all observations

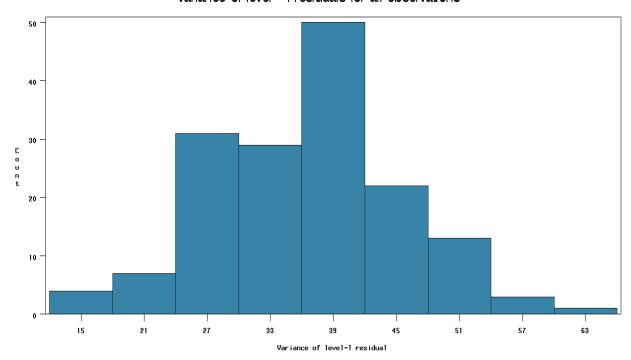


Figure 3. Distribution of the variance of level-1 residuals for all observations

### Plot of level-1 residuals\*predicted values

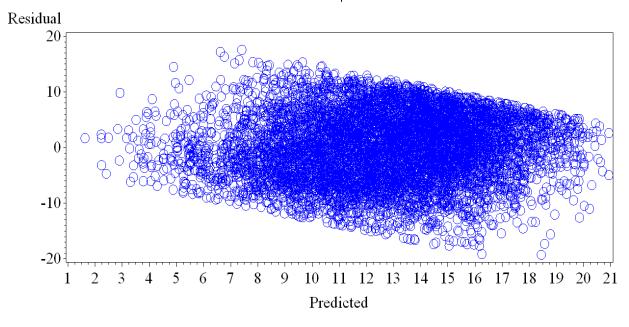


Figure 4. Level-1 residual\*predicted value for all observations

# Homogeneity of variance plot of level-2 errors

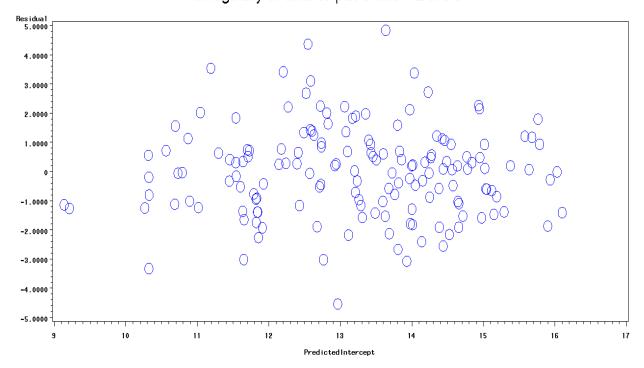


Figure 5. Level-2 residual\*predicted intercept value for all level-2 units

# Homogeneity of variance plot of level-2 errors

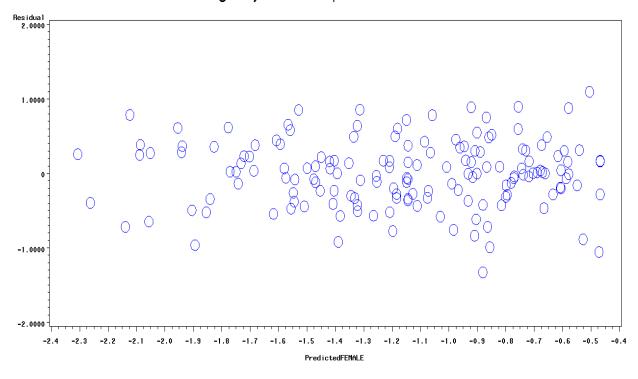


Figure 6. Level-2 residual\*predicted variance component for level-1 variable 'female' for all level-2 units

# Distribution of Mahalanobis distances for multivariate outlier analysis

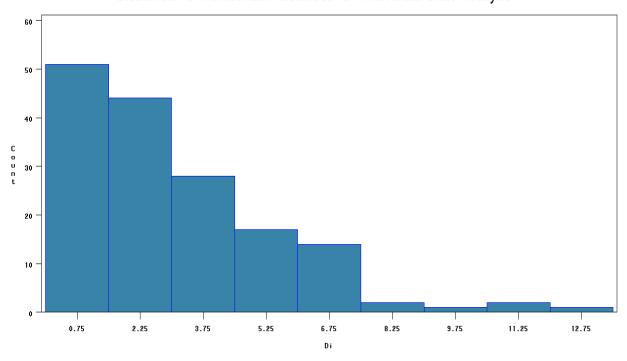


Figure 7. Distribution of Mahalanobis distance values for each level-2 unit

Level-2 Unit	Skewness	Kurtosis	Shapiro-Wil	lk p-value	Variance	Std Dev	N
Overall	-0.1464	-0.5220	0.0321	0.0100	35.6292	5.9690	7185
1224	0.2481	-1.1449	0.9375	0.0143	54.4537	7.3793	47
1288	-0.4288	-0.6287	0.9475	0.2206	46.0958	6.7894	25
1296	0.6568	1.3472	0.9562	0.0711	28.5501	5.3432	48
1308	-0.3487	-0.5644	0.9483	0.3423	38.2406	6.1839	20
1317	0.1423	-0.8975	0.9604	0.1052	29.4795	5.4295	48
1358	-0.4556	0.2699	0.9683	0.4938	25.4989	5.0496	30
1374	-0.3089	-0.7325	0.9599	0.3471	61.3113	7.8302	28
1433	-0.4066	-0.7341	0.9572	0.1883	13.8575	3.7226	35
1436	-0.7585	0.1641	0.9476	0.0450	17.8752	4.2279	44
1461	-1.4369	2.4512	0.8717	0.0011	34.2199	5.8498	33
1462	0.5766	-0.4901	0.9467	0.0139	41.0545	6.4074	57
1477	-0.4332	-0.7634	0.9522	0.0171	50.5179	7.1076	62
1499	0.6850	0.0731	0.9523	0.0337	32.1495	5.6701	53
1637	0.7102	-0.0295	0.9374	0.1050	45.9021	6.7751	27
1906	-0.5982	-0.1637	0.9573	0.0561	40.8894	6.3945	53
1909	-0.2498	-1.3750	0.9193	0.0333	34.4495	5.8694	28
1942	-0.9756	0.7643	0.8972	0.0084	30.7441	5.5447	29
1946	0.0692	-1.1729	0.9503	0.0843	39.2686	6.2665	39
2030	-0.0596	-0.7441	0.9765	0.4578	38.4415	6.2001	47
2208	-0.2361	-1.1965	0.9445	0.0087	33.6639	5.8021	60
2277	0.5186	-0.3404	0.9664	0.0916	33.4067	5.7799	61
2305	-0.5484	-0.0718	0.9647	0.0541	26.2418	5.1227	67
2336	-0.5344	-0.2465	0.9653	0.1742	32.5628	5.7064	47

Figure 8. Partial output from MIXED\_DX normality summary table for level-1 residuals: Overall and for each level-2 unit

Levenes test of homogeneity of variance of level-1 residuals											
The GLM Procedure											
Dependent Variable: Absolute_resid (Absolute value of level-1 residual)											
		Sum of									
Source	D	F S	quares Me	ean Square F	- Value	Pr > F					
Model	15	9 3269	.56971	20.56333	1.85	<.0001					
Error	702	5 77957	.87122	11.09721							
Corrected To	otal 718	7184 81227.44092									
	R-Square Coef	f Var	Root MSE A	Absolute_resid Mean							
	0.040252 67.	55124	3.331247	4.931437							
Source	D	F Typ	e I SS Me	ean Square F	- Value	Pr > F					
SCHOOLID	15	9 3269.	569707	20.563331	1.85	<.0001					
Source	D	F Type	III SS Me	ean Square F	Value	Pr > F					
SCHOOLID	15	9 3269.	569707	20.563331	1.85	<.0001					

Figure 9. MIXED\_DX output for Levene's homogeneity of variance test of level-1 residuals

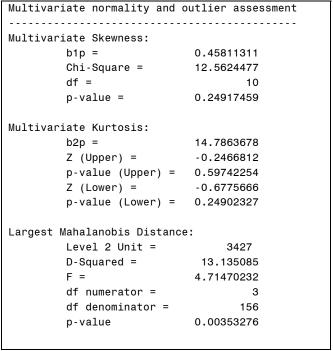


Figure 10. MIXED\_DX output for level-2 multivariate normality and outlier assessment

Ranked influence diagnostics														
						Ham	.cu inii	uchoc u	1 agrico ci	.00				
											С	С		
										М	0	0		
	s						С	С		D	V	V		
	С					M	0	0	С	F	R	Т		
	Н					D	V	V	0	F	а	r		
	0	f			Р	F	R	Т	0	I	t	а		
	0	1	N	Ι	R	F	а	r	k	T	i	С	R	
0	L	а	0	t	Е	I	t	а	D	S	0	е	М	R
b	I	g	b	е	S	T	i	С	С	С	С	С	S	L
S	D	S	S	r	S	S	0	е	Р	Р	Р	Р	Е	D
	3427		49							0.60558				
	8367		14							0.30125				
	2277		61							0.28665				
	2305		67							0.12935				
	6074		56							0.13011				
	2655		52							0.14074				
	6990		53							0.20090				
	3533		48							0.32961				
	3716		41							0.13249				
	8628		61							0.22709				
	2526		57							0.10099				
	2639 7345		42 56							0.03170 0.15451				
	7345		56 54							0.15451				
	7688 8193		54 43							0.13235				
	8627		43 53							0.03279				
10	0027	4	ეე		2343.90	0.01002	1.1410	0.1304	0.03230	0.03279	1.0007	0.0038	0.03346	0.1939

Figure 11. Partial output from MIXED DX ranked influence diagnostics summary table for each level-2 unit

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#### MACRO MIXED DX

```
%macro MIXED DX (Modell = modelstuff, Dims = datstuff, solnF = Fixed, solnR =
L2Resid. Level1 = L1Resid, Influence = influence, min_nj = 5, pr = 90);
Important information about the types of models that are supported by
macro MIXED DX:
* +-----+·
a. MIXED DX has been tested for use in SAS 9.1.3 and 9.2
b. MIXED DX utilizes the following SAS components: Base SAS, SAS/STAT,
SAS/GRAPH and SAS/IMI
c. MIXED_DX works for 2-level linear multilevel models estimated in PROC MIXED
d. MIXED DX works correctly for models that have converged and have positive
e. MIXED_DX is designed to read model effect names up to 32 characters in length. If a
user has a n effect that is longer than 32 characters, the 'NAMELEN=' option in
PROC MIXED must be updated accordingly.
f. If a user's model contains any interactions, they must be created on the model
statement (e.g., model v=x1 x2 w1 w2 x1*w1)
g. Users cannot use the class statement to create dummy variables in PROC MIXED.
That is, any dummy variables used in the model statement need to be created prior
to the MIXED procedure (e.g., in a data step).
h. Users need to specify their desired value for MIN NJ, which defines the smallest
level-2 unit to be included in the within-unit box-and-whisker plots and normality
```

level-2 unit to be included in the within-unit box-and-whisker plots and normality assessments. The default is set to 5.Even if this subanalysis is not wanted, the user must override the default value of 5 and specify some value for MIN\_NJ such as min\_nj = 1.

i. Users need to specify their desired value for PR, which defines the threshold percentile rank of influence diagnostics that trigger the flagging of level-2 units. The default is set to 90.

As you use the macro, please feel free to send feedback, comments, and/or suggestions to, Bethany A. Bell, at babell@sc.edu.

The complete MIXED\_DX SAS program file is available for download from <a href="http://www.ed.sc.edu/bell/">http://www.ed.sc.edu/bell/</a>

\*/
\* +----
Define the variable &SUBJ whose value is the clustering variable name from PROC MIXED

%global subi:

data getsubj; set &Modell; if Descr = 'Subject Effect'; call symput('SUBJ', Value);

proc sort data = &Level1; by &subj; proc means noprint data = &Level1; by &subj;

var Resid:

output out = L1Summary mean = MN\_Resid  $Var = Var_Resid N = Nj$ ; data L1Summary; set L1summary; label  $Var_Resid = Var_Resid = Var_Resid$ 

```
* +------+
  Obtain level-1 residual means, variances, and sample sizes for each unit
  where Ni >= &min ni
data L1Summary2; set L1Summary; if Nj GE &min_nj;
  Organizing the data to generate box-plots for the overall level-1 residual and for
  level-2 units where Ni >= &min ni
**Applying format to subject variable to denote Overall Residual;
proc format: value subilbl 0 = 'Overall': value $ subilbl '0' = 'Overall':
data L1ResidAll: set &Level1: &subi = 0:
data box; set &Level1 L1ResidAll;
         &subj=trim(left(&subj));
         format &subi subilbl .:
proc sort data = box; by &subj;
proc sort data = L1summary; by &subj;
data box2; merge box L1summary; by &subj;
         if &subj = 0 or Nj ge &min_nj;
         format &subj subjlbl.;
proc boxplot data=box;
         plot resid*&subi/vaxis = axis1 boxstyle = schematic cframe = steel
         cboxes = black
         cboxfill = white idsymbol = circle idcolor = bbl boxwidth = 2;
         symbol v = * color = bbl f='times new roman' h=1.5:
         axis1 label=(f='times new roman' h=2.5 a=90 r=0 'Level-1 Residual')
         value=(f='times new roman' h=2.5):
         title 'Overall level-1 residual and level-1 residual for each level-2 unit';
proc boxplot data=box2:
         plot resid*&subi/vaxis = axis1 boxstyle = schematic cframe = steel
         choxes = black
         cboxfill = white idsymbol = circle idcolor = bbl boxwidth = 2;
         symbol v = * color = bbl f='times new roman' h=1.5;
         axis1 label=(f='times new roman' h=2.5 a=90 r=0 'Level-1 Residual')
         value=(f='times new roman' h=2.5);
         title "Overall level-1 residual and level-1 residual for each level-2 unit
         where Nj >= &min_nj";
* +------
 Creating a normality summary statistics table for the overall level-1 residual and
 each level-2 unit
proc univariate data = box normal noprint; var resid; by &subi;
output out=table n=nT skewness=skew kurtosis=ku normal=SW var=varT std=stdT
probn=pvalue;
data normal_table;
         set table:
         file print header=h notitles;
put @1 &subj @25 skew 9.4 @35 ku 9.4 @45 SW 10.4 @60 pvalue 6.4 @70 varT 9.4
@80 stdT 9.4 @90 nT 5.0:
```

H: put @1 'Normality summary statistics for the overall residual and each level-2 unit'//

```
@1 'Level-2 Unit' @26 'Skewness' @36 'Kurtosis' @47 'Shapiro-Wilk' @60 'p-value'
@71 'Variance' @82 'Std Dev' @93 'N'/:
* +------
 Creating a normality summary statistics table for the overall level-1 residual and
 each level-2 unit where Nj >= &min_nj
proc univariate data = box2 normal noprint; var resid; by &subi;
output out=table2 n=nT skewness=skew kurtosis=ku normal=SW var=varT std=stdT
probn=pvalue;
data normal table2;
        set table2:
        file print header=h notitles:
put @1 &subi @25 skew 9.4 @35 ku 9.4 @45 SW 10.4 @60 pvalue 6.4 @70 varT 9.4
@80 stdT 9.4 @90 nT 5.0;
return:
H: put @1 "Normality summary statistics for the overall residual and each level-2 unit
where Ni >= &min ni"//
@1 'Level-2 Unit' @26 'Skewness' @36 'Kurtosis' @47 'Shapiro-Wilk' @60 'p-value'
@71 'Variance' @82 'Std Dev' @93 'N'/-
  Histograms for overall level-1 residual and for each level-2 unit
* <del>4</del>-----<del>-</del>
proc univariate data = L1summary normal plot; var var_resid; id &subj;
        histogram var resid /vscale=count cfill=steel;
title 'Variance of level-1 residuals for all observations':
*+------
Histograms for overall level-1 residual and for level-2 units where Ni >= &min_ni
proc univariate data = L1summary2 normal plot; var var resid; id &subj;
        histogram var resid /vscale=count cfill=steel:
title "Variance of level-1 residuals for each level-2 unit where Nj >= &min_nj";
  Obtaining the absolute value of level-1 residuals and then running a one-way
  ANOVA to test for equal variances, which is analogous to Levenes Test
data Levene; set &Level1;
        Absolute_resid = abs(resid);
        label Absolute resid = '(Absolute value of level-1 residual)':
        title 'Levenes test of homogeneity of variance of level-1 residuals';
proc glm data=levene; class &subj; model Absolute resid=&subj;
 +-------
  Assessment of level-1 residual normality and equal variance through the scatterplot
symbol font =, color=blue interpol=none height=2.5 v=circle;
proc gplot data = &Level1;
        plot resid*pred /vaxis = axis1 haxis =axis2:
        title1 'Plot of level-1 residuals*predicted values';
        axis1 label=(f='times new roman' h=2.5 'Residual')
                 value=(f='times new roman' h=2.5);
                 axis2 label=(f='times new roman' h=2.5 'Predicted')
                 value=(f='times new roman' h=2.5);
```

```
* +------
  Define the variable &dat whose value is the dataset name from PROC MIXED
data getdat; set &Modell; if Descr = 'Data Set'; call symput('DAT', Value);
* +------
Define the variable &n2 whose value is the number of level-2 units from PROC MIXED
data getn2; set &Dims; if Descr = 'Subjects'; call symput('n2', Value);
 Make a little data set containing names of the main effects used in PROC MIXED
data preds; set &solnF; keep effect; if effect='Intercept' then delete;
       if index(effect, '*')>0 then delete,
  Count the number of main effects used in PROC MIXED
data null : set preds end=lastrec:
       if lastrec then do:
               call symput('totalvars', n );
run:
  Make a macro variable to calculate predicted values (pred1, pred2, etc.) for each
  main effect in PROC MIXED
 +-----<sup>+</sup>
%do nn=1 %to &totalvars
       %global pred&nn;
       data _null_; set preds; if &nn=_n_; call symput("pred&nn",effect); run;
%end:
 Make a mini-macro to generate the list of main effects in PROC MIXED
%macro create predlist.
       %do k=1 %to &totalvars;
               &&pred&k
       %end:
%mend create predlist:
 Compute means and variances of each main effect variable for each level-2 unit
* <del>| -----</del>
proc sort data = &dat: by &subi:
proc means data = &dat nway noprint; class &subj;
       var %create predlist.
       output out=m mean=;
       output out=s std=;
```

```
* +-----+
  Compute the sum of the within unit stds for each main effect variable
proc means data=s nway noprint:
       var %create predlist.
       output out=ss sum=:
 Make data set containing names of the level-2 variables
       (those where within unit std = 0)
proc transpose data=ss out=sss:
data sss2: set sss: if COL1=0 and NAME NE 'TYPE':
data sss3; set sss2; rename NAME = Effect;
data sss4; set sss3; lev2=1; keep effect lev2;
  Count the number of level-1 predictors with random slopes
data null; set &SolnR end=lastrec;
        if lastrec then do:
               call symput('totalrand', n );
               call symput('numlev1', n /&n2);
        end:
run:
 Make data set containing names of the level-1 variables with random slopes
data lev11; set &solnR (FIRSTOBS=1 OBS=&numlev1);
data lev12; set lev11; keep effect;
data lev13: set lev12:
        effecttxt=cats("",effect,""");
        lev1r=1;
        inter1=cats("*",effect); inter2=cats(effect,"*");
        lenint1=lenath(inter1): lenint2=lenath(inter2): lenint3=lenint1+1:
 Make a macro variable (rand1, rand2, etc.) for each random effect in PROC MIXED
* +------
%do mm=1 %to &numlev1:
        %global rand&mm;
        %global int1&mm;
        %global int2&mm;
        %global len1&mm:
        %global len2&mm;
        %global len3&mm;
        data _null_;
                set lev13:
               if % mm = n;
               call symput("rand&mm",Effecttxt);
                call symput("int1&mm",inter1);
                call symput("int2&mm",inter2);
               call symput("len1&mm",lenint1);
               call symput("len2&mm".lenint2):
```

```
call symput("len3&mm".lenint3):
%end:
proc sort data = m: by &subi:
For each random effect: Make data set containing level-2 variables, level-2 errors, and
· <del>_</del>____+·
%do ii=1 %to &numlev1;
For each random effect: Make a prediction equation, compute predicted values and
plot the level-2 residuals with the predicted values
       %if &ii=1 %then %do;
               data eq1; set &solnF;
               data eq2; set sss4;
               proc sort data=eq1; by effect;
               proc sort data=eq2; by effect;
               data eq4; merge eq1 eq2; by effect; if Effect=&&rand&ii or lev2=1;
               data eq5; set eq4;
                      term=cats("+",estimate,"*",effect);
                      if effect='Intercept' then term=cats("1*".estimate):
data _null_; set eq5 end=lastrec; if lastrec then do;
       call symput('totalterms', n );
               end:
               run:
       %do z=1 %to &totalterms:
       %global term&z;
data null; set eq5; if &z= n; call symput("term&z",term); run;
       %end:
%macro create_eqs;
       %do w=1 %to &totalterms;
        &&term&w
       %end:
%mend create eqs;
data r1; set &solnR; if Effect=&&randⅈ
proc sort data=r1; by &subj;
data lev2dat: merge r1 m; by &subi; rename estimate = Residual;
data lev2datp; set lev2dat; Predicted=%create eqs;
label predicted=Predicted&&randⅈ
*+-----+
Assessment of level-2 residual normality and equal variance through scatterplots
 for each level-2 effect
* 4------
 If no variance in the predicted values, do not produce a scatter plot
* +------
proc means noprint data = lev2datp;
       var predicted:
       output out = check var var = predicted variance;
data back; if n = 1 then set check var; set lev2datp; retain predicted variance;
       if predicted variance = 0 then do:
```

```
predicted = .: residual = .:
          end:
proc gplot data=back;
          plot residual*predicted:
          title "Homogeneity of variance plot of level-2 errors";
%end:
%if &ii>1 %then %do:
data eq1: set &solnF: keep effect estimate:
data eq2; set sss4;
data eq3; set lev13; keep effect lev1r;
proc sort data=eq1: by effect:
proc sort data=eq2; by effect;
proc sort data=eq3; by effect;
data eq3; merge eq1 eq2 eq3; by effect;
data eq4; set eq3;
          inter=cats(&&rand&ii."*"):
          inter2=cats("*",&&rand&ii);
          length effect2 $&&len1ⅈ
          effect2=effect:
          length effect5 $&&len2ⅈ
          length effect6 $&&len2ⅈ
          effect3=reverse(effect):
          effect4=cats(effect3);
          effect5=effect4;
          effect6=reverse(effect5);
data eq5: set eq4: if effect=&&rand&ii or inter=effect2 or inter2=effect6:
data eq6; set eq5;
if index(effect, '*')>0 then do;
          if inter=effect2 then do;
                    effect7=substr(effect, &&len3&ii):
          end;
          if inter2=effect6 then do;
                    effect8=cats(reverse(effect));
                    effect9=substr(effect8, &&len3&ii):
                    effect7=cats(reverse(effect9));
          end:
end.
term=cats("+".estimate."*".effect7):
if effect=&&rand&ii then term=cats("1*",estimate);
run:
data _null_; set eq6 end=lastrec;
          if lastrec then do:
          call symput('totalterms', n );
run:
%do z=1 %to &totalterms:
          %global term&z;
          data null; set eg6; if &z= n; call symput("term&z",term); run;
%end:
%macro create egs:
%do w=1 %to &totalterms;
          &&term&w
```

```
%end:
%mend create eqs;
         data r1: set &solnR: if Effect=&&rand&ii:
proc sort data=r1: by &subi:
data lev2dat; merge r1 m; by &subj;
         rename estimate = Residual;
data lev2datp: set lev2dat: Predicted=%create eqs:
         label predicted=Predicted&&rand&ii:
 If no variance in the predicted values, do not produce a scatter plot
proc means noprint data = lev2datp:
         var predicted;
         output out = check_var var = predicted_variance;
data back: if n = 1 then set check var: set lev2datp: retain predicted variance:
         if predicted variance = 0 then do:
                   predicted = .; residual = .;
         end:
proc gplot data=back;
         plot residual*predicted;
         title "Homogeneity of variance plot of level-2 errors";
%end;
%end:
  Assessment of univariate and multivariate normality for level-2 residuals
* +------
proc sort data = &solnR; by effect &subj;
proc univariate data = &solnR normal plot; by effect;
         var estimate; id &subj;
         histogram estimate /vscale=count cfill=steel:
         title 'Distribution of level-2 residuals':
proc means noprint data = &solnR; by effect; var estimate;
         output out = geteffects mean = dummyvar;
data geteffects; set geteffects; keep effect;
proc iml
         use geteffects;
         read all var {effect} into effects:
         use &solnR;
         read all var {estimate} into allresids;
         read all var {&SUBJ} into allsubj;
         N obs = NROW(allresids)/NROW(effects):
         y1 = J(N \text{ obs,NROW(effects),0});
         startrow = 1; endrow = N obs;
         do col = 1 to NROW(effects);
                   v1[.col] = allresids[startrow:endrow.1]:
                   startrow = endrow+1;
                   endrow = endrow + N Obs;
         end:
         subjlist = allsubj[1:N Obs,1];
         n1=nrow(y1); p1=ncol(y1);
         ybar1 = 1/n1 * y1 * J(N1,1);
```

```
v1barm = repeat(vbar1.1.n1):
s1 = \frac{1}{(n1-1)} y1 (i(n1)-\frac{1}{n1} (i(n1)) y1;
d1 = (v1)^{-v1}barm)^{*inv(s1)^{*}(v1)^{-v1}barm):
D1i2 = vecdiag(d1):
sighat1 = \frac{1}{n1} \cdot \frac{1}{i(n1) - \frac{1}{n1}} \cdot \frac{1}{i(n1)} \cdot \frac{1}{i(n1)
g1 = (y1)-y1barm)*inv(sighat1)*(y1)-y1barm);
b1p = SUM(q1##3)/n1##2:
b2p = TRACE(g1##2)/n1;
chiskew = (((p1+1)*(n1+1)*(n1+3))/(6*((n1+1)*(p1+1)-6)))*b1p;
dfskew = (1/6)*p1*(p1+1)*(p1+2);
pskew = 1 - probchi(chiskew.dfskew):
zupper = (b2p - p1*(p1+2))/sqrt(8*p1*(p1+2)/n1);
zlower = (b2p - (p1*(p1+2)*(n1+p1+1))/n1)/sqrt(8*p1*(p1+2)/(n1-1));
pupper = 1 - probnorm(zupper);
plower = probnorm(zlower):
d1i2max=d1i2[1,1];
d1maxsubj = subilist[1,1];
do i = 1 to n1;
                           if d1i2[i,1] >= d1i2max then do;
                                                       d1i2max=d1i2[i,1];
                                                       d1maxsubj = subjlist[i,1];
                            end:
end:
Fout = ((n1-p1-1)/p1)*((1/(1-(n1*d1i2max/(n1-1)**2)))-1);
df1=p1;
df2=n1-p1-1:
pout= 1 - probf(Fout,df1,df2);
file print;
                                                        @1 'Multivariate normality and outlier assessment' /
                           put
                                                        @1'-----'/
                                                        @1 'Multivariate Skewness:' /
                                                        @10 'b1p =' @30 b1p best10. /
                                                         @10 'Chi-Square =' @30 chiskew best10. /
                                                         @10 'df = ' @30 dfskew best10. /
                                                         @10 'p-value =' @30 pskew best10. //
                                                        @1 'Multivariate Kurtosis: ' /
                                                        @10 'b2p =' @30 b2p best10./
                                                        @10 'Z (Upper) = ' @30 Zupper best10. /
                                                        @10 'p-value (Upper) = ' @30 pupper best10. /
                                                        @10 'Z (Lower) = ' @30 Zlower best10. /
                                                        @10 'p-value (Lower) = ' @30 plower best10. //
                                                        @1 'Largest Mahalanobis Distance:' /
                                                        @10 'Level 2 Unit =' @30 d1maxsubj /
                                                        @10 'D-Squared = ' @30 d1i2max best10. /
                                                        @10 'F = ' @30 fout best10. /
                                                        @10 'df numerator = ' @30 df1 best10. /
                                                        @10 'df denominator =' @30 df2 best10. /
                                                        @10 'p-value' @30 pout best10.;
create v1 from D1i2 (|COLNAME={'Di'}|);
append from D1i2:
create v2 from subilist (|COLNAME={'Level 2 Unit'}|);
append from subilist:
```

```
data v3: merge v1 v2:
        proc sort data = v3; by descending Di;
        proc print noobs: var Level 2 Unit Di:
                 title 'Mahalanobis distance values for level-2 units':
proc univariate data = v3 normal plot; var Di; id Level 2 Unit;
        histogram Di /vscale=count cfill=steel;
        title 'Distribution of Mahalanobis distances for multivariate outlier analysis':
* +------
  Code to create a ranked summary table of the oodles of influence diagnostics
        created in SAS
proc rank data=&influence groups=100 out=rinfluence:
        var press CookD MDFFITS COVRATIO COVTRACE CookDCP MDFFITSCP
        COVRATIOCP COVTRACECP RMSE RLD:
        ranks rpress rCookD rMDFFITS rCOVRATIO rCOVTRACE rCookDCP
        rMDFFITSCP rCOVRATIOCP
        rCOVTRACECP rRMSE rRLD;
**create flags to count 'influential' markers for sorting;
data rinfluence; set rinfluence;
        if (rpress qt &pr) then press flag = 1;
        if (rCookD gt &pr) then CookD flag = 1;
        if (rMDFFITS at &pr) then MDFFITS flag = 1:
        if (rCOVRATIO It (100-&pr)) then COVRATIO_flag = 1;
        if (rCOVTRACE gt &pr) then COVTRACE flag = 1;
        if (rCookDCP gt &pr) then CookDCP flag = 1;
        if (rMDFFITSCP at &pr) then MDFFITSCP flag = 1:
        if (rCOVRATIOCP gt &pr) then COVRATIOCP_flag = 1;
        if (rCOVTRACECP gt &pr) then COVTRACECP flag = 1;
        if (rRMSE gt &pr) then RMSE_flag = 1;
        if (rRLD at &pr) then RLD flag = 1:
        array flag(1:11) press_flag CookD_flag MDFFITS_flag COVRATIO_flag
COVTRACE flag
                   CookDCP flag
                                     MDFFITSCP flag
                                                          COVRATIOCP flag
COVTRACECP_flag RMSE_flag RLD_flag;
        flags=0:
        do i=1 to 11;
                 if flag(i)=1 then flags=flags+1;
        end:
proc sort data = rinfluence: by descending flags:
proc print data = rinfluence;
        var &subj flags nobs iter cookd mdffits covratio covtrace CookDCP
MDFFITSCP COVRATIOCP COVTRACECP PRESS RMSE RLD;
        title 'Ranked influence diagnostics':
        run:
%mend MIXED DX;
* +------
Example of how users need to set up their PROC MIXED code for use with the
MIXED DX macro. Specifically, the MIXED DX macro is set up to read output
generated from 2-level models using the specific options and output data files as
specified below. Users only need to update the code where indicated by
'USER......'.
```