Assessing local fit in confirmatory factor models by approximating probabilities

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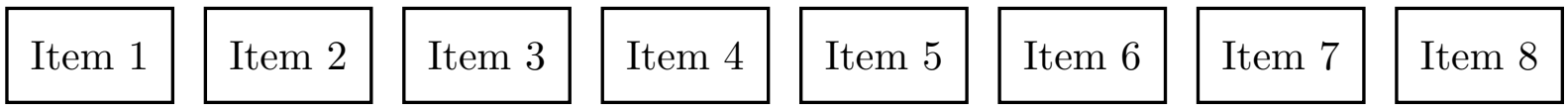
A Road-map

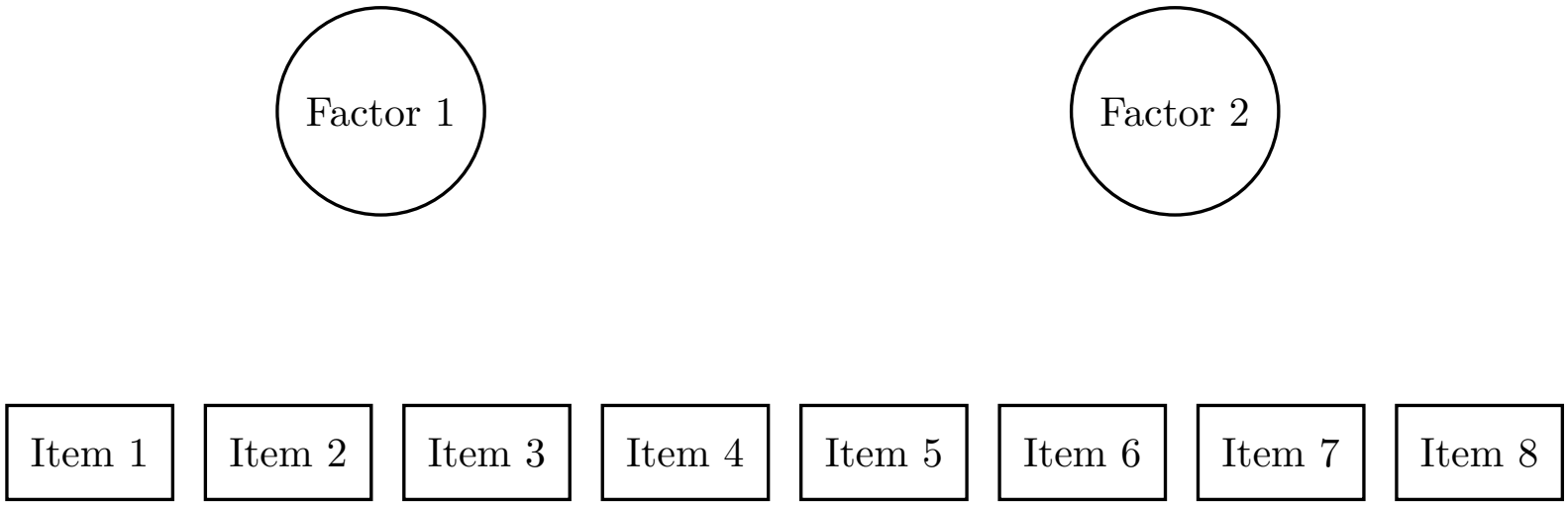
Test Construction and Validity

- Confirmatory Factor Analysis is commonly used in test construction.
- Aim is to provide some construct validity evidence through the measurement model proposed (Bandalos & Gerstner, 2016).
  - Is there evidence that items commonly measure a construct(s)?
  - How do items groups into subscales or is there evidence of multidimensionality in some items?
  - These are a couple of potential questions we aim to address in the test evaluation process.

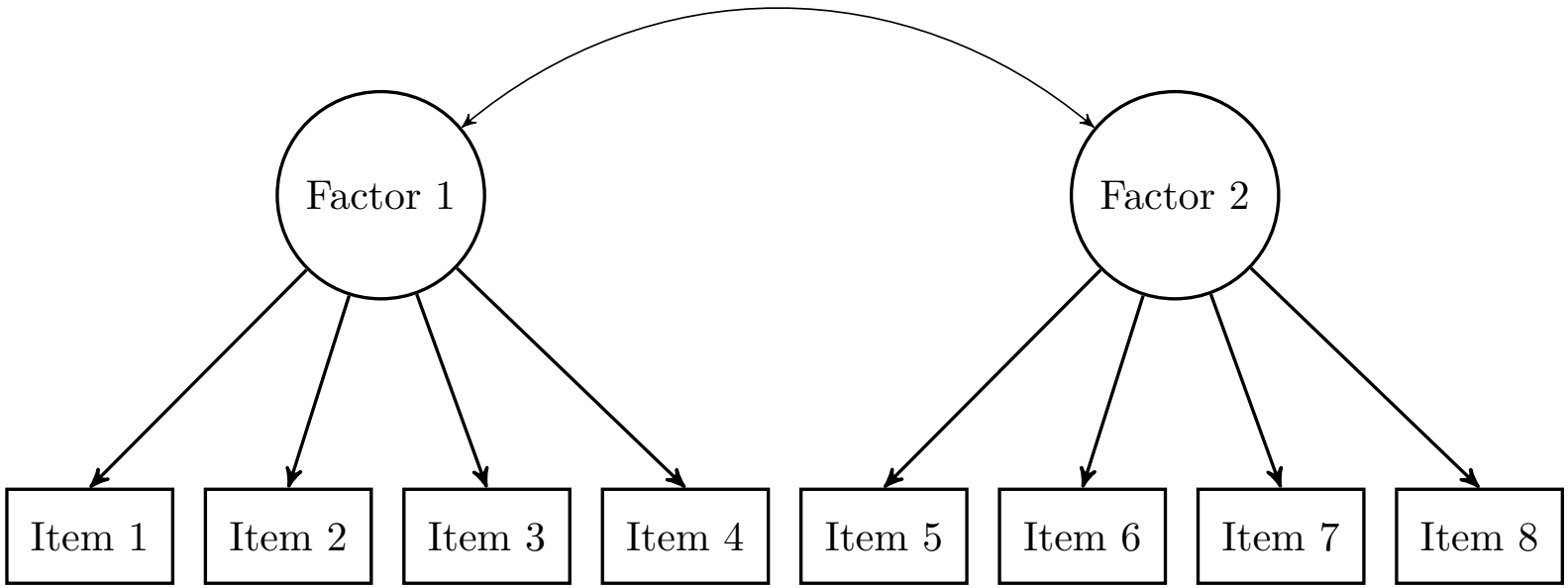
Confirmatory Factor Analysis



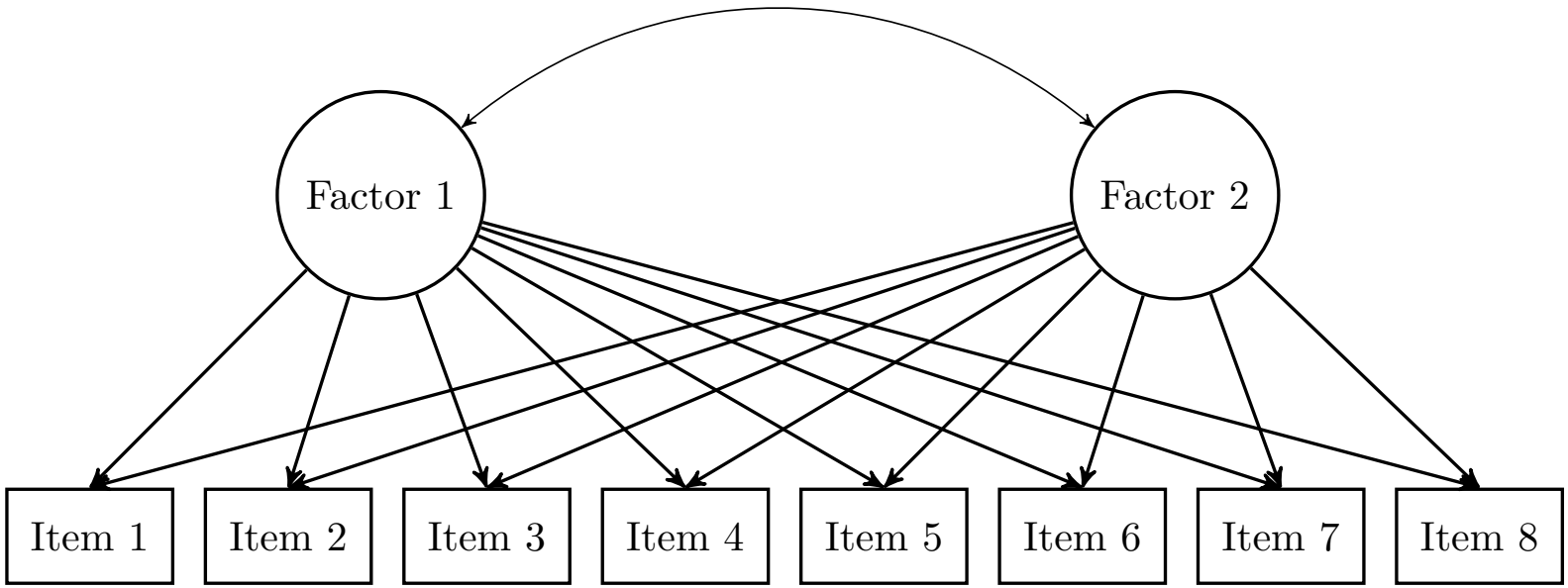
Confirmatory Factor Analysis



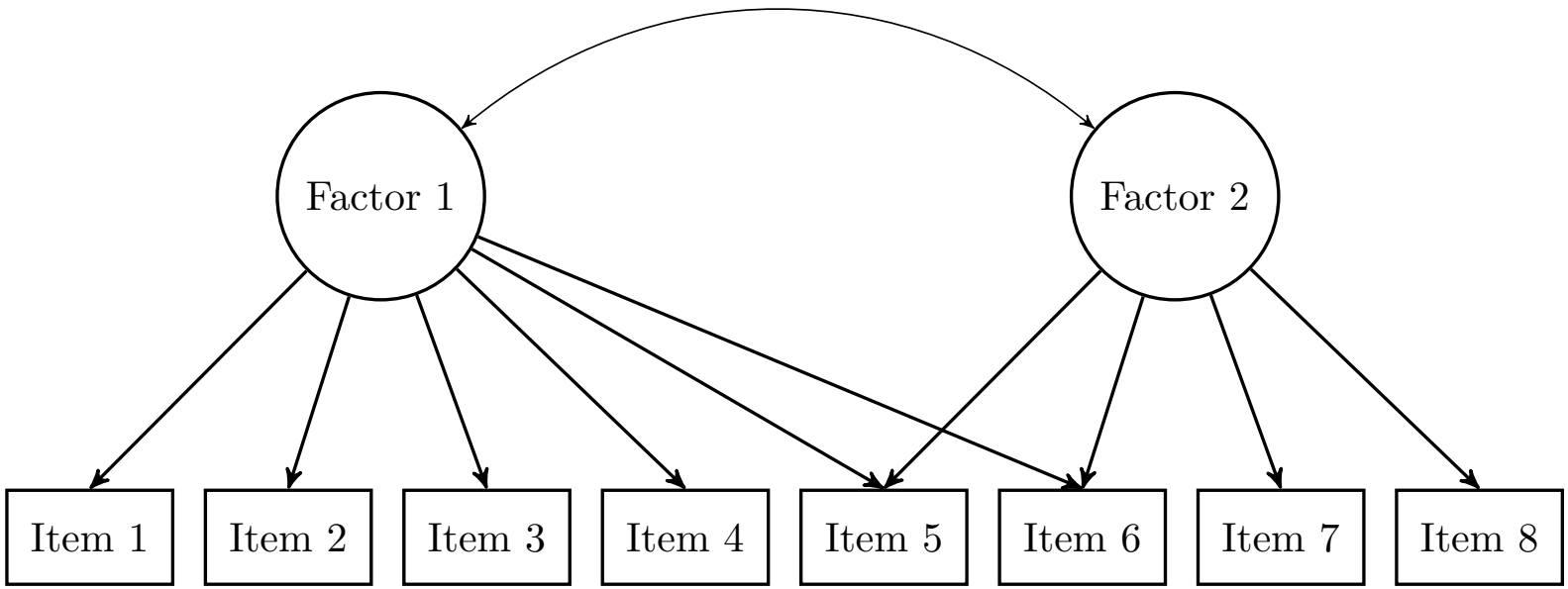
Confirmatory Factor Analysis



Confirmatory Factor Analysis



Confirmatory Factor Analysis



CFA Model Modifcation

How do you justifiably modify your measurement model?
(pause - this isn't rhetorical)

Global Assessment

A *global assessment* of model fit is traditional day by investigating the overall relationship between

$$\textit{Observed} \quad - \quad \textit{Expected}$$

$$\mathbf{S} - \boldsymbol{\Sigma}(\hat{\theta})$$

Global Assessment Cont.

Global assessment is traditionally accomplished using

- χ^2-test of model fit (Bollen, 1989) (need p. #)
- Comparative Fit Index (CFI; Bentler, 1990)
- Room Mean Square Error of Approximation (RMSEA; Browne & Cudeck, 1992)
- Standardized root mean square residual (SRMR; Bentler, 1995; Jöreskog & Sörbom, 1981; Maydeu-Olivares et al., 2018)
- among others as well.

A potential pitfall is that these approaches only provide a general overall assessment of model fit and does not inform us where or which relationships are not captured by our model (Steiger, 2007).

Local Assessment

A *local assessment* of model fit refers to assessing model subcomponents or individuals relationship for lack of fit or contribution to inference

$$s_{ij} - \sigma_{ij}(\hat{\theta})$$

for observed covariance between items i and j or the contribution of a parameter θ to the model

Local Assessment Cont.

Often, local fit assessment results in suggested modifications to the proposed model. Potential modifications include

- Removing paths or potentially unnecessary relationships
- Adding paths or covariances to account for relationships

Some Approaches to Model Modification

- Modification Indices (Kaplan, 1989; Sörbom, 1989)
- Wald Tests (Buse, 1982; Wald, 1943)
- Likelihood Ratio Tests (Buse, 1982; Neyman & Pearson, 1928)
- Model Implied Instrumental Variables (Bollen, 1995, 2019)

And now...

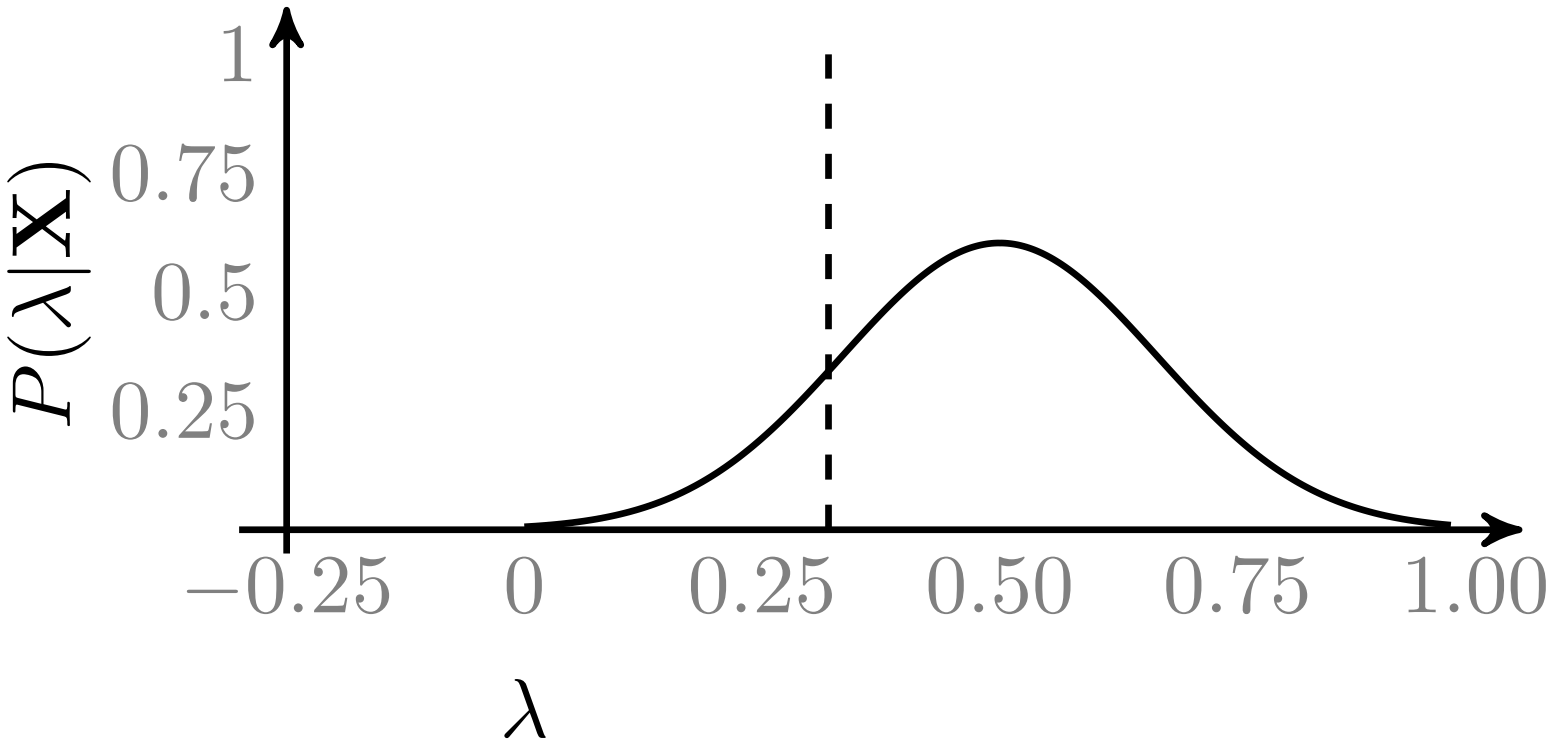
Some Approaches to Model Modification

Region of Practical Equivalence Probability Approximation
or
ROPE Probability Approximation

ROPE Probability Approximation

- The goal is to provide an interpretable metric to evaluate the parameter space of a selected model.
- The logic is to evaluate the probability that a parameter is outside a region of the parameter space we (as researchers) determine to provide little inferential benefit.
- Taken from exploratory factor analysis, we could define all factor loadings (standardized) that are below 0.32 to too low and our new approach approximates the probability that the loading is greater than 0.32.

ROPE Probability Approximation Cont.



Probability Definition

- The ROPE Probability can be expressed as $\Pr(|\theta| \geq \theta^* \mid \mathbf{X})$.
- For factor loadings, the probability statement is reduced to $\Pr(|\lambda| \geq 0.32 \mid \mathbf{X})$.
- For error covariances, the cutoff could be defined by a correlation of 0.25.

Estimation

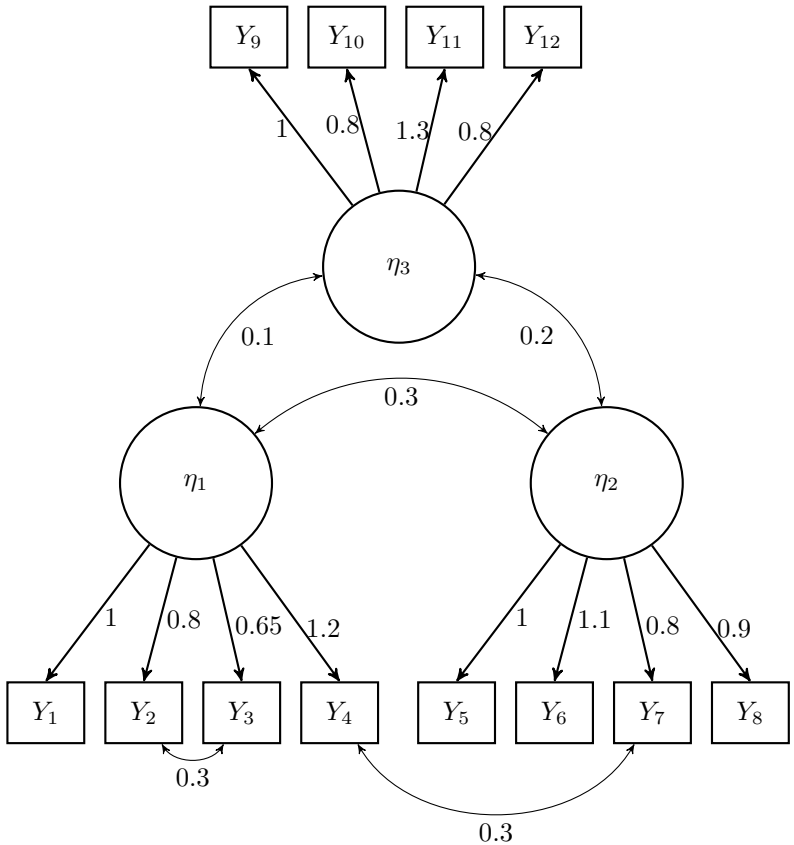
- The probability is condition on the data ($\mathbf{X}$).
- Traditional frequentist approaches to inference conditions on the parameter, so a Bayesian approach is taken.
- To side-step computational complexities, we utilized Laplace's method to approximate the posterior for each parameter.
- Briefly, Laplace's method is a Taylor series expansion to approximate any function by a sum of derivatives.

$$f(x) = \sum_{k=0}^{\infty} (\theta - \hat{\theta})^k f^{(k)}(\hat{\theta})$$
$$\approx \sum_{k=0}^{2} (\theta - \hat{\theta})^k f^{(k)}(\hat{\theta}) + O_2$$

Research Questions

1. Does this new approach provide a noticeably higher probability for parameters from the data generating model compared to true-zero paths?
2. How does the empirical sampling distribution used in the approximation compare to the sampling distribution of the parameters from the true model?

The Population Model



A random sample of 300 was drawn from this population which was adapted from (Bollen, 1989, 2019).

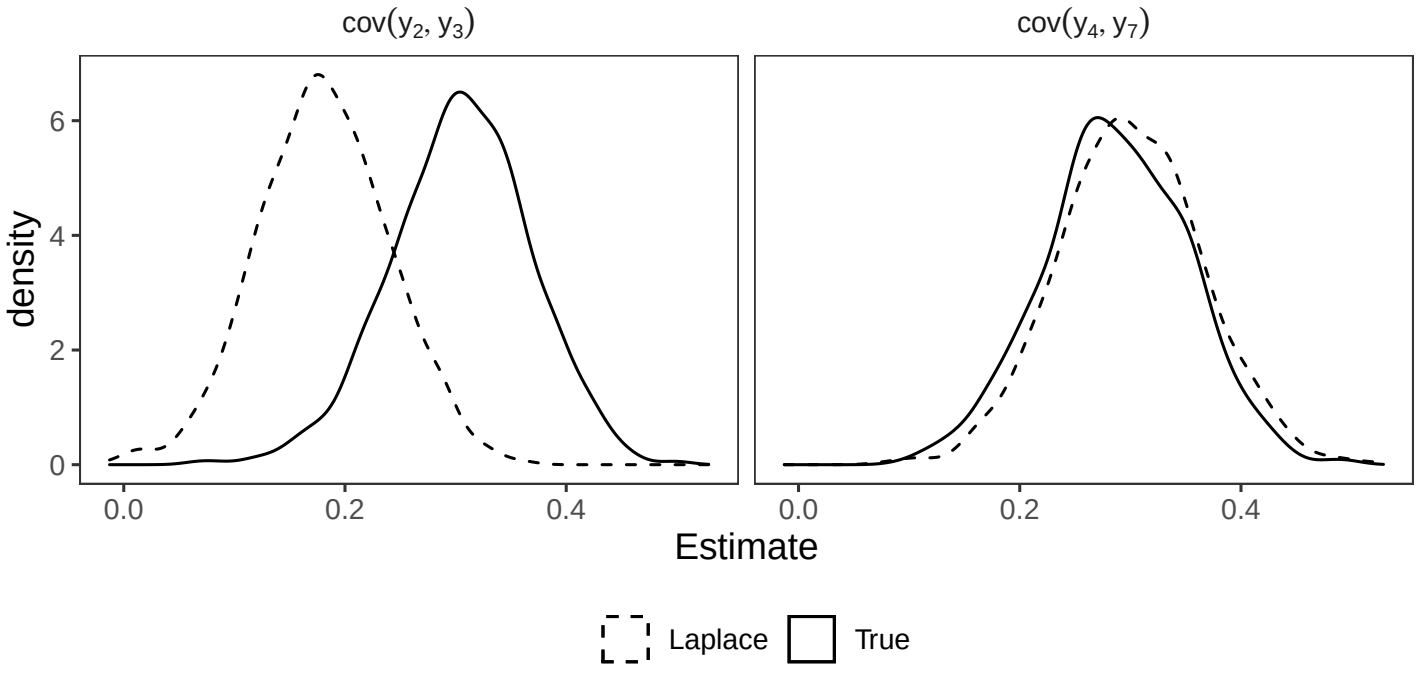
Hypothesized Model Fit

- Model was fit using `lavaan` (Rosseel, 2012).
- Model fit summary $\chi^2(51) = 72.8, p = 0.024$
- Model does not "fit" these data according to a coarse view.
- Now, using the ROPE Probability Approximation, we aim to identify where in the model potential modification should be considered.

Model Modification Results

Parameter (θ)	$\Pr(\mid \theta \mid \geq \mathit{cutoff})$
$\mathrm{cov}(y_7, y_4)$	0.810
$\mathrm{cov}(y_3, y_2)$	0.461
$\mathrm{cov}(y_9, y_4)$	0.233
$\mathrm{cov}(y_4, y_1)$	0.160
$\vdots$	
λ_{19}	0.005

Sampling Distribution of Error Covariances



Conclusions

- The proposed local fit method is an efficient approach to transforming local fit assessment into a probabilistic framework.
- The probability is an intuitive approach to assessing whether paths should be modified.
- More work needs to be done to evaluate how this approach works in non-simulated datasets.

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