# Math 3B: Lecture 4

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October 5, 2018

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- Graphing using calculus

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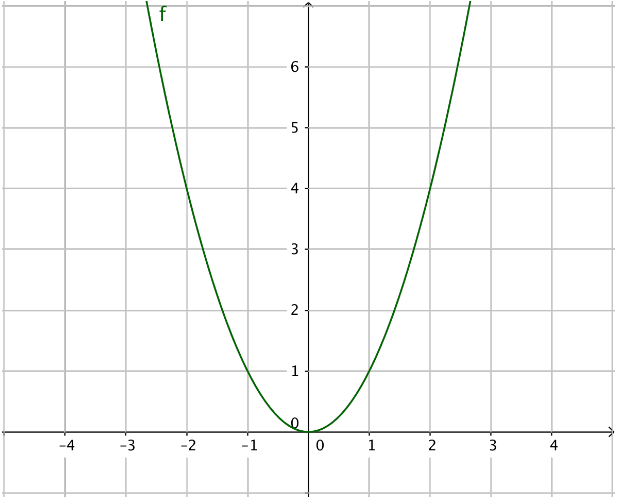
$f : \mathbb{R} \longrightarrow \mathbb{R}; x \mapsto x^2$ has a min at $x = 0$



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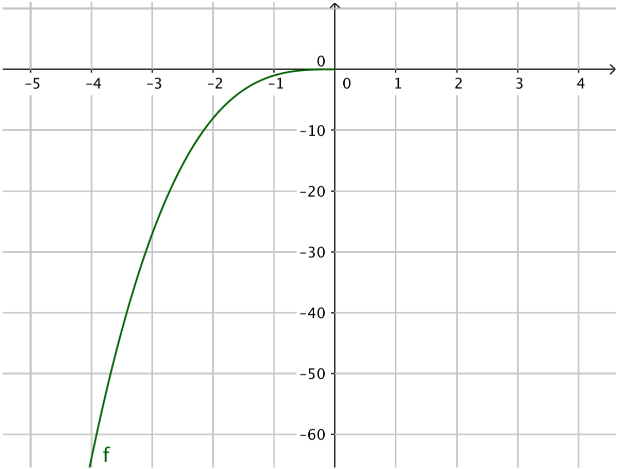
$f : (-\infty, 0\,] \longrightarrow \mathbb{R}; f(x) = x^3$ has a max at $x = 0$



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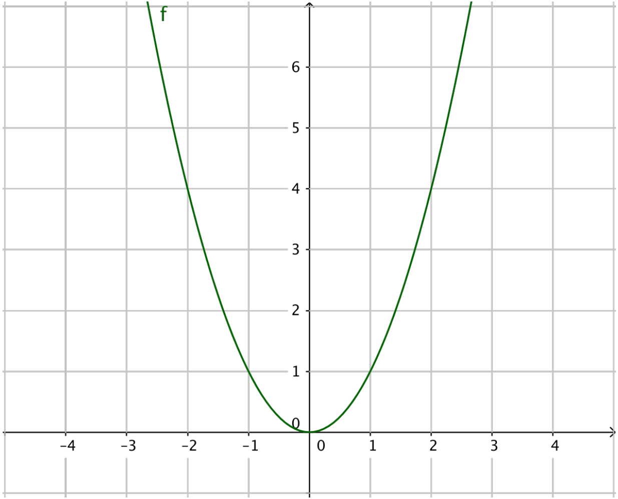
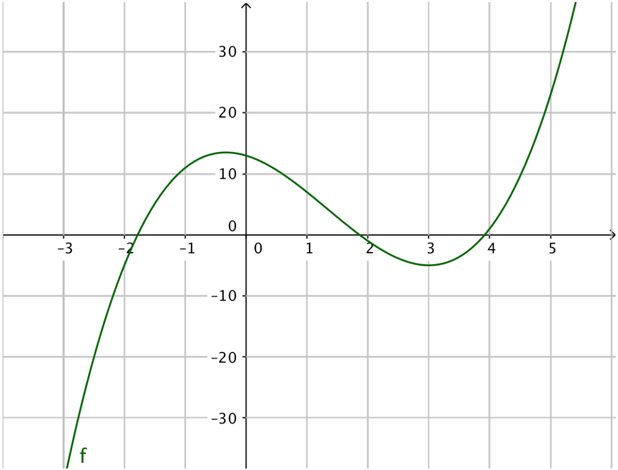


# Example of a local maximum

$$f : \mathbb{R} \longrightarrow \mathbb{R}; f(x) = x^3 - 4x^2 - 3x + 13 \text{ has a local max at } x = -\frac{1}{3}$$



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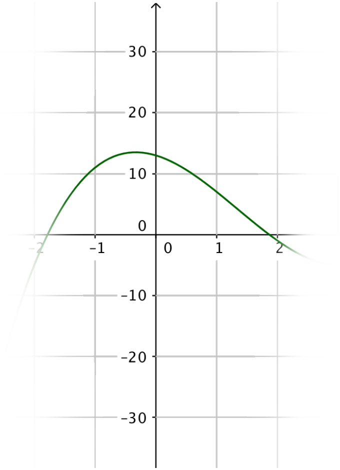

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- $f(x) = \sin x$ has a critical point at $x = \frac{\pi}{2}$ (since $f'(x) = \cos x$)
- $f(x) = e^x$ doesn't have any critical points since $f'(x) = e^x$ can never be zero

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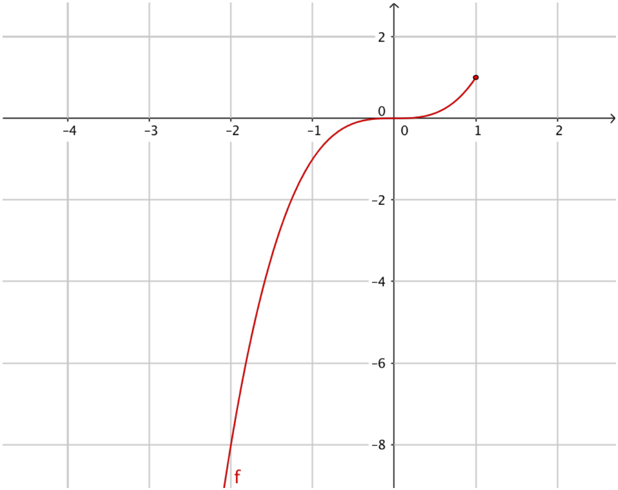
## Example

$f : (-\infty, 1] \longrightarrow \mathbb{R}; f(x) = x^3$ has critical points at

$$x = 0 \text{ and } 1$$

# Example

$f'(x) = 3x^2$ so $f'(0) = 0$ and $f'(1)$ is undefined.



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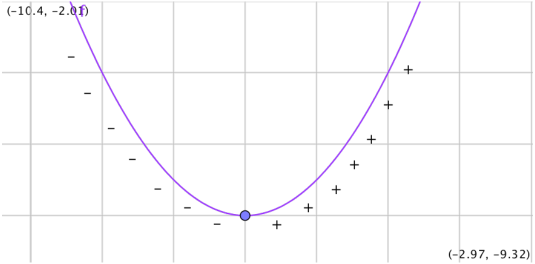

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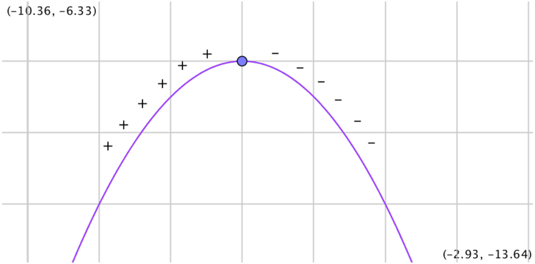

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Note: If $f''(a) = 0$ then we cannot conclude anything! E.g $x^3$ or $x^4$.