This week you will get practice applying integration to problems about accumulated change. All of these problems require approximating using smaller and smaller "subintervals" and interpreting this as an integral.

Homework: The homework will be due on Friday 4 November, at 2pm, the *start* of the lecture. It will consist of questions:

1, 5, and 8.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. Use the fundamental theorem of calculus and the interpretation of the definite integral as an area to find the general antiderivative of the function $f(x) = |x|$.

2. (5.8-20) Analysts speculate that patients will enter a new clinic at a rate of $300 + 100 \sin \frac{\pi t}{6}$ individuals per month. Moreover, the likelihood an individual is in the clinic t months later is e^{-t}. Find the number of patients in the clinic one year from now.

3. (5.8-21) A patient receives a continuous drug infusion at a rate of 10 mg/h. Studies have shown that t hours after injection, the fraction of drug remaining in a patients body is e^{-2t}. If the patient initially has 5 mg of drug in her bloodstream, then what is the amount of drug in the patients bloodstream 24 hours later?

Solution: First we can look at the 5 mg of the drug that is already in the patients bloodstream. After 24 hours, the question states that there will be

$$5e^{-48} \text{ mg}$$

left.

Now we estimate the amount of drug in the bloodstream due to the continuous infusion by dividing the time period $t = 0$ to $t = 24$ into n evenly sized pieces. Let $\Delta t = \frac{24}{n}$ be the length of each of these subintervals. Let t_k be the starting time of the k^{th} interval (where we count from $k = 0$), so that $t_k = \frac{24k}{n}$.

If we concentrate on the k^{th} interval, then during this time, $10\Delta t$ mg of drug will have been injected into the patient. If we assume that the interval is small enough (i.e. n is large) then at time $t = 24$, this amount of the drug will have been in the patients bloodstream for $24 - t_k$ hours. Using the formula from the question, this means there will be

$$10\Delta t e^{-2(24-t_k)} \text{ mg}$$

of the drug left in the system at $t = 24$. Adding all these contributions together and letting $n \to \infty$, we get that the total amount of the drug in the patients bloodstream at $t = 24$ is

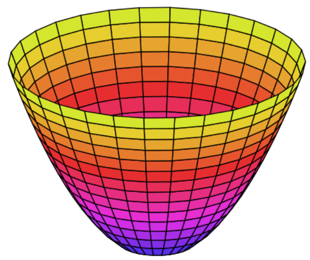
$$D = 5e^{-48} + \lim_{n\to\infty} \sum_{k=0}^{n-1} 10\Delta t e^{-2(24-t_k)} = 5e^{-48} + \lim_{n\to\infty} \Delta t \sum_{k=0}^{n-1} 10 e^{-2(24-t_k)}$$

We recognise this as a Riemann sum and thus

$$D = 5e^{-48} + \int_0^{24} 10e^{-2(24-t)}\, \mathrm{d}t = 5e^{-48} + \left[5e^{-2(24-t)}\right]_0^{24} = 5$$

4. (5.8-24) The administrators of a town estimate that the fraction of people who will still be residing in the town t years from now is given by the function $S(t) = e^{-0.04t}$. The current population is $20,000$ people and new people are arriving at a rate of 500 per year.
   (a) What will be the population size 10 years from now?
   (b) What will be the population size 100 years from now?

5. If we rotate the graph of $y = x^2$ around the y-axis, we obtain a 3D shape that looks like a bowl (see the picture below). If we take the part of this bowl that exists below $y = h$, how much volume does it contain?



Thanks to Wikipedia for the picture! Hint: I have drawn some extra diagrams (see the class website), you may find them helpful. It would be a good idea to review the example of cone from class.

6. (5.6-28) A bucket weighing 75 lb when filled and 10 lb when empty is pulled up the side of a 100 ft building. How much more work in foot-pounds is done in pulling up the full bucket than the empty bucket?

   Hint: this doesn't involve a Riemann sum, it is just applying the definition of "work".

7. (5.6-29) A 20-ft rope weighting 0.4 lb/ft hangs over the edge of a building 100 ft high. How much work is done in pulling the rope to the top of the building? Assume that the top of the rope is flush with the top of the building, and the lower end of the rope is swinging freely.

8. Various radioactive materials are used in medical diagnostic techniques. A company which produces these radioactive materials would like to store its waste materials in a special storage facility which can hold up to 100 kg of radioactive materials. The radioactive waste has a half-life of 5 months. This means, if we start with M kg of waste, after t months, only

$$Me^{-0.2t\ln(2)} \text{ kg}$$

   will remain.
   (a) If the company adds waste to the storage facility at a rate of a kg per month, write a function for the amount of material in the storage facility x months after the company opens the storage facility.
   (b) What is the maximum rate at which the company can add waste, so they never exceed the capacity of the storage facility?

9. (5.8-32) Determine the length of a rectangular trench you can dig with the energy gained from eating one Milky Way bar (270 Cal). Assume that you convert the energy gained from the food with 10% efficiency and that the trench is 1 meter wide and 1 meter deep. Assume the density of soil is $1,000$ kg/m^3.