

# Midterm 2 practice 1

## UCLA: Math 115A, Winter 2020

*Instructor:* Noah White

*Date:*

*Version:* practice

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
Total:	20	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (1 point) Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be the linear map given by  $T(a, b, c) = (a + b - c, b + c)$ . Consider the bases  $B = \{(1, 1, 1), (1, 0, -1), (1, 0, 1)\}$  and  $C = \{(2, 1), (1, 2)\}$ . The matrix  $[T]_B^C$  is

A.  $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$

B.  $\frac{1}{3} \begin{pmatrix} 0 & 5 & -1 \\ 3 & -4 & 2 \end{pmatrix}$

C.  $\begin{pmatrix} 0 & -2 & 2 \\ 1 & -1 & 2 \end{pmatrix}$

D.  $\frac{1}{5} \begin{pmatrix} 0 & -2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

(b) (1 point) For an arbitrary linear combination  $T : V \rightarrow W$  and bases  $B = \{v_1, \dots, v_n\}$ ,  $C = \{w_1, \dots, w_m\}$  of  $V$  and  $W$ , the  $i^{\text{th}}$  column of the matrix  $[T]_B^C$  is

A.  $T(v_i)$

B.  $[T(v_i)]_B$

C.  $[w_i]_C$

D.  $[T(v_i)]_C$

(c) (1 point) Consider the linear map  $T : \mathbb{R}_1[x] \rightarrow \mathbb{R}_1[x]$  given by  $T(a + bx) = (a + b) + (a - b)x$ . Which of the following is a true statement.

A.  $T$  has an eigenvalue of 2.

B.  $T$  is diagonalizable.

C. The only eigenvalue of  $T$  is  $-1$ .

D.  $T$  has infinitely many eigenvalues.

(d) (1 point) If  $\dim V = 6$ , and  $W$  is a subspace such that  $\dim W = 3$  then

- A.  $\dim V/W = 1$
- B.  $\dim V/W = 2$
- C.  $\dim V/W = 3$
- D.  $\dim V/W = 4$

(e) (1 point) The linear map  $T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$  defined by  $T(a + bx + cx^2) = (a + c) - (b - c)x + bx^2$  is invertible. What is  $T^{-1}(1 - x + x^2)$ ?

- A.  $1 + x + x^2$
- B.  $2x - x^2$
- C.  $1 + x$
- D.  $x^2$

2. (a) (2 points) Define what it means for two vector space  $V$  and  $W$  to be isomorphic.
- (b) (3 points) Prove that, if  $V$  and  $W$  are finite dimensional, then  $V$  and  $W$  are isomorphic if and only if  $\dim V = \dim W$ .

3. Let  $T : V \longrightarrow V$  be a linear transformation for a vector space  $V$  over  $\mathbb{C}$ . Define  $T^n$  to be the linear map obtained by repeatedly applying  $T$ ,  $n$  times. E.g.  $T^3(v) = T(T(T(v)))$ .
- (a) (2 points) Suppose  $T^n = 0$  for some  $n$ . Show that the only eigenvalue of  $T$  is zero.
  - (b) (3 points) Prove that, if  $T^2 = 0$  if and only if  $\text{im } T \subseteq \ker T$ .

4. Let  $T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$  be a linear map such that

$$[T]_B^B = \frac{1}{2} \begin{pmatrix} 2 & 6 & -2 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

where  $B = \{1, (x-1)^2, (x+1)^2\}$

- (a) (1 point) Is  $T$  an isomorphism? *Hint: This should be a very easy, and quick calculation. If you are spending more than 30 seconds on it, you are missing something important.*
- (b) (2 points) Let  $E = \{1, x, x^2\}$ . Find the change of basis matrix  $[\text{id}]_E^B$ .
- (c) (2 points) Calculate  $[T]_E^E$  and  $T^6(x)$ .

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