Midterm 2 practice

UCLA: Math 31B, Spring 2017

Instructor: Noah White
Date:
Version: practice

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Discussion section: _____

Question	Points	Score
1	8	
2	10	
3	15	
4	7	
Total:	40	

1. Calculate the following limits using any technique you like.

   (a) (5 points)

   $$\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^{x^2}$$

   (b) (3 points)

   $$\lim_{x\to\infty}\left(1+\frac{1}{x}+e^{\frac{1}{x}}\right)^{\arctan x}$$

2. For each of the following improper integrals say whether it converges or diverges. If the integral converges, you should say what value it converges to.

   (a) (5 points)

   $$\int_{-\frac{\pi}{2}}^{0} \frac{\cos x}{\sin x}\, dx$$

   (b) (5 points)

   $$\int_{1}^{\infty} \frac{\ln x}{x^2}\, dx$$

3. For each of the following series say whether it converges or diverges. You do NOT need to justify your answer.

   Grading scheme: 0 points for wrong, 1 point for no response, 3 points for correct.

   (a) (3 points) $\sum_{n=1}^{\infty} \frac{1}{n}$.

   (b) (3 points) $\sum_{n=1}^{\infty} \frac{(n+3)^2}{n^4}$.

   (c) (3 points) $\sum_{n=45}^{\infty} \frac{(-2)^n+8^n}{3^{2n}}$.

   (d) (3 points) $\sum_{n=1}^{\infty} a_n$ where the sequence of partial sums $(s_N)_{N=1}^{\infty}$ is described by

   $$s_N = \sum_{n=1}^{N} \frac{1}{\sqrt{n}}.$$

   (e) (3 points) $\sum_{n=1}^{\infty} a_n$ where the sequence of partial sums $(s_N)_{N=1}^{\infty}$ is described by

   $$s_N = \frac{1}{\sqrt{N}}.$$

4. (7 points) Let (F_n) be the *Fibonacci* sequence, that is

$$F_0 = 0, F_1 = 1, \text{ and } F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2.$$

Consider the sequence (R_n) for $n \geq 1$ defined by $R_n = \frac{F_{n+1}}{F_n}$, i.e.

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{5}, \dots$$

You may assume that (R_n) converges to a limit L such that $1 < L < 2$. Find L.

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