Math 3B: Lecture 17

Noah White

February 22, 2017

Newton's Law of Cooling

Isaac Newton stated that

Newton's Law of Cooling

The temperature T of a body changes at a rate propotional to the difference between the ambient temperature A and T.

Newton's Law of Cooling

Isaac Newton stated that

Newton's Law of Cooling

The temperature T of a body changes at a rate propotional to the difference between the ambient temperature A and T.

Newton's Law of Cooling

Isaac Newton stated that

Newton's Law of Cooling

The temperature T of a body changes at a rate propotional to the difference between the ambient temperature A and T.

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(A - T)$$

General solution

$$T(t) = A - Ce^{-kt}.$$

Example 2

An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

Example 2

An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

Example 2

An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

- The solution is given by

$$T(t) = 70 - Ce^{-kt}$$

Example 2

An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

- The solution is given by

$$T(t) = 70 - Ce^{-kt}$$

- We know $T(0) = 90$ and $T(20) = 86$.

Example 2

An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

- The solution is given by

$$T(t) = 70 - Ce^{-kt}$$

- We know $T(0) = 90$ and $T(20) = 86$.
- Thus

$$90 = 70 - C \quad \text{so} \quad C = -20.$$

Example 2

- $T(t) = 70 + 20e^{-kt}$

Example 2

- $T(t) = 70 + 20e^{-kt}$
- To find k we use $T(20) = 86$

$$86 = 70 + 20e^{-20k}$$

Example 2

- $T(t) = 70 + 20e^{-kt}$
- To find k we use $T(20) = 86$

$$86 = 70 + 20e^{-20k}$$

- Thus

$$e^{-20k} = \frac{86-70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20}\ln\left(\frac{4}{5}\right) \approx -0.01.$$

Example 2

- $T(t) = 70 + 20e^{-kt}$
- To find k we use $T(20) = 86$

$$86 = 70 + 20e^{-20k}$$

- Thus

$$e^{-20k} = \frac{86 - 70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20} \ln\left(\frac{4}{5}\right) \approx -0.01.$$

- The model is thus $T(t) = 70 + 20e^{-0.01t}$. We want to solve

$$75 = 70 + 20e^{-0.01t}.$$

Example 2

- $T(t) = 70 + 20e^{-kt}$
- To find k we use $T(20) = 86$

$$86 = 70 + 20e^{-20k}$$

- Thus

$$e^{-20k} = \frac{86-70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20}\ln\left(\frac{4}{5}\right) \approx -0.01.$$

- The model is thus $T(t) = 70 + 20e^{-0.01t}$. We want to solve

$$75 = 70 + 20e^{-0.01t}.$$

- Rearranging we get $20e^{-0.01t} = 5$ i.e.

Example 2

- $20e^{-0.01t} = 5$ becomes

$$e^{-0.01t} = \frac{1}{4}$$

Example 2

- $20e^{-0.01t} = 5$ becomes

$$e^{-0.01t} = \frac{1}{4}$$

- Applying a logarithm

$$-0.01t = \ln\left(\frac{1}{4}\right)$$

Example 2

- $20e^{-0.01t} = 5$ becomes

$$e^{-0.01t} = \frac{1}{4}$$

- Applying a logarithm

$$-0.01t = \ln\left(\frac{1}{4}\right)$$

- So we get

$$t = -100\ln\left(\frac{1}{4}\right) \approx 138 = 2 \text{ hours } 18 \text{ minutes.}$$