# Midterm 1 practice 3

## UCLA: Math 115A, Spring 2020

*Instructor:* Noah White
*Date:*

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ____________________

ID number: ____________________

**Discussion section (please circle):**

| Question | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| Total: | 20 | |

**Question 1** is multiple choice. Indicate your answers in the table below. *The following three pages will not be graded, your answers must be indicated here.*

| *Part* | A | B | C | D |
|---|---|---|---|---|
| (a) | | | | |
| (b) | | | | |
| (c) | | | | |
| (d) | | | | |
| (e) | | | | |

☐ I wish to opt out of having my exam graded using Gradescope.

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) If $V$ is a vector space over the field $\mathbb{C}$ and $v \in V$ then

$$(2-3)\cdot(v+w)+(5-3)\cdot v+w$$

   equals

   A. 1
   B. $v$
   C. 0
   D. $2v$

   The following two questions concern the subsets

$$A = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \;\middle|\; \lambda a^2 + 2(c - a^2) = 0 \right\} \subseteq \mathbb{R}^3$$

$$B = \{\, p \in \mathbb{R}[x] \mid p' = \lambda \,\} \subseteq \mathbb{R}[x]$$

   for some $\lambda \in \mathbb{R}$.

   (b) (1 point) Which of the following is a true statement.

   A. Both $A$ and $B$ are subspaces regardless of the value of $\lambda \in \mathbb{R}$.
   B. $A$ is not a subspace for any $\lambda$ and $B$ is a subspace when $\lambda = 0$.
   C. Both are subspaces when $\lambda = 0$.
   D. $A$ is a subspace when $\lambda = 2$.

   (c) (1 point) When $\lambda = 0$, the subspace $B$ has dimension

   A. 1
   B. 2
   C. 3
   D. 4

(d) (1 point) Let $\mathbb{F}$ be one of the fields $\mathbb{Z}_p$ for $p = 2, 3, 5, 7$. Consider the vectors

$$\begin{pmatrix} [1] \\ [1] \end{pmatrix} \text{ and } \begin{pmatrix} [-1] \\ [5] \end{pmatrix}.$$

For which fields are the two vectors linearly dependent?

A. Only for $\mathbb{Z}_2$.
B. Only for $\mathbb{Z}_5$
C. For $\mathbb{Z}_2$ and $\mathbb{Z}_3$.
D. For $\mathbb{Z}_2$ and $\mathbb{Z}_5$.

(e) (1 point) Fix $\lambda \in \mathbb{R}$. Consider the subspaces

$$U = \left\{ \begin{pmatrix} -a \\ \lambda a \end{pmatrix} \,\middle|\, a \in \mathbb{R} \right\} \subset \mathbb{R}^2$$

$$W = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \,\middle|\, a + b = 0 \right\} \subset \mathbb{R}^2$$

When is $U \oplus W = \mathbb{R}^2$? (i.e. when is $\mathbb{R}^2$ a direct sum of these two subspaces).

A. For any $\lambda \in \mathbb{R}$.
B. Never.
C. When $\lambda \neq 1$.
D. When $\lambda \neq 0$.

2. Give (simple) examples of all of the following situations.

   (a) (2 points) An infinite dimensional vector space $V$ over $\mathbb{C}$ and an infinite dimensional subspace $W$ such that $W \neq V$.

   (b) (2 points) A linearly dependant subset $\{v_1, v_2, v_3\} \subseteq V$ consisting of 3 elements such that no element is a scalar multiple of another (i.e. $v_i \neq \lambda v_j$ for any $\lambda \in \mathbb{C}$ and $i \neq j$).

   (c) (1 point) A basis for $W$.

3. (5 points) Let $\mathbb{C}_3[x]$ be the vector space consisting of polynomials of degree less than 3 (i.e constant, linear and quadratic polynomials only). Let $S = \{1 + x, x - x^2, 1 + x + x^2\} \subset \mathbb{C}_3[x]$. Prove or disprove that $S$ is a basis of $\mathbb{C}_3[x]$.

4. Let $V$ be a vector space over a field $\mathbb{F}$ and $W$ be a subspace.

   (a) (2 points) Consider the map $\pi : V \longrightarrow V/W$ given by $\pi(v) = v + W$. Show that $\pi$ is a linear map.

   (b) (3 points) Let $V = \mathbb{R}^3$ and

$$W = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \in V \;\middle|\; a - b = 0, b - c = 0, c - a = 0 \right\}$$

   Find the dimension of $W$ and $V/W$.

This page has been left intentionally blank. You may use it as scratch paper. It will not be graded unless indicated very clearly here and next to the relevant question.