Final exam practice

UCLA: Math 3B, Fall 2016

Instructor: Noah White
Date: Monday, November 28, 2016
Version: practice.

- This exam has 7 questions, for a total of 84 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section: ______________________________

Question	Points	Score
1	20	
2	15	
3	11	
4	12	
5	10	
6	8	
7	8	
Total:	84	

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) The function $f(x) = e^{-x} - e^{x}$ is

   A. Always increasing.
   B. Always decreasing.
   C. Always concave up.
   D. Always concave down.

   (b) (2 points) The function $f(x) = \frac{x^2}{x-1}$ has a

   A. Horizontal asymptote at $y = 1$.
   B. Vertical asymptote at $x = 2$.
   C. Slanted asymptote with gradient -1.
   D. Slanted asymptote with gradient 1.

   (c) (2 points) The function $f(x) = 4x - \frac{1}{1-x}$ has a critical point at

   A. $x = 0$
   B. $x = 0.5$
   C. $x = -1$
   D. $x = -0.5$

   (d) (2 points) The function $f(x) = 1 - e^{-(2x-4)^2}$ has a

   A. minimum at $x = 2$.
   B. maximum at $x = 2$.
   C. minimum $x = 0$.
   D. maximum $x = 0$.

   (e) (2 points) The definite integral $\int_{-2}^{1} 3x^2 + 1 \, \mathrm{d}x$ has a value of

   A. 12.
   B. maximum at 4.
   C. minimum 0.
   D. maximum -2.

(f) (2 points) The definite integral $\int_{-2}^{1} \sin^3 x \cos x \, \mathrm{d}x$ has a value of

A. 0.
B. maximum at $\frac{1}{4}$.
C. minimum 0.5.
D. maximum 2π.

(g) (2 points) The solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = 2y$ when $y(0) = 3$ has

A. $y(1) = 2e^2$
B. $y(0.5) = 3e$.
C. $y(1) = 3e$.
D. $y(0.5) = 3e^{0.5}$.

(h) (2 points) The solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = 2te^{-y}$ when $y(0) = 1$ has

A. $y(1) = \ln 4$
B. $y(0.5) = \ln 2$.
C. $y(1) = \ln 2$.
D. $y(0.5) = \ln 4$.

(i) (2 points) The differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = y(1-y)(y-3)$ has a

A. unstable equilibrium at $y = 2$.
B. unstable equilibrium at $y = 0$.
C. stable equilibrium at $y = 3$.
D. stable equilibrium at $y = 1$.

(j) (2 points) The differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = \ln(y)$ has a

A. unstable equilibrium at $y = 0$.
B. unstable equilibrium at $y = e^{-1}$.
C. stable equilibrium at $y = e$.
D. stable equilibrium at $y = 1$.

2. Let $f(x) = \frac{1}{1+e^{-2x}}$. Note that $f'(x) = \frac{2e^{-2x}}{(1+e^{-2x})^2}$ and $f''(x) = -\frac{4e^{-2x}(e^{-2x}-1)}{(1+e^{-2x})^3}$.

   (a) (2 points) Find the x and y intercepts of $f(x)$.

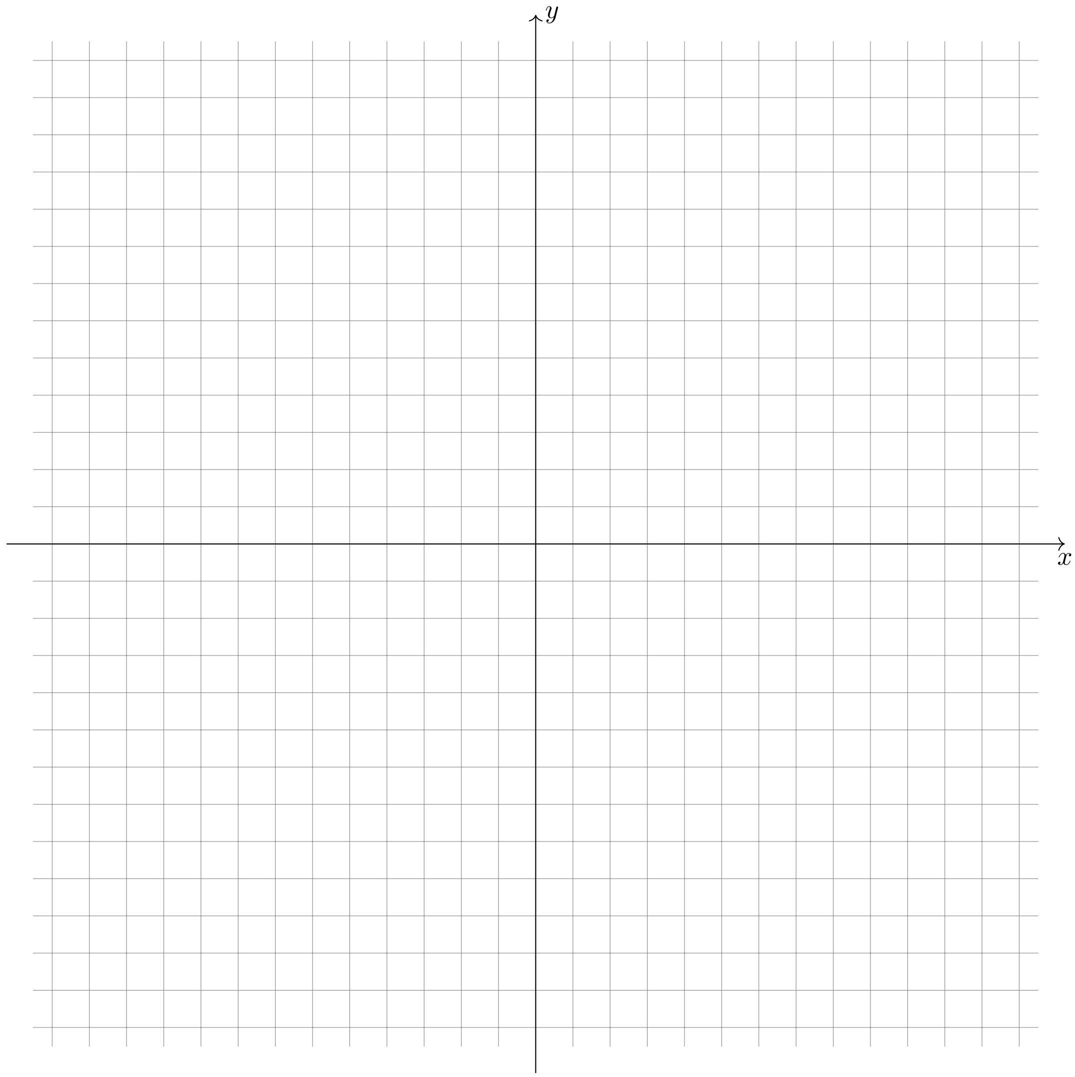
   (b) (1 point) Does $f(x)$ have any horizontal asymptotes? If so what are they?

   (c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

   (d) (2 points) For what x is the first derivative $f'(x)$ positive?

(e) (2 points) For what x is the second derivative $f''(x)$ positive?

(f) (3 points) On the graph provided, sketch $f(x)$



(g) (4 points) List the local maximums and minimums of $f'(x)$ (note: this question if asking about the extrema of the derivative of f!)

3. A heavy chain hangs over a ledge. The chain is 4 m in length and weighs a total of 10 kg. Attached to the end of the chain is a full bucket weighing 20 kg. In this question we will calculate the work needed to pull the chain and bucket up to the top of the ledge. Recall that

$$W = Fd$$

that is, the work done, is the force needed multiplied by the distance moved. You may assume the acceleration due to gravity is 10 m/s^2.

(a) (1 point) If we ignore the weight of the chain, how much work needs to be done to pull the bucket up?

(b) (1 point) Suppose we divide the chain into n intervals, of constant length Δh. What is Δh in terms of n?

(c) (1 point) Let $h = 0$ be the top of the chain and $h = 4$ be the bottom. Let h_k be the depth of the top of the k^{th} interval of chain below the ledge (where the first interval is the 0^{th}), thus $h_0 = 0$. What is h_k in terms of Δh and k?

(d) (2 points) How much does the k^{th} interval, of chain weigh? Your answer should be in terms of Δh.

(e) (2 points) If n is large enough, we can say approximately that the k^{th} interval of chain needs to be pulled up h_k m. Using this approximation, how much work is needed to be done to pull up the k^{th} interval of chain?

(f) (2 points) Write a Riemann sum which represents the total amount of work needed to pull the chain and bucket up.

(g) (2 points) Use an integral to evaluate the Riemann sum above.

4. Solve the following differential equations. If no initial condition is given, find the general solution.

   (a) (4 points) $\frac{\mathrm{d}y}{\mathrm{d}t} = 2 - y$.

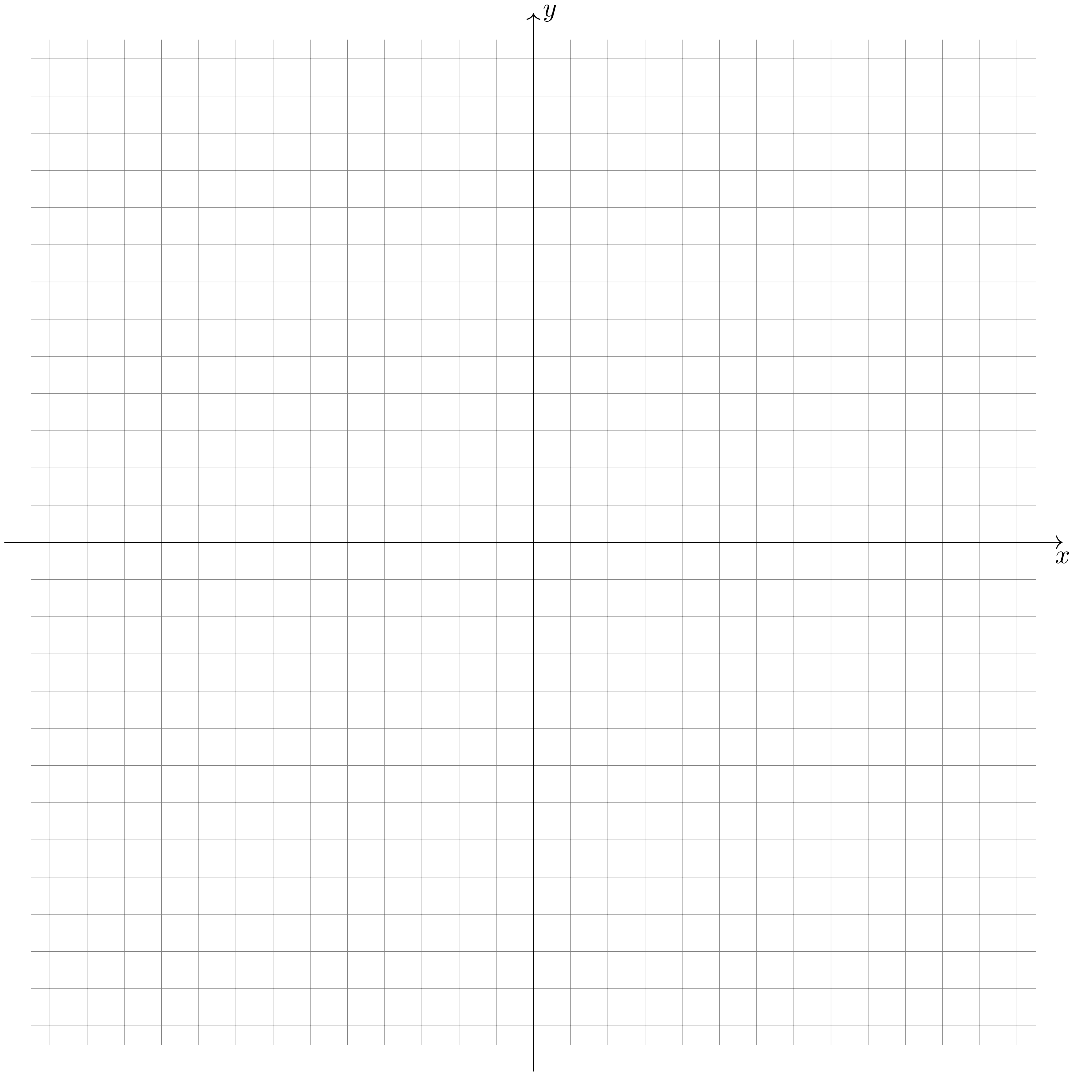
   (b) (4 points) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2(1 - 2t)$ where $y(0) = 1$.

   (c) (4 points) $\frac{\mathrm{d}y}{\mathrm{d}t} = e^{-y} \sin y$ where $y(0) = 0$.

5. In this question we will investigate the behaviour of the solutions of

$$\frac{\mathrm{d}y}{\mathrm{d}t} = a - (\ln y)^2 - 2$$

(a) (4 points) Draw a bifurcation diagram for this equation with parameter a. Make sure to label the regions of your diagram with up/down arrows according to the direction of the derivative.



(b) (2 points) Draw a phase diagram when $a = 4$ and sketch the solution if $y(0) = 1$.

(c) (2 points) Draw a phase diagram when $a = 2$ and sketch the solution if $y(0) = 0.5$.

(d) (2 points) Draw a phase diagram when $a = 1$ and sketch the solution if $y(0) = 1$.

6. (8 points) A new internet service provider (ISP) opens. They have horrible customer service so they calculate that the probability a given customer is still a customer in t months time is

$$e^{-0.2t}.$$

In order to remain profitable, the ISP must have approximately 3000 customers at any given time (in the long term). How many customers must the ISP gain per month in order to remain profitable?

7. (8 points) A straight, and very narrow steel bar is to be carried along a 1 meter wide corridor. The corridor makes a 90 degree turn. What is the longest length of steel pipe that can be carried down the corridor and around the corner?

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