Math 3B: Lecture 23

Noah White

November 30, 2018

Announcements

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- Final homework: q3 from PS10

Bifurcation

Often in real life situations we would like to study a system that includes an unknown parameter

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y, a)$$

The behaviour of the solution depends on a!

Example

We have been studying populations growing logistically. We also considered their behaviour under harvesting, but suppose we don't know exactly how many are harvested and we want to understand the effect of different harvesting rates.

$$= N(1 - N) - h$$

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- The bahviour of the solution, depends on the equilibria and their stability!
- Draw a bifurcation diagram
- The bifurcation diagram tells us how the phase line changes for different parameters.

Bifurcation diagram

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (y - a)(y - 3)$$

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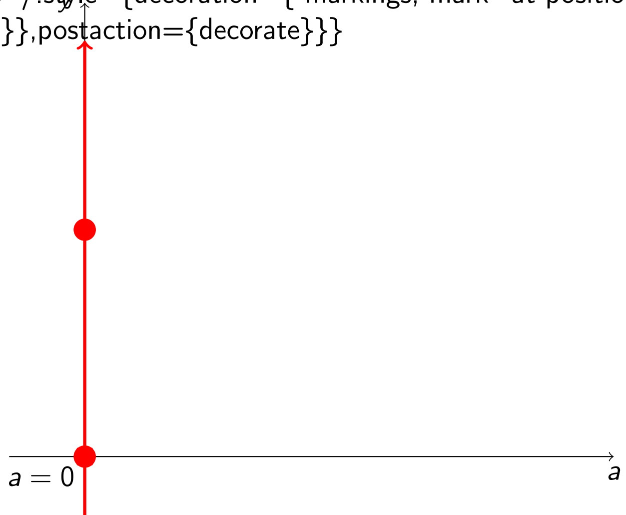

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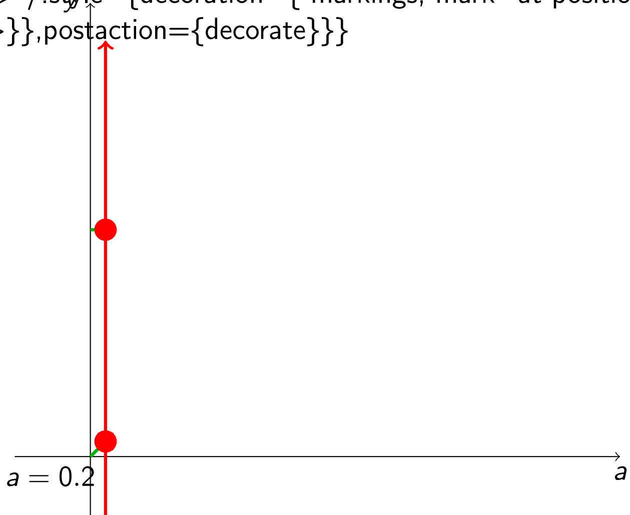

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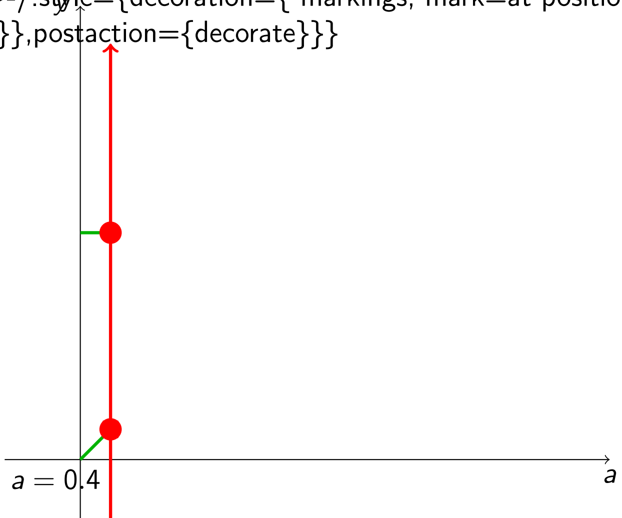

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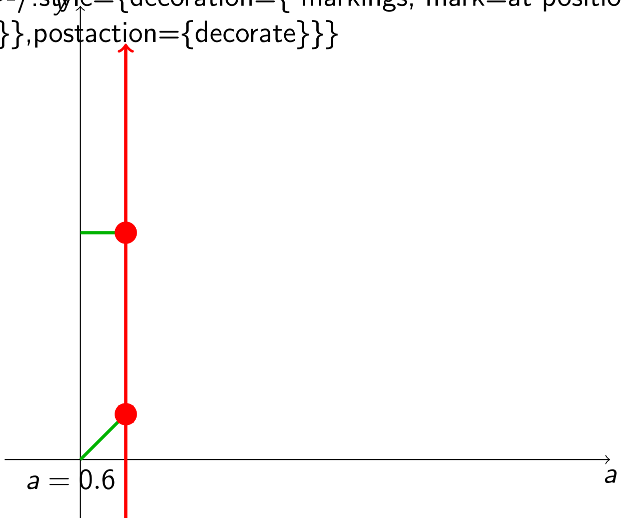

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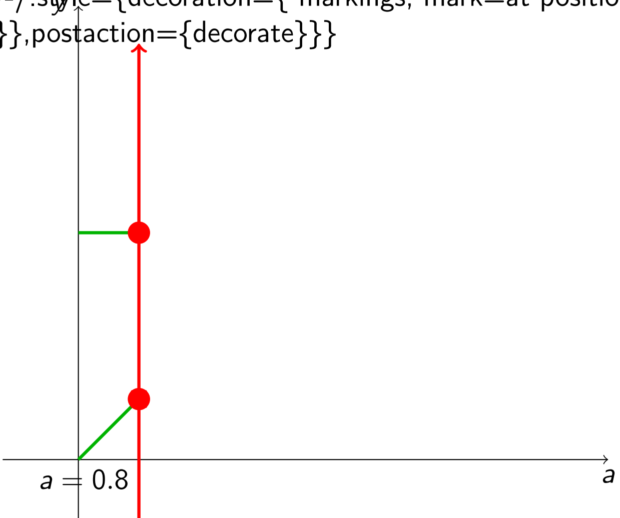

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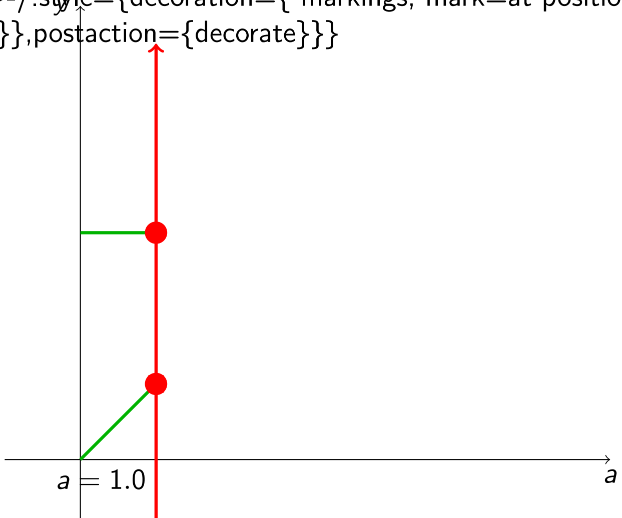

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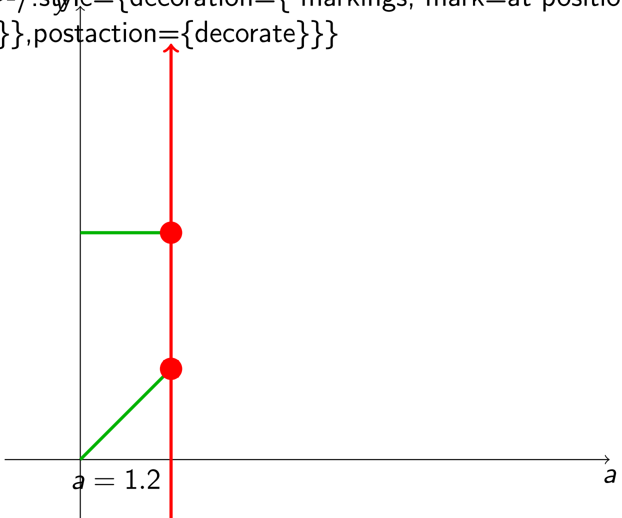

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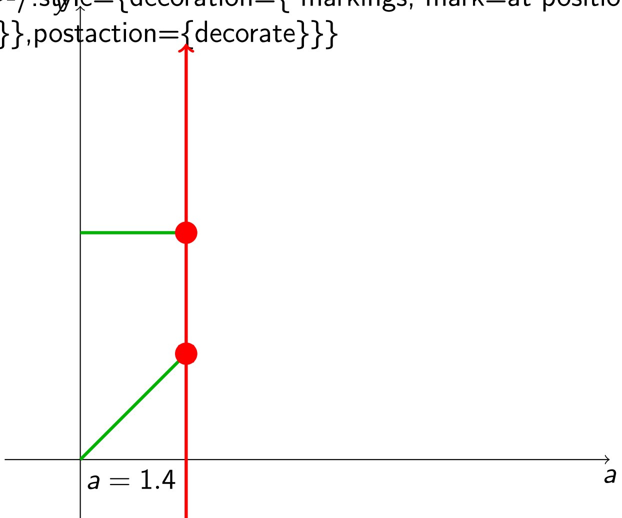

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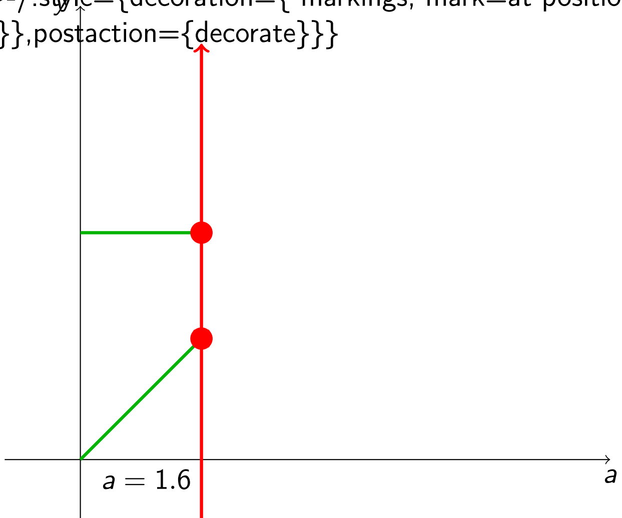

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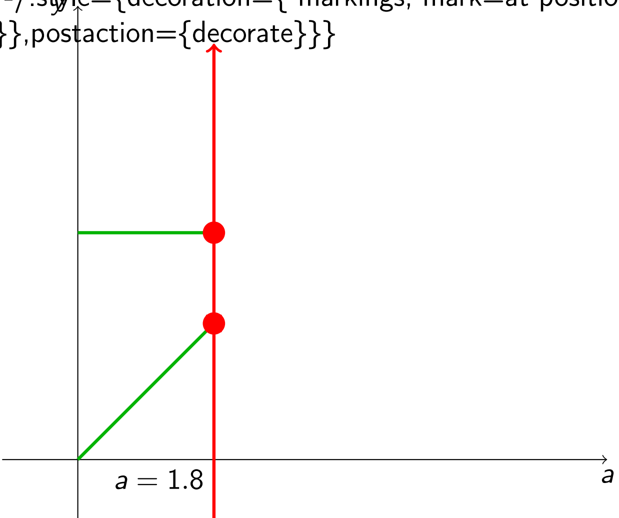

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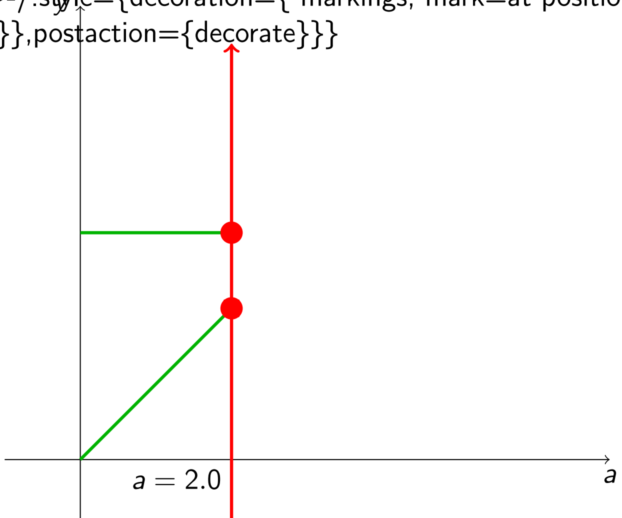

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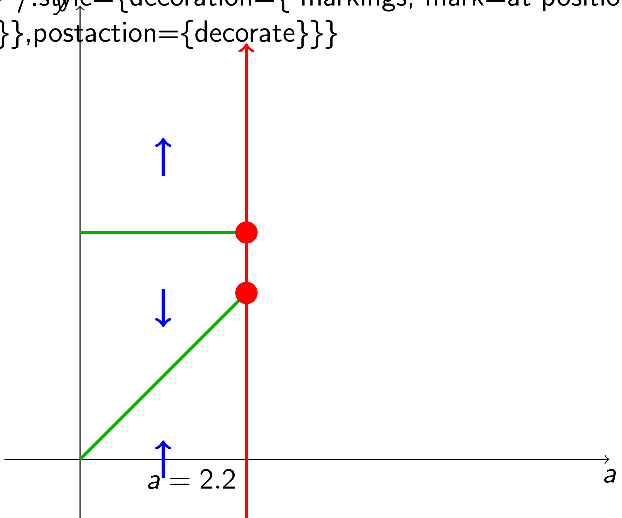

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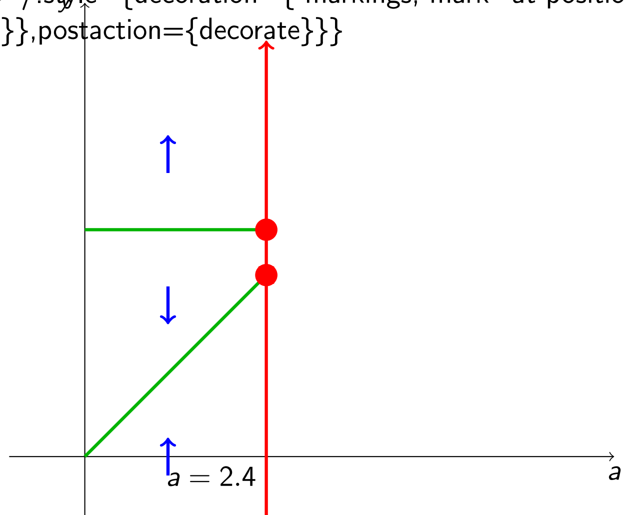

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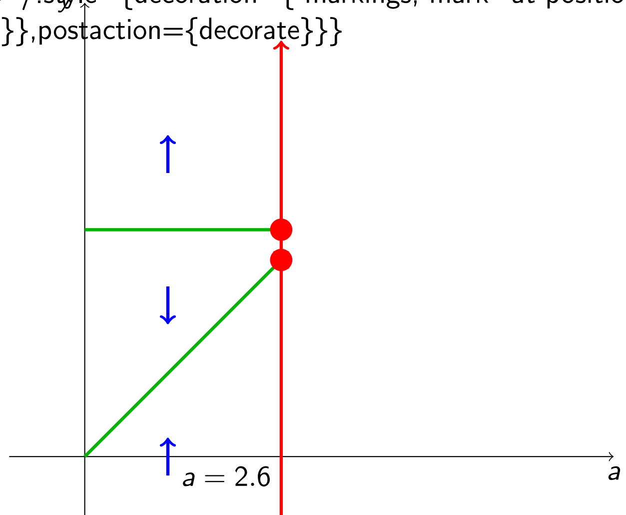

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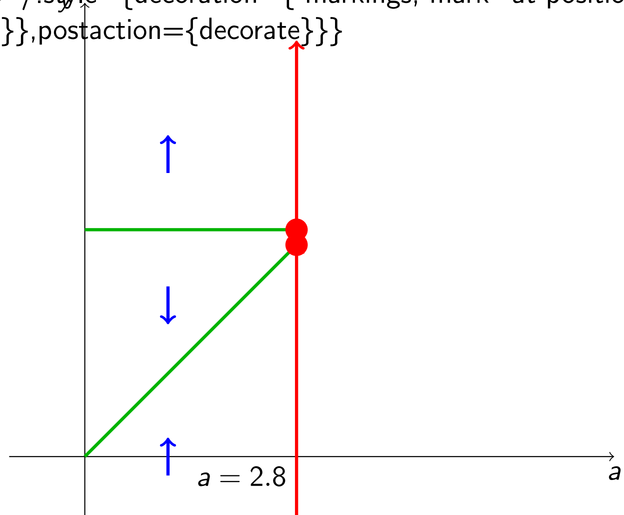

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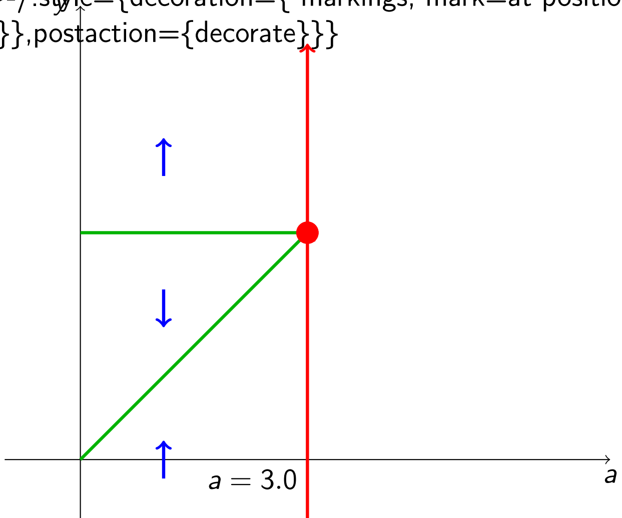

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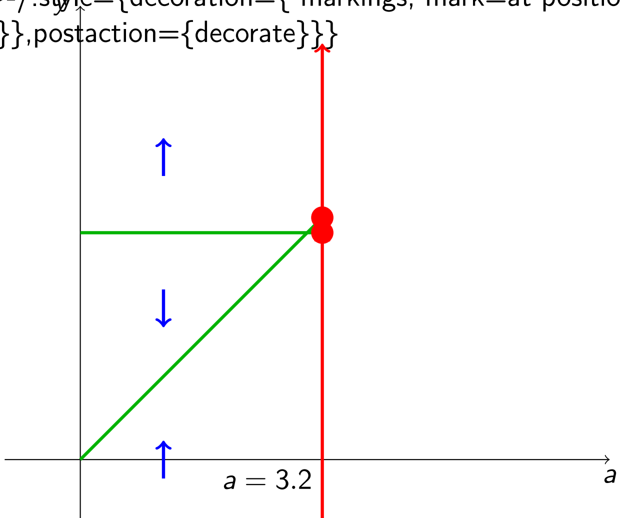

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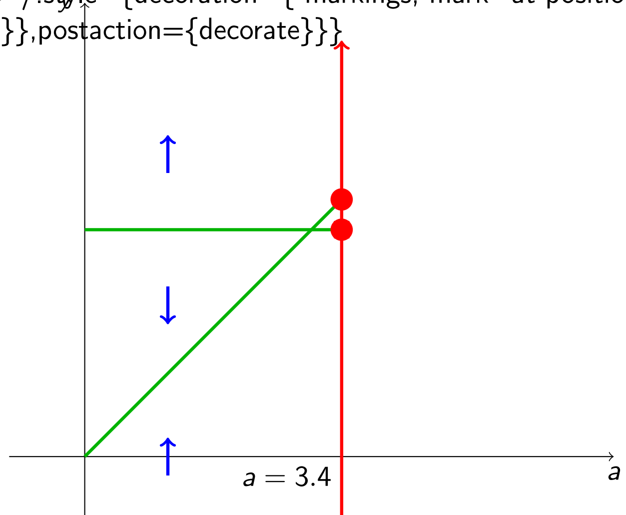

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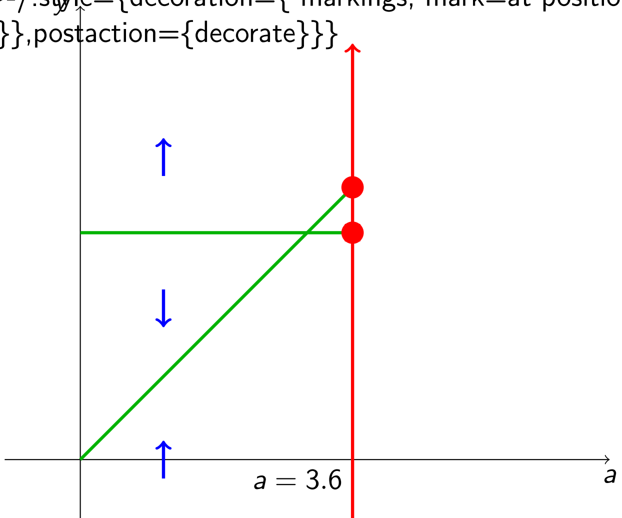

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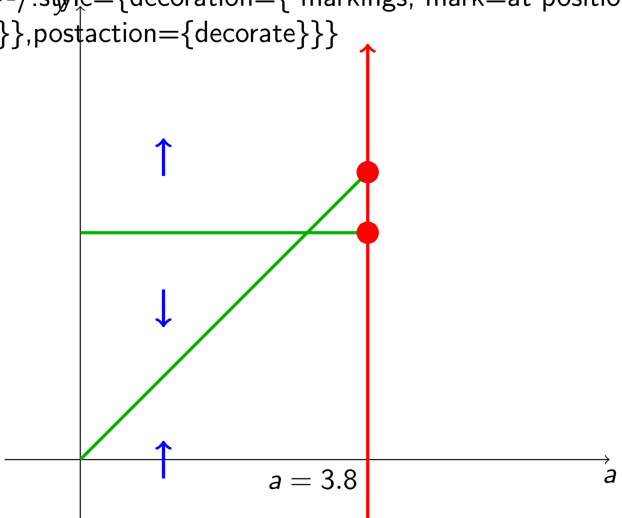

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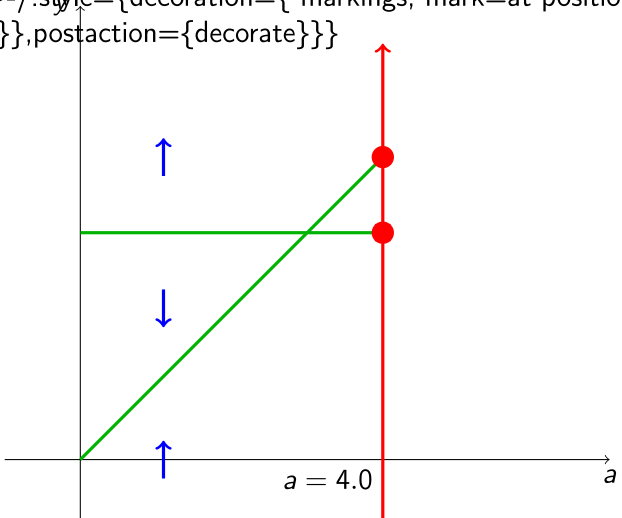

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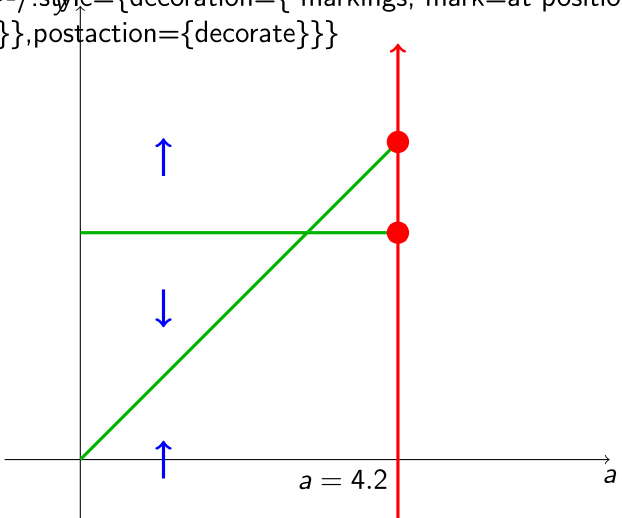

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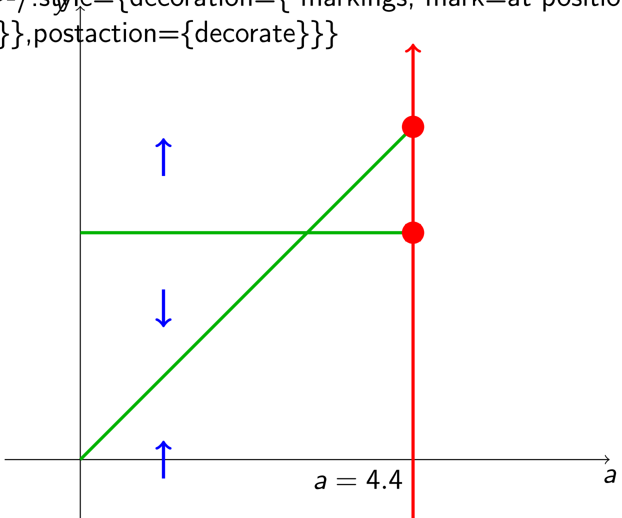

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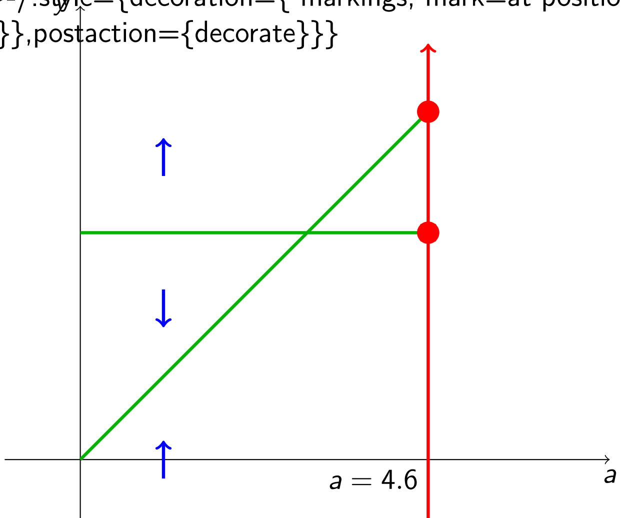

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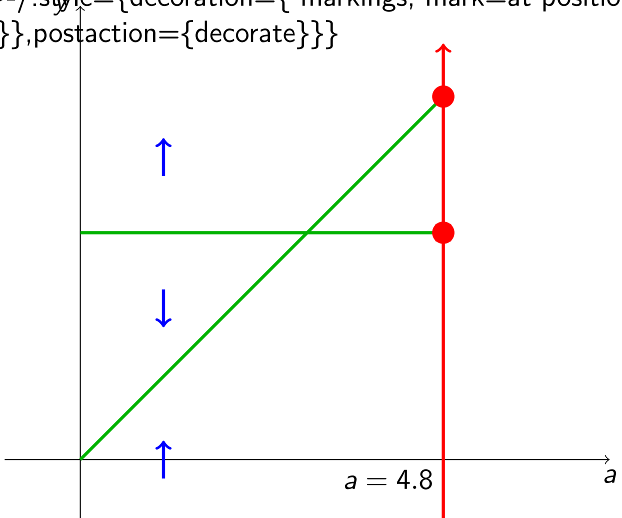

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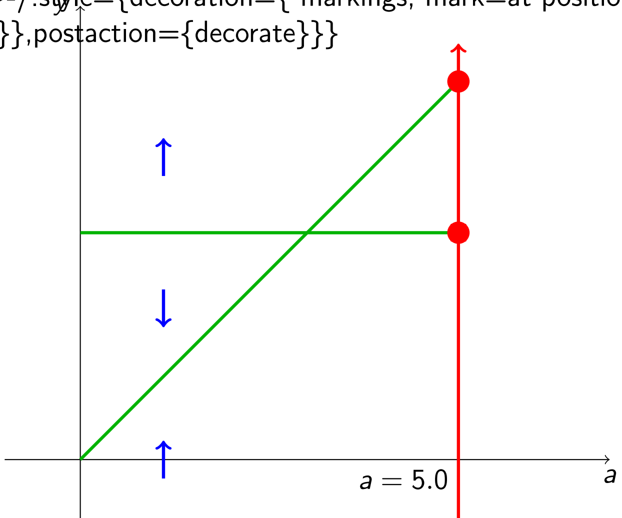

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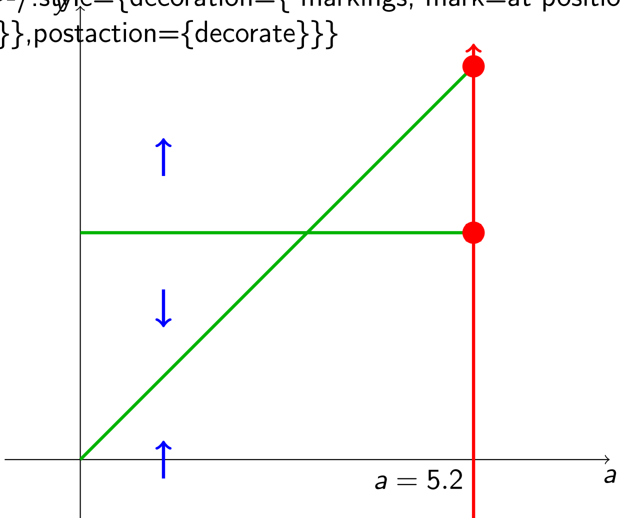

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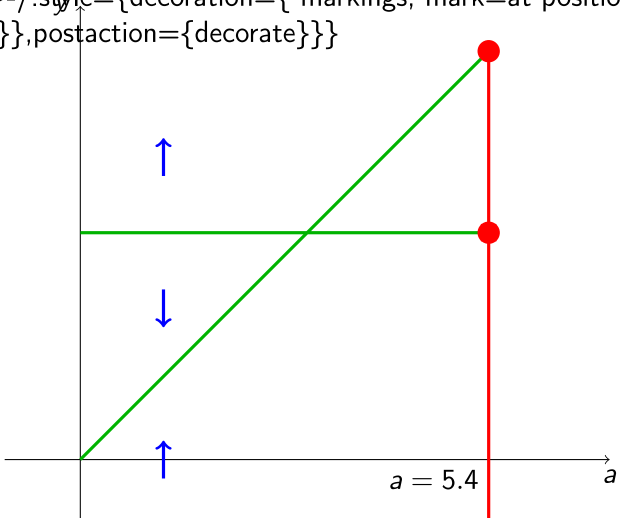

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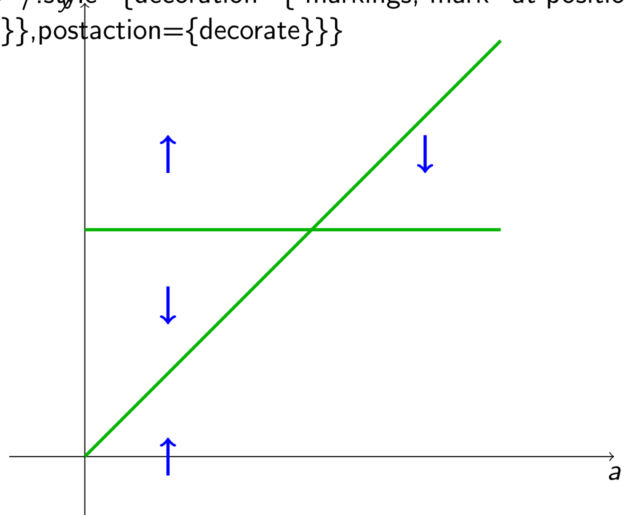

Recipe to draw the bifurcation diagram

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(a, y)$$

- Draw the axes for the *ay*-plane (*y* vertical axis)

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- Draw the axes for the ay-plane (y vertical axis)
- draw the points (a, y) such that $f(a, y) = 0$
- label the regions according to whether $f(a, y)$ is positive or negative.

Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = a - 3 + (2 - y)^2$$

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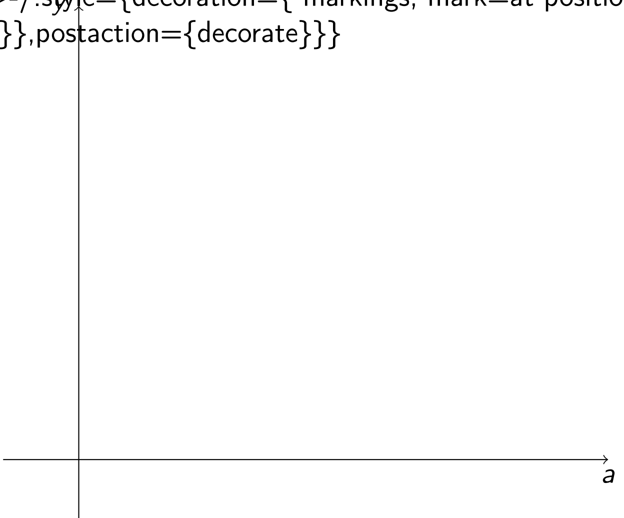

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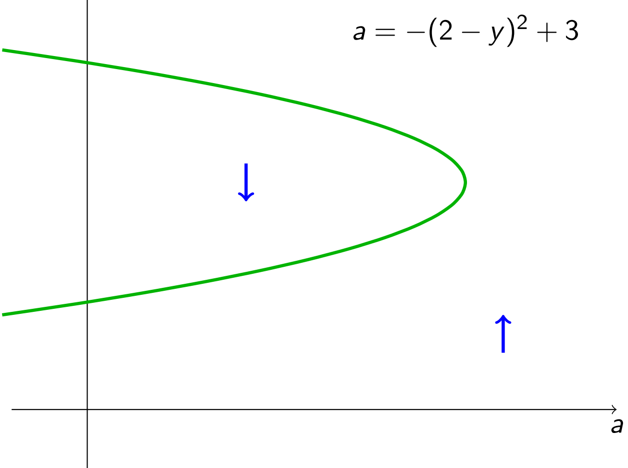

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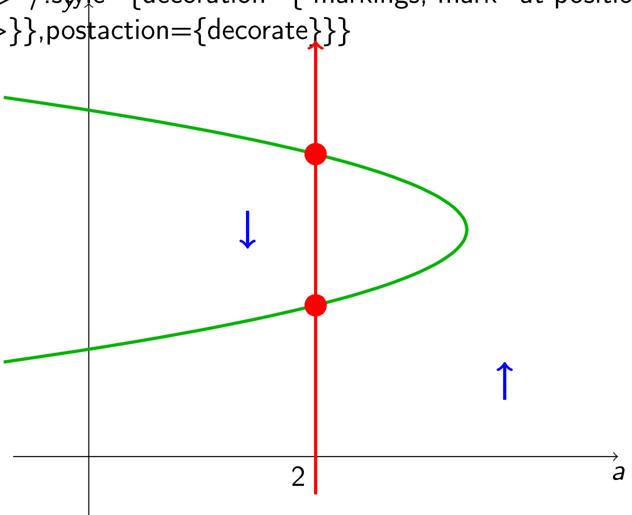

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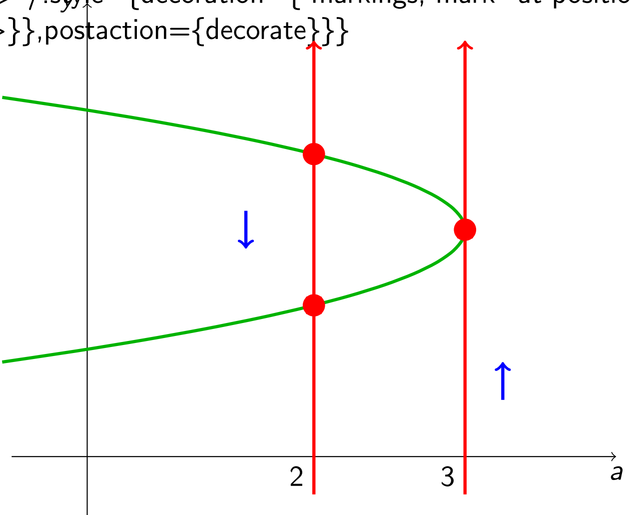

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