

This week you will get practice drawing and understanding bifurcation diagrams.

\*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

**Homework:** The homework will be due on Friday 11 March, at 8am, the *start* of the lecture. It will consist of

question 1, 2(e) from PS 9, and question 2 from PS 10.

1. (6.6) Sketch the bifurcation diagrams for the equations in the following.

- (a) (6.6-7)  $\frac{dy}{dt} = ay - y^2$
- (b) (6.6-10)  $\frac{dy}{dt} = 1 - ay^2$
- (c) (6.6-11)  $\frac{dy}{dt} = \sin y + a$
- (d) (6.6-12)  $\frac{dy}{dt} = y^2 - ay + 2$  for  $a > 0$ .

2. The fisheries management department is considering opening up an area for recreational fishing. The area has a population of abalone. The department would like to set an overall cap on the number of abalone that can be taken per year. It is known that the population of abalone grow logistically with a birth rate of 0.5 and a carrying capacity of 5000. If the department sets a cap of  $A$  individuals to be taken per year, it expects the actual number of abalone taken will be  $\pm 10\%$  of this. Prior to allowing harvesting, the population of abalone is in equilibrium at 5000 individuals. What is the maximum cap that the department could set, without endangering the population of abalone?

3. (6.6) Consider the model

$$\frac{dy}{dt} = \frac{ay^2}{k^2 + y^2} - cy$$

of an autocatalytic gene from question 5. In each of the following cases, two of the parameters are specified. Sketch a bifurcation diagram with respect to the third parameter.

- (a) (6.6-13)  $k = 1$ ,  $c = 2$  with  $a$  as the bifurcation parameter.
  - (b) (6.6-14)  $k = 2$ ,  $c = 1$  with  $a$  as the bifurcation parameter.
  - (c) (6.6-15)  $a = 10$ ,  $k = 1$  with  $c$  as the bifurcation parameter.
  - (d) (6.6-16)  $a = 10$ ,  $c = 1$  with  $k^2$  as the bifurcation parameter.
4. (6.6-40) Suppose the growth rate of a whale population at density  $N$  (individuals per million square kilometers of ocean), harvested at a rate  $h$ , is given by

$$\frac{dN}{dt} = 0.07N \left( \frac{N}{10} - 1 \right) \left( 1 - \frac{N}{80} \right) - h$$

where the units of  $t$  are years.

- (a) Sketch a bifurcation diagram with respect to the parameter  $h$  as it varies over the interval  $[0, 8]$ .
- (b) If  $h = vN$ , then sketch a bifurcation diagram with respect to the parameter  $v$  as it varies over the interval  $[0, 0.12]$ .