

Math 3B: Lecture 13

Noah White

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Problem set and last time

- Problem set 5 is online

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- Quiz 3 next week

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Last time

- Properties of integrals

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- Properties of integrals
- Integration by parts

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- Properties of integrals
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Today

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Last time

- Properties of integrals
- Integration by parts

Today

- Polynomial long division

How to factorize polynomials

The normal method for factorizing a polynomial $p(x)$ is to find a root α and then writing

$$p(x) = q(x)(x - \alpha).$$

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The normal method for factorizing a polynomial $p(x)$ is to find a root α and then writing

$$p(x) = q(x)(x - \alpha).$$

What if we want to divide a polynomial $p(x)$ by another polynomial $q(x)$? We want to write

$$p(x) = q(x)d(x) + r(x)$$

for a polynomial $d(x)$ (the **divisor**) and a **remainder** $r(x)$.

Long division

We know how to do this with numbers! Long division.

$$\begin{array}{r} 176 \\ 34 \overline{) 6000} \\ \underline{3400} \\ 2600 \\ \underline{2380} \\ 220 \\ \underline{204} \\ 16 \end{array}$$

Long division

We know how to do this with numbers! Long division.

$$\begin{array}{r} 176 \\ 34 \overline{) 6000} \\ \underline{3400} \\ 2600 \\ \underline{2380} \\ 220 \\ \underline{204} \\ 16 \end{array}$$

$$\text{So } 6000 = 34 \cdot 176 + 16$$

Why?

Lets rewrite the equation $p(x) = q(x)d(x) + r(x)$

$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

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E.g.

$$\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 1} = x + \frac{1}{(x + 1)^2}.$$

Why?

Lets rewrite the equation $p(x) = q(x)d(x) + r(x)$

$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

E.g.

$$\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 1} = x + \frac{1}{(x + 1)^2}.$$

The left hand side is difficult to integrate. The right hand side is easy!

How?

$$x + 3 \overline{) x^2 + 5x + 4}$$

How?

$$\begin{array}{r} x \\ x+3 \overline{) x^2 + 5x + 4} \end{array}$$

How?

$$\begin{array}{r} x \\ x+3 \overline{) x^2 + 5x + 4} \\ \underline{-x^2 - 3x} \end{array}$$

How?

$$\begin{array}{r} x \\ x+3 \overline{) x^2 + 5x + 4} \\ \underline{-x^2 - 3x} \\ 2x + 4 \end{array}$$

How?

$$\begin{array}{r} x+2 \\ x+3 \overline{) x^2+5x+4} \\ \underline{-x^2-3x} \\ 2x+4 \end{array}$$

How?

$$\begin{array}{r} x+2 \\ x+3 \overline{) x^2+5x+4} \\ \underline{-x^2-3x} \\ 2x+4 \\ \underline{-2x-6} \end{array}$$

How?

$$\begin{array}{r} x+2 \\ x+3 \overline{) x^2+5x+4} \\ \underline{-x^2-3x} \\ 2x+4 \\ \underline{-2x-6} \\ -2 \end{array}$$

How?

$$\begin{array}{r} x+2 \\ x+3 \overline{) x^2+5x+4} \\ \underline{-x^2-3x} \\ 2x+4 \\ \underline{-2x-6} \\ -2 \end{array}$$

So

$$\frac{x^2 + 5x + 4}{x + 3} = x + 2 - \frac{2}{x + 3}.$$

Example 1

$$x - 3 \overline{) x^3 - 12x^2 - 42}$$

Example 1

$$\begin{array}{r} x^2 \\ x-3 \overline{) x^3 - 12x^2 } \\ - 12x^2 \\ - 42 \end{array}$$

Example 1

$$\begin{array}{r} x^2 \\ x-3 \overline{) x^3 - 12x^2 - 42} \\ \underline{-x^3 + 3x^2} \end{array}$$

Example 1

$$\begin{array}{r} x^2 \\ x-3 \overline{) x^3 - 12x^2 } \\ \underline{-x^3 + 3x^2} \\ -9x^2 \end{array}$$

Example 1

$$\begin{array}{r} x^2 - 9x \\ x - 3 \overline{) x^3 - 12x^2 - 42} \\ \underline{- x^3 + 3x^2} \\ - 9x^2 \end{array}$$

Example 1

$$\begin{array}{r} x^2 - 9x \\ x - 3 \overline{) x^3 - 12x^2 - 42} \\ \underline{-x^3 + 3x^2} \\ -9x^2 - 42 \\ \underline{9x^2 - 27x} \\ -42 \end{array}$$

Example 1

$$\begin{array}{r} x^2 - 9x \\ x - 3 \overline{) x^3 - 12x^2 - 42} \\ \underline{-x^3 + 3x^2} \\ -9x^2 - 42 \\ \underline{9x^2 - 27x} \\ -27x - 42 \end{array}$$

Example 1

$$\begin{array}{r} x^2 - 9x - 27 \\ x - 3 \overline{) x^3 - 12x^2 - 42} \\ \underline{-x^3 + 3x^2} \\ -9x^2 - 42 \\ \underline{9x^2 - 27x} \\ -27x - 42 \end{array}$$

Example 1

$$\begin{array}{r}
 x^2 - 9x - 27 \\
 x-3 \overline{) x^3 - 12x^2 - 42} \\
 \underline{-x^3 + 3x^2} \\
 -9x^2 \\
 \underline{9x^2 - 27x} \\
 -27x - 42 \\
 \underline{27x} \\
 -81
 \end{array}$$

Example 1

$$\begin{array}{r}
 x^2 - 9x - 27 \\
 x-3 \overline{) x^3 - 12x^2 } \\
 \underline{-x^3 + 3x^2} \\
 -9x^2 \\
 \underline{9x^2 - 27x} \\
 -27x - 42 \\
 \underline{27x - 81} \\
 -123
 \end{array}$$

Example 1

$$\begin{array}{r} x^2 - 9x - 27 \\ x-3 \overline{) x^3 - 12x^2 } \\ \underline{-x^3 + 3x^2} \\ -9x^2 \\ \underline{9x^2 - 27x} \\ -27x - 42 \\ \underline{27x - 81} \\ -123 \end{array}$$

So

$$\frac{x^3 - 12x^2 - 42}{x - 3} = x^2 - 9x - 27 - \frac{123}{x - 3}.$$

Example 2

$$x^2 + 1 \overline{) x^3 - x^2 + x - 1}$$

Example 2

$$x^2 + 1 \overline{) \begin{array}{r} x^3 - x^2 + x - 1 \end{array}}$$

Example 2

$$\begin{array}{r} x \\ \hline x^2 + 1) x^3 - x^2 + x - 1 \\ - x^3 - x \\ \hline x - 1 \end{array}$$

Example 2

$$\begin{array}{r} x^2 + x - 1 \\ x^2 + 1 \overline{) x^3 - x^2 + x - 1} \\ \underline{- x^3} - x - 1 \\ - x - 1 \end{array}$$

Example 2

$$\begin{array}{r} x - 1 \\ \hline x^2 + 1) x^3 - x^2 + x - 1 \\ \underline{- x^3} - x \\ - x^2 - 1 \\ - 1 \end{array}$$

Example 2

$$\begin{array}{r} x - 1 \\ \hline x^2 + 1) x^3 - x^2 + x - 1 \\ \underline{- x^3} - x \\ - x^2 \\ \underline{x^2} \\ - 1 \\ \underline{+ 1} \end{array}$$

Example 2

$$\begin{array}{r} x - 1 \\ \hline x^2 + 1) x^3 - x^2 + x - 1 \\ \underline{- x^3} - x \\ - x^2 - 1 \\ \underline{+ 1} \\ 0 \end{array}$$

Example 2

$$\begin{array}{r} x - 1 \\ \hline x^2 + 1) x^3 - x^2 + x - 1 \\ \underline{- x^3} - x \\ - x^2 - 1 \\ \underline{x^2} + 1 \\ \underline{ 0} \end{array}$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

Example 3

$$x^2 + x + 1 \overline{) x^3 - 1}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) x^3 } \\ \underline{x^3 } \\ 0x^2 + 0x + 0 \\ \underline{0x^2 + 0x + 0} \\ 0x + 0 \\ \underline{0x + 0} \\ 0 \end{array}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) \begin{array}{r} x^3 - 1 \\ - x^3 - x^2 - x \\ \hline \end{array}} \end{array}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) \begin{array}{r} x^3 \\ - x^3 - x^2 - x \\ \hline - x^2 - x - 1 \end{array}} \end{array}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) \begin{array}{r} x^3 \\ - x^3 - x^2 - x \\ \hline - x^2 - x - 1 \end{array}} \end{array}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) \begin{array}{r} x^3 \\ - x^3 - x^2 - x \\ \hline - x^2 - x - 1 \\ x^2 + x + 1 \\ \hline \end{array}} \end{array}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) \begin{array}{r} x^3 \\ - x^3 - x^2 - x \\ \hline - x^2 - x - 1 \\ x^2 + x + 1 \\ \hline 0 \end{array}} \end{array}$$

Example 3

$$\begin{array}{r} x^2 + x + 1 \overline{) \begin{array}{r} x^3 \\ - x^3 - x^2 - x \\ \hline - x^2 - x - 1 \\ x^2 + x + 1 \\ \hline 0 \end{array}} \end{array}$$

So

$$\frac{x^3 - 1}{x^2 + x + 1} = x - 1.$$

Example 4

$$3x - 1 \overline{) 2x^3 - 4x^2 + 1}$$

Example 4

$$\begin{array}{r} \frac{2}{3}x^2 \\ 3x-1\overline{) 2x^3 - 4x^2 + 1} \end{array}$$

Example 4

$$\begin{array}{r} - \frac{2}{3}x^2 \\ \underline{ 2x^3 - 4x^2} \\ -2x^3 + \frac{2}{3}x^2 \end{array}$$

Example 4

$$\begin{array}{r} \frac{2}{3}x^2 \\ 3x-1) \\ \underline{-2x^3 + \frac{2}{3}x^2} \\ -\frac{10}{3}x^2 \end{array}$$

Example 4

$$\begin{array}{r} \phantom{3x-1) } \frac{2}{3}x^2 - \frac{10}{9}x \\ \hline 3x-1) 2x^3 - 4x^2 \\ \phantom{3x-1) } - 2x^3 + \frac{2}{3}x^2 \\ \hline \phantom{3x-1) } -\frac{10}{3}x^2 \end{array}$$

Example 4

$$\begin{array}{r} \frac{2}{3}x^2 - \frac{10}{9}x \\ 3x - 1 \overline{) 2x^3 - 4x^2 + 1} \\ \underline{- 2x^3 + \frac{2}{3}x^2} \\ -\frac{10}{3}x^2 \\ \underline{ \frac{10}{3}x^2 - \frac{10}{9}x} \end{array}$$

Example 4

$$\begin{array}{r}
 \frac{2}{3}x^2 - \frac{10}{9}x \\
 \hline
 3x - 1 \bigg) \quad 2x^3 - 4x^2 \qquad \qquad + 1 \\
 \underline{- 2x^3 + \frac{2}{3}x^2} \qquad \qquad \qquad \\
 \qquad \qquad - \frac{10}{3}x^2 \\
 \qquad \qquad \underline{\frac{10}{3}x^2 - \frac{10}{9}x} \\
 \qquad \qquad \qquad - \frac{10}{9}x + 1
 \end{array}$$

Example 4

$$\begin{array}{r} \frac{2}{3}x^2 - \frac{10}{9}x - \frac{10}{27} \\ 3x-1) + 1 \\ \underline{-2x^3 + \frac{2}{3}x^2} \\ -\frac{10}{3}x^2 \\ \underline{\frac{10}{3}x^2 - \frac{10}{9}x} \\ \phantom{\frac{10}{3}x^2 - \frac{10}{9}x} -\frac{10}{9}x + 1 \end{array}$$

Example 4

$$\begin{array}{r}
 \frac{2}{3}x^2 - \frac{10}{9}x - \frac{10}{27} \\
 \hline
 3x - 1 \big) \quad 2x^3 - 4x^2 \qquad \qquad + 1 \\
 \underline{- 2x^3 + \frac{2}{3}x^2} \qquad \qquad \qquad \\
 \qquad \qquad \qquad - \frac{10}{3}x^2 \\
 \qquad \qquad \qquad \underline{\frac{10}{3}x^2 - \frac{10}{9}x} \\
 \qquad \qquad \qquad \qquad \qquad - \frac{10}{9}x + 1 \\
 \qquad \qquad \qquad \qquad \qquad \underline{\frac{10}{9}x - \frac{10}{27}}
 \end{array}$$

$$\begin{array}{r}
 \frac{2}{3}x^2 - \frac{10}{9}x - \frac{10}{27} \\
 \hline
 3x - 1) \quad 2x^3 - 4x^2 \qquad \qquad + 1 \\
 \quad - 2x^3 + \frac{2}{3}x^2 \\
 \quad \hline
 \qquad - \frac{10}{3}x^2 \\
 \qquad \quad \frac{10}{3}x^2 - \frac{10}{9}x \\
 \qquad \quad \hline
 \qquad \qquad - \frac{10}{9}x + 1 \\
 \qquad \qquad \quad \frac{10}{9}x - \frac{10}{27} \\
 \qquad \qquad \quad \hline
 \qquad \qquad \qquad \frac{17}{27}
 \end{array}$$

Example 4

$$\begin{array}{r}
 \frac{2}{3}x^2 - \frac{10}{9}x - \frac{10}{27} \\
 \hline
 3x - 1 \big) \quad 2x^3 - 4x^2 \qquad \qquad + 1 \\
 \underline{- 2x^3 + \frac{2}{3}x^2} \qquad \qquad \qquad \\
 \qquad \qquad - \frac{10}{3}x^2 \\
 \qquad \qquad \underline{\frac{10}{3}x^2 - \frac{10}{9}x} \\
 \qquad \qquad \qquad \qquad - \frac{10}{9}x + 1 \\
 \qquad \qquad \qquad \qquad \underline{\frac{10}{9}x - \frac{10}{27}} \\
 \qquad \qquad \qquad \qquad \qquad \frac{17}{27}
 \end{array}$$

So

$$\frac{2x^3 - 4x^2 + 1}{3x - 1} = \frac{2}{3} \left(x^2 - \frac{5}{3}x - \frac{5}{9} \right) - \frac{17}{27(3x - 1)}.$$

Example 5

$$\begin{array}{r} x^2 - 2x + 5 \overline{) x^4 - x^2 - 4} \end{array}$$

Example 5

$$\begin{array}{r} x^2 - 2x + 5 \overline{) x^4 - x^2 + x - 4} \\ x^4 \\ \hline - x^2 + x - 4 \end{array}$$

Example 5

$$\begin{array}{r} x^2 \\ x^2 - 2x + 5 \overline{) \quad x^4 \quad \quad - x^2 \quad + x \quad - 4} \\ \underline{-x^4 + 2x^3 - 5x^2} \end{array}$$

Example 5

$$\begin{array}{r} \phantom{x^2 - 2x + 5) } x^2 \\ x^2 - 2x + 5 \overline{) } \\ \underline{- x^4 + 2x^3 - 5x^2} \\ 2x^3 - 6x^2 \end{array}$$

Example 5

$$\begin{array}{r} x^2 - 2x + 5 \overline{) \begin{array}{r} x^4 - x^2 + x - 4 \\ - x^4 + 2x^3 - 5x^2 \\ \hline 2x^3 - 6x^2 + x \end{array}} \end{array}$$

Example 5

$$\begin{array}{r} x^2 - 2x + 5 \overline{) \begin{array}{r} x^4 - x^2 + x - 4 \\ - x^4 + 2x^3 - 5x^2 \\ \hline 2x^3 - 6x^2 + x \\ - 2x^3 + 4x^2 - 10x \\ \hline \end{array}} \end{array}$$

Example 5

$$\begin{array}{r}
 x^2 2x \\
 x^2 - 2x + 5 \overline{) x^4 x^2 x 4} \\
 \underline{ x^4 2x^3 5x^2} x 4 \\
 2x^3 6x^2 x 4 \\
 \underline{ 2x^3 4x^2 10x} 4 \\
 - 2x^2 9x 4
 \end{array}$$

Example 5

$$\begin{array}{r} \phantom{x^2 - 2x + 5) } \\ \phantom{x^2 - 2x + 5) } \\ \hline x^2 - 2x + 5) \\ \phantom{x^2 - 2x + 5) } x^4 \\ \phantom{x^2 - 2x + 5) } - x^4 + 2x^3 - 5x^2 \\ \hline \phantom{x^2 - 2x + 5) } 2x^3 - 6x^2 \\ \phantom{x^2 - 2x + 5) } - 2x^3 + 4x^2 - 10x \\ \hline \phantom{x^2 - 2x + 5) } - 2x^2 - 9x - 4 \end{array}$$

Example 5

$$\begin{array}{r} \phantom{x^2 - 2x + 5) } \\ \phantom{x^2 - 2x + 5) } \\ \hline x^2 - 2x + 5) \\ \phantom{x^2 - 2x + 5) } x^4 \\ \phantom{x^2 - 2x + 5) } - x^4 + 2x^3 - 5x^2 \\ \hline \phantom{x^2 - 2x + 5) } 2x^3 - 6x^2 \\ \phantom{x^2 - 2x + 5) } - 2x^3 + 4x^2 - 10x \\ \hline \phantom{x^2 - 2x + 5) } - 2x^2 - 9x - 4 \\ \phantom{x^2 - 2x + 5) } 2x^2 - 4x + 10 \\ \hline \end{array}$$

Example 5

$$\begin{array}{r} x^2 + 2x - 2 \\ x^2 - 2x + 5 \overline{) x^4 } \\ \underline{- x^4 + 2x^3 - 5x^2} \\ 2x^3 - 6x^2 \\ \underline{- 2x^3 + 4x^2 - 10x} \\ - 2x^2 - 9x - 4 \\ 2x^2 - 4x + 10 \\ \underline{} \\ - 13x + 6 \end{array}$$

Example 5

$$\begin{array}{r} \\ x^2 - 2x + 5 \overline{) } \\ \underline{- x^4 + 2x^3 - 5x^2} \\ 2x^3 - 6x^2 \\ \underline{- 2x^3 + 4x^2 - 10x} \\ - 2x^2 - 9x - 4 \\ 2x^2 - 4x + 10 \\ \underline{ - 13x + 6} \end{array}$$

So

$$\frac{x^4 - x^2 + x - 4}{x^2 - 2x + 5} = x^2 + 2x - 2 + \frac{-13x + 6}{x^2 - 2x + 5}.$$