Math 3B: Lecture 10

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February 3, 2017

Introduction

Last time

- Properties of the definite integral

This time

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Last time

- Properties of the definite integral
- The fundamental theorem of calculus

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- Properties of the definite integral
- The fundamental theorem of calculus
- Substitution

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- Properties of the definite integral
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- Integration by parts

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Last time

- Properties of the definite integral
- The fundamental theorem of calculus
- Substitution

This time

- Integration by parts
- Polynomial long division

Midterm discussion

- Average score was 21.5.

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Grade	Range
A	26+
B	21-25
C	14-20

Example

Question

Find an antiderivative of $f(x) = |x|$?

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Solution

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Solution

- The FTC tells us that

$$F(x) = \int_a^x f(t)\,\mathrm{d}t$$

  is an antiderivative for any choice of a.

Example

Question

Find an antiderivative of $f(x) = |x|$?

Solution

- The FTC tells us that

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  is an antiderivative for any choice of a.
- Lets use $a = 0$.

Example

Question

Find an antiderivative of $f(x) = |x|$?

Solution

- The FTC tells us that

  $$F(x) = \int_a^x f(t)\,\mathrm{d}t$$

  is an antiderivative for any choice of a.
- Lets use $a = 0$.
- How should we calculate $F(x)$?

Example

Use the defintition!

$$F(x) = \int_0^x |t| \ \mathrm{d}t$$

is the area under $|t|$!

`./abs.png`

Example

If $x \leq 0$ then

$$F(x) = \int_0^x |t| \ \mathrm{d}t = -\int_x^0 |t| \ \mathrm{d}t$$

is the negative of the area under $|t|$!

`./abs2.png`

Example

In summary

$$F(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x \geq 0 \\ -\frac{1}{2}x^2 & \text{if } x \leq 0 \end{cases}$$

or

$$F(x) = \frac{1}{2}x\,|x|$$

The product rule

Just like integration by substitution reverses the chain rule, integration by parts "reverses" the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

The product rule

Just like integration by substitution reverses the chain rule, integration by parts "reverses" the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

writen another way

$$(uv)' = u'v + uv'$$

Integration by parts

$$(uv)' = u'v + uv'$$

Integration by parts

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Lets integrate both sides

$$\int (uv)' \, \mathrm{d}x = \int u'v \, \mathrm{d}x + \int uv' \, \mathrm{d}x$$

Integration by parts

$$(uv)' = u'v + uv'$$

Lets integrate both sides

$$\int (uv)' \,\mathrm{d}x = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

By the fundamental theorem of calculus

$$uv = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

Integration by parts

$$(uv)' = u'v + uv'$$

Lets integrate both sides

$$\int (uv)' \,\mathrm{d}x = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

By the fundamental theorem of calculus

$$uv = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

Rearranging. . .

Integration by parts

The integration by parts formula

$$\int uv' \,\mathrm{d}x = uv - \int u'v \,\mathrm{d}x$$

Integration by parts

The integration by parts formula

$$\int uv' \, \mathrm{d}x = uv - \int u'v \, \mathrm{d}x$$

Alternative statement

$$\int u \, \mathrm{d}v = uv - \int v \, \mathrm{d}u$$

Examples

One the board. . .

How to factorize polynomials

The normal method for factorizing a polynomial $p(x)$ is to find a root α and then writing

$$p(x) = q(x)(x - \alpha).$$

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The normal method for factorizing a polynomial $p(x)$ is to find a root α and then writing

$$p(x) = q(x)(x - \alpha).$$

What if we want to devide a polynomial $p(x)$ by another polynomial $q(x)$? We want to write

$$p(x) = q(x)d(x) + r(x)$$

for a polynomial $d(x)$ (the devisor) and a remainder $r(x)$.

Long devision

We know how to do this with numbers! Long devision.

$$\begin{array}{r}
176 \\
34\,\overline{\smash{)}6000} \\
\underline{3400} \\
2600 \\
\underline{2380} \\
220 \\
\underline{204} \\
16
\end{array}$$

Long devision

We know how to do this with numbers! Long devision.

$$\begin{array}{r} 176 \\ 34\,\overline{\smash{)}6000} \\ \underline{3400} \\ 2600 \\ \underline{2380} \\ 220 \\ \underline{204} \\ 16 \end{array}$$

So $6000 = 34 \cdot 176 + 16$

Why?

Lets rewrite the equation $p(x) = q(x)d(x) + r(x)$

$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

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E.g.

$$\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 1} = x + \frac{1}{(x+1)^2}.$$

Why?

Lets rewrite the equation $p(x) = q(x)d(x) + r(x)$

$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

E.g.

$$\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 1} = x + \frac{1}{(x+1)^2}.$$

The left hand side is difficult to integrate. The right hand side is easy!

How?

$$x + 3\,\overline{)\quad x^2 + 5x + 4}$$

So

$$\frac{x^2 + 5x + 4}{x + 3} = x + 2 - \frac{2}{x + 3}.$$

}

How?

$$\begin{array}{r} x \\ x+3\overline{\smash{\big)}\quad x^2+5x+4} \end{array}$$

So

$$\frac{x^2+5x+4}{x+3} = x+2-\frac{2}{x+3}.$$

}

How?

$$\begin{array}{rl} & x\phantom{{}+4} \\ x+3\big) & \overline{x^2+5x+4} \\ & \underline{-x^2-3x}\phantom{{}+4} \end{array}$$

So

$$\frac{x^2+5x+4}{x+3} = x+2-\frac{2}{x+3}.$$

}

How?

$$\begin{array}{rrrr}
 & & x & \\ \cline{2-4}
x+3\big) & x^2 & +5x & +4 \\
 & -x^2 & -3x & \\ \cline{2-3}
 & & 2x & +4
\end{array}$$

So

$$\frac{x^2+5x+4}{x+3} = x+2-\frac{2}{x+3}.$$

}

How?

$$\begin{array}{rl}
 & x + 2 \\
\cline{2-2}
x + 3 \big) & x^2 + 5x + 4 \\
 & \underline{- x^2 - 3x} \\
 & 2x + 4
\end{array}$$

So

$$\frac{x^2 + 5x + 4}{x + 3} = x + 2 - \frac{2}{x + 3}.$$

}

How?

$$\begin{array}{rl} & x+2 \\ x+3\big) & \overline{\;x^2+5x+4} \\ & \underline{-x^2-3x} \\ & 2x+4 \\ & \underline{-2x-6} \end{array}$$

So

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How?

$$\begin{array}{rr} & x+2 \\ \cline{2-2} x+3\big) & x^2+5x+4 \\ & -x^2-3x \\ \cline{2-2} & 2x+4 \\ & -2x-6 \\ \cline{2-2} & -2 \end{array}$$

So

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How?

$$\begin{array}{rr} & x+2 \\ \cline{2-2} x+3\big) & x^2+5x+4 \\ & \underline{-x^2-3x} \\ & 2x+4 \\ & \underline{-2x-6} \\ & -2 \end{array}$$

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Example 1

$$x-3\,\overline{\big)\quad x^3-12x^2\qquad\qquad -42}$$

So

$$\frac{x^3-12x^2-42}{x-3}=x^2-9x-27-\frac{123}{x-3}.$$

}

Example 1

$$\begin{array}{r l}
 & \qquad\qquad x^2 \\ \cline{2-2}
x - 3 \big) & x^3 - 12x^2 \qquad\qquad - 42
\end{array}$$

So

$$\frac{x^3 - 12x^2 - 42}{x - 3} = x^2 - 9x - 27 - \frac{123}{x - 3}.$$

}

Example 1

$$
\begin{array}{r|llll}
 & & x^2 & & \\ \cline{2-5}
x-3 & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
\end{array}
$$

So

$$\frac{x^3-12x^2-42}{x-3} = x^2-9x-27-\frac{123}{x-3}.$$

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Example 1

$$\begin{array}{r l} & x^2 \\ \cline{2-2} x-3\big) & x^3 - 12x^2 \qquad\qquad - 42 \\ & -x^3 + 3x^2 \\ \cline{2-2} & -9x^2 \end{array}$$

So

$$\frac{x^3 - 12x^2 - 42}{x-3} = x^2 - 9x - 27 - \frac{123}{x-3}.$$

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$$\begin{array}{rrrrr}
 & & x^2 & -9x & \\ \cline{2-5}
x-3\big) & x^3 & -12x^2 & & -42 \\
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\multicolumn{1}{r}{} & & x^2 & -9x & \\
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\cline{2-3}
\multicolumn{1}{r}{} & & -9x^2 & & \\
\multicolumn{1}{r}{} & & 9x^2 & -27x & \\
\cline{3-4}
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 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\ \cline{3-4}
 & & & -27x & -42 \\
 & & & 27x & -81 \\ \cline{4-5}
\end{array}
$$

So

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 & -\,x^3 & +\,3x^2 & & \\ \cline{2-3}
 & & -\,9x^2 & & \\
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\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & x^2 & -9x & -27 \\ \cline{2-5}
x-3 & x^3 & -12x^2 & & -42 \\
\multicolumn{1}{r}{} & -x^3 & +3x^2 & & \\ \cline{2-3}
\multicolumn{1}{r}{} & & -9x^2 & & \\
\multicolumn{1}{r}{} & & 9x^2 & -27x & \\ \cline{3-4}
\multicolumn{1}{r}{} & & & -27x & -42 \\
\multicolumn{1}{r}{} & & & 27x & -81 \\ \cline{4-5}
\multicolumn{1}{r}{} & & & & -123
\end{array}
$$

{ So

$$\frac{x^3 - 12x^2 - 42}{x-3} = x^2 - 9x - 27 - \frac{123}{x-3}.$$

}

Example 2

$$x^2 + 1\big)\overline{\quad x^3 - x^2 + x - 1}$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

}

Example 2

$$\begin{array}{r} x \\ x^2+1\overline{)\quad x^3-x^2+x-1} \end{array}$$

So

$$\frac{x^3-x^2+x-1}{x^2+1}=x-1.$$

}

Example 2

$$
\begin{array}{rl}
 & \phantom{x^3 - x^2 + {}}x \\
x^2+1 \big) & \overline{\;x^3 - x^2 + x - 1} \\
 & \underline{-x^3 \qquad\quad -x\phantom{{}-1}}
\end{array}
$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

}

Example 2

$$\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & & x & \\ \cline{2-5}
x^2+1) & x^3 & -x^2 & +x & -1 \\
\multicolumn{1}{r}{} & -x^3 & & -x & \\ \cline{2-4}
\multicolumn{1}{r}{} & & -x^2 & & -1
\end{array}$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

}

Example 2

$$\begin{array}{rrrrr}
 & & & x & -1 \\ \cline{2-5}
x^2+1\big) & x^3 & -x^2 & +x & -1 \\
 & -x^3 & & -x & \\ \cline{2-4}
 & & -x^2 & & -1
\end{array}$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

}

Example 2

$$
\begin{array}{rrrrr}
 & & & x & -\,1 \\ \cline{2-5}
x^2+1\big) & x^3 & -\,x^2 & +\,x & -\,1 \\
 & -\,x^3 & & -\,x & \\ \cline{2-4}
 & & -\,x^2 & & -\,1 \\
 & & x^2 & & +\,1 \\ \cline{3-5}
\end{array}
$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

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Example 2

$$
\begin{array}{rrrrr}
 & & & x & -\,1 \\ \cline{2-5}
x^2+1\big) & x^3 & -\,x^2 & +\,x & -\,1 \\
 & -\,x^3 & & -\,x & \\ \cline{2-4}
 & & -\,x^2 & & -\,1 \\
 & & x^2 & & +\,1 \\ \cline{3-5}
 & & & & 0
\end{array}
$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

}

Example 2

$$\begin{array}{rl}
 & \phantom{-x^3-x^2+{}}x-1\\
\cline{2-2}
x^2+1 \big) & x^3-x^2+x-1\\
 & \underline{-x^3\phantom{{}-x^2}-x\phantom{{}-1}}\\
 & -x^2\phantom{{}+x}-1\\
 & \underline{\phantom{x^3-{}}x^2\phantom{{}+x}+1}\\
 & \phantom{x^3-x^2+x-{}}0
\end{array}$$

{ So

$$\frac{x^3-x^2+x-1}{x^2+1}=x-1.$$

}

Example 3

$$x^2+x+1\overline{\smash{)}\;x^3 \qquad\qquad\qquad -1}$$

So

$$\frac{x^3-1}{x^2+x+1}=x-1.$$

}

Example 3

$$\begin{array}{r} x \\ x^2+x+1\overline{)\;\; x^3 \qquad\qquad\qquad -1} \end{array}$$

So

$$\frac{x^3-1}{x^2+x+1} = x-1.$$

}

Example 3

$$\begin{array}{rl} & \phantom{-x^3 - x^2 - {}}x \\ \cline{2-2} x^2+x+1\big) & x^3 \qquad\qquad\quad -1 \\ & -x^3 - x^2 - x \\ \cline{2-2} \end{array}$$

So

$$\frac{x^3-1}{x^2+x+1} = x-1.$$

}

Example 3

$$
\begin{array}{rrrrr}
 & & & x & \\ \cline{2-5}
x^2+x+1 \big) & x^3 & & & -1 \\
 & -x^3 & -x^2 & -x & \\ \cline{2-4}
 & & -x^2 & -x & -1
\end{array}
$$

So

$$\frac{x^3-1}{x^2+x+1} = x-1.$$

}

Example 3

$$
\begin{array}{rrrrr}
 & & & x & -\,1 \\ \cline{2-5}
x^2+x+1\big) & x^3 & & & -\,1 \\
 & -\,x^3 & -\,x^2 & -\,x & \\ \cline{2-4}
 & & -\,x^2 & -\,x & -\,1
\end{array}
$$

So

$$\frac{x^3-1}{x^2+x+1}=x-1.$$

}

Example 3

$$
\begin{array}{r@{}l}
 & \hphantom{x^3-x^2-}\,x-1 \\
x^2+x+1\big) & \overline{\;x^3 \qquad\qquad\qquad -1} \\
 & \underline{-x^3-x^2-x\qquad} \\
 & \qquad\;\; -x^2-x-1 \\
 & \qquad\;\; \underline{\;\;\;x^2+x+1}
\end{array}
$$

So

$$\frac{x^3-1}{x^2+x+1} = x-1.$$

}

Example 3

$$
\begin{array}{rr}
 & x - 1 \\ \cline{2-2}
x^2 + x + 1 \big) & x^3 \qquad\qquad - 1 \\
 & - x^3 - x^2 - x \qquad \\ \cline{2-2}
 & - x^2 - x - 1 \\
 & x^2 + x + 1 \\ \cline{2-2}
 & 0
\end{array}
$$

So

$$\frac{x^3 - 1}{x^2 + x + 1} = x - 1.$$

}

Example 3

$$
\begin{array}{r r}
 & x - 1 \\ \cline{2-2}
x^2 + x + 1 \big) & x^3 \qquad\qquad\qquad - 1 \\
 & \underline{-x^3 - x^2 - x \qquad} \\
 & -x^2 - x - 1 \\
 & \underline{x^2 + x + 1} \\
 & 0
\end{array}
$$

{ So

$$\frac{x^3 - 1}{x^2 + x + 1} = x - 1.$$

}

Example 4

$$3x-1\overline{\big)\;2x^3 \quad -4x^2 \qquad\qquad +1}$$

So

$$\frac{2x^3-4x^2+1}{3x-1} = \frac{2}{3}\left(x^2-\frac{5}{3}x-\frac{5}{9}\right) - \frac{17}{27(3x-1)}.$$

}

Example 4

$$\begin{array}{r l} & \qquad\qquad \tfrac{2}{3}x^2 \\ \cline{2-2} 3x-1 \big) & \; 2x^3 \quad -4x^2 \qquad\qquad\quad +1 \end{array}$$

So

$$\frac{2x^3-4x^2+1}{3x-1} = \frac{2}{3}\left(x^2-\frac{5}{3}x-\frac{5}{9}\right)-\frac{17}{27(3x-1)}.$$

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\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & & \\
\cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
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\cline{2-3}
\end{array}
$$

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}

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$$
\begin{array}{rllll}
 & & \frac{2}{3}x^2 & & \\ \cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & &
\end{array}
$$

So

$$\frac{2x^3-4x^2+1}{3x-1}=\frac{2}{3}\left(x^2-\frac{5}{3}x-\frac{5}{9}\right)-\frac{17}{27(3x-1)}.$$

}

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & \\
\cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\
\cline{2-3}
 & & -\frac{10}{3}x^2 & &
\end{array}
$$

So

$$\frac{2x^3 - 4x^2 + 1}{3x - 1} = \frac{2}{3}\left(x^2 - \frac{5}{3}x - \frac{5}{9}\right) - \frac{17}{27(3x-1)}.$$

}

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\begin{array}{rllll}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & \\ \cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
\end{array}
$$

So

$$\frac{2x^3 - 4x^2 + 1}{3x - 1} = \frac{2}{3}\left(x^2 - \frac{5}{3}x - \frac{5}{9}\right) - \frac{17}{27(3x-1)}.$$

}

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & \\
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3x-1 \big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\
\cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\
\cline{3-4}
 & & & -\frac{10}{9}x & +1
\end{array}
$$

So

$$\frac{2x^3-4x^2+1}{3x-1} = \frac{2}{3}\left(x^2-\frac{5}{3}x-\frac{5}{9}\right)-\frac{17}{27(3x-1)}.$$

}

Example 4

$$\begin{array}{rllll}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & -\frac{10}{27} \\ \cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1
\end{array}$$

So

$$\frac{2x^3-4x^2+1}{3x-1}=\frac{2}{3}\left(x^2-\frac{5}{3}x-\frac{5}{9}\right)-\frac{17}{27(3x-1)}.$$

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 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1 \\
 & & & \frac{10}{9}x & -\frac{10}{27} \\ \cline{4-5}
\end{array}
$$

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 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1 \\
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}

Example 5

$$x^2-2x+5\big)\overline{\quad x^4 \qquad\qquad -x^2 \quad +x \quad -4}$$

So

$$\frac{x^4-x^2+x-4}{x^2-2x+5}=x^2+2x-2+\frac{-13x+6}{x^2-2x+5}.$$

}

Example 5

$$\begin{array}{rl} & \qquad\qquad\qquad\qquad x^2 \\ x^2 - 2x + 5 \big) & \overline{\; x^4 \qquad\qquad\quad - x^2 \quad + x \quad - 4} \end{array}$$

So

$$\frac{x^4 - x^2 + x - 4}{x^2 - 2x + 5} = x^2 + 2x - 2 + \frac{-13x + 6}{x^2 - 2x + 5}.$$

}

Example 5

$$\begin{array}{r|l} & x^2 \\ \cline{2-2} x^2-2x+5 \big) & \;\; x^4 \qquad\qquad -x^2 \quad +x \quad -4 \\ & -x^4+2x^3-5x^2 \\ \cline{2-2} \end{array}$$

So

$$\frac{x^4-x^2+x-4}{x^2-2x+5} = x^2+2x-2+\frac{-13x+6}{x^2-2x+5}.$$

}

Example 5

$$\begin{array}{rrrrrr}
 & & & x^2 & & \\ \cline{2-6}
x^2-2x+5 \big) & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\ \cline{2-4}
 & & 2x^3 & -6x^2 & +x &
\end{array}$$

So

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Example 5

$$
\begin{array}{r|rrrrr}
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\cline{2-6}
x^{2}-2x+5) & x^{4} & & -x^{2} & +x & -4 \\
 & -x^{4} & +2x^{3} & -5x^{2} & & \\
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 & & -2x^3 & +4x^2 & -10x & \\ \cline{3-5}
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 & & & -2x^2 & -9x & -4
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Example 5

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 & & & x^2 & +2x & -2 \\ \cline{2-6}
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