Final exam practice 2

UCLA: Math 3B, Fall 2017

Instructor: Noah White
Date:

- This exam has 7 questions, for a total of 80 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Discussion section (please circle):

Day/TA	Kevin	Bohyun	Ryan
Tuesday	3A	3C	3E
Thursday	3B	3D	3F

Question	Points	Score
1	12	
2	12	
3	12	
4	10	
5	12	
6	14	
7	8	
Total:	80	

Questions 1 and 2 are multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following four pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

Question 2.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) The function $f(x) = e^{x+\cos x}$ is

   A. Always increasing.
   B. Always decreasing.
   C. Always concave up.
   D. Always concave down.

   (b) (2 points) The function $f(x) = e^x - e^{-x}$ is

   A. Always increasing.
   B. Always decreasing.
   C. Always concave down.
   D. Always concave up.

   (c) (2 points) The function $f(x) = \frac{4x^3-x}{x^3+x}$ has a

   A. Horizontal asymptote at $y = 4$.
   B. Vertical asymptote at $x = 1$.
   C. Slanted asymptote with slope -1.
   D. Slanted asymptote with slope 1.

(d) (2 points) The function $f(x) = \frac{2x^2-x}{x+1}$ has a

A. Horizontal asymptote at $y = 1$.

B. No vertical asymptotes.

C. A vertical asymptote at $x = 0$.

D. Slanted asymptote with slope 2.

(e) (2 points) The function $f(x) = e^{\sin x}$ has a critical point at

A. $x = \pi/2$

B. $x = 0$

C. $x = \pi$

D. $x = -\pi$

(f) (2 points) The function $f(x) = x - \ln(x^4)$ has a

A. minimum at $x = 4$.

B. maximum at $x = 4$.

C. minimum $x = 1$.

D. maximum $x = 1$.

2. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) The definite integral $\int_1^e \frac{1}{x}\,\mathrm{d}x$ has a value of

   A. e.
   B. 0.
   C. 1.
   D. π.

   (b) (2 points) The definite integral $\int_{\pi/2}^{\pi} x \sin x\,\mathrm{d}x$ has a value of

   A. $\pi/2$.
   B. $\pi - 1$.
   C. π.
   D. $-\pi$.

   (c) (2 points) The solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = 4y$ when $y(0) = 2$ has

   A. $y(1) = 3e^{-2}$
   B. $y(0.5) = 2e$.
   C. $y(1) = 3e$.
   D. $y(0.5) = 2e^2$.

(d) (2 points) The solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = e^{-y}(2t+1)$ when $y(0) = 0$ has

A. $y(\pi) = \ln 2$

B. $y(-\pi/2) = 0$.

C. $y(4) = \ln 2$.

D. $y(1) = \ln 3$.

(e) (2 points) The differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = (1-y)\left(\frac{y}{9} - 1\right)$ has a

A. stable equilibrium at $y = 9$.

B. unstable equilibrium at $y = 1/9$.

C. stable equilibrium at $y = 1$.

D. unstable equilibrium at $y = 3$.

(f) (2 points) The differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - (y-3)^3$ has a

A. stable equilibrium at $y = -2$.

B. unstable equilibrium at $y = -1$.

C. stable equilibrium at $y = 4$.

D. unstable equilibrium at $y = 4$.

3. Let $f(x) = \frac{1}{e^{2x}+1}$. Note that $f'(x) = -\frac{2e^{2x}}{(e^{2x}+1)^2}$ and $f''(x) = \frac{4e^{2x}(e^{2x}-1)}{(e^{2x}+1)^3}$.

   (a) (2 points) Does $f(x)$ cross the x and y axes? If so, where?

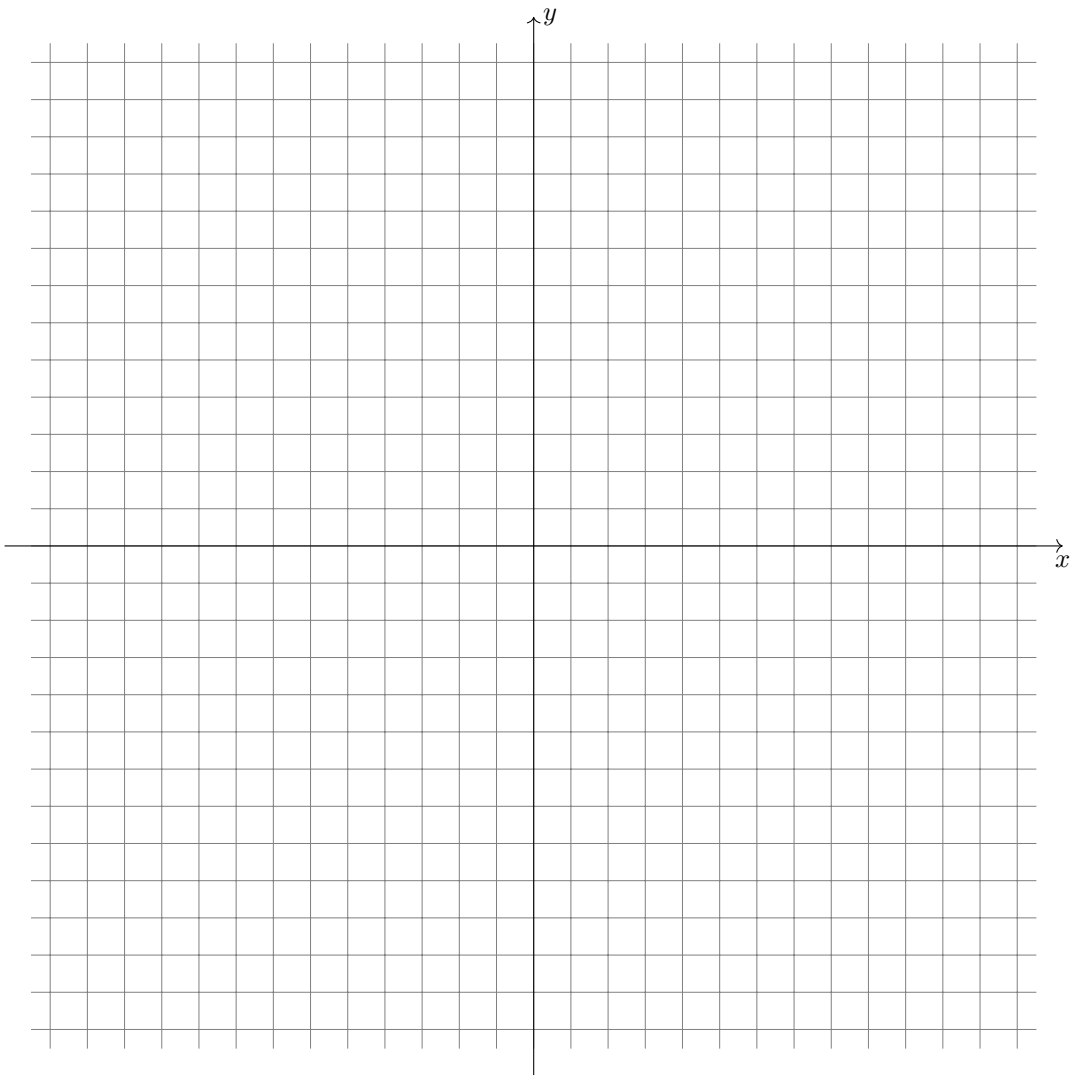
   (b) (2 points) Does $f(x)$ have any horizontal asymptotes? If so what are they?

   (c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

   (d) (2 points) For what x is the first derivative $f'(x)$ positive?

(e) (2 points) For what x is the second derivative $f''(x)$ positive?

(f) (3 points) On the graph provided, sketch $f(x)$



4. A new invasive species of fish has recently been found of the coast of Australia in the Great Barrier Reef. Government researchers determine that the species of fish are migrating to the Reef at a rate of $30,000$ individuals per year. They also estimate that the growth in population due to births will be roughly exponential; this means, for every one fish of this species at the Reef, t years later there will be

$$e^{0.5t}$$

fish. Initially the Reef contained no individuals from this species.

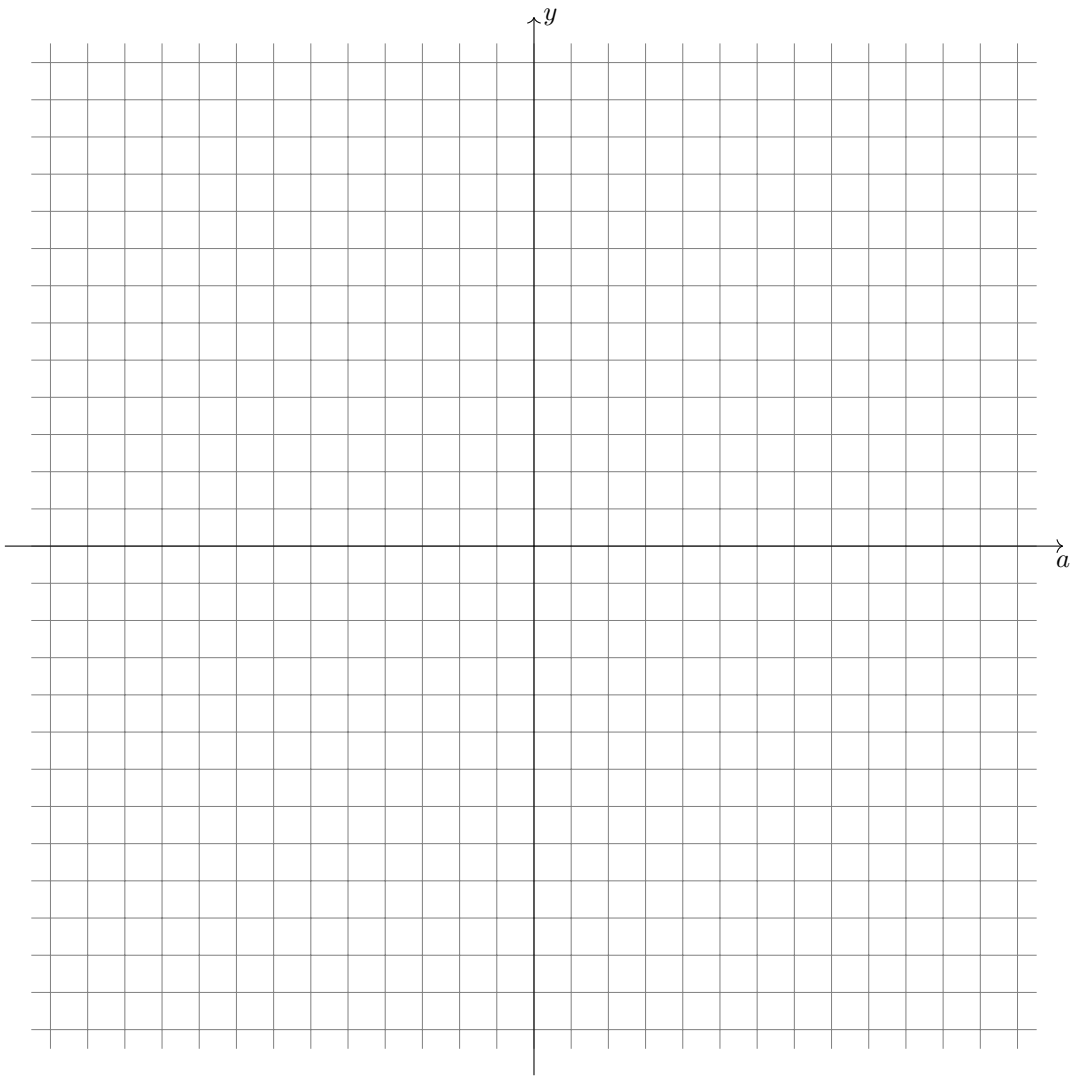
(a) (6 points) Write a Riemann sum which represents the total number of fish from this species in the Reef in 10 years. Be sure to define any symbols that you use (e.g. t_k, Δt, etc).

(b) (4 points) Use an integral to evaluate the Riemann sum above.

5. In this question we will investigate the behaviour of the solutions of

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (y-1)(1-ay)$$

(a) (4 points) Draw a bifurcation diagram for this equation with parameter a. Make sure to label the regions of your diagram with up/down arrows according to the direction of the derivative.



(b) (2 points) Draw a phase diagram when $a = 3$ and sketch the solution if $y(0) = 0.5$.

(c) (2 points) Draw a phase diagram when $a = 1$ and sketch the solution if $y(0) = 0.5$.

(d) (2 points) Draw a phase diagram when $a = 0$ and sketch the solution if $y(0) = 0.5$.

(e) (2 points) The differential equation above has an a equilibrium solution of $y = 1$ for any value of a. For what a is this equilibrium stable?

6. A patient is receiving a continuous injection of a drug with a half life of 5 hours. The drug is being administered at 2 mg/h.

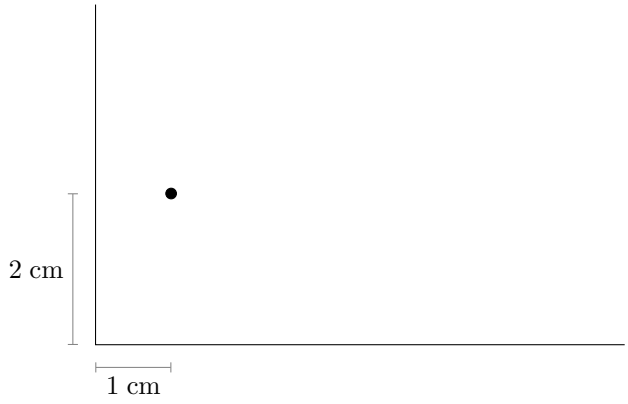
   (a) (4 points) Write a differential equation modelling the total amount of drug $y(t)$ (in milligrams) in the patient's body.

   (b) (3 points) Solve the differential equation. Note that initially the patient does not have any drug in their system.

(c) (1 point) If the patient has 5 L of blood, what is the concentration of the drug on their bloodstream in the long term?

(d) (6 points) Now suppose that the drug is being administered at an unknown rate. You may assume that it still has a half life of 5 hours. After 10 hours, doctors find that the concentration of the drug is 3 mg/L. If the patient has 5 L of blood, at what rate was the drug being administered? Please give an exact value. No decimals.

7. (8 points) A large, rectangular piece of material has a small stain in a corner, 1 cm from one edge and 2 cm from the other. We would like to make a single, straight cut to remove the stained corner. What is the area of the smallest piece we could remove?



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