Math 115 A

- We have all learnt how useful vectors and matrices are.
- In this course we will identify what parts of the structure of vectors/matrices make them so useful.
- We will call any collection of objects with the same properties a vector space
- Linear algebra is the study of vector spaces

Def (very vague) A vector space is any collection of objects which we can add together and multiply by scalars.

(for now a scalar is either a real number or a complex number.)

- A much more formal and precise definition will be given later.
- Instead, lets learn some examples that we will come back to again and again.

Examples

1 (i) $\mathbb{R}^n = \{$ column vectors with n coords $\}$.

we can add:

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ \vdots \\ a_n+b_n \end{pmatrix}.$$

and scalar multiply

$$\lambda \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} \lambda a_1 \\ \lambda a_2 \\ \vdots \\ \lambda a_n \end{pmatrix}$$

(ii) $\sum_n = \{$ column vectors with n coords that add to zero, e.g. (if $n=3$) $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\}$.

If we have two vectors

$$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}, \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} \in \sum_n$$

so $a_1 + \dots + a_n = b_1 + \dots + b_n = 0$ ~~thus~~ $a_1 + b_1 + \dots + a_n + b_n = 0$

so

$$\begin{pmatrix} a_1+b_1 \\ \vdots \\ a_n+b_n \end{pmatrix} \in \sum_n$$

so we can add. Similarly, scalar mult work too.

2. $\text{Mat}_{m\times n}(\mathbb{C}) = \{ m \times n \text{ matrices with entries in } \mathbb{C} \}$

We can add and scalar mult matrices as usual.

3(i) $\underline{\ell} = \{$ infinite sequences $(a_0, a_1, \dots)$ with entries in $\mathbb{C} \}$.

$(a_0, a_1, \dots) + (b_0, b_1, \dots) = (a_0 + b_0, a_1 + b_1, \dots)$

(ii) $\underline{\ell_c} = \{$ infinite series with only finitely many nonzero terms $\}$.

If $(a_0, a_1, \dots)$, $(b_0, b_1, \dots)$ only have fin. many non zero terms, so does $(a_0, a_1, \dots) + (b_0, b_1, \dots)$

(iii) $\underline{\ell_{\to 0}} = \{$ infinite series $(a_0, \dots)$ s.t. $\lim_{n \to 0} a_n = 0 \}$.

note that $\lim_{n\to 0} (a_n + b_n) = 0$ if $\lim_{n\to 0} a_n = \lim_{n \to 0} b_n = 0$.

and $\lim_{n \to 0} \lambda a_n = 0$.

4(i) $\mathcal{C}(\mathbb{R}) = \{$ real valued function, continuous, in one variable, eg x^2, $\sin(x) - e^{x^3}$, etc $\}$.

If $f(x), g(x)$ are cts then so if $f(x) + g(x)$ and $\lambda f(x)$.

(ii) $\mathbb{R}[x] = \{$ polynomials in one variable $\}$

sum of polynomials is a polynomial.

scalar multiple of a polynomial is a polynomial.

(iii) $\mathbb{R}(x) = \{$ rational functions, i.e. ~~qo~~ quotients of polynomials $\frac{p(x)}{q(x)} \}$.

again, sums and scalar multiples of rational functions, are rational functions

e.g. $$\frac{x^2 - 1}{x} + \frac{1}{x+2} = \frac{(x^2-1)(x+2) + x}{x(x+2)}$$