# Math 3B: Lecture 12

Noah White

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- I will provide some context for the grades (average, distribution, comments)

# Last time

- Fundamental theorem of calculus

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- Integration by substitution

# Properties of definite integrals

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## Adding areas

$$\int_a^c f(x)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_b^c f(x)\,\mathrm{d}x$$

# More properties of definite integrals

## Additivity

$$\int_a^b f(x) + g(x)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x$$

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$$\int_a^b f(x) + g(x)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_a^b g(x)\,\mathrm{d}x$$

## Lineararity (scalars factor out)

$$\int_a^b \alpha f(x)\,\mathrm{d}x = \alpha \int_a^b f(x)\,\mathrm{d}x$$

# Scalars factor out

$$\int_a^b \alpha f(x)\,\mathrm{d}x = \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n-1} \alpha f(a + k\Delta x)$$

# Scalars factor out

$$\begin{aligned}\int_a^b \alpha f(x)\,\mathrm{d}x &= \lim_{n\to\infty} \frac{b-a}{n}\sum_{k=0}^{n-1} \alpha f(a+k\Delta x)\\ &= \lim_{n\to\infty} \alpha\frac{b-a}{n}\sum_{k=0}^{n-1} f(a+k\Delta x)\end{aligned}$$

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$$\int \alpha f(x) \, \mathrm{d}x = \alpha \int f(x) \, \mathrm{d}x$$

# The product rule

Just like integration by substitution reverses the chain rule, integration by parts "reverses" the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

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writen another way

$$(uv)' = u'v + uv'$$

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Lets integrate both sides

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By the fundamental theorem of calculus

$$uv = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

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Rearranging. . .

# Integration by parts

## The integration by parts formula

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## Alternative statement

$$\int u \, \mathrm{d}v = uv - \int v \, \mathrm{d}u$$