This week on the problem set we will see examples of triple integrals. Often the most difficult part of these problems will be the setup. Once we have an iterated integral, that calculation is "easy". We will also have questions on polar coordinates and integration in polar coordinates. The last few questions deal with coordinate change maps and Jacobians.

*Note:* A few of the questions below refer to figures in the textbook. You don't actually need the figures but they might help.

*Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3rd Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. (16.3) Evaluate $\iiint_{\mathcal{B}} f(x,y,z)\,\mathrm{d}V$ for the specified function $f$ and box $\mathcal{B}$.
   (a) (16.3.3) $f(x,y,z) = xe^{y-2z}$; $0 \le x \le 2, 0 \le y \le 1, 0 \le z \le 1$
   (b) (16.3.5) $f(x,y,z) = (x-y)(y-z)$; $[0,1] \times [0,3] \times [0,3]$
   (c) (16.3.6) $f(x,y,z) = \dfrac{z}{x}$; $1 \le x \le 3, 0 \le y \le 2, 0 \le z \le 4$
   (d) (16.3.7) $f(x,y,z) = (x+z)^3$; $[0,a] \times [0,b] \times [0,c]$

2. (16.3) Evaluate $\iiint_{\mathcal{W}} f(x,y,z)\,\mathrm{d}V$ for the function $f$ and region $\mathcal{W}$ specified.
   (a) (16.3.9) $f(x,y,z) = x+y$; $\mathcal{W} : y \le z \le x, 0 \le y \le x, 0 \le x \le 1$
   (b) (16.3.12) $f(x,y,z) = x$; $\mathcal{W} : x^2 + y^2 \le z \le 4$
   (c) (16.3.13) $f(x,y,z) = e^z$; $\mathcal{W} : x + y + z \le 1, x \ge 0, y \ge 0, z \ge 0$
   (d) (16.3.14) $f(x,y,z) = z$; $\mathcal{W} : x^2 \le y \le 2, 0 \le x \le 1, x - y \le z \le x + y$

3. (16.3.15) Calculate the integral of $f(x,y,z) = z$ over the region $\mathcal{W}$ in Figure 11 below the hemisphere of radius 3 and lying over the triangle $\mathcal{D}$ in the $xy$-plane bounded by $x = 1, y = 0$ and $x = y$.

4. (16.3.20) Find the volume of the solid in $\mathbb{R}^3$ bounded by $y = x^2, x = y^2, z = x + y + 5$ and $z = 0$.

5. (16.3.23) Evaluate $\iiint_{\mathcal{W}} xz\,\mathrm{d}V$; where $\mathcal{W}$ is the domain bounded by the elliptic cylinder $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ and the sphere $x^2 + y^2 + z^2 = 16$ in the first octant $x \ge 0, y \ge 0, z \ge 0$ (Figure 14).

6. (16.3.31) Let $\mathcal{W}$ be the region bounded by $z = 1 - y^2, y = x^2$, and the plane $z = 0$. Calculate the volume of $\mathcal{W}$ as a triple integral in the order $\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

7. (12.3.8) What is the slope of the line $\theta = \dfrac{3\pi}{5}$?

8. (12.3) Convert to an equation in rectangular coordinates.
   (a) (12.3.11) $r = 7$
   (b) (12.3.12) $r = \sin\theta$
   (c) (12.3.15) $r = \dfrac{1}{\cos\theta - \sin\theta}$

9. (12.3) Convert to an equation in polar coordinates of the form $r = f(\theta)$.
   (a) (12.3.19) $y = x^2$
   (b) (12.3.20) $xy = 1$

(c) (12.3.21) $e^{\sqrt{x^2+y^2}} = 1$

10. (12.3.30) Sketch $r = 3\cos\theta - 1$ (see Example 9)

11. (16.4) Sketch the region $\mathcal{D}$ indicated and integrate $f(x, y)$ over $\mathcal{D}$ using polar coordinates.
    (a) (16.4.1) $f(x, y) = \sqrt{x^2 + y^2}, x^2 + y^2 \leq 2$
    (b) (16.4.5) $f(x, y) = y(x^2 + y^2)^{-1}; y \geq \frac{1}{2}, x^2 + y^2 \leq 1$

12. (16.4) Sketch the region of integration and evaluate by changing to polar coordinates
    (a) (16.4.10) $\displaystyle\int_0^4 \int_0^{\sqrt{16-x^2}} \tan^{-1} \frac{y}{x} \, \mathrm{d}y \, \mathrm{d}x$
    (b) (16.4.13) $\displaystyle\int_{-1}^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) \, \mathrm{d}y \, \mathrm{d}x$

13. (16.4.21) Find the volume of the wedge-shaped region (Figure 18) contained in the cylinder $x^2 + y^2 = 9$, above bounded by the plane $z = x$ and below by the $xy$-plane.

14. (16.4.24) Evaluate $\displaystyle\iint_{\mathcal{D}} x\sqrt{x^2 + y^2} \, \mathrm{d}A$, where $\mathcal{D}$ is the shaded region enclosed by the lemniscate curve $r^2 = \sin 2\theta$ in Figure 21.

15. (16.6.3) Let $G(u, v) = (u^2, v)$. Is $G$ one-to-one? If not determine a domain on which $G$ is one-to-one. Find the image under $G$ of:
    (a) The $u$- and $v$-axes
    (b) The rectangle $\mathcal{R} = [-1, 1] \times [-1, 1]$
    (c) The line segment joining $(0, 0)$ and $(1, 1)$
    (d) The triangle with vertices $(0, 0), (0, 1)$ and $(1, 1)$

16. (16.6) Compute the Jacobian (at the point, if specified)
    (a) (16.6.13) $G(u, v) = (3u + 4v, u - 2v)$
    (b) (16.6.15) $G(r, t) = (r \sin t, r - \cos t), (r, t) = (1, \pi)$
    (c) (16.6.16) $G(u, v) = (v \ln u, u^2 v^{-1})$

17. (16.6.19) Find a linear mapping $G$ that maps $[0, 1] \times [0, 1]$ to the parallelogram in the $xy$-plane spanned by the vectors $\langle 2, 3\rangle$ and $\langle 4, 1\rangle$.