Midterm 1

UCLA: Math 3B, Winter 2017

Instructor: Noah White
Date: 30 January 2017

- This exam has 5 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: Solutions.

ID number: ______

Discussion section (please circle):

Day/TA	Max	Yuejiao	Jeanine
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	6	
2	12	
3	9	
4	5	
5	8	
Total:	40	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)	X			
(b)				X
(c)	X			
(d)				X
(e)		X		
(f)				X

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) The function $f(x) = \frac{1}{1+x}$ has

   A. a vertical asymptote at $x = -1$.
   B. a horizontal asymptote at $y = 1$.
   C. no asymptotes.
   D. a slanted asymptote.

   (b) (1 point) The function $f(x) = \frac{1}{x} - x$ is

   A. increasing when $x > 0$ and decreasing when $x < 0$.
   B. increasing when $x < 0$ and decreasing when $x > 0$.
   C. always increasing.
   D. always decreasing.

   (c) (1 point) The function $f(t) = t - 2 \ln t$ has a

   A. local minimum at $t = 2$.
   B. local maximum at $t = 2$.
   C. local minimum at $t = 1$.
   D. local maximum at $t = 1$.

(d) (1 point) The function $f(x) = 2\ln(e^x + 1) - x$ has

A. a horizontal asymptote at $y = 1$.

B. a vertical asymptote at $x = 0$.

C. no asymptotes.

D. a slanted asymptote of $y = x$.

Hint: For this question you may use the fact that $\lim_{x\to\infty} f(x) = \infty$ *and* $f'(x) = \frac{e^x-1}{e^x+1}$

(e) (1 point) The function $h(t) = 1 - \sin x + x$ is an antiderivative of

A. $x - \cos +0.5x^2$

B. $1 - \cos x$

C. $1 - \sin x$

D. $x + \cos x - x^2$

(f) (1 point) The definite integral $\int_{-1}^{1} 2xe^{x^2}\, dx$

A. 2

B. $\frac{1}{2}(e-1)$

C. $e - 1$

D. 0

2. Let $f(x) = \frac{1}{x^2-x-2}$. Note that $f'(x) = \frac{1-2x}{(x-2)^2(x+1)^2}$ and $f''(x) = \frac{6(x^2-x+1)}{(x-2)^3(x+1)^3}$.

(a) (2 points) Find the x and y intercepts of $f(x)$.

$f(0) = -\frac{1}{2}$ y-int: $y = -\frac{1}{2}$

$f(x) \neq 0$ so no x-ints

(b) (1 point) Does $f(x)$ have any horizontal asymptotes? If so what are they?

$\lim_{x \to \pm\infty} f(x) = 0$ so hor. asymptotes at $y=0$ in both directions.

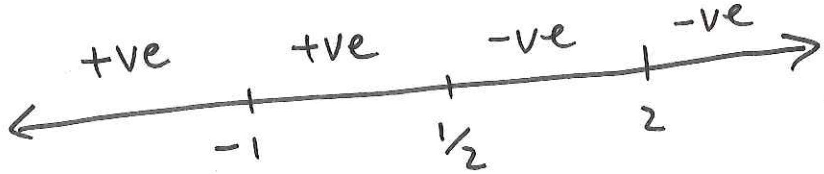
(c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

$f(x)$ undef. at $x = 2, -1$ yes, vert. asymptotes @ $x = 2, -1$.

$\left(\lim_{x \to 2^+} f(x) = \infty, \lim_{x \to 2^-} f(x) = -\infty, \lim_{x \to -1^+} f(x) = -\infty, \lim_{x \to -1^-} f(x) = +\infty\right)$

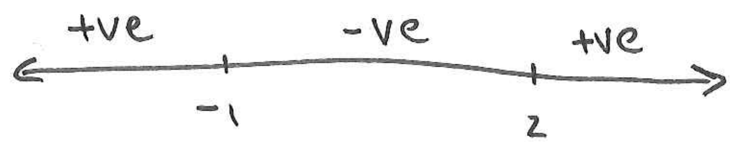
(d) (2 points) For what x is the first derivative $f'(x)$ positive?

$f''(x) = 0$ when $x = \frac{1}{2}$, undef $x = 2, -1$

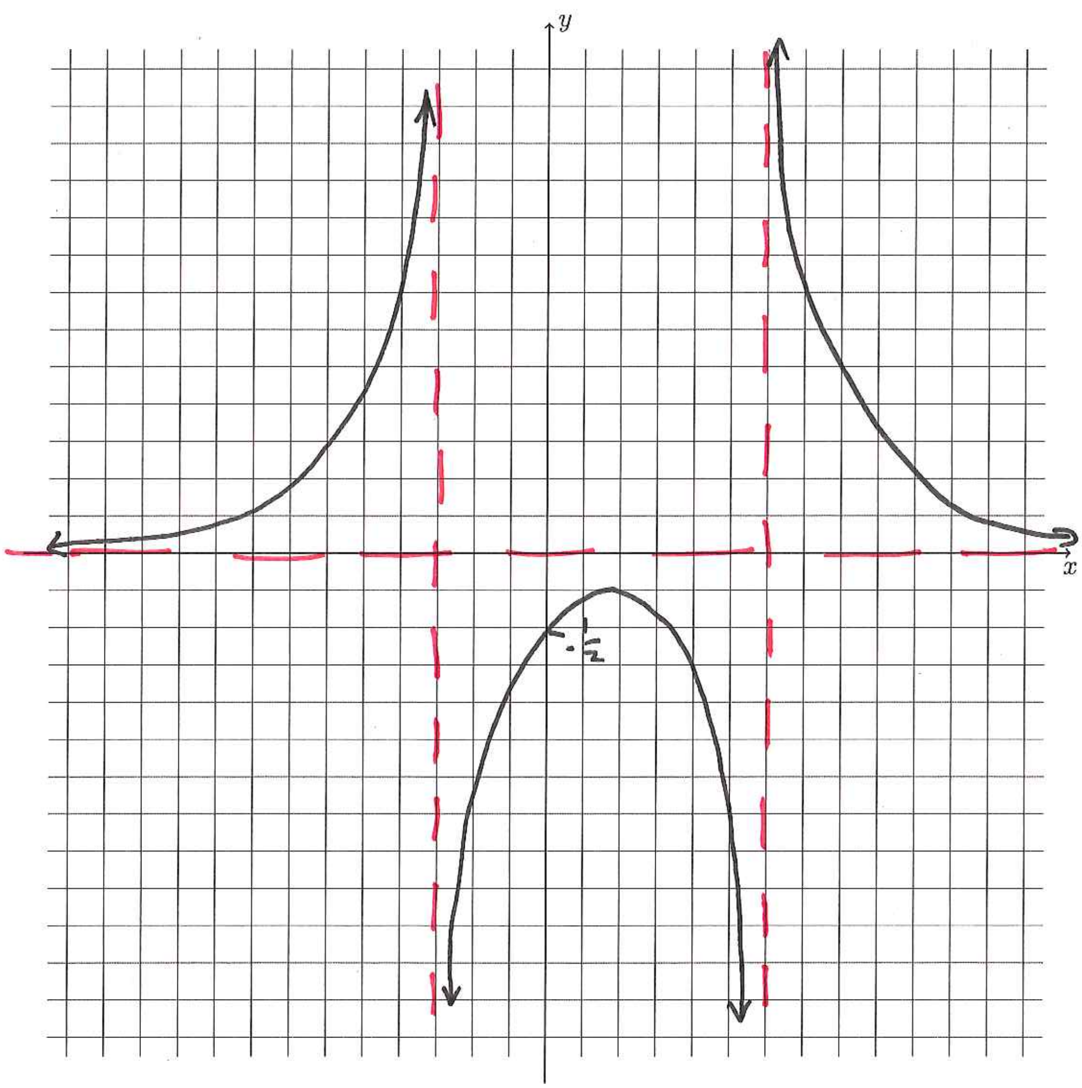


(e) (2 points) For what x is the second derivative $f''(x)$ positive?

$f''(x) \neq 0$ ~~when x~~: since $x^2 - x + 1 \neq 0$ (discriminant $= -3$). f'' undef. $x = 2, -1$.



(f) (4 points) On the graph provided, sketch $f(x)$



3. A company would like to advertise its product in two media markets (market A and market B). It is known experimentally that a spend of $\$x$ thousand dollars in market A will result in

$$R_A(x) = 18\ln(1+x)$$

thousand extra customers and a spend of $\$y$ thousand in market B will result in

$$R_B(x) = 8\ln(1+x)$$

thousand new customers.

(a) (4 points) If the company has a marketing budget of \$10 thousand and they wish to spend it all, how many new customers will they attract in total? (Let $\$x$ thousand be the amount spent in market A.)

x = spend in A

y = spend in B

so $x + y = 10$

Total new customers:

$$T = R_A(x) + R_B(y)$$

But $y = 10 - x$ so

$$T(x) = R_A(x) + R_B(10-x)$$

$$= 18\ln(1+x) + 8\ln(11-x)$$

(b) (5 points) How much should the company spend in market A in order to maximise the number of new customers they attract?

$$T'(x) = \frac{18}{1+x} - \frac{8}{11-x}$$

Find crit points, $T'(x) = 0$

$$\frac{18}{1+x} = \frac{8}{11-x}$$

$$18(11-x) = 8(1+x)$$

$$9(11-x) = 4(1+x)$$

$$99 - 9x = 4 + 4x$$

$$95 = 13x$$

$$x = 95/13.$$

Domain is $[0, 10]$. Use closed int. test

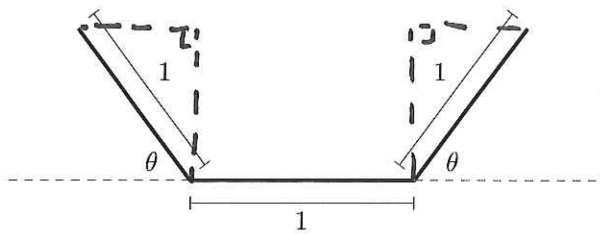
$T(95/13) \approx 48.6$

$T(0) \simeq 19.2$

$T(10) \approx 43.2$

so $x = 95/13$ is a global max.

5. A trough is being built for feeding animals. The cross section of the is pictured below.



Each side, and the base of the trough are 1 foot wide. The sides are bent up from the horizontal, at an angle θ.

(a) (3 points) What is the area of the cross section?

Break into two triangles + rectangle

height = $\sin\Theta$

base of triangle = $\cos\Theta$

$$A = \text{Area} = 2\left(\tfrac{1}{2}\sin\Theta\cos\Theta\right) + 1\cdot\sin\Theta = \sin\Theta + \sin\Theta\cos\Theta.$$

(b) (5 points) What angle should the sides be bent to, in order that the trough can hold as much food as possible? You may use the fact that $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$.

Maximise A on the domain $[0, \pi/2]$

$$A'(\Theta) = \cos\Theta + \cos^2\Theta - \sin^2\Theta = 2\cos^2\Theta + \cos\Theta - 1$$

Find crit points: factorise quadratic

$$(2\cos\Theta - 1)(\cos\Theta + 1) = 0$$

so $\cos\Theta = \frac{1}{2}$ or -1

so $\Theta = \frac{\pi}{3}$ or π ← not in domain

End points $\Theta = 0, \frac{\pi}{2}$, use closed int. method

$A(0) = 0 \quad A(\frac{\pi}{2}) = 1 \quad A(\frac{\pi}{3}) = \frac{3\sqrt{3}}{4} > 1$

so $\Theta = \frac{\pi}{3}$ is the global max.

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