This week on the problem set you will get practice with independant random variables. Especially challenging questions, or questions that are not appropriate for an exam, are indicated with one or more asterisks.

1. From the textbook, chapter 2, problems 39, 41, 42*, 45*.
2. From the supplementary problems, chapter 2, problem 20, 21.
3. If X and Y are independent geometric random variables with parameters p_1 and p_2 respectively, compute $\mathbf{P}(X_1 < X_2)$ and $\mathbf{P}(X_1 = X_2)$.
4. We have a biased coin (probability of heads equal to p). In the first stage of the experiment we keep tossing it until we get a heads, and we remember how many times we had to toss it (say k times). In the 2nd stage of the experiment we will toss the same coin the same number of times (k times). What is the probability that in the 2nd stage we get no heads?
5. Let X and Y be independent geometric random variables, both with parameter p. Find the probability mass function of $X + Y$.
6. We have a fair coin and we will keep tossing it until we observe n consecutive heads. What is the expected number of tosses? It would be nice to try to simplify your answer using mathematical induction.
   Hint: condition on the events that the first tails that appears is in the 1st toss, 2nd toss, ..., n-th toss and the event that there are no tails in the first n tosses.
7. Let X and Y be independent random variables each of which attains any value between 1 and n with probability $1/n$. Compute $\mathbf{E}(|X - Y|)$ and simplify your answer.
8. Let X and Y be independent random variables and let both have expectation equal to zero. Show that $\mathrm{var}(XY) = \mathrm{var}(X)\,\mathrm{var}(Y)$.
9. If X and Y are independent random variables and $\mathbb{E}(X) = 0$ show that
   $$\mathbb{E}((X - Y)^2) = \mathbb{E}((X + Y)^2).$$
   Does $\mathbb{E}((X - Y)^3) = \mathbb{E}((X + Y)^3)$ also have to hold?
10. Let X and Y be independent random variables with the same probability mass function. Assume that both of them have well-defined expectation and variance and that X and Y are not constant random variables. Show that $X + 2Y$ and $X - Y$ are not independent.
    Hint: recall a theorem which says that if X and Y are independent then "something has to hold". Then show that this something doesn't hold for X and Y.