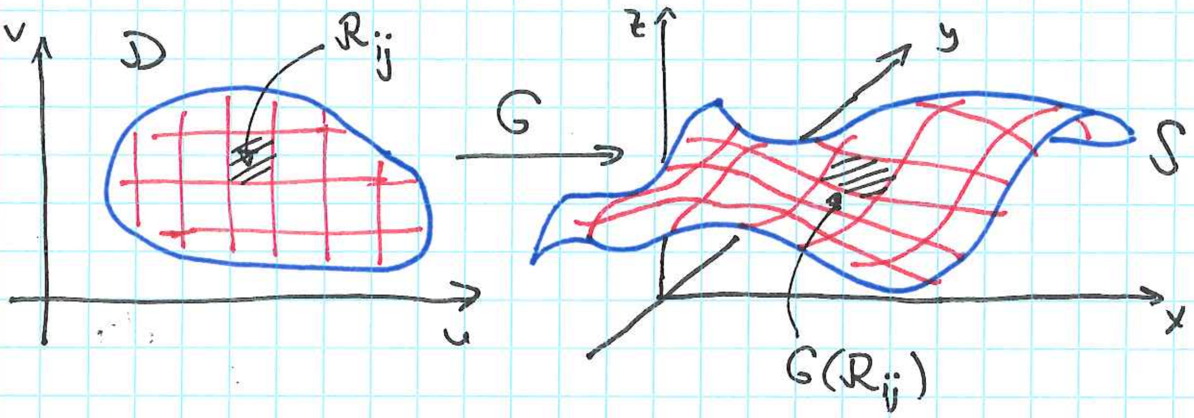
Lecture 17

- Suppose we have a parametrised surface S



- Divide the domain D into rectangles R_{ij}
- Choose points $P_{ij} \in R_{ij}$.
- If f is a function defined on the surface S then

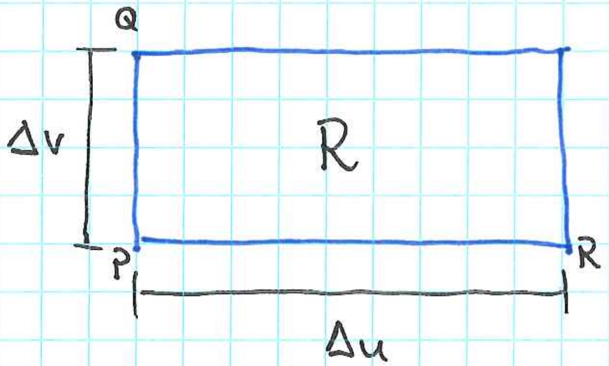
$$\lim_{\|P\| \to 0} \sum_{i,j=1}^{\dots} f(G(P_{ij})) \cdot \text{Area}(G(R_{ij}))$$

~~Questio~~ is a good definition for a surface integral!

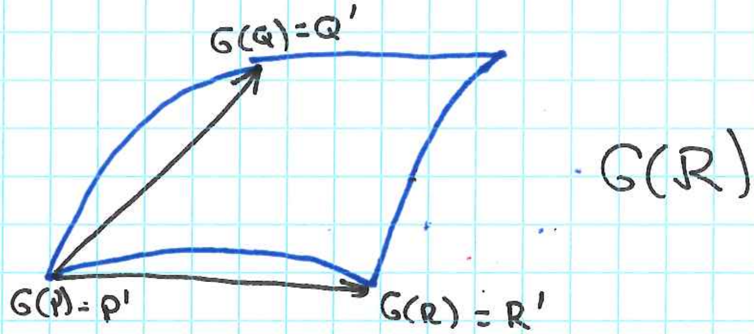
- Question; how can we compute, approximate

$$\text{Area}\left(G(R_{ij})\right) \; ?$$

- Consider a small rectangle $\mathcal{R} \subseteq (uv)$-plane, side lengths $\Delta u, \Delta v$:

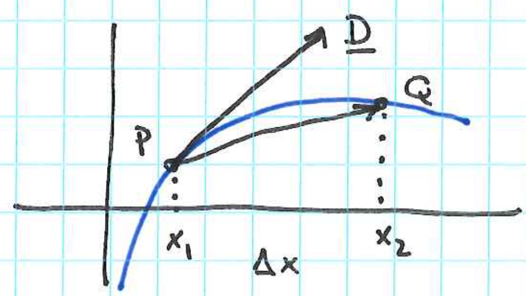


- Under G, this is transformed into a "curved parallelogram":



- ~~Area~~ $\text{Area}(G(\mathcal{R})) \approx \| \overrightarrow{P'R'} \times \overrightarrow{P'Q'} \|$

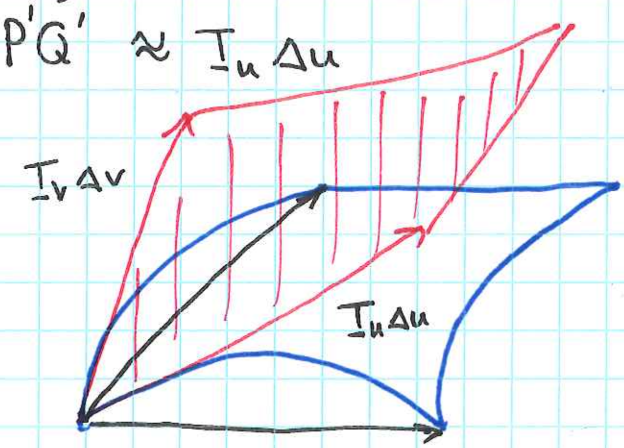
- <u>Recall</u> If we have a 1D curve $y = f(x)$



$$\overrightarrow{PQ} \approx \underline{D} = \left(1, \frac{df}{dx}\Big|_{x=x_1} \right) \Delta x$$

- Thus $\overrightarrow{P'R'} \approx \cancel{\frac{\partial G}{\partial}}\ \underline{T}_v\, \Delta v$

$\overrightarrow{P'Q'} \approx \underline{T}_u\, \Delta u$



So

$$\text{Area}(G(R)) \approx \|\underline{T}_u \times \underline{T}_v\|\, \Delta u\, \Delta v$$

$$\approx \|\underline{T}_u \times \underline{T}_v\|\, \text{Area}(R_{ij})$$

$$= \|\underline{N}(u,v)\|\, \text{Area}(R_{ij})$$

- Hence

$$\lim_{\|P\| \to 0} \sum f(G(P_{ij}))\, \text{Area}(G(R_{ij}))$$

$$= \lim_{\|P\| \to 0} \sum f(G(P_{ij}))\, \|\underline{N}(u_{ij}, v_{ij})\|\, \text{Area}(R_{ij})$$

Thus

Def

$$\iint_S f(x\ y\ z)\, dS = \iint_D f(G(u,v))\, \|\underline{N}(u\,v)\|\, dA_{uv}$$