# Math 3B: Lecture 5

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# Definition (local maximum)

A function $f : D \longrightarrow R$ has a local maximum at $a$ if

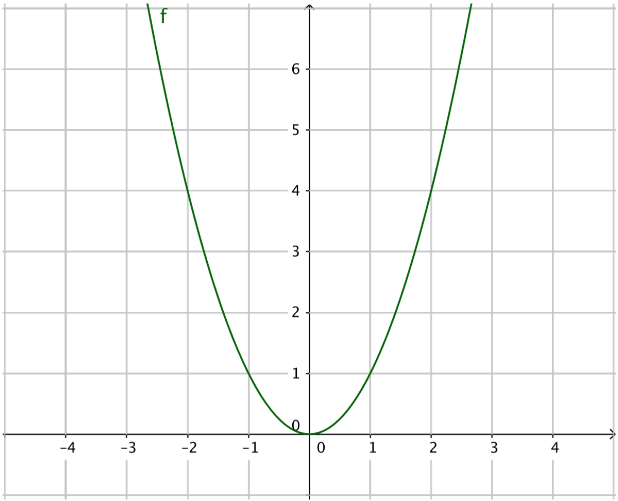
$$f(x) \leq f(a) \quad \text{for all } x \text{ near } a$$

# Definition (local minimum)

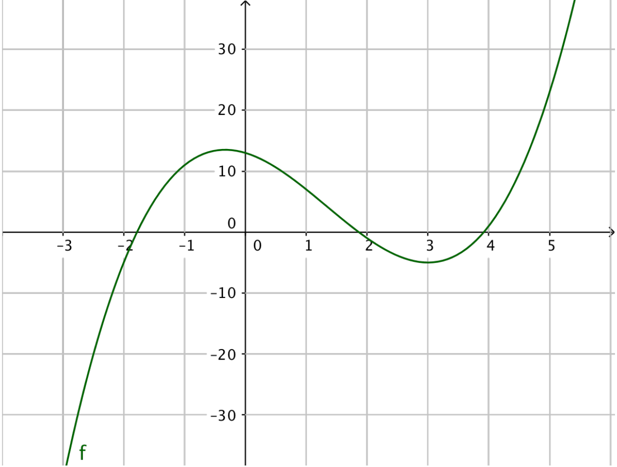
A function $f : D \longrightarrow R$ has a local minimum at $a$ if

$$f(x) \geq f(a) \quad \text{for all } x \text{ near } a$$

$f : \mathbb{R} \longrightarrow \mathbb{R}; x \mapsto x^2$ has a min at $x = 0$




$f : \mathbb{R} \longrightarrow \mathbb{R}; f(x) = x^3 - 4x^2 - 3x + 13$ has a local max at $x = -\frac{1}{3}$

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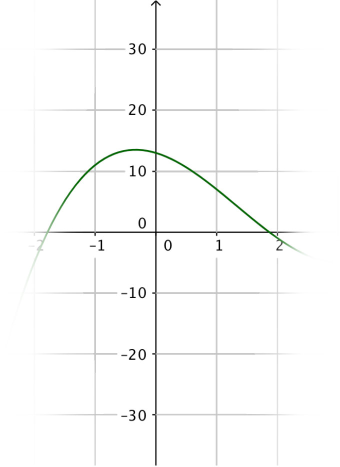

30
20
10
0
-10
-20
-30
-2
-1
0
1
2

# Definition (Critical point)

A function $f(x)$ has a critical point at $x = a$ if $f'(a) = 0$ or if $f'(a)$ is undefined.

# Examples

- $f(x) = x^2$ has a critical point at $x = 0$ (since f'(x) = 2x)

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# Examples

- $f(x) = x^2$ has a critical point at $x = 0$ (since f'(x) = 2x)
- $f(x) = \sin x$ has a critical point at $x = \frac{\pi}{2}$ (since $f'(x) = \cos x$)
- $f(x) = e^x$ doesn't have any critical points since $f'(x) = e^x$ can never be zero

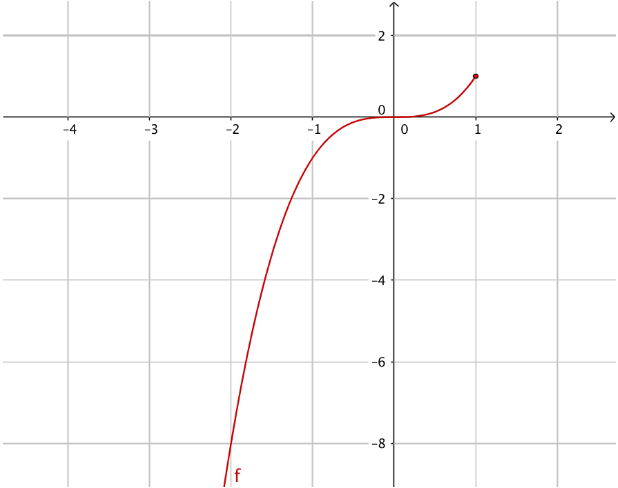
Local maximums and minimums (extrema) occur at

# Example

$f : (-\infty, 1] \longrightarrow \mathbb{R}; f(x) = x^3$ has critical points at

$$x = 0 \text{ and } 1$$

$f'(x) = 3x^2$ so $f'(0) = 0$ and $f'(1)$ is undefined.




Suppose $x = a$ is a critical point for the function $f(x)$.

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- If $f'(x) < 0$ for $x$ less than and close to $a$, and

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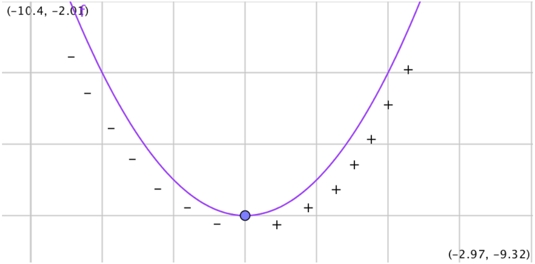

Suppose $x = a$ is a critical point for the function $f(x)$.

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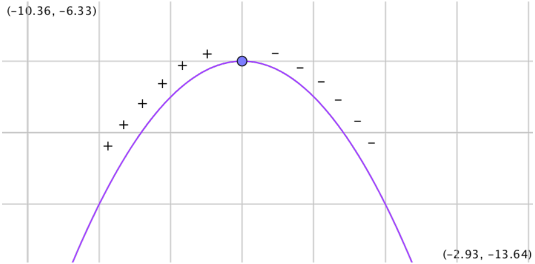

Suppose $x = a$ is a critical point of the function $f(x)$

# Second derivative test

If

- $f''(a) > 0$ then $f$ has a minimum at $a$

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- $f''(a) < 0$ then $f$ has a maximum at $a$

Note: If $f''(a) = 0$ then we cannot conclude anything! E.g $x^3$ or $x^4$.

We have a function $f : D \longrightarrow R$. How do we find all local/global extrema?

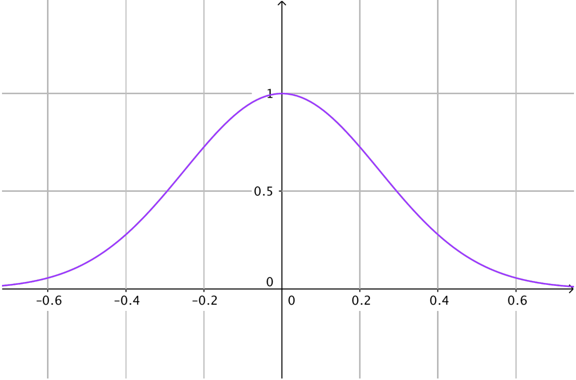
To find the global extrema of $f(x)$ defined on a closed interval $[a, b]$:

To find the global extrema of $f(x)$ defined on a open interval $(a, b)$:
Note: $a$ could be $-\infty$ and $b$ could be $\infty$.

2. Find the limits

$$L = \lim_{x \to a^+} f(x) \quad \text{and} \quad M = \lim_{x \to b^-} f(x)$$

3. Evaluate $f(x)$ at all the critical points
4. The smallest value is the global min unless $L$ is smaller, in which case there is no global min
5. The largest value is the global max unless $M$ is larger, in which case there is no global max


1
0.5
0
−0.6
−0.4
−0.2
0
0.2
0.4
0.6