

This week on the problem set you will get practice thinking about potential functions and calculating line integrals.

Homework: The homework will be due on Wednesday 27 May. It will consist of questions 3, 4 and 5 below. *Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3rd Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. (Section 17.4) 2, 3, 5, 8, 9, 10, 13, 14, 17, 18, 27, 30, 34, 37, 40, 41*, 46* 48*. (questions are the same in previous versions)
2. (Section 17.5) 1, 6, 7, 12, 17, 18, 21, 22, 31*, 35. (questions are the same in previous versions)
3. Consider the line segment $(x, 0, 0)$ where $x \in [-1, 1]$ in \mathbb{R}^3 . Imagine this line segment moving up with its centre on the z -axis, rotating parallel to the xy -plane at constant speed. It completes one full revolution when it gets to $z = 2\pi$. What surface area is swept out by the rotating line segment? You may wish to use the fact that

$$\frac{d}{dt} \left(t\sqrt{1+t^2} + \sinh^{-1}(t) \right) = 2\sqrt{1+t^2}$$

and that $\sinh^{-1} t$ is an odd function and $\sinh^{-1}(1) = \ln(1 + \sqrt{2})$.

4. The velocity vector field of a fluid is given by $\mathbf{F}(x, y, z) = \frac{\langle x, y, z \rangle}{x^2 + y^2 + z^2}$ measured in meters per second. What is the volume of fluid flowing each second through the open cylinder of radius 1 and height 1 centered along the z -axis, i.e. the cylinder $C = \{(x, y, z) : x^2 + y^2 = 1, 0 \leq z \leq 1\}$, with outward orientation?
5. Let $\mathbf{F} \left\langle y(ye^{x+y^2} - 1) + x^2, 2y(1 + y^2)e^{x+y^2} + x \right\rangle$ and let \mathcal{C} be the portion of $y = 1 - x^2$ above the x -axis, oriented left to right.

(a) Parameterise \mathcal{C} and write $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ as a single integral. Do not try and evaluate.

(b) Now let \mathcal{L} be the straight line from $(-1, 0)$ to $(1, 0)$ oriented left to right and let \mathcal{D} be the region bounded by the x -axis and \mathcal{C} . Use Green's theorem to relate the integrals of \mathbf{F} over \mathcal{C} and \mathcal{L} to an integral over \mathcal{D} . Use this to evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

(c) **Path (almost)-independence for non-conservative vector fields.** More generally, suppose \mathcal{C}_1 and \mathcal{C}_2 are two oriented (nonintersecting) curves with the same endpoints, and \mathbf{F} is a vector field that is defined everywhere on the region \mathcal{D} between \mathcal{C}_1 and \mathcal{C}_2 . If \mathbf{F} is conservative we know that $\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r} - \int_{\mathcal{C}_2} \mathbf{F} \cdot d\mathbf{r} = 0$. If \mathbf{F} is not conservative, what is

$$\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r} - \int_{\mathcal{C}_2} \mathbf{F} \cdot d\mathbf{r}?$$

*The questions marked with an asterisk are more difficult or are of a form that would not appear on an exam. Nonetheless they are worth thinking about as they often test understanding at a deeper conceptual level.