Math 3B: Lecture 4

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January 18, 2017

AAP tutoring

- AAP Peer Learning Facilitators available for Math 3B.

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- Sign-ups start Wednesday (1/10/17) and the first session begins the following Wednesday (1/18/17).

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- If you are not a part of AAP, applications for this program are found in Campbell Hall!

Last time

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- Graphing using calculus

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- Slanted asymptotes

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Note: You have homework due this Friday. It should be neatly presented and be written in full sentances. You will receive points for presentation.

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are all different functions!

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Definition (Global maximum)

A function $f : D \longrightarrow R$ has a global maximum at a if

$$f(x) \leq f(a) \quad \text{for all } x \in D$$

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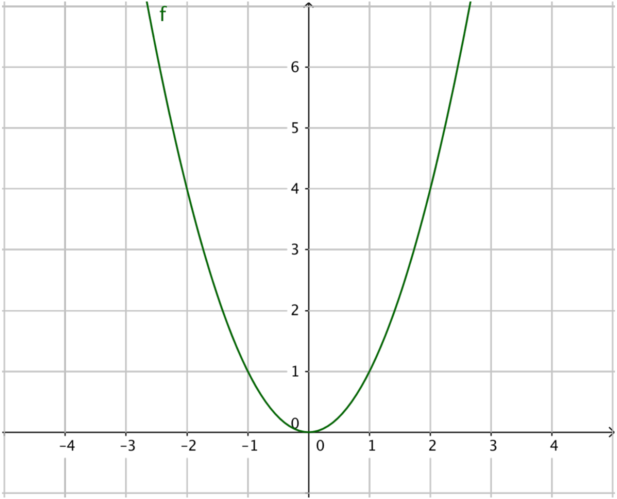
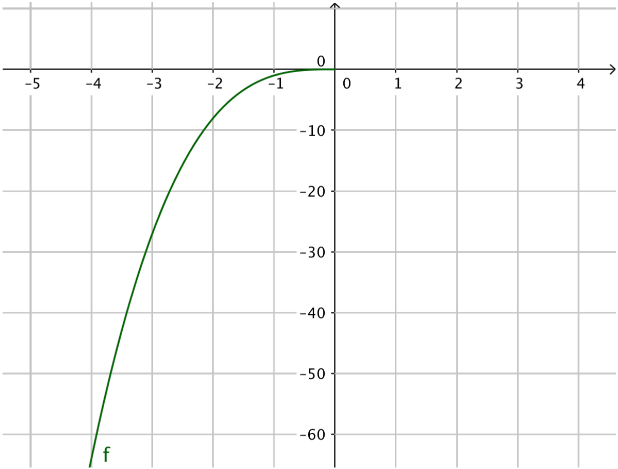
Example of a global minimum

$f : \mathbb{R} \longrightarrow \mathbb{R}; x \mapsto x^2$ has a min at $x = 0$



Example of a global maximum

$f : (-\infty, 0\,] \longrightarrow \mathbb{R}; f(x) = x^3$ has a max at $x = 0$



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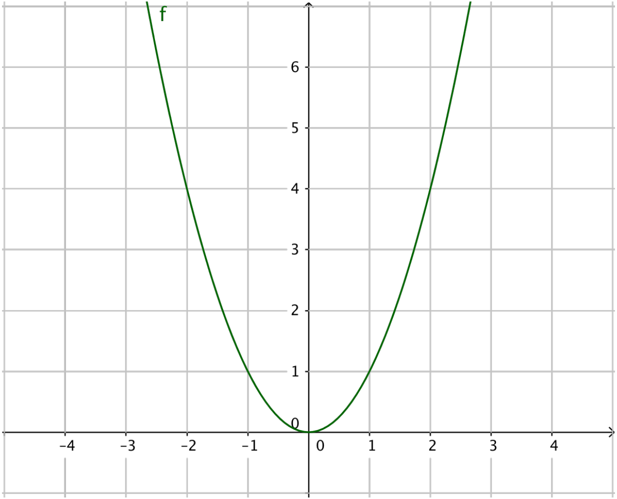
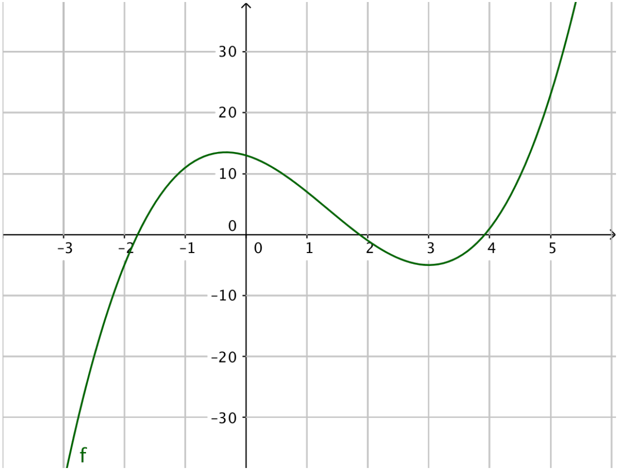


Example of a local maximum

$$f : \mathbb{R} \longrightarrow \mathbb{R}; f(x) = x^3 - 4x^2 - 3x + 13 \text{ has a local max at } x = -\frac{1}{3}$$



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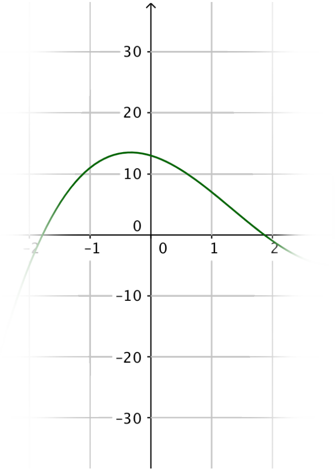

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- $f(x) = \sin x$ has a critical point at $x = \frac{\pi}{2}$ (since $f'(x) = \cos x$)
- $f(x) = e^x$ doesn't have any critical points since $f'(x) = e^x$ can never be zero

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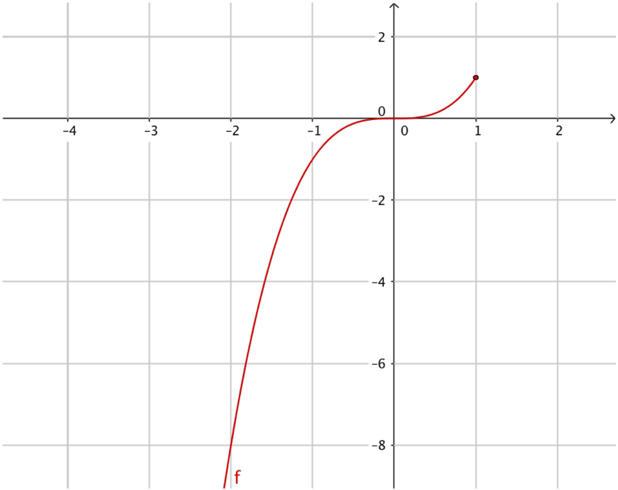
Example

$f : (-\infty, 1] \longrightarrow \mathbb{R}; f(x) = x^3$ has critical points at

$$x = 0 \text{ and } 1$$

Example

$f'(x) = 3x^2$ so $f'(0) = 0$ and $f'(1)$ is undefined.



First derivative test

Suppose $x = a$ is a critical point for the function $f(x)$.

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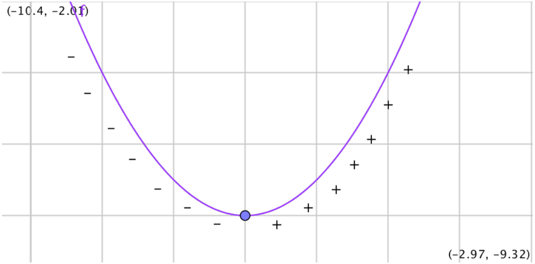

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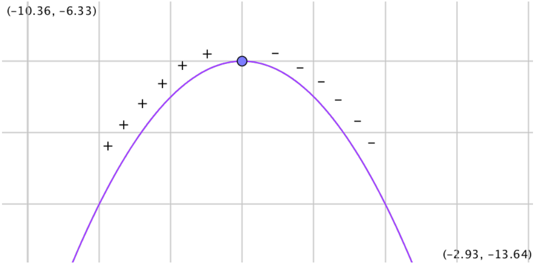

Second derivative test

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Note: If $f''(a) = 0$ then we cannot conclude anything! E.g x^3 or x^4.

Recipe for classifying maxima and minima

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5. The lartgest value is the global max unless M is larger, in which case there is no global max

Example

$f(x)$ defined on $(-\infty, \infty)$ with

$$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 0$$