

Math 32B: Problem Set 2

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This week on the problem set we will see examples of integrals over more general regions.

You will only need to hand in a small selection of the questions for homework, however I recommend that you at least attempt them all by the end of the quarter as some may appear on exams!

Homework: due Friday 10 April, uploaded to Gradescope before 11:59pm. It will consist of questions 3, 4, 5, and 6 below.

Note that the references to the textbook are for the 4th edition, *late transcendentals* version. Any differences between the 3rd and 4th editions is noted in parentheses.

1. From 16.2 in the textbook: 4, 8, 14, 20, 21, 23, 29, 31, 45, 48, 49 (Question 21 is different in the two versions, but both are fine.).
2. From 16.3 in the textbook: 3, 5, 6, 7.

3. Consider an integral over the domain \mathcal{D} that is the part of the first quadrant bounded by $y = -(x-1)^2 + 1$ and $y = 1/x$. We can write an integral over this domain as: $\int_1^{\frac{1+\sqrt{5}}{2}} \int_{1/x}^{-(x-1)^2+1} f(x, y) dy dx$. Change the order of integration to write this as an integral where you integrate in the order $dx dy$.

4. Consider the function $E(s) = \int_0^s e^{-x^2} dx$. This is an incredibly important function in applied mathematics (and therefore physics, chemistry, etc). Unfortunately it is impossible to express the antiderivative of e^{-x^2} in terms of functions you already know. So how can we calculate $E(s)$? It turns out, that its value at infinity,

$$E(\infty) := \lim_{s \rightarrow \infty} E(s) = \int_0^\infty e^{-x^2} dx,$$

can be calculated using a trick which this question will guide you through. In fact, we will calculate $E(\infty)^2$.

- (a) Express $E(\infty)^2 = \left(\int_0^\infty e^{-x^2} dx \right) \left(\int_0^\infty e^{-y^2} dy \right)$ as a double integral and therefore as an iterated integral, in the order $dx dy$. Make sure to describe the region in \mathbb{R}^2 we are integrating over precisely. *Hint: consider the separation of variables formula.*
 - (b) Use the change of variables $t = x/y$ to transform the inner integral. Express $E(\infty)^2$ as an iterated integral in the order $dy dt$.
 - (c) Evaluate the iterated integral.
 - (d) Determine whether $E(\infty)$ is positive or negative. Find the value of $E(\infty)$.
 - (e) Explain why this method does not allow you to calculate $E(s)$ for more general $s < \infty$.
5. Find the volume of the region bounded by $y = 1 - x^2$, $z + y = 1$, $y = 0$ and $4z + 4y + x = 12$.
 6. Compute the integral $\iiint_{\mathcal{W}} xy dV$ where \mathcal{W} is the part of the first octant inside the elliptical cylinder $(x/2)^2 + (z/3)^2 = 1$ and inside the ellipsoid $(x/4)^2 + (y/4)^2 + (z/5)^2 = 1$.