

Math 3B: Lecture 6

Noah White

October 11, 2017

Introduction

Last time

- Examples of maxima/minima problems

Quiz 3

Introduction

Last time

- Examples of maxima/minima problems
- Maximising functions with constraints

Quiz 3

Introduction

Last time

- Examples of maxima/minima problems
- Maximising functions with constraints
- Competing economies

Quiz 3

Introduction

Last time

- Examples of maxima/minima problems
- Maximising functions with constraints
- Competing economies

Quiz 3

- Arrive on time

Introduction

Last time

- Examples of maxima/minima problems
- Maximising functions with constraints
- Competing economies

Quiz 3

- Arrive on time
- You will have 10 mins

Introduction

Last time

- Examples of maxima/minima problems
- Maximising functions with constraints
- Competing economies

Quiz 3

- Arrive on time
- You will have 10 mins
- Look at questions 1-8 on PS3

Differential equations (motivation)

A **differential equation** is an equation that involves derivatives of an unknown function.

$$\frac{d^2y}{dx^2} = y - 3y^2$$

or

$$x^2y'' + xy' + x^2y = 0$$

Differential equations (motivation)

A **differential equation** is an equation that involves derivatives of an unknown function.

$$\frac{d^2y}{dx^2} = y - 3y^2$$

or

$$x^2y'' + xy' + x^2y = 0$$

The **challenge** is to find all the functions $y = f(x)$ (or even just one) that satisfy a given equation.

Newton's second law (motivation)

The original differential equation!

$$F = ma$$

Newton's second law (motivation)

The original differential equation!

$$F = ma$$

If $h(t)$ measures the height of an object (maybe an apple?) above the earth then

$$a = h''(t)$$

Newton's second law (motivation)

The original differential equation!

$$F = ma$$

If $h(t)$ measures the height of an object (maybe an apple?) above the earth then

$$a = h''(t)$$

The force due to gravity is roughly $-10m$ Newtons, so

$$-10m = mh''(t)$$

Population growth (motivation)

A very rudimentary (but sometimes surprisingly accurate) way to model population growth is

Population growth (motivation)

A very rudimentary (but sometimes surprisingly accurate) way to model population growth is

Assumption

The rate of growth of a population is proportional to its current size

Population growth (motivation)

A very rudimentary (but sometimes surprisingly accurate) way to model population growth is

Assumption

The rate of growth of a population is proportional to its current size

Population growth (motivation)

A very rudimentary (but sometimes surprisingly accurate) way to model population growth is

Assumption

The rate of growth of a population is proportional to its current size

If $P(t)$ is the population at time t :

$$\frac{dP}{dt} = rP(t)$$

Some more examples of differential equations

$$\frac{dy}{dx} = f(x)$$

$$\frac{dy}{dx} = y(1 - y)$$

$$y'' = \sqrt{a^2 - (y')^2}$$

$$\frac{dy}{dt} = k(A - y)^2$$

Antiderivatives

We will be concentrating on solving differential equations of the form

$$\frac{dy}{dx} = f(x)$$

Antiderivatives

We will be concentrating on solving differential equations of the form

$$\frac{dy}{dx} = f(x)$$

A solution $y = F(x)$ is called an **antiderivative** of $f(x)$.

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2$$

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2 + 4$$

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2 + 8$$

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2 + C$$

Example 2

Question

What is the antiderivative of $f(x) = x^3 + 4x - 1$?

Example 2

Question

What is the antiderivative of $f(x) = x^3 + 4x - 1$?

Solution

$$F(x) = \frac{1}{4}x^4$$

Example 2

Question

What is the antiderivative of $f(x) = x^3 + 4x - 1$?

Solution

$$F(x) = \frac{1}{4}x^4 + 2x^2$$

Example 2

Question

What is the antiderivative of $f(x) = x^3 + 4x - 1$?

Solution

$$F(x) = \frac{1}{4}x^4 + 2x^2 - x$$

Example 2

Question

What is the antiderivative of $f(x) = x^3 + 4x - 1$?

Solution

$$F(x) = \frac{1}{4}x^4 + 2x^2 - x + C$$

Example 3

Question

What is the antiderivative of $f(x) = e^{2x}$?

Example 3

Question

What is the antiderivative of $f(x) = e^{2x}$?

Solution

$$F(x) = \frac{1}{2}e^{2x}$$

Example 4

Question

What is the antiderivative of $f(x) = \frac{1}{x}$ (for $x > 0$)?

Example 4

Question

What is the antiderivative of $f(x) = \frac{1}{x}$ (for $x > 0$)?

Solution

$$F(x) = \ln x$$

Example 5

Question

What is the antiderivative of $f(x) = \frac{1}{(1+x)^2}$?

Example 5

Question

What is the antiderivative of $f(x) = \frac{1}{(1+x)^2}$?

Solution

Note that $f(x) = (1+x)^{-2}$. So

$$F(x) = \frac{1}{1+x}$$

Example 5

Question

What is the antiderivative of $f(x) = \frac{1}{(1+x)^2}$?

Solution

Note that $f(x) = (1+x)^{-2}$. So

$$F(x) = -\frac{1}{1+x}$$

Example 6

Question

What is the antiderivative of $f(x) = 2x \cos x^2$?

Example 6

Question

What is the antiderivative of $f(x) = 2x \cos x^2$?

Solution

$$F(x) = \sin x^2$$

Example 7

Question

What is the antiderivative of $f(x) = \frac{1}{\sqrt{x}}$?

Example 7

Question

What is the antiderivative of $f(x) = \frac{1}{\sqrt{x}}$?

Solution

$$f(x) = x^{-\frac{1}{2}}$$

Example 7

Question

What is the antiderivative of $f(x) = \frac{1}{\sqrt{x}}$?

Solution

$$f(x) = x^{-\frac{1}{2}}$$

Example 7

Question

What is the antiderivative of $f(x) = \frac{1}{\sqrt{x}}$?

Solution

$$f(x) = x^{-\frac{1}{2}}$$

$$F(x) = x^{\frac{1}{2}}$$

Example 7

Question

What is the antiderivative of $f(x) = \frac{1}{\sqrt{x}}$?

Solution

$$f(x) = x^{-\frac{1}{2}}$$

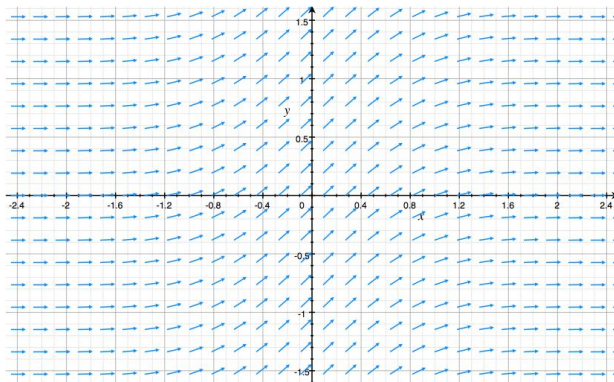
$$F(x) = 2x^{\frac{1}{2}}$$

Slope fields

In some cases it is impossible to find the antiderivative (without special functions). E.g.

$$f(x) = e^{-x^2}$$

But we can still (approximately) graph the antiderivative! First we draw the slope field



Slope fields (how to draw)

How to draw a slope field for

$$\frac{dy}{dx} = f(x)$$

1. Draw the xy -plane.

Slope fields (how to draw)

How to draw a slope field for

$$\frac{dy}{dx} = f(x)$$

1. Draw the xy -plane.
2. At every point (x, y) what would the slope of $y = F(x)$ be if it passed through that point?

Slope fields (how to draw)

How to draw a slope field for

$$\frac{dy}{dx} = f(x)$$

1. Draw the xy -plane.
2. At every point (x, y) what would the slope of $y = F(x)$ be if it passed through that point?
3. Answer given by differential equation above, slope is $f(x)$

Slope fields (how to draw)

How to draw a slope field for

$$\frac{dy}{dx} = f(x)$$

1. Draw the xy -plane.
2. At every point (x, y) what would the slope of $y = F(x)$ be if it passed through that point?
3. Answer given by differential equation above, slope is $f(x)$
4. Draw a small arrow with slope $f(x)$ and the point (x, y)

Slope fields (how to draw)

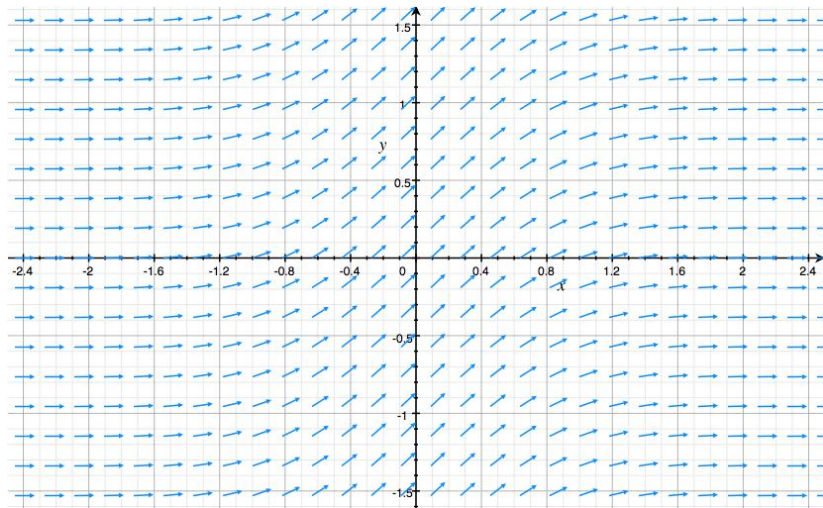
How to draw a slope field for

$$\frac{dy}{dx} = f(x)$$

1. Draw the xy -plane.
2. At every point (x, y) what would the slope of $y = F(x)$ be if it passed through that point?
3. Answer given by differential equation above, slope is $f(x)$
4. Draw a small arrow with slope $f(x)$ and the point (x, y)
5. Do this for a grid of points on the xy -plane.

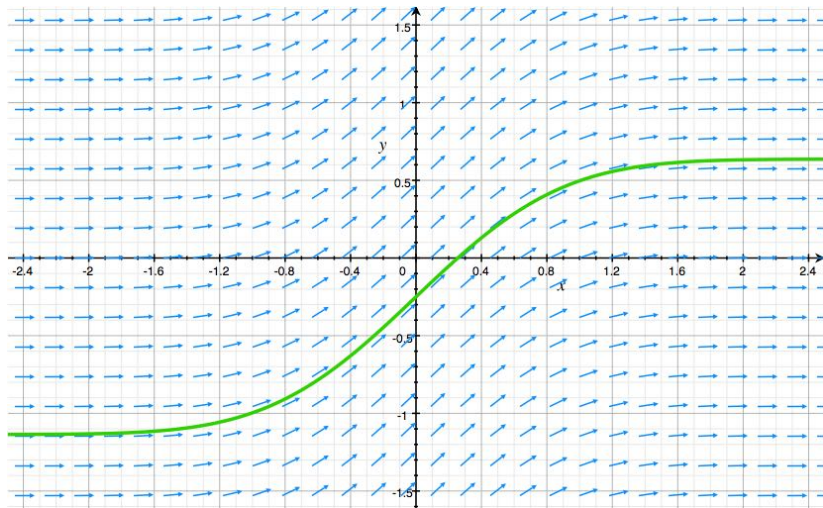
Example 1

$$f(x) = e^{-x^2}$$



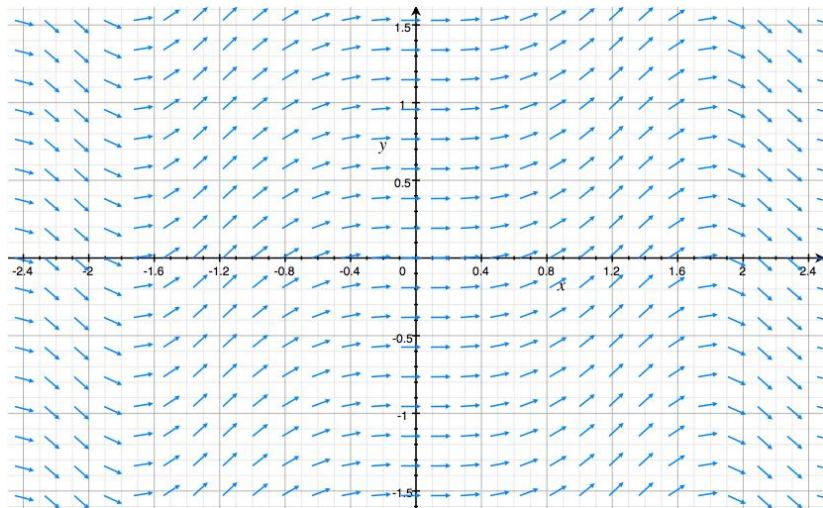
Example 1

$$f(x) = e^{-x^2}$$



Example 2

$$f(x) = \sin(x^2)$$



Example 2

$$f(x) = \sin(x^2)$$

