This week on the problem set we will review the basics of vectors and calculus of vector valued functions and functions with multiple inputs. This should be familiar to you from Math 32A or other equivalent calculus courses you have taken. Make sure you can do these types of routine questions. Problems 8 to 14 will give you some practice with double integrals.

*Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3rd Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. Calculate the following dot products of vectors.
   (a) $(1,2,3)\cdot(3,2,1)$
   (b) $(1,1)\cdot(1,-1)$
   (c) $(1,0,1)\cdot(0,4,0)$
   (d) $(-1,2,1)\cdot(2,1,-1)$
2. Are the following vectors orthogonal to each other?
   (a) $(1,0,0)$ and $(0,0,1)$
   (b) $(1,1)$ and $(2,2)$
   (c) $(1,0,3,-2,1,1)$ and $(0,-3,1,2,2,-1)$
   (d) $(-1,2,1)$ and $(2,1,-1)$
3. Calculate the following cross products.
   (a) $(1,2,3)\times(3,2,1)$
   (b) $(1,1,1)\times(1,-1,1)$
   (c) $(1,0,1)\times(0,4,0)$
   (d) $(-1,2,1)\times(-2,4,2)$
4. Does the vector $(-1,-4,1)$ lie in the plane spanned by the vectors $(1,0,-2)$ and $(0,4,1)$?
5. Calculate the tangent lines at $t=0$ to the curves described by the following vector valued functions.
   (a) $f(t)=(\cos t, t, \sin t)$
   (b) $f(t)=(t,t^2,t^3)$
6. Calculate the tangent planes at $(0,0)$ to the graph of the following functions.
   (a) $f(x,y)=\sin x+\sin y$
   (b) $g(x,y)=x^2+y^2-2y+1$
7. Calculate the following partial/directional derivatives for $f(x,y)=\sin x+\sin y$.
   (a) $\partial_x f$
   (b) $\partial_{\mathbf{u}} f$ for $\mathbf{u}=(1,1)$
   (c) $\partial_{\mathbf{u}} f$ for $\mathbf{u}=(-1,-1)$
   (d) $\partial_{\mathbf{u}} f$ for $\mathbf{u}=(1,-1)$
8. (16.1.7) Let $\mathcal{R}=[0,1]\times[0,1]$. Estimate $\iint_{\mathcal{R}}(x+y)\,\mathrm{d}A$ by computing two different Riemann sums, each with at least six rectangles.
9. (16.1.8) Evaluate $\iint_{\mathcal{R}} 4\,\mathrm{d}A$ where $\mathcal{R}=[2,5]\times[4,7]$.

10. (16.1.9) Evaluate $\iint_{\mathcal{R}} (15 - 3x)\, \mathrm{d}A$ where $\mathcal{R} = [0, 5] \times [0, 3]$, and sketch the corresponding solid region.

11. (16.1) Use symmetry to evaluate the double integral.

    (a) (16.1.15) $\iint_{\mathcal{R}} x^3\, \mathrm{d}A$ where $\mathcal{R} = [-4, 4] \times [0, 5]$.

    (b) (16.1.17) $\iint_{\mathcal{R}} \sin x\, \mathrm{d}A$ where $\mathcal{R} = [0, 2\pi] \times [0, 2\pi]$.

12. (16.1) In the following exercises evaluate the iterated integral.

    (a) (16.1.19) $\int_1^3 \int_0^2 x^3 y\, \mathrm{d}y\, \mathrm{d}x$

    (b) (16.1.23) $\int_{-1}^1 \int_0^\pi x^2 \sin y\, \mathrm{d}y\, \mathrm{d}x$

13. (16.1) In the following exercises evaluate the double integral.

    (a) (16.1.39) $\iint_{\mathcal{R}} \cos x \sin 2y\, \mathrm{d}A$, $\mathcal{R} = \left[0, \frac{\pi}{2}\right] \times \left[0, \frac{\pi}{2}\right]$

    (b) (16.1.41) $\iint_{\mathcal{R}} e^x \sin y\, \mathrm{d}A$, $\mathcal{R} = [0, 2] \times \left[0, \frac{\pi}{4}\right]$

14. (16.1.47) Evaluate $\int_0^1 \int_0^1 \frac{y}{1 + xy}\, \mathrm{d}y\, \mathrm{d}x$, *Hint: Change the order of integration.*