This week on the problem set you will get practice thinking about potential functions and calculating line integrals.

Homework: The homework will be due on Wednesday 27 May. It will consist of questions $3, 4$ and 5 below. *Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3rd Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. (Section 17.4) 2, 3, 5, 8, 9, 10, 13, 14, 17, 18, 27, 30, 34, 37, 40, 41*, 46* 48*. (questions are the same in previous versions)

2. (Section 17.5) 1, 6, 7, 12, 17, 18, 21, 22, 31*, 35. (questions are the same in previous versions)

3. Consider the line segment $(x, 0, 0)$ where $x \in [-1, 1]$ in $\mathbb{R}^3$. Imagine this line segment moving up with its centre on the z-axis, rotating parallel to the xy-plane at constant speed. It completes one full revolution when it gets to $z = 2\pi$. What surface area is swept out by the rotating line segment? You may wish to use the fact that
$$\frac{d}{dt}\left(t\sqrt{1+t^2} + \sinh^{-1}(t)\right) = 2\sqrt{1+t^2}$$
and that $\sinh^{-1} t$ is an odd function and $\sinh^{-1}(1) = \ln(1+\sqrt{2})$.

4. The velocity vector field of a fluid is given by $\mathbf{F}(x, y, z) = \dfrac{\langle x, y, z\rangle}{x^2 + y^2 + z^2}$ measured in meters per second. What is the volume of fluid flowing each second through the open cylinder of radius 1 and height 1 centered along the z-axis, i.e. the cylinder $C = \{(x, y, z) : x^2 + y^2 = 1, 0z \leq 1\}$?

5. Let $\mathbf{F}\left\langle y(ye^{x+y^2} - 1) + x^2, 2y(1+y^2)e^{x+y^2} + x\right\rangle$ and let $\mathcal{C}$ be the portion of $y = 1 - x^2$ oriented left to right.
   (a) Parameterise $\mathcal{C}$ and write $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ as a single integral. Do not try and evaluate.
   (b) Now let $\mathcal{L}$ be the straight line from $(-1, 0)$ to $(1, 0)$ oriented left to right and let $\mathcal{D}$ be the region bounded by the x-axis and $\mathcal{C}$. Use Green's theorem to relate the integrals of $\mathbf{F}$ over $\mathcal{C}$ and $\mathcal{L}$ to an integral over $\mathcal{D}$. Use this to evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.
   (c) **Path (almost)-independence for non-conservative vector fields.** More generally, suppose $\mathcal{C}_1$ and $\mathcal{C}_2$ are two oriented curves with the same endpoints, and $\mathbf{F}$ is a vector field that is defined everywhere on the region $\mathcal{D}$ between $\mathcal{C}_1$ and $\mathcal{C}_2$. If $\mathbf{F}$ is conservative we know that $\displaystyle\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r} - \int_{\mathcal{C}_2} \mathbf{F} \cdot d\mathbf{r} = 0$. If $\mathbf{F}$ is not conservative, what is
$$\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r} - \int_{\mathcal{C}_2} \mathbf{F} \cdot d\mathbf{r}?$$

*The questions marked with an asterisk are more difficult or are of a form that would not appear on an exam. Nonetheless they are worth thinking about as they often test understanding at a deeper conceptual level.