Math 3B: Lecture 26

Noah White

November 23, 2016

Announcements

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- Exam behaviour...

Autonomous equations

Deafinition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

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The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

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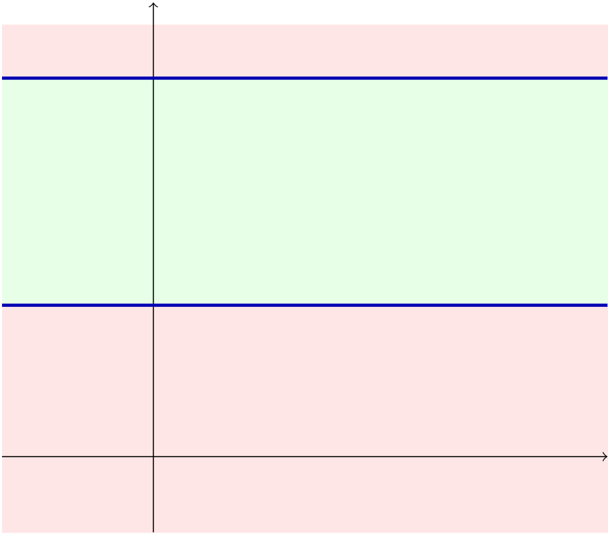
We want points (t, y) such that $f(y) = 0$.

- Suppose $f(a) = 0$.
- Then (t, a) is on the nullcline, for any t.
- So the line $y = a$ is part of the nullcline, whenever $f(a) = 0$.

Slope fields and nullclines for autonomous systems

Thus our slope field and nullclines look something like



Phase lines/diagram

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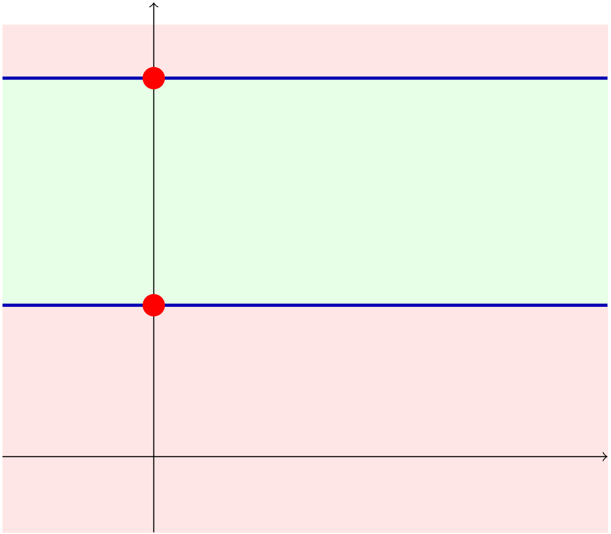

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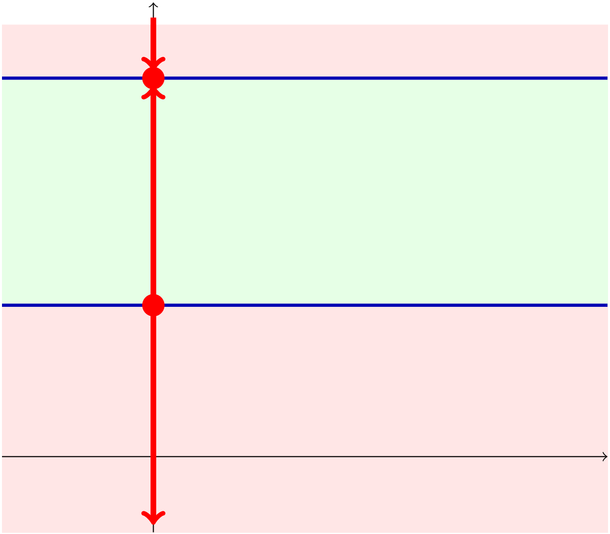

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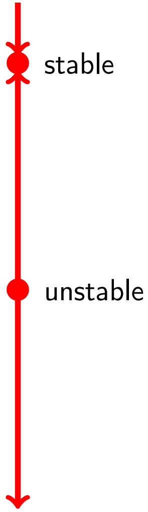

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- It is semistable if the arrows point in the same direction.

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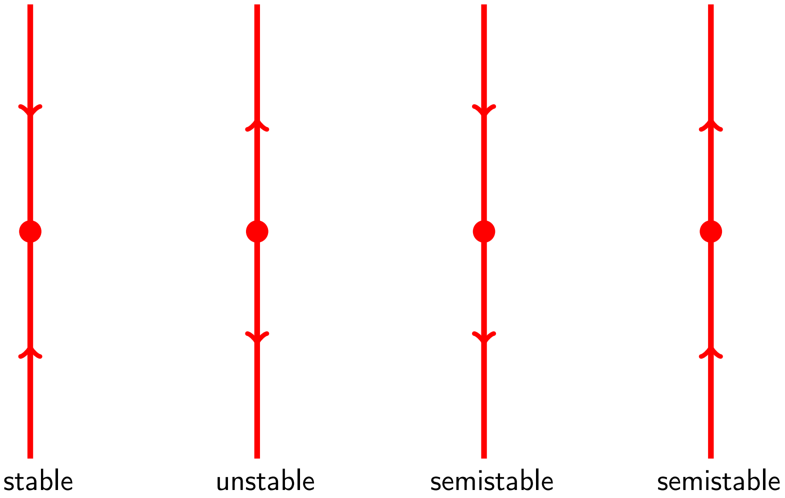

Example

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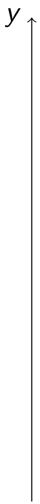

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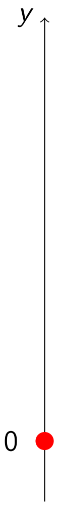

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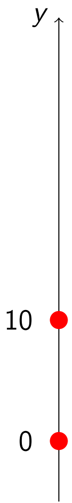

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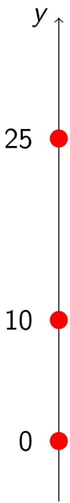

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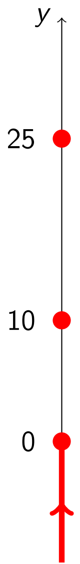

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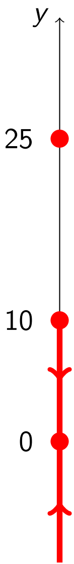

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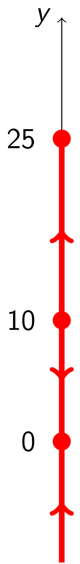

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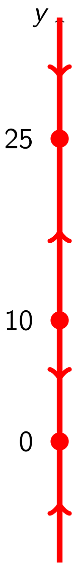

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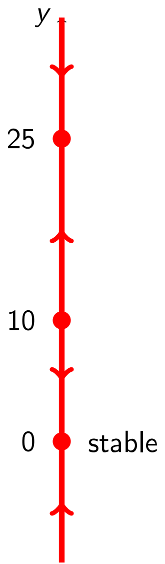

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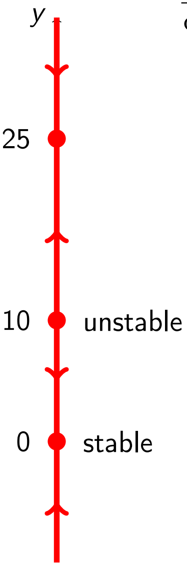

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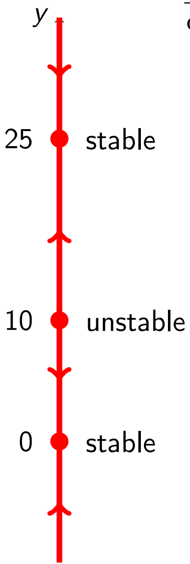

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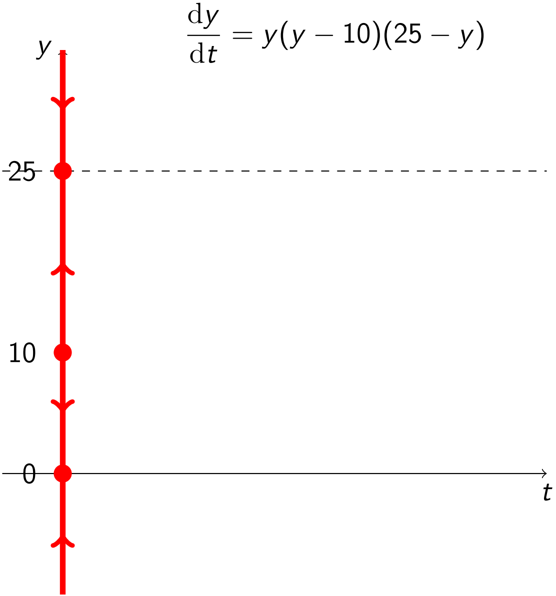

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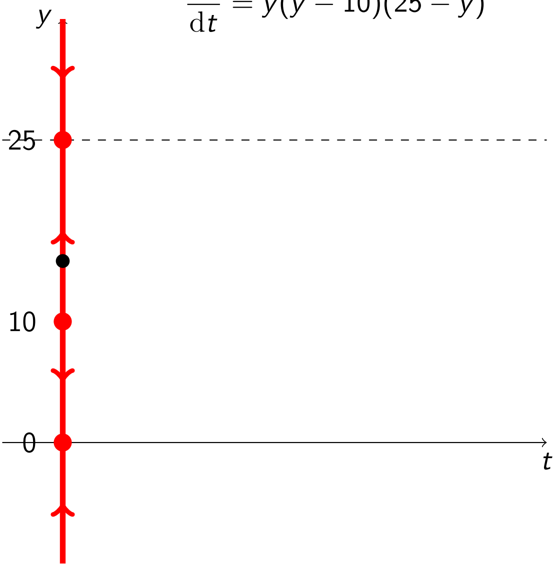

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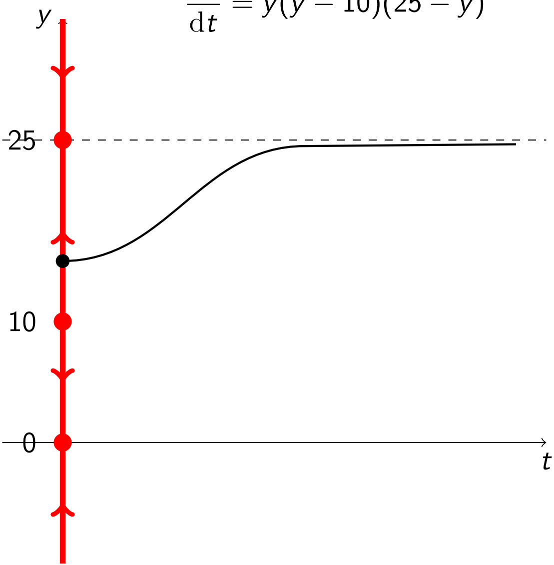

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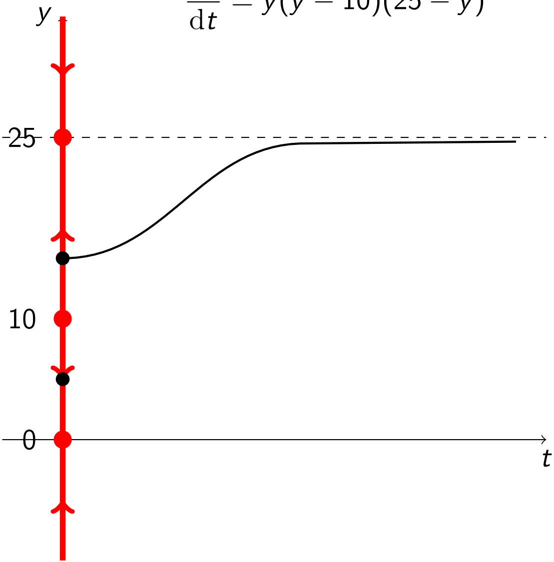

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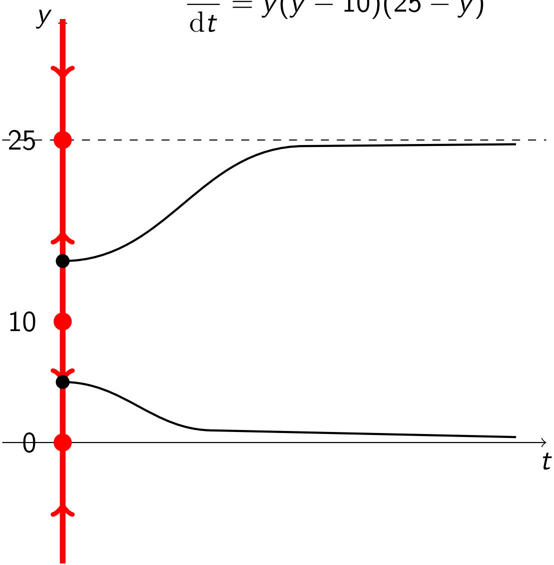

Classifying equilibria using derivatives

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- stable if $f'(a) < 0$

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- stable if $f'(a) < 0$
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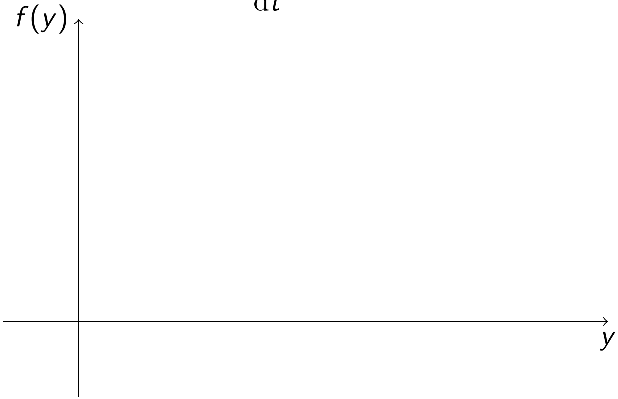
- indeterminate if $f'(a) = 0$

Why?

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$



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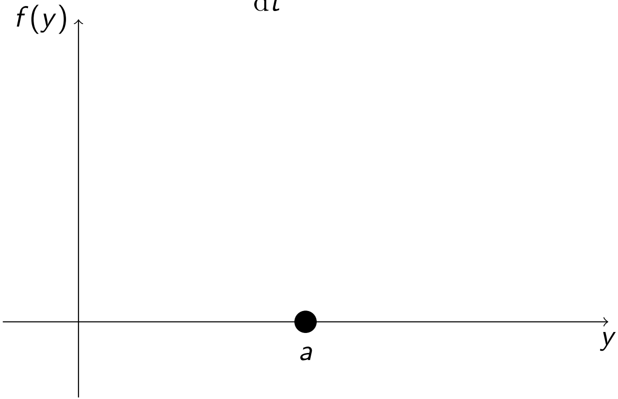

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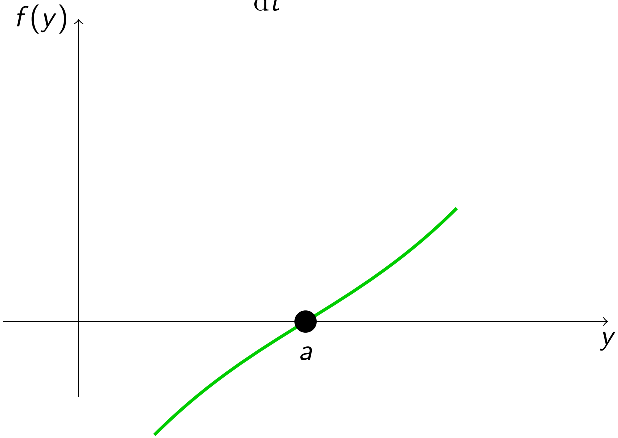

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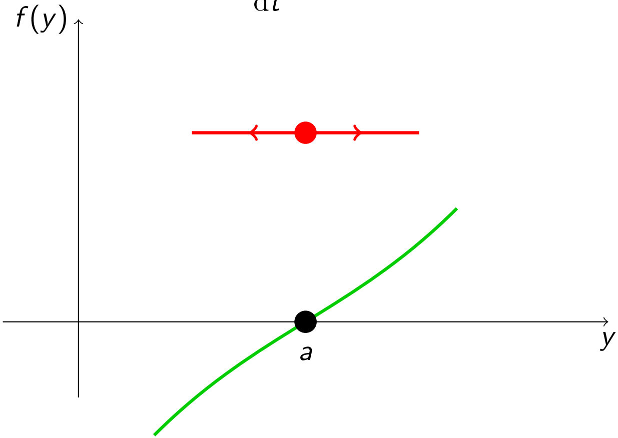

Birfurcation

Often in real life situations we would like to study a system that includes an unknown parameter

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y, a)$$

The behaviour of the solution depends on a!

Example

The queen Conch population we have been studying grows logistically, they are also harvested but we don't know exactly how many are harvested.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = N(1 - N) - h$$

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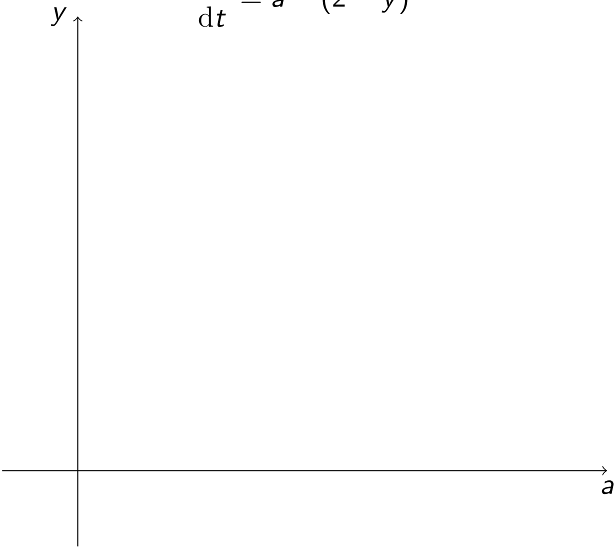
- Plot $f(y, a) = 0$ on the y-a coordinate plane

Bifuration diagram

$$\frac{\mathrm{d}y}{\mathrm{d}t} = a - (2 - y)^2$$



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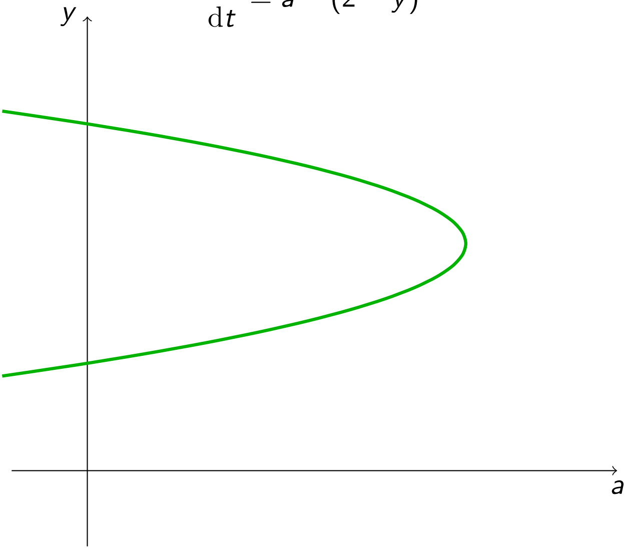

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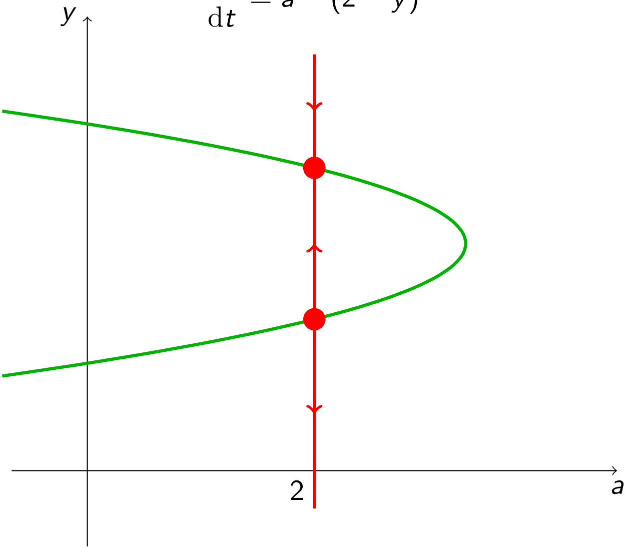

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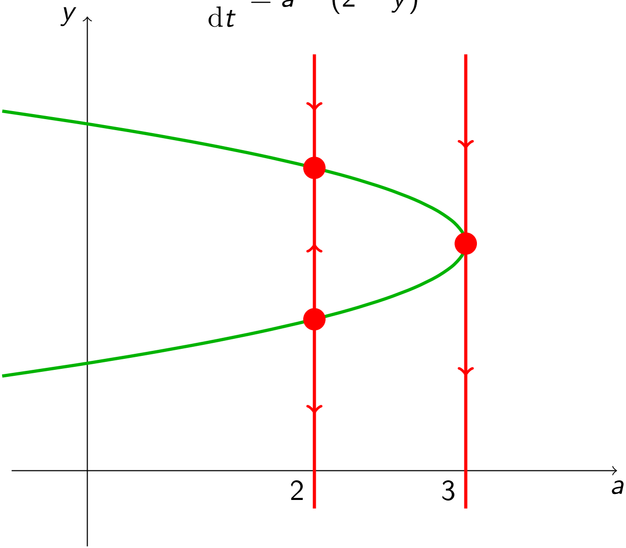

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