Midterm 1

UCLA: Math 3B, Fall 2018

Instructor: Noah White
Date: 22 October 2018

- This exam has 3 questions, for a total of 30 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: Solutions

ID number: ______

Discussion section (please circle):

Day/TA	Ben	Ryan
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	9	
2	11	
3	10	
Total:	30	

Question 1 is multiple choice. Indicate your answers in the table below. *The following three pages will not be graded, your answers must be indicated here.*

Part	A	B	C	D
(a)	X			
(b)		X		
(c)	X			
(d)		X		
(e)			X	
(f)				X
(g)			X	
(h)		X		
(i)	X			

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) The function $f(x) = e^x$ has

   A. a horizontal asymptote at $y = 0$.
   B. a vertical asymptote at $x = 1$.
   C. no asymptotes.
   D. a slanted asymptote with positive slope.

   (b) (1 point) The function $g(x) = (1 + x^2)^{-1}$ has a critical point at

   A. $x = e^{-2}$.
   B. $x = 0$.
   C. $x = 1$.
   D. $x = -1$.

   (c) (1 point) The function $f(x) = \ln(x^2 - 4x + 5)$ has a

   A. local minimum at $x = 2$.
   B. local maximum at $x = 2$.
   C. local maximum at $x = 1$.
   D. local minimum at $x = 1$.

(d) (1 point) An antiderivative of $h(t) = 2t\sin(t^2)$ is given by

A. $\sin(t^2) + 3$
B. $1 - \cos(t^2)$
C. $2t\cos(2t)$
D. $2 + \sin(t^2)$

(e) (1 point) The area $\int_1^3 3 - x^2 \, dx$ can be expressed as the limit as $n \to \infty$ of

A. $\sum_{k=1}^{n} \left(\frac{6}{n} + \frac{4k^2}{n^3} \right)$

B. $\sum_{k=1}^{n} \left(\frac{2}{n} + \frac{k}{n^2} \right)$

C. $\sum_{k=1}^{n} \left(\frac{4}{n} - \frac{8k}{n^2} - \frac{8k^2}{n^3} \right)$

D. $\sum_{k=1}^{n} \left(\frac{2}{n} - \frac{2k}{n^2} - \frac{k^2}{n^3} \right)$

(f) (1 point) Evaluate the definite integral $\int_1^{e^\pi} x^{-1}\sin(\ln x) \, dx$

A. 1
B. π
C. 0
D. 2

(g) (1 point) The function $g(x) = 2e^x - x^2$ has

A. a single local minimum.

B. at least two local minimums.

C. no critical points.

D. a critical point when $x = 0$.

(h) (1 point) Evalute the definite integral $\int_1^2 15x\sqrt{x-1}\,dx$

A. $44\sqrt{2} - 16$

B. 16

C. 2

D. $11\sqrt{2}$

(i) (1 point) Consider the function $f(x) = \max\{0, 2x\}$. An antiderivative of $f(x)$ is given by

A. $x \cdot \max\{0, x\}$

B. $\max\{0, x^2\}$

C. $\max\{0, x\} + 1$

D. x^2

2. Let $f(x) = \frac{x}{\sqrt{x^4+1}}$. Note that $f'(x) = \frac{1-x^4}{(x^4+1)^{3/2}}$ and $f''(x) = \frac{2x^3(x^4-5)}{(x^4+1)^{5/2}}$.

(a) (2 points) Find the x and y intercepts of $f(x)$.

$f(0) = 0$

thus the interceps are

$x = 0 \quad y = 0$

(b) (1 point) Does $f(x)$ have any horizontal asymptotes? If so what are they?

$$\lim_{x \to \pm\infty} \frac{x}{\sqrt{x^4+1}} = \lim_{x \to \pm\infty} \frac{\frac{1}{x}}{\sqrt{1 + 1/x^4}} = 0$$

hor. asymptote @ $y = 0$

(c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

No

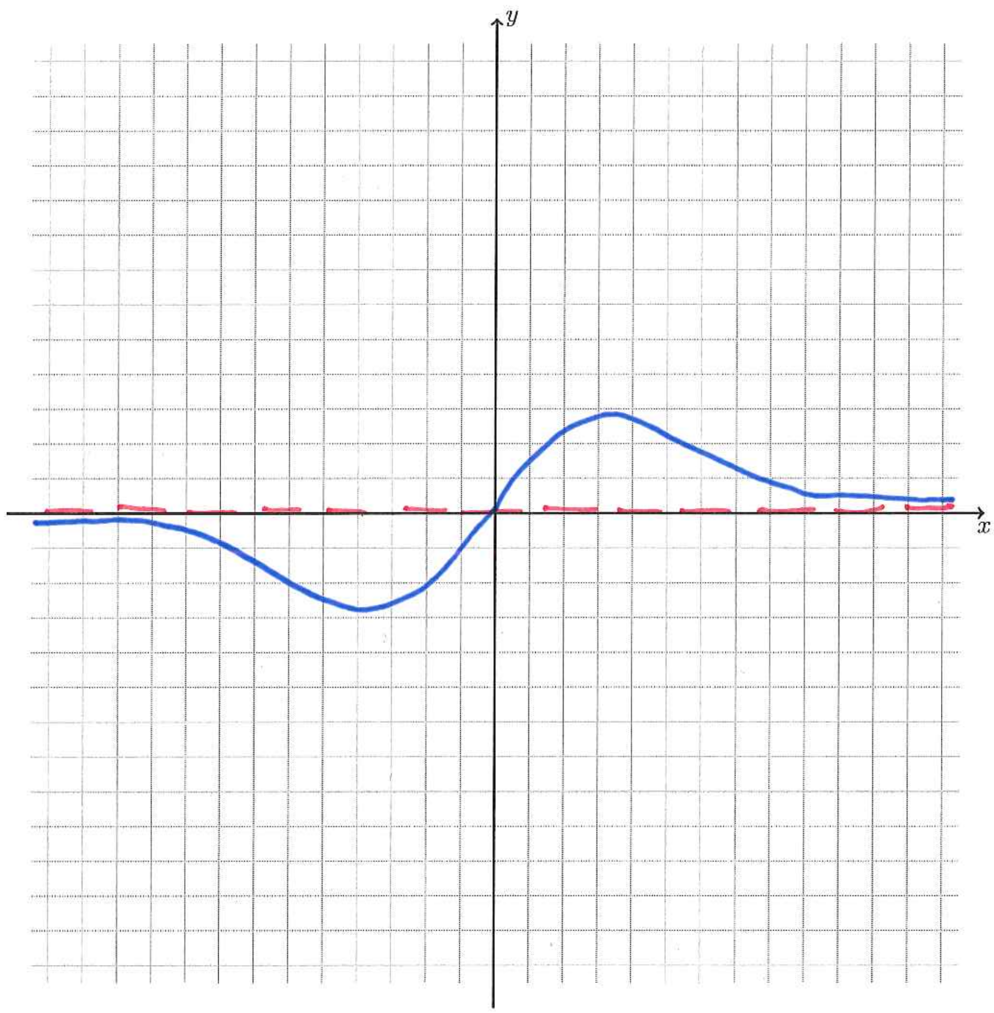
(d) (2 points) For what x is the first derivative $f'(x)$ positive?

$1 - x^4 > 0 \iff 1 > x^4$

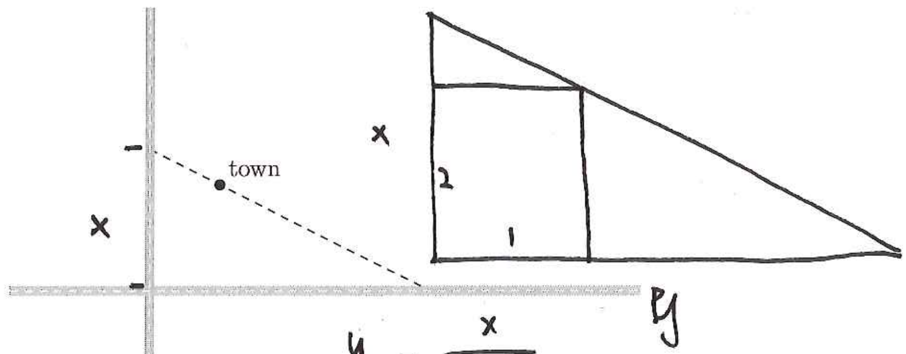
(e) (2 points) For what x is the second derivative $f''(x)$ positive?

$$2x^3(x^4-5) = 0 \Rightarrow x = 0, \pm\sqrt[4]{5}$$

(f) (3 points) On the graph provided, sketch $f(x)$



3. Two straight freeways intersect at right angles. The freeways run North-South and East-West. One mile East, and two miles North of the intersection is a town. A *straight* road is to be built from the North-South freeway, through the town and then onwards to the East-West freeyway.



$$y_1 = \frac{x}{x-2}$$

(a) (3 points) Let x be the distance from the intersection of the two freeways, to where the road branches off the North-South freeway. What is the length of the new road?

$$L(x) = \sqrt{x^2 + \frac{x^2}{(x-2)^2}} \qquad \underline{x > 2}$$

(b) (7 points) How far North of the intersection should the road begin in order to minimise the length of the new road?

First find the critical points of L:

$$L'(x) = \frac{1}{2L(x)} \cdot 2x\left(1 - \frac{2}{(x-2)^3}\right) = 0$$

so $x = 0$ or $(x-2)^3 = 2$ ie $x = 2 + \sqrt[3]{2}$

not in domain

note $3 < 2 + \sqrt[3]{2} < 4$ (since $1 < \sqrt[3]{2} < 2$).

and $L'(3) = -\frac{3}{L(3)} < 0$ $L'(4) = \frac{9/4}{L(4)} > 0$

so $2 + \sqrt[3]{2}$ is a ~~mint~~ minimum.

Since $\lim_{x \to 2} L(x) = \lim_{x \to \infty} L(x) = \infty$ it is a global minimum

$$x = 2 + \sqrt[3]{2}.$$