Math 3B: Lecture 5

Noah White

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Last time

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- global and local extrema

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- critical points

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- global and local extrema
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- how to find local extrema

Recipe for classifying maxima and minima

We have a function $f : D \longrightarrow R$. How do we find all local/global extrema?

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4. The largest value is the global max

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5. The largest value is the global max unless L or M are larger, in which case there is no global max

Example

$f(x)$ defined on $(-\infty, \infty)$ with

$$L = \lim_{x \to -\infty} f(x) = M = \lim_{x \to \infty} f(x) = 0$$