Math 3B: Lecture 15

Noah White

February 15, 2017

Checking solutions

The most straighforward way of checking a function $y = f(x)$ is a solution to a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(x, y)$$

is to simply plug it in to both sides.

Checking solutions

The most straighforward way of checking a function $y = f(x)$ is a solution to a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(x, y)$$

is to simply plug it in to both sides.

Example

The function $y = e^{\sin x}$ is a solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = y \cos x$. To check note that

Checking solutions

The most straightforward way of checking a function $y = f(x)$ is a solution to a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(x, y)$$

is to simply plug it in to both sides.

Example

The function $y = e^{\sin x}$ is a solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = y \cos x$. To check note that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{\sin x} \cos x$$

Checking solutions

The most straightforward way of checking a function $y = f(x)$ is a solution to a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(x, y)$$

is to simply plug it in to both sides.

Example

The function $y = e^{\sin x}$ is a solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = y \cos x$. To check note that

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= e^{\sin x} \cos x \\ y \cos x &= e^{\sin x} \cos x \end{aligned}$$

Reminder: Implicit differentiation

If we have an equation relating variables y and x, e.g.

$$x^2 + y^2 = 1$$

we can differentiate implicitly by applying $\frac{\mathrm{d}}{\mathrm{d}x}$ to both side.

Reminder: Implicit differentiation

If we have an equation relating variables y and x, e.g.

$$x^2 + y^2 = 1$$

we can differentiate implicitly by applying $\frac{\mathrm{d}}{\mathrm{d}x}$ to both side.

Example

In the above example we get

Reminder: Implicit differentiation

If we have an equation relating variables y and x, e.g.

$$x^2 + y^2 = 1$$

we can differentiate implicitly by applying $\frac{\mathrm{d}}{\mathrm{d}x}$ to both side.

Example

In the above example we get

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2 + y^2) = \frac{\mathrm{d}}{\mathrm{d}x}1$$

Reminder: Implicit differentiation

If we have an equation relating variables y and x, e.g.

$$x^2 + y^2 = 1$$

we can differentiate implicitly by applying $\frac{\mathrm{d}}{\mathrm{d}x}$ to both side.

Example

In the above example we get

$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2 + y^2) = \frac{\mathrm{d}}{\mathrm{d}x}1$$
$$\frac{\mathrm{d}}{\mathrm{d}x}(x^2) + \frac{\mathrm{d}}{\mathrm{d}x}(y^2) = 0$$

Reminder: Implicit differentiation

If we have an equation relating variables y and x, e.g.

$$x^2 + y^2 = 1$$

we can differentiate implicitly by applying $\frac{\mathrm{d}}{\mathrm{d}x}$ to both side.

Example

In the above example we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}(x^2 + y^2) &= \frac{\mathrm{d}}{\mathrm{d}x}1 \\ \frac{\mathrm{d}}{\mathrm{d}x}(x^2) + \frac{\mathrm{d}}{\mathrm{d}x}(y^2) &= 0 \\ 2x + 2y'y &= 0\end{aligned}$$

Another example

Lets differentiate

$$3x + \cos y = xy$$

To do this we apply $\frac{\mathrm{d}}{\mathrm{d}x}$ to both sides:

Another example

Lets differentiate

$$3x + \cos y = xy$$

To do this we apply $\frac{\mathrm{d}}{\mathrm{d}x}$ to both sides:

$$\frac{\mathrm{d}}{\mathrm{d}x}(3x + \cos y) = \frac{\mathrm{d}}{\mathrm{d}x}(xy)$$

Another example

Lets differentiate

$$3x + \cos y = xy$$

To do this we apply $\frac{\mathrm{d}}{\mathrm{d}x}$ to both sides:

$$\frac{\mathrm{d}}{\mathrm{d}x}(3x + \cos y) = \frac{\mathrm{d}}{\mathrm{d}x}(xy)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(3x) + \frac{\mathrm{d}}{\mathrm{d}x}(\cos y) = y\frac{\mathrm{d}}{\mathrm{d}x}(x) + x\frac{\mathrm{d}}{\mathrm{d}x}(y)$$

Another example

Lets differentiate

$$3x + \cos y = xy$$

To do this we apply $\frac{\mathrm{d}}{\mathrm{d}x}$ to both sides:

$$\frac{\mathrm{d}}{\mathrm{d}x}(3x + \cos y) = \frac{\mathrm{d}}{\mathrm{d}x}(xy)$$
$$\frac{\mathrm{d}}{\mathrm{d}x}(3x) + \frac{\mathrm{d}}{\mathrm{d}x}(\cos y) = y\frac{\mathrm{d}}{\mathrm{d}x}(x) + x\frac{\mathrm{d}}{\mathrm{d}x}(y)$$
$$3 - y' \sin y = y + xy'.$$

Another example

Lets differentiate

$$3x + \cos y = xy$$

To do this we apply $\frac{\mathrm{d}}{\mathrm{d}x}$ to both sides:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}(3x + \cos y) &= \frac{\mathrm{d}}{\mathrm{d}x}(xy) \\
\frac{\mathrm{d}}{\mathrm{d}x}(3x) + \frac{\mathrm{d}}{\mathrm{d}x}(\cos y) &= y\frac{\mathrm{d}}{\mathrm{d}x}(x) + x\frac{\mathrm{d}}{\mathrm{d}x}(y) \\
3 - y' \sin y &= y + xy'.
\end{aligned}$$

Note

We can rearrange this to get

$$y' = \frac{3 - y}{x + \sin y}$$

a differential equation. Whatever y is, as long as it obeys the above relation, it is a solution to this ODE!

Separation of variables

Separation of variables is a very powerful technique for solving ODEs. It works, whenever we can write out ODE in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(y)g(x)$$

Separation of variables

Separation of variables is a very powerful technique for solving ODEs. It works, whenever we can write out ODE in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(y)g(x)$$

It works by

1. moving all the y's over to the left:

$$\frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x} = g(x)$$

Separation of variables

Separation of variables is a very powerful technique for solving ODEs. It works, whenever we can write out ODE in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(y)g(x)$$

It works by

1. moving all the y's over to the left:
$$\frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x} = g(x)$$
2. integrating both sides (with respect to x):
$$\int \frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x = \int g(x)\,\mathrm{d}x.$$

Separation of variables

Separation of variables is a very powerful technique for solving ODEs. It works, whenever we can write out ODE in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(y)g(x)$$

It works by

1. moving all the y's over to the left:
$$\frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x} = g(x)$$
2. integrating both sides (with respect to x):
$$\int \frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x = \int g(x)\,\mathrm{d}x.$$
3. we can use the integration by substitution formula to rewrite the left hand side:
$$\int \frac{1}{f(y)}\,\mathrm{d}y = \int g(x)\,\mathrm{d}x.$$

Separation of variables

Separation of variables is a very powerful technique for solving ODEs. It works, whenever we can write out ODE in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(y)g(x)$$

It works by

1. moving all the y's over to the left:
$$\frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x} = g(x)$$
2. integrating both sides (with respect to x):
$$\int \frac{1}{f(y)}\frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x = \int g(x)\,\mathrm{d}x.$$
3. we can use the integration by substitution formula to rewrite the left hand side:
$$\int \frac{1}{f(y)}\,\mathrm{d}y = \int g(x)\,\mathrm{d}x.$$
4. solve for y!

Examples

On the board...