

This week on the problem set you will get practice at calculating integrals using substitution and integration by parts.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (5.3) Express the limits as definite integrals of the form $\int_0^1 f(x) \, dx$.

- (a) (5.3.1) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i}{n^2}$
- (b) (5.3.5) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 - \frac{i^2}{n^2}\right) \frac{1}{n}$
- (c) (5.3.6) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\frac{\pi i}{n} - \pi\right) \frac{\pi}{n}$

2. (5.3) Express the definite integrals as limits of Riemann sums.

- (a) (5.3.8) $\int_{-1}^1 (x^2 - x) \, dx$
- (b) (5.3.9) $\int_0^1 e^x \, dx$
- (c) (5.3.11) $\int_{-1}^1 |x| \, dx$

3. (5.5) Calculate the following integrals using substitution.

- (a) (5.5.12) $\int \frac{x}{\sqrt{x^2+1}} \, dx$
- (b) (5.5.14) $\int \sin^3 t \cos t \, dt$
- (c) (5.5.16) $\int \frac{z^3}{\sqrt{z^4+12}} \, dz$
- (d) (5.5.19) $\int_1^2 \frac{e^{1/x}}{x^2} \, dx$
- (e) (5.5.23) $\int_1^2 x\sqrt{x-1} \, dx$
- (f) (5.5.24) $\int_0^2 (e^x - e^{-x})^2 dx$

4. (5.5-30) Suppose an environmental study indicates that the ozone level, L , in the air above a major metropolitan center is changing at a rate modeled by the function

$$L'(t) = \frac{0.24 - 0.03t}{\sqrt{36 + 16t - t^2}}$$

parts per million per hour (ppm/h) t hours after 7:00 A.M.

- (a) Express the ozone level $L(t)$ as a function of t if L is 4 ppm at 7:00 A.M.
 - (b) Find the time between 7:00 A.M. and 7:00 P.M. when the highest level of ozone occurs. What is the highest level? (*Note: part b has been changed slightly from what is written in the textbook.*)
5. The circle $x^2 + (y+1)^2 = 4$ has area 4π . What is the area of the portion of the circle lying above the x axis?

You may use the fact that

$$\int \sqrt{1-t^2} \, dt = \frac{1}{2} \left(t\sqrt{1-t^2} + \sin^{-1} t \right) + C.$$

6. (5.6) Calculate the following integrals using integration by parts.

- (a) (2) $\int e^t \sin t \, dt$
- (b) (6) $\int x^2 \ln x \, dx$

(c) (9) $\int \sin x \cos x \, dx$

(d) (14) $\int_0^\pi x \sin x \, dx$

(e) (16) $\int_1^e x^3 \ln x \, dx$

7. Use the fundamental theorem of calculus and the interpretation of the definite integral as an area to find a formula for the general antiderivative of the function $f(x) = |x|$.
8. Use the fundamental theorem of calculus and the interpretation of the definite integral as an area to find a formula for the general antiderivative of the function $f(x) = \frac{1}{x}$.