Math 3B: Lecture 4

Noah White

September 30, 2016

Problem set 2 and homework

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- The homework will be problems 2(a), 3, and 8.

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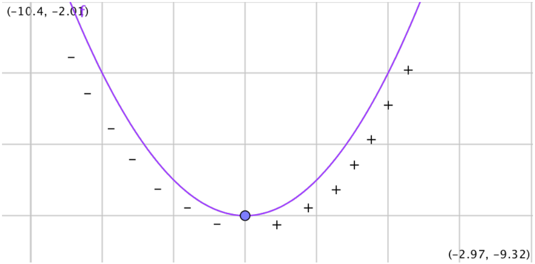

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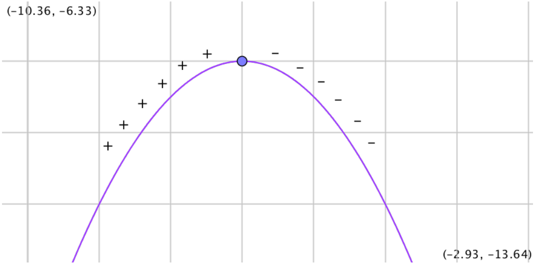

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Note: If $f''(a) = 0$ then we cannot conclude anything! E.g x^3 or x^4.

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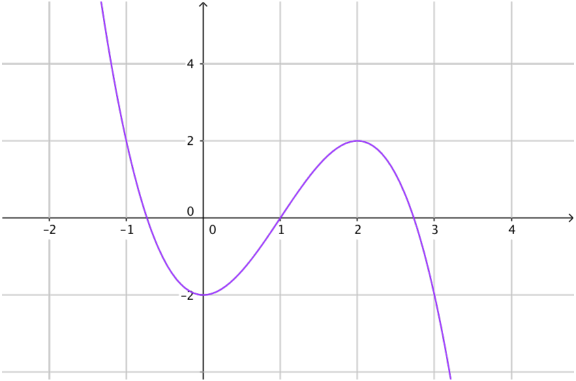

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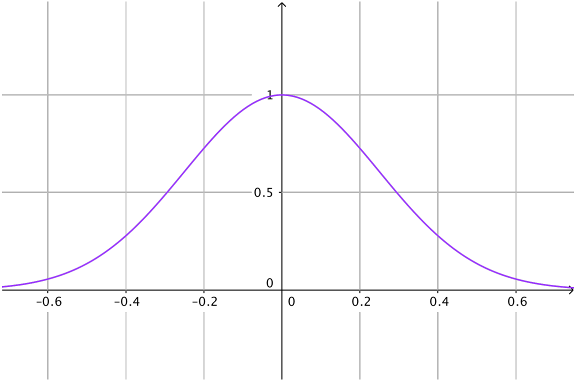

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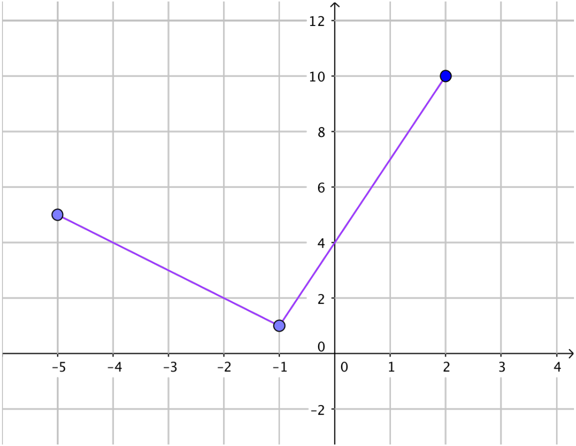

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We would like to find the value of x which maximises this function!

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The second derivative $A''(x) = -2$ is always negative so this must be a maximum! Thus the dimensions for the rectangle with the largest area are

$$x = y = \frac{M}{4}$$

Catalina island

Question

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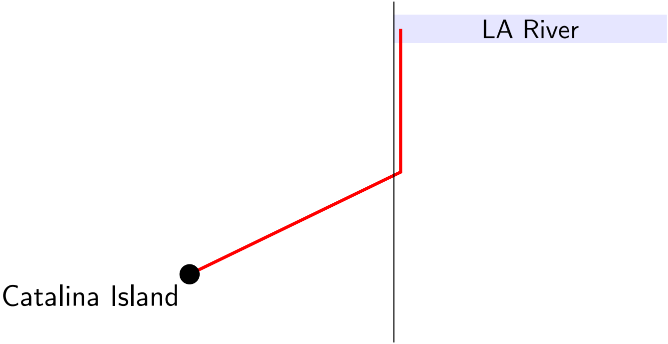

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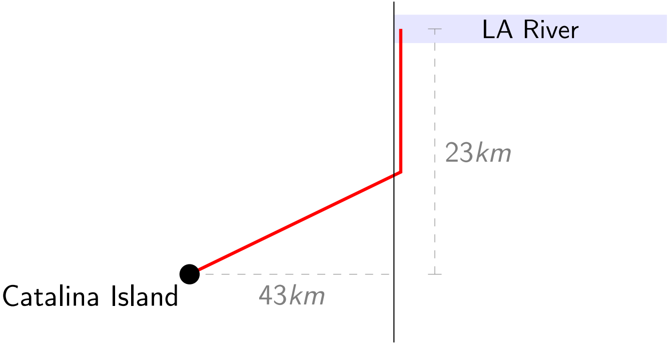

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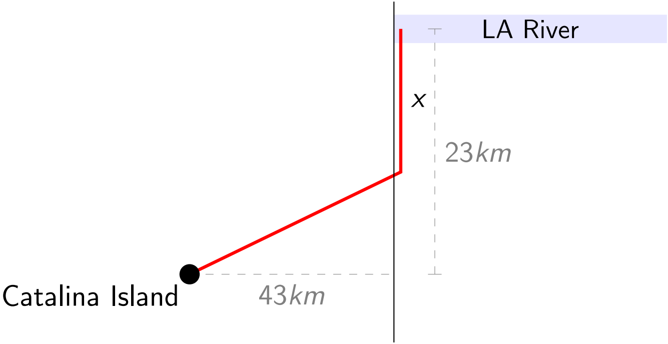

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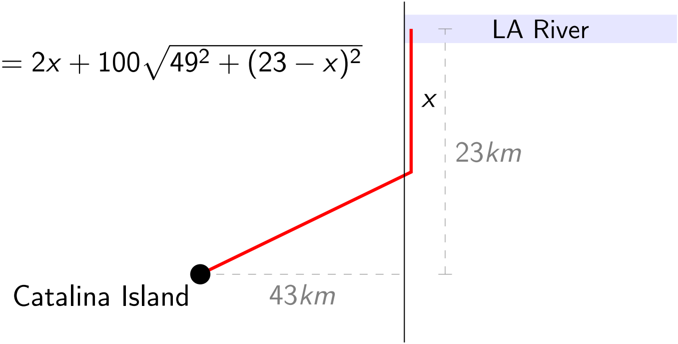

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So the solutions are

$$x = 23 - \frac{43}{3\sqrt{11}} \quad \text{and} \quad 23 + \frac{43}{3\sqrt{11}}$$