

# Midterm 1 practice 1

## UCLA: Math 115A, Winter 2020

*Instructor:* Noah White

*Date:*

*Version:* practice

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
Total:	20	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (1 point) If  $V$  is a vector space over the field  $\mathbb{R}$  and  $v \in V$  then

$$(2 - 3) \cdot v + (5 - 3) \cdot v$$

equals

- A. 1
- B.  $v$
- C. 0
- D.  $2v$

The following two questions concern the subsets

$$A = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \mid a_1 \neq \lambda \right\} \subseteq \mathbb{R}^3$$
$$B = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \mid a_1 + a_2 + a_3 = \lambda \right\} \subseteq \mathbb{R}^3$$

for some  $\lambda \in \mathbb{R}$ .

- (b) (1 point) Which of the following is a true statement?
- A. Both  $A$  and  $B$  are subspaces regardless of the value of  $\lambda \in \mathbb{R}$ .
  - B. Only  $A$  is a subspace.
  - C. Both are subspaces when  $\lambda = 0$ .
  - D. Only  $B$  is a subspace when  $\lambda = 0$ .
- (c) (1 point) When  $\lambda = 0$ , the subspace  $B$  has dimension
- A. 1
  - B. 2
  - C. 3
  - D. 4

(d) (1 point) Let  $V$  be a vector space and  $W$  a subspace. Consider the quotient space  $V/W$ . Which of the following is true?

- A.  $V/W$  is a subspace of  $V$ .
- B.  $v + W = w + W$  for any  $v \in V$  and some  $w \in W$ .
- C.  $-v + W$  is the additive inverse of  $v + W$ .
- D.  $1 + W$  is the zero element.

(e) (1 point) Which of the following definitions, makes  $p : V \rightarrow V/W$  into a surjective linear map?

- A.  $p(v) = W$
- B.  $p(v) = 0 + W$
- C.  $p(v) = 2v + W$
- D.  $p(v) = 2v$

2. Give (simple) examples of all of the following situations.
- (a) (2 points) A vector space  $V$  and a subspace  $W$  where  $\dim W \geq 2$  and  $\dim V \geq 3$ .
  - (b) (2 points) A basis for your example  $V$  above such that a subset of this basis is a basis for  $W$ .
  - (c) (1 point) A basis for  $V/W$ .

3. Consider the following maps. Prove or disprove that they are linear and find the dimension of the kernel (nullspace).
- (a) (2 points)  $T : \mathbb{C}_2[x] \longrightarrow \mathbb{C}$  given by  $T(p) = p(2)$  (i.e. evaluate the polynomial at two). Recall that  $\mathbb{C}_n[x]$  is the set of polynomials of degree at most  $n$ .
- (b) (3 points)  $E : \text{Mat}_{2 \times 2}(\mathbb{R}) \longrightarrow \text{Mat}_{2 \times 2}(\mathbb{R})$  defined by

$$E(M) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot M - M \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

4. Let  $\mathcal{F}(\mathbb{R}, \mathbb{R})$  be the space of all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$ .
- (a) (1 point) Is the subset  $\{ f \in \mathcal{F}(\mathbb{R}, \mathbb{R}) \mid f(0) = 1 \} \subseteq \mathcal{F}(\mathbb{R}, \mathbb{R})$  a subspace? Justify your answer.
  - (b) (4 points) Let  $\mathcal{O}$  and  $\mathcal{E}$  be the subspaces of odd and even functions in  $\mathcal{F}(\mathbb{R}, \mathbb{R})$ . Prove that  $\mathcal{F}(\mathbb{R}, \mathbb{R}) = \mathcal{O} \oplus \mathcal{E}$ . (Recall that a function is even if  $f(-x) = f(x)$  for all  $x \in \mathbb{R}$ , or odd if  $f(-x) = -f(x)$  for all  $x \in \mathbb{R}$ ).

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