This week you will get practice solving separable differential equations, as well as some practice with linear models

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (6.2) Solve the following differential equations.
   (a) $\frac{dy}{dt} = 5y$
   (b) $\frac{dy}{dt} = -y$
   (c) $\frac{dy}{dx} = -3y$
   (d) $\frac{dy}{dx} = 0.2y$
   (e) (6.2-17) $\frac{dy}{dt} = y^3$
   (f) (6.2-18) $\frac{dy}{dt} = y \sin t$
   (g) (6.2-20) $\frac{dy}{dt} = \frac{t}{y}$
   (h) (6.2-24) $\frac{dy}{dx} = \frac{x}{y}\sqrt{1+x^2}$
   (i) (6.2-26) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$
   (j) (6.2-30) $\frac{dy}{dt} = yt$ with $y(1) = -1$
   (k) (6.2-32) $\frac{dy}{dt} = e^{-y}t$ with $y(-2) = 0$
   (l) (6.2-34) $\frac{dy}{dt} = ty^2 + 3t^2y^2$ with $y(-1) = 2$
   (m) $\frac{dy}{dx} = y \sin x + \frac{y}{(x+1)^2}$ with $y(0) = 1$
   (n) $\frac{dy}{dx} = \frac{x}{y}e^{-x^2}$ with $y(0) = 1$
   (o) $\frac{dy}{dx} = y + ye^x$ with $y(0) = e$

2. (6.2-44) Populations may exhibit seasonal growth in response to seasonal fluctuations in resource availability. A simple model accounting for seasonal fluctuations in the abundance N of a population is

$$\frac{dN}{dt} = (R + \cos t)N$$

   where R is the average per-capita growth rate and t is measured in years.
   (a) Assume $R = 0$ and find a solution to this differential that satisfies $N(0) = N_0$. What can you say about $N(t)$ at $t \to \infty$?
   (b) Assume $R = 1$ (more generally $R > 0$) and find a solution to this differential that satisfies $N(0) = N_0$. What can you say about $N(t)$ at $t \to \infty$?
   (c) Assume $R = -1$ (more generally $R < 0$) and find a solution to this differential that satisfies $N(0) = N_0$. What can you say about $N(t)$ at $t \to \infty$?

3. (6.3-25) In 1990 the gross domestic product (GDP) of the United States was \$5,464 billion. Suppose the growth rate from 1989 to 1990 was 5.08%. Predict the GDP in 2003.

   (Hint: You should assume that the percentage growth rate is constant - not very realistic!)

4. (6.3-28) According to the Department of Health and Human Services, the annual growth rate in the number of marriages per year in 1990 in the United States was 9.8% and there were 2,448,000 marriages that year. How many marriages will there be in 2004 if the annual growth rate in the number of marriages per year is constant?

5. (6.3-30) Calculate the infusion rate in milligrams per hour required to maintain a long-term drug concentration of 50 mg/L (i.e., the rate of change of drug in the body equals zero when the concentration is 50 mg/L). Assume that the half-life of the drug is 3.2 hours and that the patient has 5 L of blood.

6. (6.3-31) Calculate the infusion rate in milligrams per hour required to maintain a desired drug concentration of 2 mg/L. Assume the patient has 5.6 L of blood and the half-life of the drug is 2.7 hours.

7. (6.3-34) A drug is given at an infusion rate of 50 mg/h. The drug concentration value determined at 3 hours after the start of the infusion is 8 mg/L. Assuming the patient has 5 L of blood, estimate the half-life of this drug.

8. (6.3-37) After one hydrodynamic experiment, a tank contains 300 L of a dye solution with a dye concentration of 2 g/L. To prepare for the next experiment, the tank is to be rinsed with water flowing in at a rate of 2 L/min, with the well-stirred solution flowing out at the same rate. Write an equation that describes the amount of dye in the container. Be sure to identify variables and their units.

9. (6.3-38) At midnight the coroner was called to the scene of the brutal murder of Casper Cooly. The coroner arrived and noted that the air temperature was 70° F and Cooly's body temperature was 85° F. At 2a.m., she noted that the body had cooled to 76° F. The police arrested Cooly's business partner Tatum Twit and charged her with the murder. She has an eyewitness who said she left the theater at 11p.m. Does her alibi help?

10. (*Note: this question is a challenge! It would be too difficult for an exam*) A cylindrical water tank, 2 meters in diameter and 5 meters tall, has a small hole in its base of radius 0.05 meters. From the *Bernoulli principle* in fluid dynamics one can derive the fact that if the tank is filled to a level of h meters then the water is flowing out of the hole at a rate of

$$A\sqrt{2gh}\ \mathrm{m}^3/\mathrm{s}$$

where A is the area (in meters squared) of the hole and g is acceleration due to gravity (you may assume $g = 10\ \mathrm{m/s}^2$). Rainwater is caught by a guttering system and is pouring into the tank at a constant rate of $I\ \mathrm{m}^3/\mathrm{s}$.

   (a) Write a differential equation that describes the change in the volume of water (in m^3/s) held by the tank, over time.

   (b) Find the equilibrium solution for this equation (leave your answer in terms of I and π).

   (c) If the tank is initially filled up to the 3 meter mark, describe how the volume of the tank behaves over the long term, for different values of I.

   (d) Solve the differential equation assuming that $I = 0$ (i.e. it is not raining).

   (e) Under the above assumptions, how long would it take for the tank to drain? Here we will declare that the tank is drained once it contains less than $0.001\ \mathrm{m}^3$ of water.

   (f) Solve the differential equation assuming that $I = 0.5$ but leave the answer as an implicit function (do not try to solve for $V(t)$).

11. A river flows into a small lake and another river flows out of the lake such that the lake has a constant volume of $2000\ \mathrm{m}^3$ (the rate of water flowing in equals the rate of water flowing out). The river flowing into the lake contains a pollutant present at $0.5\ \mathrm{mg/m}^3$. In this question you will model the total amount of pollutant, $y(t)$, present at time t (Note that $y(t)$ is the total amount of pollutant in the lake and not a concentration).

   (a) Assume that the river flowing in, flows at a constant rate of $20\ \mathrm{m}^3/\mathrm{h}$. At what rate is the pollutant flowing into the lake (in mg/h)?

   (b) Under the above assumption, write a differential equation describing the change in the level of pollution in the lake.

   (c) Assuming that initially there is no pollutant in the lake, solve this differential equation.

(d) Now assume that there is some seasonal variability and that the river flowing in (and thus also the river flowing out), flow at a rate of $40\sin^2 t$ m^3/h. Write and solve a differential equation to model this situation, assuming there is initially no pollution in the lake.

(e) Compare the long term behaviour of the two solutions.