Midterm 1 practice

UCLA: Math 31B, Spring 2017

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Date:
Version: practice

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. (a) (4 points) Calculate $\frac{d}{dx}\left[e^{e^{(x^2+2)}}\right]$

   (b) (6 points) Calculate $\int \frac{1}{x(\ln x)^2}\ dx$.

Solution:

(a) $e^{e^{(x^2+2)}}e^{(x^2+2)}(2x)$.

(b) Let $u = \ln x$. Then $du = \frac{1}{x}\ dx$. So

$$\int \frac{1}{x(\ln x)^2}\ dx = \int \frac{1}{u^2}\ du = -\frac{1}{u} + c = -\frac{1}{\ln x} + c.$$

2. (a) (3 points) Let $f(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3 + \frac{11}{24}x^4 + \frac{7}{120}x^5$.
What are the Taylor polynomials $T_3(x)$ and $T_7(x)$ for $f(x)$ centered at 0?

(b) (7 points) Let $T_n(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots + \frac{(-1)^{n-1}}{n}(x-1)^n$ be the n-th Taylor polynomial for $\ln x$ centered at 1.
Find an n such that

$$\left|\ln\left(\frac{1}{2}\right) - T_n\left(\frac{1}{2}\right)\right| < \frac{1}{10^{10}}.$$

Solution:

(a) $T_3(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3$, $T_7(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3 + \frac{11}{24}x^4 + \frac{7}{120}x^5$.

(b) We'll use Taylor's Error Bound for $f(u) = \ln u$, $a = 1$, $x = \frac{1}{2}$, and n unspecified.
For u between a and x, i.e. $\frac{1}{2} \le u \le 1$, we have

$$|f^{(n+1)}(u)| = \left|\frac{n!}{u^{n+1}}\right| \le \frac{n!}{(\frac{1}{2})^{n+1}} = 2^{n+1}n!$$

so we can take $K = 2^{n+1}n!$ and Taylor's Error Bound says that

$$\left|\ln\left(\frac{1}{2}\right) - T_n\left(\frac{1}{2}\right)\right| \le \frac{2^{n+1}n!|\frac{1}{2}-1|^{n+1}}{(n+1)!} = \frac{1}{n+1}.$$

We see that

$$\left|\ln\left(\frac{1}{2}\right) - T_{10^{10}}\left(\frac{1}{2}\right)\right| \le \frac{1}{10^{10}+1} < \frac{1}{10^{10}}$$

so $n = 10^{10}$ works.

3. For (a)-(c), give the value or say, "undefined."
   (a) (1 point) $\tan(\arctan(2)) =$
   (b) (1 point) $\sin(\arcsin(2)) =$
   (c) (2 points) $\arctan(\tan(\frac{7\pi}{3})) =$
   (d) (6 points) Suppose $a \neq 0$.
   Calculate the following indefinite integral as I did in class (using a u-substitution and the knowledge of fundamental integrals which relate to inverse trigonometric functions).

$$\int \frac{1}{a^2 + x^2}\, dx$$

Solution:

(a) 2.

(b) Undefined.

(c) $\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$.

(d) Let $u = \frac{x}{a}$. Then $x = au$ so that $a^2 + x^2 = a^2 + a^2u^2$ and $dx = adu$. Thus,

$$\begin{aligned} \int \frac{1}{a^2 + x^2}\, dx &= \int \frac{a}{a^2 + a^2u^2}\, du \\ &= \frac{1}{a} \int \frac{1}{1 + u^2}\, du = \frac{1}{a}\arctan(u) + c = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + c. \end{aligned}$$

4. (10 points) Calculate the following indefinite integral

$$\int \frac{6x^3 + 3x^2 + 9x - 8}{(x^2-1)(x^2+4)}\, dx.$$

[The numbers have been chosen so that they work out well; they are all whole numbers.

In the method of partial fractions, I found looking at the x^3-coefficient useful.

You'll get points for spotting the correct partial fraction decomposition, and displaying knowledge of the relevant integrals. Notice that you did one of these integrals in 3.d).]

Solution: Clearing denominators on

$$\frac{6x^3 + 3x^2 + 9x - 8}{(x^2-1)(x^2+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$$

gives

$$6x^3 + 3x^2 + 9x - 8 = A(x+1)(x^2+4) + B(x-1)(x^2+4) + (Cx+D)(x^2-1).$$

Setting $x = 1$ gives $10 = 10A$ so that $A = 1$.

Setting $x = -1$ gives $-20 = -10A$ so that $B = 2$.

Setting $x = 0$ gives $-8 = 4A - 4B - D$ so that $D = 4A - 4B + 8 = 4$.

Looking at the x^3 coefficient gives $A + B + C = 6$ so that $C = 3$.

So

$$\begin{aligned}\int \frac{6x^3 + 3x^2 + 9x - 8}{(x^2-1)(x^2+4)}\, dx &= \int \frac{1}{x-1} + \frac{2}{x+1} + \frac{3x}{x^2+4} + \frac{4}{x^2+4}\, dx \\ &= \ln|x-1| + 2\ln|x+1| + \frac{3}{2}\ln(x^2+4) + 2\arctan\left(\frac{x}{2}\right) + c.\end{aligned}$$