Midterm 1 practice

UCLA: Math 170A, Fall 2017

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Date:
Version: practice

- This exam has 5 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	0	
Total:	20	

1. Let A, B, C be events is some probability space. Find expressions for the following events in terms of A, B, C and set operations (e.g $\cap, \cup, ^c$).

   (a) (1 point) Only A occurs.

   Solution:
   $$A \cap (B \cup C)^c$$

   (b) (1 point) Both A and B but not C occur.

   Solution:
   $$(A \cap B) \cap C^c$$

   (c) (1 point) At least one event occurs.

   Solution:
   $$A \cup B \cup C$$

   (d) (1 point) One and no more occurs.

   Solution:
   $$[A \cap (B \cup C)^c] \cup [B \cap (A \cup C)^c] \cup [C \cap (A \cup B)^c]$$

   (e) (1 point) Not more than two occur.

   Solution:
   $$(A \cap B \cap C)^c$$

2. There is a box with r red balls and b blue balls, where $r, b \geq 4$. Three balls are drawn without replacement. For $i = 1, 2, 3$, let A_i be the event that the i-th ball is blue. Let B be the event that all three balls are blue.

   (a) (1 point) Find $\mathbb{P}(A_1)$.

   Solution:
   $$\mathbb{P}(A_1) = b/(r + b)$$

   (b) (1 point) Find $\mathbb{P}(A_3)$.

   Solution: We consider the disjoint events that the requence of balls is bbb, rrb, rbb or brb. Since we can sum these probabilities we get
   $$\mathbb{P}(A_3) = \frac{b(b-1)(b-2) + r(r-1)b + 2rb(b-1)}{(r+b)(r+b-1)(r+b-2)}.$$
   Turns out this simplifies to
   $$\frac{b}{r+b}.$$
   Woah! Why in the world is this true? Turns out there is a much better way of thinking of this problem. Lets imagine that our experiment is actually picking $r + b$ balls out of the box and

recording the order that we observe the colours. Then, each sequence of colors is equaly likely. This is a good sample space, now we can count. How many requences are there in total? Easy, we just choose at what positions the blue balls come out at:

$$\binom{r+b}{b}.$$

Now we want to know how many sequences there are with a blue ball in the 3rd position. Well to create such a sequence, simply put a blue ball in the 3rd position, and choose where the remaining $b-1$ blue balls go:

$$\binom{r+b-1}{b-1}.$$

Thus

$$\mathbb{P}(A_3) = \frac{\binom{r+b-1}{b-1}}{\binom{r+b}{b}} = \frac{b}{r+b}.$$

(c) (1 point) Find $\mathbb{P}(B)$.

Solution: Using the same argument as above, we simply count the number of sequences where we fix the first three balls as blue, so we only need to choose where the remaining $b-3$ balls are placed:

$$\mathbb{P}(B) = \frac{\binom{r+b-3}{b-3}}{\binom{r+b}{b}} = \frac{b(b-1)(b-2)}{(r+b)(r+b-1)(r+b-2)}.$$

(d) (2 points) Mark each of the following as True or False. Do not provide justification. One point for each correct answer + one point if all your answers are correct (i.e if you aren't sure of one, it might be better to leave it blank).

- TRUE FALSE A_1 and A_2 are independant.

  Solution: FALSE

- TRUE FALSE A_1, A_2 and A_3 are independant.

  Solution: FALSE

- TRUE FALSE A_1 and B are independant.

  Solution: FALSE

3. (5 points) Suppose that each student in a class of 40 students has birthday occurring on each day of the year with equal probability and independently of the other students. What is the probability that there are no shared birthdays?

4. (5 points) There are three museums in a town. Each is open with probability 0.4 and closed with probability 0.6 independently of the other museums. Two of the museums are located on the street A and one on the street B. A tourist chooses either street A or street B with equal probability and walks down the whole length of the street he chose. He will enter every **open** museum he encounters on his way. Given that he entered exactly one museum, what is the probability he chose to walk down the street A?

Solution: Use the Bayes' rule here. Let A be the event the tourist chooses to take street A and B he chooses to take street B. Let C be the event he entered exactly one museum.

Given that he takes the street B he will enter exactly one museum if and only if the museum on the street B is open. This has probability 0.6, so $\mathbf{P}(C|B) = 0.4$.

Given that he takes the street A he will enter exactly one museum if and only if exactly one of the two museums on the street A is open. The probability of this is $2 \cdot 0.6 \cdot 0.4 = 0.48$. Therefore, $\mathbf{P}(C|A) = 0.48$.

Since $\mathbf{P}(A) = \mathbf{P}(B) = 0.5$ use the Bayes' rule

$$\mathbf{P}(A|C) = \frac{\mathbf{P}(A)\mathbf{P}(C|A)}{\mathbf{P}(A)\mathbf{P}(C|A) + \mathbf{P}(B)\mathbf{P}(C|B)} = \frac{0.5 \cdot 0.48}{0.5 \cdot 0.48 + 0.5 \cdot 0.4} = \frac{6}{11}.$$

5. (0 points) *Bonus** If you have n lines in the plane $\mathbb{R}^2$ that only intersect in pairs (i.e. there is no point where 3 or more lines intersect simultaneously), how many regions do these lines divide the plane into?

   **The real exam will not have a bonus question. This is maybe too hard for an exam and maybe slightly outside the scope of the course, so it is just a question to entertain those of you who are curious. Show me your solutions.*

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