This week on the problem set we will see examples of integrals over more general regions.

You will only need to hand in a small selection of the questions for homework, however I recommend that you at least attempt them all by the end of the quarter as some may appear on exams!

Homework: due Friday 10 April, uploaded to Gradescope before 11:59pm. It will consist of questions $3, 4, 5$, and 6 below.
Note that the references to the textbook are for the 4^{th} edition, *late transcendentals* version. Any differences between the 3^{rd} and 4^{th} editions is noted in parentheses.

1. From 16.2 in the textbook: 4, 8, 14, 20, 21, 23, 29, 31, 45, 48, 49 (Question 21 is different in the two versions, but both are fine.).

2. From 16.3 in the textbook: 3, 5, 6, 7.

3. Consider an integral over the domain $\mathcal{D}$ that is the part of the first quadrant bounded by $y = -(x-1)^2+1$ and $y = 1/x$. We can write an integral over this domain as: $\displaystyle\int_1^{\frac{1+\sqrt{5}}{2}} \int_{1/x}^{-(x-1)^2+1} f(x,y)\, dy\, dx$. Change the order of integration to write this as an integral where you integrate in the order $dx\, dy$.

4. Consider the function $E(s) = \displaystyle\int_0^s e^{-x^2}\, dx$. This is an incredibly important function in applied mathematics (and therefore physics, chemistry, etc). Unfortunately it is impossible to express the antiderivative of e^{-x^2} in terms of functions you already know. So how can we calculate $E(s)$? It turns out, that its value at infinity,
$$E(\infty) := \lim_{s\to\infty} E(s) = \int_0^\infty e^{-x^2}\, dx,$$
cal be calculated using a trick which this question will guide you through. In fact, we will calculate $E(\infty)^2$.
   (a) Express $E(\infty)^2 = \left(\displaystyle\int_0^\infty e^{-x^2}\, dx\right)\left(\displaystyle\int_0^\infty e^{-y^2}\, dy\right)$ as a double integral and therefore as an iterated integral, in the order $dx\, dy$. Make sure to describe the region in $\mathbb{R}^2$ we are integrating over precisely. *Hint: consider the separation of variables formula.*
   (b) Use the change of variables $t = x/y$ to transform the inner integral. Express $E(\infty)^2$ as an iterated integral in the order $dy\, dt$.
   (c) Evaluate the iterated integral.
   (d) Determine whether $E(\infty)$ is positive or negative. Find the value of $E(\infty)$.
   (e) Explain why this method does not allow you to calculate $E(s)$ for more general $s < \infty$.

5. Find the volume of the region bounded by $y = 1 - x^2$, $z + y = 1, y = 0$ and $4z + 4y + x = 12$.

6. Compute the integral $\displaystyle\iiint_{\mathcal{W}} xy\, dV$ where $\mathcal{W}$ is the part of the first octant inside the elliptical cyclinder $(x/2)^2 + (z/3)^2 = 1$ and inside the ellipsoid $(x/4)^2 + (y/4)^2 + (z/5)^2 = 1$.