Math 3B: Lecture 21

Noah White

March 6, 2017

Last time

Last time

- Slope fields

Reminder: Quiz 5 this week!

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- Slope fields
- Autonomous equations

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- Slope fields
- Autonomous equations
- Slope fields for autonomous equations

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Last time

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- Slope fields
- Autonomous equations
- Slope fields for autonomous equations
- Examples involving populations and harvesting

Reminder: Quiz 5 this week!

Eulers method

Often it is impossible to solve a differential equation. E.g.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

(the *Riccati equation*) has no solutions that can be written in terms of usual functions like $\sin x$, e^x, etc.

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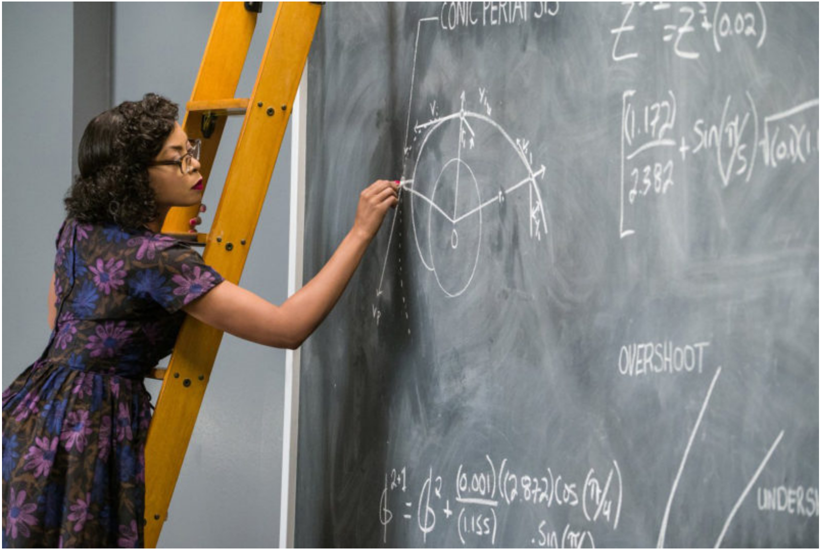
(the *Riccati equation*) has no solutions that can be written in terms of usual functions like $\sin x$, e^x, etc.

We want a method to estimate $y(t)$ is we know that $y(t_0) = y_0$.

Eulers method

Let's use Eulers method!



Idea behind Eulers method

Suppose $y(t)$ is a solution to

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

and that $y(t_0) = y_0$.

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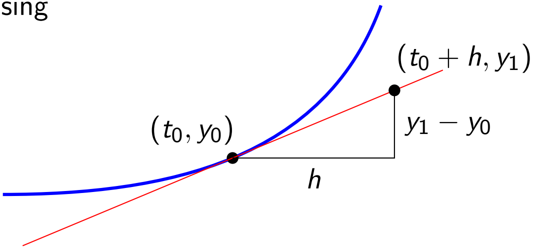
If h is a small number (e.g. $h = 0.1$), then we approximate $y(t_0 + h)$ using



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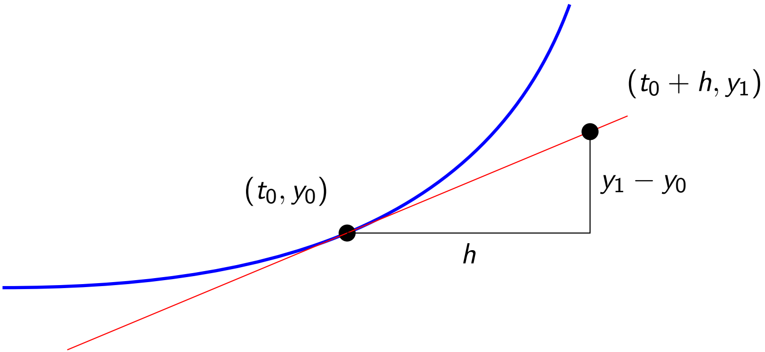
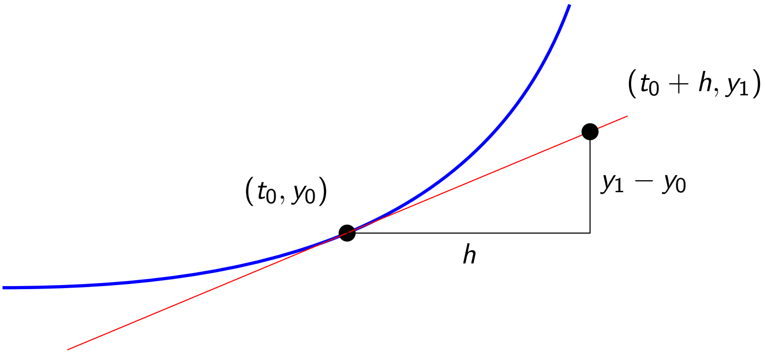


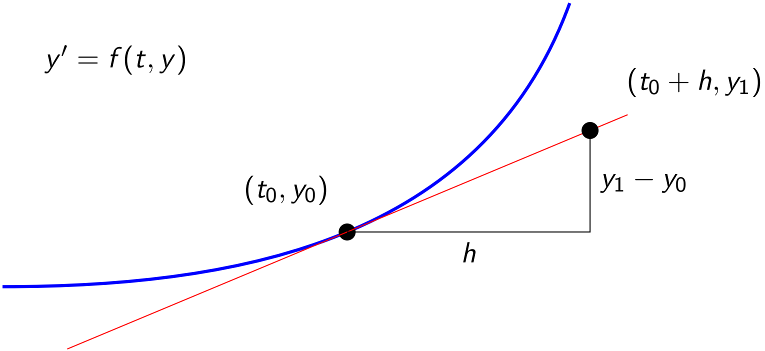
$$y(t_0 + h) \approx y_1$$

Idea behind Eulers method



$$y'(t_0) = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{h}$$

Idea behind Eulers method



$$y(t_0 + h) \approx y_1 = y_0 + hy'(t_0)$$
$$= y_0 + hf(t_0, y_0)$$

Idea behind Eulers method

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

If we know that the solution satisfies $y(t_0) = y_0$ then

- let h be a small step forward in time

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- we can get an approximate value for the solution at $t = t_0 + h = t_1$

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If we know that the solution satisfies $y(t_0) = y_0$ then

- let h be a small step forward in time
- we can get an approximate value for the solution at $t = t_0 + h = t_1$
- i.e. $y(t_1) \approx y_1$ where

$$y_1 = y_0 + hf(t_0, y_0)$$

Eulers method

To carry out Eulers method, we simply repeat this a number of times!

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

Given an initial value $y(t_0) = y_0$. To approximate $y(t)$ at $t = a$ follow the steps:

- Choose an increment h

Eulers method

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$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

Given an initial value $y(t_0) = y_0$. To approximate $y(t)$ at $t = a$ follow the steps:

- Choose an increment h
- set $t_1 = t_0 + h$

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- set $t_2 = t_1 + h$

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- keep repeating until $t_n \approx a$

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- Choose an increment h
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- set $t_2 = t_1 + h$
- set $y_2 = y_1 + hf(t_1, y_1)$
- keep repeating until $t_n \approx a$
- then $y(a) \approx y_n$.

An example

We will aprroximate $y(2)$, where y obeys

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	
1			
2			
3			
4			

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Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1			
2			
3			
4			

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Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	
3			
4			

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$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3			
4			

An example

We will aprroximate $y(2)$, where y obeys

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3	1.5	0.78	
4			

An example

We will aprroximate $y(2)$, where y obeys

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3	1.5	0.78	$y_4 = 0.78 + 0.5 \cdot (0.78^2 + 1.5)$
4			

An example

We will aprroximate $y(2)$, where y obeys

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3	1.5	0.78	$y_4 = 0.78 + 0.5 \cdot (0.78^2 + 1.5)$
4	2.0	1.84	