Math 3B: Lecture 22

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November 16, 2016

Announcements

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- Lectures 11/28 and 11/30 change. Vote on Piazza

Slope fields

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$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

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Key tool

Slope fields. At every point on the *yt*-plane we draw a small line segment (a vector) with slope $f(y, t)$.

Examples

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If we want to draw a slope field, we cannot actually draw a line segment for every point. Instead we pick a grid of points in the plane.

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Examples

Lets use Geogebra! Here is the command we will use:

`SlopeField[f(x,y)]` will produce a slope field for the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$$

Sketching solutions

Using the slope field we can sketch rough pictures of the solution, given a starting point (an initial condition).

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- is the solution ever above to below a certain value?

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Examples

Lets use Geogebra again.

Nullclines

Definition

The nullcline for $\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$ is the set of points (t, y) where $f(t, y) = 0$

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Examples

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Drawing slope fields by hand

Drawing slope fields by hand can be difficult! But we can use the nullclines to get an approximate picture

Examples

Lets draw some on the board.

Autonomous equations

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An ODE of the form

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The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

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- Suppose $f(a) = 0$.

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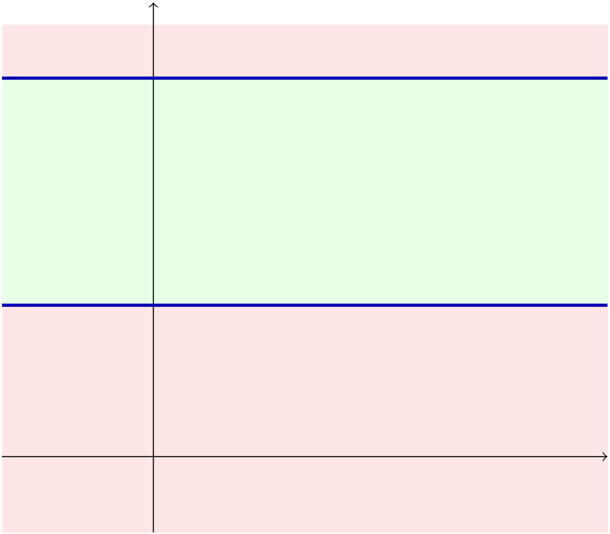
- Suppose $f(a) = 0$.
- Then (t, a) is on the nullcline, for any t.
- So the line $y = a$ is part of the nullcline, whenever $f(a) = 0$.

Slope fields and nullclines for autonomous systems

Thus our slope field and nullclines look something like



Phase lines/diagram

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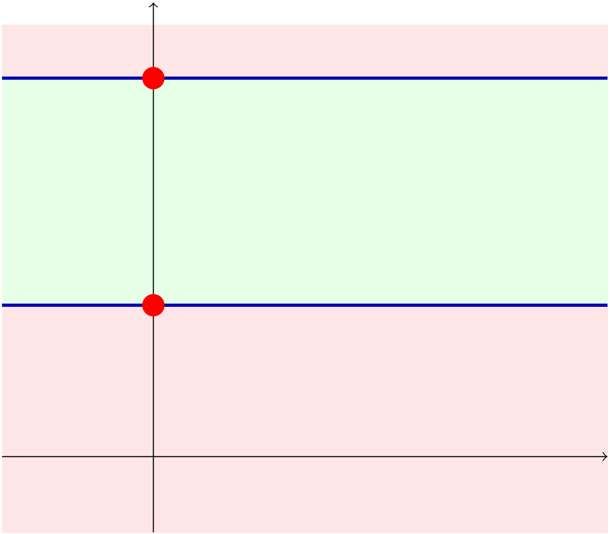

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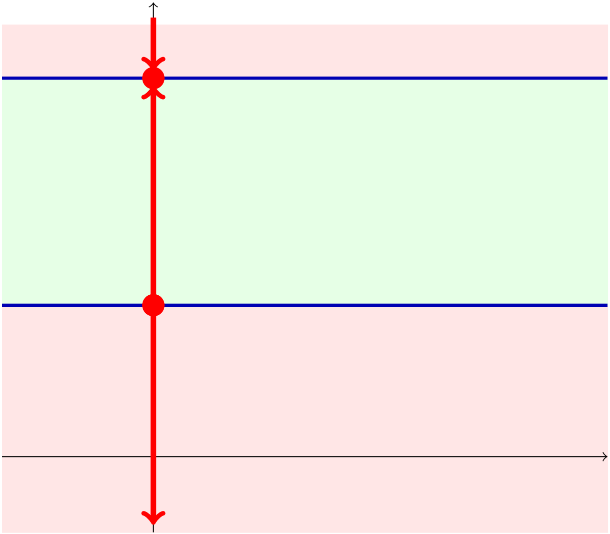

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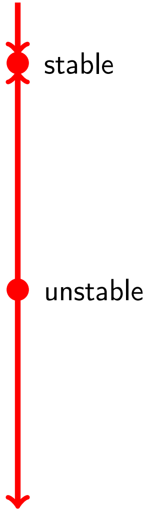

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- It is unstable if the two arrows are pointing away from it.
- It is semistable if the arrows point in the same direction.