This week you will get practice drawing and understanding slope fields, making qualitative statements about solutions using them and some practice applying Euler's method.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (6.4.33) Consider the differential equation
$$\frac{dy}{dt} = \frac{1}{t}$$
   (a) verify that $y(t) = \ln t$ is a solution to this differential equation satisfying $y(1) = 0$.
   (b) Use Euler's method to approximate $y(2) = \ln 2$ with $h = 0.5$.

2. (6.5) Draw phase lines, classify the equilibria, and sketch a solution satisfying the specified initial value for the equations in the following.
   (a) (6.5-2) $\frac{dy}{dt} = 2 - 3y$, $y(0) = 2$
   (b) (6.5-5) $\frac{dy}{dt} = y(y - 10)(20 - y)$, $y(0) = 9$
   (c) (6.5-6) $\frac{dy}{dt} = y(y - 5)(25 - y)$, $y(0) = 7$
   (d) (6.5-7) $\frac{dy}{dt} = \sin y$, $y(0) = 0.1$
   (e) (6.5-10) $\frac{dy}{dt} = y^3 - 4y$, $y(0) = 0.1$

3. (6.5-33) To account for the effect of a generalist predator (with a type II functional response) on a population, ecologists often write differential equations of the form
$$\frac{dN}{dt} = 0.1N \left(1 - \frac{N}{1,000}\right) - \frac{10N}{1 + N}$$

   (a) Sketch the phase line for this system.
   (b) Discuss how the fate of the population depends on its initial abundance.

   Hint: dont worry about what the first sentence means, you dont need to know where the differential equation comes from.

4. (6.5-39) Consider a population of clonally reproducing individuals consisting of two genotypes, a and A, with per capita growth rates, r_a and r_A, respectively. If N_a and N_A denote the densities of genotypes a and A, then
$$\frac{dN_a}{dt} = r_a N_a \qquad \frac{dN_A}{dt} = r_A N_A$$
   Also, let $y = \frac{N_a}{N_a + N_A}$ be the fraction of individuals in the population that are genotype a. Show that y satisfies
$$\frac{dy}{dt} = (r_a - r_A)y(1 - y)$$

5. (6.5-40) In the Hawk-Dove replicator equation
$$\frac{dy}{dt} = \frac{y}{2}(1 - y)(C(1 - y) - V)$$

   if the value $V > 0$ is specified, then find the range of values of C (in terms of V) that will ensure a polymorphism exists (i.e., find conditions that ensure the existence of an equilibrium $0 < y^* < 1$ that is stable).

   (Hint: you do not need to know anything about the Hawk-Dove Replicator - though it is very interesting! - all you need to know is that V is a constant and C is a parameter. A polymorphism is a stable equilibrum between zero and one.)

6. (6.5-41) Production of pigments or other protein products of a cell may depend on the activation of a gene. Suppose a gene is *autocatalytic* and produces a protein whose presence activates greater production of that protein. Let y denote the amount of the protein (say, micrograms) in the cell. A basic model for the rate of this self-activation as a function of y is

$$A(y) = \frac{ay^b}{k^b + y^b} \text{ micrograms/minute}$$

where a represents the maximal rate of protein production, $k > 0$ is a "half saturation" constant, and $b \geq 1$ corresponds to the number of protein molecules required to active the gene. On the other hand, proteins in the cell are likely to degrade at a rate proportional to y, say cy. Putting these two components together, we get the following differential equation model of the protein concentration dynamics:

$$\frac{dy}{dt} = \frac{ay^b}{k^b + y^b} - cy$$

(a) Verify that $\lim_{y \to \infty} A(y) = a$ and $A(k) = a/2$.

(b) Verify that $y = 0$ is an equilibrium for this model and determine under what conditions it is stable.

(Hint: the definition of autocatalytic is given in the question, it is a gene that produces a protein whose presence activate greater production of that protein.)

7. (6.5-42) Consider the model of an autocatalytic gene in Problem 41 with $b = 1$, $k > 0$, $a > 0$, and $c > 0$.

(a) Sketch the phase line for this model when $ck > a$.

(b) Sketch the phase line for this model when $ck < a$.