

# Final exam practice 1

## UCLA: Math 3B, Winter 2019

*Instructor:* Noah White

*Date:*

- This exam has 7 questions, for a total of 80 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

Discussion section (please circle):

Day/TA	Louis	Matthew
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	12	
2	12	
3	12	
4	10	
5	12	
6	14	
7	8	
Total:	80	

Questions 1 and 2 are multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following four pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

**Question 2.**

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) The function  $f(x) = e^{x+\sin x}$  is

- A. Always increasing.
- B. Always decreasing.
- C. Always concave up.
- D. Always concave down.

(b) (2 points) The function  $f(x) = e^{-x^2}$  is

- A. Always increasing.
- B. Always decreasing.
- C. Always concave up.
- D. None of the above.

(c) (2 points) The function  $f(x) = \frac{4x-x^2}{1+x^2}$  has a

- A. Horizontal asymptote at  $y = -1$ .
- B. Vertical asymptote at  $x = 1$ .
- C. Slanted asymptote with slope  $-1$ .
- D. Slanted asymptote with slope  $1$ .

- (d) (2 points) The function  $f(x) = \frac{6e^x + 5x^2}{2x - 2e^x}$  has a
- A. Horizontal asymptote at  $y = 1$ .
  - B. No horizontal asymptotes.
  - C. Horizontal asymptote at  $y = 0$ .
  - D. Horizontal asymptote at  $y = -3$ .
- (e) (2 points) The function  $f(x) = \cos x$  has a critical point at
- A.  $x = 0$
  - B.  $x = \pi/2$
  - C.  $x = -\pi/2$
  - D.  $x = 1$
- (f) (2 points) The function  $f(x) = e^{x-1} + e^{3-x}$  has a
- A. minimum at  $x = 2$ .
  - B. maximum at  $x = 2$ .
  - C. minimum  $x = 0$ .
  - D. maximum  $x = 0$ .

2. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) The definite integral  $\int_{-\pi}^{\pi} \cos x \, dx$  has a value of

- A. 2.
- B. 1.
- C. 0.
- D.  $\pi$ .

(b) (2 points) The definite integral  $\int_1^e x \ln x \, dx$  has a value of

- A. 0.
- B.  $\frac{1}{2}e^2$ .
- C.  $\frac{1}{4}(e^2 + 1)$ .
- D.  $e$ .

(c) (2 points) The solution of the differential equation  $\frac{dy}{dt} = -2y$  when  $y(0) = 3$  has

- A.  $y(1) = 2e^2$
- B.  $y(0.5) = 3e$ .
- C.  $y(1) = 3e$ .
- D.  $y(0.5) = 3e^{-1}$ .

(d) (2 points) The solution of the differential equation  $\frac{dy}{dt} = e^y \cos t$  when  $y(0) = 0$  has

- A.  $y(\pi) = \ln 2$
- B.  $y(-\pi/2) = 0$ .
- C.  $y(1) = \ln 2$ .
- D.  $y(-\pi/2) = -\ln 2$ .

(e) (2 points) The differential equation  $\frac{dy}{dt} = (1 - y) \left( \frac{y}{9} - 1 \right)$  has a

- A. stable equilibrium at  $y = 9$ .
- B. unstable equilibrium at  $y = 1/9$ .
- C. stable equilibrium at  $y = 1$ .
- D. unstable equilibrium at  $y = 3$ .

(f) (2 points) The differential equation  $\frac{dy}{dt} = 3 - \sqrt{5 + y^2}$  has a

- A. stable equilibrium at  $y = -2$ .
- B. unstable equilibrium at  $y = -1$ .
- C. stable equilibrium at  $y = 2$ .
- D. unstable equilibrium at  $y = 0$ .

3. Let  $f(x) = \frac{e^x+1}{e^{2x}-1}$ . Note that  $f'(x) = -\frac{e^x}{(e^x-1)^2}$  and  $f''(x) = \frac{e^x(e^x+1)}{(e^x-1)^3}$ .

(a) (2 points) Does  $f(x)$  cross the  $x$  and  $y$  axes? If so, where?

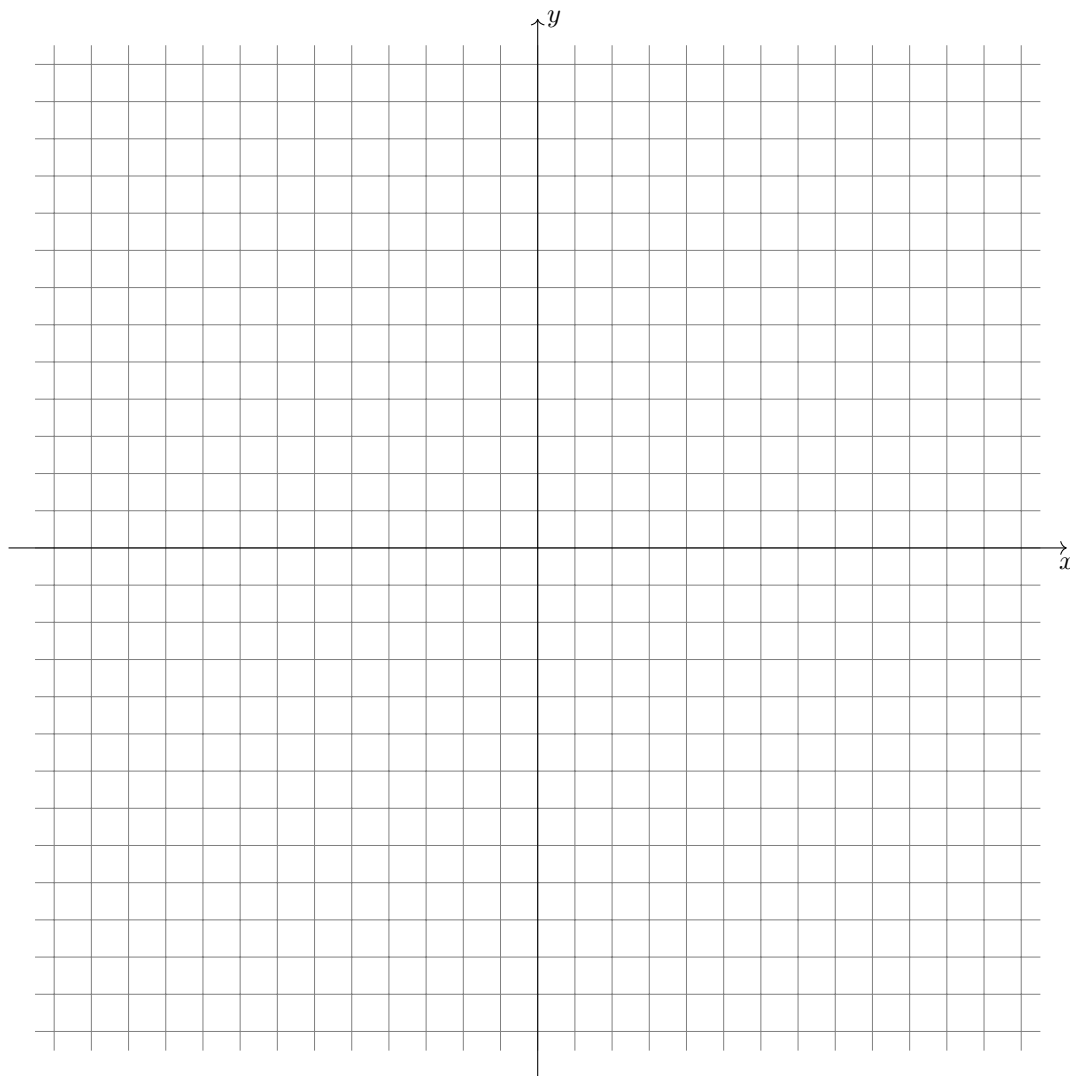
(b) (2 points) Does  $f(x)$  have any horizontal asymptotes? If so what are they?

(c) (1 point) Does  $f(x)$  have any vertical asymptotes? If so what are they?

(d) (2 points) For what  $x$  is the first derivative  $f'(x)$  positive?

(e) (2 points) For what  $x$  is the second derivative  $f''(x)$  positive?

(f) (3 points) On the graph provided, sketch  $f(x)$





4. A new nature reserve is being planned. The planning officials estimate that there will be a steady stream of deer immigrating to the reserve, at a rate of 300 deer per year. They also estimate that in the early years, the growth in population due to births will be roughly exponential; this means, for every one deer in the reserve at a given point,  $t$  years later there will be

$$0.2e^{0.5t}$$

deer. Initially the reserve contains no deer at all.

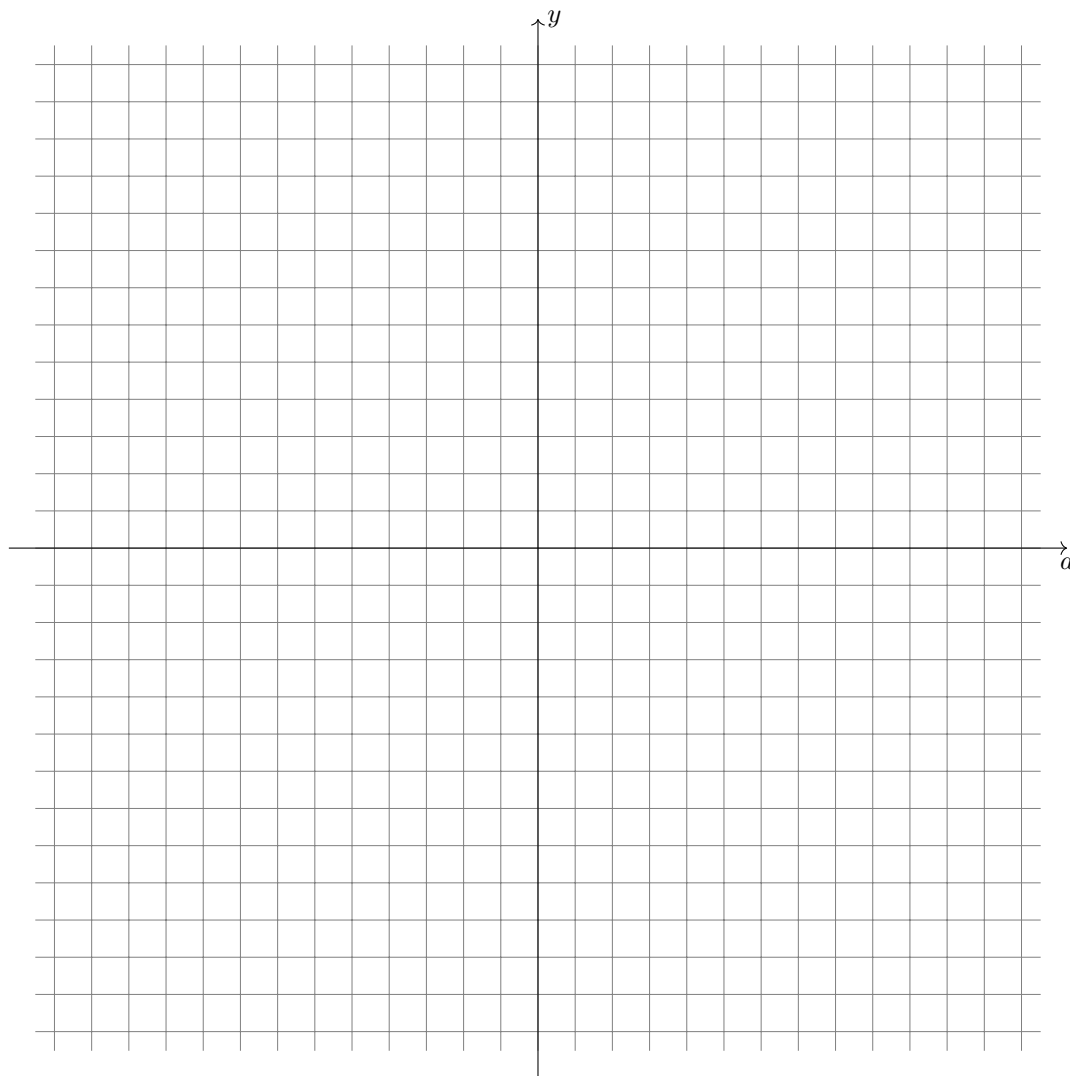
- (a) (6 points) Write a Riemann sum which represents the total number of deer in the reserve in 10 years. Be sure to define any symbols that you use (e.g.  $t_k$ ,  $\Delta t$ , etc).

- (b) (4 points) Use an integral to evaluate the Riemann sum above.

5. In this question we will investigate the behaviour of the solutions of

$$\frac{dy}{dt} = (a + y^2 - 3y)(y - 2)$$

- (a) (4 points) Draw a bifurcation diagram for this equation with parameter  $a$ . Make sure to label the regions of your diagram with up/down arrows according to the direction of the derivative.



(b) (2 points) Draw a phase diagram when  $a = 0$  and sketch the solution if  $y(0) = 1$ .

(c) (2 points) Draw a phase diagram when  $a = 2$  and sketch the solution if  $y(0) = 1.5$ .

(d) (2 points) Draw a phase diagram when  $a = 2.25$  and sketch the solution if  $y(0) = 3$ .

(e) (2 points) The differential equation above has an equilibrium solution of  $y = 2$  for any value of  $a$ . For what  $a$  is this equilibrium stable?

6. A saltwater pool has a volume of 100,000 liters. Fresh water is being pumped in using a hose at 1000 L/h (liters per hour) and is draining out through an outlet in the bottom of the pool at the same rate. You should assume that the fresh and saltwater mix immediately.
- (a) (3 points) Write a differential equation modelling the total amount of salt (in grams) in the pool.

- (b) (2 points) If the initial concentration of salt is 3 g/L, solve the differential equation.

(c) (3 points) How long does it take to fall below 0.1 g/L?

(d) (3 points) Now suppose that the water coming from the hose is saltwater containing  $a$  g/L of salt. Write a differential equation modelling the amount of salt in the pool over time and solve it. (You should still assume that the initial concentration of salt in the pool is 3 g/L).

- (e) (3 points) What concentration of salt should the water coming out of the hose have (i.e. what should  $a$  be) in order for the pool to achieve a concentration of 3.4 g/L in 12 hours?

7. (8 points) What point  $(x, y)$  on the parabola  $y = x^2$  is closest to the point  $(16, 0.5)$ ?



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