Math 3B: Lecture 5

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Definition (local maximum)

A function $f : D \longrightarrow R$ has a local maximum at a if

$$f(x) \leq f(a) \quad \text{for all } x \text{ near } a$$

Definition (local minimum)

A function $f : D \longrightarrow R$ has a local minimum at a if

$$f(x) \geq f(a) \quad \text{for all } x \text{ near } a$$

$f : \mathbb{R} \longrightarrow \mathbb{R}; x \mapsto x^2$ has a min at $x = 0$

./parab.png

$f : \mathbb{R} \longrightarrow \mathbb{R}; f(x) = x^3 - 4x^2 - 3x + 13$ has a local max at $x = -\frac{1}{3}$

./cubic2.png

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./cubic2-covered.png

Definition (Critical point)

A function $f(x)$ has a critical point at $x = a$ if $f'(a) = 0$ or if $f'(a)$ is undefined.

Examples

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Examples

- $f(x) = x^2$ has a critical point at $x = 0$ (since f'(x) = 2x)
- $f(x) = \sin x$ has a critical point at $x = \frac{\pi}{2}$ (since $f'(x) = \cos x$)
- $f(x) = e^x$ doesn't have any critical points since $f'(x) = e^x$ can never be zero

Local maximums and minimums (extrema) occur at

Example

$f : (-\infty, 1] \longrightarrow \mathbb{R}; f(x) = x^3$ has critical points at

$$x = 0 \text{ and } 1$$

$f'(x) = 3x^2$ so $f'(0) = 0$ and $f'(1)$ is undefined.

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Suppose $x = a$ is a critical point for the function $f(x)$.

First derivative test (minimums)

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./first-der.png

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./first-der2.png

Suppose $x = a$ is a critical point of the function $f(x)$

Second derivative test

If

- $f''(a) > 0$ then f has a minimum at a

Second derivative test

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Second derivative test

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- $f''(a) > 0$ then f has a minimum at a
- $f''(a) < 0$ then f has a maximum at a

Note: If $f''(a) = 0$ then we cannot conclude anything! E.g x^3 or x^4.

We have a function $f : D \longrightarrow R$. How do we find all local/global extrema?

To find the global extrema of $f(x)$ defined on a closed interval $[a, b]$:

To find the global extrema of $f(x)$ defined on a open interval (a, b):
Note: a could be $-\infty$ and b could be ∞.

2. Find the limits

$$L = \lim_{x \to a^+} f(x) \quad \text{and} \quad M = \lim_{x \to b^-} f(x)$$

3. Evaluate $f(x)$ at all the critical points
4. The smallest value is the global min unless L is smaller, in which case there is no global min
5. The largest value is the global max unless M is larger, in which case there is no global max

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