Midterm 2 practice 2

UCLA: Math 32B, Fall 2019

 ${\it Instructor:}\ {\rm Noah\ White}$

Date:

- This exam has 4 questions, for a total of 29 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name:		
ID number:		

Question	Points	Score
1	9	
2	8	
3	7	
4	5	
Total:	29	

Here are some formulas that you may find useful as some point in the exam.

$$\int \cos^2 x \, dx = \frac{1}{2} (x + \cos x \sin x)$$
$$\int \sin^2 x \, dx = \frac{1}{2} (x - \cos x \sin x)$$
$$\int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x$$

Spherical coordinates are given by

$$x(\rho, \theta, \phi) = \rho \cos \theta \sin \phi$$
$$y(\rho, \theta, \phi) = \rho \sin \theta \sin \phi$$
$$z(\rho, \theta, \phi) = \rho \cos \phi$$

The Jacobian for the change of coordinates is $J = \rho^2 \sin \phi$.

1. Let \mathcal{E} be the solid region defined by

$$x^2 + y^2 + z^2 \le a, \quad x, y, z \ge 0,$$

for a fixed constant a > 0.

(a) (2 points) Find the volume of $\mathcal E$ as an iterated integral.

(b) (2 points) Find the volume of \mathcal{E} .

(c) (3 points) Let $V = \text{Vol}(\mathcal{E})$. Express $C_x = \frac{1}{V} \iiint_{\mathcal{E}} x \ dV$, $C_y = \frac{1}{V} \iiint_{\mathcal{E}} y \ dV$, and $C_z = \frac{1}{V} \iiint_{\mathcal{E}} z \ dV$ as iterated integrals.

(d) (2 points) Evaluate C_z .

2. Consider the helix C, given by the parameterisation

$$\mathbf{r}(t) = \left(\cos t, \sin t, \frac{1}{2\pi}t\right) \quad t \in [0, 4\pi],$$

so that $\mathcal C$ is oriented with the z coordinate increasing.

(a) (4 points) Compute the length of C.

(b) (4 points) Compute the work done by the field

$$\mathbf{F}(x,y,z) = \langle z^2, 2yz^2, 2z(x+y^2) - e^z \rangle$$

on a particle constrained to move on the curve C.

3. For this question consider the vector field

$$\mathbf{F}(x,y) = \frac{1}{r^2} \langle y(r^2 - 1), x(r^2 + 1) \rangle,$$

where $r = \sqrt{x^2 + y^2}$. This vector field is defined everywhere apart from the origin.

(a) (4 points) Is **F** conservative on the domain described above? Justify your answer.

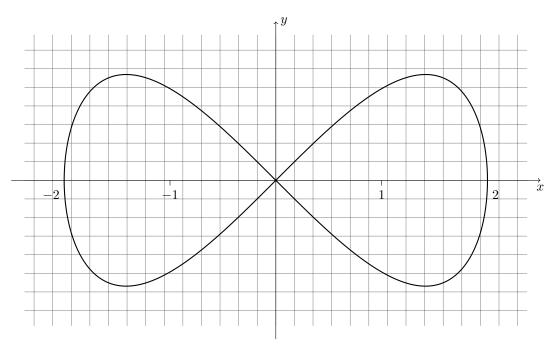
(b) (1 point) Give a domain on which \mathbf{F} is conservative.

(c) (2 points) Calculate the line integral $\frac{1}{2}$

$$\int_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$$

where C is the ellipse $\frac{(x-4)^2}{2} + y^2 = 1$, oriented in the counter clockwise direction.

4. In this question assume that **E** is a vector field defined on the whole plane, apart from the points $(\pm 1, 0)$. Suppose that $\nabla \times \mathbf{E} = 0$. The function $\mathbf{r}(t) = (2\cos t, \sin 2t)$ for $t \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$ defines the curve \mathcal{C} on the graph below



- (a) (1 point) Indicate on the above graph, the orientation of the curve.
- (b) (4 points) Let \mathcal{A} and \mathcal{B} be the circles, radius $\frac{1}{2}$, and center (1,0) and (-1,0) respectively, both oriented counter clockwise. Suppose that

$$\int_{\mathcal{A}} \mathbf{E} \cdot d\mathbf{r} = 2 \quad \text{and} \quad \int_{\mathcal{B}} \mathbf{E} \cdot d\mathbf{r} = 1.$$

What is $\int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{r}$? Justify your answer.

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