

# Midterm 2 practice 2

## UCLA: Math 32B, Fall 2019

*Instructor:* Noah White

*Date:*

- This exam has 5 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

| Question | Points | Score |
|----------|--------|-------|
| 1        | 9      |       |
| 2        | 8      |       |
| 3        | 7      |       |
| 4        | 5      |       |
| 5        | 11     |       |
| Total:   | 40     |       |

Here are some formulas that you may find useful as some point in the exam.

$$\int \cos^2 x \, dx = \frac{1}{2} (x + \cos x \sin x)$$

$$\int \sin^2 x \, dx = \frac{1}{2} (x - \cos x \sin x)$$

$$\int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x$$

Spherical coordinates are given by

$$x(\rho, \theta, \phi) = \rho \cos \theta \sin \phi$$

$$y(\rho, \theta, \phi) = \rho \sin \theta \sin \phi$$

$$z(\rho, \theta, \phi) = \rho \cos \phi$$

The Jacobian for the change of coordinates is  $J = \rho^2 \sin \phi$ .

1. Let  $\mathcal{E}$  be the solid region defined by

$$x^2 + y^2 + z^2 \leq a, \quad x, y, z \geq 0,$$

for a fixed constant  $a > 0$ . Suppose the region has a constant mass density of  $\delta(x, y, z) = 1$ .

- (a) (2 points) Express the total mass of  $\mathcal{E}$  as an iterated integral.

- (b) (2 points) Find the total mass of  $\mathcal{E}$ .

(c) (3 points) Express the coordinates of the center of mass of  $\mathcal{E}$  as an iterated triple integral.

(d) (2 points) Find the  $z$  coordinate of the center of mass.

2. Consider the helix  $\mathcal{C}$ , given by the parameterisation

$$\mathbf{r}(t) = \left( \cos t, \sin t, \frac{1}{2\pi}t \right) \quad t \in [0, 4\pi],$$

so that  $\mathcal{C}$  is oriented with the  $z$  coordinate increasing.

- (a) (4 points) Compute the length of  $\mathcal{C}$ .

(b) (4 points) Compute the work done by the field

$$\mathbf{F}(x, y, z) = \langle z^2, 2yz^2, 2z(x + y^2) - e^z \rangle$$

on a particle constrained to move on the curve  $\mathcal{C}$ .

3. For this question consider the vector field

$$\mathbf{F}(x, y) = \frac{1}{r^2} \langle y(r^2 - 1), x(r^2 + 1) \rangle,$$

where  $r = \sqrt{x^2 + y^2}$ . This vector field is defined everywhere apart from the origin.

- (a) (4 points) Is  $\mathbf{F}$  conservative on the domain described above? Justify your answer.

- (b) (1 point) Give a domain on which  $\mathbf{F}$  is conservative.

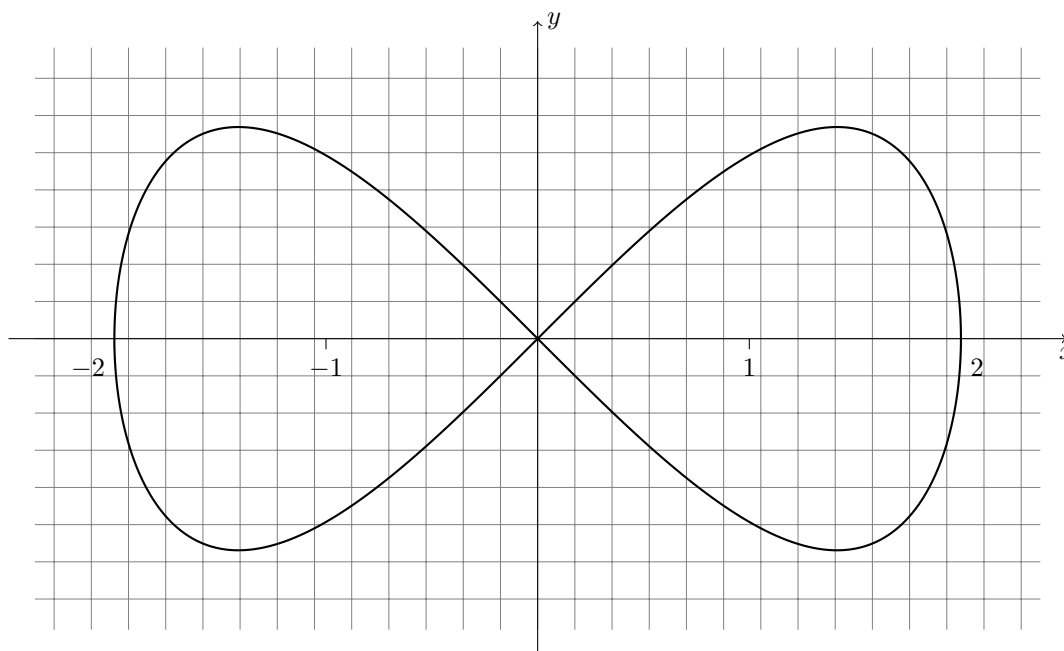
(c) (2 points) Calculate the line integral

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$$

where  $\mathcal{C}$  is the ellipse  $\frac{(x-4)^2}{2} + y^2 = 1$ , oriented in the counter clockwise direction.



4. In this question assume that  $\mathbf{E}$  is a vector field defined on the whole plane, apart from the points  $(\pm 1, 0)$ . Suppose that  $\nabla \times \mathbf{E} = 0$ . The function  $\mathbf{r}(t) = (2 \cos t, \sin 2t)$  for  $t \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$  defines the curve  $\mathcal{C}$  on the graph below



- (a) (1 point) Indicate on the above graph, the orientation of the curve.
- (b) (4 points) Let  $\mathcal{A}$  and  $\mathcal{B}$  be the circles, radius  $\frac{1}{2}$ , and center  $(1, 0)$  and  $(-1, 0)$  respectively, both oriented counter clockwise. Suppose that

$$\int_{\mathcal{A}} \mathbf{E} \cdot d\mathbf{r} = 2 \quad \text{and} \quad \int_{\mathcal{B}} \mathbf{E} \cdot d\mathbf{r} = 1.$$

What is  $\int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{r}$ ? Justify your answer.

5. The *hyperboloid* is Noah's favorite surface. It is given by the equation  $x^2 + y^2 - z^2 = 1$ .

(a) (3 points) Find a parameterisation

$$G(s, \theta) = (x(s, \theta), y(s, \theta), z(s, \theta)) \quad (s, \theta) \in \mathbb{R} \times [0, 2\pi]$$

for the hyperboloid. *Hint: Let  $z = s$ .*

(b) (5 points) Express the surface area of the hyperboloid between  $z = a$  and  $z = -a$  as an iterated integral.

*(extra working room for part (b))*

- (c) (3 points) Calculate the surface area. You may use the formula  $\int \sqrt{1+x^2} \, dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \ln(x + \sqrt{1+x^2})$ .

This page has been left intentionally blank. You may use it as scratch paper. It will not be graded unless indicated very clearly here and next to the relevant question.

This page has been left intentionally blank. You may use it as scratch paper. It will not be graded unless indicated very clearly here and next to the relevant question.