

Math 3B: Lecture 21

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March 4, 2019

Autonomous equations

Definition

An ODE of the form

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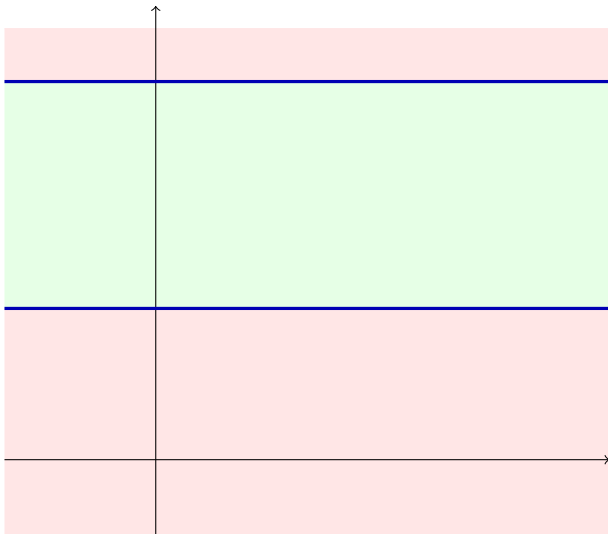
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We want points (t, y) such that $f(y) = 0$.

- Suppose $f(a) = 0$.
- Then (t, a) is on the nullcline, for **any** t .
- So the line $y = a$ is part of the nullcline, whenever $f(a) = 0$.

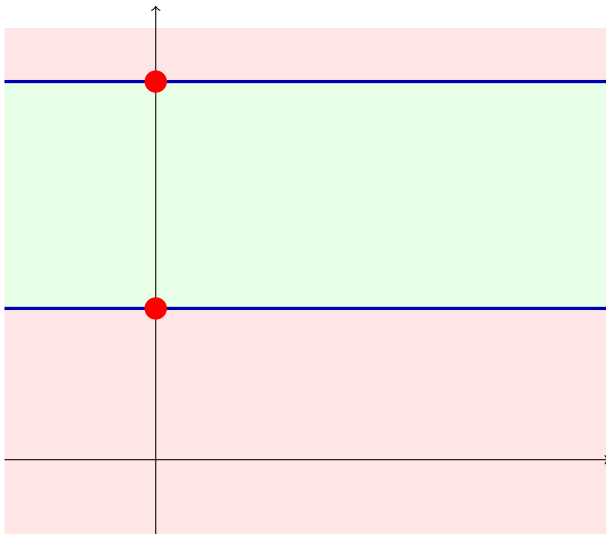
Slope fields and nullclines for autonomous systems

Thus our slope field and nullclines look something like



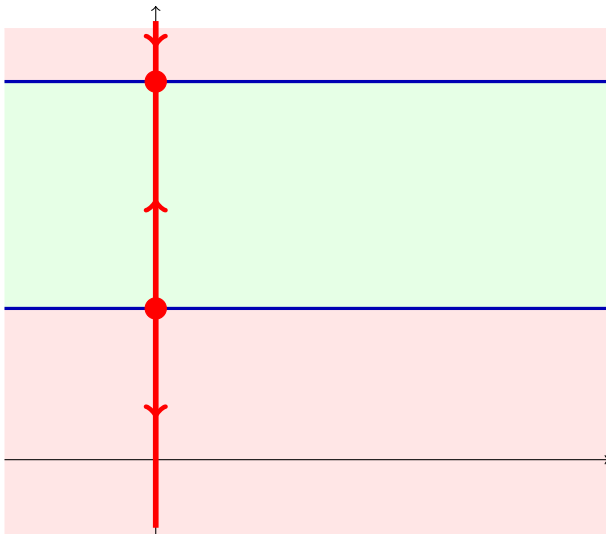
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- An equilibrium is **stable** if the two arrows are pointing towards it.
- It is **unstable** if the two arrows are pointing away from it.
- It is **semistable** if the arrows point in the same direction.

Phase lines



stable



unstable



semistable




semistable

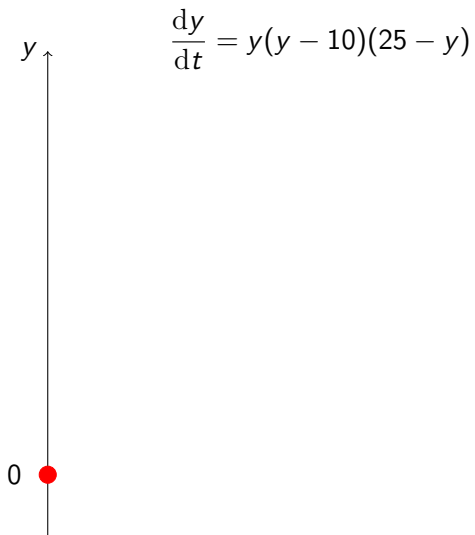
Example

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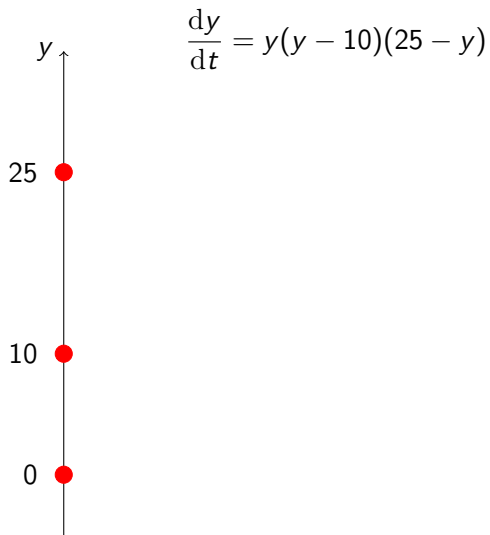
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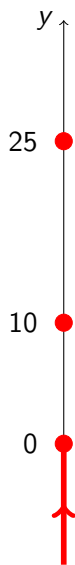


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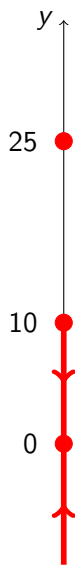
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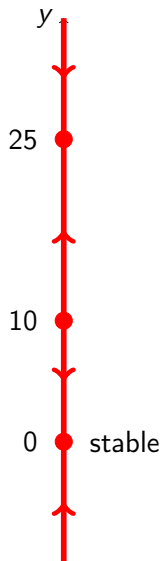
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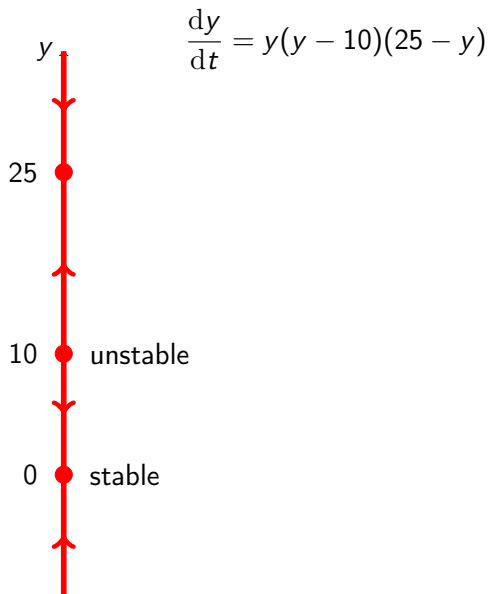


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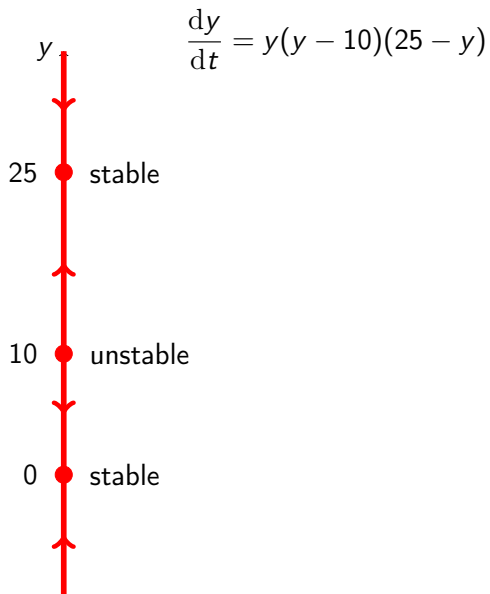
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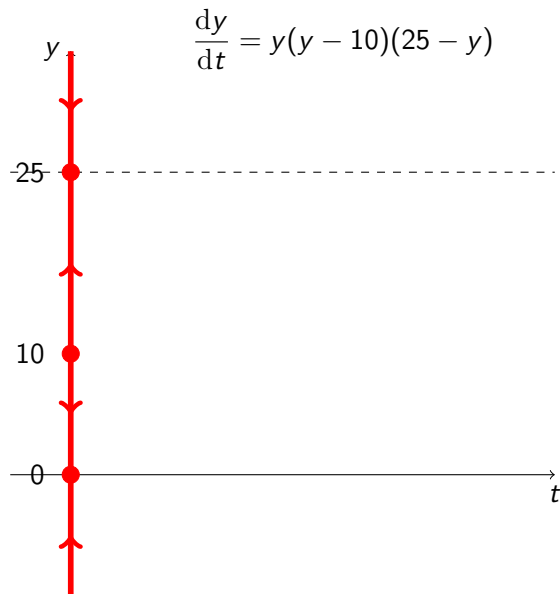
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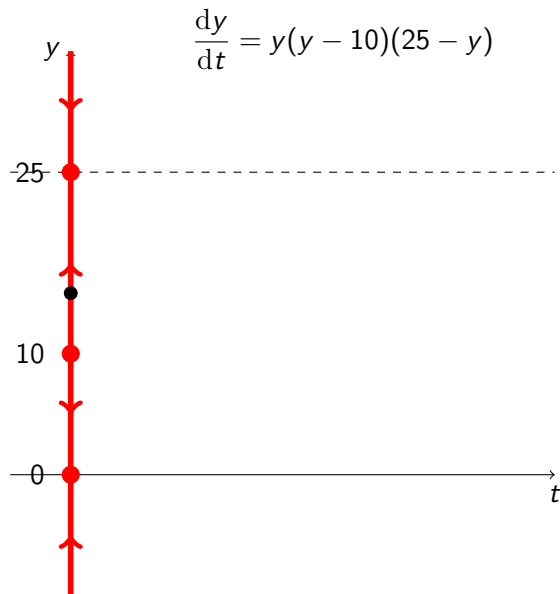
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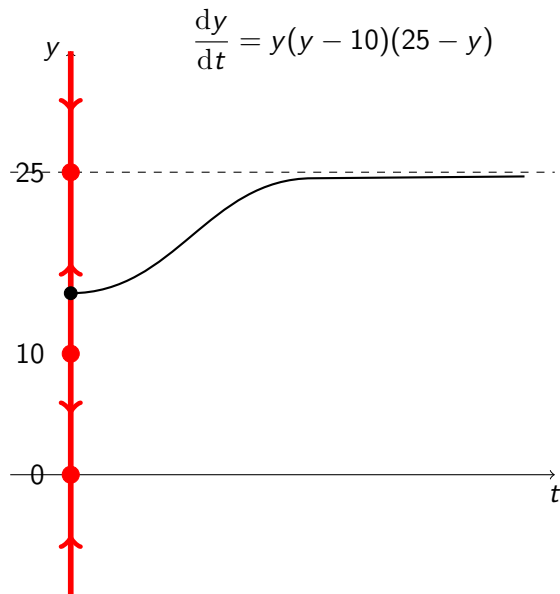
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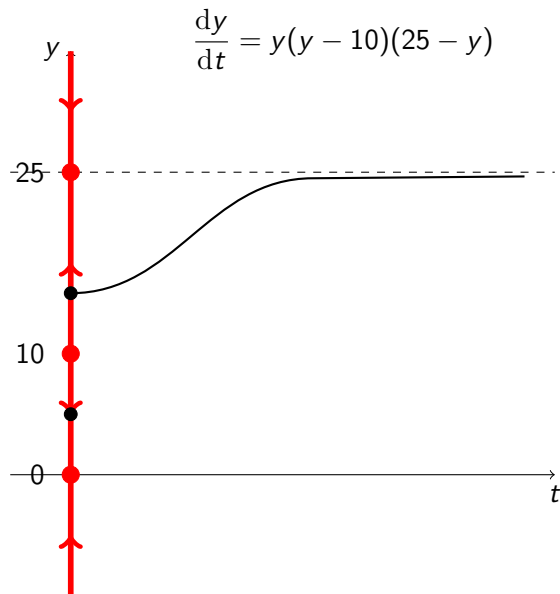
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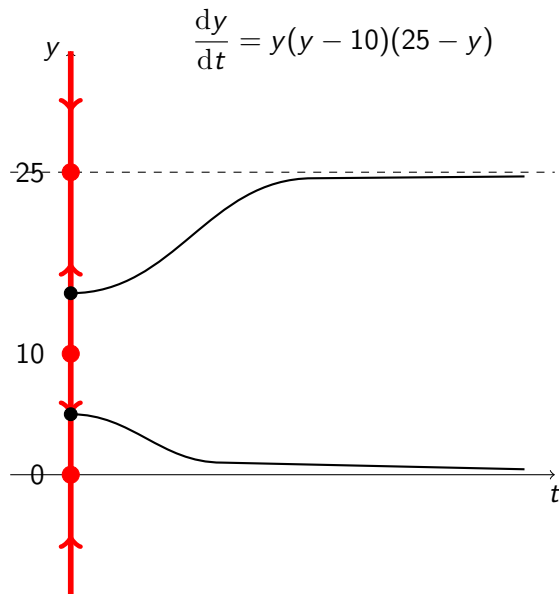
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Classifying equilibria using derivatives

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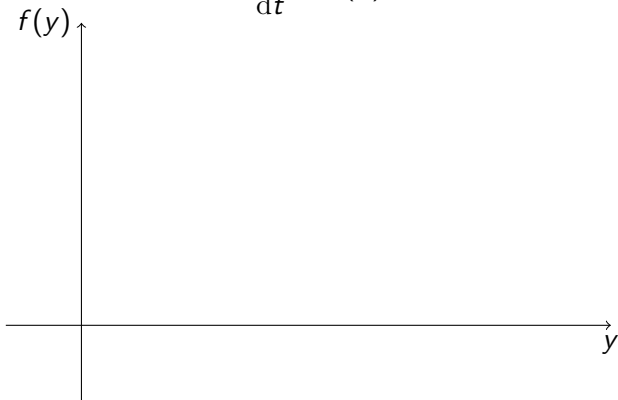
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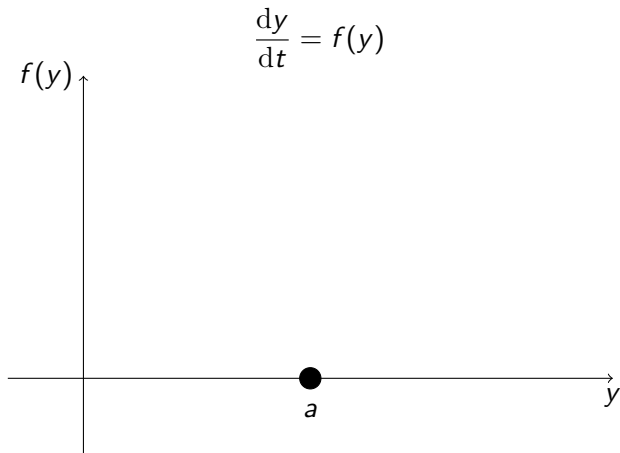
- **stable** if $f'(a) < 0$
- **unstable** if $f'(a) > 0$
- **indeterminate** if $f'(a) = 0$

Why?

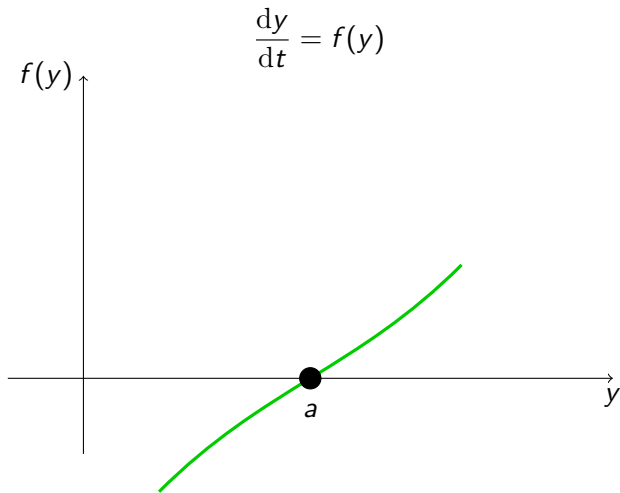
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