

# Midterm 1

## UCLA: Math 3B (practice 1), Winter 2019

*Instructor:* Noah White

*Date:*

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

Discussion section (please circle):

Day/TA	Louis	Matthew
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	10	
2	12	
3	10	
4	8	
Total:	40	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

- (a) (2 points) The function  $f(x) = \frac{1}{1+x}$  has
- A. a vertical asymptote at  $x = -1$ .
  - B. a horizontal asymptote at  $y = 1$ .
  - C. no asymptotes.
  - D. a slanted asymptote.

- (b) (2 points) The function  $f(x) = \frac{1}{x} - x$  is
- A. increasing when  $x > 0$  and decreasing when  $x < 0$ .
  - B. increasing when  $x < 0$  and decreasing when  $x > 0$ .
  - C. always increasing.
  - D. always decreasing.

- (c) (2 points) The function  $f(t) = t - 2 \ln t$  has a
- A. local minimum at  $t = 2$ .
  - B. local maximum at  $t = 2$ .
  - C. local minimum at  $t = 1$ .
  - D. local maximum at  $t = 1$ .

(d) (2 points) The function  $f(x) = 2 \ln(e^x + 1) - x$  has

- A. a horizontal asymptote at  $y = 1$ .
- B. a vertical asymptote at  $x = 0$ .
- C. no asymptotes.
- D. a slanted asymptote of  $y = x$ .

*Hint: For this question you may use the fact that  $\lim_{x \rightarrow \infty} f(x) = \infty$  and  $f'(x) = \frac{e^x - 1}{e^x + 1}$*

(e) (2 points) The function  $h(t) = 1 - \sin x + x$  is an antiderivative of

- A.  $x - \cos + 0.5x^2$
- B.  $1 - \cos x$
- C.  $1 - \sin x$
- D.  $x + \cos x - x^2$

2. Let  $f(x) = \frac{1}{x^2-x-2}$ . Note that  $f'(x) = \frac{1-2x}{(x-2)^2(x+1)^2}$  and  $f''(x) = \frac{6(x^2-x+1)}{(x-2)^3(x+1)^3}$ .

(a) (2 points) Find the  $x$  and  $y$  intercepts of  $f(x)$ .

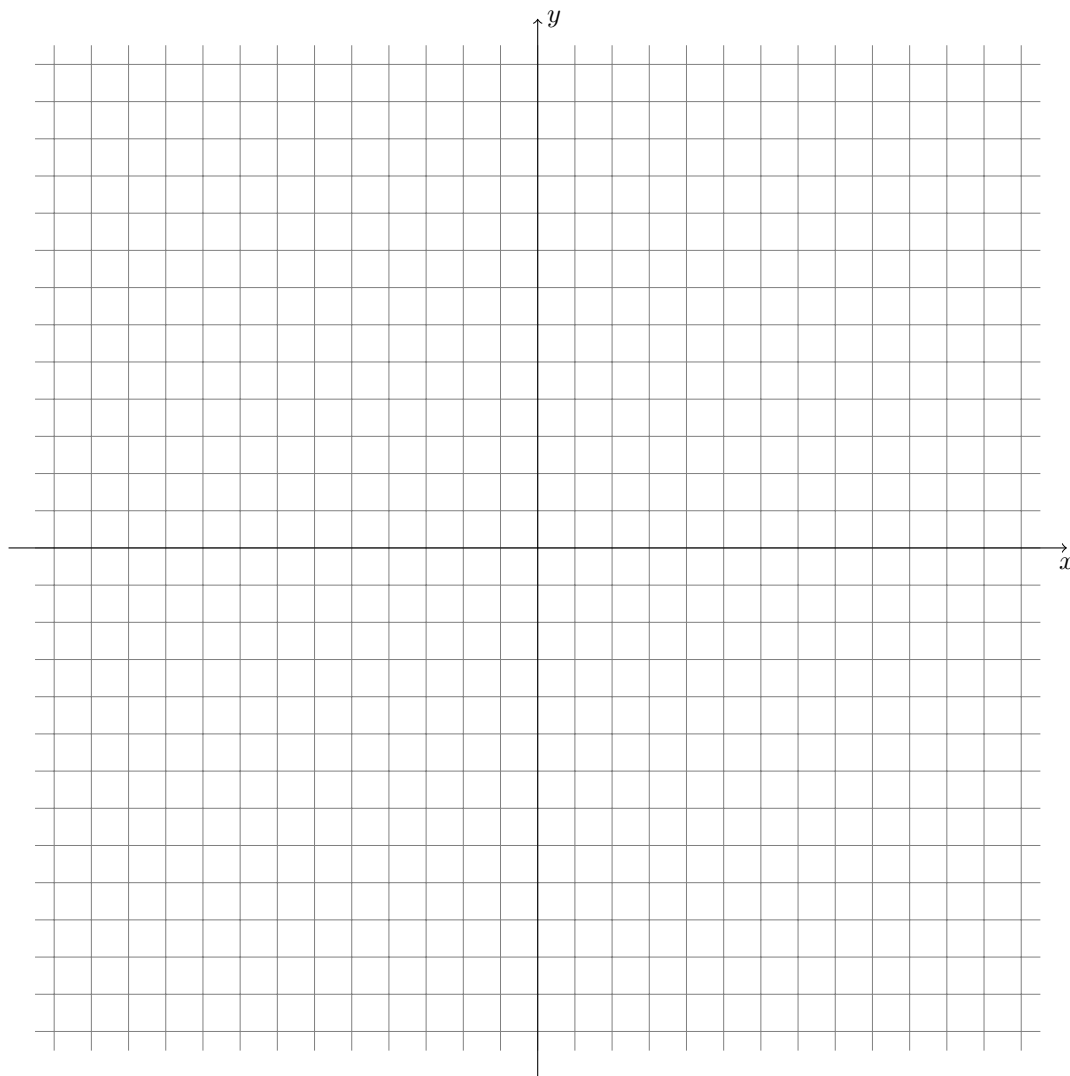
(b) (1 point) Does  $f(x)$  have any horizontal asymptotes? If so what are they?

(c) (1 point) Does  $f(x)$  have any vertical asymptotes? If so what are they?

(d) (2 points) For what  $x$  is the first derivative  $f'(x)$  positive?

(e) (2 points) For what  $x$  is the second derivative  $f''(x)$  positive?

(f) (4 points) On the graph provided, sketch  $f(x)$



3. A company would like to advertise its product in two media markets (market  $A$  and market  $B$ ). It is known experimentally that a spend of  $\$x$  thousand dollars in market  $A$  will result in

$$R_A(x) = 18 \ln(1 + x)$$

thousand extra customers and a spend of  $\$y$  thousand in market  $B$  will result in

$$R_B(x) = 8 \ln(1 + x)$$

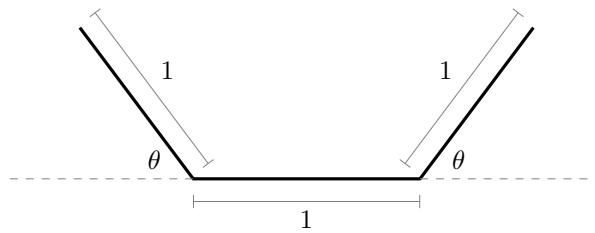
thousand new customers.

- (a) (5 points) If the company has a marketing budget of \$10 thousand and they wish to spend it all, how many new customers will they attract in total? (Let  $\$x$  thousand be the amount spent in market  $A$ .)

- (b) (5 points) How much should the company spend in market  $A$  in order to maximise the number of new customers they attract?



4. A trough is being built for feeding animals. The cross section of the is pictured below.



Each side, and the base of the trough are 1 foot wide. The sides are bent up from the horizontal, at an angle  $\theta$ .

- (a) (3 points) What is the area of the cross section?
- (b) (5 points) What angle should the sides be bent to, in order that the trough can hold as much food as possible? You may use the fact that  $\cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$ .

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