

Midterm 1 practice 2

UCLA: Math 31B, Spring 2017

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Date:

Version: practice

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Discussion section: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. (a) (4 points) Calculate $\frac{d}{dx} \left[e^{e^{(x^2+2)}} \right]$
(b) (6 points) Calculate $\int \frac{1}{x(\ln x)^2} dx$.

Solution:

(a) $e^{e^{(x^2+2)}} e^{(x^2+2)} (2x)$.

(b) Let $u = \ln x$. Then $du = \frac{1}{x} dx$. So

$$\int \frac{1}{x(\ln x)^2} dx = \int \frac{1}{u^2} du = -\frac{1}{u} + c = -\frac{1}{\ln x} + c.$$

2. (a) (3 points) Let $f(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3 + \frac{11}{24}x^4 + \frac{7}{120}x^5$.
What are the Taylor polynomials $T_3(x)$ and $T_7(x)$ for $f(x)$ centered at 0?
- (b) (2 points) Find the n^{th} Taylor polynomial about 1 of the function $\ln x$.
- (c) (5 points) Let $T_n(x)$ be the n -th Taylor polynomial for $\ln x$ centered at 1.
Find an n such that

$$\left| \ln\left(\frac{1}{2}\right) - T_n\left(\frac{1}{2}\right) \right| < \frac{1}{10^{10}}.$$

Solution:

(a) $T_3(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3$, $T_7(x) = 4 + \frac{3}{1}x + \frac{9}{2}x^2 + \frac{5}{6}x^3 + \frac{11}{24}x^4 + \frac{7}{120}x^5$.

(b) $T_n(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \dots + \frac{(-1)^{n-1}}{n}(x-1)^n$

(c) We'll use Taylor's Error Bound for $f(u) = \ln u$, $a = 1$, $x = \frac{1}{2}$, and n unspecified.

For u between a and x , i.e. $\frac{1}{2} \leq u \leq 1$, we have

$$|f^{(n+1)}(u)| = \left| \frac{n!}{u^{n+1}} \right| \leq \frac{n!}{(\frac{1}{2})^{n+1}} = 2^{n+1}n!$$

so we can take $K = 2^{n+1}n!$ and Taylor's Error Bound says that

$$\left| \ln\left(\frac{1}{2}\right) - T_n\left(\frac{1}{2}\right) \right| \leq \frac{2^{n+1}n! \left|\frac{1}{2} - 1\right|^{n+1}}{(n+1)!} = \frac{1}{n+1}.$$

We see that

$$\left| \ln\left(\frac{1}{2}\right) - T_{10^{10}}\left(\frac{1}{2}\right) \right| \leq \frac{1}{10^{10} + 1} < \frac{1}{10^{10}}$$

so $n = 10^{10}$ works.

3. For (a)-(c), give the value or say, “undefined.”

(a) (1 point) $\tan(\arctan(2)) =$

(b) (1 point) $\sin(\arcsin(2)) =$

(c) (2 points) $\arctan(\tan(\frac{7\pi}{3})) =$

(d) (6 points) Suppose $a \neq 0$.

Calculate the following indefinite integral as I did in class (using a u -substitution and the knowledge of fundamental integrals which relate to inverse trigonometric functions).

$$\int \frac{1}{a^2 + x^2} dx$$

Solution:

(a) 2.

(b) Undefined.

(c) $\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$.

(d) Let $u = \frac{x}{a}$. Then $x = au$ so that $a^2 + x^2 = a^2 + a^2u^2$ and $dx = adu$. Thus,

$$\begin{aligned} \int \frac{1}{a^2 + x^2} dx &= \int \frac{a}{a^2 + a^2u^2} du \\ &= \frac{1}{a} \int \frac{1}{1 + u^2} du = \frac{1}{a} \arctan(u) + c = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + c. \end{aligned}$$

4. (10 points) Calculate the following indefinite integral

$$\int \frac{6x^3 + 3x^2 + 9x - 8}{(x^2 - 1)(x^2 + 4)} dx.$$

[The numbers have been chosen so that they work out well; they are all whole numbers.

In the method of partial fractions, I found looking at the x^3 -coefficient useful.

You'll get points for spotting the correct partial fraction decomposition, and displaying knowledge of the relevant integrals. Notice that you did one of these integrals in 3.d).]

Solution: Clearing denominators on

$$\frac{6x^3 + 3x^2 + 9x - 8}{(x^2 - 1)(x^2 + 4)} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{Cx + D}{x^2 + 4}$$

gives

$$6x^3 + 3x^2 + 9x - 8 = A(x + 1)(x^2 + 4) + B(x - 1)(x^2 + 4) + (Cx + D)(x^2 - 1).$$

Setting $x = 1$ gives $10 = 10A$ so that $A = 1$.

Setting $x = -1$ gives $-20 = -10A$ so that $B = 2$.

Setting $x = 0$ gives $-8 = 4A - 4B - D$ so that $D = 4A - 4B + 8 = 4$.

Looking at the x^3 coefficient gives $A + B + C = 6$ so that $C = 3$.

So

$$\begin{aligned} \int \frac{6x^3 + 3x^2 + 9x - 8}{(x^2 - 1)(x^2 + 4)} dx &= \int \frac{1}{x - 1} + \frac{2}{x + 1} + \frac{3x}{x^2 + 4} + \frac{4}{x^2 + 4} dx \\ &= \ln|x - 1| + 2\ln|x + 1| + \frac{3}{2}\ln(x^2 + 4) + 2\arctan\left(\frac{x}{2}\right) + c. \end{aligned}$$