

Math 3B: Lecture 23

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March 10, 2017

Birfurcation

Often in real life situations we would like to study a system that includes an unknown parameter

$$\frac{dy}{dt} = f(y, a)$$

The behaviour of the solution depends on a !

Example

We have been studying populations growing logistically. We also considered their behaviour under harvesting, but suppose we don't know exactly how many are harvested and we want to understand the effect of different harvesting rates.

$$\frac{dN}{dt} = N(1 - N) - h$$

Bifurcation

- We would like to study **how** the behaviour of the solution depends on the parameter.

#- Plot $f(y, a) = 0$ on the y - a coordinate plane

Bifurcation

- We would like to study **how** the behaviour of the solution depends on the parameter.
- The behaviour of the solution, depends on the equilibria and their stability!

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Bifurcation

- We would like to study **how** the behaviour of the solution depends on the parameter.
- The behaviour of the solution, depends on the equilibria and their stability!
- Draw a **bifurcation diagram**

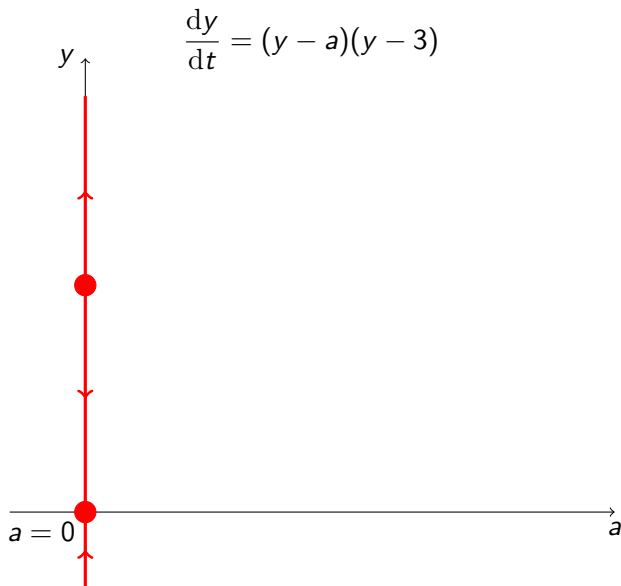
#- Plot $f(y, a) = 0$ on the y - a coordinate plane

Bifurcation

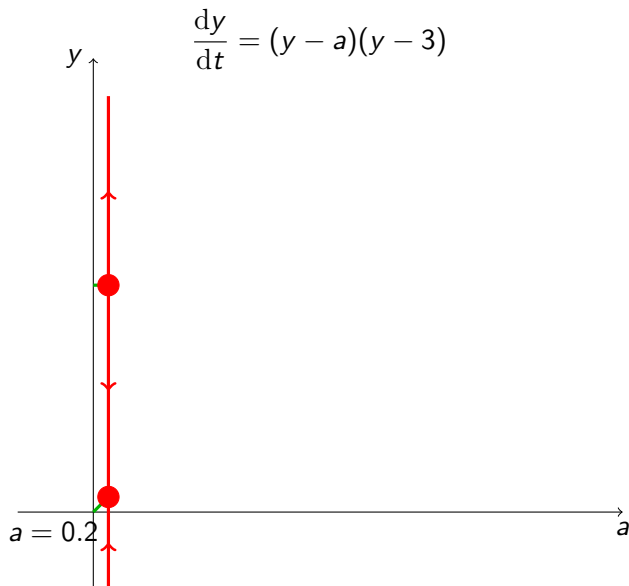
- We would like to study **how** the behaviour of the solution depends on the parameter.
- The behaviour of the solution, depends on the equilibria and their stability!
- Draw a **bifurcation diagram**
- The bifurcation diagram tells us how the **phase line** changes for different parameters.

#- Plot $f(y, a) = 0$ on the y - a coordinate plane

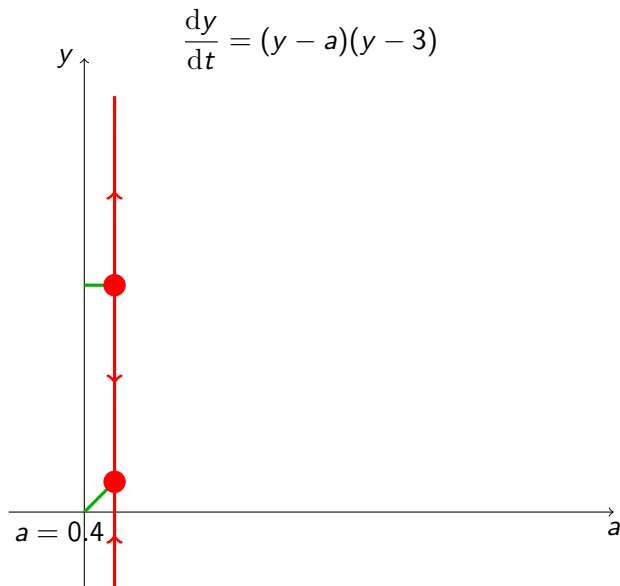
Bifurcation diagram



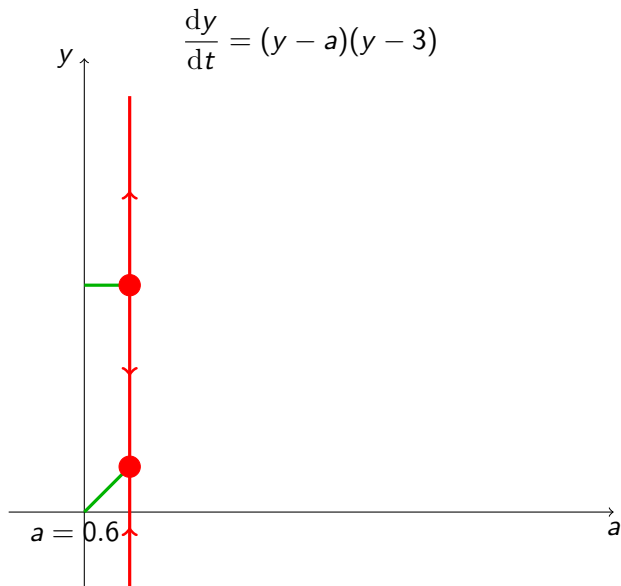
Bifurcation diagram



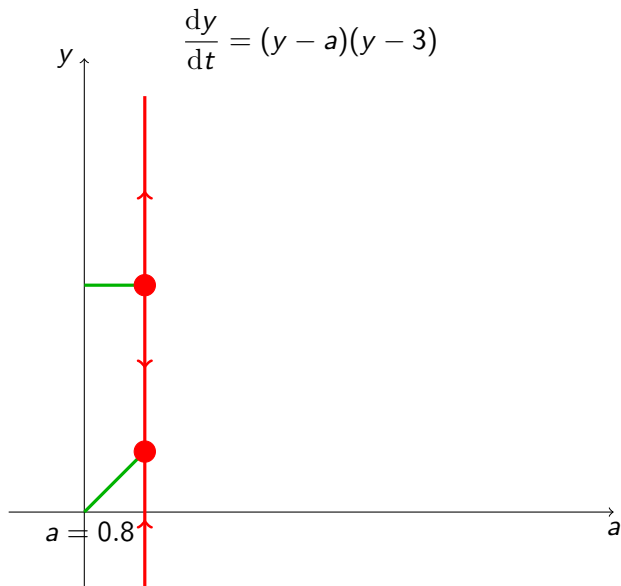
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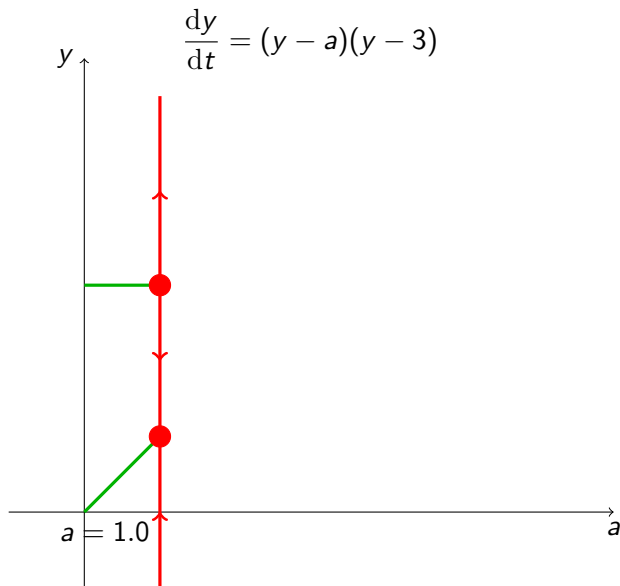
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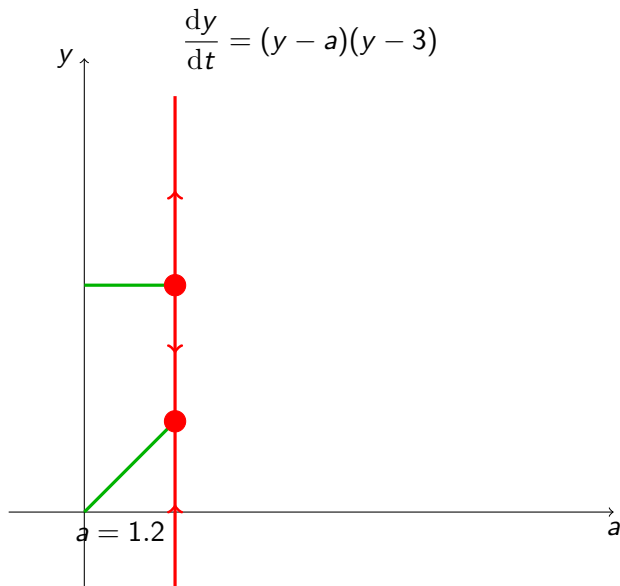
Bifurcation diagram



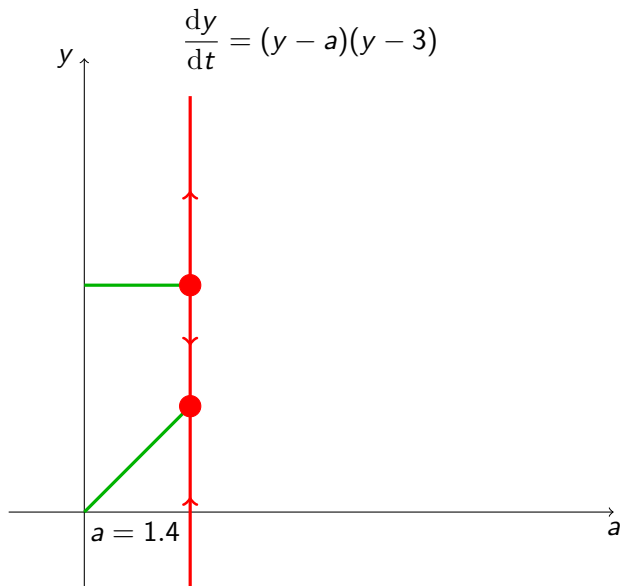
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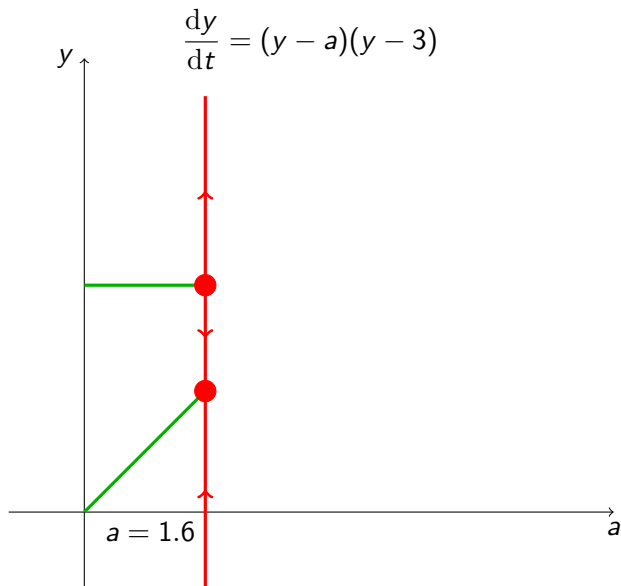
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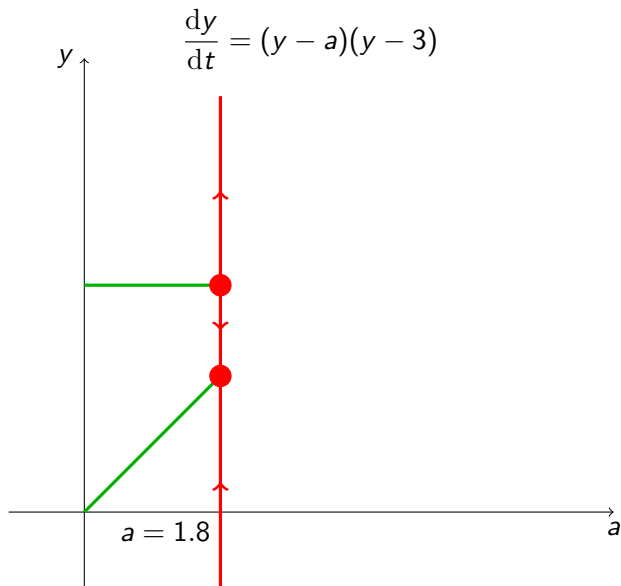
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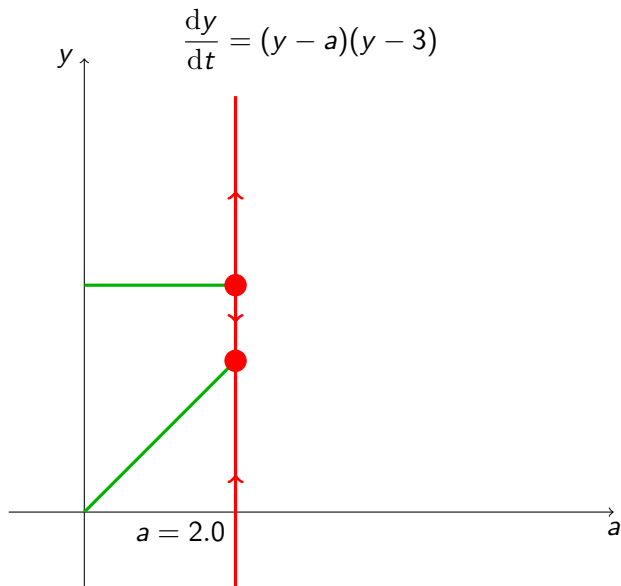
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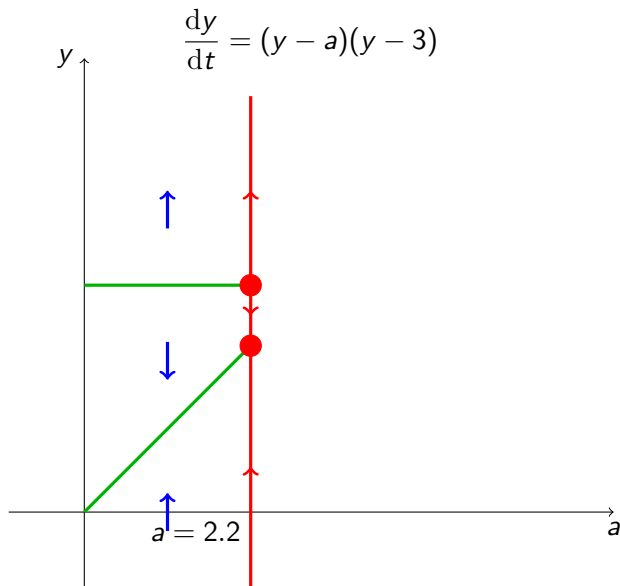
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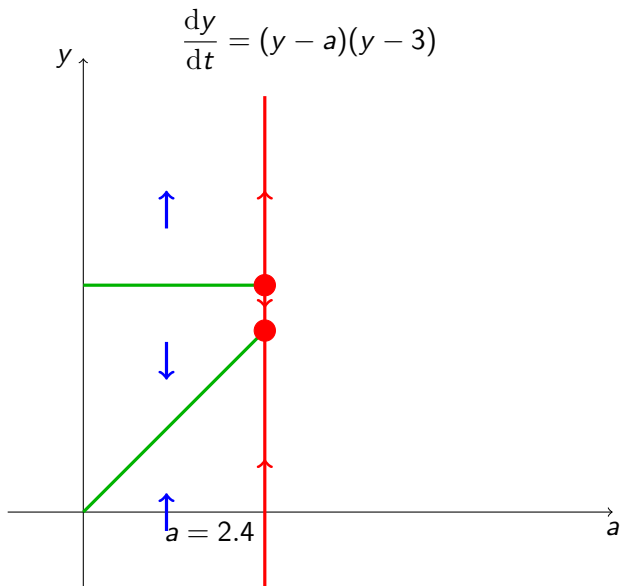
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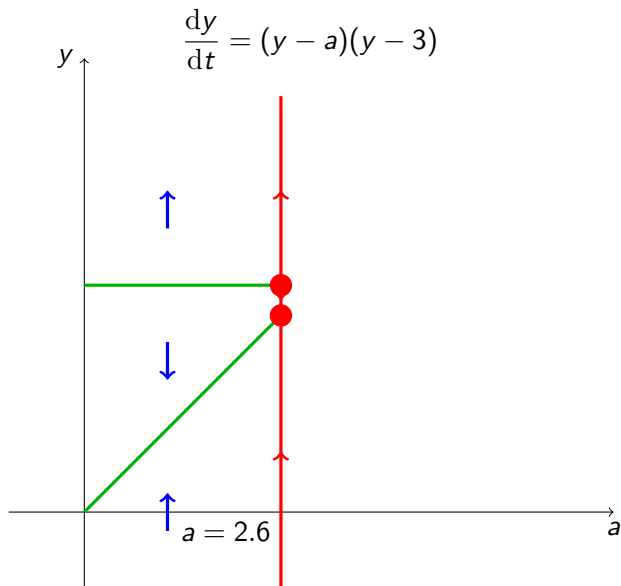
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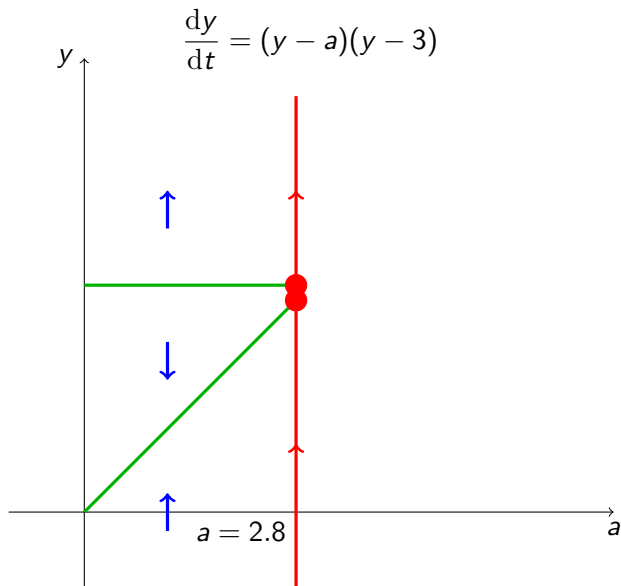
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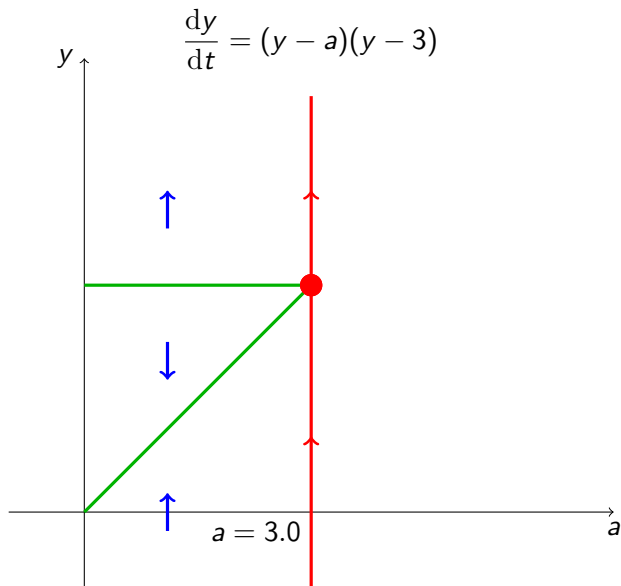
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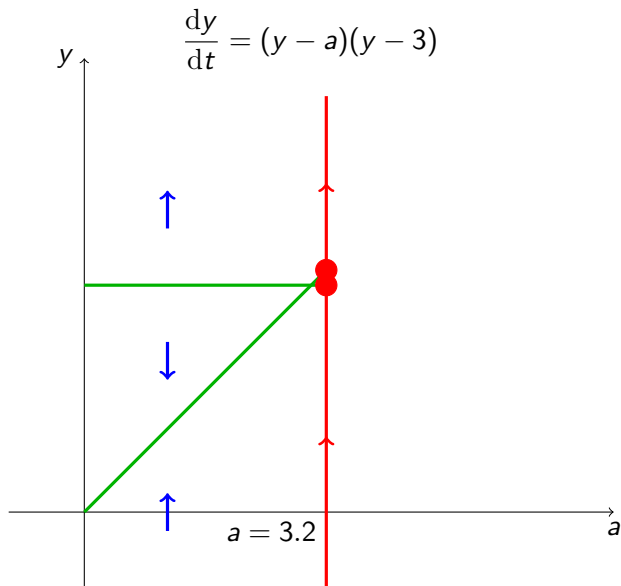
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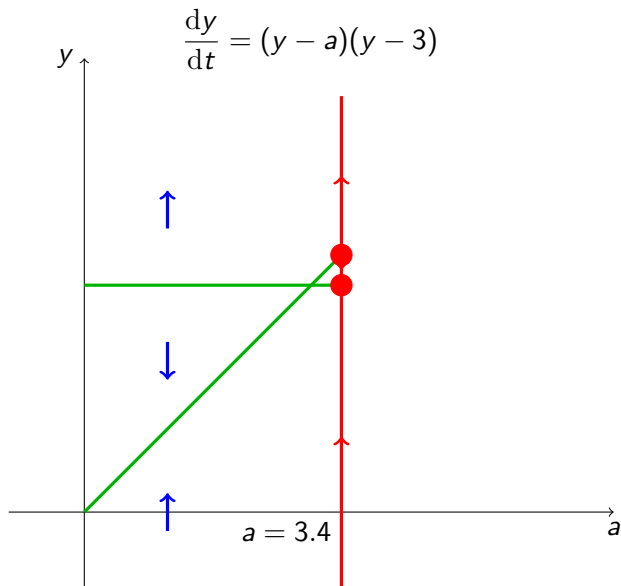
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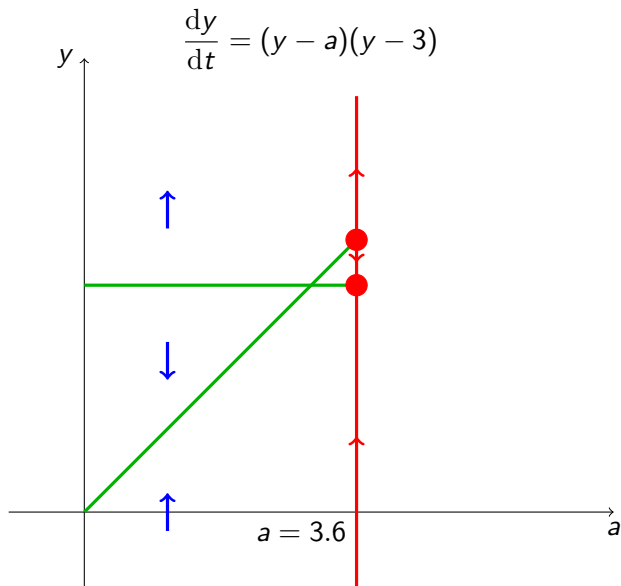
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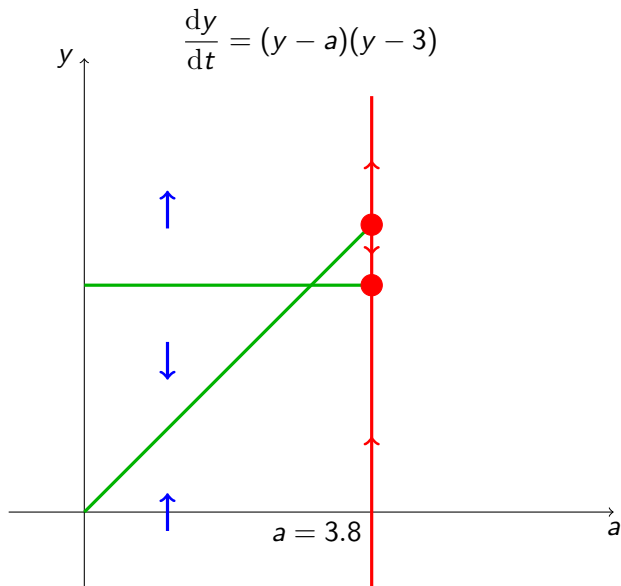
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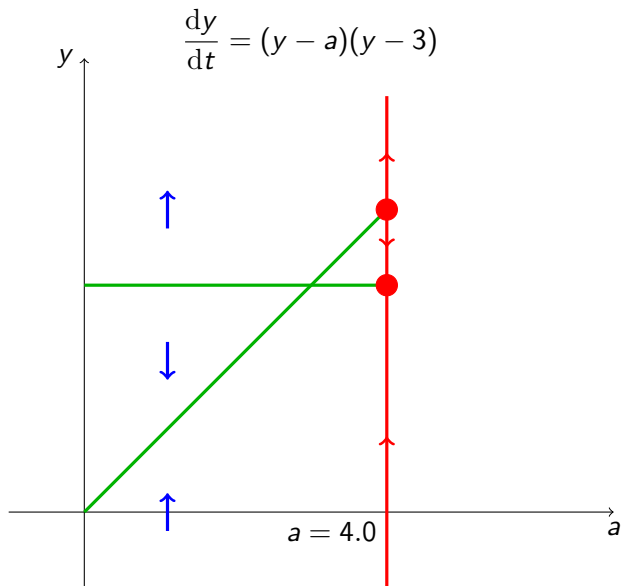
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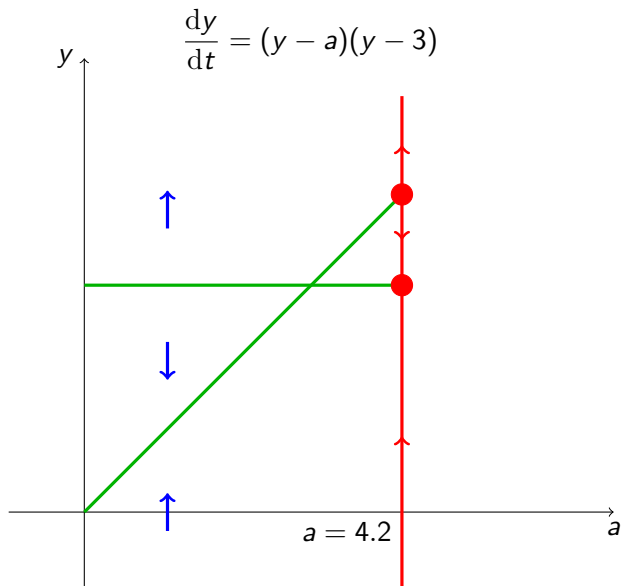
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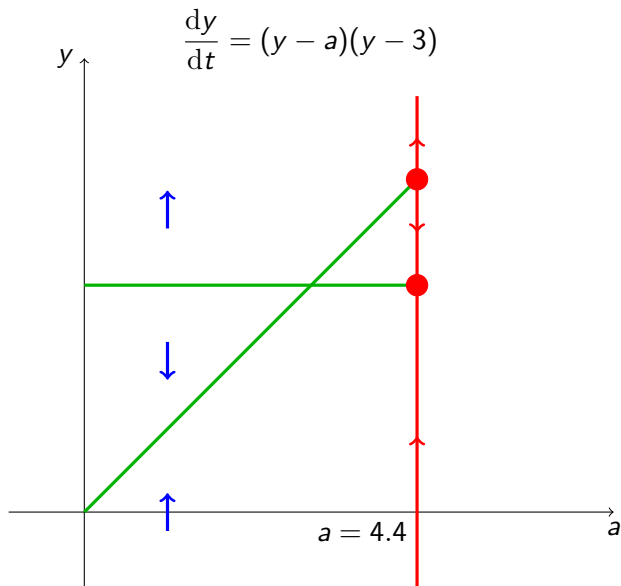
Bifurcation diagram



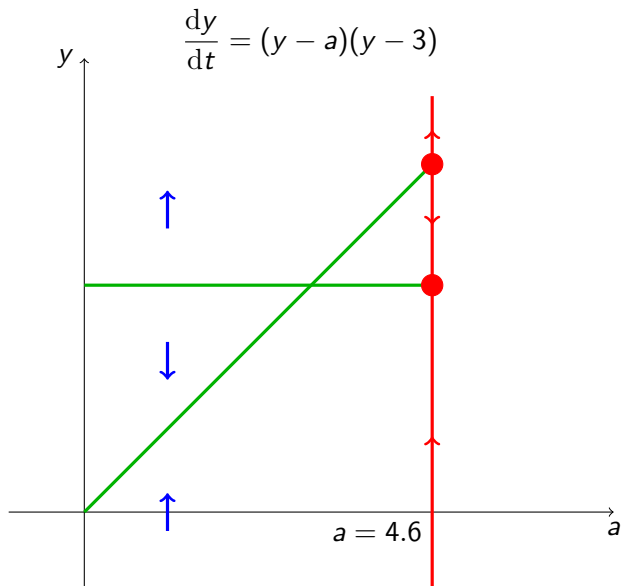
Bifurcation diagram



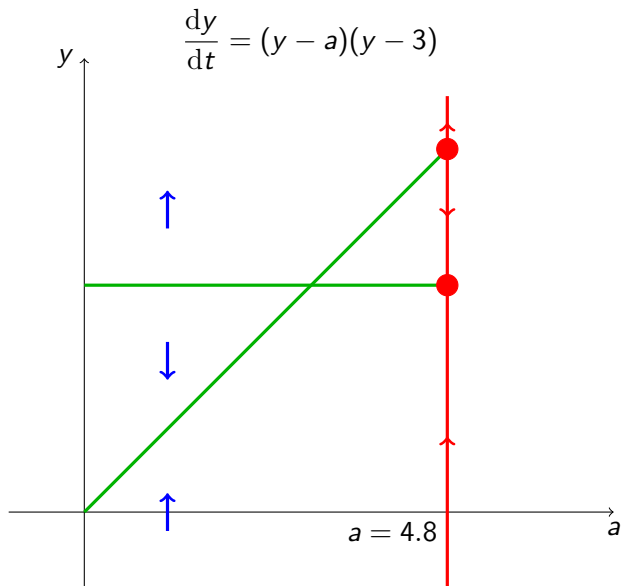
Bifurcation diagram



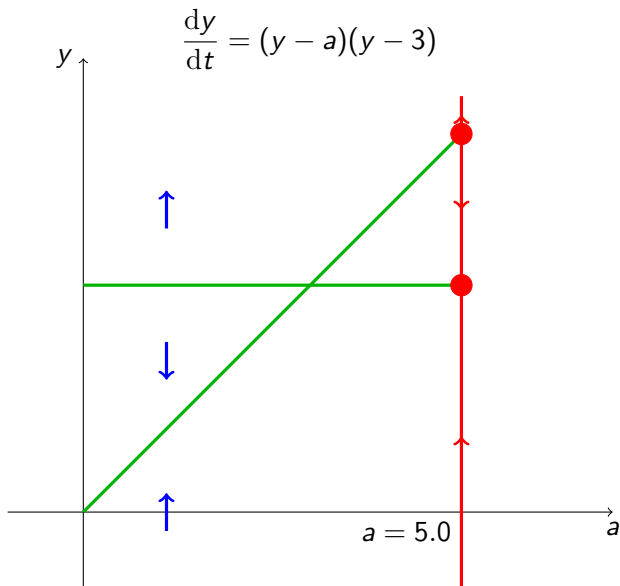
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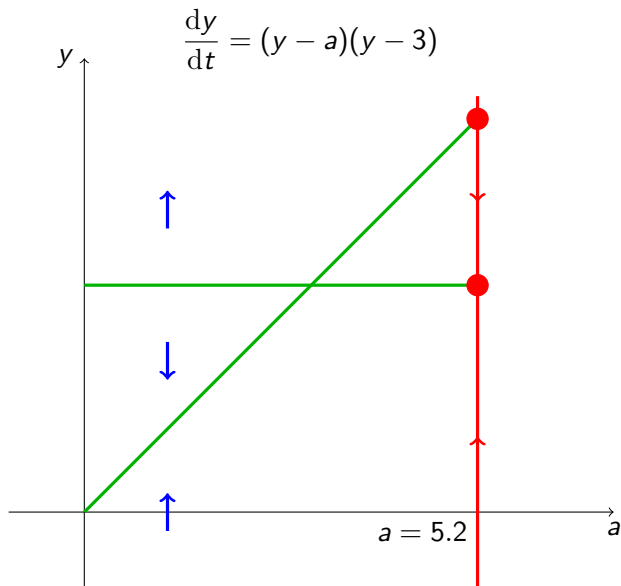
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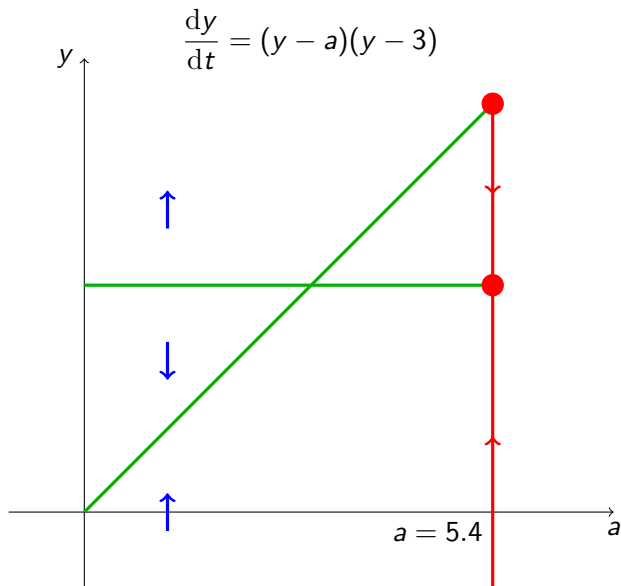
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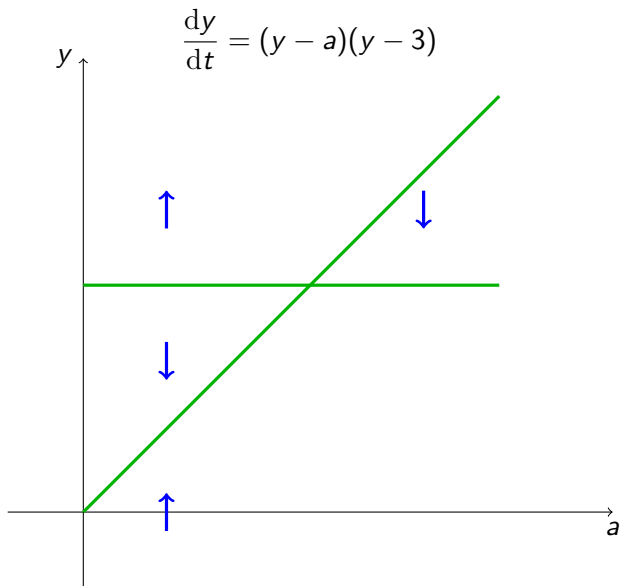
Bifurcation diagram



Bifurcation diagram



Bifurcation diagram



Recipe to draw the bifurcation diagram

$$\frac{dy}{dt} = f(a, y)$$

- Draw the axes for the ay -plane (y vertical axis)

Recipe to draw the bifurcation diagram

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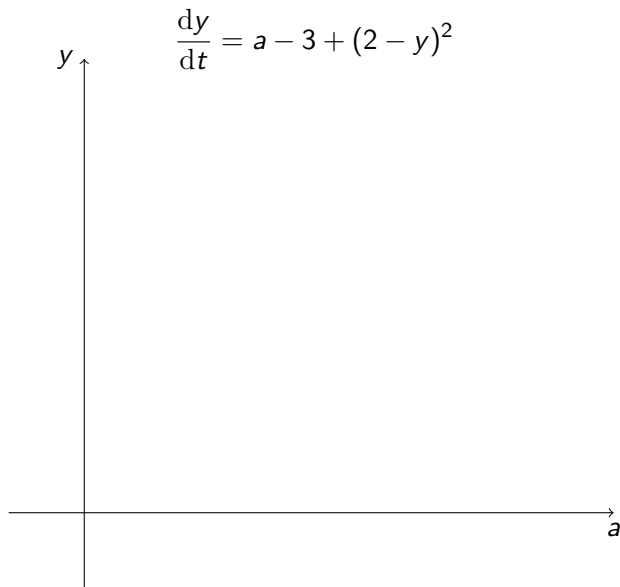
- Draw the axes for the ay -plane (y vertical axis)
- draw the points (a, y) such that $f(a, y) = 0$

Recipe to draw the bifurcation diagram

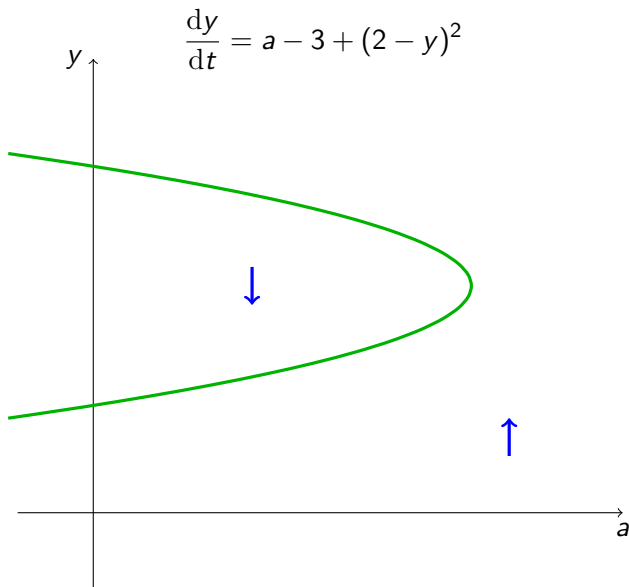
$$\frac{dy}{dt} = f(a, y)$$

- Draw the axes for the ay -plane (y vertical axis)
- draw the points (a, y) such that $f(a, y) = 0$
- label the regions according to whether $f(a, y)$ is positive or negative.

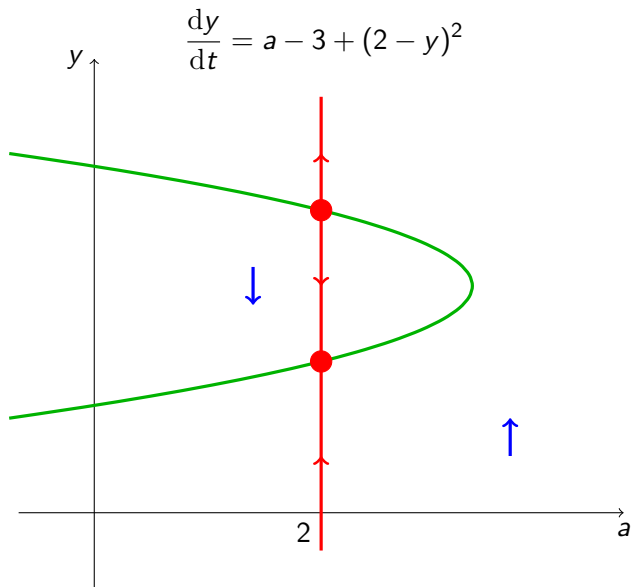
Example



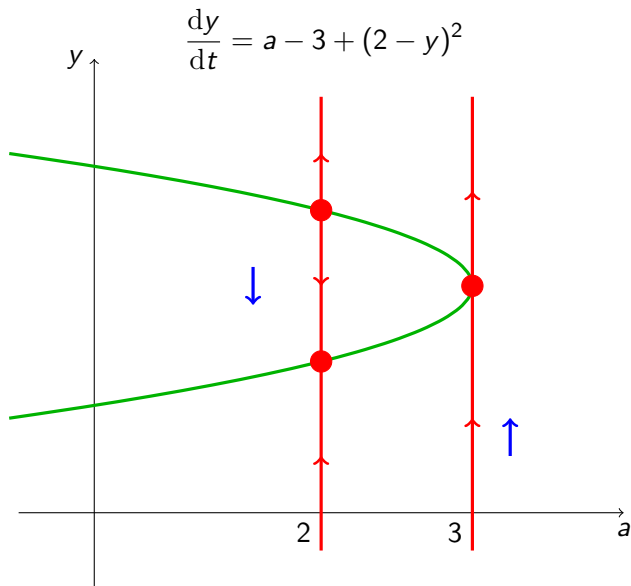
Example



Example



Example



Example

