Math 3B: Lecture 17

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Modelling using differential equations

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Today we will see an number of real world situations

- The goal is to write down a function $y(t)$ that describes something we are interested in (e.g. population/mass/etc)

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- as some other variable changes (usually time)

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- The goal is to write down a function $y(t)$ that describes something we are interested in (e.g. population/mass/etc)
- as some other variable changes (usually time)
- We can't do this directly, but we can write down an ODE that y satisfies instead.

Example 1

The senario

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  $$bN(t) \text{ births per year, for some } b$$

- Number of deaths is proportional to the total number of people. So

  $$dN(t) \text{ deaths per year, for some } d$$

Example 1

The total change in population at time t is

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}t} &= \text{ births at } t - \text{ deaths at } t \\ &= bN(t) - dN(t) \\ &= (b-d)N(t).\end{aligned}$$

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In real life we would determine b and d experimentally. Let $r = b - d$. the instinsic growth rate. So our model is

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN.$$

and we know $N(0) = 100$.

Behaviour of solutions

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN.$$

Case 1: $r = 0$

The population never grows or shrinks, it always stays the same (so $N(t) = 100$ for all t).

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The population is increasing indefinitely.

Case 3: $r < 0$

The population is decreasing indefinitely.

Solution to a simple ODE

Theorem

For any constant a, if y is a solution to the ODE

$$\frac{\mathrm{d}y}{\mathrm{d}x} = ay$$

then y is given by

$$y = Ce^{ax}$$

for some constant C.

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Next time

We will see why, but for now we can verify it is actually a solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x} Ce^{ax} = C\frac{\mathrm{d}}{\mathrm{d}x} e^a x = Cae^{ax} = ay.$$

Back to example 1

We know our population model was governed by

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$$100 = Ce^{(b-d)} \quad \text{so} \quad C = 100e^{(d-b)}.$$

Logistic growth

The previous example is a good first approximation but it is not very realistic in the long term. Usually there are constrains, e.g amount of space, food, etc.

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$$(d \qquad\qquad) N(t).$$

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Where $K = r/k$.

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$$\begin{aligned} \frac{\mathrm{d}N}{\mathrm{d}t} &= bN - (d + kN)N \\ &= (b - d - kN)N = (r - kN)N \\ &= r\left(1 - \frac{kN}{r}\right)N = r\left(1 - \frac{N}{K}\right)N \end{aligned}$$

Where $K = r/k$.

Logistic growth

The equation

$$\frac{\mathrm{d}N}{\mathrm{d}t} = r\left(1 - \frac{N}{K}\right)N$$

is called the Logistic equation and K is the carrying capacity.

Behaviour of logistic growth

Assume that $r > 0$ and $K > 0$.

$$\frac{\mathrm{d}N}{\mathrm{d}t} = r\left(1 - \frac{N}{K}\right)N$$

Case 1. $N(0) = 0$

In this case the growth rate is 0 initially, so N(t) does not increase or decrease, so remains 0.

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Key takeaway

Both $N(t) = 0$ and $N(t) = K$ are solutions to the ODE. They are called equalibrium solutions.

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Case 3. $0 \leq N(0) \leq K$

In this case, N is initially increasing and so becomes more positive, slowing down as it gets close to K.

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In this case, N is initially increasing and so becomes more positive, slowing down as it gets close to K.

Case 4. $N(0) \geq K$

In this case N is initially decreasing but decreases slower and slower as it gets close to K.

Checking solutions

The most straighforward way of checking a function $y = f(x)$ is a solution to a differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = g(x, y)$$

is to simply plug it in to both sides.

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The function $y = e^{\sin x}$ is a solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = y \cos x$. To check note that

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$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= e^{\sin x} \cos x \\ y \cos x &= e^{\sin x} \cos x \end{aligned}$$

Separation of variables

Separation of variables is a very powerful technique for solving ODEs. It works, whenever we can write out an ODE in the form

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$$\int \frac{1}{f(y)}\,\mathrm{d}y = \int g(x)\,\mathrm{d}x.$$
4. solve for y!

Examples

On the board. . .

Linear models

Definition

A first order ODE is linear if it is of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = a + by$$

for constants a and b.

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Examples

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ay, \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = -\lambda y.$$

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A mixing model describes the concentration of something over time, if

- rate in is constant

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Something could mean (for example)

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- concentration of a drug in bloodstream

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Note

Something could mean (for example)

- concentration of a drug in bloodstream
- pollutant in water supply

General solution

Using separation of variables, we can show that the general solution to

$$\frac{\mathrm{d}y}{\mathrm{d}t} = a - by$$

is

$$y(t) = \frac{a}{b} - Ce^{-bt}$$

where C is an arbitrary constant.

Example 1

A drug with a half-life of 2 hours is injected into the bloodstream with an infusion rate of 10 mg/h. Determine the concentration $y(t)$ at time t.

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- Starting with M mg, after t hours there will be

$$M\left(\frac{1}{2}\right)^{t/2} = Me^{-0.5t\ln(2)} \text{ mg left}$$

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- Starting with M mg, after t hours there will be

$$M\left(\frac{1}{2}\right)^{t/2} = Me^{-0.5t\ln(2)} \text{ mg left}$$

- Thus the rate at which the drug is leaving (at time t) is given by

$$0.5\ln(2)Me^{-0.5t\ln(2)} = 0.5\ln(2)(\text{current concentration}) \ \text{ mg/h}.$$

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$$0 = \frac{20}{\ln(2)} - C \approx 28.9 - C$$

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- Since there was initially no drug in the bloodstream, $y(0) = 0$,

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- Thus at time t the concentration is

$$y(t) = 28.9 - 28.9e^{-0.3t} = 28.9\left(1 - e^{-0.3t}\right)$$

Newton's Law of Cooling

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The temperature T of a body changes at a rate propotional to the difference between the ambient temperature A and T.

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$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(A - T)$$

General solution

$$T(t) = A - Ce^{-kt}.$$

Example 2

An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

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An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

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An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

- The solution is given by

$$T(t) = 70 - Ce^{-kt}$$

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Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

- The solution is given by

$$T(t) = 70 - Ce^{-kt}$$

- We know $T(0) = 90$ and $T(20) = 86$.

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An object takes 20 minutes to cool from 90° to 86° in a room which is 70°. At what time will it be 75°?

Solution

- The temp is described by the equation

$$\frac{\mathrm{d}T}{\mathrm{d}t} = k(70 - T).$$

- The solution is given by

$$T(t) = 70 - Ce^{-kt}$$

- We know $T(0) = 90$ and $T(20) = 86$.
- Thus

$$90 = 70 - C \quad \text{so} \quad C = -20.$$

Example 2

- $T(t) = 70 + 20e^{-kt}$

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$$86 = 70 + 20e^{-20k}$$

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$$86 = 70 + 20e^{-20k}$$

- Thus

$$e^{-20k} = \frac{86 - 70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20} \ln\left(\frac{4}{5}\right) \approx -0.01.$$

Example 2

- $T(t) = 70 + 20e^{-kt}$
- To find k we use $T(20) = 86$

$$86 = 70 + 20e^{-20k}$$

- Thus

$$e^{-20k} = \frac{86-70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20}\ln\left(\frac{4}{5}\right) \approx -0.01.$$

- The model is thus $T(t) = 70 + 20e^{-0.01t}$. We want to solve

$$75 = 70 + 20e^{-0.01t}.$$

Example 2

- $T(t) = 70 + 20e^{-kt}$
- To find k we use $T(20) = 86$

$$86 = 70 + 20e^{-20k}$$

- Thus

$$e^{-20k} = \frac{86 - 70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20}\ln\left(\frac{4}{5}\right) \approx -0.01.$$

- The model is thus $T(t) = 70 + 20e^{-0.01t}$. We want to solve

$$75 = 70 + 20e^{-0.01t}.$$

- Rearranging we get $20e^{-0.01t} = 5$ i.e.

Example 2

- $20e^{-0.01t} = 5$ becomes

$$e^{-0.01t} = \frac{1}{4}$$

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- Applying a logarithm

$$-0.01t = \ln\left(\frac{1}{4}\right)$$

Example 2

- $20e^{-0.01t} = 5$ becomes

$$e^{-0.01t} = \frac{1}{4}$$

- Applying a logarithm

$$-0.01t = \ln\left(\frac{1}{4}\right)$$

- So we get

$$t = -100\ln\left(\frac{1}{4}\right) \approx 138 = 2 \text{ hours } 18 \text{ minutes.}$$