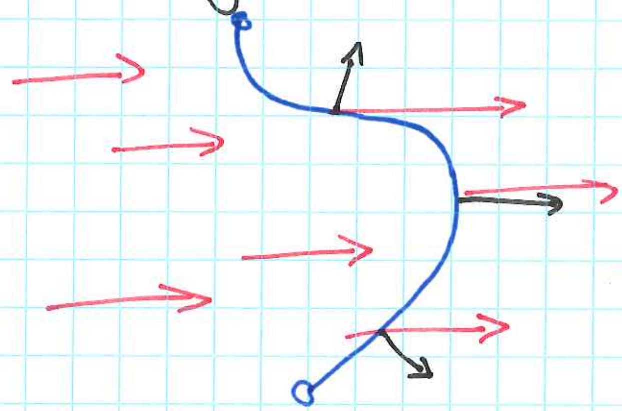
Lecture 19 + 20

Flux through a surface.

- We have met the integral which (in 2D) calculates the flux of a vector field $\underline{F}$ through a curve $\mathcal{C}$



- The flux in this case is

$$\int_{\mathcal{C}} \underline{F} \cdot \underline{n} \, ds$$

the integral of the normal component of the v.f.

- The analogous situation for curves in 3D is not well defined since we can't canonically choose a normal vector.

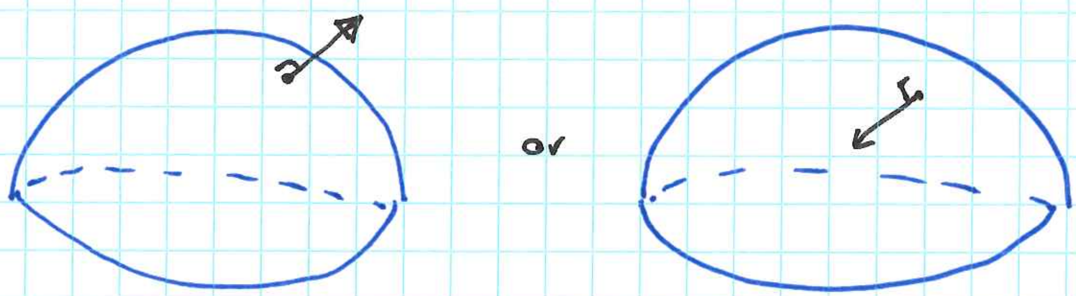
- We can however talk about Flux through a surface

- Just like our curves needed to be oriented in 2D, so do our surfaces.

Oriented surfaces

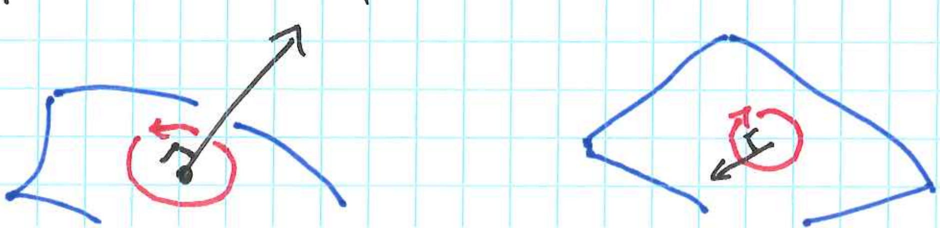
Def A orientation for a surface S is a choice of smoothly varying normal vectors

E.g.



Rmk Some surfaces cannot be oriented! Eg. mobius strip (see demonstration in class!)

- Given a normal vector to our surface we can use the "right hand rule" to assing a ~~ff~~ circular "flow" around each point



This explains the word orientation

Vector surface integral

If S is an oriented surface, $\underline{F}$ a vector field and $\underline{n}$ the unit normal to S then

Def The flux of $\underline{F}$ through S, or the vector surface integral is

$$\iint_S \underline{F} \cdot d\underline{S} = \iint_S (\underline{F} \cdot \underline{n})\, dS$$

Parametrised surfaces

- If we have a parametrisation for S

$$G(u,v) = \big(x(u\,v)\ \ y(u\,v)\ z(u\,v)\big)$$

then it comes with a normal vector

$$\underline{N} = \underline{T}_u \times \underline{T}_v.$$

- If this matches the orientation on S we call it a oriented parametrisation

$$\left(\text{ie if } \frac{\underline{N}}{\|\underline{N}\|} = \underline{n} \right)$$

- If we have such a param. then

$$\iint_S \underline{F} \cdot d\underline{S} = \iint_S (\underline{F} \cdot \underline{n})\, dS$$

$$= \iint_S \underline{F} \cdot \frac{\underline{N}}{\|\underline{N}\|}\, dS$$

$$= \iint_D (\underline{F} \cdot \underline{N}) \frac{1}{\|\underline{N}\|} \cdot \|\underline{N}\|\, dA_{uv}$$

$$= \iint_D \underline{F} \cdot \underline{N}\, dA$$

- This is what we use in practice.