Math 3B: Lecture 2

Noah White

October 1, 2018

AAP tutoring

- AAP Peer Learning Facilitators available for Math 3B.

AAP tutoring

- AAP Peer Learning Facilitators available for Math 3B.
- Sign-ups start Wednesday (10/3/18) and the first session begins the following week.

AAP tutoring

- AAP Peer Learning Facilitators available for Math 3B.
- Sign-ups start Wednesday (10/3/18) and the first session begins the following week.
- If you are not a part of AAP, applications for this program are found in Campbell Hall!

Last time

Last time, we spoke about

- The syllabus

Last time

Last time, we spoke about

- The syllabus
- Problem sets, homework, and quizzes

Last time

Last time, we spoke about

- The syllabus
- Problem sets, homework, and quizzes
- Campuswire

Last time

Last time, we spoke about

- The syllabus
- Problem sets, homework, and quizzes
- Campuswire
- Differentiation of common functions

Last time

Last time, we spoke about

- The syllabus
- Problem sets, homework, and quizzes
- Campuswire
- Differentiation of common functions
- Product rule

Last time

Last time, we spoke about

- The syllabus
- Problem sets, homework, and quizzes
- Campuswire
- Differentiation of common functions
- Product rule
- Chain rule

Last time

Last time, we spoke about

- The syllabus
- Problem sets, homework, and quizzes
- Campuswire
- Differentiation of common functions
- Product rule
- Chain rule
- Remember: I will be away until next Monday (Jens Eberhadt taking lectures)

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively
- Better understanding of derivatives

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

1. $P(t) = \frac{t \ln t}{t-1}$

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

1. $P(t) = \frac{t \ln t}{t-1}$
2. $P(t) = \frac{t}{t+1}$

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

1. $P(t) = \frac{t \ln t}{t-1}$
2. $P(t) = \frac{t}{t+1}$
3. $P(t) = \frac{t}{t+e^t}$

The ingredients

In order to sketch a function accurately we need a few ingredients

The ingredients

In order to sketch a function accurately we need a few ingredients

- The x and y intercepts

The ingredients

In order to sketch a function accurately we need a few ingredients

- The x and y intercepts
- Horizontal asymptotes

The ingredients

In order to sketch a function accurately we need a few ingredients

- The x and y intercepts
- Horizontal asymptotes
- Vertical asymptotes

The ingredients

In order to sketch a function accurately we need a few ingredients

- The x and y intercepts
- Horizontal asymptotes
- Vertical asymptotes
- Slanted asymptotes

The ingredients

In order to sketch a function accurately we need a few ingredients

- The x and y intercepts
- Horizontal asymptotes
- Vertical asymptotes
- Slanted asymptotes
- The regions of increase/decrease of the first derivative

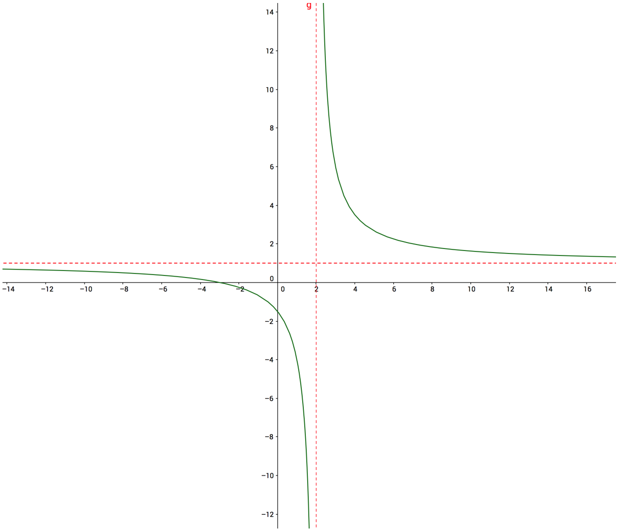
The ingredients

In order to sketch a function accurately we need a few ingredients

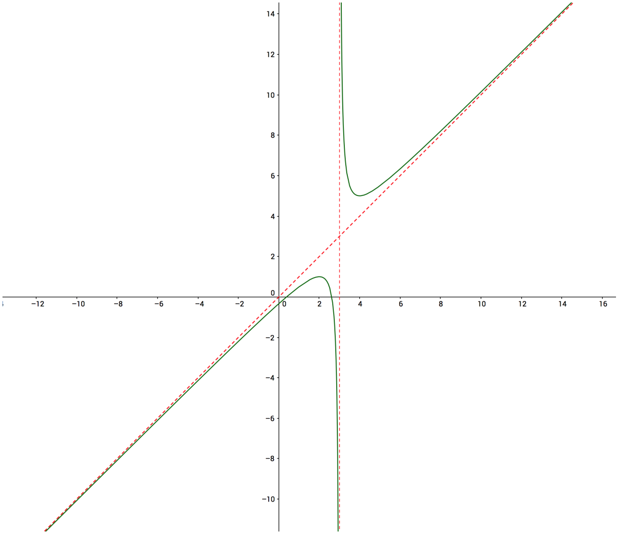
- The x and y intercepts
- Horizontal asymptotes
- Vertical asymptotes
- Slanted asymptotes
- The regions of increase/decrease of the first derivative
- The regions of increase/decrease of the second derivative

Asymptotes

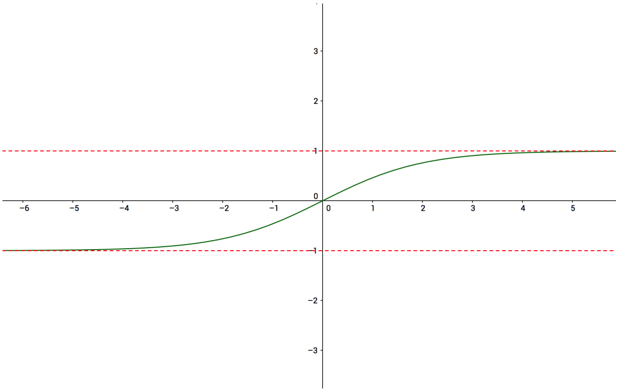
An asmptote is a line which the function approches. Some examples:



Asymptotes



Asymptotes



Finding horizontal asymptotes

These are the easiest asymptotes to find. Suppose you have a function $f(x)$

Finding horizontal asymptotes

These are the easiest asymptotes to find. Suppose you have a function $f(x)$

- Calculate $\lim_{x \to \infty} f(x)$

Finding horizontal asymptotes

These are the easiest asymptotes to find. Suppose you have a function $f(x)$

- Calculate $\lim_{x \to \infty} f(x)$
- Calculate $\lim_{x \to -\infty} f(x)$

Finding horizontal asymptotes

These are the easiest asymptotes to find. Suppose you have a function $f(x)$

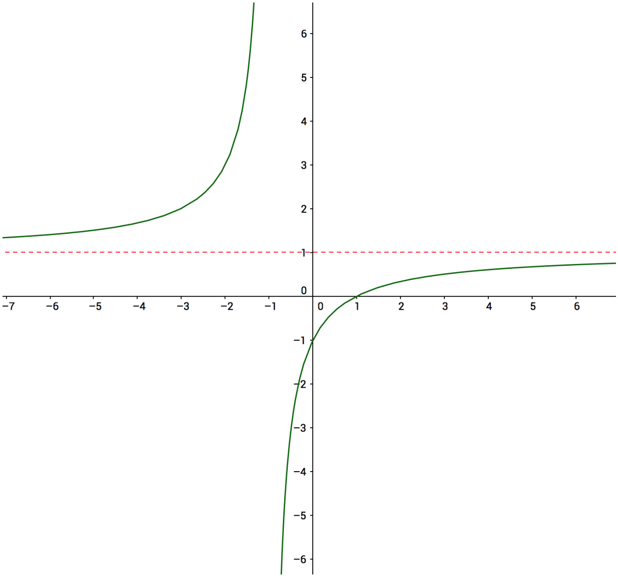
- Calculate $\lim\limits_{x \to \infty} f(x)$
- Calculate $\lim\limits_{x \to -\infty} f(x)$

Example

Say $f(x) = \frac{x-1}{x+1}$. In this case

$$\lim_{x \to \pm\infty} \frac{x-1}{x+1} = 1$$

Finding horizontal asymptotes



More examples

Example

Say $f(t) = \frac{t}{t+1}$. In this case

$$\lim_{t \to \pm\infty} \frac{t}{t+1} = 1$$

More examples

Example

Say $f(t) = \frac{t}{t+1}$. In this case

$$\lim_{t \to \pm\infty} \frac{t}{t+1} = 1$$

Example

Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

More examples

Example

Say $f(t) = \frac{t}{t+1}$. In this case

$$\lim_{t\to\pm\infty} \frac{t}{t+1} = 1$$

Example

Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t\to\infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

Example

Say $f(t) = \frac{t}{t+e^t}$. In this case

$$\lim_{t\to\infty} \frac{t}{t+e^t} = 0$$

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

1. $P(t) = \frac{t \ln t}{t-1}$

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

1. $P(t) = \frac{t \ln t}{t-1}$
2. $P(t) = \frac{t}{t+1}$

Building intuition

Here is an example where a good understanding of a functions behaviour is useful:

You are a biologist studying the population of a species of fish recently introduced to a new ecosystem. Currently they exist in small number, but you want to model their poputation over time. You know the population should increase quickly and then stabalise. What functions would make good candidates for a population model?

1. $P(t) = \frac{t \ln t}{t-1}$
2. $P(t) = \frac{t}{t+1}$
3. $P(t) = \frac{t}{t+e^t}$

Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approches a finite number. I.e. Is there a real number a so that

$$\lim_{x \to a^+} f(x) = \pm\infty$$

or

$$\lim_{x \to a^-} f(x) = \pm\infty$$

Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approches a finite number. I.e. Is there a real number a so that

$$\lim_{x\to a^+} f(x) = \pm\infty$$

or

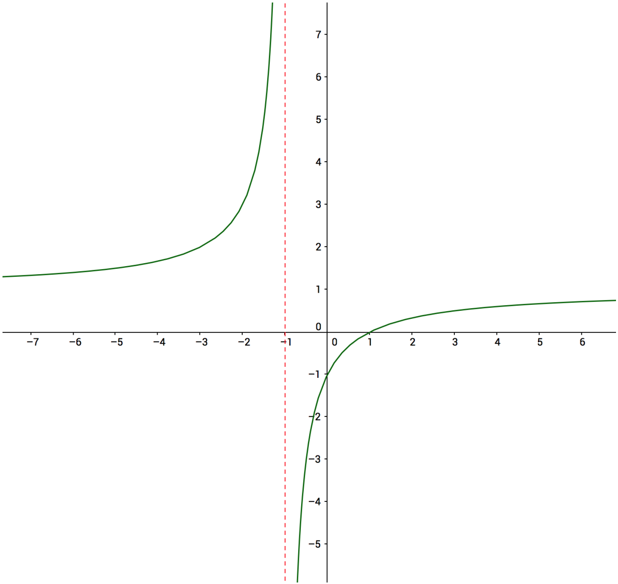
$$\lim_{x\to a^-} f(x) = \pm\infty$$

Example

$f(x) = \frac{x-1}{1+x}$, we have

$$\lim_{x\to -1^+} \frac{x-1}{1+x} = -\infty \quad \text{and} \quad \lim_{x\to -1^-} \frac{x-1}{1+x} = \infty$$

Finding verticle asymptotes

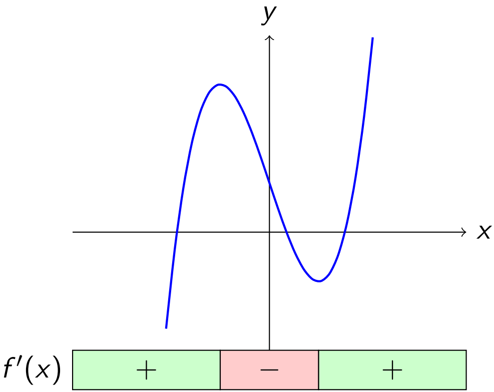


Finding slanted asymptotes

Lets come back to this...

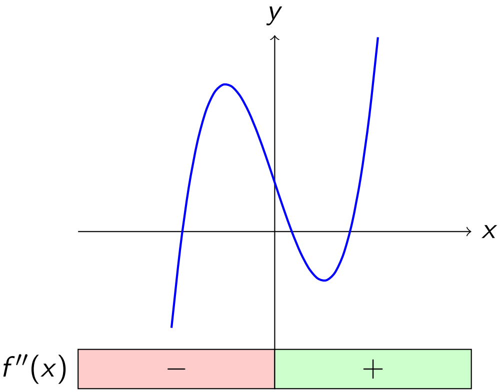
The first derivative

The first derivative tells us is the function going up or down?



The second derivative

The second derivative tells us is the function concave up or down?



Example time

... On the board.