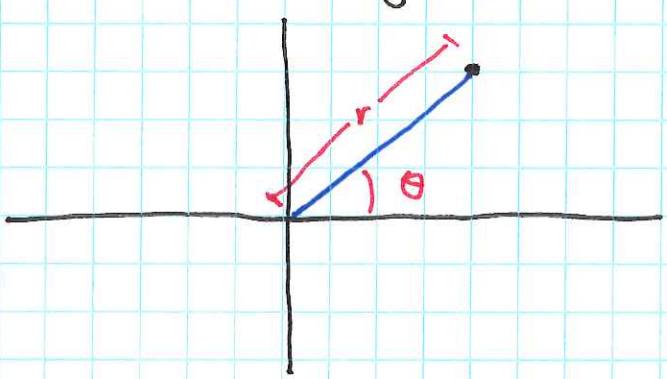
# Lecture 5

## 1 Polar coordinates

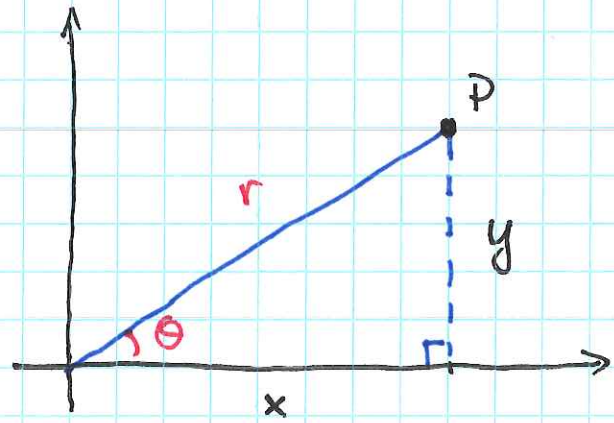
- So far we have been using the horizontal ($x$) and vertical ($y$) measurements to represent points on the plane.
- Sometimes these are not the ~~more~~ most convenient representation
- Alternatively: a point in the plane can be determined by
  * distance from the origin ($r$)
  * angle between $x$-axis and line connecting the point to the origin ($\theta$)




Rmk: The $(x,y)$ representation of a point is unique however the polar representation $(r,\theta)$ is not

e.g. $(r,\theta) = (r, \theta + 2\pi)$

- We can convert between the two representations



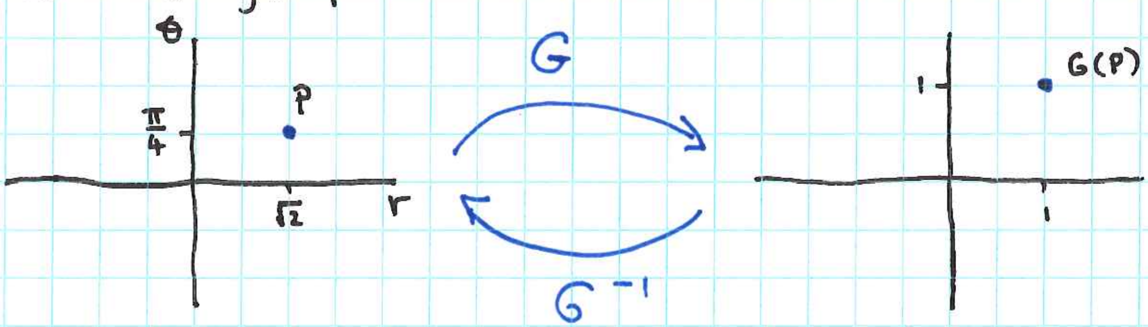

- $x = r\cos\theta \qquad y = r\sin\theta$

$r = \sqrt{x^2+y^2} \qquad \tan\theta = \dfrac{y}{x}$

(or $\theta = \frac{\pi}{2}$ if $x=0$ and $y>0$

$\theta = \frac{3\pi}{2}$ if $x=0$ and $y<0$ )

- This gives us a map between the $(r\ \theta)$-plane and the $(x\ y)$-plane:




$$G(r\ \theta) = (r\cos\theta,\ r\sin\theta)$$
$$G^{-1}(x\ y) = \left(\sqrt{x^2+y^2},\ \tan^{-1} = \frac{y}{x}\right)$$

- Thus we have a formula for converting a function $F(x\ y)$ to polar coordinates:

$$F(r\ \theta) = F(G(r,\theta))$$
$$= F(r\cos\theta, r\sin\theta)$$

Ex. $F(x\ y) = x^2 + y^2$

$$F(r\ \theta) = r^2\cos^2\theta + r^2\sin^2\theta = r^2$$

- We can also convert equations into polar coordinates.

Ex $x^2+y^2=4 \implies r^2\cos^2\theta + r^2\sin^2\theta = 4$

$\implies r = 2$

- Some equations are much simpler in polar coords

2) Integrals in polar coordinates