This week on the problem set you will get practice at calculating integrals using substitution, integration by parts and using partial fractions. Many of these are routine but some are quite difficult!

Homework: The homework will be due on Friday 3 February, at 8am, the *start* of the lecture. It will consist of questions:

$$1(c),\ 2(a) \text{ and } 6$$

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (5.3) Express the limits as definite integrals of the form $\int_0^1 f(x)\,\mathrm{d}x$.
   (a) (5.3.1) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n^2}$
   (b) (5.3.5) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(1-\frac{i^2}{n^2}\right)\frac{1}{n}$
   (c) (5.3.6) $\lim_{n\to\infty}\sum_{i=1}^{n}\sin\left(\frac{\pi i}{n}-\pi\right)\frac{\pi}{n}$
2. (5.3) Express the definite integrals as limits of Riemann sums.
   (a) (5.3.8) $\int_{-1}^{1}(x^2-x)\,\mathrm{d}x$
   (b) (5.3.9) $\int_0^1 e^x\,\mathrm{d}x$
   (c) (5.3.11) $\int_{-1}^{1}|x|\,\mathrm{d}x$
3. (5.5) Calculate the following integrals using substitution.
   (a) (5.5.12) $\int\frac{x}{\sqrt{x^2+1}}\,\mathrm{d}x$
   (b) (5.5.14) $\int\sin^3 t\cos t\,\mathrm{d}t$
   (c) (5.5.16) $\int\frac{z^3}{\sqrt{z^4+12}}\,\mathrm{d}z$
   (d) (5.5.19) $\int_1^2\frac{e^{1/x}}{x^2}\,\mathrm{d}x$
   (e) (5.5.23) $\int_1^2 x\sqrt{x-1}\,\mathrm{d}x$
   (f) (5.5.24) $\int_0^2(e^x-e^{-x})^2dx$
4. (5.5-30) Suppose an environmental study indicates that the ozone level, L, in the air above a major metropolitan center is changing at a rate modeled by the function

$$L'(t)=\frac{0.24-0.03t}{\sqrt{36+16t-t^2}}$$

   parts per million per hour (ppm/h) t hours after 7:00 A.M.
   (a) Express the ozone level $L(t)$ as a function of t if L is 4 ppm at 7:00 A.M.
   (b) Find the time between 7:00 A.M. and 7:00 P.M. when the highest level of ozone occurs. What is the highest level? *(Note: part b has been changed slightly from what is written in the textbook.)*
5. The circle $x^2+(y+1)^2=4$ has area 4π. What is the area of the portion of the circle lying above the x axis?

   You may use the fact that

$$\int\sqrt{1-t^2}\,\mathrm{d}t=\frac{1}{2}\left(t\sqrt{1-t^2}+\sin^{-1}t\right)+C.$$

6. (5.6-36) Assume that after t hours on the job, a factory worker can produce $100te^{0.5t}$ units per hour. How many units does the worker produce during the first 3 hours?