This week on the problem set you will get practice applying the law of total probability, Bayes' law and the concept of independance to problems. Especially challenging questions, or questions that are not appropriate for an exam, are indicated with one or more asterisks.

1. From the textbook, chapter 1, problems 3, 4, 5, 7, 9*.
2. From the supplementary problems, chapter 1, problem 2.
3. Let X be a Binomial random variable with parameters 50 and 0.2. What is the probability of the event that $X \leq 5$ (you don't have to simplify the expression).
4. In a certain soccer tournament you are playing once with each of the other nine teams. In every match you get 3 points if you win, 1 point for a draw and 0 points if you lose. For each match the probability you win is 0.5, the probability you draw is 0.2 and the probability you lose is 0.3, independently of the results of all other matches. What is the probability you finish the tournament with at least 20 points?
5. If X is a Bernoulli random variable with parameter p, show that $Y = 1 - X$ is also a Bernoulli random variable. What is it's parameter?
6. If X is a Binomial random variable with parameters n and p, show that $Y = n - X$ is also a Binomial random variable. What are the parameters of Y?
7. Let X be a Binomial random variable with parameters n and $1/2$. Find the probability mass function of the random variable Y which can have values 0 and 1 and is defined as the remainder when we divide X by 2. Show that this probability mass function does not depend on n.
8. Let a, b and n be positive integers such that $n \leq a$ and $n \leq b$. Construct a probabilistic model (that is describe a random experiment) and a random variable X whose probability mass function is

$$p(k) = \frac{\binom{a}{k}\binom{b}{n-k}}{\binom{a+b}{n}},$$

for $k = 0, 1, 2, \ldots, n$.