This weeks problem set focuses on examples of vector spaces and subspaces. It is expecially important to become comfortable with these since they will be used throughout the course to test our understanding. A question marked with a † is difficult and probably too hard for an exam (though still illustrates a useful point). A question marked with a * is especially important.

1. From section 1.2, problems 1, 4, 8, 10, 11, 15, 16.

2. From section 1.3, problems 1, 6, 7, $8a, c, d$, 11, 15, 20^*, 23, 24^*, $30^\dagger$, 31^*.

3.* Define the symmetric difference of two sets to be $S \uplus T = (S \cup T) \setminus (S \cap T)$. Show that the power set $\mathcal{P}(S)$ is a vector space over $\mathbb{Z}_2$ with addition given by $\uplus$.

4.† For the prime 5, fill in the following tables of sums and products of elements in $\mathbb{Z}_5$. In addition, find the multiplicative inverses of each element.

$+$	$[0]$	$[1]$	$[2]$	$[3]$	$[4]$
$[0]$					
$[1]$					
$[2]$					
$[3]$					
$[4]$					

$\cdot$	$[0]$	$[1]$	$[2]$	$[3]$	$[4]$
$[0]$					
$[1]$					
$[2]$					
$[3]$					
$[4]$					

5. Convince yourself that

    (a) $\mathbb{C}$ is a vector space over $\mathbb{R}$

    (b) $\mathbb{R}$ is a vector space over $\mathbb{Q}$