

# Midterm 2 practice 2

## UCLA: Math 3B, Winter 2019

*Instructor:* Noah White

*Date:*

- This exam has 3 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

Discussion section (please circle):

Day/TA	Louis	Matthew
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	13	
2	14	
3	13	
Total:	40	

1. A large hole is filled with water. The hole has the shape of an upside down cone, with a depth of 5 meters at its deepest and a diameter of 20 meters at ground level. In this question we will calculate how much work is needed to pump all of the water out of the hole. You may assume that water has a density of 1000 kilograms per meter cubed and that the acceleration due to gravity is  $10 \text{ m/s}^2$ .
  - (a) (1 point) Denote by  $x$  the depth below ground in meters, so that  $x = 0$  is ground level and  $x = 5$  is the bottom of the conical hole. We divide the hole into  $n$  horizontal slices. Let  $\Delta x$  be the thickness of each slice. What is  $\Delta x$  in terms of  $n$ ?
  - (b) (1 point) Let  $x_k$  be the depth of the bottom of the  $k^{\text{th}}$  slice. What is  $x_k$  in terms of  $n$  and  $k$ ?
  - (c) (2 points) The base of the  $k^{\text{th}}$  slice is a circle. What is its radius (leave your answer in terms of  $x_k$ )?

- (d) (1 point) If we assume that  $n$  is large, then each slice is approximately a cylinder with height  $\Delta x$  and radius given by your answer to part (c). What is the volume of this cylinder? Leave your answer in terms of  $\Delta x$  and  $x_k$ .

- (e) (2 points) Approximately how much work is needed to raise the water in the  $k^{\text{th}}$  slice to ground level?

(f) (3 points) Write a Riemann sum that calculates the amount of work needed to drain the hole.

(g) (3 points) Use an integral to evaluate this sum.

2. (a) (3 points) Use long division to write the following fraction in the form  $d(x) + \frac{r(x)}{q(x)}$  where the degree of  $r(x)$  is less than the degree of  $q(x)$ .

$$\frac{p(x)}{q(x)} = \frac{x^4 - 2x^2 - 4x + 1}{x^3 + x^2 - x - 1}$$

- (b) (4 points) Calculate the integral  $\int \frac{x^4 - 2x^2 - 4x + 1}{x^3 + x^2 - x - 1} dx$ .

- (c) (3 points) Convert the following Riemann sum into an integral of the form  $\int_0^4 f(x) \, dx$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{n} \sqrt{1 + 32 \frac{k^2}{n^2}}.$$

- (d) (4 points) Solve the differential equation  $\frac{dy}{dt} = te^t \sqrt{1+y}$  if  $y(0) = -0.75$ .

3. Oxygen-15 is a radioactive isotope used in medicine. It has a half life of 120 seconds. During production, a solution containing 0.1 mg/ml of Oxygen-15 is added to a flask at a rate of 2 ml/s. Initially the flask contains 100 ml of pure water (i.e. initially there is no Oxygen-15 in the flask).

(a) (1 point) How much Oxygen-15 (in mg) is being added, per second to the flask?

(b) (2 points) Write a differential equation describing the total amount  $y(t)$  of Oxygen-15 (in mg) in the flask at time  $t$ .

(c) (2 points) Solve the differential equation from part (b).

(d) (3 points) What is the concentration of Oxygen-15 (in mg/ml) in the flask at time  $t$ ?



- (e) (5 points) Now suppose that, at the same time it is being filled, the flask is also being drained at a rate of 1 ml/s. Write a differential equation describing the total amount  $y(t)$  of Oxygen-15 (in mg) in the flask at time  $t$  (you do not need to solve it).

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