Lecture 16

1. Parametrized surfaces

– A **parametrised surface**, is the image of $D \subseteq \mathbb{R}^2$ under a ~~pa~~ map

$$G : D \longrightarrow \mathbb{R}^3$$

where

$$G(u,v) = \left(\cancel{G} x(u,v), y(u,v), z(u,v) \right)$$

Ex A cylinder of radius 1, $x^2+y^2=1$ is parametrized by $G(\theta, z) = (\cos\theta, \sin\theta, z)$ where $D = [0, 2\pi] \times \mathbb{R}$

Ex A sphere of radius R

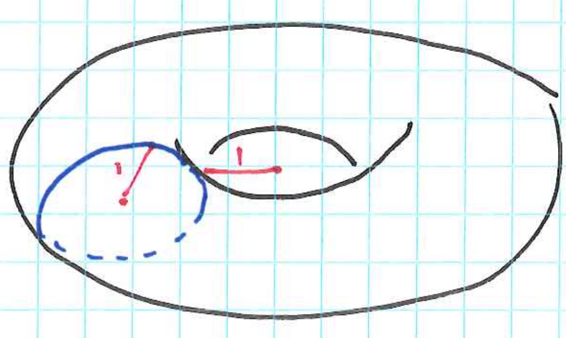
$$G(\theta, \phi) = \left(R\cos\theta \sin\phi, R\sin\theta\sin\phi, R\cos\phi \right)$$

$D = [0, 2\pi] \times [0\ \pi]$

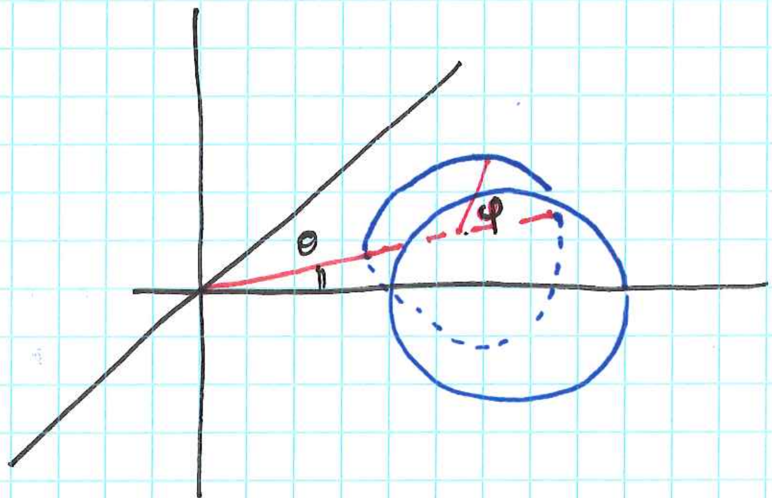
Exercise parametrise the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Ex A torus (inner radius 1, outer radius 3)



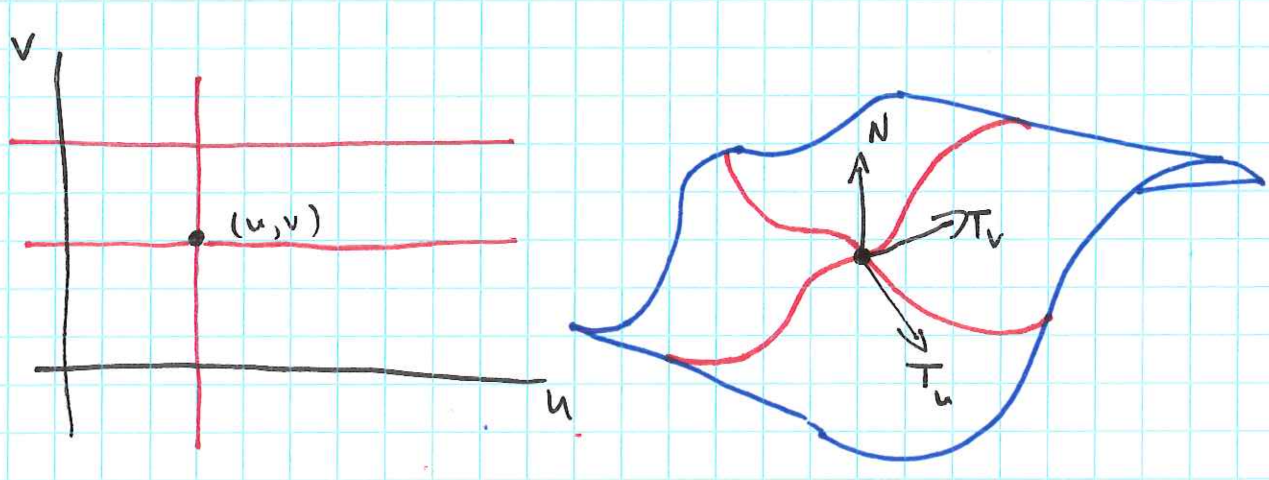
To parametrise let ~~φ~~ θ be angle from x axis and φ angle on rotated circle



$$G(\theta, \varphi) = \left((2+\cos\varphi)\cos\theta,\ (2+\cos\varphi)\sin\theta,\ \sin\varphi \right).$$

- We will assume G has cont. partial derivatives.

- If $G(u,v) = \left(x(u,v), y(u,v), z(u,v) \right)$ we would like to understand the tangent vectors at a point $G(u,v) \in \mathbb{R}^3$.



- Tangent vectors are given by

$$\underline{T}_u = (x_u \quad y_u \quad z_u)$$

$$\underline{T}_v = (x_v \quad y_v \quad z_v)$$

- The Normal to the surface is

$$\underline{N}(u,v) = \underline{T}_u \times \underline{T}_v$$

<u>Def</u> The parametrized surface is regular if

$$\underline{N}(u\,v) \neq 0$$

for all $(u,v) \in D$

<u>Ex</u> For the torus

$$\underline{T}_\theta = \left(-(2+\cos\varphi)\sin\theta,\ (2+\cos\varphi)\cos\theta,\ 0\right)$$

$$\underline{T}_\varphi = \left(-\sin\varphi\cos\theta,\ -\sin\varphi\sin\theta,\ \cos\varphi\right)$$

$$\underline{N}(u,v) = \Big((2+\cos\varphi)\cos\varphi\cos\theta,\ (2+\cos\varphi)\cos\varphi\sin\theta,$$
$$(2+\cos\varphi)\sin\varphi\sin^2\theta + (2+\cos\varphi)\sin\varphi\cos^2\theta \Big)$$

$$= \cancel{\big((2+\cos\varphi)\cos\varphi\cos\theta}$$

$$= (2+\cos\varphi)\left\langle \cos\varphi\cos\theta,\ \cos\varphi\sin\theta,\ \sin\varphi \right\rangle$$

So the torus is regular.