Midterm 2 practice

UCLA: Math 3B, Fall 2016

Instructor: Noah White
Date: Monday, November 21, 2016
Version: practice.

- This exam has 3 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section: ______________________________

Question	Points	Score
1	14	
2	13	
3	13	
Total:	40	

1. A hemispherical hole needs to be dug in order to plant a tree. The hole should be 2 m in diameter. Recall that the volume of a sphere, radius r, is given by

$$V = \frac{4}{3}\pi r^3.$$

(a) (1 point) What is the total volume of the hole?

(b) (1 point) Suppose we divide the hole into n slices, horizontally, of constant thickness Δh. What is Δh in terms of n?

(c) (1 point) Let $h = 0$ be the top of the hole and $h = 1$ be the bottom. Let h_k be the depth of the top of the k^{th} slice (where the first slice is the 0^{th} slice), this $h_0 = 0$. What is h_k in terms of Δh and k?

(d) (2 points) At a depth of $h = h_k$, what is the radius of the hole?

(e) (2 points) If n is large enough, we can approximate each slice by a cylinder of height Δh and radius as in part (d). Using this approximation, what is the volume of of the k^{th} slice?

(f) (3 points) The work needed to lift m kg of material, d meters up is given by

$$W = 10md$$

where we have approximated the acceleration due to gravity as 10 m/s^2. How much work needs to be done in order to lift the k^{th} slice out of the hole if we assume 1 m^3 of dirt weighs 1000 kg?

(g) (2 points) Write a Riemann sum which represents the total amount of work needed to dig the dirt out of the hole.

(h) (2 points) Use an integral to evaluate the Riemann sum above.

2. Solve the following differential equations. If no initial condition is given, find the general solution.

   (a) (4 points) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{2x+4}$.

   (b) (4 points) $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y}{2t+4}$ where $y(1) = 0$.

   (c) (5 points) $\frac{\mathrm{d}y}{\mathrm{d}x} = e^{x+y}$ where $y(0) = 0$.

3. A river flows into a small lake and another river flows out of the lake such that the lake has a constant volume of 2000 m^3 (the rate of water flowing in equals the rate of water flowing out). The river flowing into the lake contains a pollutant present at 0.5 g/m^3. In this question you will model the total amount of pollutant, $y(t)$, present at time t (Note that $y(t)$ is the total amount of pollutant in the lake and not a concentration).

   (a) (1 point) Assume that the river flowing in, flows at a constant rate of 20 m^3/h. At what rate is the pollutant flowing into the lake (in mg/h)?

   (b) (3 points) Under the above assumption, write a differential equation describing the change in the level of pollution in the lake.

(c) (3 points) Assuming that initially there is no pollutant in the lake, solve this differential equation.

(d) (5 points) Now assume that there is some seasonal variability and that the river flowing in (and thus also the river flowing out), flow at a rate of $40 \sin^2 t$ m^3/h. Write and solve a differential equation to model this situation, assuming there is initially no pollution in the lake.

(e) (1 point) Compare the long term behaviour of the two solutions.

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