Midterm 2 practice 1

UCLA: Math 3B, Winter 2019

Instructor: Noah White
Date:
Version: practice

- This exam has 3 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section (please circle):

Day/TA	Louis	Matthew
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	13	
2	14	
3	13	
Total:	40	

1. The Department of Homeland Security receives 500 visa applications per day. Applications take varying amounts of time to process, depending on the type and complexity of the request. Once an application is approved or denied it is termed *resolved*. Experimentally, auditors determined that if an application is currently unresolved then the probability that it will remain unresolved after t days is

$$e^{-0.01t}.$$

The department currently has $10,000$ unresolved applications. In this question we will work out how many unresolved applications the department has one year from now.

(a) (1 point) Of the $10,000$ currently unresolved applications, how many will remain unresolved in one years time (assume a year has 365 days)?

(b) (1 point) We divide the time period of 1 year into n subintervals. Let Δt be the length in days of each subinterval. What is Δt in terms of n?

(c) (2 points) Let $t = 0$ be the start of the time period so that $t = 365$ is one year later. Let t_k be the end of the k^{th} subinterval. What is t_k in terms of n and k?

(d) (1 point) How many visa applications does the Department receive during the k^{th} subinterval (in terms of Δt)?

(e) (2 points) Assume that all of these applications arrive at the department at the end of the k^{th} subinterval. How many of these remain unresolved one year later when $t = 365$? Your answer should be in terms of Δt and t_k.

(f) (3 points) Write a Riemann sum that calculates the total number of unresolved applications the Department will have in one year. Remember to include those remaining from the original $10,000$.

(g) (3 points) Use an integral to evaluate this sum.

2. (a) (3 points) Use long division to write the following fraction in the form $d(x) + \frac{r(x)}{q(x)}$ where the degree of $r(x)$ is less than the degree of $q(x)$.

$$\frac{p(x)}{q(x)} = \frac{3x^4 + x^2 - 10}{x^2 - 1}$$

(b) (4 points) Calculate the integral $\int \frac{3x^4+x^2-10}{x^2-1}\,\mathrm{d}x$.

(c) (3 points) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = 2yt$ when $y(0) = 4$.

(d) (4 points) Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{te^t}{2y}$ if $y(1) = 1$.

3. Chlorine is added to swimming pools to keep the water free from hazardous bacteria. It is recommended that the level of chlorine in a pool be kept at about 2 mg/L (grams per liter). However, chlorine, will *degas* from the water over time. This means the chlorine will leave the water. In fact the half-life of chlorine in water is about 4 hours.

   You have just purchased a fancy new chlorinator for your pool. This device pumps water out of the pool and pumps it back in, with an extra a mg/L of chlorine. It pumps water at a rate of 100 L/h. You can control the rate a by adjusting the settings on the chlorinator. Your pool has a volume of $75,000$ L.

   (a) (2 points) How much chlorine (in mg) is being added, per hour by the chlorinator? Leave your answer in terms of a.

   (b) (4 points) Write a differential equation describing the total level $y(t)$ of chlorine (in mg) in the pool at time t.

(c) (3 points) What should a be to ensure that over the long term, the pool has roughly 2 mg/L of chlorine?

(d) (4 points) Assume that the pool initially contains 0.5 mg/L of chlorine. Solve the differential equation.

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