# Midterm 1 practice 3

## UCLA: Math 3B, Winter 2019

*Instructor:* Noah White
*Date:*

- This exam has 3 questions, for a total of 36 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section (please circle):

| Day/TA | Louis | Matthew |
|---|---|---|
| Tuesday | 1A | 1C |
| Thursday | 1B | 1D |

| Question | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 12 | |
| Total: | 36 | |

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

| *Part* | A | B | C | D |
|---|---|---|---|---|
| (a) | | | | |
| (b) | | | | |
| (c) | | | | |
| (d) | | | | |
| (e) | | | | |

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) The function $f(x) = \frac{1}{1+x^2}$ has

   A. a vertical asymptote at $x = -1$.

   **B. a horizontal asymptote at $y = 0$.**

   C. no asymptotes.

   D. a slanted asymptote.

   (b) (2 points) The function $g(x) = x - \sin x$ has a critical point at

   A. $x = \pi/2$.

   B. $x = 2$.

   C. $x = \pi$.

   **D. $x = 0$.**

   (c) (2 points) The function $f(x) = \frac{1}{5-4x+x^2}$ has a

   A. local minimum at $x = 2$.

   **B. local maximum at $x = 2$.**

   C. local minimum at $x = 1$.

   D. local maximum at $x = 1$.

(d) (2 points) An antiderivative of $h(t) = 2e^{2x} - 4x$ is given by

A. $2x^2 - \cos x^2$

B. $2x^2 - 2e^{2x}$

**C.** $e^{2x} - 2x^2 + \frac{5}{11}$

D. $4e^{2x} - 4$

(e) (2 points) The area $\displaystyle\int_2^3 \ln x \, \mathrm{d}x$ can be expressed as the limit as $n \to \infty$ of

A. $\displaystyle\sum_{k=1}^{n} \ln\left(2 + \frac{k}{n}\right)$

B. $\displaystyle\sum_{k=1}^{n} \frac{2}{n} + \frac{k}{n^2}$

**C.** $\displaystyle\frac{1}{n}\sum_{k=1}^{n} \left[\ln(2n+k) - \ln n\right]$

D. $\displaystyle\sum_{k=1}^{n} \frac{k}{n^2}$

(f) (2 points) Evalute the definite integral $\displaystyle\int_0^{\pi} \cos(x - \frac{\pi}{2}) \, \mathrm{d}x$

A. 1

**B.** 2

C. $\pi$

D. 0

2. Let $f(x) = \frac{1}{1+e^{2x}}$. Note that $f'(x) = \frac{-2e^{2x}}{(1+e^{2x})^2}$ and $f''(x) = \frac{-4e^{2x}(1-e^{2x})}{(1+e^{2x})^3}$.

(a) (2 points) Find the $x$ and $y$ intercepts of $f(x)$.

**Solution:** No $x$-intercepts, $y$-intercept at $y = 0.5$.

(b) (2 points) Does $f(x)$ have any horizontal asymptotes? If so what are they?

**Solution:** We need to evaluate

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{1+e^{2x}} = 0.$$

and

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{1+e^{2x}} = 1.$$

So we have horizontal asymptote in the positive direction at $y = 0$ and in the negative direction at $y = 1$.

(c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

**Solution:** The denominator of $f(x)$ can never be zero so the function is always well defined, thus no vertical asymptotes.

(d) (2 points) For what $x$ is the first derivative $f'(x)$ positive?

**Solution:** The denominator is always positive, and the numerator is always negative. Thus $f'(x)$ is negative for all $x$.

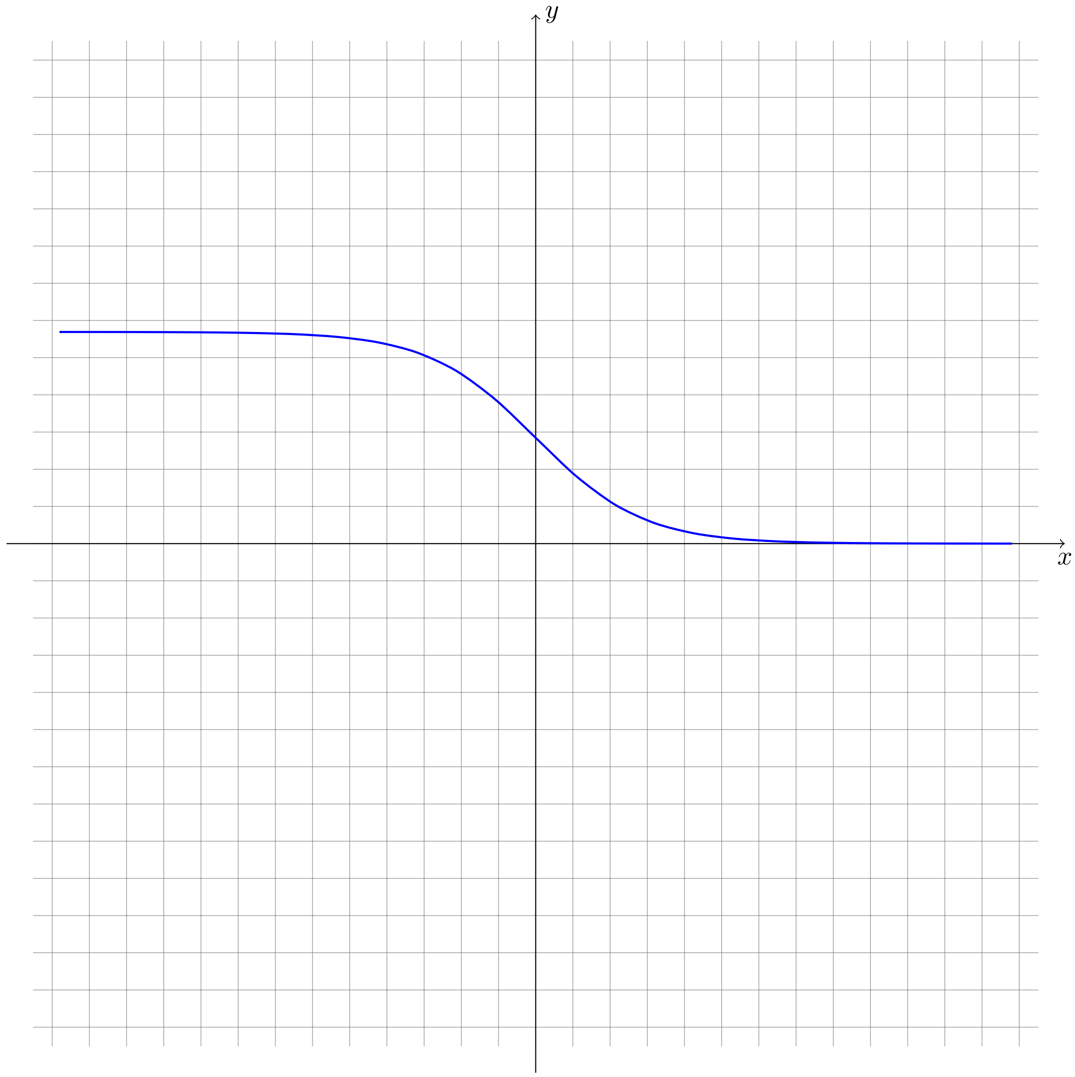
(e) (2 points) For what $x$ is the second derivative $f''(x)$ positive?

**Solution:** Again, the denominator is always positive. The numerator is positive when

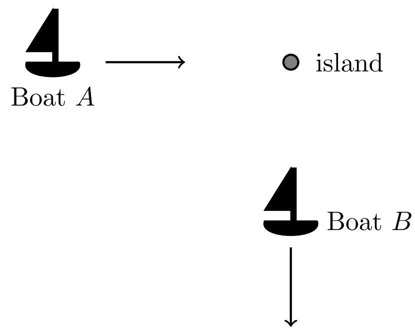
$$e^{2x} > 1$$

i.e. when $2x > 0$ thus when $x > 0$.

(f) (3 points) On the graph provided, sketch $f(x)$



3. Two boats are travelling to and from an island in straight lines, as indicated below. Boat $A$ is heading due east at a constant speed of 1 $m/h$ and at time $t = 0$ is 3 miles from the island. Boat $B$ is heading due south at 2 $m/h$ and at time $t = 0$ is at the island. Both boats stop travelling after boat $A$ reaches the island.




(a) (5 points) Write down an expression for the distance $s(t)$ between the boats after $t$ hours have elapsed.

**Solution:** After $t$ hours, the distance from $A$ to the island is $3 - t$ miles and the distance from $B$ to the island is $2t$ miles. Thus the distance between them (using Pythagoras' theorem) is

$$d(t) = \sqrt{(3-t)^2 + 4t^2} = \sqrt{9 - 6t + 5t^2}.$$

(b) (2 points) What is a sensible domain for $s(t)$?

**Solution:** The domain for this function is $[0, 3]$ (since otherwise $A$ reaches the island). Answers like $[0, \infty)$ were also accepted.

(c) (5 points) At what point in time, are the boats closest together?

**Solution:** The derivative is

$$d'(t) = \frac{-3 + 5t}{\sqrt{9 - 6t + 5t^2}}$$

which is only zero when $t = \frac{3}{5}$. It is defined everywhere since the quadratic on the bottom never vanishes (this is easy to see since $6^2 - 4 \cdot 9 \cdot 5 < 0$, or from the physical situation since it is obvious that the boats will never be in the same place.)

Thus we only have a single critical point $t = 3/5$ on our closed interval, so using the closed interval method we just need to compare the values

$$d(0) = 3, \quad d(3/5) = \frac{6}{\sqrt{5}}, \quad d(3) = 6.$$

Since $6/\sqrt{3}$ is the smallest value (easily seen using a calculator or noticing that $\sqrt{5} > 2$ so $6/\sqrt{5} < 6/2 = 3$) we can conclude that $t = 3/5$ is the minimum (i.e. 36 minutes after starting they are closest together).

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