

Midterm 2 practice 2

UCLA: Math 32B, Fall 2019

Instructor: Noah White

Date:

- This exam has 5 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Question	Points	Score
1	9	
2	8	
3	7	
4	5	
5	11	
Total:	40	

Here are some formulas that you may find useful as some point in the exam.

$$\int \cos^2 x \, dx = \frac{1}{2} (x + \cos x \sin x)$$

$$\int \sin^2 x \, dx = \frac{1}{2} (x - \cos x \sin x)$$

$$\int \sin x \cos x \, dx = \frac{1}{2} \sin^2 x$$

Spherical coordinates are given by

$$x(\rho, \theta, \phi) = \rho \cos \theta \sin \phi$$

$$y(\rho, \theta, \phi) = \rho \sin \theta \sin \phi$$

$$z(\rho, \theta, \phi) = \rho \cos \phi$$

The Jacobian for the change of coordinates is $J = \rho^2 \sin \phi$.

1. Let \mathcal{E} be the solid region defined by

$$x^2 + y^2 + z^2 \leq a, \quad x, y, z \geq 0,$$

for a fixed constant $a > 0$. Suppose the region has a constant mass density of $\delta(x, y, z) = 1$.

- (a) (2 points) Express the total mass of \mathcal{E} as an iterated integral.

- (b) (2 points) Find the total mass of \mathcal{E} .

(c) (3 points) Express the coordinates of the center of mass of \mathcal{E} as an iterated triple integral.

(d) (2 points) Find the z coordinate of the center of mass.

2. Consider the helix \mathcal{C} , given by the parameterisation

$$\mathbf{r}(t) = \left(\cos t, \sin t, \frac{1}{2\pi}t \right) \quad t \in [0, 4\pi],$$

so that \mathcal{C} is oriented with the z coordinate increasing.

- (a) (4 points) Compute the length of \mathcal{C} .

(b) (4 points) Compute the work done by the field

$$\mathbf{F}(x, y, z) = \langle z^2, 2yz^2, 2z(x + y^2) - e^z \rangle$$

on a particle constrained to move on the curve \mathcal{C} .

3. For this question consider the vector field

$$\mathbf{F}(x, y) = \frac{1}{r^2} \langle y(r^2 - 1), x(r^2 + 1) \rangle,$$

where $r = \sqrt{x^2 + y^2}$. This vector field is defined everywhere apart from the origin.

- (a) (4 points) Is \mathbf{F} conservative on the domain described above? Justify your answer.

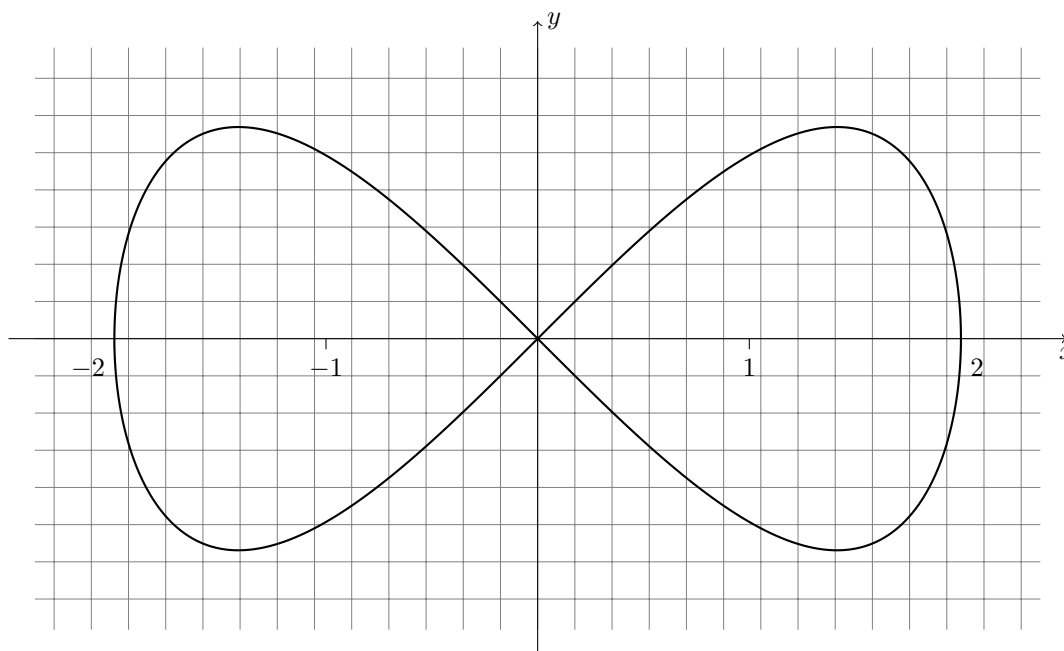
- (b) (1 point) Give a domain on which \mathbf{F} is conservative.

- (c) (2 points) Calculate the line integral

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$$

where \mathcal{C} is the ellipse $\frac{(x-4)^2}{2} + y^2 = 1$, oriented in the counter clockwise direction.

4. In this question assume that \mathbf{E} is a vector field defined on the whole plane, apart from the points $(\pm 1, 0)$. Suppose that $\nabla \times \mathbf{E} = 0$. The function $\mathbf{r}(t) = (2 \cos t, \sin 2t)$ for $t \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$ defines the curve \mathcal{C} on the graph below



- (a) (1 point) Indicate on the above graph, the orientation of the curve.
- (b) (4 points) Let \mathcal{A} and \mathcal{B} be the circles, radius $\frac{1}{2}$, and center $(1, 0)$ and $(-1, 0)$ respectively, both oriented counter clockwise. Suppose that

$$\int_{\mathcal{A}} \mathbf{E} \cdot d\mathbf{r} = 2 \quad \text{and} \quad \int_{\mathcal{B}} \mathbf{E} \cdot d\mathbf{r} = 1.$$

What is $\int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{r}$? Justify your answer.

5. The *hyperboloid* is Noah's favorite surface. It is given by the equation $x^2 + y^2 - z^2 = 1$.

(a) (3 points) Find a parameterisation

$$G(s, \theta) = (x(s, \theta), y(s, \theta), z(s, \theta)) \quad (s, \theta) \in \mathbb{R} \times [0, 2\pi]$$

for the hyperboloid. *Hint: Let $z = s$.*

(b) (5 points) Express the surface area of the hyperboloid between $z = a$ and $z = -a$ as an iterated integral.

(extra working room for part (b))

- (c) (3 points) Calculate the surface area. You may use the formula $\int \sqrt{1+x^2} \, dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \ln(x + \sqrt{1+x^2})$.

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