Math 3B: Lecture 7

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October 12, 2018

Midterm information

Midterm 1

- Practice midterms online

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- arrive on time

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- calculators, cheat sheets

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- expected average, grades

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These problems involve finding the area under some curve.

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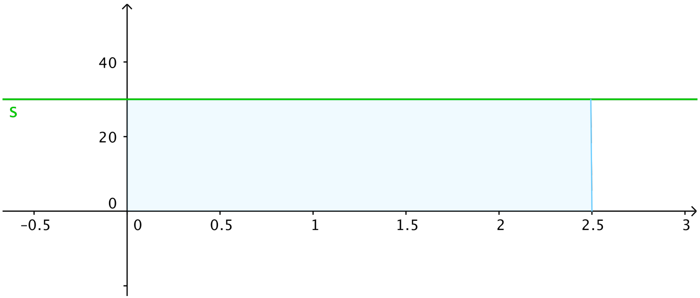
Solution

We model the car's speed using the function $s(t) = 30$. So we can see that the area under this curve



is the distance travelled (75 miles)

Example 2

If a car accellerates for 20 seconds at a rate of $2m/s^2$ and then decelerates for 30 seconds at a rate of $1m/s^2$, how far has it travelled?

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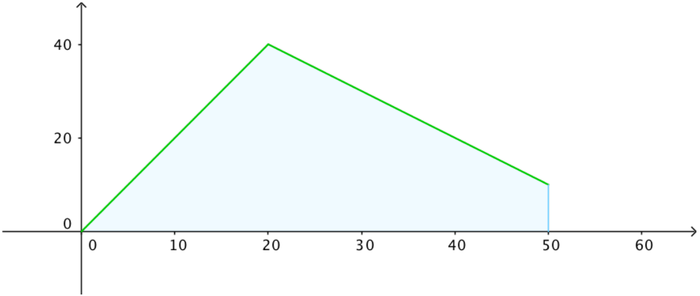
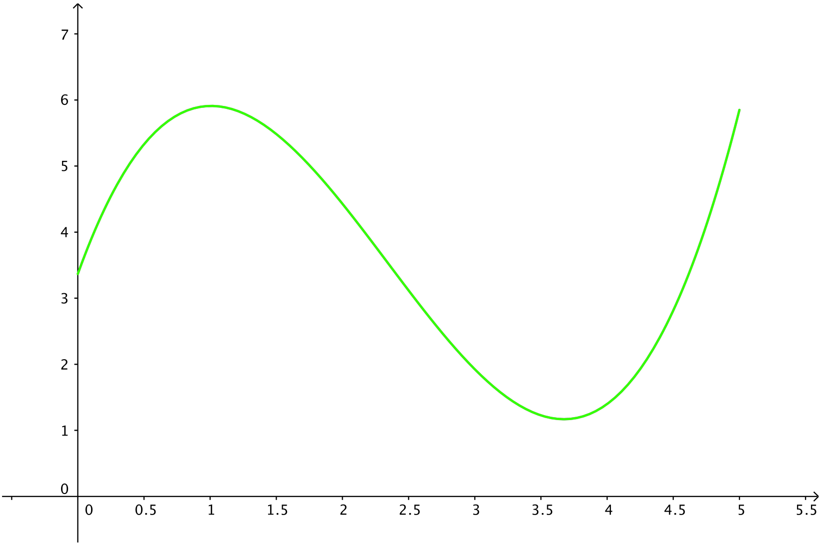
Solution

The car's speed is given by $s(t) = 2t$ when $0 \leq t \leq 20$ and $s(t) = 60 - t$ when $20 \leq t \leq 50$. So the graph looks like



More complicated rates of change

Suppose we have a car whose speed is descibed by the following curve. How far has it travelled in this time?



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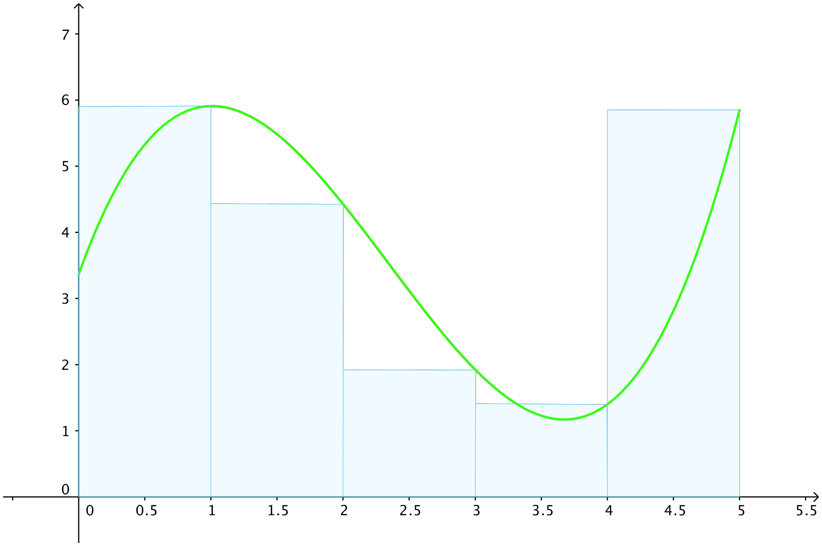

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- How do we find the total amount by which this changes between $t = a$ and $t = b$?
- Answer: area under $f(t)$ between a and b.

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We would like to calculate the area between a function $f(x)$ and the x-axis, between $x = a$ and $x = b$.

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(Too hard to draw, lets look at an animation)

The definite integral

Defintion

The definite integral of a function $f(x)$ is defined to be

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \Delta x \sum_{k=1}^{n} f(a + k\Delta x)$$

where $\Delta x = \frac{b-a}{n}$.

The fundamental theorem of calculus

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_a^x f(t)\,\mathrm{d}t = f(x)$$

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Note

- $F(x) = \int_a^x f(t)\,\mathrm{d}t$ is a function of x.
- every input x produces a number as an output.

A consequence (corrollary)

Corollary

For any antiderivative $F(x)$ of $f(x)$

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Why?

Well $F(x) = \int_a^x f(t)\,\mathrm{d}t + C$ for some a and C. So

$$\begin{aligned} F(b) - F(a) &= \int_a^b f(t)\,\mathrm{d}t + C - \int_a^a f(t)\,\mathrm{d}t - C \\ &= \int_a^b f(t)\,\mathrm{d}t \end{aligned}$$

Example 1

Question

Evaluate the definite integral

$$\int_0^1 x^2 - 4 \, \mathrm{d}x$$

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Solution

An antiderivative of $x^2 - 4$ is $\frac{1}{3}x^3 - 4x$ so

$$\begin{aligned}\int_0^1 x^2 - 4 \, \mathrm{d}x &= \frac{1}{3} \cdot 1^3 - 4 - \frac{1}{3} \cdot 0^3 + 4 \cdot 0 \\ &= \frac{1}{3} - 4 = -\frac{11}{3}\end{aligned}$$

Example 2

Question

Evaluate the definite integral

$$\int_0^{\pi} \sin x \, \mathrm{d}x$$

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Solution

An antiderivative of $\sin x$ is $-\cos x$ so

$$\begin{aligned}\int_0^{\pi} \sin x \, \mathrm{d}x &= -\cos \pi + \cos 0 \\ &= -(-1) + 1 = 2\end{aligned}$$