This week on the problem set you will get practice at integration by parts, polynomial long division, using the partial fractions method and applying these to integrals. There are lots and lots of questions! You don't need to do all of them, only enough to convince yourself you are comfortable with them!

Homework: The first homework will be due on Friday 13 October, at 8am, the *start* of the lecture. It will consist of questions

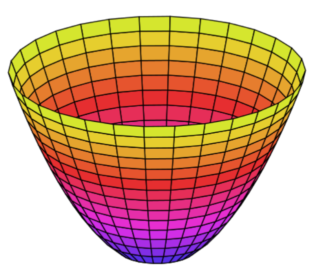
7 on problem set 4, and 3on problem set 5

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (5.6) Calculate the following integrals using integration by parts.
   (a) (2) $\int e^t \sin t \, \mathrm{d}t$
   (b) (6) $\int x^2 \ln x \, \mathrm{d}x$
   (c) (9) $\int \sin x \cos x \, \mathrm{d}x$
   (d) (14) $\int_0^\pi x \sin x \, \mathrm{d}x$
   (e) (16) $\int_1^e x^3 \ln x \, \mathrm{d}x$
2. (5.8-20) Analysts speculate that patients will enter a new clinic at a rate of $300 + 100 \sin \frac{\pi t}{6}$ individuals per month. Moreover, the likelihood an individual is in the clinic t months later is e^{-t}. Find the number of patients in the clinic one year from now.
3. (5.8-21) A patient receives a continuous drug infusion at a rate of 10 mg/h. Studies have shown that t hours after injection, the fraction of drug remaining in a patients body is e^{-2t}. If the patient initially has 5 mg of drug in her bloodstream, then what is the amount of drug in the patients bloodstream 24 hours later?
4. (5.8-24) The administrators of a town estimate that the fraction of people who will still be residing in the town t years from now is given by the function $S(t) = e^{-0.04t}$. The current population is $20,000$ people and new people are arriving at a rate of 500 per year.
   (a) What will be the population size 10 years from now?
   (b) What will be the population size 100 years from now?
5. If we rotate the graph of $y = x^2$ around the y-axis, we obtain a 3D shape that looks like a bowl (see the picture below). If we take the part of this bowl that exists below $y = h$, how much volume does it contain?



Thanks to Wikipedia for the picture!

6. (5.6-28) A bucket weighing 75 lb when filled and 10 lb when empty is pulled up the side of a 100 ft building. How much more work in foot-pounds is done in pulling up the full bucket than the empty bucket?

   Hint: this doesn't involve a Riemann sum, it is just applying the definition of "work".

7. (5.6-29) A 20-ft rope weighing 0.4 lb/ft hangs over the edge of a building 100 ft high. How much work is done in pulling the rope to the top of the building? Assume that the top of the rope is flush with the top of the building, and the lower end of the rope is swinging freely.

   Warning: the answer in the textbook is given in slightly strange units! See the text for an explanation of the units they are using. You don't need to know this though.

8. A helicopter rescues a sailor from a shipwreck out at sea. The rescue is achieved by lowering a rope (of weight 0.1 kg/m) to the sailor and then pulling the sailor up. If the Helicopter is hovering at 100 m above the water and the sailor weighs 75 kg, how much work is done pulling the sailor to safety?

9. Various radioactive materials are used in medical diagnostic techniques. A company which produces these radioactive materials would like to store its waste materials in a special storage facility which can hold up to 100 kg of radioactive materials. The radioactive waste has a half-life of 5 months. This means, if we start with M kg of waste, after t months, only

$$Me^{-0.2t\ln(2)} \text{ kg}$$

   will remain.

   (a) If the company adds waste to the storage facility at a rate of a kg per month, write a function for the amount of material in the storage facility x months after the company opens the storage facility.

   (b) What is the maximum rate at which the company can add waste, so they never exceed the capacity of the storage facility?

10. (5.8-32) Determine the length of a rectangular trench you can dig with the energy gained from eating one Milky Way bar (270 Cal). Assume that you convert the energy gained from the food with 10% efficiency and that the trench is 1 meter wide and 1 meter deep. Assume the density of soil is $1,000$ kg/m^3.

11. Use any method to evaluate the following integrals.

   (a) $\int x\sqrt{x+1}\,\mathrm{d}x$

   (b) $\int_1^2 \frac{t}{(t^2+1)\ln(t^2+1)}\,\mathrm{d}t$

   (c) $\int \frac{\ln x}{x^5}\,\mathrm{d}x$

   (d) $\int \ln x\,\mathrm{d}x$

   (e) $\int (\ln x)^2\,\mathrm{d}x$

   (f) $\int_1^e (\ln x)^3\,\mathrm{d}x$

   (g) $\int e^{6x}\sin(e^{3x})\,\mathrm{d}x$

   (h) $\int_0^1 \frac{t^3 e^{t^2}}{(t^2+1)^2}\,\mathrm{d}t$

   (i) $\int \frac{2z+3}{z^2-9}\,\mathrm{d}z$

   (j) $\int \frac{x^2+x-1}{(x^2-1)}\,\mathrm{d}x$ (*Hint: see partial fraction on PS6*)

   (k) $\int \frac{e^x}{(e^x-1)(e^x+3)}\,\mathrm{d}x$ (*Hint: see partial fraction on PS6*)

   (l) $\int_0^{\pi/3} e^t \sin t\,\mathrm{d}t$

   (m) $\int e^{\sqrt{x}}\,\mathrm{d}x$

   (n) $\int \frac{1}{\cos x}\,\mathrm{d}x$ (quite challenging)

   (o) $\int (\sin x)^2\,\mathrm{d}x$ (quite challenging)

12. Divide $p(x)$ by $q(x)$ and express the quotient as a divisor plus a remainder.

   (a) $p(x) = 2x^3 + 4x^2 - 5$, $q(x) = x + 3$

   (b) $p(x) = 15x^4 - 3x^2 - 6x$, $q(x) = 3x + 6$

(c) $p(x) = 2x^4 - 5x^3 + 6x^2 + 3x - 2$, $q(x) = x - 2$
(d) $p(x) = 5x^4 + 2x^3 + x^2 - 3x + 1$, $q(x) = x + 2$
(e) $p(x) = x^6$, $q(x) = x - 1$
(f) $p(x) = x^3 - 5x^2 + x - 15$, $q(x) = x^2 - 1$
(g) $p(x) = x^3 - 2x^2 - 5x + 7$, $q(x) = x^2 + x - 6$
(h) $p(x) = x^3 + 3x^2 - 6x - 7$, $q(x) = x^2 + 2x - 8$
(i) $p(x) = 2x^3 - 8x^2 + 8x - 4$, $q(x) = 2x^2 - 4x + 2$
(j) $p(x) = 3x^4 - x^3 - 2x^2 + 5x - 1$, $q(x) = x + 1$
(k) $p(x) = 4x^5 + 7x^4 - 9x^3 + 2x^2 - x + 3$, $q(x) = x^2 - 4x + 3$
(l) $p(x) = 4x^5 + 7x^4 - 9x^3 + 2x^2 - x + 3$, $q(x) = x^3 + x^2 - 5x + 3$
(m) Make up your own! Pick random polynomials and divide!

13. Use the method of partial fractions to break up these rational functions.
(a) $\frac{2}{(x-2)x}$
(b) $\frac{5}{(x-2)(x+3)}$
(c) $\frac{7}{(x+6)(x-1)}$
(d) $\frac{5x}{(x-1)(x+4)}$
(e) $\frac{x}{(x+1)(x+2)}$
(f) $\frac{12x-6}{(x-3)(x+3)}$
(g) $\frac{x-1}{(x+2)(x+1)}$
(h) $\frac{1}{x^2-x-6}$
(i) $\frac{11}{x^2-3x-28}$
(j) $\frac{10}{x^2+2x-24}$
(k) $\frac{4x}{x^2+6x+5}$
(l) $\frac{3x}{x^2-7x+10}$
(m) $\frac{1}{x^3-2x^2-5x+6}$
(n) $\frac{4x^2-x}{x^3-4x^2-x+4}$

14. Use the method of partial fractions to break up these rational functions.
(a) $\frac{x}{(x+1)^2}$
(b) $\frac{2x-1}{(x+3)^2}$
(c) $\frac{1-3x}{(x-1)^2}$
(d) $\frac{1+3x}{(x-2)^2}$
(e) $\frac{2x^2}{(x-1)^3}$
(f) $\frac{x-1}{(x-2)^3}$
(g) $\frac{x-3}{(x+2)^2(x-2)}$
(h) $\frac{x}{(x-1)(x+3)^2}$

15. Integrate the functions in question 13 and question 14.

16. Calculate $\int \frac{p(x)}{q(x)}\,\mathrm{d}x$ for each part of question 12.