

This week you will get practice with slope fields.

\*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

**Homework:** The homework will be due on Friday 1 MarchDecember, at 8am, the *start* of the lecture. It will consist of questions

question 3 and question 4.

1. (6.4) Sketch the slope fields and a few solutions for the differential equations given

(a) (6.4.12)  $\frac{dy}{dt} = y(4 - y)(y - 2)$

(b) (6.4.14)  $\frac{dy}{dt} = t^2 - y$

(c) (6.4.16)  $\frac{dy}{dt} = y^2 + t^2 - 1$

(d) (6.4.17)  $\frac{dy}{dt} = -\frac{y}{t}$

*Hint: feel free to use technology, just make sure you know how to draw a solution if you are given a slope field.*

2. (6.4) Sketch the slope fields and the solution passing through the specified point for the differential equations given

(a) (6.4.19)  $\frac{dy}{dt} = t^2 - y^2, (t, y) = (0, 0)$

(b) (6.4.20)  $\frac{dy}{dt} = 1.5y(1 - y), (t, y) = (0, 0.1)$

(c) (6.4.21)  $\frac{dy}{dt} = \sqrt{\frac{t}{y}}, (t, y) = (4, 1)$

(d) (6.4.22)  $\frac{dy}{dt} = y^2\sqrt{t}, (t, y) = (9, -1)$

3. (6.4.37) A population subject to seasonal fluctuations can be described by the logistic equation with an oscillating carrying capacity. Consider, for example,

$$\frac{dP}{dt} = P \left( 1 - \frac{P}{100 + 50 \sin 2\pi t} \right)$$

Although it is difficult to solve this differential equation, it is easy to obtain a qualitative understanding.

- (a) Sketch a slope field over the region  $0 \leq t \leq 5$  and  $0 \leq P \leq 200$ .  
(b) Sketch solutions that satisfy  $P(0) = 0$ ,  $P(0) = 10$ , and  $P(0) = 200$ , use technology if you like.  
(c) Comment on the behaviour of the solutions.
4. (6.4.40) A population, in the absence of harvesting, exhibits the following growth

$$\frac{dN}{dt} = N \left( \frac{N}{100} - 1 \right) \left( 1 - \frac{N}{1000} \right)$$

where  $N$  is abundance and  $t$  is time in years.

- (a) Write an equation that corresponds to harvesting the population at a rate of 0.5% per day.  
(b) Sketch the slope field for the differential equation you found in part a; by sketching solutions, describe how the fate of the population depends on its initial abundance.