This week you will get practice drawing and understanding bifurcation diagrams.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

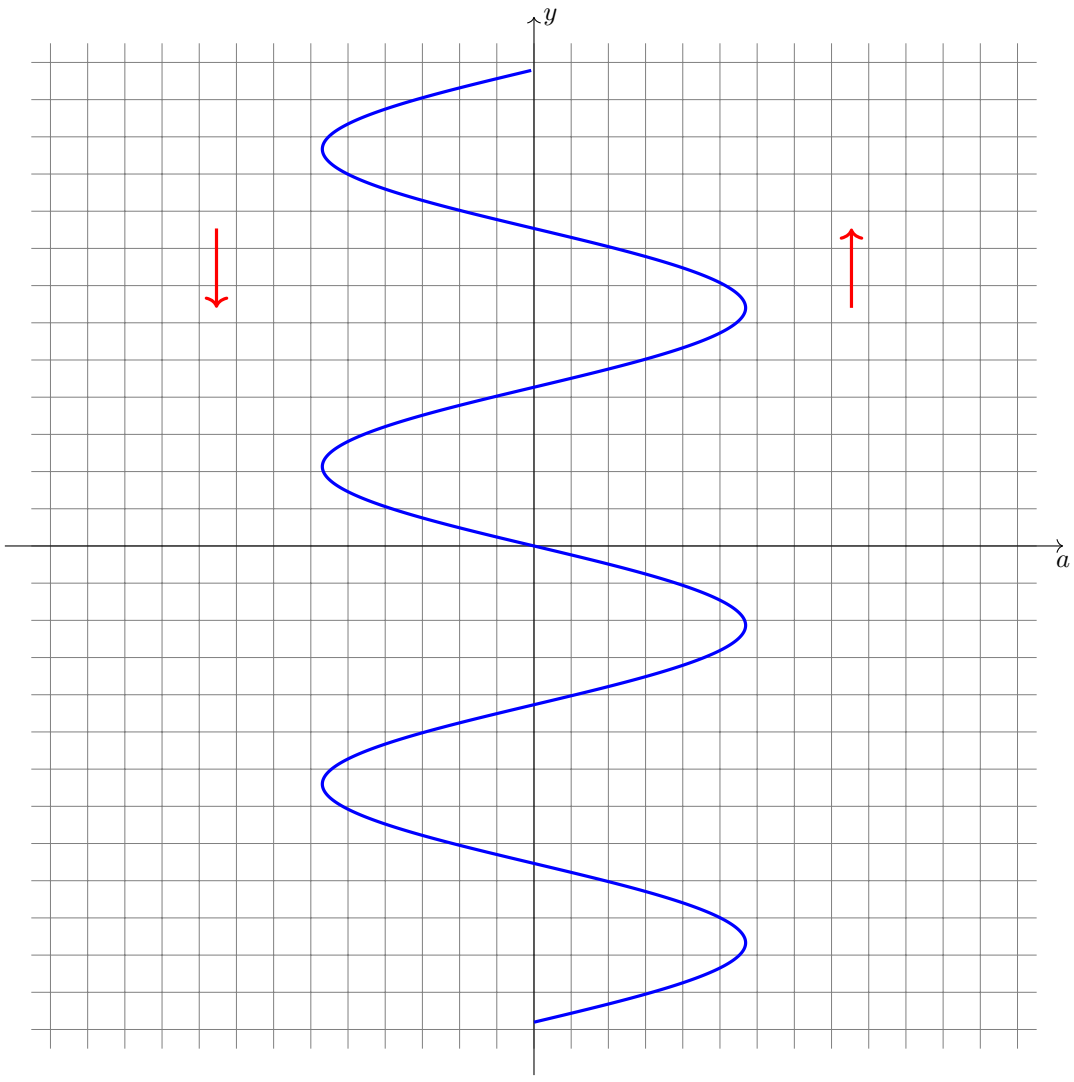
and refer to the section and question number in the textbook.

Homework: The homework will be due on Friday 1 MarchDecember, at 8am, the *start* of the lecture. It will consist of

question 3

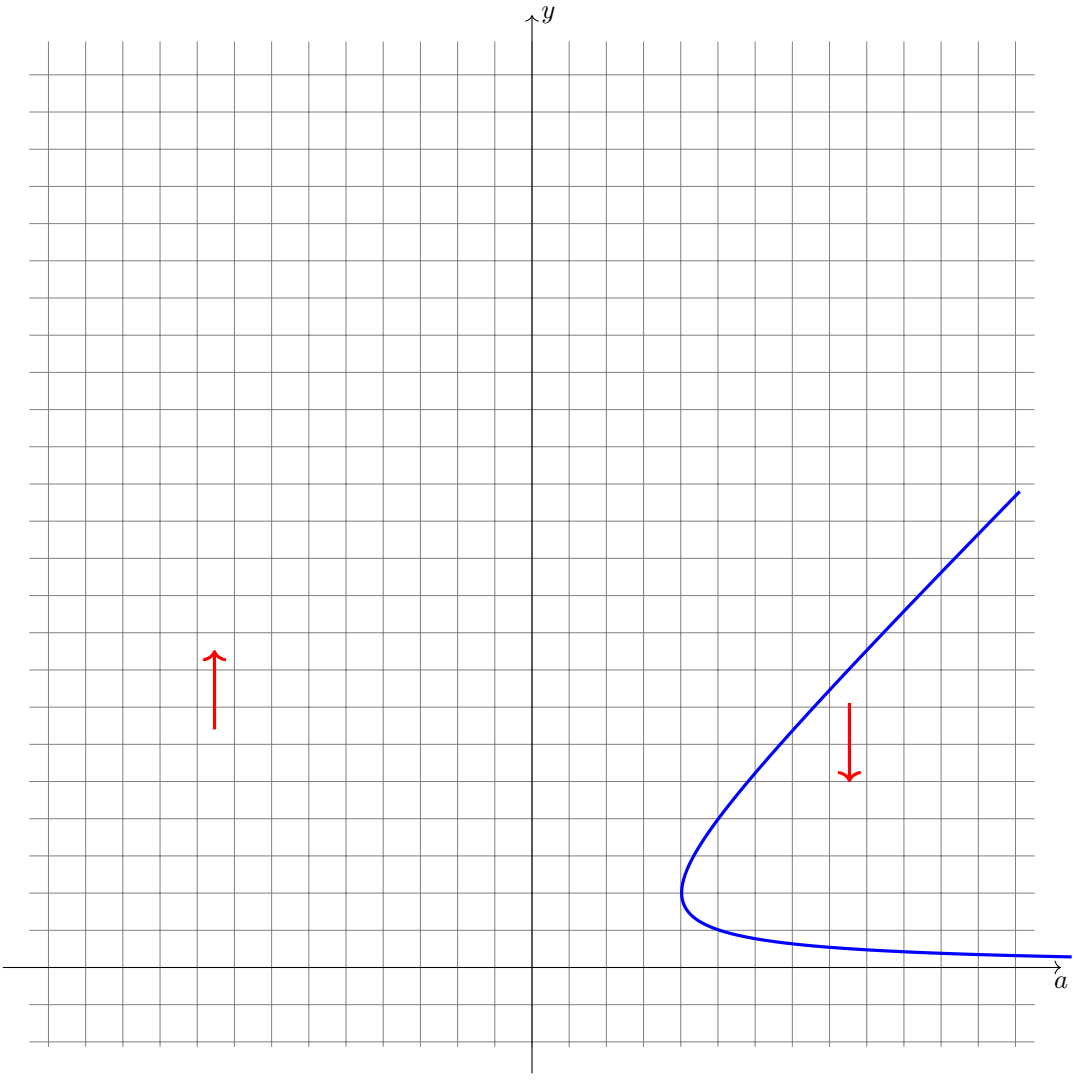
1. (6.6) Sketch the bifurcation diagrams for the equations in the following.
   (a) (6.6-7) $\frac{\mathrm{d}y}{\mathrm{d}t} = ay - y^2$
   (b) (6.6-10) $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - ay^2$
   (c) (6.6-11) $\frac{\mathrm{d}y}{\mathrm{d}t} = \sin y + a$

Solution:



(d) (6.6-12) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 - ay + 2$ for $a > 0$.

Solution:



2. (6.6) Consider the model

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{ay^2}{k^2 + y^2} - cy$$

of an autocatalytic gene from question 5. In each of the following cases, two of the parameters are specified. Sketch a bifurcation diagram with respect to the third parameter.

(a) (6.6-13) $k = 1$, $c = 2$ with a as the bifurcation parameter.

(b) (6.6-14) $k = 2$, $c = 1$ with a as the bifurcation parameter.

(c) (6.6-15) $a = 10$, $k = 1$ with c as the bifurcation parameter.

(d) (6.6-16) $a = 10$, $c = 1$ with k^2 as the bifurcation parameter.

3. (6.6-40) Suppose the growth rate of a whale population at density N (individuals per million square kilometers of ocean), harvested at a rate h, is given by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = 0.07N\left(\frac{N}{10} - 1\right)\left(1 - \frac{N}{80}\right) - h$$

where the units of t are years.

(a) Sketch a bifurcation diagram with respect to the parameter h as it varies over the interval $[0, 8]$.

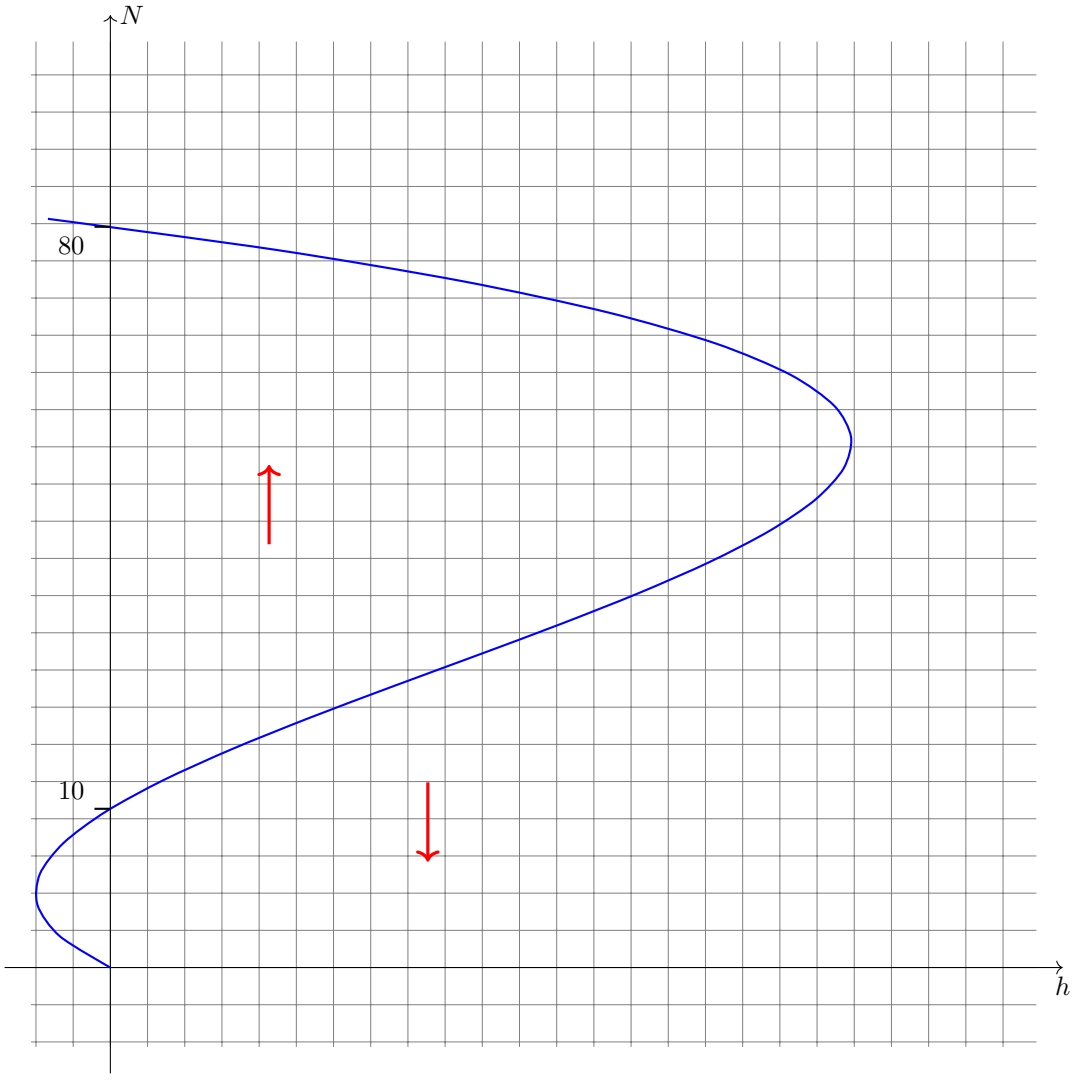
Solution: We want to sketch the points that satisfy

$$0.07N\left(\frac{N}{10}-1\right)\left(1-\frac{N}{80}\right)-h=0$$

i.e. where

$$h=0.07N\left(\frac{N}{10}-1\right)\left(1-\frac{N}{80}\right)$$

To do this we can sketch it in the hN-axis and then flip to the Nh-axis:



The direction of the arrows is found by subbing in points from each region (e.g. $(0, 40)$ and $(1, 0)$) into

$$0.07N\left(\frac{N}{10}-1\right)\left(1-\frac{N}{80}\right)-h$$

and determining whether it is positive or negative.

(b) If $h = vN$, then sketch a bifurcation diagram with respect to the parameter v as it varies over the interval $[0, 0.12]$.

Solution: We want to sketch the points that satisfy

$$0.07N\left(\frac{N}{10} - 1\right)\left(1 - \frac{N}{80}\right) - vN = 0$$

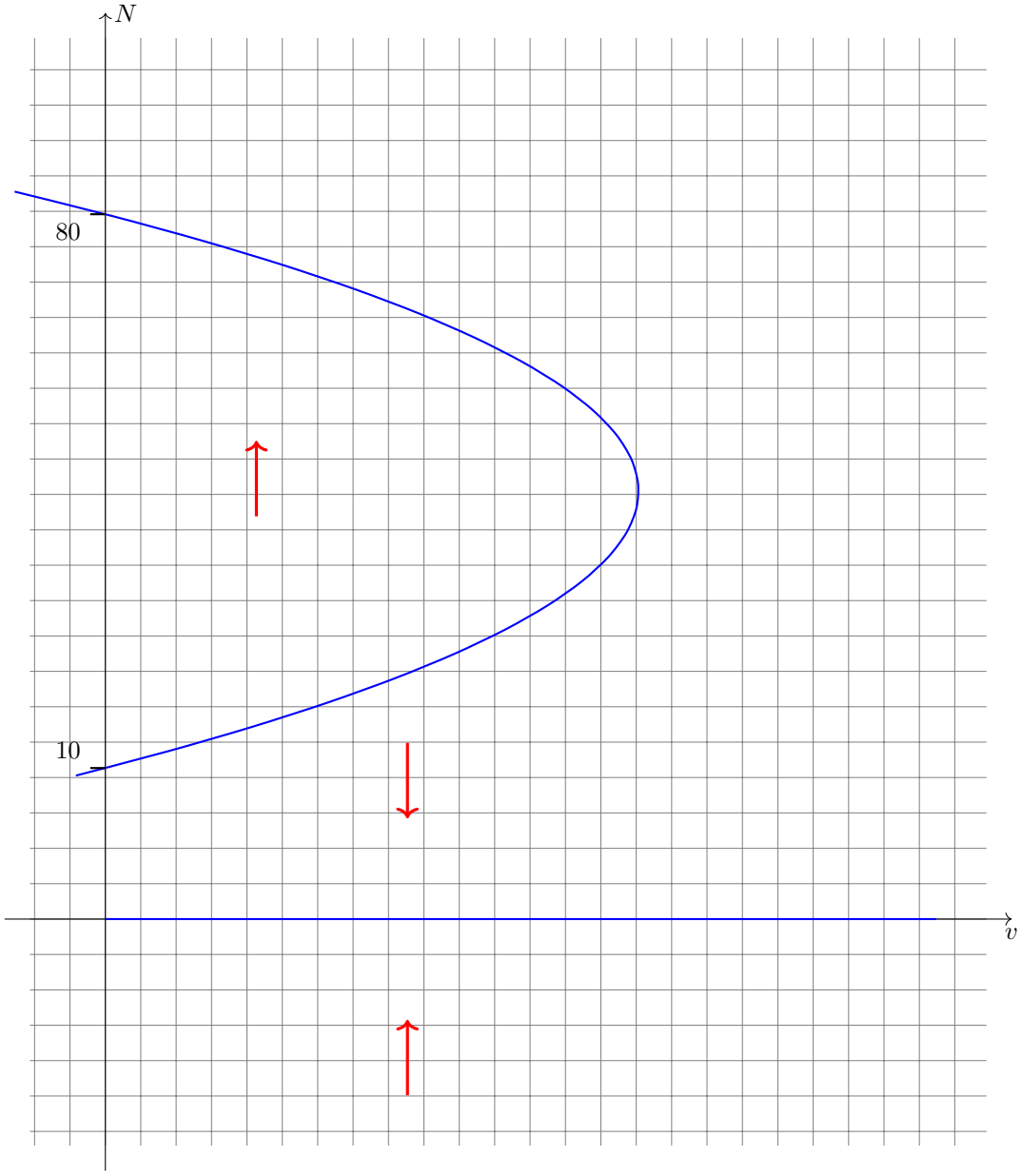
i.e. where

$$\left[0.07\left(\frac{N}{10} - 1\right)\left(1 - \frac{N}{80}\right) - v\right] N = 0$$

Thus we want to sketch the points where either $N = 0$ or

$$v = 0.07\left(\frac{N}{10} - 1\right)\left(1 - \frac{N}{80}\right).$$

To do this we can first sketch the line $N = 0$ and then the sidewards parabola:



The direction of the arrows is found by subbing in points from each region (e.g. $(0, 40)$, $(0, 5)$ and $(0, -1)$) into

$$0.07N\left(\frac{N}{10} - 1\right)\left(1 - \frac{N}{80}\right) - vN$$

and determining whether it is positive or negative.