Math 3B: Lecture 22

Noah White

November 28, 2018

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

Important property

The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

Important property

The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

Important property

The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

We want points (t, y) such that $f(y) = 0$.

- Suppose $f(a) = 0$.

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

Important property

The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

We want points (t, y) such that $f(y) = 0$.

- Suppose $f(a) = 0$.
- Then (t, a) is on the nullcline, for any t.

Autonomous equations

Definition

An ODE of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

i.e. where the right hand side does not depend on t, is called autonomous

Important property

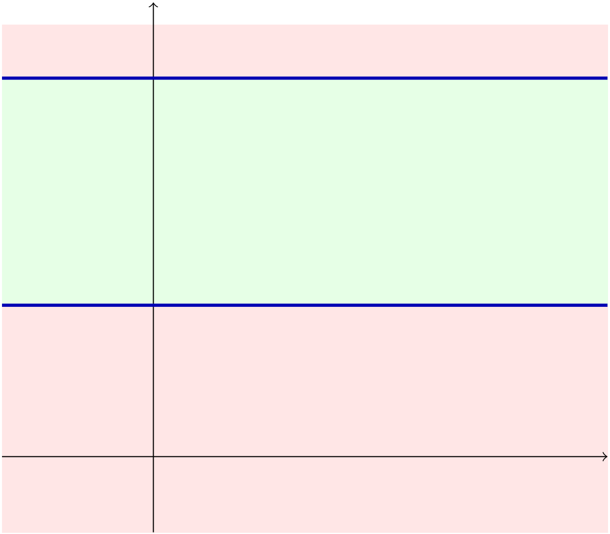
The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

We want points (t, y) such that $f(y) = 0$.

- Suppose $f(a) = 0$.
- Then (t, a) is on the nullcline, for any t.
- So the line $y = a$ is part of the nullcline, whenever $f(a) = 0$.

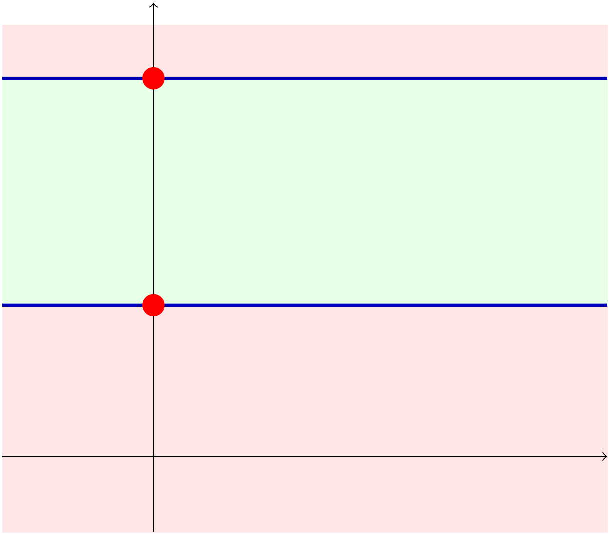
Slope fields and nullclines for autonomous systems

Thus our slope field and nullclines look something like



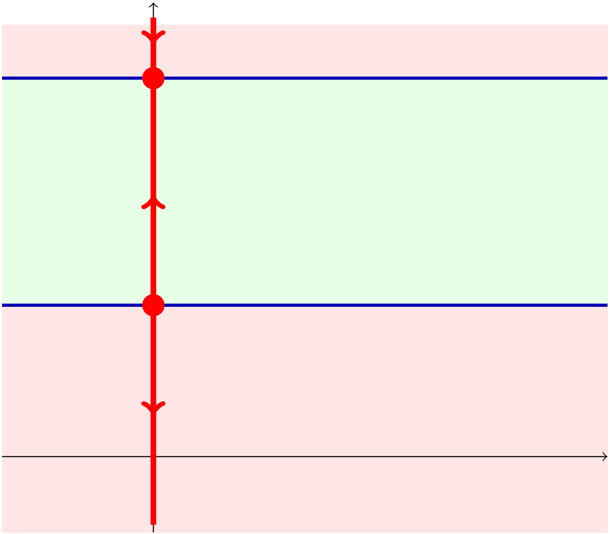
Phase lines/diagram

Thus our slope field and nullclines look something like



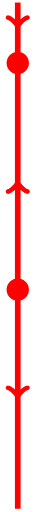
Phase lines/diagram

Thus our slope field and nullclines look something like



Phase lines/diagram

Thus our slope field and nullclines look something like



Phase lines

Recipe to draw phase lines

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live
3. Draw up arrows on intervals between dots where the derivative is positive

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live
3. Draw up arrows on intervals between dots where the derivative is positive
4. Draw down arrows on intervals between dots where the derivative is negative

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live
3. Draw up arrows on intervals between dots where the derivative is positive
4. Draw down arrows on intervals between dots where the derivative is negative

Definition

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live
3. Draw up arrows on intervals between dots where the derivative is positive
4. Draw down arrows on intervals between dots where the derivative is negative

Definition

- An equalibrium is stable if the two arrows are pointing towards it.

Phase lines

Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live
3. Draw up arrows on intervals between dots where the derivative is positive
4. Draw down arrows on intervals between dots where the derivative is negative

Definition

- An equalibrium is stable if the two arrows are pointing towards it.
- It is unstable if the two arrows are pointing away from it.

Phase lines

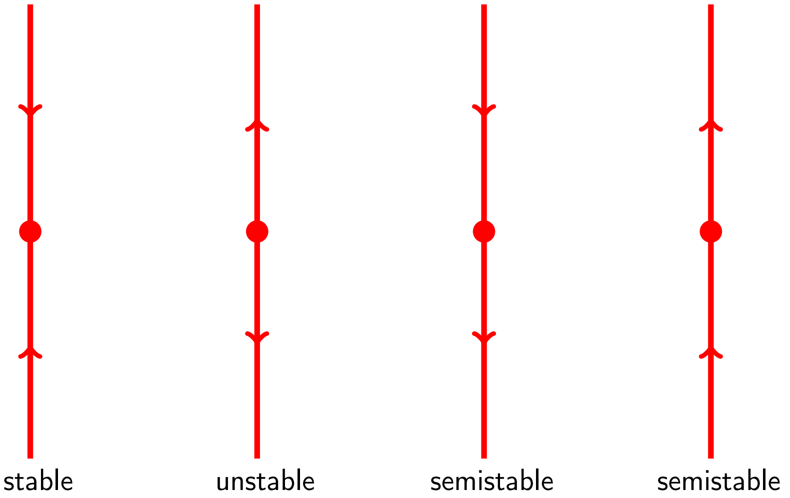
Recipe to draw phase lines

1. Draw a vertical corresponding to y axis
2. Draw dots where equilibrim solutions live
3. Draw up arrows on intervals between dots where the derivative is positive
4. Draw down arrows on intervals between dots where the derivative is negative

Definition

- An equalibrium is stable if the two arrows are pointing towards it.
- It is unstable if the two arrows are pointing away from it.
- It is semistable if the arrows point in the same direction.

Phase lines

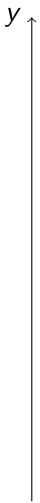


Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y - 10)(25 - y)$$

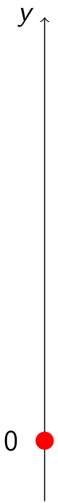
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

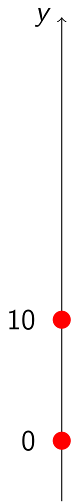


Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



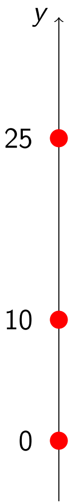
Example



$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

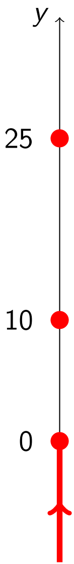
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

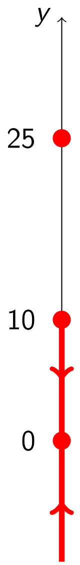


Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

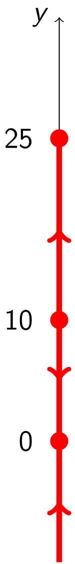


Example



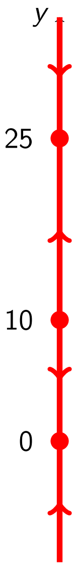
$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

Example



$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

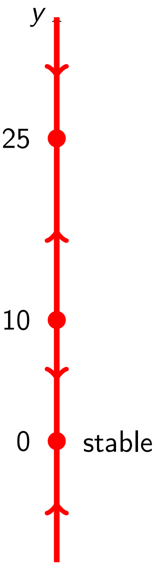
Example



$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

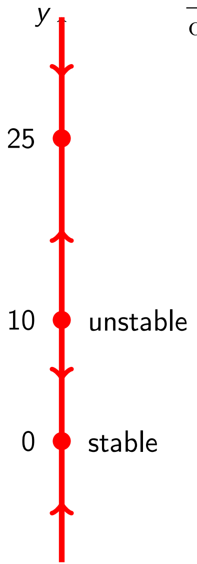
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



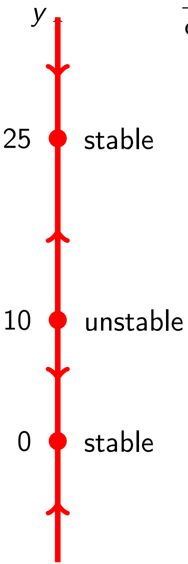
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



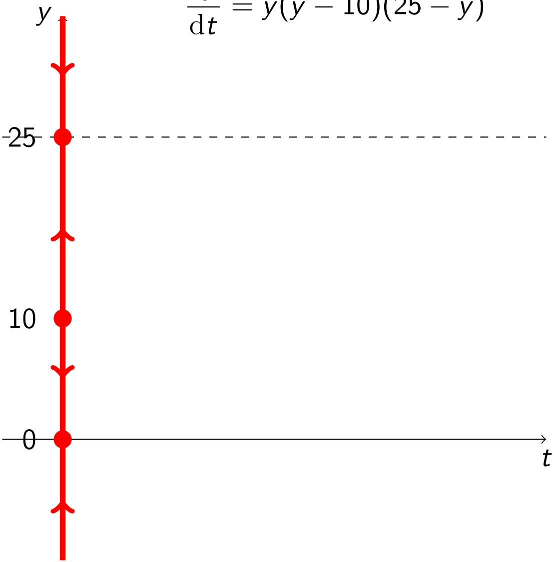
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



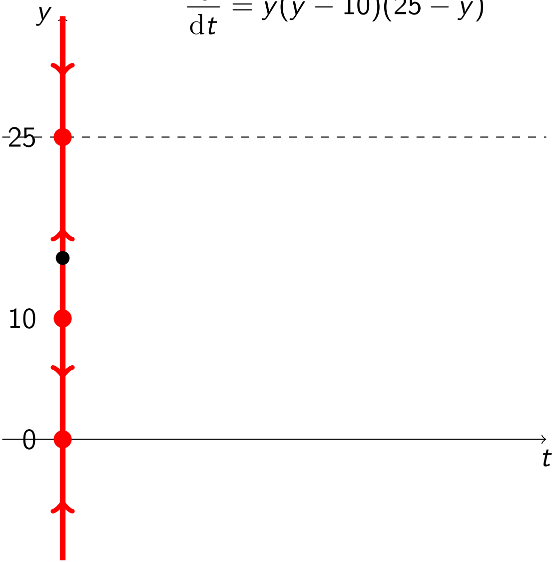
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



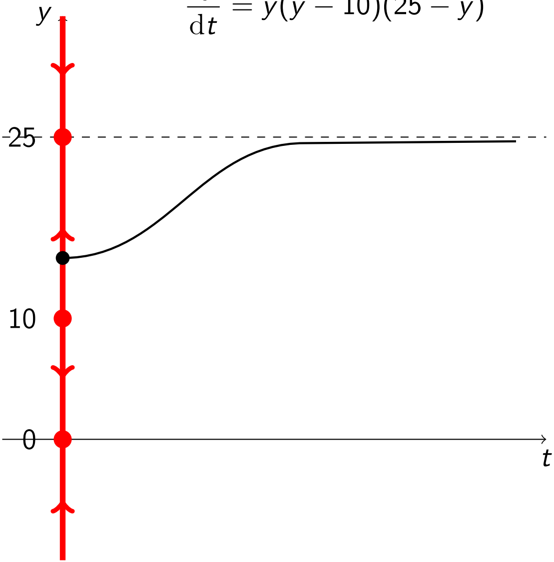
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



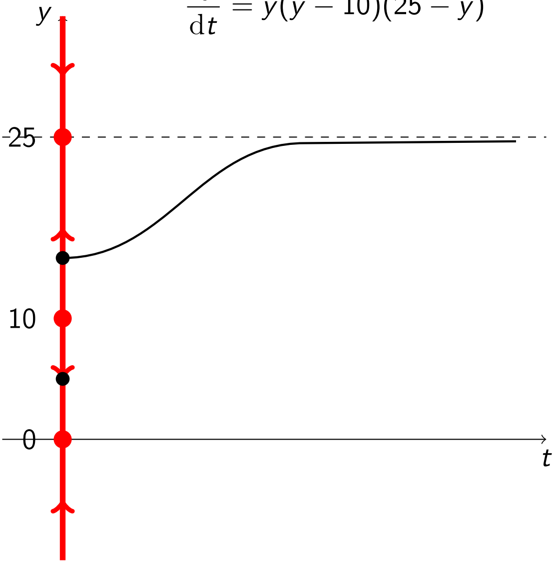
Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$

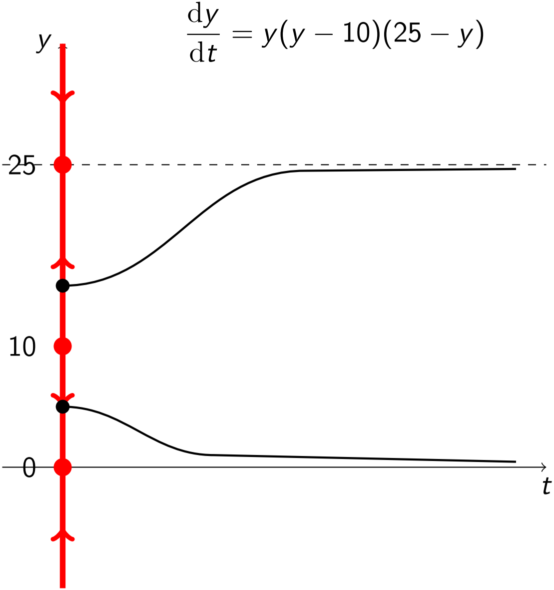


Example

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y(y-10)(25-y)$$



Example



Classifying equilibria using derivatives

Classification of equilibria

If a is an equillibrium of

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

(i.e. $f(a) = 0$) then a is

Classifying equilibria using derivatives

Classification of equilibria

If a is an equillibrium of

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

(i.e. $f(a) = 0$) then a is

- stable if $f'(a) < 0$

Classifying equilibria using derivatives

Classification of equilibria

If a is an equillibrium of

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

(i.e. $f(a) = 0$) then a is

- stable if $f'(a) < 0$
- unstable if $f'(a) > 0$

Classifying equilibria using derivatives

Classification of equilibria

If a is an equillibrium of

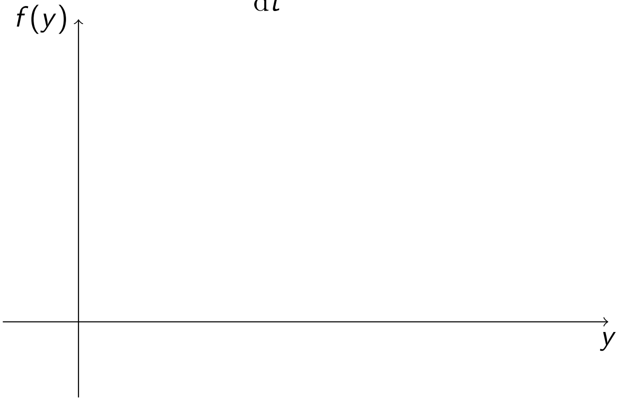
$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

(i.e. $f(a) = 0$) then a is

- stable if $f'(a) < 0$
- unstable if $f'(a) > 0$
- indeterminate if $f'(a) = 0$

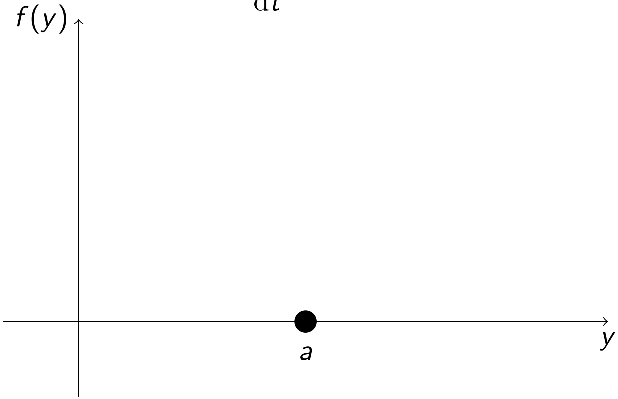
Why?

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$



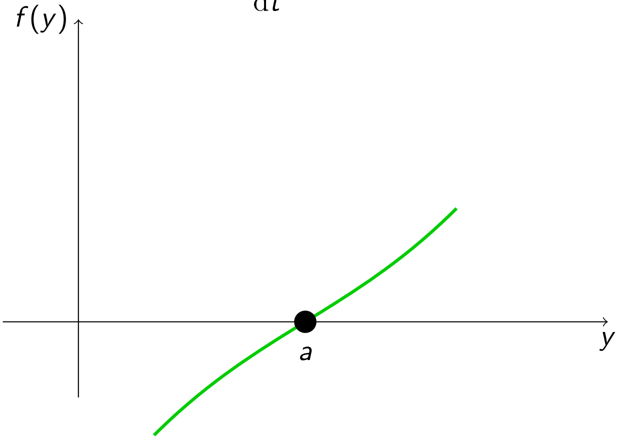
Why?

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$



Why?

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$



Why?

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$