Math 3B: Lecture 7

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Antiderivatives

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$$\frac{dy}{dx} = f(x)$$

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A solution $y = F(x)$ is called an antiderivative of $f(x)$.

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

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What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2$$

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What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2 + 4$$

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What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2 + 8$$

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

Solution

$$F(x) = x^2 + C$$

Example 2

Question

What is the antiderivative of $f(x) = x^3 + 4x - 1$?

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What is the antiderivative of $f(x) = x^3 + 4x - 1$?

Solution

$$F(x) = \frac{1}{4}x^4$$

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Solution

$$F(x) = \frac{1}{4}x^4 + 2x^2 - x + C$$

Example 3

Question

What is the antiderivative of $f(x) = e^{2x}$?

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Solution

$$F(x) = \quad e^{2x}$$

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What is the antiderivative of $f(x) = e^{2x}$?

Solution

$$F(x) = \frac{1}{2}e^{2x}$$

Example 4

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What is the antiderivative of $f(x) = \frac{1}{x}$ (for $x > 0$)?

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What is the antiderivative of $f(x) = \frac{1}{x}$ (for $x > 0$)?

Solution

$$F(x) = \ln x$$

Example 5

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What is the antiderivative of $f(x) = \frac{1}{(1+x)^2}$?

Solution

Note that $f(x) = (1+x)^{-2}$. So

$$F(x) = \quad \frac{1}{1+x}$$

Example 5

Question

What is the antiderivative of $f(x) = \frac{1}{(1+x)^2}$?

Solution

Note that $f(x) = (1+x)^{-2}$. So

$$F(x) = -\frac{1}{1+x}$$

Example 6

Question

What is the antiderivative of $f(x) = 2x \cos x^2$?

Example 6

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What is the antiderivative of $f(x) = 2x \cos x^2$?

Solution

$$F(x) = \sin x^2$$

Example 7

Question

What is the antiderivative of $f(x) = \frac{1}{\sqrt{x}}$?

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Solution

$$f(x) = x^{-\frac{1}{2}}$$

$$F(x) = 2x^{\frac{1}{2}}$$

The general antiderivative/indefinite integral

In general, if $F(x)$ is any antiderivative of $f(x)$, then $F(x) + C$ is an antiderivative for any constant C

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We call $F(x) + C$ the general antiderivative.

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Indefinite integral

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Example

$$\int \frac{1}{x^2}\, dx = -\frac{1}{x} + C$$

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Solution

We use the substitution $u = x^2 + 1$, so $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$, we can write the integral

$$\int 2 \cdot \sqrt{x^2+1} \cdot 2x\;dx = 2\int \sqrt{u}\;du$$

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$$\begin{aligned}\int 2 \cdot \sqrt{x^2+1} \cdot 2x\, dx &= 2\int \sqrt{u}\, du \\ &= 2 \cdot \frac{2}{3} u^{\frac{3}{2}} + C\end{aligned}$$

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Solution

We use the substitution $u = x^2 + 1$, so $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$, we can write the integral

$$\begin{aligned}
\int 2 \cdot \sqrt{x^2+1} \cdot 2x\,dx &= 2\int \sqrt{u}\,du \\
&= 2 \cdot \frac{2}{3}u^{\frac{3}{2}} + C \\
&= \frac{4}{3}(x^2+1)^{\frac{3}{2}} + C
\end{aligned}$$