This week you will get practice drawing and understanding bifurcation diagrams.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (6.6) Sketch the bifurcation diagrams for the equations in the following.
   (a) (6.6-7) $\frac{\mathrm{d}y}{\mathrm{d}t} = ay - y^2$
   (b) (6.6-10) $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - ay^2$
   (c) (6.6-11) $\frac{\mathrm{d}y}{\mathrm{d}t} = \sin y + a$
   (d) (6.6-12) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 - ay + 2$ for $a > 0$.
2. (6.6) Consider the model
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{ay^2}{k^2 + y^2} - cy$$
   of an autocatalytic gene from question 5. In each of the following cases, two of the parameters are specified. Sketch a bifurcation diagram with respect to the third parameter.
   (a) (6.6-13) $k = 1$, $c = 2$ with a as the bifurcation parameter.
   (b) (6.6-14) $k = 2$, $c = 1$ with a as the bifurcation parameter.
   (c) (6.6-15) $a = 10$, $k = 1$ with c as the bifurcation parameter.
   (d) (6.6-16) $a = 10$, $c = 1$ with k^2 as the bifurcation parameter.
3. (6.6-40) Suppose the growth rate of a whale population at density N (individuals per million square kilometers of ocean), harvested at a rate h, is given by
$$\frac{\mathrm{d}N}{\mathrm{d}t} = 0.07N\left(\frac{N}{10} - 1\right)\left(1 - \frac{N}{80}\right) - h$$
   where the units of t are years.
   (a) Sketch a bifurcation diagram with respect to the parameter h as it varies over the interval $[0, 8]$.
   (b) If $h = vN$, then sketch a bifurcation diagram with respect to the parameter v as it varies over the interval $[0, 0.12]$.