This week on the problem set you will get practice calculating joint PMFs and covariance. Especially challenging questions, or questions that are not appropriate for an exam, are indicated with one or more asterisks.

1. From the textbook, chapter 2, problems 24, 26, 32, 37.

2. From the supplementary problems, chapter 2, problem 15, 18, 21.

3. We have a biased coin (probability of heads is equal to p). We toss it first for n times and let X denote the number of heads that came up. Then we toss it for another n times. Let Y denote the number of times heads came up in all $2n$ tosses. Find the range, joint PMF and covariance of (X, Y). Compute $p_{X|Y}(k|l)$ for any (k, l) in the range of (X, Y).

4. We have two fair coins, one is red and the other one is blue. We start tossing them simultaneously, and repeat this indefinitely. Let X denote the number tosses up to (and including) the time when at least one heads appeared. Let Y denote the number of tosses up to (and including) the time that heads appeared on the red coin. Find the range, joint PMF, and the covariance of (X, Y). Compute $p_{X|Y}(k|l)$ for any (k, l) in the range of (X, Y).

5. Recall Problem 7 from the last week homework (n people coming to your party). Compute the variance of the number of presents you receive.

6. Let (X, Y) be a pair of random variables whose range is $\{(k, l)|1 \leq k \leq 10, 1 \leq l \leq 10\}$.
   (a) Express $p_{X|Y}(k|l)$ in terms of the values of the joint PMF $p_{X,Y}$.
   (b) Express $p_{X|Y}(k|l)$ in terms of the values of $p_{Y|X}(l|k)$ and the marginal PMFs p_X and p_Y.

7. Recall question 7 from Problem Set 2. Assume that the loser in the game pays the winner the amount $\$x$, which is defined as follows: x is the difference between the highest card of the winner and the highest card of the loser. Your friend tells you that his higher card is k, and wants you to tell him if, given this information, the conditional expectation of his income in the game is positive or negative.
   (a) Find the expression for the conditional expectation of the your friends income.
   (b) Try to simplify the expression and determine for which values of k it is positive and for which values of k it is negative.