Math 3B: Lecture 8

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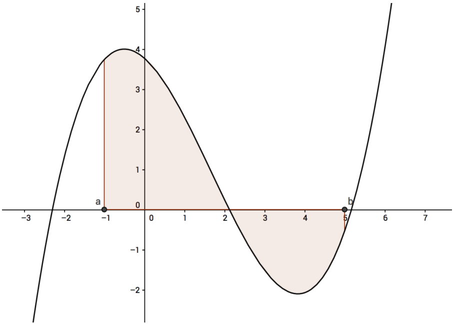
The definite integral

Defintion

The definite integral of a function $f(x)$ is defined to be

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n\to\infty} \Delta x \sum_{k=1}^{n} f(a + k\Delta x)$$

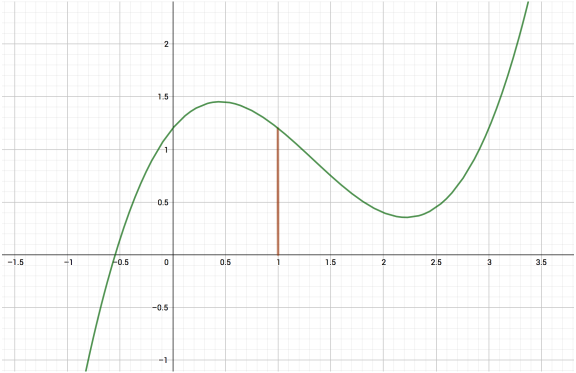
where $\Delta x = \frac{b-a}{n}$.



Properties of definite integrals

Zero area

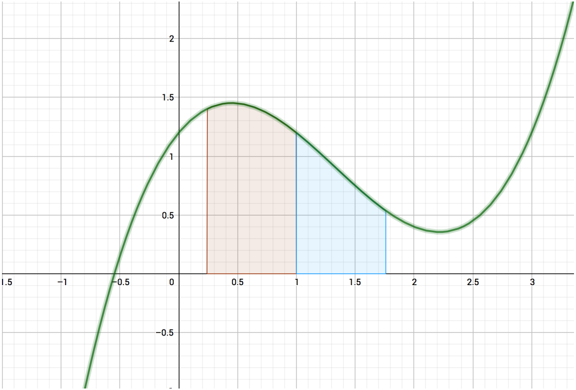
$$\int_a^a f(x)\, \mathrm{d}x = 0$$



Properties of definite integrals

Adding areas

$$\int_a^c f(x)\,\mathrm{d}x = \int_a^b f(x)\,\mathrm{d}x + \int_b^c f(x)\,\mathrm{d}x$$



More properties of definite integrals

Reversing the area

$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$$

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Lineararity (scalars factor out)

$$\int_a^b \alpha f(x)\,\mathrm{d}x = \alpha\int_a^b f(x)\,\mathrm{d}x$$

The fundamental theorem of calculus

Theorem

For any a,

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_a^x f(t) \, \mathrm{d}t = f(x)$$

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Note

- $F(x) = \int_a^x f(t)\,\mathrm{d}t$ is a function of x.
- every input x produces a number as an output.

A consequence (corrollary)

Corollary

For any antiderivative $F(x)$ of $f(x)$

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a)$$

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Why?

Well $F(x) = \int_a^x f(t)\,\mathrm{d}t + C$ for some a and C. So

$$\begin{aligned} F(b) - F(a) &= \int_a^b f(t)\,\mathrm{d}t + C - \int_a^a f(t)\,\mathrm{d}t - C \\ &= \int_a^b f(t)\,\mathrm{d}t \end{aligned}$$

Example 1

Question

Evaluate the definite integral

$$\int_0^1 x^2 - 4 \, \mathrm{d}x$$

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Solution

An antiderivative of $x^2 - 4$ is $\frac{1}{3}x^3 - 4x$ so

$$\begin{aligned}\int_0^1 x^2 - 4 \, \mathrm{d}x &= \frac{1}{3} \cdot 1^3 - 4 - \frac{1}{3} \cdot 0^3 + 4 \cdot 0 \\ &= \frac{1}{3} - 4 = -\frac{11}{3}\end{aligned}$$