Math 3B: Lecture 10

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February 3, 2017

Introduction

Last time

- Properties of the definite integral

This time

Introduction

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- Properties of the definite integral
- The fundamental theorem of calculus

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- Properties of the definite integral
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- Substitution

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- Integration by parts

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Last time

- Properties of the definite integral
- The fundamental theorem of calculus
- Substitution

This time

- Integration by parts
- Polynomial long division

The product rule

Just like integration by substitution reverses the chain rule, integration by parts "reverses" the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

The product rule

Just like integration by substitution reverses the chain rule, integration by parts "reverses" the product rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

writen another way

$$(uv)' = u'v + uv'$$

Integration by parts

$$(uv)' = u'v + uv'$$

Integration by parts

$$(uv)' = u'v + uv'$$

Lets integrate both sides

$$\int (uv)' \, \mathrm{d}x = \int u'v \, \mathrm{d}x + \int uv' \, \mathrm{d}x$$

Integration by parts

$$(uv)' = u'v + uv'$$

Lets integrate both sides

$$\int (uv)' \, \mathrm{d}x = \int u'v \, \mathrm{d}x + \int uv' \, \mathrm{d}x$$

By the fundamental theorem of calculus

$$uv = \int u'v \, \mathrm{d}x + \int uv' \, \mathrm{d}x$$

Integration by parts

$$(uv)' = u'v + uv'$$

Lets integrate both sides

$$\int (uv)' \,\mathrm{d}x = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

By the fundamental theorem of calculus

$$uv = \int u'v \,\mathrm{d}x + \int uv' \,\mathrm{d}x$$

Rearranging. . .

Integration by parts

The integration by parts formula

$$\int uv' \, \mathrm{d}x = uv - \int u'v \, \mathrm{d}x$$

Integration by parts

The integration by parts formula

$$\int uv' \,\mathrm{d}x = uv - \int u'v \,\mathrm{d}x$$

Alternative statement

$$\int u \,\mathrm{d}v = uv - \int v \,\mathrm{d}u$$

Examples

One the board. . .

How to factorize polynomials

The normal method for factorizing a polynomial $p(x)$ is to find a root α and then writing

$$p(x) = q(x)(x - \alpha).$$

How to factorize polynomials

The normal method for factorizing a polynomial $p(x)$ is to find a root α and then writing

$$p(x) = q(x)(x - \alpha).$$

What if we want to devide a polynomial $p(x)$ by another polynomial $q(x)$? We want to write

$$p(x) = q(x)d(x) + r(x)$$

for a polynomial $d(x)$ (the devisor) and a remainder $r(x)$.

Long devision

We know how to do this with numbers! Long devision.

$$\begin{array}{r@{\,}r}
 & 176 \\
\cline{2-2}
34 \big) & 6000 \\
 & 3400 \\
\cline{2-2}
 & 2600 \\
 & 2380 \\
\cline{2-2}
 & 220 \\
 & 204 \\
\cline{2-2}
 & 16
\end{array}$$

Long devision

We know how to do this with numbers! Long devision.

$$\begin{array}{r} 176 \\ 34 \,\big)\overline{\,6000} \\ \underline{3400} \\ 2600 \\ \underline{2380} \\ 220 \\ \underline{204} \\ 16 \end{array}$$

So $6000 = 34 \cdot 176 + 16$

Why?

Lets rewrite the equation $p(x) = q(x)d(x) + r(x)$

$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

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$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

E.g.

$$\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 1} = x + \frac{1}{(x+1)^2}.$$

Why?

Lets rewrite the equation $p(x) = q(x)d(x) + r(x)$

$$\frac{p(x)}{q(x)} = d(x) + \frac{r(x)}{q(x)}.$$

E.g.

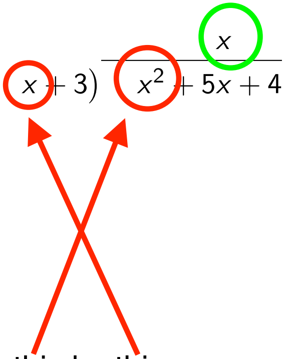
$$\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 1} = x + \frac{1}{(x+1)^2}.$$

The left hand side is difficult to integrate. The right hand side is easy!

How?

$$x + 3\big)\overline{\quad x^2 + 5x + 4}$$

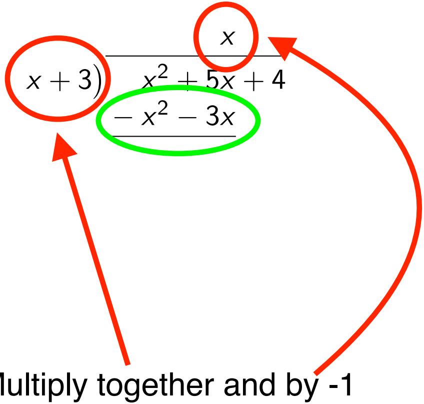
How?



$$\begin{array}{r} x \\ x+3 \overline{\smash{)}\,x^2+5x+4} \end{array}$$

Divide this by this

How?



$$\begin{array}{rl} & x \\ \cline{2-2} x+3) & x^2+5x+4 \\ & \underline{-x^2-3x} \end{array}$$

Multiply together and by -1

How?

$$
\begin{array}{rr}
 & x \\ \cline{2-2}
x+3 \big) & x^2+5x+4 \\
 & -x^2-3x \\ \cline{2-2}
 & 2x+4
\end{array}
$$

How?

$$\begin{array}{rr} & x+2 \\ \cline{2-2} x+3\big) & x^2+5x+4 \\ & -x^2-3x \\ \cline{2-2} & 2x+4 \end{array}$$

How?

$$
\begin{array}{rr}
 & x+2 \\ \cline{2-2}
x+3 \big) & x^2+5x+4 \\
 & -x^2-3x \\ \cline{2-2}
 & 2x+4 \\
 & -2x-6 \\ \cline{2-2}
\end{array}
$$

How?

$$\begin{array}{rr}
 & x + 2 \\
\cline{2-2}
x + 3\big) & x^2 + 5x + 4 \\
 & -\, x^2 - 3x \phantom{{}+4} \\
\cline{2-2}
 & 2x + 4 \\
 & -\, 2x - 6 \\
\cline{2-2}
 & -\, 2
\end{array}$$

How?

$$\begin{array}{rr} & x+2 \\ \cline{2-2} x+3\big) & x^2+5x+4 \\ & -x^2-3x \\ \cline{2-2} & 2x+4 \\ & -2x-6 \\ \cline{2-2} & -2 \end{array}$$

So

$$\frac{x^2+5x+4}{x+3} = x+2-\frac{2}{x+3}.$$

Example 1

$$x - 3\,\overline{)\quad x^3 - 12x^2 \qquad\qquad\quad - 42}$$

Example 1

$$\begin{array}{rl} & \qquad\quad x^2 \\ x-3\big) & \overline{\quad x^3 - 12x^2 \qquad\qquad - 42} \end{array}$$

Example 1

$$\begin{array}{rll}
 & x^2 & \\
x-3 \big) & \overline{x^3 - 12x^2} & \overline{\qquad\qquad - 42} \\
 & \underline{-x^3 + 3x^2} &
\end{array}$$

Example 1

$$
\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & x^2 & & \\
\cline{2-5}
x-3 \big) & x^3 & -12x^2 & & -42 \\
\multicolumn{1}{r}{} & -x^3 & +3x^2 & & \\
\cline{2-3}
\multicolumn{1}{r}{} & & -9x^2 & &
\end{array}
$$

Example 1

$$\begin{array}{rrrrr}
 & & x^2 & -9x & \\ \cline{2-5}
x-3\big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
 & & -9x^2 & &
\end{array}$$

Example 1

$$
\begin{array}{rrrrr}
 & & x^2 & -9x & \\ \cline{2-5}
x-3\big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\ \cline{3-4}
\end{array}
$$

Example 1

$$
\begin{array}{rrrrr}
 & & x^2 & -9x & \\ \cline{2-5}
x-3 \big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\ \cline{3-4}
 & & & -27x & -42
\end{array}
$$

Example 1

$$\begin{array}{rrrrr}
 & & x^2 & -9x & -27 \\ \cline{2-5}
x-3\big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\ \cline{3-4}
 & & & -27x & -42
\end{array}$$

Example 1

$$\begin{array}{rrrrr}
 & & x^2 & -9x & -27 \\ \cline{2-5}
x-3\big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\ \cline{3-4}
 & & & -27x & -42 \\
 & & & 27x & -81 \\ \cline{4-5}
\end{array}$$

Example 1

$$
\begin{array}{rrrrr}
 & & x^2 & -9x & -27 \\ \cline{2-5}
x-3\big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\ \cline{2-3}
 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\ \cline{3-4}
 & & & -27x & -42 \\
 & & & 27x & -81 \\ \cline{4-5}
 & & & & -123
\end{array}
$$

Example 1

$$
\begin{array}{rrrrr}
 & & x^2 & -9x & -27 \\
\cline{2-5}
x-3\,\big) & x^3 & -12x^2 & & -42 \\
 & -x^3 & +3x^2 & & \\
\cline{2-3}
 & & -9x^2 & & \\
 & & 9x^2 & -27x & \\
\cline{3-4}
 & & & -27x & -42 \\
 & & & 27x & -81 \\
\cline{4-5}
 & & & & -123
\end{array}
$$

So

$$\frac{x^3-12x^2-42}{x-3} = x^2-9x-27-\frac{123}{x-3}.$$

Example 2

$$x^2 + 1\big)\overline{\quad x^3 - x^2 + x - 1}$$

Example 2

$$\begin{array}{r} x \phantom{{}-1} \\ x^2+1 \overline{\smash{)}\; x^3 - x^2 + x - 1} \end{array}$$

Example 2

$$
\begin{array}{rrrrr}
 & & & x & \\
\cline{2-5}
x^2+1\big) & x^3 & -x^2 & +x & -1 \\
 & -x^3 & & -x & \\
\cline{2-4}
\end{array}
$$

Example 2

$$
\begin{array}{rrrrr}
 & & & x & \\ \cline{2-5}
x^2+1\big) & x^3 & -\,x^2 & +\,x & -\,1 \\
 & -\,x^3 & & -\,x & \\ \cline{2-4}
 & & -\,x^2 & & -\,1
\end{array}
$$

Example 2

$$\begin{array}{rrrrr}
 & & & x & -\,1 \\ \cline{2-5}
x^2+1\big) & x^3 & -\,x^2 & +\,x & -\,1 \\
 & -\,x^3 & & -\,x & \\ \cline{2-4}
 & & -\,x^2 & & -\,1
\end{array}$$

Example 2

$$\begin{array}{rrrrr}
 & & & x & -1 \\ \cline{2-5}
x^2+1) & x^3 & -x^2 & +x & -1 \\
 & -x^3 & & -x & \\ \cline{2-4}
 & & -x^2 & & -1 \\
 & & x^2 & & +1 \\ \cline{3-5}
\end{array}$$

Example 2

$$
\begin{array}{rrrrr}
 & & & x & -1 \\ \cline{2-5}
x^2+1\big) & x^3 & -x^2 & +x & -1 \\
 & -x^3 & & -x & \\ \cline{2-4}
 & & -x^2 & & -1 \\
 & & x^2 & & +1 \\ \cline{3-5}
 & & & & 0
\end{array}
$$

Example 2

$$
\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & & x & -1 \\
\cline{2-5}
x^2 + 1\big) & x^3 & -x^2 & +x & -1 \\
\multicolumn{1}{r}{} & -x^3 & & -x & \\
\cline{2-4}
\multicolumn{1}{r}{} & & -x^2 & & -1 \\
\multicolumn{1}{r}{} & & x^2 & & +1 \\
\cline{3-5}
\multicolumn{1}{r}{} & & & & 0
\end{array}
$$

So

$$\frac{x^3 - x^2 + x - 1}{x^2 + 1} = x - 1.$$

Example 3

$$x^2 + x + 1 \overline{\big)\quad x^3 \qquad\qquad\qquad - 1}$$

Example 3

$$\begin{array}{rl} & x \\ x^2 + x + 1 \big) & \overline{\quad x^3 \qquad\qquad\qquad -1} \end{array}$$

Example 3

$$
\begin{array}{rrrrr}
 & & & x & \\ \cline{2-5}
x^2 + x + 1 \big) & x^3 & & & -\,1 \\
 & -\,x^3 & -\,x^2 & -\,x & \\ \cline{2-4}
\end{array}
$$

Example 3

$$
\begin{array}{r@{\,}rrrr}
 & & & x & \\ \cline{2-5}
x^2+x+1\big) & x^3 & & & -1 \\
 & -x^3 & -x^2 & -x & \\ \cline{2-4}
 & & -x^2 & -x & -1
\end{array}
$$

Example 3

$$
\begin{array}{rlrrrr}
 & & & & x & -1 \\ \cline{3-6}
x^2+x+1 & \big) & x^3 & & & -1 \\
 & & -x^3 & -x^2 & -x & \\ \cline{3-5}
 & & & -x^2 & -x & -1
\end{array}
$$

Example 3

$$
\begin{array}{rrrrr}
 & & & x & -\,1 \\ \cline{2-5}
x^2+x+1\big) & x^3 & & & -\,1 \\
 & -\,x^3 & -\,x^2 & -\,x & \\ \cline{2-4}
 & & -\,x^2 & -\,x & -\,1 \\
 & & x^2 & +\,x & +\,1 \\ \cline{3-5}
\end{array}
$$

Example 3

$$
\begin{array}{r@{\;}r}
 & x - 1 \\ \cline{2-2}
x^2 + x + 1 \big) & x^3 \qquad\qquad - 1 \\
 & \underline{- x^3 - x^2 - x } \\
 & - x^2 - x - 1 \\
 & \underline{x^2 + x + 1} \\
 & 0
\end{array}
$$

Example 3

$$
\begin{array}{rr}
 & x - 1 \\ \cline{2-2}
x^2 + x + 1 \big) & x^3 \qquad\qquad\qquad - 1 \\
 & \underline{- x^3 - x^2 - x} \phantom{{}-1} \\
 & - x^2 - x - 1 \\
 & \underline{x^2 + x + 1} \\
 & 0
\end{array}
$$

So

$$
\frac{x^3 - 1}{x^2 + x + 1} = x - 1.
$$

Example 4

$$3x - 1 \overline{)\quad 2x^3 \quad - 4x^2 \qquad\qquad\quad + 1}$$

Example 4

$$
\begin{array}{rllll}
 & & \frac{2}{3}x^2 & & \\ \cline{2-5}
3x-1 \big) & 2x^3 & -4x^2 & & +1
\end{array}
$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & & \\
\cline{2-5}
3x-1 \big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\
\cline{2-3}
\end{array}
$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & & \\ \cline{2-5}
3x-1 \big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & &
\end{array}
$$

Example 4

$$\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & \\ \cline{2-5}
3x-1 \big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & &
\end{array}$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & \\
\cline{2-5}
3x-1\,\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\
\cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\
\cline{3-4}
\end{array}
$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & \\
\cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\
\cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\
\cline{3-4}
 & & & -\frac{10}{9}x & +1
\end{array}
$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & -\frac{10}{27} \\ \cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1
\end{array}
$$

Example 4

$$\begin{array}{r rrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & -\frac{10}{27} \\ \cline{2-5}
3x-1 \big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1 \\
 & & & \frac{10}{9}x & -\frac{10}{27} \\ \cline{4-5}
\end{array}$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & -\frac{10}{27} \\ \cline{2-5}
3x-1\big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1 \\
 & & & \frac{10}{9}x & -\frac{10}{27} \\ \cline{4-5}
 & & & & \frac{17}{27}
\end{array}
$$

Example 4

$$
\begin{array}{rrrrr}
 & & \frac{2}{3}x^2 & -\frac{10}{9}x & -\frac{10}{27} \\ \cline{2-5}
3x-1 \big) & 2x^3 & -4x^2 & & +1 \\
 & -2x^3 & +\frac{2}{3}x^2 & & \\ \cline{2-3}
 & & -\frac{10}{3}x^2 & & \\
 & & \frac{10}{3}x^2 & -\frac{10}{9}x & \\ \cline{3-4}
 & & & -\frac{10}{9}x & +1 \\
 & & & \frac{10}{9}x & -\frac{10}{27} \\ \cline{4-5}
 & & & & \frac{17}{27}
\end{array}
$$

So

$$\frac{2x^3-4x^2+1}{3x-1} = \frac{2}{3}\left(x^2-\frac{5}{3}x-\frac{5}{9}\right) - \frac{17}{27(3x-1)}.$$

Example 5

$$x^2 - 2x + 5 \overline{\Big) \quad x^4 \qquad\qquad - x^2 \quad + x \quad - 4}$$

Example 5

$$
\begin{array}{rlllll}
 & & & x^2 & & \\ \cline{2-6}
x^2-2x+5 \big) & x^4 & & -x^2 & +x & -4
\end{array}
$$

Example 5

$$
\begin{array}{r|rrrrr}
 & & & x^2 & & \\ \cline{2-6}
x^2-2x+5 & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\ \cline{2-4}
\end{array}
$$

Example 5

$$
\begin{array}{rlllll}
 & & & x^2 & & \\ \cline{2-6}
x^2-2x+5\big) & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\ \cline{2-4}
 & & 2x^3 & -6x^2 & +x &
\end{array}
$$

Example 5

$$\begin{array}{rrrrrr}
 & & & x^2 & +2x & \\
\cline{2-6}
x^2-2x+5\big) & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\
\cline{2-4}
 & & 2x^3 & -6x^2 & +x &
\end{array}$$

Example 5

$$\begin{array}{r|rrrrr}
\multicolumn{1}{r}{} & & & x^2 & +2x & \\
\cline{2-6}
x^2-2x+5\big) & x^4 & & -x^2 & +x & -4 \\
\multicolumn{1}{r}{} & -x^4 & +2x^3 & -5x^2 & & \\
\cline{2-4}
\multicolumn{1}{r}{} & & 2x^3 & -6x^2 & +x & \\
\multicolumn{1}{r}{} & & -2x^3 & +4x^2 & -10x & \\
\cline{3-5}
\end{array}$$

Example 5

$$\begin{array}{rrrrrr}
 & & & x^2 & +2x & \\ \cline{2-6}
x^2-2x+5\big) & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\ \cline{2-4}
 & & 2x^3 & -6x^2 & +x & \\
 & & -2x^3 & +4x^2 & -10x & \\ \cline{3-5}
 & & & -2x^2 & -9x & -4
\end{array}$$

Example 5

$$
\begin{array}{rrrrrr}
 & & & x^2 & +\,2x & -\,2 \\ \cline{2-6}
x^2 - 2x + 5\big) & x^4 & & -\,x^2 & +\,x & -\,4 \\
 & -\,x^4 & +\,2x^3 & -\,5x^2 & & \\ \cline{2-4}
 & & 2x^3 & -\,6x^2 & +\,x & \\
 & & -\,2x^3 & +\,4x^2 & -\,10x & \\ \cline{3-5}
 & & & -\,2x^2 & -\,9x & -\,4
\end{array}
$$

Example 5

$$
\begin{array}{rrrrrr}
 & & & x^2 & +2x & -2 \\ \cline{2-6}
x^2-2x+5\big) & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\ \cline{2-4}
 & & 2x^3 & -6x^2 & +x & \\
 & & -2x^3 & +4x^2 & -10x & \\ \cline{3-5}
 & & & -2x^2 & -9x & -4 \\
 & & & 2x^2 & -4x & +10 \\ \cline{4-6}
\end{array}
$$

Example 5

$$
\begin{array}{rrrrrr}
 & & & x^2 & +2x & -2 \\ \cline{2-6}
x^2-2x+5\big) & x^4 & & -x^2 & +x & -4 \\
 & -x^4 & +2x^3 & -5x^2 & & \\ \cline{2-4}
 & & 2x^3 & -6x^2 & +x & \\
 & & -2x^3 & +4x^2 & -10x & \\ \cline{3-5}
 & & & -2x^2 & -9x & -4 \\
 & & & 2x^2 & -4x & +10 \\ \cline{4-6}
 & & & & -13x & +6
\end{array}
$$

Example 5

$$
\begin{array}{rrrrrr}
 & & & x^2 & +\,2x & -\,2 \\
\cline{2-6}
x^2-2x+5\big) & x^4 & & -\,x^2 & +\,x & -\,4 \\
 & -\,x^4 & +\,2x^3 & -\,5x^2 & & \\
\cline{2-4}
 & & 2x^3 & -\,6x^2 & +\,x & \\
 & & -\,2x^3 & +\,4x^2 & -\,10x & \\
\cline{3-5}
 & & & -\,2x^2 & -\,9x & -\,4 \\
 & & & 2x^2 & -\,4x & +\,10 \\
\cline{4-6}
 & & & & -\,13x & +\,6
\end{array}
$$

So

$$\frac{x^4-x^2+x-4}{x^2-2x+5} = x^2+2x-2+\frac{-13x+6}{x^2-2x+5}.$$