Math 3B: Lecture 17

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Checking solutions

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$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= e^{\sin x} \cos x \\ y \cos x &= e^{\sin x} \cos x \end{aligned}$$

Reminder: Implicit differentiation

If we have an equation relating variables y and x, e.g.

$$x^2 + y^2 = 1$$

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$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}(x^2 + y^2) &= \frac{\mathrm{d}}{\mathrm{d}x}1 \\ \frac{\mathrm{d}}{\mathrm{d}x}(x^2) + \frac{\mathrm{d}}{\mathrm{d}x}(y^2) &= 0 \\ 2x + 2y'y &= 0 \end{aligned}$$

Another example

Lets differentiate

$$3x + \cos y = xy$$

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3 - y' \sin y = y + xy'.
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Note

We can rearrange this to get

$$y' = \frac{3 - y}{x + \sin y}$$

a differential equation. Whatever y is, as long as it obeys the above relation, it is a solution to this ODE!

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$$\int \frac{1}{f(y)}\,\mathrm{d}y = \int g(x)\,\mathrm{d}x.$$
4. solve for y!

Examples

On the board...