Midterm 1 practice 4

UCLA: Math 3B, Winter 2019

Instructor: Noah White
Date:

- This exam has 3 questions, for a total of 30 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Discussion section (please circle):

Day/TA	Louis	Matthew
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	9	
2	11	
3	10	
Total:	30	

Question 1 is multiple choice. Indicate your answers in the table below. *The following three pages will not be graded, your answers must be indicated here.*

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				
(i)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) The function $f(x) = e^x$ has

   A. a horizontal asymptote at $y = 0$.
   B. a vertical asymptote at $x = 1$.
   C. no asymptotes.
   D. a slanted asymptote with positive slope.

   (b) (1 point) The function $g(x) = (1 + x^2)^{-1}$ has a critical point at

   A. $x = e^{-2}$.
   B. $x = 0$.
   C. $x = 1$.
   D. $x = -1$.

   (c) (1 point) The function $f(x) = \ln(x^2 - 4x + 5)$ has a

   A. local minimum at $x = 2$.
   B. local maximum at $x = 2$.
   C. local maximum at $x = 1$.
   D. local minimum at $x = 1$.

(d) (1 point) An antiderivative of $h(t) = 2t\sin(t^2)$ is given by

A. $\sin(t^2) + 3$

B. $1 - \cos(t^2)$

C. $2t\cos(2t)$

D. $2 + \sin(t^2)$

(e) (1 point) The area $\displaystyle\int_1^3 3 - x^2 \, \mathrm{d}x$ can be expressed as the limit as $n \to \infty$ of

A. $\displaystyle\sum_{k=1}^{n} \left(\frac{6}{n} + \frac{4k^2}{n^3} \right)$

B. $\displaystyle\sum_{k=1}^{n} \left(\frac{2}{n} + \frac{k}{n^2} \right)$

C. $\displaystyle\sum_{k=1}^{n} \left(\frac{4}{n} - \frac{8k}{n^2} - \frac{8k^2}{n^3} \right)$

D. $\displaystyle\sum_{k=1}^{n} \left(\frac{2}{n} - \frac{2k}{n^2} - \frac{k^2}{n^3} \right)$

(f) (1 point) Evaluate the definite integral $\displaystyle\int_1^{e^\pi} x^{-1}\sin(\ln x) \, \mathrm{d}x$

A. 1

B. π

C. 0

D. 2

(g) (1 point) The function $g(x) = 2e^x - x^2$ has

A. a single local minimum.
B. at least two local minimums.
C. no critical points.
D. a critical point when $x = 0$.

(h) (1 point) Evalute the definite integral $\int_1^2 15x\sqrt{x-1}\,\mathrm{d}x$

A. $44\sqrt{2} - 16$
B. 16
C. 2
D. $11\sqrt{2}$

(i) (1 point) Consider the function $f(x) = \max\{0, 2x\}$. An antiderivative of $f(x)$ is given by

A. $x \cdot \max\{0, x\}$
B. $\max\{0, x^2\}$
C. $\max\{0, x\} + 1$
D. x^2

2. Let $f(x) = \frac{x}{\sqrt{x^4+1}}$. Note that $f'(x) = \frac{1-x^4}{(x^4+1)^{3/2}}$ and $f''(x) = \frac{2x^3(x^4-5)}{(x^4+1)^{5/2}}$.

   (a) (2 points) Find the x and y intercepts of $f(x)$.

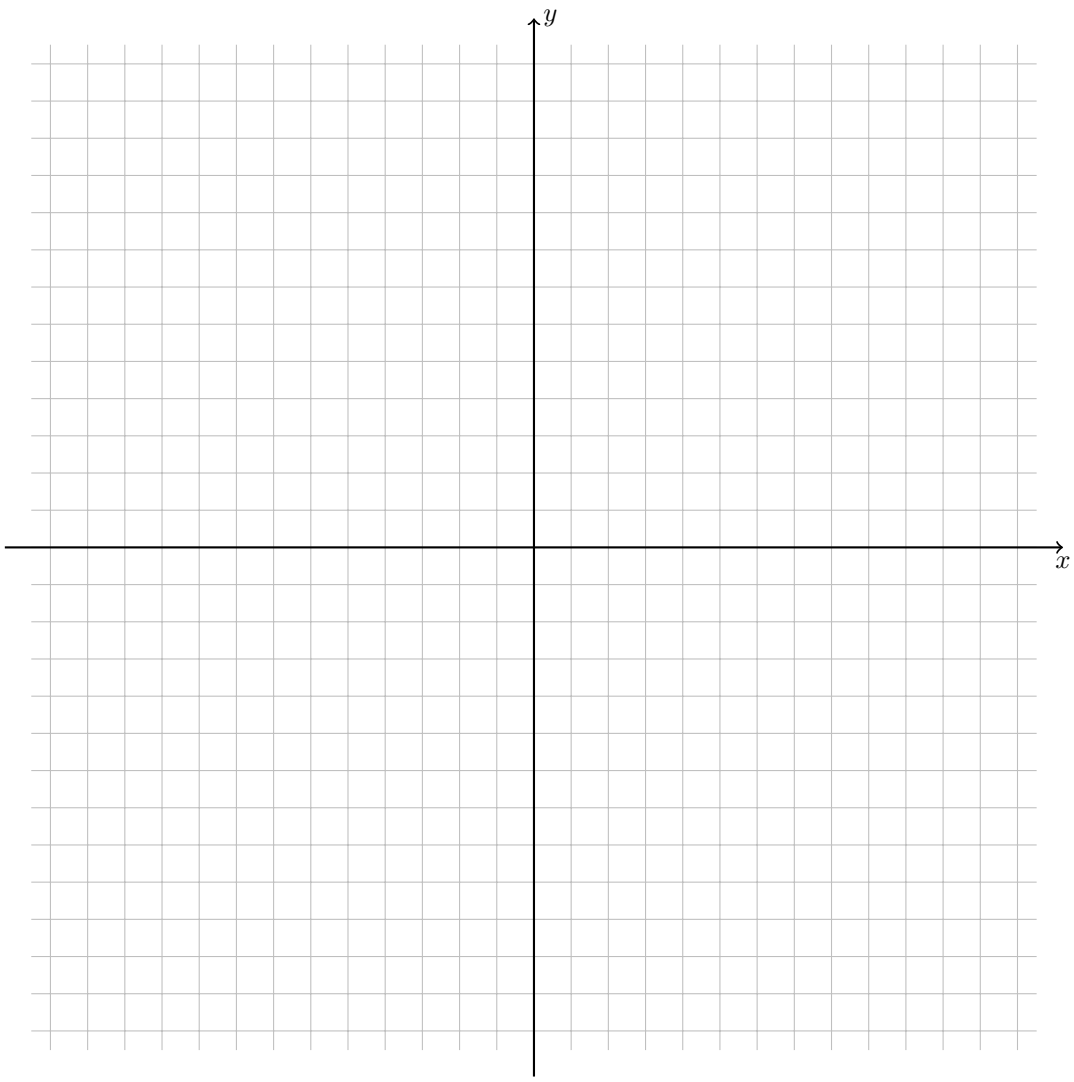
   (b) (1 point) Does $f(x)$ have any horizontal asymptotes? If so what are they?

   (c) (1 point) Does $f(x)$ have any vertical asymptotes? If so what are they?

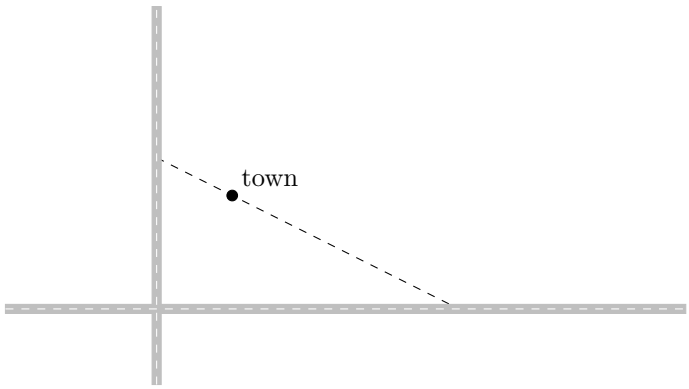
   (d) (2 points) For what x is the first derivative $f'(x)$ positive?

(e) (2 points) For what x is the second derivative $f''(x)$ positive?

(f) (3 points) On the graph provided, sketch $f(x)$



3. Two straight freeways intersect at right angles. The freeways run North-South and East-West. One mile East, and two miles North of the intersection is a town. A *straight* road is to be built from the North-South freeway, through the town and then onwards to the East-West freeyway.



(a) (3 points) Let x be the distance from the intersection of the two freeways, to where the road branches off the North-South freeway. What is the length of the new road?

(b) (7 points) How far North of the intersection should the road begin in order to minimise the length of the new road?

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