Midterm 2

UCLA: Math 3B, Fall 2017

Instructor: Noah White
Date: 20 November, 2017
Version: practice

- This exam has 3 questions, for a total of 36 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section (please circle):

Day/TA	Kevin	Bohyun	Ryan
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	12	
2	12	
3	12	
Total:	36	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) If the function $y(t)$ satisfies the differential equation $\frac{\mathrm{d}y}{\mathrm{d}t} = y \cos t$ and $y(0) = 1$ then

   A. $y(1) = 1$
   B. $y(\pi) = 1$**.**
   C. $y(\pi) = 0$.
   D. $y(1) = 0$.

   (b) (2 points) The definite integral $\displaystyle\int_0^1 2xe^x \ \mathrm{d}x$ is equal to

   A. e.
   B. 1.
   C. 0.
   D. 2.

   (c) (2 points) In the partial fraction expansion $\dfrac{x^2 + 9x + 10}{(x^2 - 4)(x + 2)} = \dfrac{A}{x + 2} + \ldots,$

   A. $A = 1$.
   B. $A = -1$**.**
   C. $A = 0.25$.
   D. $A = 2$.

(d) (2 points) The function $y(t) = t(1 + \ln t)$ is a solution to the ODE

A. $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 + y$

B. $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y}{t}$

C. $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 + \frac{y}{t}$

D. $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{t}{t-y}$

(e) (2 points) A radioactive substance has a half-life of $\ln 4$ days. If $y(t)$ descibes the total amount of substance after t days, then

A. $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{\ln 2}{2}y$

B. $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -2y$

C. $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -0.5y$

D. $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{0.5}{\ln 2}y$

(f) (2 points) A population $P(t)$ obeys the differetial equation $\frac{\mathrm{d}P}{\mathrm{d}t} = (P^2 - 5P)(9 - P)$. What is the eventual fate of the population if $0 < P(0) < 5$?

A. The population grows forever.

B. The population dies out, i.e. $P(t)$ is eventually close to zero.

C. The population stabilises at or close to 5.

D. The population stabilises at or close to 9.

2. A long chain is lowered into a 100 meter deep mine shaft. The chain weights 5 kg per meter. At the bottom, a miner attaches a bucket full of ore, the bucket and ore weigh 30 kg together. In the parts below, you will calculate how much work is done pulling the chain and bucket to the surface. You may assume that the acceleration due to gravity is 10 m/s^2.

   (a) (2 points) If we ignore the weight of the chain, how much work is done bringing the full bucket to the surface?

   Solution: The bucket has a mass of 30 kg, so the force needed to lift it is 300 Newtons, thus the work is force times distance, i.e. $30,000$ Joules.

   (b) (1 point) Divide the 100 meter length of chain hanging in the mine shaft into n subintervals. Let Δx be the length of each subinterval. What is Δx in terms of n?

   Solution:
   $$\Delta x = \frac{100}{n}$$

   (c) (1 point) Let x measure depth below the ground, so $x = 0$ is the top of the mine shaft and $x = 100$ is the bottom. Let x_k be the bottom of the k^{th} subinterval. What is x_k in terms of n and k?

   Solution:
   $$t_k = k\frac{100}{n} = k\Delta x$$

(d) (1 point) How much does the k^{th} subinterval of chain weigh (in terms of Δx)?

Solution:

$$5\Delta x$$

(e) (2 points) How much work is done lifting just the k^{th} subinterval of chain to ground level? Your answer should be in terms of Δx and x_k.

Solution: We need to lift the subinterval of chain a distance of x_k meters, and we need to apply a force of $50\Delta x$ Newtons, i.e. the work done is

$$50 x_k \Delta x.$$

(f) (3 points) Write a Riemann sum that calculates the total amount of work done lifting the entire chain and bucket to ground level. Dont forget to add the contribution due to the bucket!

Solution: We sum all the contributions from all subinterval and take a limit as $n \to \infty$. We also add the result of part (a).

$$30,000 + \lim_{n\to\infty} \sum_{k=1}^{n} 50 x_k \Delta x.$$

(g) (2 points) Use an integral to evaluate this sum.

Solution:

$$\begin{aligned} 30000 + \int_0^{100} 50x \; dx &= 30000 + \left[25x^2\right]_0^{100} \\ &= 30000 + 250000 \\ &= 280000. \end{aligned}$$

3. A garden bed is fertalized continuously by attaching a source of nitrogen to a drip watering system. This releases 25 grams of Nitrogen per day into the soil. The plants will take up 40% of whatever Nitrogen is available in the soil.

   In addition, a layer of compost is placed on top of the bed. The compost contains 20 grams of Nitrogen per kilogram, however this is not available to the plants to use. It becomes available as the compost breaks down. It is estimated that the compost breaks down and becomes soil at a rate of 0.5 kilograms per day.

   (a) (2 points) How much Nitrogen (in mg) is being added, per day to the soil?

   Solution: 25 grams from the fertalizer and $0.5 \cdot 20 = 10$ grams from the compost, so 35 grams total.

   (b) (4 points) Write a differential equation describing the total level $y(t)$ of Nitrogen (in grams) in the soil at time t.

   Solution: The differential equation will take the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \text{rate in} - \text{rate out}.$$

   The rate in is 35 from above. Thus we need to find the rate out. The plants use 40% of the available nitrogen so the rate out is $0.40y$.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 35 - 0.45y$$

(c) (4 points) Assume that the soil initially contains 45 grams of Nitrogen total. Solve the differential equation.

Solution: Solving the DE gives

$$y(t) = \frac{35}{0.4} - Ce^{-0.4t}.$$

We use the initial condition $y(0) = 45$ to get $C = \frac{35}{0.4} - 45 = 42.5$, so

$$y(t) = 87.5 - 42.5e^{-0.4t}.$$

(d) (2 points) Suppose that in the long term (as t goes to ∞) we would like there to be approximately 70 grams of Nitrogen in the soil. Should we increase, decrease or maintain the level of Nitrogen currently being added to the soil on a daily basis?

Solution: Note that $\lim_{t\to\infty} y(t) = 87.5 > 70$ so we should *decrease* the amount of Nitrogen being added.

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