

# Final exam practice 4

## UCLA: Math 3B, Winter 2019

*Instructor:* Noah White

*Date:*

- This exam has 7 questions, for a total of 80 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable, non graphing and non integration capable calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

### Discussion section (please circle):

Day/TA	Louis	Matthew
Tuesday	1A	1C
Thursday	1B	1D

Question	Points	Score
1	12	
2	12	
3	12	
4	10	
5	12	
6	12	
7	10	
Total:	80	

**Question 1 and 2** are multiple choice. Indicate your answers in the table below. *The following three pages will not be graded, your answers must be indicated here.*

Part	A	B	C	D
1(a)				
1(b)				
1(c)				
1(d)				
1(e)				
1(f)				
2(a)				
2(b)				
2(c)				
2(d)				
2(e)				
2(f)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) The function  $f(x) = x^3 + x + 1$  is

- A. Always increasing.
- B. Always decreasing.
- C. Always concave up.
- D. Always concave down.

(b) (2 points) The function  $f(x) = \ln(e^x + 1)$  is

- A. Always increasing.
- B. Always decreasing.
- C. Always concave down.
- D. None of the above.

(c) (2 points) The function  $f(x) = \frac{2x^4}{x^3+1}$  has a

- A. Horizontal asymptote at  $y = -1$ .
- B. Vertical asymptote at  $x = 1$ .
- C. Slanted asymptote with slope  $-1$ .
- D. Slanted asymptote with slope  $2$ .

- (d) (2 points) The function  $f(x) = \frac{\sin x}{x^2+1}$  has a
- A. Horizontal asymptote at  $y = 0$ .
  - B. No horizontal asymptotes.
  - C. Horizontal asymptote at  $y = 1$ .
  - D. Horizontal asymptote at  $y = -1$ .
- (e) (2 points) The function  $f(x) = \ln(e^x + e^{-x})$  has a critical point at
- A.  $x = 0$
  - B.  $x = e$
  - C.  $x = 1$
  - D. It has no critical points
- (f) (2 points) The function  $f(x) = (x^2 - 4x + 5)^{-1}$  has a
- A. minimum at  $x = 2$ .
  - B. maximum at  $x = 2$ .
  - C. minimum  $x = 1$ .
  - D. maximum  $x = 1$ .

2. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (2 points) Suppose  $F(x)$  is an antiderivative of  $f(x) = xe^x$  such that  $F(0) = 0$ . What is  $F(1)$ ?

- A. 3.
- B. 2.
- C. 1.
- D. 0.

(b) (2 points) The indefinite integral  $\int \frac{\cos x}{\sin x + 1} dx$  is

- A.  $(\sin x + 1)^{-2} + C$ .
- B.  $\ln(\sin x + 1) + C$ .
- C.  $\ln(\cos x) + C$ .
- D.  $(\tan x + 1)^2 + C$ .

(c) (2 points) The solution of the differential equation  $\frac{dy}{dt} = 2y$  when  $y(0) = 1$  has

- A.  $y(1) = 2e^2$
- B.  $y(0.5) = e$ .
- C.  $y(1) = 2e^{-2}$ .
- D.  $y(0.5) = e^{-1}$ .

(d) (2 points) The solution of the differential equation  $\frac{dy}{dt} = \frac{e^{-y}}{(t+1)^2}$  when  $y(0) = 0$  has

- A.  $y(-2) = \ln(3)$ .
- B.  $y(-2) = \ln(2)$ .
- C.  $y(1) = \ln(3)$ .
- D.  $y(1) = \ln(2)$ .

(e) (2 points) Given a function  $f(x)$  what is the derivative of  $\int_{-x}^x f(x) dx$

- A.  $f(x)$ .
- B.  $2f(x)$ .
- C.  $f(-x)$ .
- D.  $f(x) + f(-x)$ .

(f) (2 points) The differential equation  $\frac{dy}{dt} = \ln(y^2 + 1) - \ln(5)$  has a

- A. stable equilibrium at  $y = 2$ .
- B. unstable equilibrium at  $y = 1$ .
- C. stable equilibrium at  $y = 1$ .
- D. unstable equilibrium at  $y = 2$ .

3. Let  $f(x) = \frac{x^2 - 4}{x^2 - 9}$ . Note that  $f'(x) = -\frac{10x}{(x^2 - 9)^2}$  and  $f''(x) = \frac{30(x^2 + 3)}{(x^2 - 9)^3}$ .

(a) (1 point) Does  $f(x)$  cross the  $x$  and  $y$  axes? If so, where?

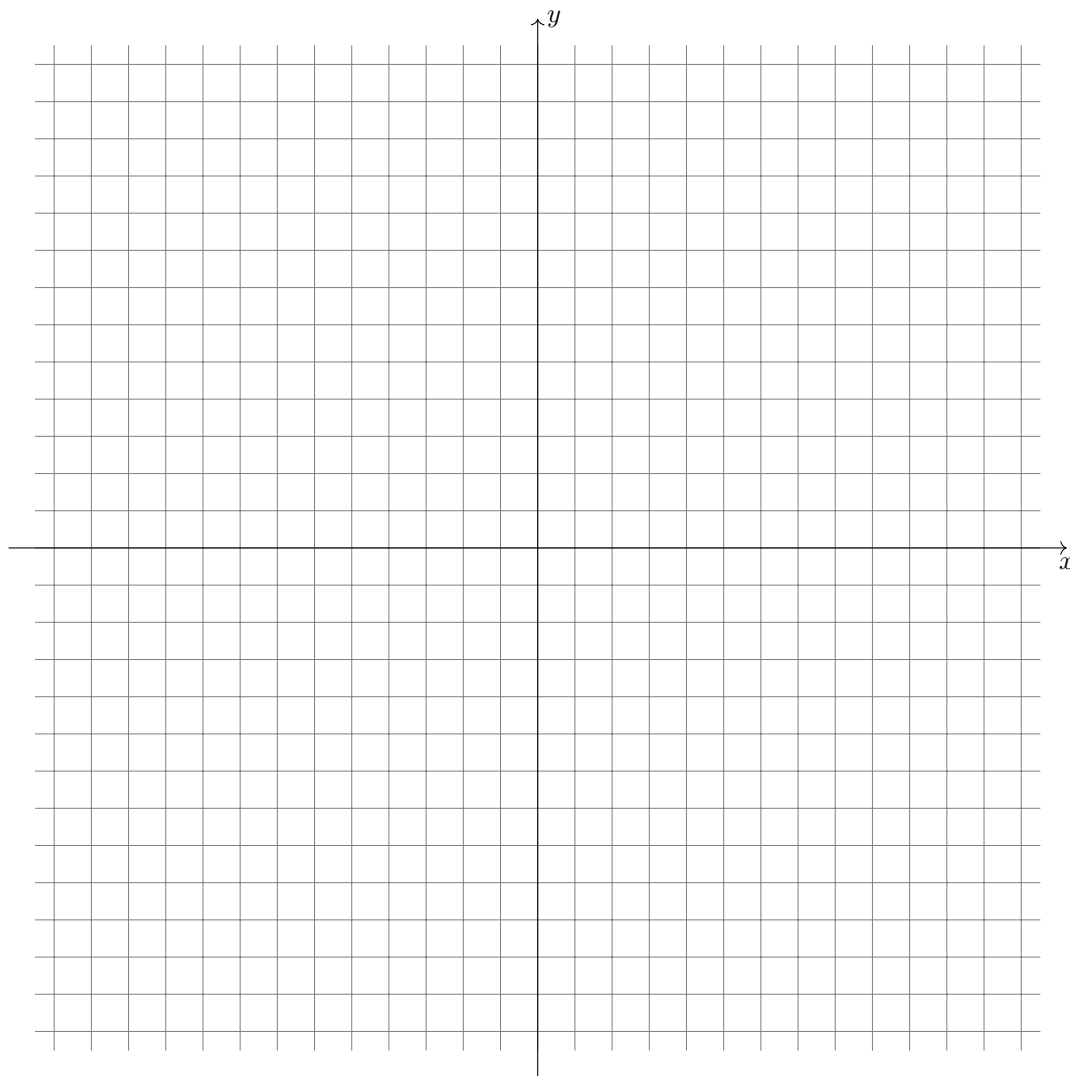
(b) (2 points) Does  $f(x)$  have any horizontal asymptotes? If so what are they?

(c) (2 points) Does  $f(x)$  have any vertical asymptotes? If so what are they?

(d) (2 points) For what  $x$  is the first derivative  $f'(x)$  positive?

(e) (2 points) For what  $x$  is the second derivative  $f''(x)$  positive?

(f) (3 points) On the graph provided, sketch  $f(x)$



4. A 100 metre long chain is hanging off a bridge. The chain weighs 0.4 kilograms per metre. You may assume that the acceleration due to gravity is  $10 \text{ m/s}^2$ .
- (a) (6 points) Write a Riemann sum which represents the total work done pulling the chain up, onto the bridge.

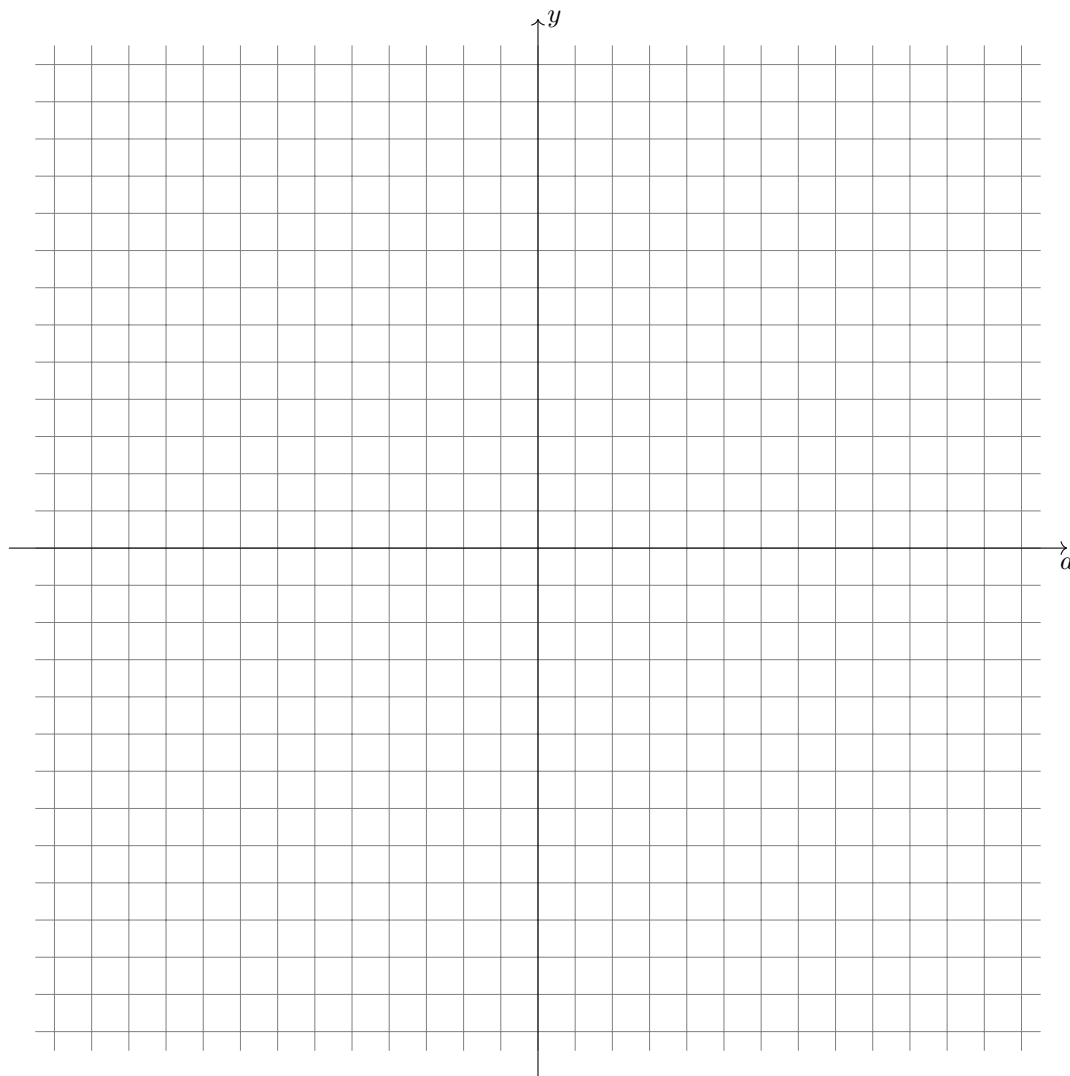
- (b) (4 points) Use an integral to evaluate the Riemann sum above.



5. In this question we will investigate the behaviour of the solutions of

$$\frac{dy}{dt} = (y - 2)(a - e^y + 1)$$

- (a) (4 points) Draw a bifurcation diagram for this equation with parameter  $a$ . Make sure to label the regions of your diagram with up/down arrows according to the direction of the derivative.



(b) (2 points) Draw a phase diagram when  $a = -1$  and sketch the solution if  $y(0) = -1$ .

(c) (2 points) Draw a phase diagram when  $a = 0$  and sketch the solution if  $y(0) = -1$ .

- (d) (2 points) Draw a phase diagram when  $a = e^2 - 1$  and sketch the solution if  $y(0) = 3$ .
- (e) (2 points) The differential equation above has an equilibrium solution of  $y = 2$  for any value of  $a$ . For what  $a$  is this equilibrium stable?

6. A solution of 4 mg/L of chlorine is being pumped into a tank at a rate of 3 L/hour. It is known that chlorine will *degas* from the water (it leaves the water) with a half-life of  $4 \ln 2$  hours.
- (a) (4 points) Write a differential equation modelling the total amount of chlorine in the tank at time  $t$ .

- (b) (3 points) If, initially, the tank contains only water with no chlorine solve the differential equation.

- (c) (1 point) How many grams of the chlorine is in the tank after 24 hours? You may leave your answer in terms of  $e$ .
- (d) (4 points) At the 24 hour mark, a tap is turned and the tank starts draining at a rate of 3 L/hour (the incoming solution continues as well). If we assume that the tank contains exactly 100 litres of water at the 24 hour mark, what is the amount of chlorine in the tank in the long term?

7. (10 points) What is the area of the largest rectangle that fits in the region bounded by the  $x$ -axis, the  $y$ -axis and the curve  $y = 8 - x^3$ ?

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