

This week on the problem set you will get practice thinking about potential functions and calculating line integrals.

\*Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3<sup>rd</sup> Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. (Section 17.1) Questions 13 – 17, 22, 26, 28, 29, 38, 42, 44, 47, 52, 56\*. (Use the following translations 4<sup>th</sup>  $\mapsto$  3<sup>rd</sup> editions: 47  $\mapsto$  45, 52  $\mapsto$  50, 56  $\mapsto$  54, otherwise the questions are the same).
2. (Section 17.2) 3, 10, 12, 13, 21, 24, 28, 43, 44, 46, 47, 54, 55, 57, 63, 64, 67. (Use the following translations 4<sup>th</sup>  $\mapsto$  3<sup>rd</sup> editions: 43  $\mapsto$  41, 44  $\mapsto$  42, 46  $\mapsto$  44, 47  $\mapsto$  45, 54  $\mapsto$  52, 55  $\mapsto$  53, 57  $\mapsto$  55, otherwise the questions are the same).
3. (17.2.24) Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$  for

$$\mathbf{F}(x, y) = \left\langle \frac{-y}{(x^2 + y^2)^2}, \frac{x}{(x^2 + y^2)^2} \right\rangle$$

and  $C$  the circle of radius  $R$  with center at the origin oriented counterclockwise.

4. (17.2.63) Let  $C$  be a curve in polar form  $r = f(\theta)$  for  $\theta_1 \leq \theta \leq \theta_2$  (see figure below), parametrised by  $\mathbf{r}(\theta) = (f(\theta) \cos \theta, f(\theta) \sin \theta)$  as in Exercise 60 (58 in 3rd ed.).
  - (a) Show that the vortex vector field  $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$  in polar coordinates is written  $\mathbf{F}(r, \theta) = r^{-1} \langle -\sin \theta, \cos \theta \rangle$ .
  - (b) Show that  $\mathbf{F} \cdot \mathbf{r}'(\theta) d\theta = d\theta$ .
  - (c) Show that  $\int_C \mathbf{F} \cdot d\mathbf{r} = \theta_2 - \theta_1$ .
5. (17.2.67) Calculate the flux of the vector field  $\mathbf{F}(x, y) = \langle e^y, 2x - 1 \rangle$  across the parabola  $y = x^2$  for  $0 \leq x \leq 1$ , oriented left to right.

\*The questions marked with an asterisk are more difficult or are of a form that would not appear on an exam. Nonetheless they are worth thinking about as they often test understanding at a deeper conceptual level.