

Math 3B: Lecture 20

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March 1, 2019

Often it is impossible to solve a differential equation. E.g.

$$\frac{dy}{dt} = y^2 + t$$

(the *Riccati equation*) has no solutions that can be written in terms of usual functions like $\sin x$, e^x , etc.

Eulers method

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(the *Riccati equation*) has no solutions that can be written in terms of usual functions like $\sin x$, e^x , etc.

We want a method to **estimate** $y(t)$ if we know that $y(t_0) = y_0$.

Eulers method

Let's use Eulers method!



Idea behind Eulers method

Suppose $y(t)$ is a solution to

$$\frac{dy}{dt} = f(t, y)$$

and that $y(t_0) = y_0$.

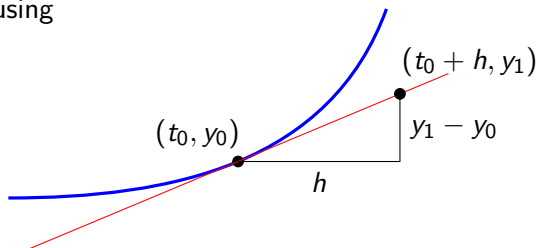
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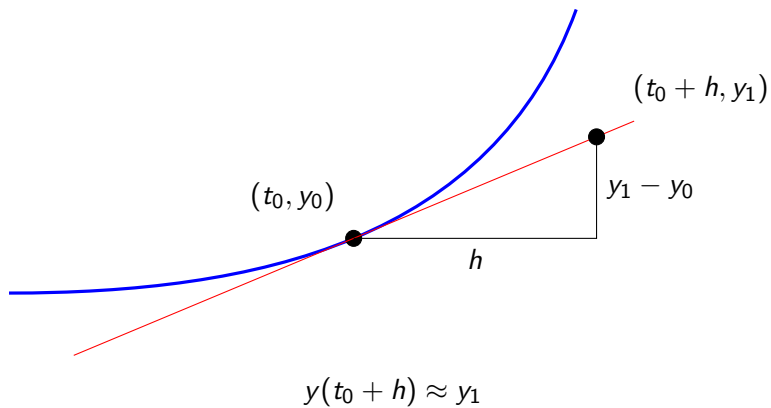
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and that $y(t_0) = y_0$.

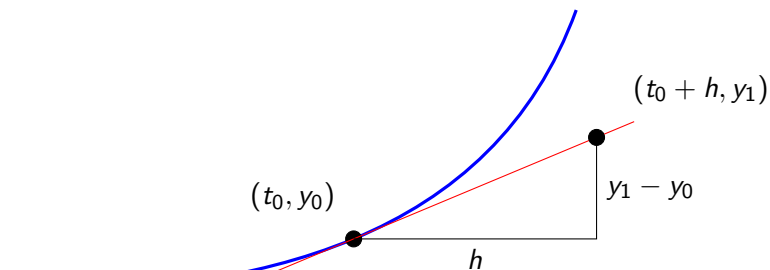
If h is a small number (e.g. $h = 0.1$), then we approximate $y(t_0 + h)$ using



Idea behind Eulers method

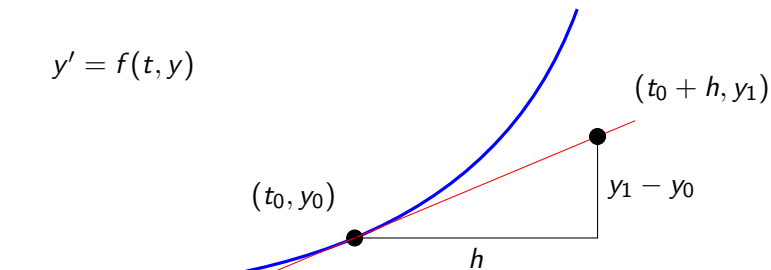


Idea behind Eulers method



$$y'(t_0) = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{h}$$

Idea behind Eulers method



$$\begin{aligned}y(t_0 + h) &\approx y_1 = y_0 + hy'(t_0) \\ &= y_0 + hf(t_0, y_0)\end{aligned}$$

Idea behind Eulers method

$$\frac{dy}{dt} = f(t, y)$$

If we know that the solution satisfies $y(t_0) = y_0$ then

- let h be a small step forward in time

Idea behind Eulers method

$$\frac{dy}{dt} = f(t, y)$$

If we know that the solution satisfies $y(t_0) = y_0$ then

- let h be a small step forward in time
- we can get an approximate value for the solution at $t = t_0 + h = t_1$

Idea behind Eulers method

$$\frac{dy}{dt} = f(t, y)$$

If we know that the solution satisfies $y(t_0) = y_0$ then

- let h be a small step forward in time
- we can get an approximate value for the solution at $t = t_0 + h = t_1$
- i.e. $y(t_1) \approx y_1$ where

$$y_1 = y_0 + hf(t_0, y_0)$$

Eulers method

To carry out Eulers method, we simply repeat this a number of times!

$$\frac{dy}{dt} = f(t, y)$$

Given an initial value $y(t_0) = y_0$. To approximate $y(t)$ at $t = a$ follow the steps:

- Choose an increment h

Eulers method

To carry out Eulers method, we simply repeat this a number of times!

$$\frac{dy}{dt} = f(t, y)$$

Given an initial value $y(t_0) = y_0$. To approximate $y(t)$ at $t = a$ follow the steps:

- Choose an increment h
- set $t_1 = t_0 + h$

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- Choose an increment h
- set $t_1 = t_0 + h$
- set $y_1 = y_0 + hf(t_0, y_0)$
- set $t_2 = t_1 + h$

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- Choose an increment h
- set $t_1 = t_0 + h$
- set $y_1 = y_0 + hf(t_0, y_0)$
- set $t_2 = t_1 + h$
- set $y_2 = y_1 + hf(t_1, y_1)$
- keep repeating until $t_n \approx a$

Eulers method

To carry out Eulers method, we simply repeat this a number of times!

$$\frac{dy}{dt} = f(t, y)$$

Given an initial value $y(t_0) = y_0$. To approximate $y(t)$ at $t = a$ follow the steps:

- Choose an increment h
- set $t_1 = t_0 + h$
- set $y_1 = y_0 + hf(t_0, y_0)$
- set $t_2 = t_1 + h$
- set $y_2 = y_1 + hf(t_1, y_1)$
- keep repeating until $t_n \approx a$
- then $y(a) \approx y_n$.

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	
1			
2			
3			
4			

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1			
2			
3			
4			

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Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2			
3			
4			

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	
3			
4			

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3			
4			

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3	1.5	0.78	
4			

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3	1.5	0.78	$y_4 = 0.78 + 0.5 \cdot (0.78^2 + 1.5)$
4			

An example

We will approximate $y(2)$, where y obeys

$$\frac{dy}{dt} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

Iter.	x	y	
0	0	0	$y_1 = 0 + 0.5 \cdot (0^2 + 0)$
1	0.5	0	$y_2 = 0 + 0.5 \cdot (0^2 + 0.5)$
2	1.0	0.25	$y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$
3	1.5	0.78	$y_4 = 0.78 + 0.5 \cdot (0.78^2 + 1.5)$
4	2.0	1.84	