This week on the problem set you will get practice at calculating integrals using substitution, integration by parts and using partial fractions. Many of these are routine but some are quite difficult!

Homework: The homework will be due on Friday 21 October, at 2pm, the *start* of the lecture. It will consist of questions:

$$4(m), \ 5, \ \text{and } 6.$$

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (5.5) Calculate the following integrals using substitution.
   (a) (12) $\int \frac{x}{\sqrt{x^2+1}}\, \mathrm{d}x$
   (b) (14) $\int \sin^3 t \cos t\, \mathrm{d}t$
   (c) (16) $\int \frac{z^3}{\sqrt{z^4+12}}\, \mathrm{d}z$
   (d) (19) $\int_1^2 \frac{e^{1/x}}{x^2}\, \mathrm{d}x$
   (e) (23) $\int_1^2 x\sqrt{x-1}\, \mathrm{d}x$
   (f) (24) $\int_0^2 (e^x - e^{-x})^2 dx$
2. (5.6) Calculate the following integrals using integration by parts.
   (a) (2) $\int e^t \sin t\, \mathrm{d}t$
   (b) (6) $\int x^2 \ln x\, \mathrm{d}x$
   (c) (9) $\int \sin x \cos x\, \mathrm{d}x$
   (d) (14) $\int_0^\pi x \sin x\, \mathrm{d}x$
   (e) (16) $\int_1^e x^3 \ln x\, \mathrm{d}x$
3. (5.6) Calculate the following integrals using partial fractions.
   (a) (17) $\int \frac{\mathrm{d}N}{N(1000-N)}$
   (b) (19) $\int \frac{x}{x(1000-x)}\, \mathrm{d}x$
   (c) (20) $\int \frac{(x+1)\, \mathrm{d}x}{(x+2)(x+3)}$
   (d) (22) $\int \frac{4}{(x+1)(x+2)(x+3)}\, \mathrm{d}x$
4. Use any method to evaluate the following integrals.
   (a) $\int x\sqrt{x+1}\, \mathrm{d}x$
   (b) $\int_1^2 \frac{t}{(t^2+1)\ln(t^2+1)}\, \mathrm{d}t$
   (c) $\int \frac{\ln x}{x^5}\, \mathrm{d}x$
   (d) $\int \ln x\, \mathrm{d}x$
   (e) $\int (\ln x)^2\, \mathrm{d}x$
   (f) $\int_1^e (\ln x)^3\, \mathrm{d}x$
   (g) $\int e^{6x} \sin(e^{3x})\, \mathrm{d}x$
   (h) $\int_0^1 \frac{t^3 e^{t^2}}{(t^2+1)^2}\, \mathrm{d}t$
   (i) $\int \frac{2x+3}{z^2-9}\, \mathrm{d}z$

(j) $\int \frac{x^2+x-1}{(x^2-1)}\ \mathrm{d}x$

(k) $\int \frac{e^x}{(e^x-1)(e^x+3)}\ \mathrm{d}x$

(l) $\int_0^{\pi/3} e^t \sin t\ \mathrm{d}t$

(m) $\int e^{\sqrt{x}}\ \mathrm{d}x$

(n) $\int \frac{1}{\cos x}\ \mathrm{d}x$ (quite challenging)

(o) $\int (\sin x)^2\ \mathrm{d}x$ (quite challenging)

5. (5.5-30) Suppose an environmental study indicates that the ozone level, L, in the air above a major metropolitan center is changing at a rate modeled by the function

$$L'(t) = \frac{0.24 - 0.03t}{\sqrt{36 + 16t - t^2}}$$

parts per million per hour (ppm/h) t hours after 7:00 A.M.

(a) Express the ozone level $L(t)$ as a function of t if L is 4 ppm at 7:00 A.M.

(b) Find the time between 7:00 A.M. and 7:00 P.M. when the highest level of ozone occurs. What is the highest level?

(Note: part b has been changed slightly from what is written in the textbook.)

6. The circle $x^2 + (y+1)^2 = 4$ has area 4π. What is the area of the portion of the circle lying above the x axis?

You may use the fact that

$$\int \sqrt{1-t^2}\ \mathrm{d}t = \frac{1}{2}\left(t\sqrt{1-t^2} + \sin^{-1} t\right) + C.$$

7. (5.6-36) Assume that after t hours on the job, a factory worker can produce $100te^{0.5t}$ units per hour. How many units does the worker produce during the first 3 hours?

8. (5.6-38) An actuary measures the probability that a person in a certain population will die at age x by the formula

$$P(x) = \lambda^2 x e^{-\lambda x}$$

where λ is a parameter such that $0 < \lambda < e$.

(a) For a given λ, find the maximum value of $P(x)$.

(b) Sketch the graph of $P(x)$.

(c) Find the area under the probability curve $y = P(x)$ for $0 \le x \le 100$, and interpret your result.

9. (5.6-39) A population P, grows at the rate

$$P'(t) = 5(t+1)\ln\sqrt{t+1}$$

thousand individuals per year at time t (in years). By how much does the population change during the eighth year?