This week on the problem set you will get practice applying and understanding Green's theorem and Stokes' theorem.

Homework: The homework will be due on Tuesday 3 December. It will consist of questions:

18.1.24, 18.1.36 and 18.2.19.

*Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 3rd Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. (Section 18.1) 3, 7, 8, 9, 12, 19, 20, 21, 23, 24 25, 29, 36*, 41, 45. (Use the following translations 4th $\mapsto$ 3rd editions: $7 \mapsto 5$, $8 \mapsto 6$, $9 \mapsto 7$, $12 \mapsto 10$, $19 \mapsto 15$, $20 \mapsto 16$, $21 \mapsto 17$, $23 \mapsto 19$, $24 \mapsto 20$, $25 \mapsto 21$, $29 \mapsto 25$, $36 \mapsto 32$, $41 \mapsto 37$, $45 \mapsto 41$ otherwise the questions are the same).

2. (Section 18.2) 5, 8, 9, 18, 19. (Use the following translations 4th $\mapsto$ 3rd editions: $18 \mapsto 16$, $19 \mapsto 17$, otherwise the questions are the same).

3. (18.1.24) Find a parametrisation of the lemniscate $(x^2 + y^2)^2 = xy$ (see Figure 23) by using $t = y/x$ as a parameter (See Exercise 23). Then use Green's theorem to find the area of one loop of the lemniscate.

4. (18.1.36) Green's Theorem leads to a convenient formula for the area of a polygon.
   (a) Let $\mathcal{C}$ be the line segment joining (x_1, y_1) to (x_2, y_2). Show that
   $$\frac{1}{2}\int_{\mathcal{C}} \langle -y, x\rangle \cdot d\mathbf{r} = \frac{1}{2}(x_1y_2 - x_2y_1).$$
   (b) Prove that the area of the polygon with vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ is equal to
   $$\frac{1}{2}\sum_{i=1}^{n}(x_iy_{i+1} - x_{i+1}y_i)$$
   where $(x_{n+1}, y_{n+1}) = (x_1, y_1)$.

5. (18.2.19) Let I be the flux of $\mathbf{F} = \langle e^y, 2xe^{x^2}, z^2\rangle$ through the upper hemisphere $\mathcal{S}$ of the unit sphere.
   (a) Let $\mathbf{G} = \langle e^y, 2xe^{x^2}, 0\rangle$. Find a vector field $\mathbf{A}$ such that $\mathrm{curl}(\mathbf{A}) = \mathbf{G}$.
   (b) Use Stokes' Theorem to show that the flux of $\mathbf{G}$ through $\mathcal{S}$ is zero. *Hint:* Calculate the circulation of $\mathbf{A}$ around $\partial\mathcal{S}$.
   (c) Calculate I. *Hint:* Use (b) to show that I is equal to the flux of $\langle 0, 0, z^2\rangle$ through $\mathcal{S}$.

*The questions marked with an asterisk are more difficult or are of a form that would not appear on an exam. Nonetheless they are worth thinking about as they often test understanding at a deeper conceptual level.