# Math 3B: Lecture 21

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March 6, 2017

# Introduction

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- PS9, question 7

# Slope fields

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Get a qualitative understanding for how a solution behaves, given an initial condition $y(t_0) = y_0$.

## Key tool

Slope fields. At every point on the *yt*-plane we draw a small line segment (a vector) with slope $f(y, t)$.

# Examples

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If we want to draw a slope field, we cannot actually draw a line segment for every point. Instead we pick a grid of points in the plane.

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## Examples

Lets use Geogebra! Here is the command we will use:

`SlopeField[f(x,y)]` will produce a slope field for the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$$

# Sketching solutions

Using the slope field we can sketch rough pictures of the solution, given a starting point (an initial condition).

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- is the solution ever above to below a certain value?

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## Examples

Lets use Geogebra again.

# Nullclines

## Definition

The nullcline for $\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$ is the set of points $(t, y)$ where $f(t, y) = 0$

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## Examples

Lets use Geogebra!

# Drawing slope fields by hand

Drawing slope fields by hand can be difficult! But we can use the nullclines to get an approximate picture

## Examples

Lets draw some on the board.

# Eulers method

Often it is impossible to solve a differential equation. E.g.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

(the *Riccati equation*) has no solutions that can be written in terms of usual functions like $\sin x$, $e^x$, etc.

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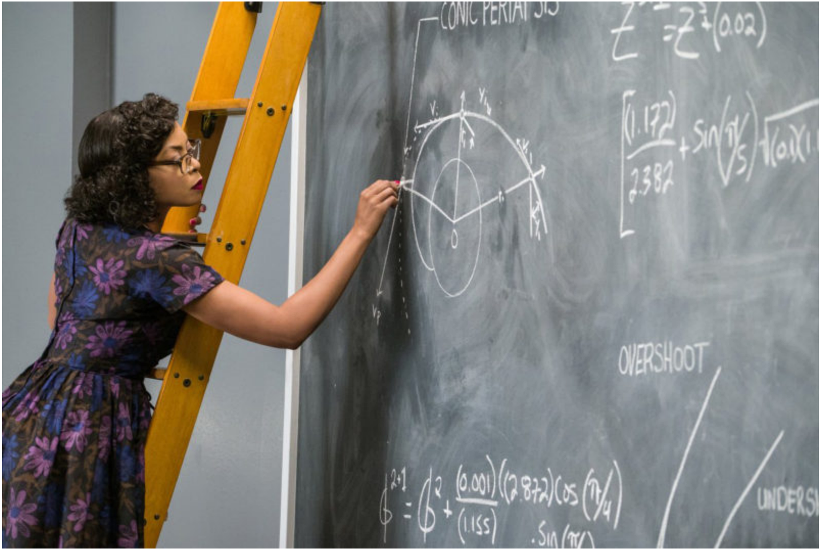
We want a method to estimate $y(t)$ is we know that $y(t_0) = y_0$.

# Eulers method

Let's use Eulers method!



# Idea behind Eulers method

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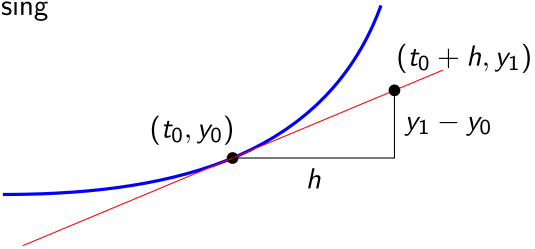
If $h$ is a small number (e.g. $h = 0.1$), then we approximate $y(t_0 + h)$ using



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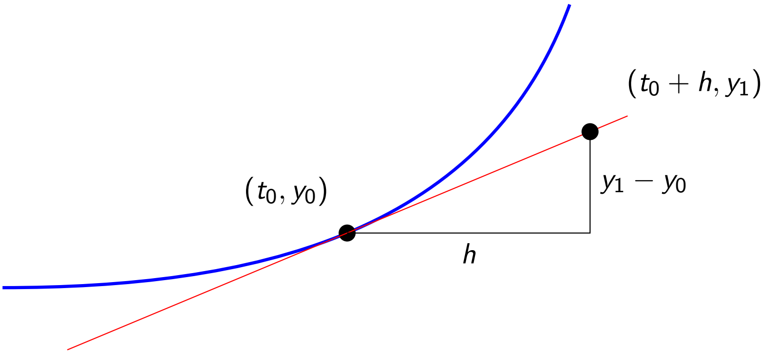


$$y(t_0 + h) \approx y_1$$

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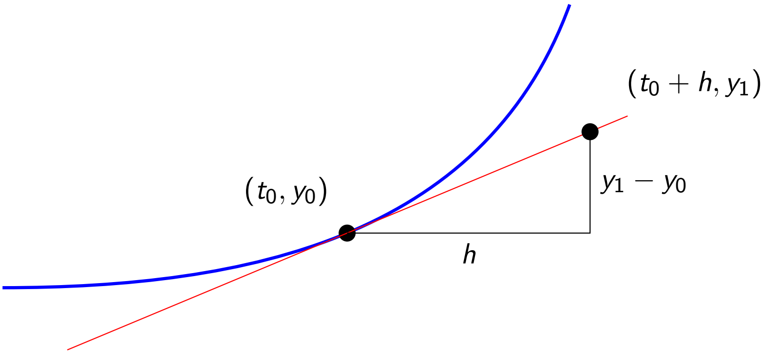


$$y'(t_0) = \frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{h}$$

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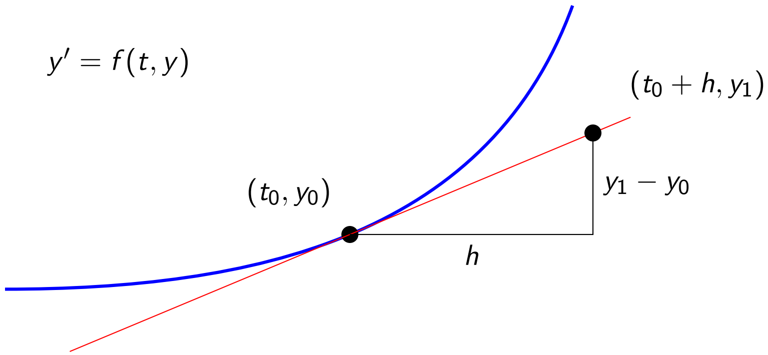


$$\begin{aligned} y(t_0 + h) \approx y_1 &= y_0 + hy'(t_0) \\ &= y_0 + hf(t_0, y_0) \end{aligned}$$

# Idea behind Eulers method

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

If we know that the solution satisfies $y(t_0) = y_0$ then

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- let $h$ be a small step forward in time
- we can get an approximate value for the solution at $t = t_0 + h = t_1$
- i.e. $y(t_1) \approx y_1$ where

$$y_1 = y_0 + hf(t_0, y_0)$$

# Eulers method

To carry out Eulers method, we simply repeat this a number of times!

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

Given an initial value $y(t_0) = y_0$. To approximate $y(t)$ at $t = a$ follow the steps:

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- keep repeating until $t_n \approx a$
- then $y(a) \approx y_n$.

# An example

We will aprroximate $y(2)$, where $y$ obeys

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t$$

and $y(0) = 0$. Let $h = 0.5$.

| Iter. | $x$ | $y$ | |
|---|---|---|---|
| 0 | 0 | 0 | |
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |

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| 2 | 1.0 | 0.25 | $y_3 = 0.25 + 0.5 \cdot (0.25^2 + 1)$ |
| 3 | 1.5 | 0.78 | |
| 4 | | | |

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| 3 | 1.5 | 0.78 | $y_4 = 0.78 + 0.5 \cdot (0.78^2 + 1.5)$ |
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| 3 | 1.5 | 0.78 | $y_4 = 0.78 + 0.5 \cdot (0.78^2 + 1.5)$ |
| 4 | 2.0 | 1.84 | |

# Autonomous equations

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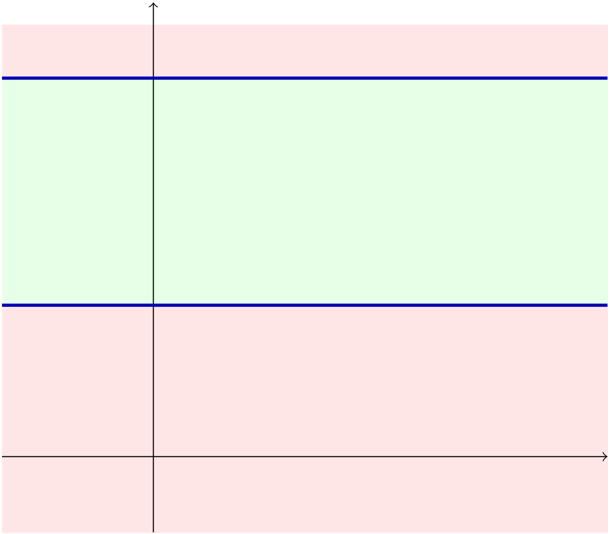
The nullclines of an autonomous equation are horizontal straight lines! Nullclines = equilibrium solutions

We want points $(t, y)$ such that $f(y) = 0$.

- Suppose $f(a) = 0$.
- Then $(t, a)$ is on the nullcline, for any t.
- So the line $y = a$ is part of the nullcline, whenever $f(a) = 0$.

Thus our slope field and nullclines look something like



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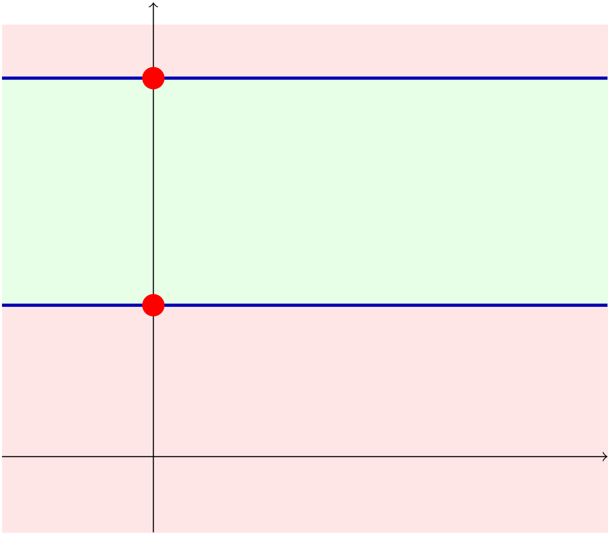

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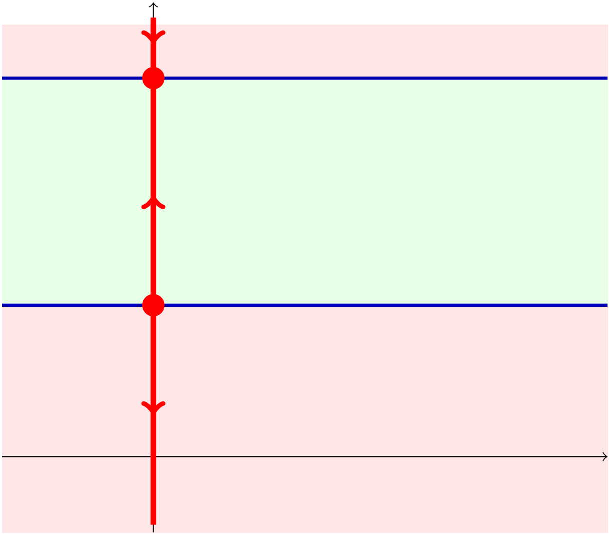

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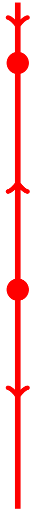

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- It is unstable if the two arrows are pointing away from it.
- It is semistable if the arrows point in the same direction.

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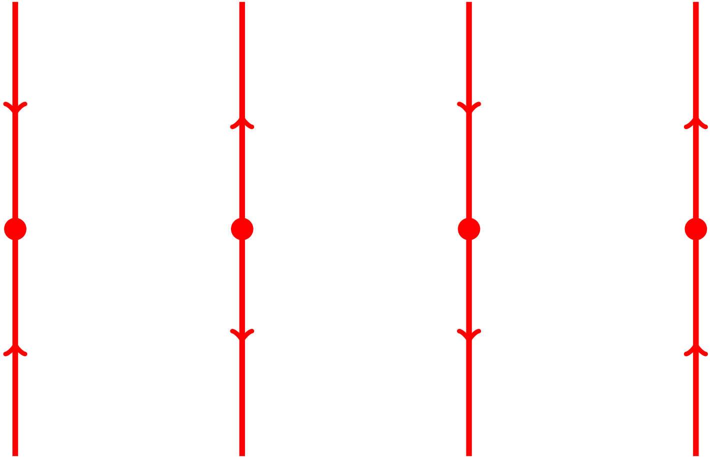

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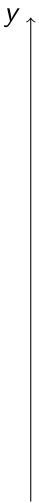

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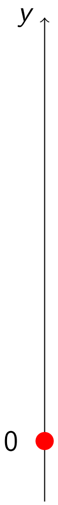

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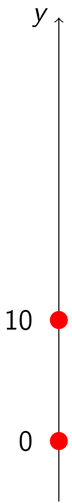

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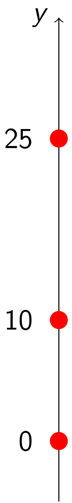

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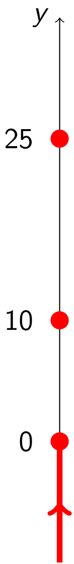

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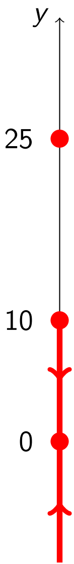

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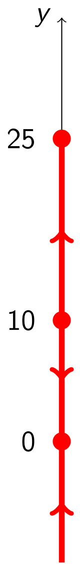

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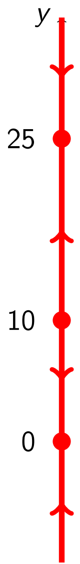

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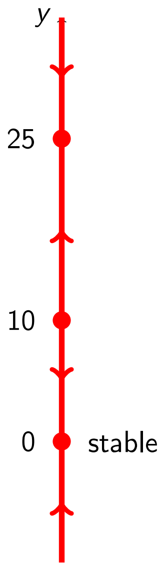

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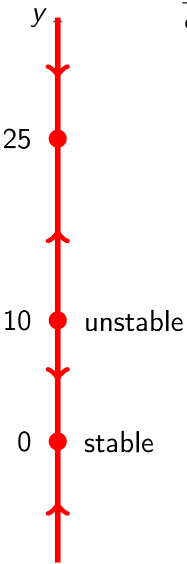

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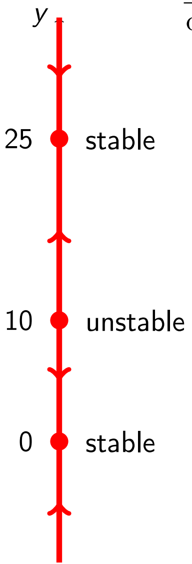

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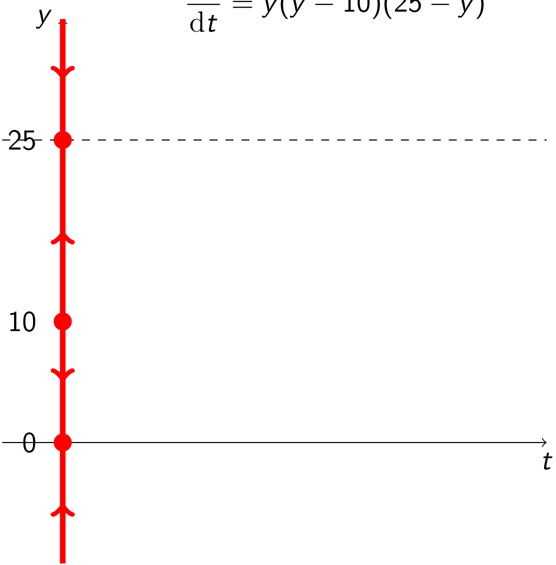

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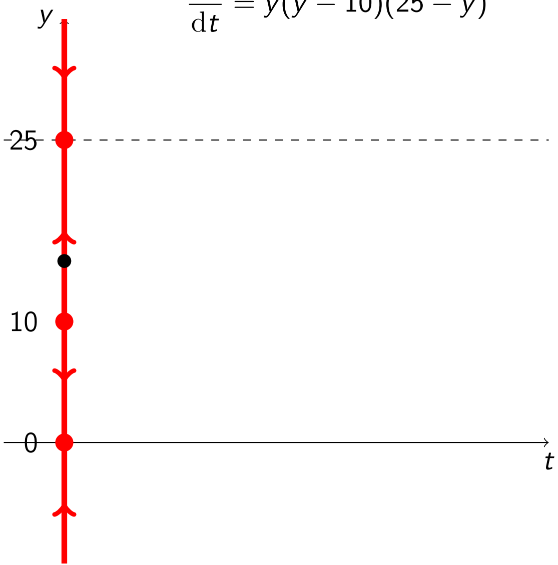

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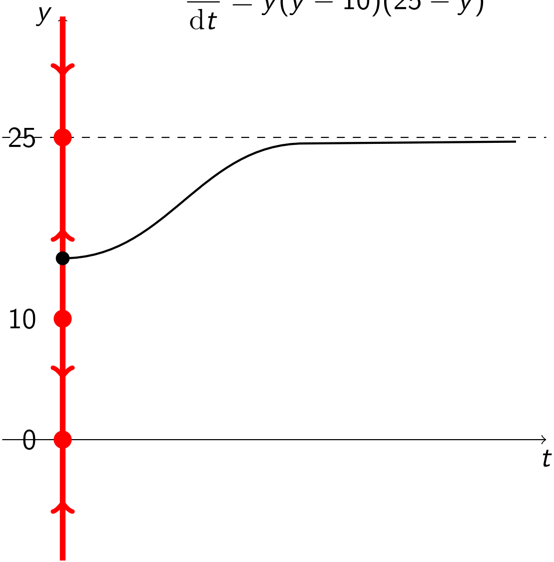

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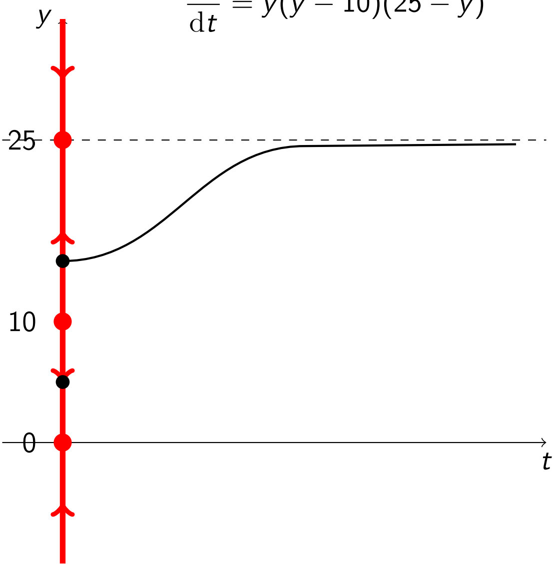

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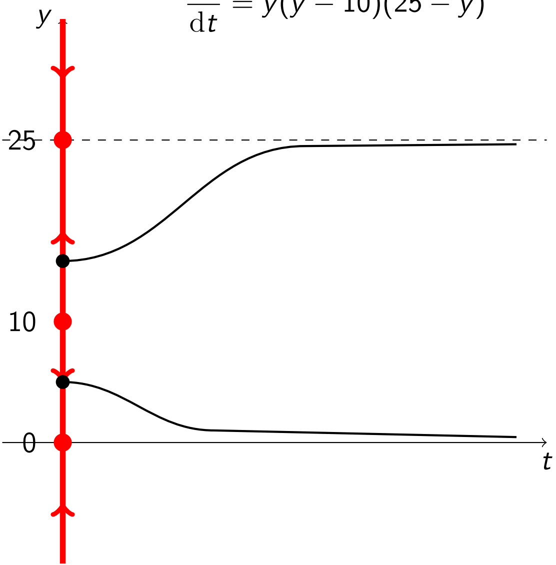

# Classifying equilibria using derivatives

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If $a$ is an equillibrium of

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

(i.e. $f(a) = 0$) then $a$ is

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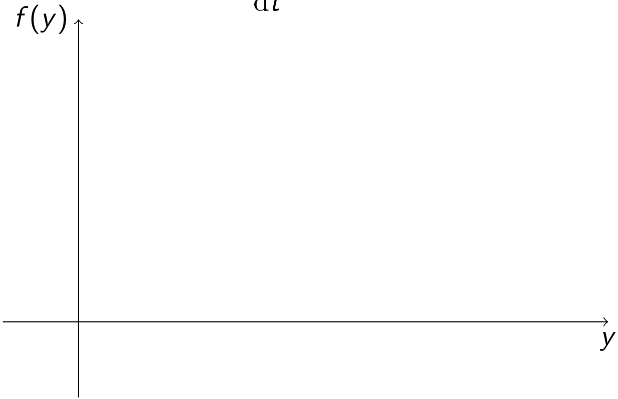
# Why?

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$



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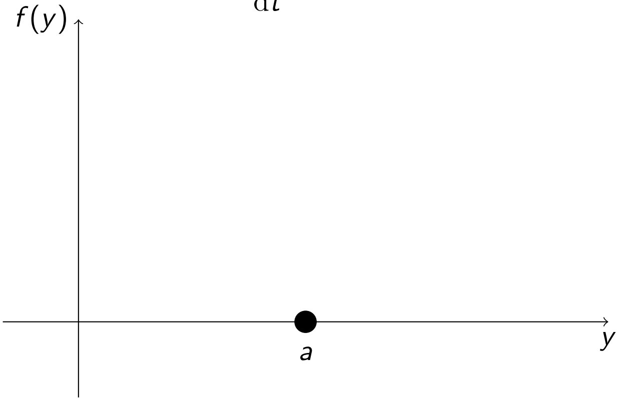

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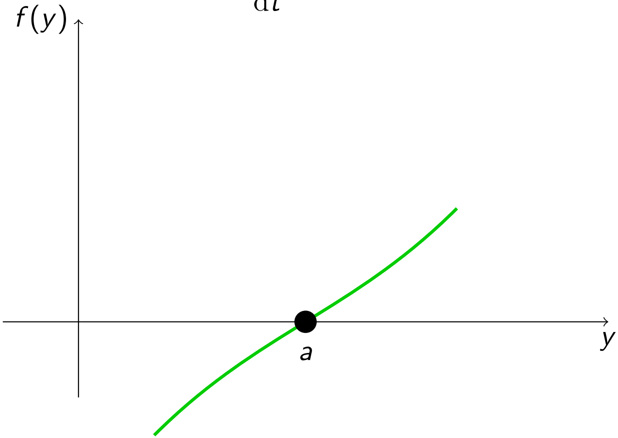

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