This week on the problem set we will review techniques for differentiating functions and evaluating limits. This should be familiar to you from Math 3A or other equivalent calculus courses you have taken. Make sure you can do these types of routine questions. Problems 7 to 11 will give you some practice graphing functions using the information provided by asymptotes and the first and second derivatives.

1. (3.1) Differentiation practice. Differentiate the following functions.
   (a) (3.1-3a) $f(x) = 3x^5$
   (b) (3.1-7) $f(x) = x^5 - 3x^2 - 1$
   (c) (3.1-9) $s(t) = 4e^t - 5t + 1$
   (d) (3.1-19) $H(w) = 2w - e^w$
   (e) (3.1-23) $f(x) = \frac{x^{1/3}}{x^2}$
   (f) (3.1-25) $h(t) = \frac{3^t+3^{-t}}{2^t}$
2. (3.2) Product and quotient rule practice. Differentiate the following functions.
   (a) (3.2-2) $p(x) = (x^2+4)(1-3x^3)$
   (b) (3.2-4) $q(x) = \frac{x+1}{1+x^2}$
   (c) (3.2-7) $f(x) = (1+x+x^2)e^x$
   (d) (3.2-14) $g(t) = \frac{1+te^t}{1+t}$
3. (3.3) Chain rule practice. Differentiate the following functions.
   (a) (3.3-9) $y = (5-x+x^4)^9$
   (b) (3.3-12) $y = e^{(x+1)^7}$
   (c) (3.3-15) $y = \ln(2x+5)$
   (d) (3.3-16) $y = xe^{-x^2}$
4. (3.4) Differentiate the following trig functions.
   (a) (3.4-1) $f(x) = \sin(x) + \cos(x)$
   (b) (3.4-7) $y = e^{-x}\sin(x)$
   (c) (3.4-16) $f(x) = \frac{\sin(x)}{1-\cos(x)}$
   (d) (3.4-20) $y = \ln(\sec(x) + \tan(x))$
5. (2.3 and 2.4) Evaluate the following limits.
   (a) (2.3-1) $\lim\limits_{x\to 3} \frac{x^2+3x-10}{3x^2+5x-7}$
   (b) (2.3-12) $\lim\limits_{s\to 1} s + \sin(\ln(s))$
   (c) (2.4-1) $\lim\limits_{x\to -\infty} e^x$
   (d) (2.4-4) $\lim\limits_{x\to 2^-} \frac{1}{x-2}$
   (e) (2.4-9) $\lim\limits_{x\to 3^+} \frac{x^2-4x+3}{x^2-6x+9}$
   (f) (2.4-15) $\lim\limits_{x\to\infty} \frac{\sin(x)}{1+x}$
6. (3.7) L'Hôpital's rule practice. Evaluate the following limits.
   (a) (3.7-3) $\lim\limits_{x\to 1} \frac{x^3-1}{x^2-1}$
   (b) (3.7-6) $\lim\limits_{x\to 0} \frac{1-\cos(x)}{x^2}$

(c) (3.7-7) $\lim_{x\to\infty} x^{-5}\ln(x)$

(d) (3.7-10) $\lim_{x\to\infty} \frac{\ln(\ln(x))}{x}$

(e) (3.7-12) $\lim_{x\to\infty} \left(1+\frac{1}{2x}\right)^{3x}$

(f) (3.7-13) $\lim_{x\to\infty} (\ln(x))^{1/x}$

7. (4.1) Graphing functions using calculus. Graph the following functions, if they involve a parameter a, graph the family of functions, demonstrating how they depend on the parameters.
   (a) (4.1-2) $y = x^2 + 5x - 3$
   (b) (4.1-6) $y = \frac{1}{x-1} + x$
   (c) (4.1-12) $y = 2e^x + e^{-x}$
   (d) (4.1-14) $y = \frac{x^2}{1+x^4}$
   (e) (4.1-16) $y = \frac{ax}{x^2+1}$
   (f) (4.1-20) $y = ax + \frac{1}{x}$

8. (4.1-24) Consider the graph of $y = \frac{e^{ax}}{1+e^a x}$. Use limits and first derivatives to determine how the shape of this curve depends on the parameter a.

9. (4.1-30) Two mathematicians, W. O. Kermack and A. G. McKendrick, showed that the weekly mortality rate during the outbreak of the Black Plague in Bombay in (1905 - 1906) can be reasonably well described by the function
$$f(t) = 890\,\mathrm{sech}^2(0.2t - 3.4) \quad \text{deaths/week}$$
where t is measured in weeks. Sketch this function using information about asymptotes and first derivatives. Recall that
$$\mathrm{sech}(x) = \frac{2}{e^x + e^{-x}}.$$

10. (4.1-33) Let f be a function that represents the weight of a fish at age t. Write a function that satisfies the following properties.
    - The weight of the fish at birth must be positive.
    - As the fish ages, the weight increases at decreasing rate.
    - No fish can grow bigger than 2 kg.

11. (4.1-34) The aerobic rate is the rate of a persons oxygen consumption and is sometimes modeled by the function A defined by
$$A(x) = 110\left(\frac{\ln(x) - 2}{x}\right)$$
for $x \geq 10$. Graph this function.