

# Midterm 1 practice 1

## UCLA: Math 32B, Spring 2020

*Instructor:* Noah White

*Date:*

*Version:* practice

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: \_\_\_\_\_

ID number: \_\_\_\_\_

Discussion section: \_\_\_\_\_

Question	Points	Score
1	9	
2	10	
3	12	
4	9	
Total:	40	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

<i>Part</i>	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				
(i)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (1 point) If  $\mathcal{R} = [0, 3] \times [-1, 4]$ , the integral  $\iint_{\mathcal{R}} 3 \, dA$  is equal to

- A. 12
- B. 0
- C. 15
- D. 45**

(b) (1 point) If  $\mathcal{R} = [0, 1] \times [-1, 1]$ , the integral  $\iint_{\mathcal{R}} y^3 \sin(x^2 y^2) \, dA$  is equal to

- A.  $\pi$
- B. 0**
- C.  $-1$
- D.  $3\pi^2$

(c) (1 point) If  $\mathcal{R} = [0, 3] \times [2, 4]$ , the integral  $\iint_{\mathcal{R}} x - y \, dA$  is equal to

- A.  $-9$**
- B. 4.5
- C.  $-4.5$
- D. 1

(d) (1 point) If  $\mathcal{B} = [0, 1] \times [0, 1] \times [-1, 1]$ , the integral  $\iiint_{\mathcal{B}} -1 \, dV$  is equal to

- A.  $-4$
- B.  $1$
- C.  $-2$**
- D.  $2$

(e) (1 point) If  $\mathcal{B} = [0, 1] \times [-1, 0] \times [0, 4]$ , the integral  $\iiint_{\mathcal{B}} ye^{y^2} \, dV$  is equal to

- A.  $e - 1$
- B.  $e^2 - 1$
- C.  $1 - e$
- D.  $2 - 2e$**

(f) (1 point) If  $\mathcal{E}$  is the region bounded by the curves  $y = 2x^2$  and  $y = 1 - x^2$ , then  $\mathcal{E}$  has the description

- A.  $-\sqrt{3} \leq x \leq \sqrt{3}, \quad 2x^2 \leq y \leq 1 - x^2$
- B.  $-1/\sqrt{3} \leq x \leq 1/\sqrt{3}, \quad 2x^2 \leq y \leq 1 - x^2$**
- C.  $-1/\sqrt{3} \leq x \leq 1/\sqrt{3}, \quad 0 \leq y \leq 1$
- D.  $-\sqrt{3} \leq x \leq \sqrt{3}, \quad 1 - x^2 \leq y \leq 2x^2$

(g) (1 point) If  $\mathcal{D}$  is the disc  $x^2 + y^2 \leq 4$ , then after changing to polar coordinates, the integral  $\iint_{\mathcal{D}} xy \, dA$  becomes

- A.  $\int_0^\pi \int_0^1 r \, dr \, d\theta$
- B.  $\int_0^{2\pi} \int_0^2 r^2 \sin 2\theta \, dr \, d\theta$
- C.  $\int_0^\pi \int_0^2 r^3 \sin 2\theta \, dr \, d\theta$
- D.**  $\int_0^{2\pi} \int_0^2 r^3 \sin \theta \cos \theta \, dr \, d\theta$

(h) (1 point) The integral of  $x^2 + y^2$  over the annulus  $4 \leq x^2 + y^2 \leq 16$  is

- A.  $6\pi$
- B.  $2\pi$
- C.  $\pi$
- D.**  $120\pi$

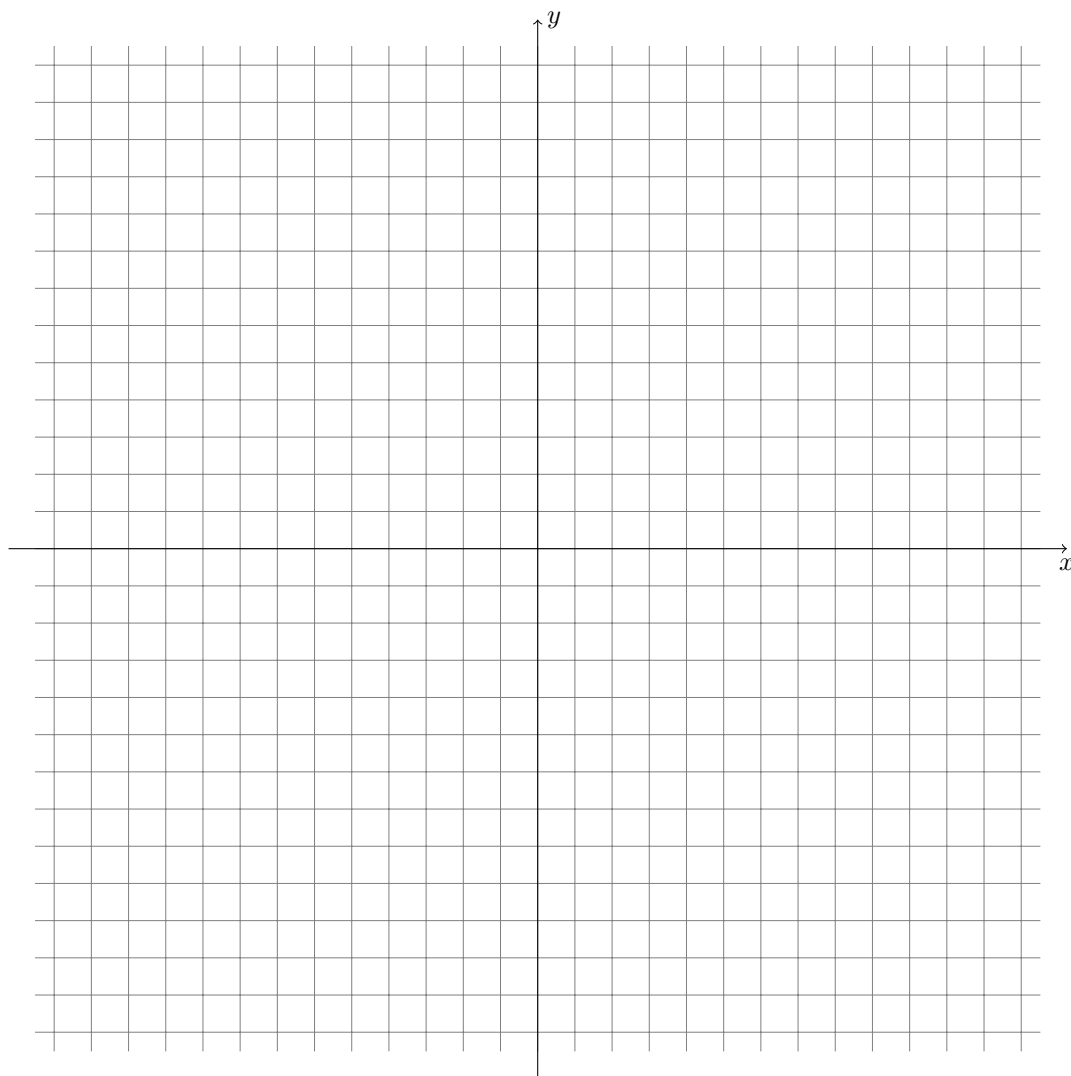
(i) (1 point) If  $\mathcal{D}$  is the region between the curves  $y = x^2$  and  $y = \sin(\frac{1}{2}\pi x)$  in the first quadrant then  $\mathcal{D}$  has the description

- A.  $0 \leq x \leq \pi, \sin(\frac{1}{2}\pi x) \leq y \leq x^2$
- B.  $0 \leq x \leq 1, \sin(\frac{1}{2}\pi x) \leq y \leq x^2$
- C.  $0 \leq x \leq \pi, 0 \leq y \leq \sin(\frac{1}{2}\pi x)$
- D.**  $0 \leq x \leq 1, x^2 \leq y \leq \sin(\frac{1}{2}\pi x)$

2. In this question we will consider the region  $\mathcal{D}$  which is the intersection of

- $x^2 + (y - 1)^2 \leq 4$ ,
- $x^2 + (y + 1)^2 \leq 4$ , and
- $x \geq 0$ .

(a) (2 points) Sketch the region  $\mathcal{D}$  on the graph provided.



(b) (2 points) Express  $\mathcal{D}$  as a vertically simple region, i.e. in the form  $a \leq x \leq b$ ,  $g_1(x) \leq y \leq g_2(x)$ .

**Solution:**  $0 \leq x \leq \sqrt{3}$ , and  $1 - \sqrt{4 - x^2} \leq y \leq -1 + \sqrt{4 - x^2}$

- (c) (2 points) Write the integral

$$\iint_{\mathcal{D}} x \, dA$$

as an iterated integral

**Solution:**

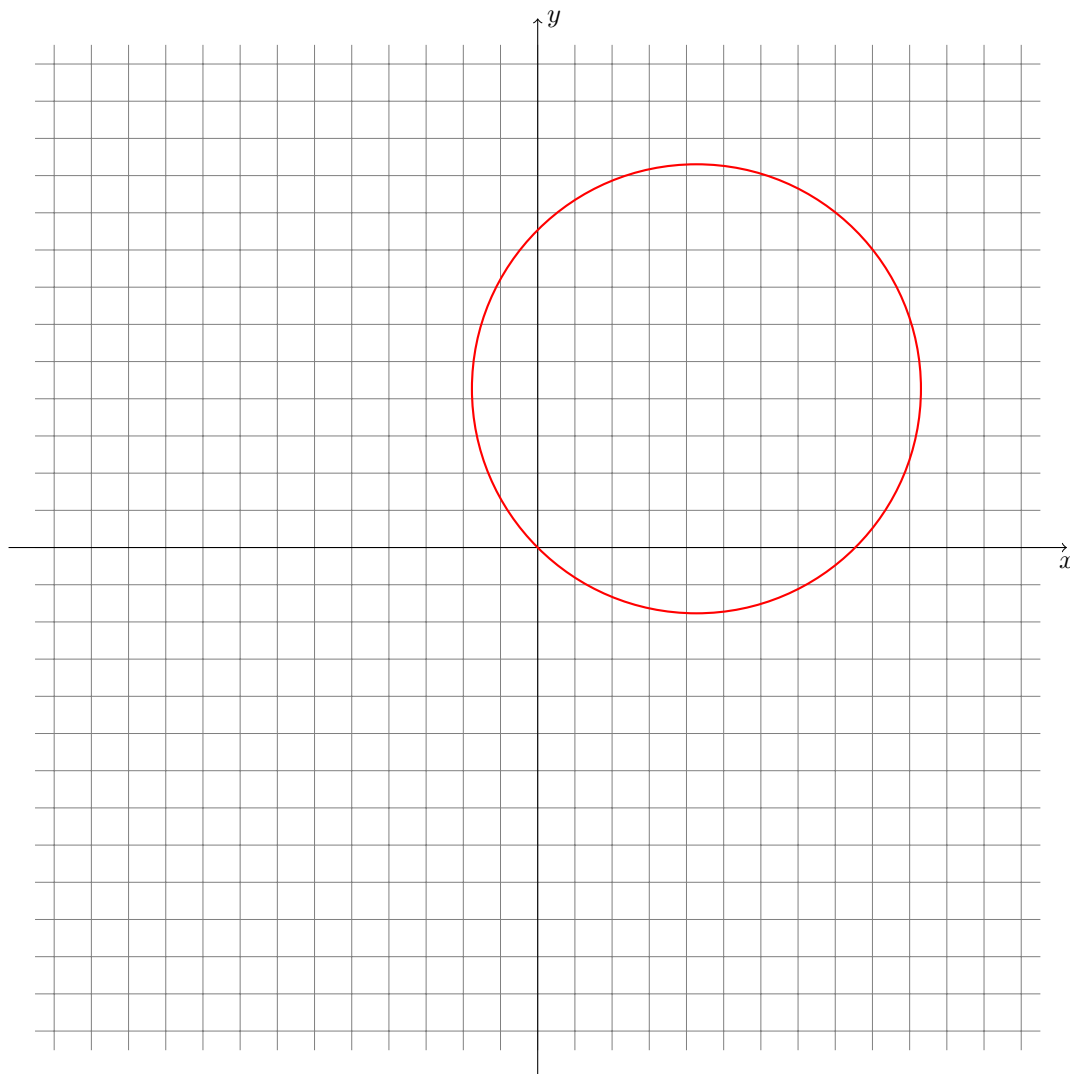
$$\int_0^{\sqrt{3}} \int_{1-\sqrt{4-x^2}}^{-1+\sqrt{4-x^2}} x \, dy \, dx$$

- (d) (4 points) Evaluate the integral in the previous part.

**Solution:**  $\frac{5}{3}$

3. In this question, consider the equation  $r = \cos \theta + \sin \theta$ .

(a) (4 points) Sketch the curve described by the above equation on the graph provided.





- (b) (4 points) Find the area bounded by this curve in the first quadrant.

**Solution:**

$$\frac{\pi}{4} + \frac{1}{2}$$

- (c) (4 points) Integrate the function  $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$  over the *entire* region (not just the first quadrant).

**Solution:**  $2\sqrt{2}$ . Note that the limits on your integral should be  $-\pi/4$  to  $3\pi/4$ , not 0 to  $2\pi$ . This is a little tricky.

4. Consider the region  $\mathcal{E}$  above the plane  $z = 4 - 2y$  and below the paraboloid  $z = 4 - x^2 - y^2$ .

(a) (4 points) Describe the region in the form

$$\mathcal{E} = \{ (x, y, z) \in \mathbb{R}^3 \mid (x, y) \in \mathcal{D}, z_1(x, y) \leq z \leq z_2(x, y) \}$$

for  $\mathcal{D}$  a region in the  $xy$ -plane. Your answer should specify what  $\mathcal{D}$  is.

**Solution:**  $\mathcal{E}$  is the region given by  $(x, y) \in \mathcal{D}$  and  $4 - 2y \leq z \leq 4 - x^2 - y^2$  where  $\mathcal{D}$  is the disk centred at  $(0, 1)$  with radius 1.

- (b) (5 points) Compute the volume of the region  $\mathcal{E}$ . *Hint: you might find the identity  $\int_0^\pi \sin^4 \theta \, d\theta = 3\pi/8$  useful*

**Solution:**

$$\frac{\pi}{2}$$

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