This week on the problem set we will review techniques for differentiating functions and evaluating limits. This should be familiar to you from Math 3A or other equivalent calculus courses you have taken. Make sure you can do these types of routine questions. Problem 7 is a challenge problem! Problems 8 to 12 will give you some practice graphing functions using the information provided by asymptotes and the first and second derivatives. Especially challenging questions are indicated with an asterisk, these may not be appropriate for exams but are good practice nonetheless.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (3.1) Differentiation practice. Differentiate the following functions.
   (a) (3.1-3a) $f(x) = 3x^5$

   Solution: $f'(x) = 15x^2$

   (b) (3.1-7) $f(x) = x^5 - 3x^2 - 1$
   (c) (3.1-9) $s(t) = 4e^t - 5t + 1$
   (d) (3.1-19) $H(w) = 2w - e^w$
   (e) (3.1-23) $f(x) = \frac{x^{1/3}}{x^2}$
   (f) (3.1-25) $h(t) = \frac{3^t + 3^{-t}}{2^t}$
2. (3.2) Product and quotient rule practice. Differentiate the following functions.
   (a) (3.2-2) $p(x) = (x^2 + 4)(1 - 3x^3)$
   (b) (3.2-4) $q(x) = \frac{x+1}{1+x^2}$
   (c) (3.2-7) $f(x) = (1 + x + x^2)e^x$
   (d) (3.2-14) $g(t) = \frac{1+te^t}{1+t}$
3. (3.3) Chain rule practice. Differentiate the following functions.
   (a) (3.3-9) $y = (5 - x + x^4)^9$
   (b) (3.3-12) $y = e^{(x+1)^7}$
   (c) (3.3-15) $y = \ln(2x + 5)$
   (d) (3.3-16) $y = xe^{-x^2}$
4. (3.4) Differentiate the following trig functions.
   (a) (3.4-1) $f(x) = \sin(x) + \cos(x)$
   (b) (3.4-7) $y = e^{-x}\sin(x)$
   (c) (3.4-16) $f(x) = \frac{\sin(x)}{1-\cos(x)}$
   (d) (3.4-20) $y = \ln(\sec(x) + \tan(x))$
5. (2.3 and 2.4) Evaluate the following limits.
   (a) (2.3-1) $\lim_{x\to 3} \frac{x^2+3x-10}{3x^2+5x-7}$
   (b) (2.3-12) $\lim_{s\to 1} s + \sin(\ln(s))$
   (c) (2.4-1) $\lim_{x\to -\infty} e^x$

(d) (2.4-4) $\lim_{x\to 2^-} \frac{1}{x-2}$

(e) (2.4-9) $\lim_{x\to 3^+} \frac{x^2-4x+3}{x^2-6x+9}$

(f) (2.4-15) $\lim_{x\to\infty} \frac{\sin(x)}{1+x}$

6. (3.7) L'Hôpital's rule practice. Evaluate the following limits.

   (a) (3.7-3) $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$

   (b) (3.7-6) $\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$

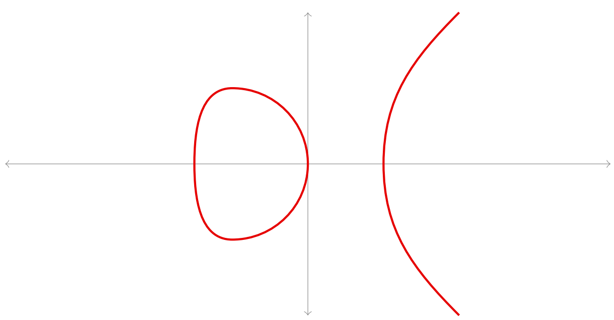
   (c) (3.7-7) $\lim_{x\to\infty} x^{-5}\ln(x)$

   (d) (3.7-10) $\lim_{x\to\infty} \frac{\ln(\ln(x))}{x}$

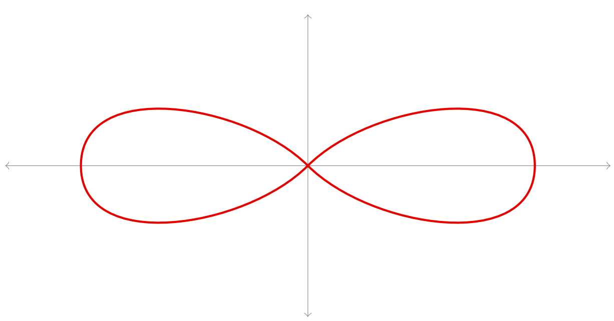
   (e) (3.7-12) $\lim_{x\to\infty} \left(1+\frac{1}{2x}\right)^{3x}$

   (f) (3.7-13) $\lim_{x\to\infty} (\ln(x))^{1/x}$

7. (*) The points (x, y) in the plane that satisfy the equation $y^2 = x^3 - x$ form a shape called an *elliptic curve* in the plane. Find the two points (x, y) at which the tangent is horizontal.



8. (**) The points (x, y) in the plane that satisfy the equation $(x^2 + y^2)^2 = 4(x^2 - y^2)$ make a ∞-shape. Find the four points (x, y) at which the tangent is horizontal.



9. (4.1) Graphing functions using calculus. Graph the following functions, if they involve a parameter a, graph the family of functions, demonstrating how they depend on the parameters.

   (a) (4.1-2) $y = x^2 + 5x - 3$

   (b) (4.1-6) $y = \frac{1}{x-1} + x$

   (c) (4.1-12) $y = 2e^x + e^{-x}$

   (d) (4.1-14) $y = \frac{x^2}{1+x^4}$

(e) (4.1-16) $y = \frac{ax}{x^2+1}$

(f) (4.1-20) $y = ax + \frac{1}{x}$

10. (4.1-24) Consider the graph of $y = \frac{e^{ax}}{1+e^{ax}}$. Use limits and first derivatives to determine how the shape of this curve depends on the parameter a.

11. (4.1-30) Two mathematicians, W. O. Kermack and A. G. McKendrick, showed that the weekly mortality rate during the outbreak of the Black Plague in Bombay in (1905 - 1906) can be reasonably well described by the function

$$f(t) = 890\,\mathrm{sech}^2(0.2t - 3.4) \quad \text{deaths/week}$$

where t is measured in weeks. Sketch this function using information about asymptotes and first derivatives. Recall that

$$\mathrm{sech}(x) = \frac{2}{e^x + e^{-x}}.$$

12. (4.1-33) Let f be a function that represents the weight of a fish at age t. Write a function that satisfies the following properties.

- The weight of the fish at birth must be positive.
- As the fish ages, the weight increases at decreasing rate.
- No fish can grow bigger than 2 kg.

13. (4.1-34) The aerobic rate is the rate of a person's oxygen consumption and is sometimes modeled by the function A defined by

$$A(x) = 110\left(\frac{\ln(x) - 2}{x}\right)$$

for $x \geq 10$. Graph this function.