Math 3B: Lecture 2

Noah White

October 1, 2018

Last time

Last time, we spoke about

- The syllabus

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- Problem sets, homework, and quizzes

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- Campuswire

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- Differentiation of common functions

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- Remember: I will be away until next Monday (Jens Eberhadt taking lectures)

Graphing using calculus: Why?

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- Understand functions qualitatively

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively
- Better understanding of derivatives

Building intuition

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The ingredients

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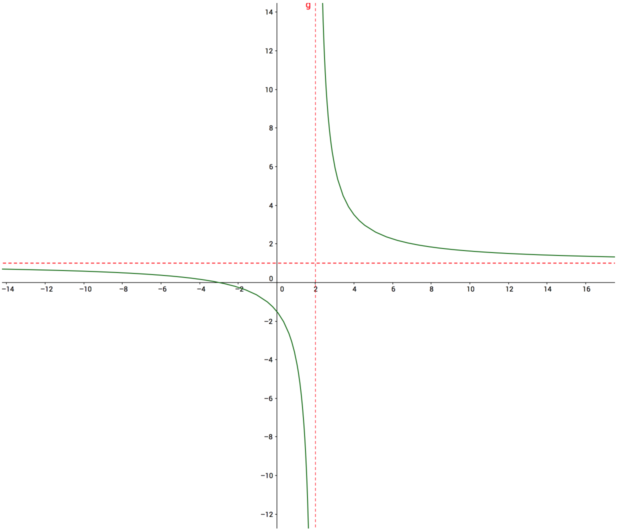
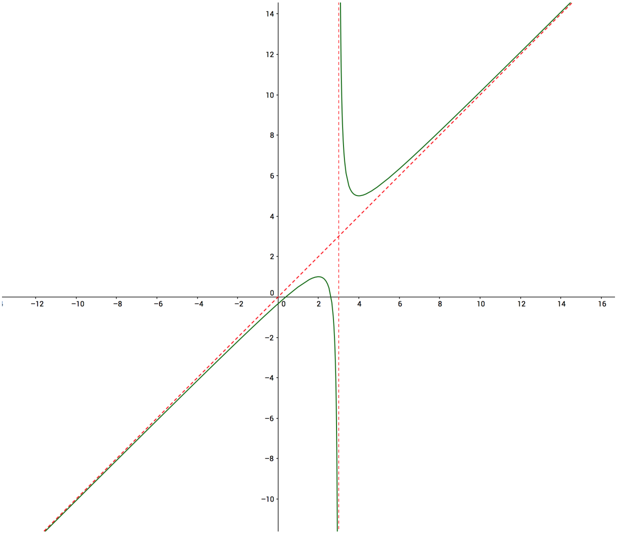
- The x and y intercepts
- Horizontal asymptotes
- Vertical asymptotes
- Slanted asymptotes
- The regions of increase/decrease of the first derivative
- The regions of increase/decrease of the second derivative

Asymptotes

An asmptote is a line which the function approches. Some examples:



Asymptotes



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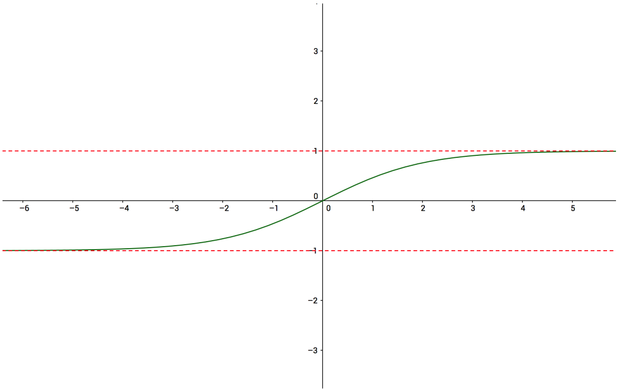

Finding horizontal asymptotes

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These are the easiest asymptotes to find. Suppose you have a function $f(x)$

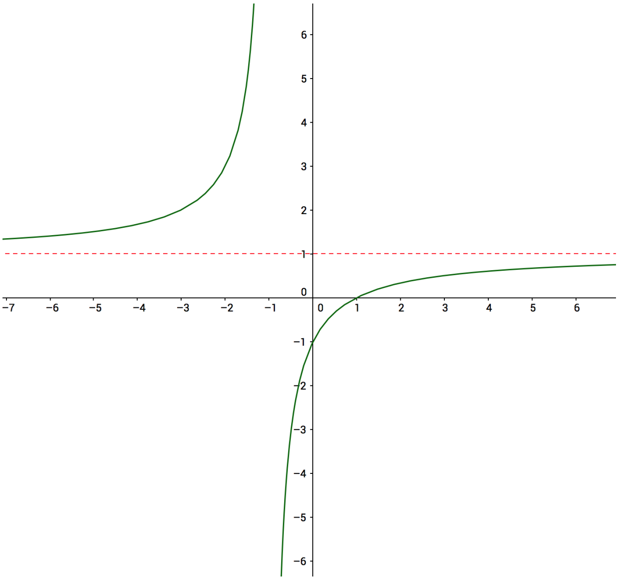
- Calculate $\lim_{x\to\infty} f(x)$
- Calculate $\lim_{x\to-\infty} f(x)$

Example

Say $f(x) = \frac{x-1}{x+1}$. In this case

$$\lim_{x\to\pm\infty} \frac{x-1}{x+1} = 1$$

Finding horizontal asymptotes



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Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

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$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

Example

Say $f(t) = \frac{t}{t+e^t}$. In this case

$$\lim_{t \to \infty} \frac{t}{t+e^t} = 0$$

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Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approches a finite number. I.e. Is there a real number a so that

$$\lim_{x \to a^+} f(x) = \pm\infty$$

or

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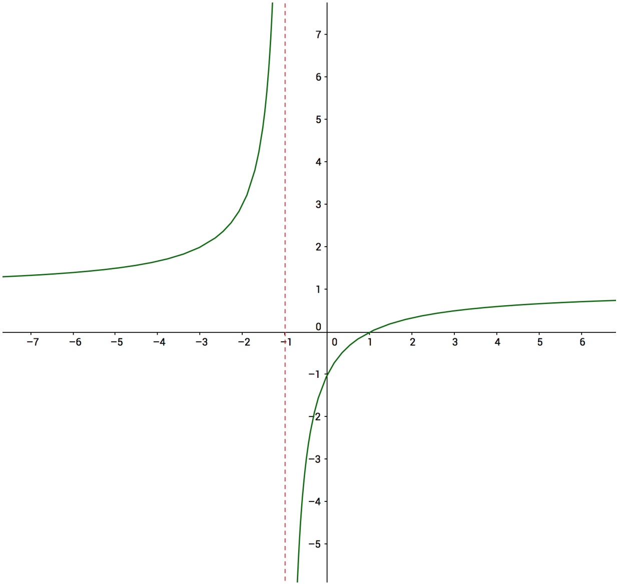
Example

$f(x) = \frac{x-1}{1+x}$, we have

$$\lim_{x \to -1^+} \frac{x-1}{1+x} = -\infty \quad \text{and} \quad \lim_{x \to -1^-} \frac{x-1}{1+x} = \infty$$

Finding verticle asymptotes

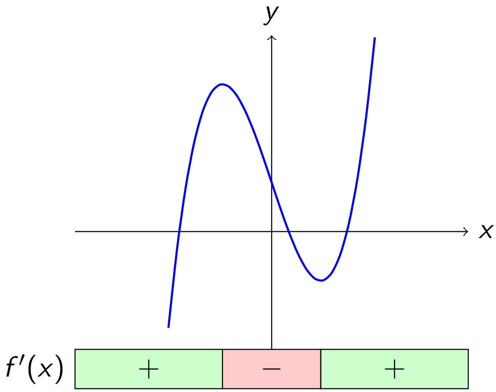


Finding slanted asymptotes

Lets come back to this...

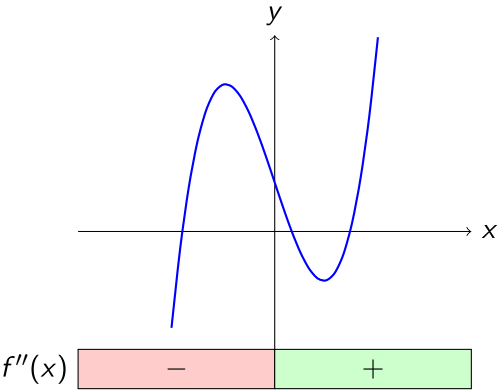
The first derivative

The first derivative tells us is the function going up or down?



The second derivative

The second derivative tells us is the function concave up or down?



Example time

... On the board.