This weeks problem set focuses on eigenvalues and eigenvectors of Matrices. A question marked with a † is difficult and probably too hard for an exam (though still illustrates a useful point). A question marked with a * is especially important.

1. From section 2.2, problems 4, 9.
2. From section 2.3, problems 12.
3. From section 2.4, problems 7, 16.
4. From section 2.5, problems 4, 8.
5. From section 5.1, problems 1, $2a, c, e$, $3a, c$, $4a, d, h$, 6, 7^*, 14^*, 15, 16, $22a$, 23.
6. From section 5.2, problems 1, $3a, d, e$, 8, 9, 10, 11, 18^*, 19, $20^\dagger$.
7. Let $\theta \in [0, 2\pi)$ and $R_\theta : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear map that takes a vector and rotates it by θ in the counterclockwise direction. Let E be the standard basis.
   (a) Find $[R_\theta]_E^E$.
   (b) Is R_θ diagonalisable?
   (c) Consider the linear map $R_\theta^{\mathbb{C}} : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ given by $R_\theta^{\mathbb{C}}(v) = [R_\theta]_E^E v$. Is this linear map diagonalisable?
8. Let U, W be subspaces of V such that $V = U \oplus W$. Let $\mathrm{pr}_U^W : V \longrightarrow V$ be the projection of V onto U along W. Is pr_U^W diagonalisable? *Hint: find a suitable basis for V.*
9. † Let V be a vector space and $E = \{ v_1, \ldots, v_n \}$ a collection of eigenvectors for a linear map $T : V \longrightarrow V$ such that the eigenvalues are all distinct. Prove that E is a linearly independent set. *Hint: use induction on n.*