This week you will get practice with slope fields.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

Homework: The homework will be due on Monday 4 March, at 8am, the *start* of the lecture. It will consist of questions

question 4 and question 5.

1. (6.4) Sketch the slope fields and a few solutions for the differential equations given
   (a) (6.4.12) $\frac{\mathrm{d}y}{\mathrm{d}t} = y(4-y)(y-2)$
   (b) (6.4.14) $\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 - y$
   (c) (6.4.16) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t^2 - 1$
   (d) (6.4.17) $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{y}{t}$

   Hint: feel free to use technology, just make sure you know how to draw a solution if you are given a slope field.

2. (6.4) Sketch the slope fields and the solution passing through the specified point for the differential equations given
   (a) (6.4.19) $\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 - y^2, (t, y) = (0, 0)$
   (b) (6.4.20) $\frac{\mathrm{d}y}{\mathrm{d}t} = 1.5y(1-y), (t, y) = (0, 0.1)$
   (c) (6.4.21) $\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{\frac{t}{y}}, (t, y) = (4, 1)$
   (d) (6.4.22) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2\sqrt{t}, (t, y) = (9, -1)$

3. (6.4.37) A population subject to seasonal fluctuations can be described by the logistic equation with an oscillating carrying capacity. Consider, for example,

$$\frac{dP}{dt} = P\left(1 - \frac{P}{100 + 50\sin 2\pi t}\right)$$

   Although it is difficult to solve this differential equation, it is easy to obtain a qualitative understanding.
   (a) Sketch a slope field over the region $0 \le t \le 5$ and $0 \le P \le 200$.
   (b) Sketch solutions that satisfy $P(0) = 0$, $P(0) = 10$, and $P(0) = 200$, use technology if you like.
   (c) Comment on the behaviour of the solutions.

4. (6.4.40) A population, in the absence of harvesting, exhibits the following growth

$$\frac{dN}{dt} = N\left(\frac{N}{100} - 1\right)\left(1 - \frac{N}{1000}\right)$$

   where N is abundance and t is time in years.
   (a) Write an equation that corresponds to harvesting the population at a rate of 0.5% per day.
   (b) Sketch the slope field for the differential equation you found in part a; by sketching solutions, describe how the fate of the population depends on its initial abundance.

5. A population of fish (measured in thousands of tons) is known to grow logistically with a carrying capacity of 100 and net birth rate 1. The population is discovered by some fishermen and they begin to harvest the population at a rate of 5 thousand tons per year. This quickly increases however, and the harvesting rate increases by 0.5 thousand tons every year. Let $y(t)$ be the size of the population (in thousands of tons) t years after harvesting begins.
   (a) Write a differential equation describing the population of fish.
   (b) What is the eventual fate of the population, and how does it depend on its initial abundance?