Midterm 1

UCLA: Math 170A, Fall 2017

Instructor: Noah White
Date: 23 October 2017
Version: 1

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ____________________

ID number: ____________________

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
Total:	20	

1. For this problem only: You do not need to show working.

   (a) (1 point) If events A and B satisfy $\mathbb{P}(A) = 0.5$ and $\mathbb{P}(A \cap B) = 0.2$, find the conditional probability $\mathbb{P}(B|A)$.

   Solution: 2/5

   (b) (1 point) What is $\mathbb{P}(\emptyset)$?

   Solution: 0

   (c) (1 point) If the probability of the event A is 0.2, the probability of the event B is 0.3 and the probability of their intersection is 0.1, what is the probability of the union of A and B?

   Solution: 0.4

   (d) (2 points) You pick two real numbers uniformaly at random between 0 and 3. Given that the smaller of your two numbers is greater than 1, what is the probabilty that your bigger number is less than 2? *Hint: think about areas.*

   Solution: 1/4

2. You have a box with 6 red balls, 6 blue balls and 6 yellow balls. Roll a fair, six sided dice. Let i be the resulting number. Then draw i balls uniformly at random from the box without replacement.

   (a) (1 point) What is the probability you draw at least two balls of the same color, given you roll a 1?

   Solution: 0

   (b) (1 point) What is the probability you draw at least two balls of the same color, given you roll a 4?

   Solution: 1

   (c) (3 points) What is the probability you draw at least two balls of the same color? *Feel free to leave your answer as a sum/product of fractions*

   Solution:
$$\mathbb{P}(\geq 2 \text{ same}) = \sum_{k=1}^{n} \mathbb{P}(k)\mathbb{P}(\geq 2 \text{ same}|k)$$
   Using the first two parts we get
$$\mathbb{P}(\geq 2 \text{ same}) = \frac{1}{6}\left(\mathbb{P}(\geq 2 \text{ same}|2) + \mathbb{P}(\geq 2 \text{ same}|3)\right) + \frac{3}{6}$$
   Now we can use $\mathbb{P}(\geq 2 \text{ same}|2) = 1 - \mathbb{P}(\text{all different}|2) = 1 - \frac{12}{17} = \frac{5}{17}$ and similarly
$$\mathbb{P}(\geq 2 \text{ same}|3) = 1 - \mathbb{P}(\text{all different}|3) = 1 - \frac{12 \cdot 6}{17 \cdot 16}$$
   so
$$\mathbb{P}(\geq 2 \text{ same}) = \frac{1}{6}\left(\frac{5}{17} + 1 - \frac{12 \cdot 6}{17 \cdot 16} + 3)\right)$$

3. (5 points) There are n different cars labeled with numbers 1 to n. These cars are parked in n consecutive parking spaces from left to right in a uniformly random order. What is the probability that the cars labeled 1 and 2 are parked next to each other (meaning that the car 2 is directly to the left or to the right of the car 1)? Simplify your answer.

Solution: Let A_1 be the event that the car 1 is parked to the left of the car 2, and A_2 the event that the car 1 is parked to the right of the car 2. These events are disjoint and we have to find the probability of their union. Let's find the probability of the first event.

There are $n!$ possibilities to park all the cars without any restrictions. How many of these permutations are those in which the car 1 is to the left of the car 2? Well first we can park the car 1 at any position except at the rightmost position, so there are $n-1$ ways to park it. Then the position of the car 2 is determined. Then we are left with $n-2$ cars to park in all other places, which can be done $(n-2)!$ ways. Therefore, there are in total $(n-1)\cdot(n-2)! = (n-1)!$ permutations of cars in which the car 1 is to the left of the car 2. Therefore,

$$\mathbf{P}(A_1) = \frac{(n-1)!}{n!} = \frac{1}{n}.$$

In the same way we get $\mathbf{P}(A_2) = \frac{1}{n}$ and so the wanted probability is $2/n$.

4. (5 points) It is known that 40% of all laptop issues are related screen problems, 30% to hard drive problems and 30% to other problems (we assume that the issue can't be related to more than cause - for example you can't have both screen and hard drive problem). Since the prices of repairs vary according to the problem, customers don't always send their laptop for repair. It is estimated that if a customer has screen problem (s)he sends the laptop to repair store with probability 0.3, if the problem is with the hard drive with probability 0.8, and for other problems with probability 0.4. If a laptop is sent to your repair store, what is the probability the issue is related to a screen problem. Simplify your answer.

Solution: Denote by B the event that a laptop is sent to repair and with A_1 the event that the laptop has a screen problem, with A_2 the event that the laptop has the hard drive problem and with A_3 the event that the laptop has another kind of problem. We are given that $\mathbb{P}(A_1) = 0.4$, $\mathbb{P}(A_2) = 0.3$ and $\mathbb{P}(A_3) = 0.3$. Moreover, we have $\mathbb{P}(B|A_1) = 0.3$, $\mathbb{P}(B|A_2) = 0.8$ and $\mathbb{P}(B|A_3) = 0.4$. We have to find $\mathbb{P}(A_1|B)$. We use Bayes' rule

$$\begin{aligned}\mathbb{P}(A_1|B) &= \frac{\mathbb{P}(A_1)\mathbb{P}(B|A_1)}{\mathbb{P}(A_1)\mathbb{P}(B|A_1) + \mathbb{P}(A_2)\mathbb{P}(B|A_2) + \mathbb{P}(A_3)\mathbb{P}(B|A_3)} \\ &= \frac{0.4 \cdot 0.3}{0.4 \cdot 0.3 + 0.3 \cdot 0.8 + 0.3 \cdot 0.4} \\ &= \frac{0.12}{0.12 + 0.24 + 0.12} \\ &= \frac{1}{4}.\end{aligned}$$

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