Math 3B: Lecture 20

Noah White

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Checking solutions

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$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= e^{\sin x} \cos x \\ y \cos x &= e^{\sin x} \cos x \end{aligned}$$

Reminder: Implicit differentiation

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$$x^2 + y^2 = 1$$

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2x + 2y'y &= 0
\end{aligned}$$

Another example

Lets differentiate

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Note

We can rearrange this to get

$$y' = \frac{3 - y}{x + \sin y}$$

a differential equation. Whatever y is, as long as it obeys the above relation, it is a solution to this ODE!

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4. solve for y!

Examples

On the board. . .