MATH 229B: LIE GROUPS AND LIE ALGEBRAS

NOAH WHITE
WINTER 2019

Class time and location: 4-5pm, MWF MS 5127 with extra problem sessions to be determined.

Course description: This course will cover the theory of semisimple Lie algebras over the complex numbers, their classification and their representation theory. We will end the course with a discussion of other objects fitting into the same classification pattern as semisimple Lie algebras.

The course will begin with some of the basic algebraic properties of Lie algebras, and we will see the correspondence to groups of rational matrices generated by reflections. This will allow us to give a very concrete classification of semisimple Lie algebras using these reflection groups, root systems and Dynkin diagrams. Next we will move on the representation theory of Lie algebras, starting with the finite dimensional representations and touching on some aspects of their infinite dimensional representations. In the final part of the course, we will attempt to demonstrate the ubiquity of this classification: in fact, many other mathematical objects are classified by the same objects. We will concentrate on the representation of quivers as the main example and time permitting, sketch some other examples such as the McKay correspondence.

References:
- W. Fulton and J. Harris, "Representation Theory",
- J. P. Serre, "Complex semi-simple Lie algebras",
- A. Kirillov, "Quiver Representations and Quiver Varieties",
- others as discussed in class.

Assessment: The course will be assessed by means of two projects. The first will be to maintain a carefully written set of explicit examples of the concepts discussed in class for a given Lie algebra. The second will be a final project, with a report and short presentation to be delivered in the final week of class. Topics for the final project will be advertised in the first week of class, and project assignments will be made in week 5, with a short abstract due in week 7. In one or both cases, there may be the opportunity to work in groups.