This week on the problem set you will get practice at calculating integrals using substitution and integration by parts.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

Homework: The second homework will be due on Monday 4 February, at 8am, the *start* of the lecture. It will consist of questions:

6 and 8

1. (5.3) Express the limits as definite integrals of the form $\int_0^1 f(x)\,\mathrm{d}x$.
   (a) (5.3.1) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2}$
   (b) (5.3.5) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{i^2}{n^2}\right) \frac{1}{n}$
   (c) (5.3.6) $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{\pi i}{n} - \pi\right) \frac{\pi}{n}$
2. (5.3) Express the definite integrals as limits of Riemann sums.
   (a) (5.3.8) $\int_{-1}^{1} (x^2 - x)\,\mathrm{d}x$
   (b) (5.3.9) $\int_0^1 e^x\,\mathrm{d}x$
   (c) (5.3.11) $\int_{-1}^{1} |x|\,\mathrm{d}x$
3. (5.5) Calculate the following integrals using substitution.
   (a) (5.5.12) $\int \frac{x}{\sqrt{x^2+1}}\,\mathrm{d}x$
   (b) (5.5.14) $\int \sin^3 t \cos t\,\mathrm{d}t$
   (c) (5.5.16) $\int \frac{z^3}{\sqrt{z^4+12}}\,\mathrm{d}z$
   (d) (5.5.19) $\int_1^2 \frac{e^{1/x}}{x^2}\,\mathrm{d}x$
   (e) (5.5.23) $\int_1^2 x\sqrt{x-1}\,\mathrm{d}x$
   (f) (5.5.24) $\int_0^2 (e^x - e^{-x})^2 dx$
4. (5.5-30) Suppose an environmental study indicates that the ozone level, L, in the air above a major metropolitan center is changing at a rate modeled by the function

$$L'(t) = \frac{0.24 - 0.03t}{\sqrt{36 + 16t - t^2}}$$

   parts per million per hour (ppm/h) t hours after 7:00 A.M.
   (a) Express the ozone level $L(t)$ as a function of t if L is 4 ppm at 7:00 A.M.
   (b) Find the time between 7:00 A.M. and 7:00 P.M. when the highest level of ozone occurs. What is the highest level? *(Note: part b has been changed slightly from what is written in the textbook.)*
5. The circle $x^2 + (y+1)^2 = 4$ has area 4π. What is the area of the portion of the circle lying above the x axis?

   You may use the fact that

$$\int \sqrt{1-t^2}\,\mathrm{d}t = \frac{1}{2}\left(t\sqrt{1-t^2} + \sin^{-1} t\right) + C.$$

6. Consider the ellipse $x^2 + 3(y+1)^2 = 4$. What is the area of the portion of the ellipse lying above the x axis?

7. (5.6) Calculate the following integrals using integration by parts.
   (a) (2) $\int e^t \sin t \, \mathrm{d}t$
   (b) (6) $\int x^2 \ln x \, \mathrm{d}x$
   (c) (9) $\int \sin x \cos x \, \mathrm{d}x$
   (d) (14) $\int_0^\pi x \sin x \, \mathrm{d}x$
   (e) (16) $\int_1^e x^3 \ln x \, \mathrm{d}x$

8. Use the fundamental theorem of calculus and the interpretation of the definite integral as an area to find a formula for the general antiderivative of the function $f(x) = \max\{0, x\}$.

9. Use the fundamental theorem of calculus and the interpretation of the definite integral as an area to find a formula for the general antiderivative of the function $f(x) = |x|$.

10. Use the fundamental theorem of calculus and the interpretation of the definite integral as an area to find a formula for the general antiderivative of the function $f(x) = \frac{1}{x}$.