Midterm 2 practice

UCLA: Math 115A, Winter 2018

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Date:
Version: practice

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
Total:	20	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be the linear map given by $T(a, b, c) = (a + b - c, b + c)$. Consider the bases $B = \{(1, 1, 1), (1, 0, -1), (1, 0, 1)\}$ and $C = \{(2, 1), (1, 2)\}$. The matrix $[T]_B^C$ is

   A. $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$

   B. $\frac{1}{5}\begin{pmatrix} 0 & -5 & 1 \\ 5 & 0 & 2 \end{pmatrix}$

   C. $\begin{pmatrix} 0 & -2 & 2 \\ 1 & -1 & 2 \end{pmatrix}$

   D. $\frac{1}{5}\begin{pmatrix} 0 & -2 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

   (b) (1 point) For an arbitrary linear combination $T : V \longrightarrow W$ and bases $B = \{v_1, \ldots, v_n\}, C = \{w_1, \ldots, w_m\}$ of V and W, the i^{th} column of the matrix $[T]_B^C$ is

   A. $T(v_i)$

   B. $[T(v_i)]_B$

   C. $[w_i]_C$

   D. $[T(v_i)]_C$

   (c) (1 point) Consider the linear map $T : \mathbb{R}_1[x] \longrightarrow \mathbb{R}_1[x]$ given by $T(a + bx) = (a + b) + (a - b)x$. Which of the following is a true statement.

   A. T has an eigenvalue of 2.

   B. T is diagonalizable.

   C. The only eigenvalue of T is -1.

   D. T has infinitely many eigenvalues.

(d) (1 point) If $\dim V = 6$, and W is a subspace such that $\dim W = 3$ then

A. $\dim V/W = 1$
B. $\dim V/W = 2$
C. $\dim V/W = 3$
D. $\dim V/W = 4$

(e) (1 point) The linear map $T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$ defined by $T(a + bx + cx^2) = (a + c) - (b - c)x + bx^2$ is invertible. What is $T^{-1}(1 - x + x^2)$?

A. $1 + x + x^2$
B. $2x - x^2$
C. $1 + x$
D. x^2

2. (a) (2 points) Define what it means for two vector space V and W to be isomorphic.

   (b) (3 points) Prove that, if V and W are finite dimensional, then V and W are isomorphic if and only if $\dim V = \dim W$.

3. Let $T : V \longrightarrow V$ be a linear transformation for a vector space V over $\mathbb{C}$. Define T^n to be the linear map obtained by repeatedly applying T, n times. E.g. $T^3(v) = T(T(T(v)))$.
   (a) (2 points) Suppose $T^n = 0$ for some n. Show that the only eigenvalue of T is zero.
   (b) (3 points) Prove that, if $T^2 = 0$ if and only if $\operatorname{im} T \subseteq \ker T$.

4. Let $T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$ be a linear map such that

$$[T]_B^B = \frac{1}{2}\begin{pmatrix} 2 & 6 & -2 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

where $B = \{1, (x-1)^2, (x+1)^2\}$

(a) (1 point) Is T an isomorphism? *Hint: This should be a very easy, and quick calculation. If you are spending more than 30 seconds on it, you are missing something important.*

(b) (2 points) Let $E = \{1, x, x^2\}$. Find the change of basis matrix $[\mathrm{id}]_E^B$.

(c) (2 points) Calculate $[T]_E^E$ and $T^6(x)$.

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