Math 3B: Lecture 2

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September 26, 2016

Last time

Last time, we spoke about

- The syllabus

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- Problem sets, homework, and quizzes

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- Differentiation of common functions

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- Product rule
- Chain rule

Graphing using calculus: Why?

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Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively
- Better understanding of derivatives

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- $P(t) = \frac{Mt}{t+e^t}$

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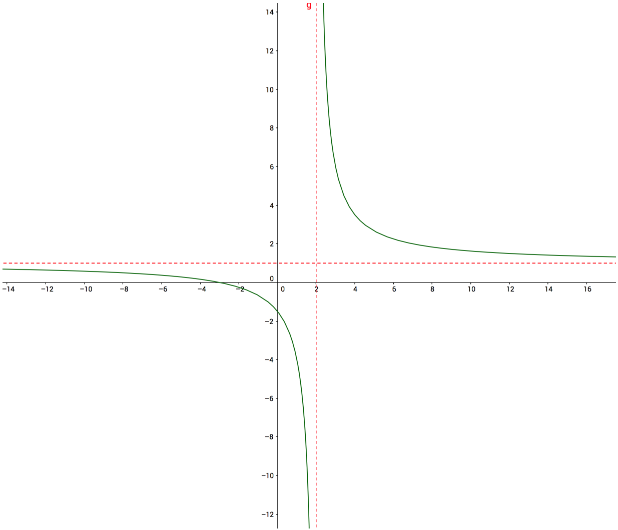
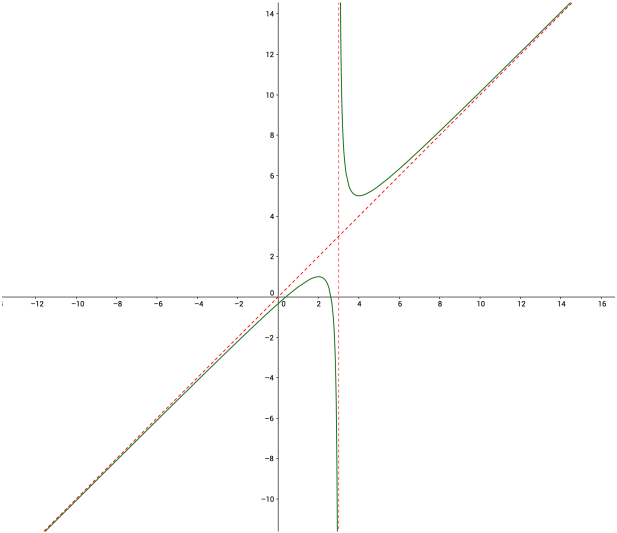
- The regions of increase/decrease of the first derivative
- The regions of increase/decrease of the second derivative

Asymptotes

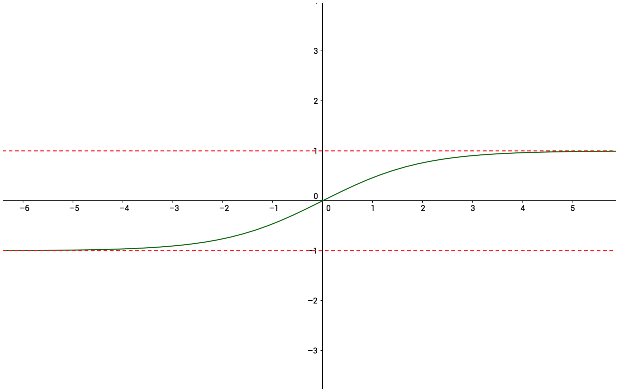
An asmptote is a line which the function approches. Some examples:



Asymptotes



Asymptotes



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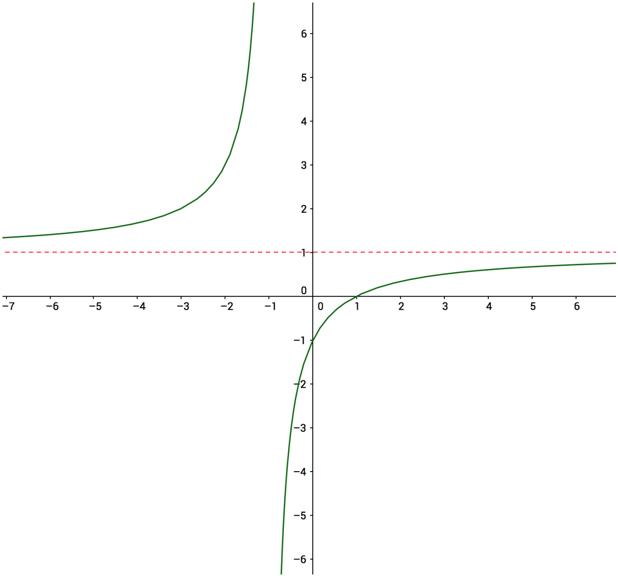
- Calculate $\lim_{x\to\infty} f(x)$
- Calculate $\lim_{x\to-\infty} f(x)$

Example

Say $f(x) = \frac{x-1}{1+x}$. In this case

$$\lim_{x\to\pm\infty} \frac{x-1}{x+1} = 1$$

Finding horizontal asymptotes



Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approches a finite number. I.e. Is there a real number a so that

$$\lim_{x \to a^+} f(x) = \pm\infty$$

or

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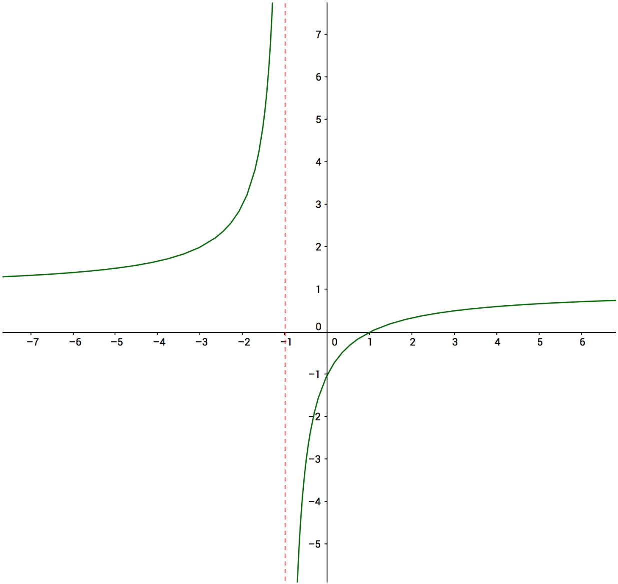
Example

$f(x) = \frac{x-1}{1+x}$, we have

$$\lim_{x\to -1^+} \frac{x-1}{1+x} = -\infty \quad \text{and} \quad \lim_{x\to -1^-} \frac{x-1}{1+x} = \infty$$

Finding verticle asymptotes

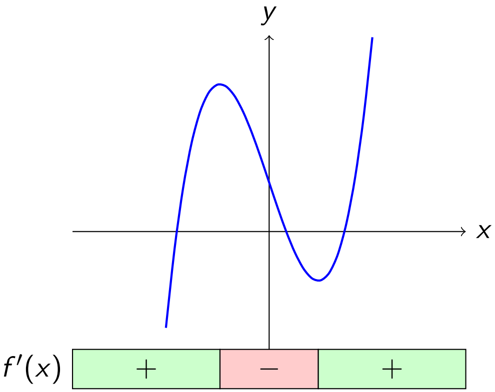


Finding slanted asymptotes

Lets come back to this. . .

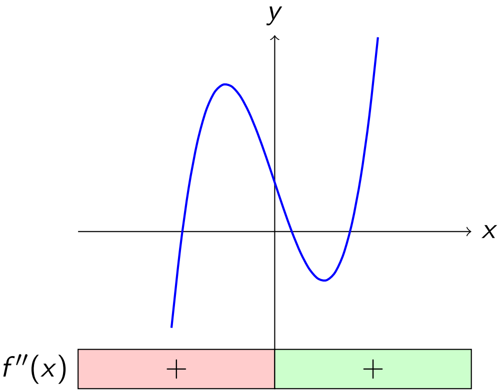
The first derivative

The first derivative tells us is the function going up or down?



The second derivative

The second derivative tells us is the function concave up or down?



Example time

... On the board.