This week on the problem set we concentrate on the concept of change of variables. Often the most difficult part of these problems will be the setup. Finding the correct change of variables is often difficult and there is no formulaic way to decide which one to use for a given integral. We also see spherical and cylindrical coordinates as a example of useful coordinate changes in three dimensions.

Homework: The second homework will be due Friday evening 25 October. It will consist of questions:

16.6.22, and 16.6.41

*Numbers in parentheses indicate the question has been taken from the textbook:

J. Rogawski, C. Adams, *Calculus, Multivariable*, 4^{th} Ed., W. H. Freeman & Company,

and refer to the section and question number in the textbook.

1. (Section 16.6) Questions 3, 4, 13, 15, 16, 18, 19, 21, 22, 31, 32, 34*, 38, 41 44*. (questions are identical in the 3^{rd} ed).
2. (Section 16.4) Questions 25, 28, 31, 37, 42, 43, 45, 48, 53, 57*. (questions are identical in the 3^{rd} ed).
3. (16.6.22) Let $G(u, v) = (u - uv, uv)$.
   (a) Show that the image of the horizontal line $v = c$ $y = \dfrac{c}{1-c}x$ if $c \neq 1$ and is the y-axis if $c = 1$.
   (b) Determine the images of the vertical lines in the uv-plane
   (c) Compute the Jacobian of G.
   (d) Observe that by the formula for the area of a triangle, the region $\mathcal{D}$ in the figure below has area $\frac{1}{2}(b^2 - a^2)$. Compute the area again, using the Change of Variables Formula applied to G.
   (e) Calculate $\displaystyle\iint_{\mathcal{D}} xy \, dx \, dy$.

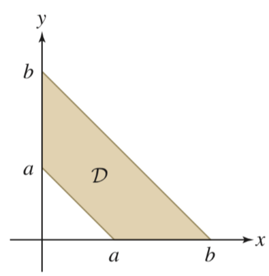

FIGURE 14

4. (16.6.41) Let $I = \displaystyle\iint_{\mathcal{D}} (x^2 - y^2) \, dx \, dy$, where

$$\mathcal{D} = \{ \, (x, y) \; : \; 2 \leq xy \leq 4, 0 \leq x - y \leq 3, x \geq 0, y \geq 0 \, \}$$

   (a) Show that the mapping $u = xy, v = x - y$ maps $\mathcal{D}$ to the rectangle $\mathcal{R} = [2, 4] \times [0, 3]$.
   (b) Compute $\partial(x, y)/\partial(u, v)$ (the Jacobian) by first computing $\partial(u, v)/\partial(x, y)$ (the Jacobian of the inverse mapping).
   (c) Use the change of variables formula to show that I is equal to the integral of $f(u, v) = v$ over $\mathcal{R}$ and evaluate.

*The questions marked with an asterisk are more difficult or are of a form that would not appear on an exam. Nonetheless they are worth thinking about as they often test understanding at a deeper conceptual level.