Midterm 2 practice 2

UCLA: Math 115A, Spring 2020

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Date:
Version: 1

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ____________________

ID number: ____________________

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
Total:	20	

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				

Clarification on notation: Let $T : V \longrightarrow W$ be a linear map. The *kernel* of T is the same thing as the *nullspace* of T, i.e. $\ker T = \mathsf{N}(T)$. Similarly the *image* of T is the same thing as the *range* of T, i.e. $\operatorname{im} T = \mathsf{R}(T)$.

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) If V is a finite dimensional vector space, with two bases, B and C then the matrix that changes B-coordinate vectors, into C-coordinate vectors is

   A. $[T]_C^B$
   B. $[T]_B^C$
   C. $[\mathrm{id}]_C^B$
   D. $[\mathrm{id}]_B^C$

   (b) (1 point) Let $E = \{1, x\}$, $C = \{x+2, x+1\}$ and $T : \mathbb{C}_1[x] \longrightarrow \mathbb{C}_1[x]$ be a linear map such that

$$[T]_E^E = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

   Then, $[T]_C^C$ is equal to

   A. $\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$

   B. $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

   C. $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

   D. $\begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix}$

(c) (1 point) Suppose $S : U \longrightarrow V$ and $T : V \longrightarrow W$ are linear maps between vector spaces, such that $T \circ S$ is the zero map (i.e. $T(S(u)) = 0$ for all $u \in U$). Which of the following is true?

A. $\ker S \subseteq \operatorname{im} T$

B. $\operatorname{im} S \subseteq \ker T$

C. $\ker T \subseteq \operatorname{im} S$

D. $\operatorname{im} T \subseteq \ker S$

(d) (1 point) Suppose $T : V \longrightarrow V$ is a linear map and λ is an eigenvalue of T. Which of the following is true?

A. λ^{-1} is also an eigenvalue of T.

B. λ is an eigenvalue of T^n for some $n \geq 1$.

C. λ^n is an eigenvalue of T^n for every $n \geq 1$.

D. λ is not an eigenvalue of T^n for any $n \geq 2$.

(e) (1 point) Let V be a vector space. What is the dimension of $\mathcal{L}(\{0\}, V)$?

A. 0

B. 1

C. $\dim V - 1$

D. $\dim V$

2. Let $T : V \longrightarrow W$ be a linear map between vector spaces.

   (a) (2 points) Define the *rank* and *nullity* of T.

   (b) (3 points) Prove the *rank-nullity theorem:* If V is finite dimensional, then $\dim V = \dim \ker T + \dim \operatorname{im} T$.

3. Consider the linear map $T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$ given by

$$T(a + bx + cx) = (a + 2b + 2c) + (b + 3c)x + 2cx^2.$$

(a) (2 points) Find the characteristic polynomial and eigenvalues of T.

(b) (2 points) For each eigenvalue, determine an eigenvector of T.

(c) (1 point) Is T diagonalisable?

4. Let V and W be vector spaces, and let $S : V \longrightarrow W$ and $T : W \longrightarrow V$ be linear maps.
   (a) (1 point) Prove that if $T \circ S$ is injective (one-to-one) then S is injective.

   (b) (2 points) Prove that if $T \circ S$ is surjective (onto) then T is surjective.

   (c) (2 points) Give an example of spaces V, W and linear maps S, T such that $T \circ S = \mathrm{id}_V$ but $S \circ T \neq \mathrm{id}_W$. *Hint: you shouldn't need to do anything complicated, there is a very simple example.*

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