

This problem set concentrates on practice with antiderivatives. You will get lots of practice finding simple antiderivatives as well as finding antiderivatives graphically using slope fields. This is what we will cover before the quiz so concentrate on these questions when preparing. You will also get some practice with problems involving accumulated change (i.e. the area under curves) and definite integrals.

1. Find the antiderivatives of the following functions

- (a) $f(x) = 2x + 4$
- (b) $p(z) = 5 - 4z$
- (c) $q(x) = x^2 + 1$
- (d) $f(t) = t^3 - 3t^2 + 2$
- (e) $f(x) = x - 3\cos x$
- (f) $q(c) = \sin c - \cos c$
- (g) $g(u) = e^u - \frac{1}{u}$ (for $x > 0$)
- (h) $f(x) = \frac{1}{2x-4}$ (for $x > 2$)

2. Find the antiderivatives of the following functions. These are trickier and may require some reversing of the chain rule.

- (a) $f(x) = 2xe^{x^2}$
- (b) $g(x) = 3x^2 \sin(x^3)$
- (c) $f(t) = \frac{t}{t^2+1}$
- (d) $p(u) = \sin(-u)$
- (e) $f(x) = \sin x (\cos x)^3$
- (f) $h(z) = z^2(z^3 - 1)^{\frac{13}{21}}$
- (g) $f(x) = x \sin(x^2 - x) - \frac{1}{2} \sin(x^2 - x)$
- (h) $f(x) = \frac{x}{\sqrt{x^2+1}}$

3. Sketch the slope field for the following differential equations and thus sketch the antiderivative with the given initial value.

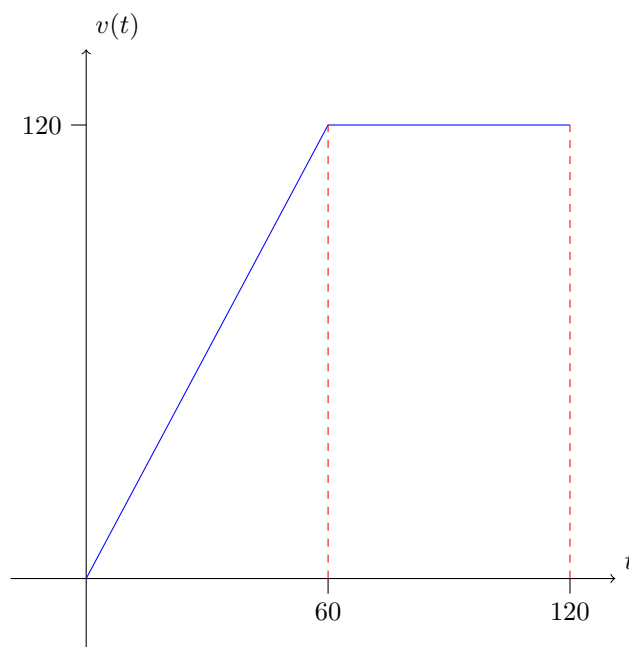
- (a) $\frac{df}{dx} = e^{x^2}$ where $f(0) = 0$
- (b) $\frac{df}{dx} = \frac{e^x}{x}$ where $f(-1) = 1$
- (c) $\frac{df}{dx} = \frac{x}{1+e^x}$ where $f(0) = 1$
- (d) $\frac{df}{dx} = \sqrt{2 + \sin x}$ where $f(2) = 1$
- (e) $\frac{df}{dx} = \sqrt{x} \sin x$ where $f(1) = 0$

4. A rocket starts from a stationary position and accelerates at a rate of $2m/s^2$ for 60 seconds and then travels at a constant speed (i.e. its acceleration is zero) for another 60 seconds. How far has the rocket travelled (assuming it is always moving in a straight line).

Solution: The velocity of the rocket is given by

$$v(t) = \begin{cases} 2t & \text{if } 0 \leq t \leq 60, \\ 120 & \text{if } 60 \leq t \leq 120. \end{cases}$$

The graph of the function is



We are asked to find the distance the rocket has travelled which is the accumulated change in the velocity of the 120 seconds the rocket has been airborne. Thus we need to find the area under the graph. We have split this into a triangle and a rectangle, for which the areas are 3600 and 7200 respectively. Thus the total distance travelled is 10800 meters.

5. Rainwater fills a tank at a rate of 4 gallons per hour. If it rains for 6 hours how much rain does the tank collect?
6. A hydroelectric dam produces 2 Megawatts of power for every million litres of water which flows through it. Over a period of 4 hours, you measure $r(t)$ million litres per second flowing through the turbines. If

$$r(t) = \sqrt{4 - (t - 2)^2} + 5,$$

how many Megawatts were produced in those 4 hours?

7. A car starts at a stationary position and accelerates at $1.5m/s^2$ for 20 seconds and then decelerates at $0.5m/s^2$ until coming to a standstill. How far did the car travel?
8. Rain adds 5 litres of water per minute to your rain tank. You check the rain tank and find that it currently has 200 litres of water in it. After 20 minutes you notice that the tank has had a hole in the bottom this whole time! Water is able to flow out of the hole at a rate of 10 litres per minute.

You immediately begin repairing the hole. During this time, water is able to drain out at a rate of $10 - \frac{1}{3}t$ litres per minute, where t is the time elapsed since you begun the repair. It takes you 30 minutes.

How much water is in the tank, 1 hour after you checked the level of the tank?

9. Evaluate the definite integrals (hint: it might be best to do questions 1 and 2 first)

(a) $\int_1^2 2x + 4 \, dx$

- (b) $\int_0^{10} 5 - 4z \, dz$
- (c) $\int_{-1}^1 x^2 + 1 \, dx$
- (d) $\int_{-1}^0 t^3 - 3t^2 + 2 \, dt$
- (e) $\int_{-\pi}^{2\pi} x - 3 \cos x \, dx$
- (f) $\int_0^\pi \sin c - \cos c \, dc$
- (g) $\int_1^3 e^u - \frac{1}{u} \, du$
- (h) $\int_{-1}^1 \frac{1}{2x-4} \, dx$
- (i) $\int_0^2 2xe^{x^2} \, dx$
- (j) $\int_0^{\sqrt[3]{\pi}} 3x^2 \sin(x^3) \, dx$
- (k) $\int_{-1}^1 \frac{t}{t^2+1} \, dt$
- (l) $\int_{-\pi}^0 \sin(-u) \, du$
- (m) $\int_{\pi/2}^\pi \sin x (\cos x)^3 \, dx$
- (n) $\int_1^2 z^2(z^3-1)^{\frac{13}{21}} \, dz$
- (o) $\int_4^5 x \sin(x^2-x) - \frac{1}{2} \sin(x^2-x) \, dx$
- (p) $\int_0^1 \frac{x}{\sqrt{x^2+1}} \, dx$