This week on the problem set you will get practice at integration by parts, polynomial long division, using the partial fractions method and applying these to integrals. There are lots and lots of questions! You don't need to do all of them, only enough to convince yourself you are comfortable with them!

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (5.6) Calculate the following integrals using integration by parts.
   (a) (2) $\int e^t \sin t \, dt$
   (b) (6) $\int x^2 \ln x \, dx$
   (c) (9) $\int \sin x \cos x \, dx$
   (d) (14) $\int_0^\pi x \sin x \, dx$
   (e) (16) $\int_1^e x^3 \ln x \, dx$

2. Use any method to evaluate the following integrals.
   (a) $\int x\sqrt{x+1} \, dx$
   (b) $\int_1^2 \frac{t}{(t^2+1)\ln(t^2+1)} \, dt$
   (c) $\int \frac{\ln x}{x^5} \, dx$
   (d) $\int \ln x \, dx$
   (e) $\int (\ln x)^2 \, dx$
   (f) $\int_1^e (\ln x)^3 \, dx$
   (g) $\int e^{6x} \sin(e^{3x}) \, dx$
   (h) $\int_0^1 \frac{t^3 e^{t^2}}{(t^2+1)^2} \, dt$
   (i) $\int \frac{2x+3}{z^2-9} \, dz$
   (j) $\int \frac{x^2+x-1}{(x^2-1)} \, dx$
   (k) $\int \frac{e^x}{(e^x-1)(e^x+3)} \, dx$
   (l) $\int_0^{\pi/3} e^t \sin t \, dt$
   (m) $\int e^{\sqrt{x}} \, dx$
   (n) $\int \frac{1}{\cos x} \, dx$ (quite challenging)
   (o) $\int (\sin x)^2 \, dx$ (quite challenging)

3. Divide $p(x)$ by $q(x)$ and express the quotient as a divisor plus a remainder.
   (a) $p(x) = 2x^3 + 4x^2 - 5$, $q(x) = x + 3$
   (b) $p(x) = 15x^4 - 3x^2 - 6x$, $q(x) = 3x + 6$
   (c) $p(x) = 2x^4 - 5x^3 + 6x^2 + 3x - 2$, $q(x) = x - 2$
   (d) $p(x) = 5x^4 + 2x^3 + x^2 - 3x + 1$, $q(x) = x + 2$
   (e) $p(x) = x^6$, $q(x) = x - 1$
   (f) $p(x) = x^3 - 5x^2 + x - 15$, $q(x) = x^2 - 1$
   (g) $p(x) = x^3 - 2x^2 - 5x + 7$, $q(x) = x^2 + x - 6$
   (h) $p(x) = x^3 + 3x^2 - 6x - 7$, $q(x) = x^2 + 2x - 8$

(i) $p(x) = 2x^3 - 8x^2 + 8x - 4$, $q(x) = 2x^2 - 4x + 2$

(j) $p(x) = 3x^4 - x^3 - 2x^2 + 5x - 1$, $q(x) = x + 1$

(k) $p(x) = 4x^5 + 7x^4 - 9x^3 + 2x^2 - x + 3$, $q(x) = x^2 - 4x + 3$

(l) $p(x) = 4x^5 + 7x^4 - 9x^3 + 2x^2 - x + 3$, $q(x) = x^3 + x^2 - 5x + 3$

(m) Make up your own! Pick random polynomials and divide!

4. Use the method of partial fractions to break up these rational functions.

   (a) $\frac{2}{(x-2)x}$

   (b) $\frac{5}{(x-2)(x+3)}$

   (c) $\frac{7}{(x+6)(x-1)}$

   (d) $\frac{5x}{(x-1)(x+4)}$

   (e) $\frac{x}{(x+1)(x+2)}$

   (f) $\frac{12x-6}{(x-3)(x+3)}$

   (g) $\frac{x-1}{(x+2)(x+1)}$

   (h) $\frac{1}{x^2-x-6}$

   (i) $\frac{11}{x^2-3x-28}$

   (j) $\frac{10}{x^2+2x-24}$

   (k) $\frac{4x}{x^2+6x+5}$

   (l) $\frac{3x}{x^2-7x+10}$

   (m) $\frac{1}{x^3-2x^2-5x+6}$

   (n) $\frac{4x^2-x}{x^3-4x^2-x+4}$

5. Use the method of partial fractions to break up these rational functions.

   (a) $\frac{x}{(x+1)^2}$

   (b) $\frac{2x-1}{(x+3)^2}$

   (c) $\frac{1-3x}{(x-1)^2}$

   (d) $\frac{1+3x}{(x-2)^2}$

   (e) $\frac{2x^2}{(x-1)^3}$

   (f) $\frac{x-1}{(x-2)^3}$

   (g) $\frac{x-3}{(x+2)^2(x-2)}$

   (h) $\frac{x}{(x-1)(x+3)^2}$

6. Integrate the functions in question 4 and question 5.

7. Calculate $\int \frac{p(x)}{q(x)} \, dx$ for each part of question 3.