This problem set concentrates on practice with antiderivatives. You will get lots of practice finding simple antiderivatives as well as finding antiderivatives graphically using slope fields. You will also get some practice with problems involving accumulated change (i.e. the area under curves) and definite integrals.

1. (4.3-33) The Statue of Liberty is 92 meters high, including the 46 meter pedestal upon which it stands. How far from the base should an individual stand to ensure that the view angle, θ, is maximized?

   (Hint: See the textbook for a picture.)

2. (4.3-35) An oil spill has fouled 200 miles of Pacific shoreline. The oil company responsible has been given fourteen days to clean up the shoreline, after which a fine will be levied in the amount of $\$10,000$/day. The local cleanup crew can scrub five miles of beach per day at a cost of \$500/day. Additional crews can be brought in at a cost of $\$18,000$, plus \$800/day for each crew. Determine how many additional crews should be brought in to minimize the total cost to the company and how much the cleanup will cost.

3. (4.3-33) Consider a cylindrical cell with radius r and height $r/2$. Assume that the cell gains energy at a rate proportional to its surface area (i.e., nutrients diffusing in from outside of the cell) and that the cell loses energy at a rate proportional to its volume (i.e., all parts of the cell are using energy). If the cell is trying to maximize its net gain of energy, determine the optimal value of r. Note: Your final expression will depend on your proportionality constants.

4. (5.1) Find the general antiderivative of the functions
   (a) (5.1-2) $f(x) = 4$
   (b) (5.1-8) $f(t) = 4t + 4t^2$
   (c) (5.2-10) $f(x) = \frac{1}{2x}$
   (d) (5.1-14) $f(x) = 4\sin(5x)$
   (e) (5.1-16) $f(x) = 14e^x$
   (f) (5.1-22) $f(u) = 6u + 3\cos u$

5. Find the antiderivatives of the following functions
   (a) $f(x) = 2x + 4$
   (b) $p(z) = 5 - 4z$
   (c) $q(x) = x^2 + 1$
   (d) $f(t) = t^3 - 3t^2 + 2$
   (e) $f(x) = x - 3\cos x$
   (f) $q(c) = \sin c - \cos c$
   (g) $g(u) = e^u - \frac{1}{u}$ (for $x > 0$)
   (h) $f(x) = \frac{1}{2x-4}$ (for $x > 2$)

6. Find the antiderivatives of the following functions. These are trickier and may require some reversing of the chain rule.
   (a) $f(x) = 2xe^{x^2}$
   (b) $g(x) = 3x^2\sin(x^3)$
   (c) $f(t) = \frac{t}{t^2+1}$
   (d) $p(u) = \sin(-u)$
   (e) $f(x) = \sin x\,(\cos x)^3$
   (f) $h(z) = z^2(z^3 - 1)^{\frac{13}{21}}$

(g) $f(x) = x\sin(x^2 - x) - \frac{1}{2}\sin(x^2 - x)$

(h) $f(x) = \frac{x}{\sqrt{x^2+1}}$

7. Sketch the slope field for the following differential equations and thus sketch the antiderivative with the given initial value.
   (a) $\frac{\mathrm{d}f}{\mathrm{d}x} = e^{x^2}$ where $f(0) = 0$
   (b) $\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{e^x}{x}$ where $f(-1) = 1$
   (c) $\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{x}{1+e^x}$ where $f(0) = 1$
   (d) $\frac{\mathrm{d}f}{\mathrm{d}x} = \sqrt{2 + \sin x}$ where $f(2) = 1$
   (e) $\frac{\mathrm{d}f}{\mathrm{d}x} = \sqrt{x}\sin x$ where $f(1) = 0$

8. A rocket starts from a stationary position and accelerates at a rate of $2m/s^2$ for 60 seconds and then travels at a constant speed (i.e. its acceleration is zero) for another 60 seconds. How far has the rocket travelled (assuming it is always moving in a straight line).

9. Rainwater fills a tank at a rate of 4 gallons per hour. If it rains for 6 hours how much rain does the tank collect?

10. A hydroelectric dam produces 2 Megawatts of power for every million litres of water which flows through it. Over a period of 4 hours, you measure $r(t)$ million litres per hour flowing through the turbines. If

$$r(t) = \sqrt{4 - (x-2)^2} + 5,$$

how many Megawatts were produced in those 4 hours?

11. A car starts at a stationary position and accelerates at $1.5m/s^2$ for 20 seconds and then decelerates at $0.5m/s^2$ until coming to a standstill. How far did the car travel?

12. Rain adds 5 litres of water per minute to your rain tank. You check the rain tank and find that it currently has 200 litres of water in it. After 20 minutes you notice that the tank has had a hole in the bottom this whole time! Water is able to flow out of the hole at a rate of 10 litres per minute.

    You immediately begin repairing the hole. During this time, water is able to drain out at a rate of $10 - \frac{1}{3}t$ litres per minute, where t is the time elapsed since you begun the repair. It takes you 30 minutes.

    How much water is in the tank, 1 hour after you checked the level of the tank?

13. Evaluate the definite integrals (hint: it might be best to do questions 1 and 2 first)
    (a) $\displaystyle\int_1^2 2x + 4 \,\mathrm{d}x$
    (b) $\displaystyle\int_0^{10} 5 - 4z \,\mathrm{d}z$
    (c) $\displaystyle\int_{-1}^{1} x^2 + 1 \,\mathrm{d}x$
    (d) $\displaystyle\int_{-1}^{0} t^3 - 3t^2 + 2 \,\mathrm{d}t$
    (e) $\displaystyle\int_{-\pi}^{2\pi} x - 3\cos x \,\mathrm{d}x$
    (f) $\displaystyle\int_0^{\pi} \sin c - \cos c \,\mathrm{d}c$

(g) $\displaystyle\int_1^3 e^u - \frac{1}{u}\,\mathrm{d}u$

(h) $\displaystyle\int_{-1}^1 \frac{1}{2x-4}\,\mathrm{d}x$

(i) $\displaystyle\int_0^2 2xe^{x^2}\,\mathrm{d}x$

(j) $\displaystyle\int_0^{\sqrt[3]{\pi}} 3x^2\sin(x^3)\,\mathrm{d}x$

(k) $\displaystyle\int_{-1}^1 \frac{t}{t^2+1}\,\mathrm{d}t$

(l) $\displaystyle\int_{-\pi}^0 \sin(-u)\,\mathrm{d}u$

(m) $\displaystyle\int_{\pi/2}^{\pi} \sin x\,(\cos x)^3\,\mathrm{d}x$

(n) $\displaystyle\int_1^2 z^2(z^3-1)^{\frac{13}{21}}\,\mathrm{d}z$

(o) $\displaystyle\int_4^5 x\sin(x^2-x) - \frac{1}{2}\sin(x^2-x)\,\mathrm{d}x$

(p) $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+1}}\,\mathrm{d}x$