

Math 3B: Lecture 2

Noah White

January 9, 2019

Last time

Last time, we spoke about

- The syllabus

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- Problem sets, homework, and quizzes

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- Campuswire

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- Chain rule

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

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- Understand functions qualitatively

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively
- Better understanding of derivatives

Building intuition

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3. $P(t) = \frac{t}{t+e^t}$

The ingredients

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- Vertical asymptotes
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- The regions of increase/decrease of the first derivative

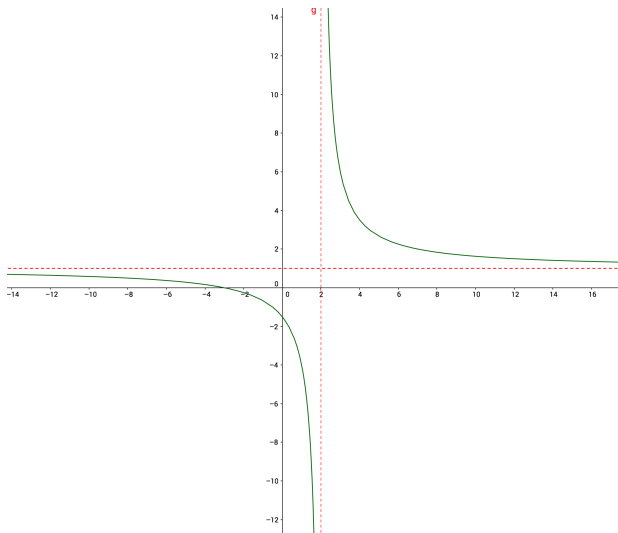
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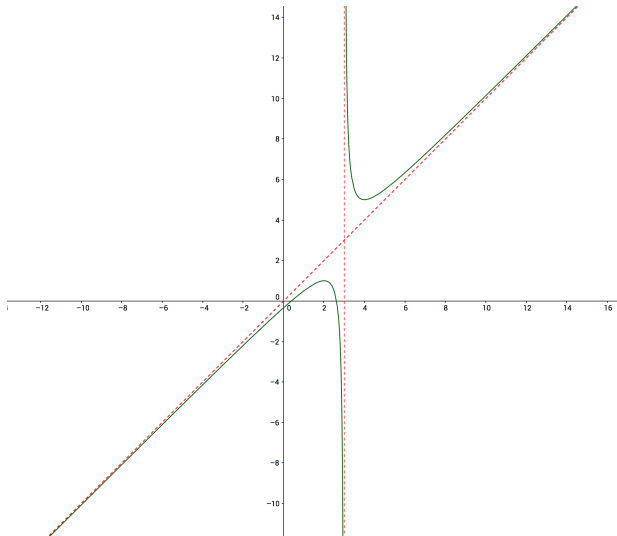
- The x and y intercepts
- Horizontal asymptotes
- Vertical asymptotes
- Slanted asymptotes
- The regions of increase/decrease of the first derivative
- The regions of increase/decrease of the second derivative

Asymptotes

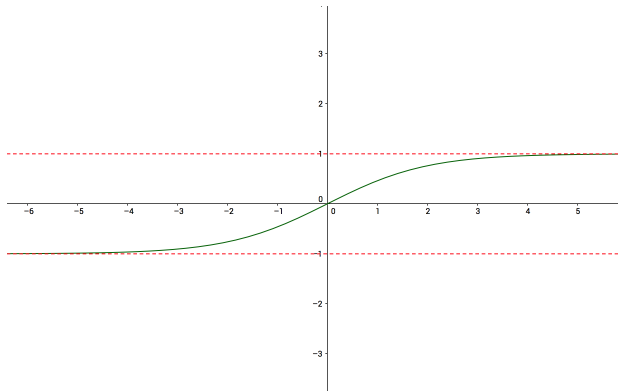
An **asymptote** is a line which the function approaches. Some examples:



Asymptotes



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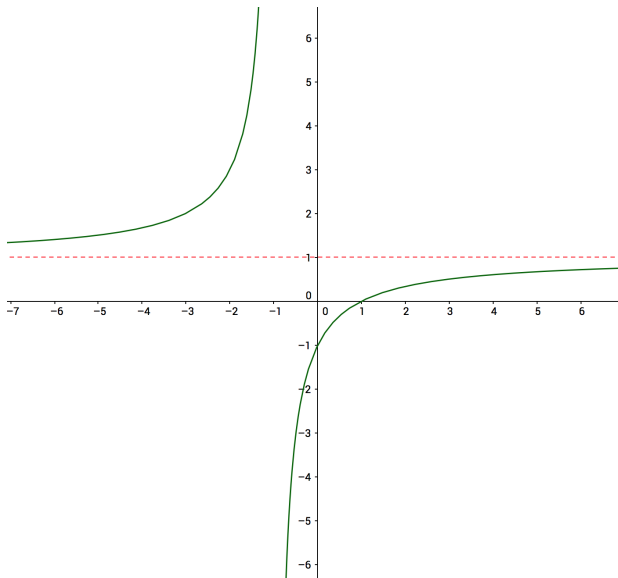
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Example

Say $f(x) = \frac{x-1}{x+1}$. In this case

$$\lim_{x \rightarrow \pm\infty} \frac{x-1}{x+1} = 1$$

Finding horizontal asymptotes



More examples

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Example

Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t \rightarrow \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

More examples

Example

Say $f(t) = \frac{t}{t+1}$. In this case

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Say $f(t) = \frac{t \ln t}{t-1}$. In this case

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No horizontal asymptotes.

Example

Say $f(t) = \frac{t}{t+e^t}$. In this case

$$\lim_{t \rightarrow \infty} \frac{t}{t+e^t} = 0$$

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Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approaches a finite number. I.e. Is there a real number a so that

$$\lim_{x \rightarrow a^+} f(x) = \pm\infty$$

or

$$\lim_{x \rightarrow a^-} f(x) = \pm\infty$$

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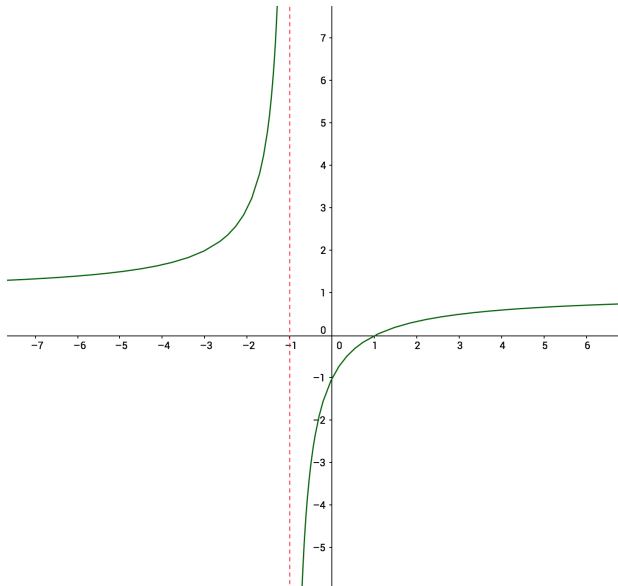
$$\lim_{x \rightarrow a^-} f(x) = \pm\infty$$

Example

$f(x) = \frac{x-1}{1+x}$, we have

$$\lim_{x \rightarrow -1^+} \frac{x-1}{1+x} = -\infty \quad \text{and} \quad \lim_{x \rightarrow -1^-} \frac{x-1}{1+x} = \infty$$

Finding verticle asymptotes

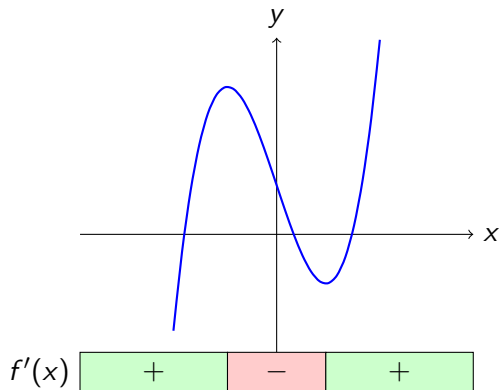


Finding slanted asymptotes

Lets come back to this...

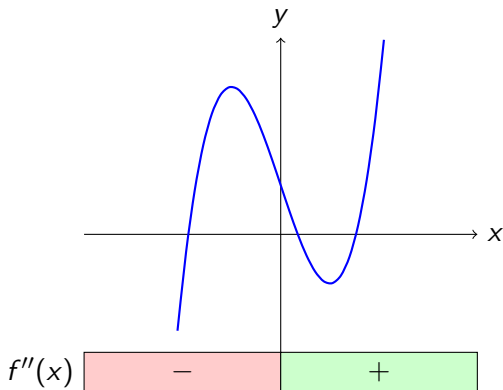
The first derivative

The first derivative tells us **is the function going up or down?**



The second derivative

The second derivative tells us **is the function concave up or down?**



Example time

... On the board.