Final exam (practice 2)

UCLA: Math 32B, Spring 2018

Instructor: Noah White
Date:

- This exam has 7 questions, for a total of 80 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section: ______________________________

Question	Points	Score
1	12	
2	12	
3	13	
4	12	
5	9	
6	8	
7	14	
Total:	80	

Questions 1 and 2 are multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

Please note! The following four pages will not be graded. You must indicate your answers here for them to be graded!

Question 1.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

Question 2.

Part	A	B	C	D
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) The iterated integral $\int_0^1 \int_3^5 1 \, \mathrm{d}x \, \mathrm{d}y$ is equal to

   A. 2
   B. 4
   C. 1
   D. -1

   (b) (2 points) Integrate $f(x, y) = \sin x$ in the region $\mathcal{R} = [-1, 1] \times [0, 4]$

   A. π
   B. $-\pi$
   C. 1
   D. 0

   (c) (2 points) Integrate the function $f(x) = 4xy$ on the triangle $\mathcal{D}$ with vertices $(0, 0), (1, 0)$ and $(0, 2)$.

   A. $2/3$
   B. 2
   C. $-4/3$
   D. 4

(d) (2 points) The Jacobian of the function $G(u, v, w) = (u^2, v - u, ve^w)$

A. ue^w

B. $(v - u)e^w$

C. we^w

D. $2uve^w$

(e) (2 points) Integrate $e^{\sqrt{x^2+y^2+z^2}}$ over the ball $x^2 + y^2 + z^2 \leq 4$.

A. $8\pi(e^2 - 1)$

B. $8\pi^2$

C. $8\pi e^2$

D. $4(e^2 - 1)$

(f) (2 points) Calculate the line integral of $f(x, y) = 1$ along the circle $x^2 + y^2 = 4$.

A. 4π.

B. 2π.

C. -4π.

D. -2π.

2. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (2 points) Suppose $f(t)$ is a function defined on all of $\mathbb{R}$. Let $\mathcal{C}$ be the oriented curve given by the parametrisation $\mathbf{r}(t) = (t, f(t))$ for $a \leq t \leq b$ (where $a < b$). The flux through $\mathcal{C}$ of the vector field $\langle 0, e^{x^2+y^2} \rangle$ is

   A. greater than and sometimes equal to zero.
   B. less than and sometimes equal to zero.
   C. always greater than zero.
   D. always less than zero.

   (b) (2 points) The vector field $r^{-2}\langle -y, x, 0 \rangle$ where $r^2 = x^2 + y^2$, has domain $\mathbb{R}^3 - \{(x, y, z) \mid x = y = 0\}$. The vector field

   A. has zero curl and is conservative.
   B. has non zero curl and is conservative.
   C. has zero curl and is not conservative.
   D. has non zero curl and is not conservative.

   (c) (2 points) Let $\varphi(x, y)$ be a scalar function defined on the plane $\mathbb{R}^2$ and let $\mathbf{F} = \nabla \varphi$. Suppose $\int_{\mathcal{C}'} \mathbf{F} \cdot d\mathbf{r} = 3$ where $\mathcal{C}'$ is the straight line from $(-3, 0)$ to $(3, 0)$. What is the value of $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ if $\mathcal{C}$ is the top half of the ellipse

   $$\frac{x^2}{9} + \frac{y^2}{2} = 1$$

   oriented counter clockwise.

   A. 0
   B. 1.
   C. 3.
   D. -3.

(d) (2 points) Consider the surface $\mathcal{S}$ parametrised by $G(u,v) = (u - v, u^2, v)$. What is the normal vector to the surface at the point $(0, 4, 2)$?

A. $\langle 2, -1, 2\rangle$

B. $\langle 8, -1, 8\rangle$.

C. $\langle 8, 1, 8\rangle$.

D. $\langle 4, -1, 4\rangle$.

(e) (2 points) Suppose $\mathbf{F}$ is an *incompressible* vector field (i.e. it has $\mathrm{div}(\mathbf{F}) = 0$). What is the (outward) flux of $\mathbf{F}$ through the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1?$$

A. 0.

B. πabc.

C. $-\pi abc$.

D. $\sqrt{a^2 + b^2 + c^2}$.

(f) (2 points) Let $\mathcal{C}$ be the unit circle, oriented counter clockwise, and $\mathcal{D}$ be the unit disk, both centered at the origin. If $\mathbf{F} = \langle \log(x^2 + y^2 + 1), \log(x^2 + y^2 + 1)\rangle$, the integral $\oint_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ is equal to

A. $\iint_{\mathcal{D}} \frac{2(x+y)}{x^2+y^2+1}\, \mathrm{d}S$

B. $-\iint_{\mathcal{D}} \frac{2(x+y)}{x^2+y^2+1}\, \mathrm{d}S$

C. $\iint_{\mathcal{D}} \frac{2(x-y)}{x^2+y^2+1}\, \mathrm{d}S$

D. $-\iint_{\mathcal{D}} \frac{2(x-y)}{x^2+y^2+1}\, \mathrm{d}S$

3. Consider the vector field

$$\mathbf{F}(x, y, z) = \frac{1}{u^2}\langle -z, z, x - y\rangle \quad \text{where } u^2 = z^2 + (x - y)^2.$$

   (a) (2 points) What is the largest domain on which $\mathbf{F}$ is defined?

   (b) (4 points) Calculate the curl of $\mathbf{F}$.

   (c) (5 points) Show that $\mathbf{F}$ is not conservative.

   (d) (2 points) Calculate $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ if $\mathcal{C}$ is the curve given by the intersection of the sphere $(x-2)^2 + (y+2)^2 + z^2 = 1$ and the surface $z = (x-2)^2$.

4. (a) (4 points) Let $\mathcal{S}$ be an oriented surface and let $\mathbf{F}$ be a vector field which has continuous partial derivatives on an open region containing $\mathcal{S}$. State Stokes' theorem for $\mathbf{F}$.

   (b) (3 points) Let $\mathbf{F} = \langle xe^{y+z} - y, ye^{y+z} + x, e^{x+y} \rangle$. Calculate the curl of $\mathbf{F}$.

   (c) (5 points) Let $\mathcal{S}$ be the surface defined by

$$\mathcal{S} = \{(x, y, z) \in \mathbb{R}^3 \mid z = e^{-(x^2+y^2)} \text{ for } z \geq \frac{1}{e}\}$$

   with upward normal, and let $\mathbf{F}$ be as above. Calculate $\iint_{\mathcal{S}} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$

5. (a) (3 points) Give *three* examples of vector fields $\mathbf{F}$ in $\mathbb{R}^2$ such that $\mathrm{curl}(\mathbf{F}) = 1$. *Hint: Recall that for 2D vector fields* $\mathrm{curl}(\mathbf{F}) = \partial_x F_2 - \partial_y F_1$.

   (b) (6 points) Use Green's theorem to find the area enclosed by the parametrised curve

$$\mathbf{r}(t) = (1 - \sin t, 1 - \cos t) \quad \text{for } t \in [0, 2\pi].$$

6. (a) (4 points) Either find a function f on $\mathbb{R}^3$ such that

$$\nabla f(x, y, z) = \langle e^z, 1, xe^z \rangle$$

or show that one cannot exist.

(b) (4 points) Either find a vector field $\mathbf{F}$ on $\mathbb{R}^3$ such that

$$\text{curl}(\mathbf{F}) = \langle y, x, z \rangle$$

or show that one cannot exist.

7. Let $\mathcal{E}$ be the solid cone given by

$$z \geq \alpha\sqrt{x^2 + y^2} \quad \text{and} \quad z \leq b,$$

such that the top of the cone has a radius of $a > 0$.

(a) (2 points) Express α in terms of a and b.

(b) (5 points) Let $\mathbf{F} = \langle y^2, xy, z\rangle$. Express the triple integral of $\text{div}(\mathbf{F})$ over $\mathcal{E}$ as a triple iterated integral.

(c) (3 points) Parametrise the boundary $\partial\mathcal{E}$ of $\mathcal{E}$ as an *oriented* surface (you may parametise this as two separate surfaces.)

(d) (4 points) use the divergence theorem to express the triple integral above as the sum of two double iterated integrals.

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