

This week you will get practice drawing and understanding bifurcation diagrams.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (6.6) Sketch the bifurcation diagrams for the equations in the following.

- (a) (6.6-7) $\frac{dy}{dt} = ay - y^2$
- (b) (6.6-10) $\frac{dy}{dt} = 1 - ay^2$
- (c) (6.6-11) $\frac{dy}{dt} = \sin y + a$
- (d) (6.6-12) $\frac{dy}{dt} = y^2 - ay + 2$ for $a > 0$.

2. (6.6) Consider the model

$$\frac{dy}{dt} = \frac{ay^2}{k^2 + y^2} - cy$$

of an autocatalytic gene from question 5. In each of the following cases, two of the parameters are specified. Sketch a bifurcation diagram with respect to the third parameter.

- (a) (6.6-13) $k = 1$, $c = 2$ with a as the bifurcation parameter.
 - (b) (6.6-14) $k = 2$, $c = 1$ with a as the bifurcation parameter.
 - (c) (6.6-15) $a = 10$, $k = 1$ with c as the bifurcation parameter.
 - (d) (6.6-16) $a = 10$, $c = 1$ with k^2 as the bifurcation parameter.
3. (6.6-40) Suppose the growth rate of a whale population at density N (individuals per million square kilometers of ocean), harvested at a rate h , is given by

$$\frac{dN}{dt} = 0.07N \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right) - h$$

where the units of t are years.

- (a) Sketch a bifurcation diagram with respect to the parameter h as it varies over the interval $[0, 8]$.

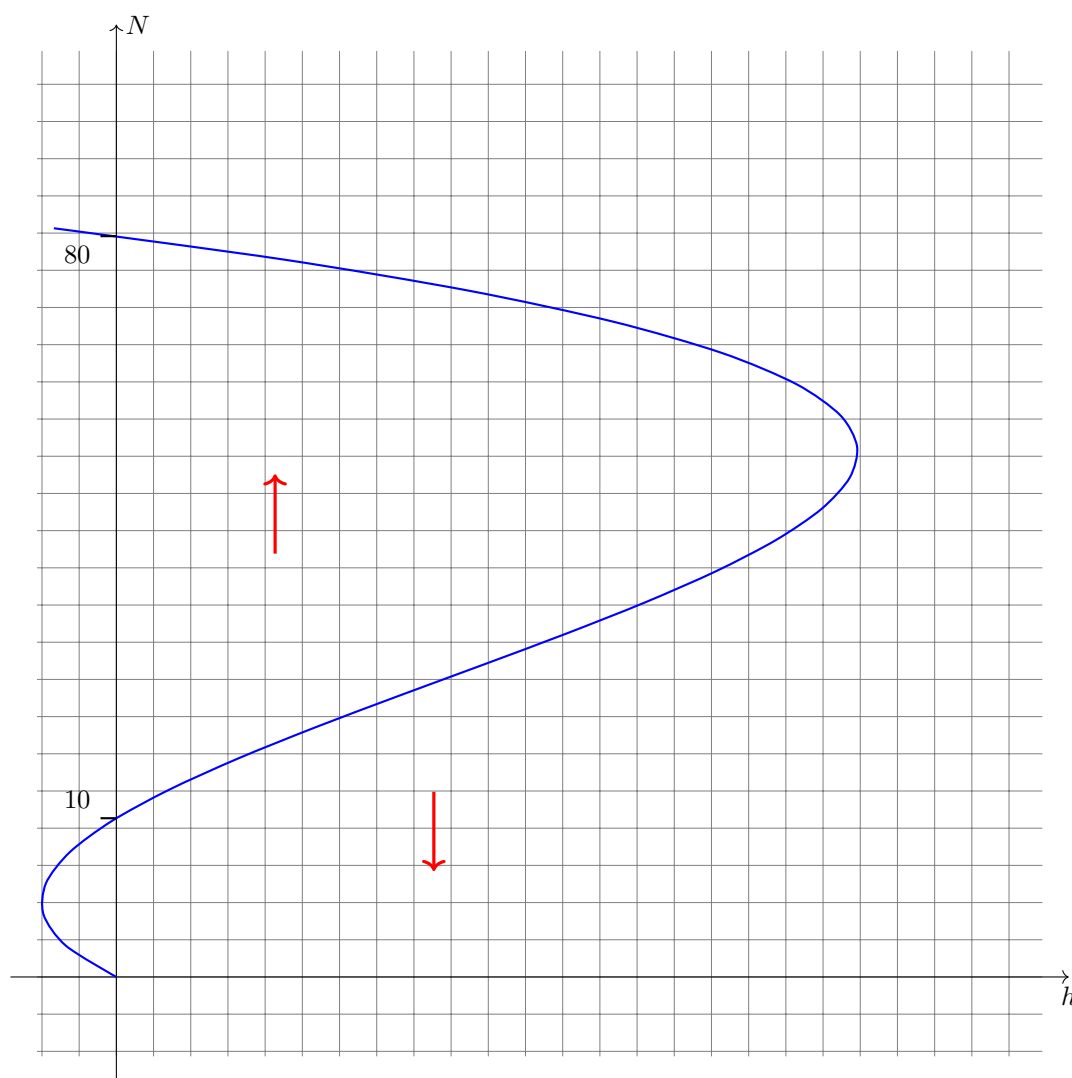
Solution: We want to sketch the points that satisfy

$$0.07N \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right) - h = 0$$

i.e. where

$$h = 0.07N \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right)$$

To do this we can sketch it in the hN -axis and then flip to the Nh -axis:



The direction of the arrows is found by subbing in points from each region (e.g. $(0, 40)$ and $(1, 0)$) into

$$0.07N \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right) - h$$

and determining whether it is positive or negative.

- (b) If $h = vN$, then sketch a bifurcation diagram with respect to the parameter v as it varies over the interval $[0, 0.12]$.

Solution: We want to sketch the points that satisfy

$$0.07N \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right) - vN = 0$$

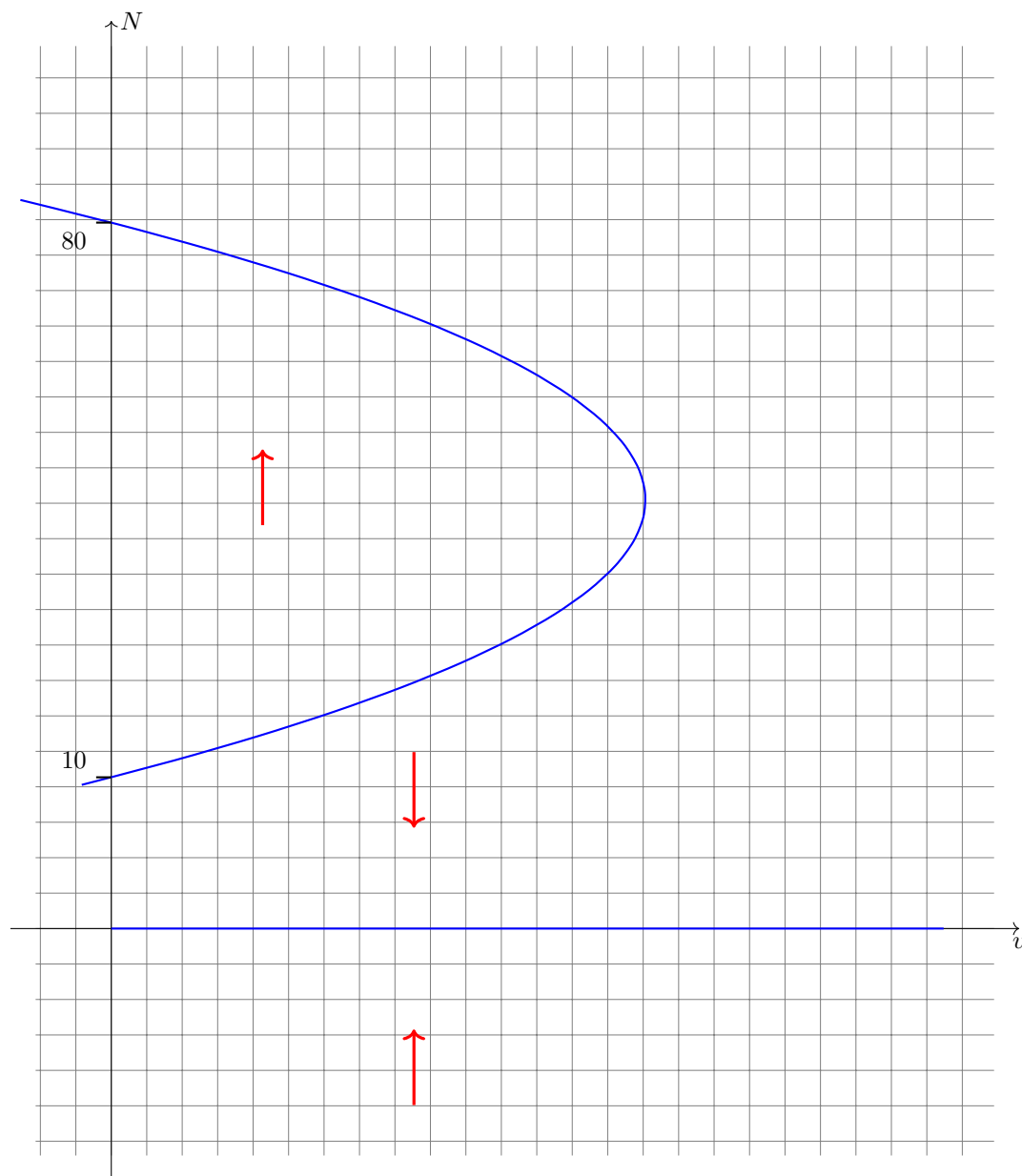
i.e. where

$$\left[0.07 \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right) - v \right] N = 0$$

Thus we want to sketch the points where either $N = 0$ or

$$v = 0.07 \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right).$$

To do this we can first sketch the line $N = 0$ and then the sideways parabola:



The direction of the arrows is found by subbing in points from each region (e.g. $(0, 40)$, $(0, 5)$ and $(0, -1)$) into

$$0.07N \left(\frac{N}{10} - 1 \right) \left(1 - \frac{N}{80} \right) - vN$$

and determining whether it is positive or negative.