# Midterm 2 practice 2

# UCLA: Math 115A, Winter 2019

*Instructor:* Noah White
*Date:*
*Version: 1*

- This exam has 4 questions, for a total of 20 points.

- Please print your working and answers neatly.

- Write your solutions in the space provided showing working.

- All final answers should be exact values. Decimal approximations will not be given credit.

- Indicate your final answer clearly.

- Full points will only be awarded for solutions with correct working.

- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

| Question | Points | Score |
|:--------:|:------:|:-----:|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| Total: | 20 | |

Question 1 is multiple choice. Once you are satisfied with your solutions, indicate your answers by marking the corresponding box in the table below.

*Please note! The following three pages will not be graded. You must indicate your answers here for them to be graded!*

**Question 1.**

| *Part* | A | B | C | D |
|--------|---|---|---|---|
| (a)    |   |   |   |   |
| (b)    |   |   |   |   |
| (c)    |   |   |   |   |
| (d)    |   |   |   |   |
| (e)    |   |   |   |   |

*Clarification on notation:* Let $T : V \longrightarrow W$ be a linear map. The *kernel* of $T$ is the same thing as the *nullspace* of T, i.e. $\ker T = \mathsf{N}(T)$. Similarly the *image* of $T$ is the same thing as the *range* of T, i.e. $\operatorname{im} T = \mathsf{R}(T)$.

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

   (a) (1 point) If $V$ is a finite dimensional vector space, with two bases, $B$ and $C$ then the matrix that changes $B$-coordinate vectors, into $C$-coordinate vectors is

   A. $[T]_C^B$
   B. $[T]_B^C$
   C. $[\mathrm{id}]_C^B$
   **D.** $[\mathrm{id}]_B^C$

   (b) (1 point) Let $E = \{1, x\}$, $C = \{x + 2, x + 1\}$ and $T : \mathbb{C}_1[x] \longrightarrow \mathbb{C}_1[x]$ be a linear map such that

   $$[T]_E^E = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

   Then, $[T]_C^C$ is equal to

   A. $\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$

   B. $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

   C. $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$

   **D.** $\begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix}$

(c) (1 point) Suppose $S : U \longrightarrow V$ and $T : V \longrightarrow W$ are linear maps between vector spaces, such that $T \circ S$ is the zero map (i.e. $T(S(u)) = 0$ for all $u \in U$). Which of the following is true?

       A. $\ker S \subseteq \operatorname{im} T$

       **B. $\operatorname{im} S \subseteq \ker T$**

       C. $\ker T \subseteq \operatorname{im} S$

       D. $\operatorname{im} T \subseteq \ker S$

(d) (1 point) Suppose $T : V \longrightarrow V$ is a linear map and $\lambda$ is an eigenvalue of $T$. Which of the following is true?

       A. $\lambda^{-1}$ is also an eigenvalue of $T$.

       B. $\lambda$ is an eigenvalue of $T^n$ for some $n \geq 1$.

       **C. $\lambda^n$ is an eigenvalue of $T^n$ for every $n \geq 1$.**

       D. $\lambda$ is not an eigenvalue of $T^n$ for any $n \geq 2$.

(e) (1 point) Let $V$ be a vector space. What is the dimension of $\mathcal{L}(\{0\}, V)$?

       **A. 0**

       B. 1

       C. $\dim V - 1$

       D. $\dim V$

2. Let $T : V \longrightarrow W$ be a linear map between vector spaces.

   (a) (2 points) Define the *rank* and *nullity* of $T$.

   > **Solution:**

   (b) (3 points) Prove the *rank-nullity theorem:* If $V$ is finite dimensional, then $\dim V = \dim \ker T + \dim \operatorname{im} T$.

   > **Solution:**

3. Consider the linear map $T : \mathbb{R}_2[x] \longrightarrow \mathbb{R}_2[x]$ given by

$$T(a + bx + cx) = (a + 2b + 2c) + (b + 3c)x + 2cx^2.$$

(a) (2 points) Find the characteristic polynomial and eigenvalues of $T$.

Solution:

(b) (2 points) For each eigenvalue, determine an eigenvector of $T$.

Solution:

(c) (1 point) Is $T$ diagonalisable?

Solution: No

4. Let $V$ and $W$ be vector spaces, and let $S : V \longrightarrow W$ and $T : W \longrightarrow V$ be linear maps.

(a) (1 point) Prove that if $T \circ S$ is injective (one-to-one) then $S$ is injective.

> **Solution:**

(b) (2 points) Prove that if $T \circ S$ is surjective (onto) then $T$ is surjective.

> **Solution:**

(c) (2 points) Give an example of spaces $V, W$ and linear maps $S, T$ such that $T \circ S = \text{id}_V$ but $S \circ T \neq \text{id}_W$. *Hint: you shouldn't need to do anything complicated, there is a very simple example.*

> **Solution:**

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