Math 3B: Lecture 7

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Antiderivatives

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$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)$$

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The solution $y = F(x)$ is called the antiderivative of $f(x)$.

Example 1

Question

What is the antiderivative of $f(x) = 2x$?

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Solution

$$F(x) = x^2$$

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$$F(x) = x^2 + 4$$

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$$F(x) = x^2 + 8$$

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Solution

$$F(x) = x^2 + C$$

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Example 3

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Solution

$$F(x) = \frac{1}{2}e^{2x}$$

Example 4

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What is the antiderivative of $f(x) = \frac{1}{x}$ (for $x > 0$)?

Solution

$$F(x) = \ln x$$

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Solution

$$F(x) = -\frac{1}{1+x}$$

Example 6

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What is the antiderivative of $f(x) = 2x \cos x^2$?

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What is the antiderivative of $f(x) = 2x\cos x^2$?

Solution

$$F(x) = \sin x^2$$

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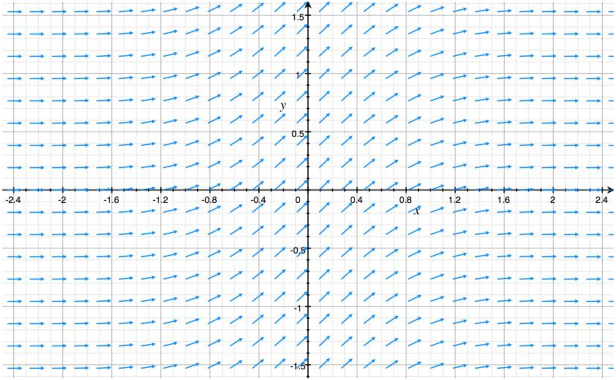
$$F(x) = 2x^{\frac{1}{2}}$$

Slope fields

In some cases it is impossible to find the antiderivative (without special functions). E.g.

$$f(x) = e^{-x^2}$$

But we can still graph the antiderivative! First we draw the slope field



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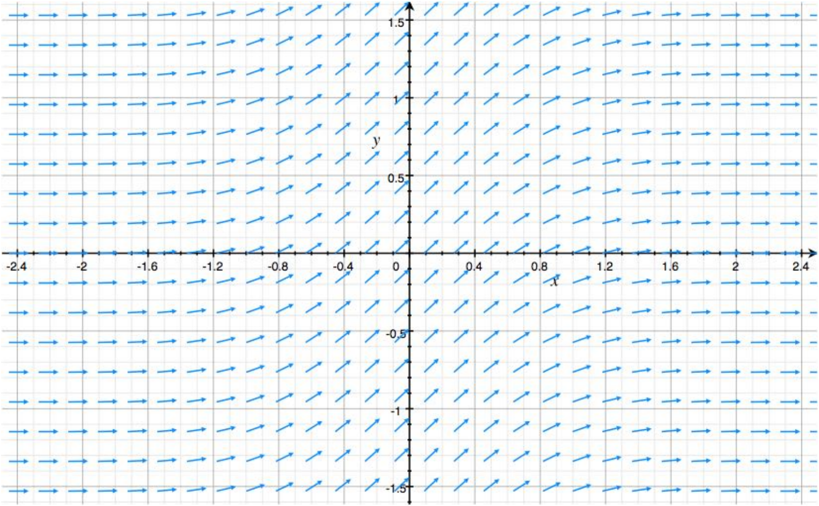

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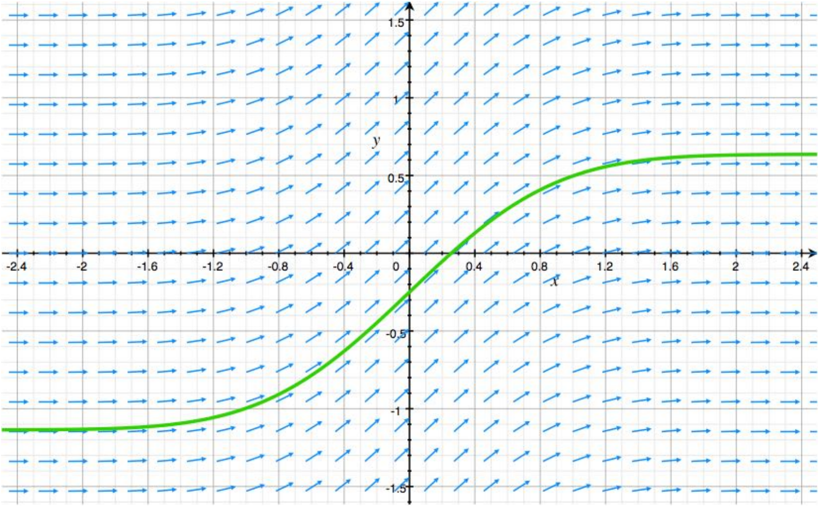
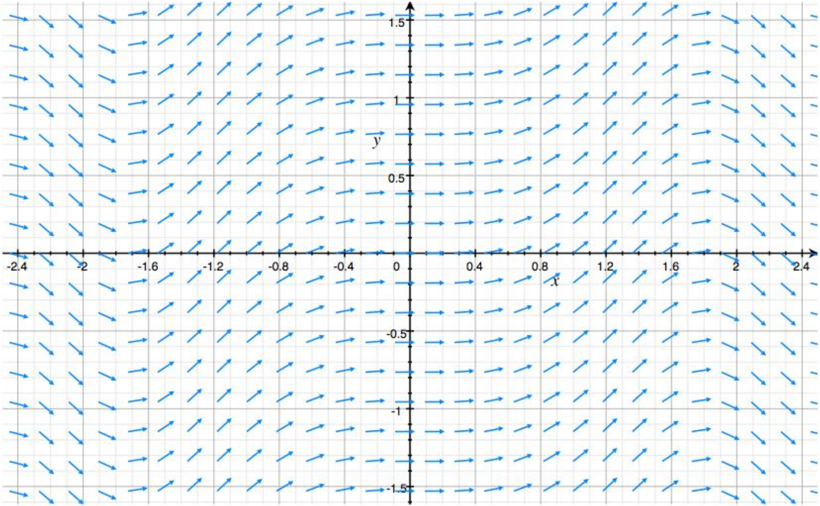


Example 2

$$f(x) = \sin(x^2)$$



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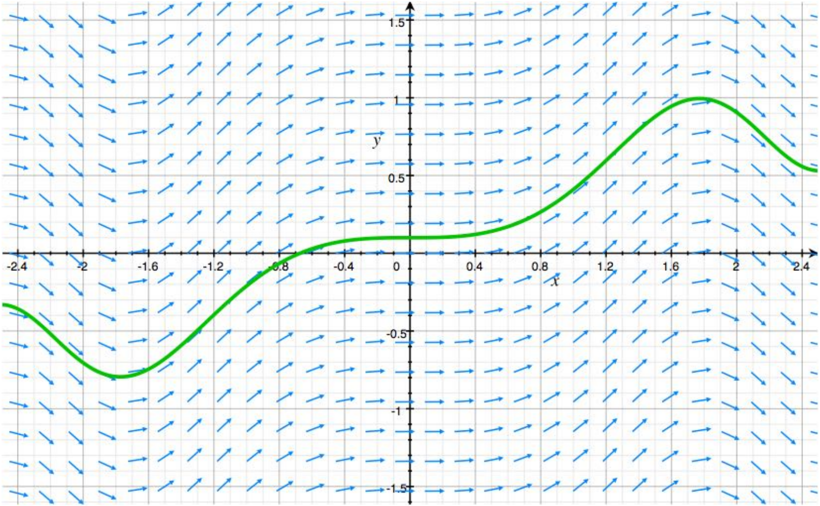

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How long would it take a 1kg watermelon to reach the ground after being from from the 335m tall Wishire Grand Center? The force on the watermelon due to gravity is a constant $-10m/s^2$.

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i.e.

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$$t = \sqrt{67} \sim 8.2$$

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These problems involve finding the area under some curve.

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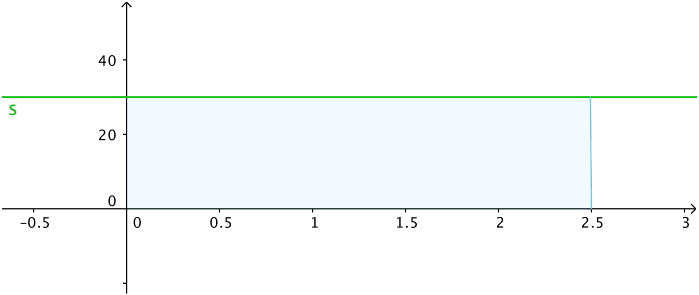
Solution

We model the car's speed using the function $s(t) = 30$. So we can see that the area under this curve



is the distance travelled (75 miles)

Example 2

If a car accellerates for 20 seconds at a rate of $2m/s^2$ and then decelerates for 30 seconds at a rate of $1m/s^2$, how far has it travelled?

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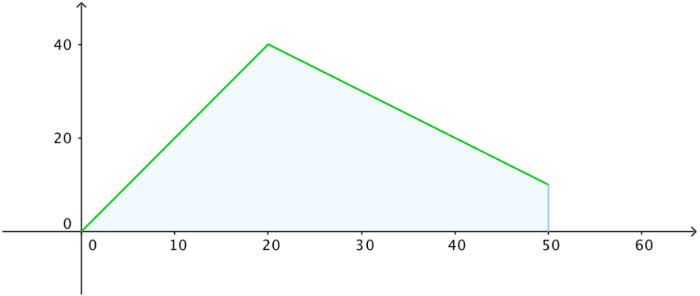
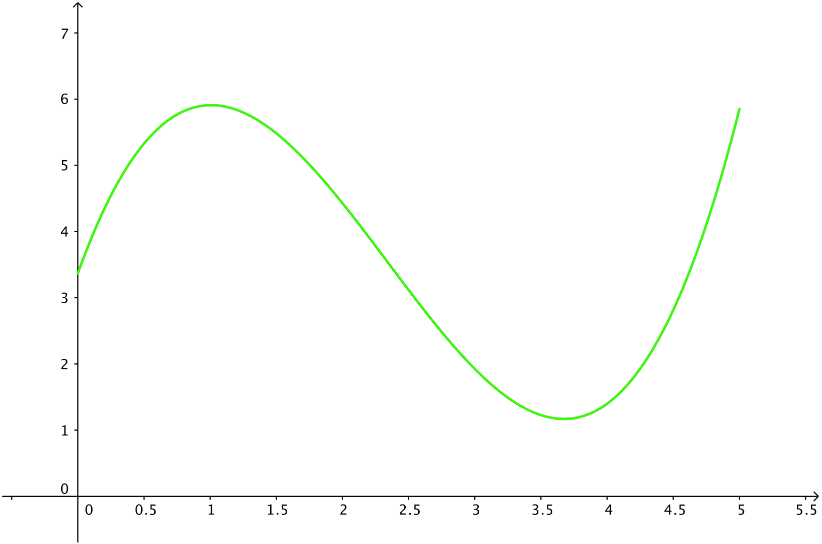
Solution

The car's speed is given by $s(t) = 2t$ when $0 \leq t \leq 20$ and $s(t) = 60 - t$ when $20 \leq t \leq 50$. So the graph looks like



More complicated areas

How do we calculate the area under more complicated curves?



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