Math 3B: Lecture 2

Noah White

January 9, 2019

Last time

Last time, we spoke about

- The syllabus

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- Problem sets, homework, and quizzes

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- Campuswire

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- Differentiation of common functions

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- Product rule

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- Product rule
- Chain rule

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

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- Building intuition

Graphing using calculus: Why?

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- Building intuition
- Understand functions qualitatively

Graphing using calculus: Why?

A common (and fair) question is: Why learn to sketch graphs when computers that do it so well?

- Building intuition
- Understand functions qualitatively
- Better understanding of derivatives

Building intuition

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In order to sketch a function accurately we need a few ingredients

The ingredients

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- The x and y intercepts

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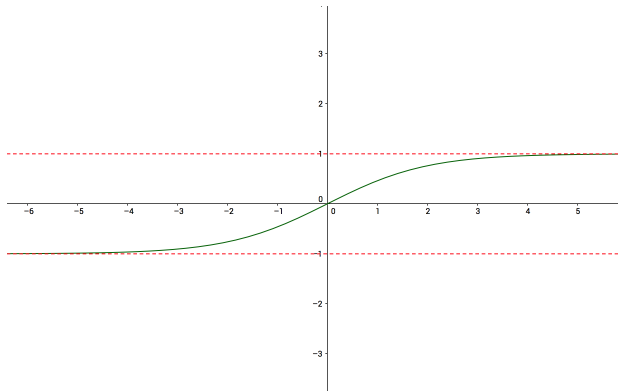- The regions where the second derivative is positive/negative

Asymptotes

An asmptote is a line which the function approches. Some examples:
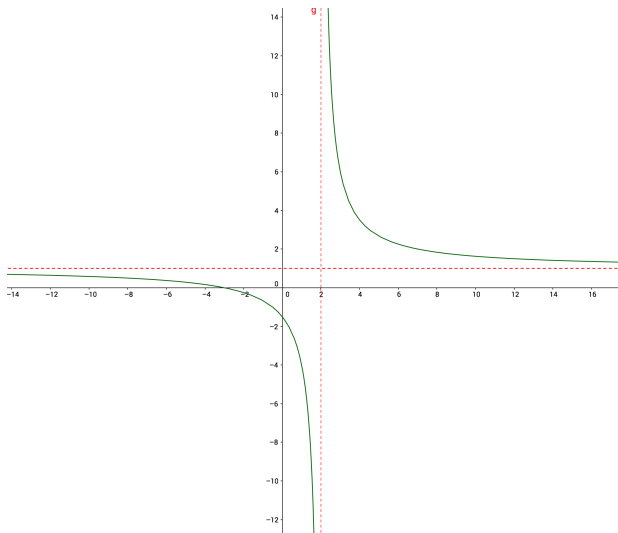
Asymptotes
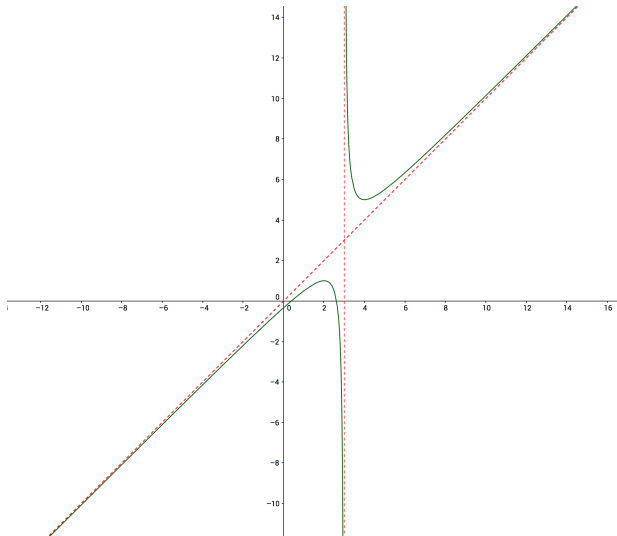
These are the easiest asymptotes to find. Suppose you have a function $f(x)$

Finding horizontal asymptotes

These are the easiest asymptotes to find. Suppose you have a function $f(x)$

- Calculate $\lim\limits_{x \to \infty} f(x)$

Finding horizontal asymptotes

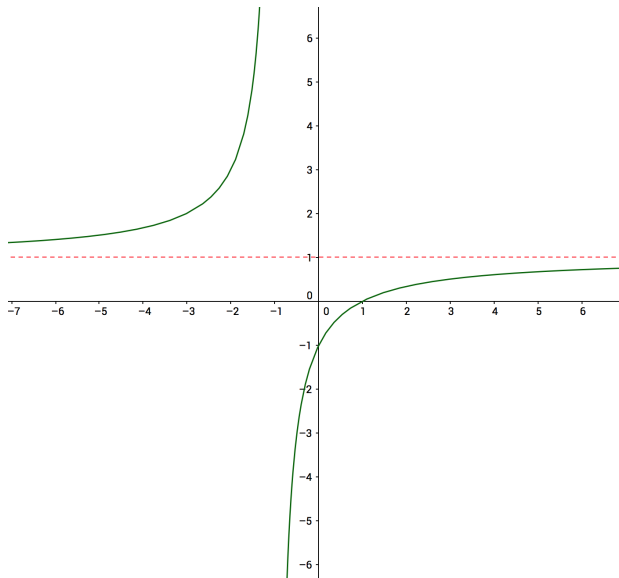These are the easiest asymptotes to find. Suppose you have a function $f(x)$

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- Calculate $\lim\limits_{x \to -\infty} f(x)$

Example
Say $f(x) = \frac{x-1}{x+1}$. In this case

$$\lim_{x \to \pm\infty} \frac{x-1}{x+1} = 1$$

Finding horizontal asymptotes

Example

Say $f(t) = \frac{t}{t+1}$. In this case

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More examples

Example

Say $f(t) = \frac{t}{t+1}$. In this case

$$\lim_{t \to \pm\infty} \frac{t}{t+1} = 1$$

Example

Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

More examples

Example

Say $f(t) = \frac{t}{t+1}$. In this case

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Say $f(t) = \frac{t \ln t}{t-1}$. In this case

$$\lim_{t \to \infty} \frac{t \ln t}{t-1} = \infty$$

No horizontal asymptotes.

Example

Say $f(t) = \frac{t}{t+e^t}$. In this case

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Finding verticle asymptotes

Verticle asymptotes happen when a function "blows up", or goes to infinity as it approches a finite number. I.e. Is there a real number *a* so that

$$\lim_{x \to a^+} f(x) = \pm\infty$$

or

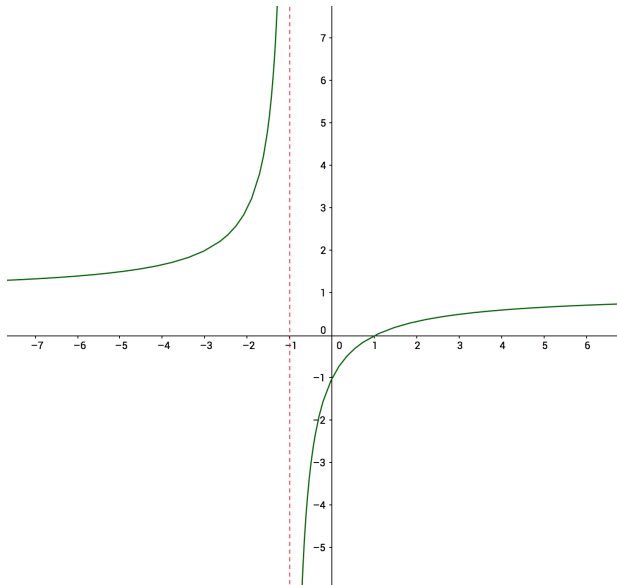$$\lim_{x \to a^-} f(x) = \pm\infty$$

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Example

$f(x) = \frac{x-1}{1+x}$, we have

$$\lim_{x \to -1^+} \frac{x-1}{1+x} = -\infty \quad \text{and} \quad \lim_{x \to -1^-} \frac{x-1}{1+x} = \infty$$

Finding verticle asymptotes

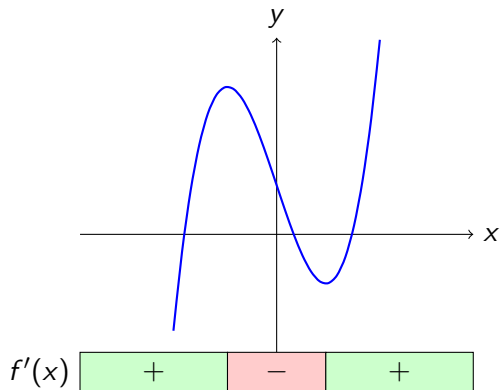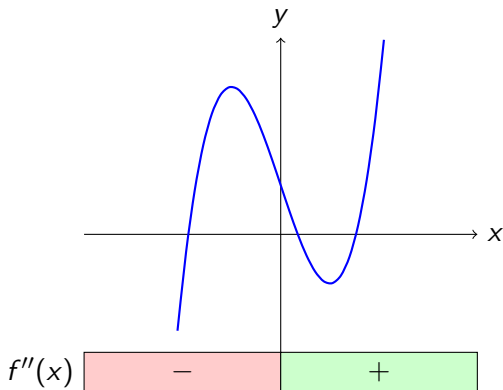Lets come back to this. . .

The first derivative

The first derivative tells us is the function going up or down?

The second derivative

The second derivative tells us is the function concave up or down?

Example time

... On the board.