

Math 3B: Lecture 21

Noah White

November 9, 2016

Math Success Program

- Free one on one tutoring sessions

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- Weekly drop in hours

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- mspucla.setmore.com

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- Due date of final homework has been changed to Wed 11/30.

Last time

- Checking solutions to differential equations.

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- Implicit differentiation.
- Separation of variables.

Linear models

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Examples

$$\frac{dy}{dt} = ay, \quad \frac{dy}{dt} = -\lambda y.$$

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- rate in is constant

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- concentration of a drug in bloodstream
- pollutant in water supply

General solution

Using separation of variables, we can show that the general solution to

$$\frac{dy}{dt} = a - by$$

is

$$y(t) = \frac{a}{b} - Ce^{-bt}$$

where C is an arbitrary constant.

Example 1

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$$M \left(\frac{1}{2} \right)^{t/2} = M e^{-0.5t \ln(2)} \text{ mg left}$$

- Thus the rate at which the drug is leaving (at time t) is given by

$$0.5 \ln(2) M e^{-0.5t \ln(2)} = 0.5 \ln(2) (\text{current concentration}) \text{ mg/h.}$$

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- Thus at time t the concentration is

$$y(t) = 28.9 - 28.9e^{-0.3t} = 28.9(1 - e^{-0.3t})$$

Newton's Law of Cooling

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$$\frac{dT}{dt} = k(A - T)$$

General solution

$$T(t) = A - Ce^{-kt}.$$

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- Thus

$$90 = 70 - C \quad \text{so} \quad C = -20.$$

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$$e^{-20k} = \frac{86 - 70}{20} = \frac{4}{5} \quad \text{so} \quad k = -\frac{1}{20} \ln \left(\frac{4}{5} \right) \approx -0.01.$$

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- Rearranging we get $20e^{-0.01t} = 5$ i.e.

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- So we get

$$t = -100 \ln\left(\frac{1}{4}\right) \approx 138 = 2 \text{ hours } 18 \text{ minutes.}$$

Von Bertalanffy growth model

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- ingests food at a rate proportional to its surface area

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- Surface area is $S = 6L^2$

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Von Bertalanffy growth model

The growth of an organism is governed by

$$\frac{dL}{dt} = k(L_{\infty} - L)$$

where k and L_{∞} are constants.