This week you will get practice with slope fields.

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

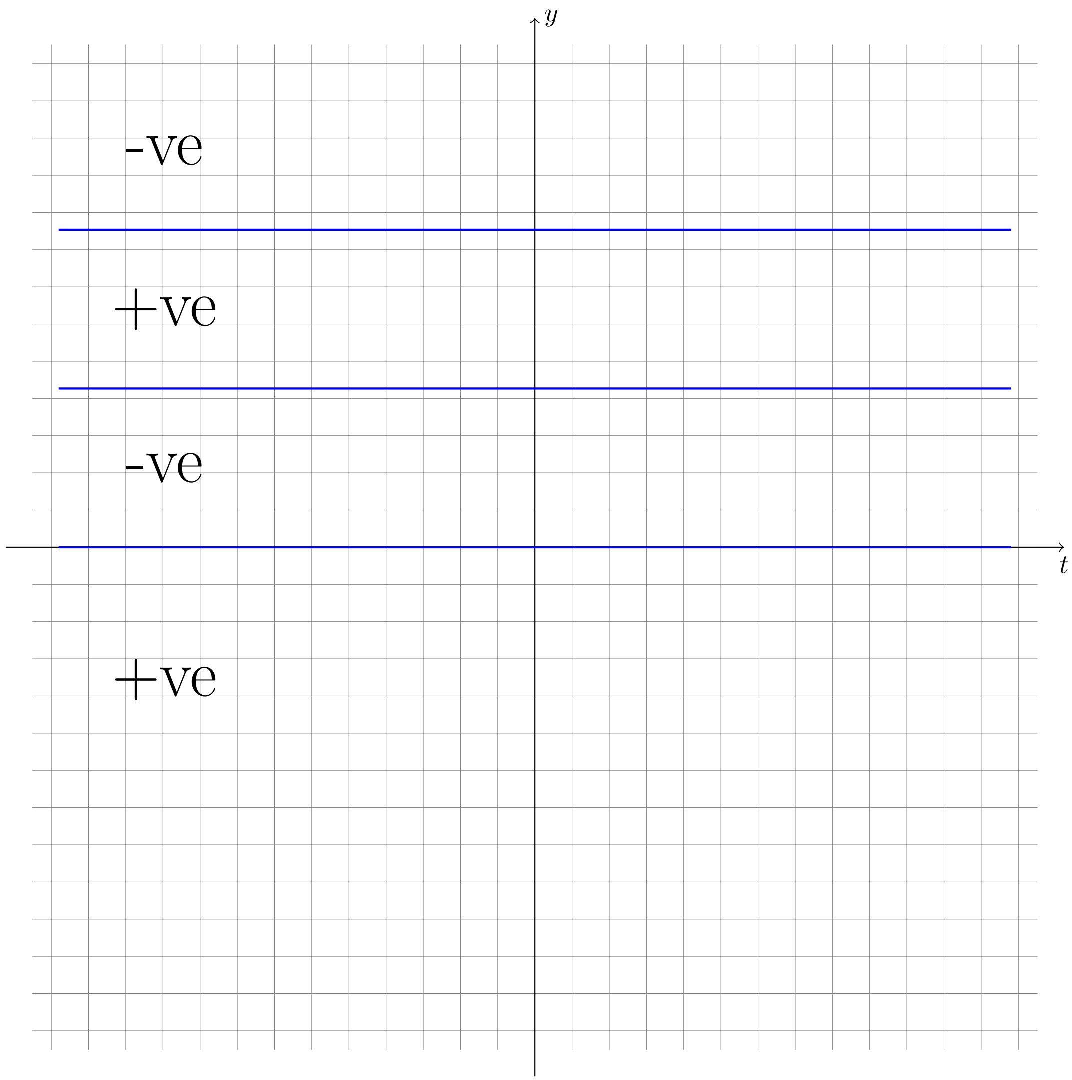
and refer to the section and question number in the textbook.

Homework: The homework will be due on Friday 1 MarchDecember, at 8am, the *start* of the lecture. It will consist of questions

question 3 and question 4.

1. (6.4) Sketch the slope fields and a few solutions for the differential equations given
   (a) (6.4.12) $\frac{\mathrm{d}y}{\mathrm{d}t} = y(4-y)(y-2)$

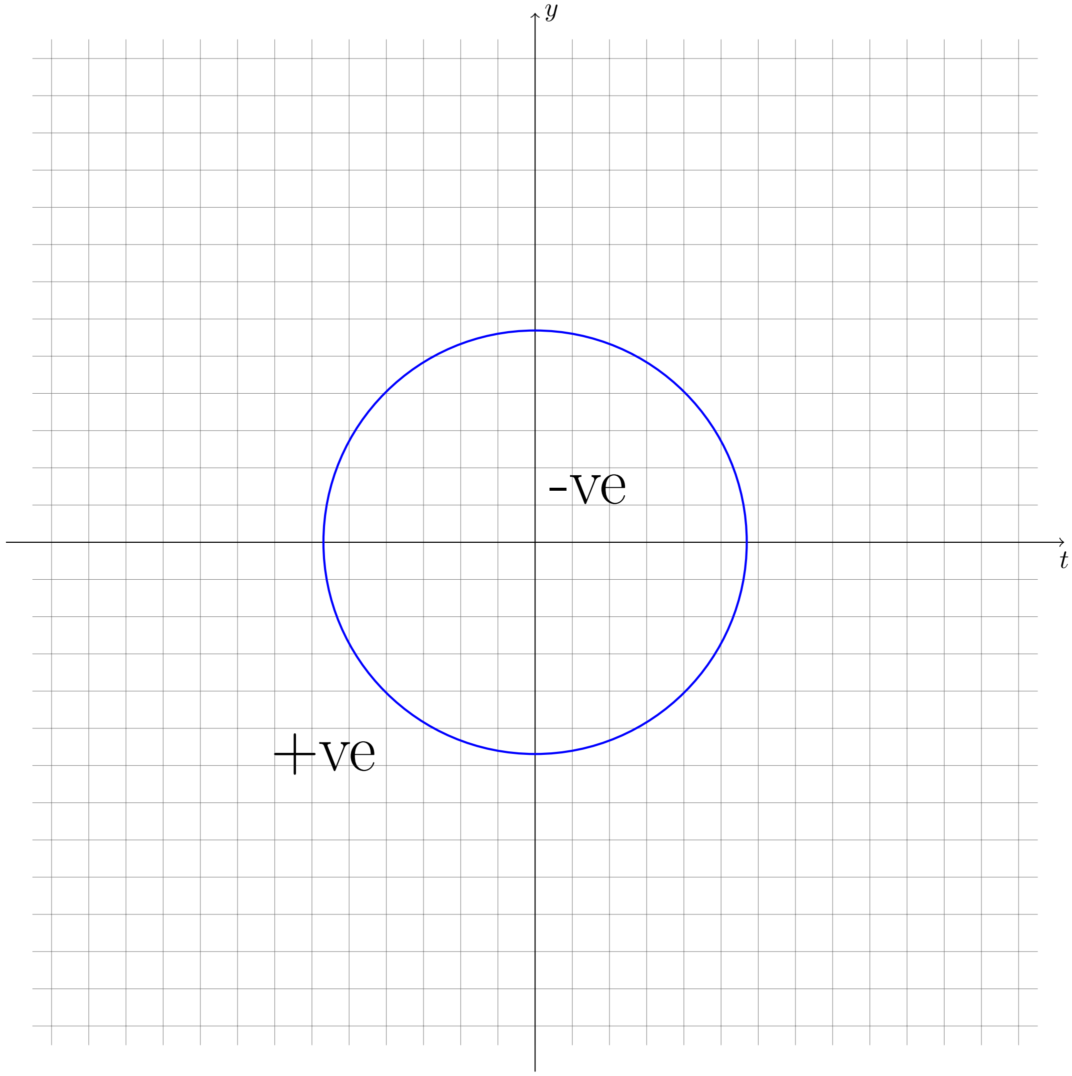
Solution:



   (b) (6.4.14) $\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 - y$

(c) (6.4.16) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 + t^2 - 1$

Solution:



(d) (6.4.17) $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{y}{t}$

Hint: feel free to use technology, just make sure you know how to draw a solution if you are given a slope field.

2. (6.4) Sketch the slope fields and the solution passing through the specified point for the differential equations given

   (a) (6.4.19) $\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 - y^2, (t, y) = (0, 0)$

   (b) (6.4.20) $\frac{\mathrm{d}y}{\mathrm{d}t} = 1.5y(1 - y), (t, y) = (0, 0.1)$

   (c) (6.4.21) $\frac{\mathrm{d}y}{\mathrm{d}t} = \sqrt{\frac{t}{y}}, (t, y) = (4, 1)$

   (d) (6.4.22) $\frac{\mathrm{d}y}{\mathrm{d}t} = y^2\sqrt{t}, (t, y) = (9, -1)$

3. (6.4.37) A population subject to seasonal fluctuations can be described by the logistic equation with an

oscillating carrying capacity. Consider, for example,

$$\frac{dP}{dt} = P\left(1 - \frac{P}{100 + 50\sin 2\pi t}\right)$$

Although it is difficult to solve this differential equation, it is easy to obtain a qualitative understanding.

(a) Sketch a slope field over the region $0 \le t \le 5$ and $0 \le P \le 200$.

(b) Sketch solutions that satisfy $P(0) = 0$, $P(0) = 10$, and $P(0) = 200$, use technology if you like.

(c) Comment on the behaviour of the solutions.

4. (6.4.40) A population, in the absence of harvesting, exhibits the following growth

$$\frac{dN}{dt} = N\left(\frac{N}{100} - 1\right)\left(1 - \frac{N}{1000}\right)$$

where N is abundance and t is time in years.

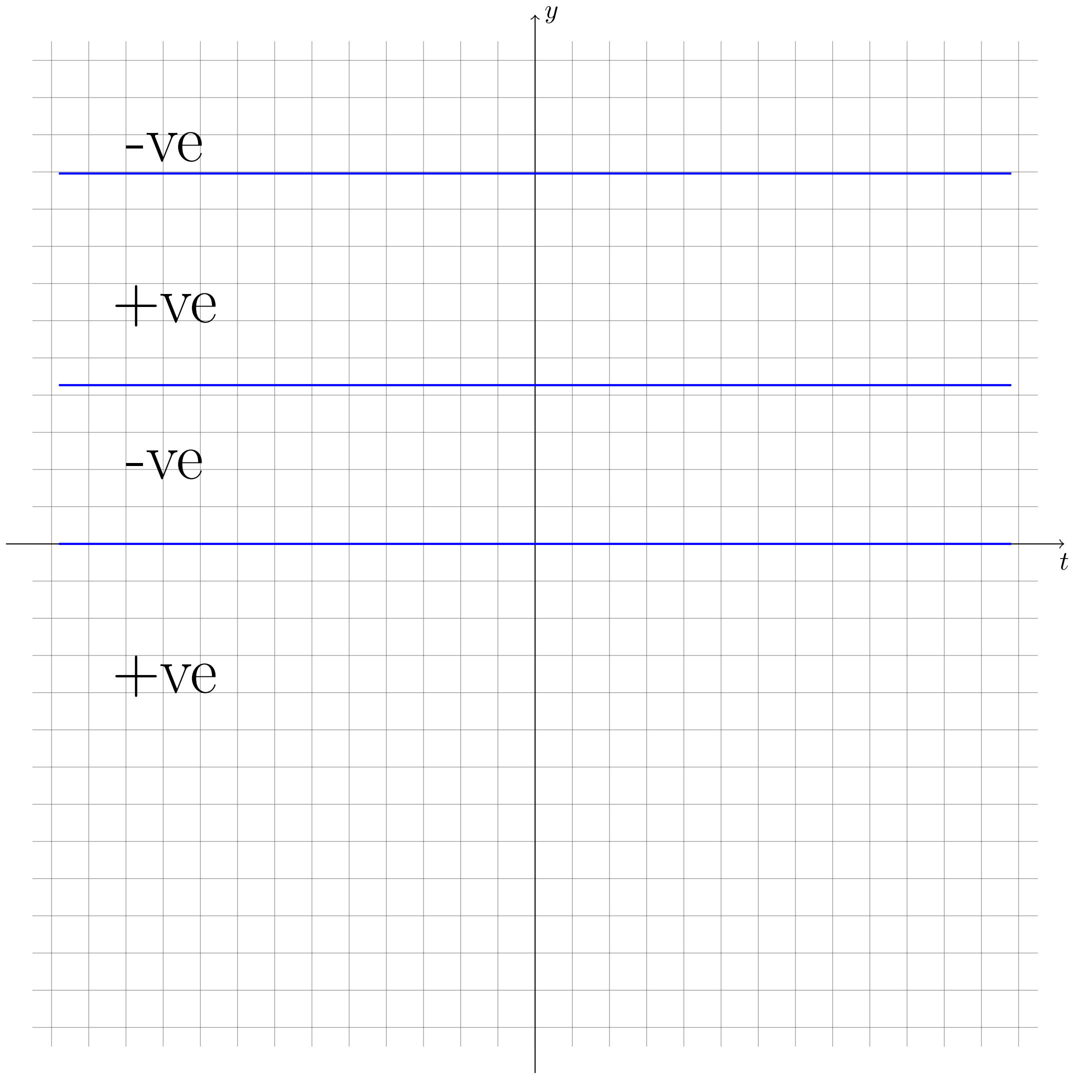
(a) Write an equation that corresponds to harvesting the population at a rate of 0.5% per day.

Solution: This means that 0.005 of the population is beign removed per day, so over a year, the total fraction of the population being removed is $365 \cdot 0.005 = 1.825 = 365/200$ so the DE becomes

$$\begin{aligned}\frac{dN}{dt} &= N\left(\frac{N}{100} - 1\right)\left(1 - \frac{N}{1000}\right) - 1.825N && (1)\\ &= N\left[\left(\frac{N}{100} - 1\right)\left(1 - \frac{N}{1000}\right) - \frac{365}{200}\right] && (2)\\ &= N\left(-\frac{N^2}{100000} + \frac{N}{100} + \frac{N}{1000} - 1 - \frac{365}{200}\right) && (3)\\ &= -\frac{N}{100000}\left(N^2 - 1100N + 282500\right) && (4)\end{aligned}$$

(b) Sketch the slope field for the differential equation you found in part a; by sketching solutions, describe how the fate of the population depends on its initial abundance.

Solution: To sketch the solution, we need to know where the right hand side is zero. One obvious place is when $N = 0$. Two others are provided by solving the quadratic equation. Approximately, the roots are $N \approx 409, 691$. Thus, the slope field looks like:



From this we can see that if the initial abundance is between 0 and 409, the population will eventually go extinct. If it is larger than 409 then the population will stabalize at 691.