This weeks problem set provides practice with diagonalisable operators and the basic properties of inner products. A question marked with a † is difficult and probably too hard for an exam (though still illustrates a useful point). A question marked with a * is especially important.

Homework 5: due Tuesday 3 December: questions 22 from Section 6.2 (see 3 below) and question 2 below.

1. From section 6.2, problems 1, $2b, g, i, k$, 5^*, 6, 7, 9, 13^*, 17^*, 22.
2. Let V be a finite dimensional inner product space over $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$.
   (a) Fix $y \in V$ and suppose $\langle x, y\rangle = 0$ for all $x \in V$. Show that $y = 0$.
   (b) Let $T : V \longrightarrow V$ be a linear map such that $\langle T(x), T(y)\rangle = \langle x, y\rangle$ for all pairs $x, y \in V$ (we call such a map an *isometry*). Prove that T is an isomorphism.
   (c) † Find all isometries $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ that have $\det T = 1$.
3. (22 from 6.2) Let $V = \mathcal{C}([0, 1], \mathbb{R})$ be the space of real valued, continuous functions on the interval $[0, 1]$ with the inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)\ dt$. Let W be the subspace spanned by the linearly independent set $\{t, \sqrt{t}\}$.
   (a) Find an orthonormal basis for W.
   (b) Let $h(t) = t^2$. Use the orthonormal basis obtained in (a) to obtain the "best" (closest) approximation of h in W.