Math 3B: Lecture 17

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November 7, 2017

Checking solutions

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$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}x} &= e^{\sin x} \cos x \\ y \cos x &= e^{\sin x} \cos x \end{aligned}$$

Reminder: Implicit differentiation

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$$x^2 + y^2 = 1$$

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Another example

Lets differentiate

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\end{aligned}$$

Note

We can rearrange this to get

$$y' = \frac{3 - y}{x + \sin y}$$

a differential equation. Whatever y is, as long as it obeys the above relation, it is a solution to this ODE!

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$$\int \frac{1}{f(y)}\,\mathrm{d}y = \int g(x)\,\mathrm{d}x.$$
4. solve for y!

Examples

On the board. . .