Midterm 2 practice

UCLA: Math 32B, Spring 2020

Instructor: Noah White
Date: May, 2018
Version: practice

- This exam has 4 questions, for a total of 26 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Question	Points	Score
1	10	
2	8	
3	8	
4	0	
Total:	26	

1. (a) (5 points) Let $\mathcal{D}$ be the region in the xy-plane above the x-axis and below the curve $y = 1 - x^2$. Compute the integrals

$$I_1 = \frac{1}{A} \iint_{\mathcal{D}} x \, dA \text{ and } I_2 = \frac{1}{A} \iint_{\mathcal{D}} y \, dA$$

where A is the area of $\mathcal{D}$.

(b) (5 points) Find a parameterisation $\mathbf{r}(t)$, of the curve that is the intersection of the surfaces $y = x^2$ and $x + y + z = 1$, oriented from $x = -4$ to $x = 4$, such that $t \in [0, 1]$ What is the velocity of the parameterisation?

2. (8 points) Consider the region $\mathcal{E}$ given by

$$0 \le z \le (y - x^2)^2, \quad x^2 \le y \le x.$$

Use the change of variables

$$x = u, y = v + u^2, z = wv^2,$$

to evaluate

$$\iiint_{\mathcal{E}} \frac{1}{y - x^2} \, \mathrm{d}V.$$

3. Let $\mathbf{F}$ be the vector field on $\mathbb{R}^3$ given by

$$\mathbf{F}(x, y, z) = (y \cos z - yze^x, x \cos z - ze^x, -xy \sin z - ye^x).$$

(a) (4 points) Show that $\mathbf{F}$ is conservative.

(b) (4 points) Find a potential function for $\mathbf{F}$.

4. Consider the vector field $\mathbf{F} = \left\langle yze^{(xyz)^2}, xze^{(xyz)^2}, xye^{(xyz)^2} + 3z^2 \right\rangle$. Let $\mathbf{C}$ be the curve given by the intersection of the cylinder $x^2 + (y-1)^2 = 1$ and the surface $y = 1 - z^2$ and $x \geq 0$, oriented upwards. Calculate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$. You may use the fact that $\int_{-1}^{1} e^{t^2}\, dt =$ *Hint: You wont be able to evaluate the integral directly. You need another method.*

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