Midterm 2 practice

UCLA: Math 32B, Fall 2019

Instructor: Noah White
Date: May, 2018
Version: practice

- This exam has 4 questions, for a total of 35 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Question	Points	Score
1	10	
2	8	
3	8	
4	9	
Total:	35	

1. (a) (5 points) Let $\mathcal{D}$ be the region in the xy-plane above the x-axis and below the curve $y = 1 - x^2$. Compute the integrals

$$I_1 = \frac{1}{A}\iint_{\mathcal{D}} x\, dA \text{ and } I_2 = \frac{1}{A}\iint_{\mathcal{D}} y\, dA$$

where A is the area of $\mathcal{D}$.

(b) (5 points) Parametrize the paraboloid and find the normal vector for this parametrisation.

$$x^2 + y^2 = 2z, \quad 0 \leq z \leq 1.$$

2. (8 points) Consider the region $\mathcal{E}$ given by

$$0 \leq z \leq (y - x^2)^2, \quad x^2 \leq y \leq x.$$

Use the change of variables

$$x = u, y = v + u^2, z = wv^2,$$

to evaluate

$$\int_{\mathcal{E}} \frac{1}{y - x^2} \, \mathrm{d}V.$$

3. Let $\mathbf{F}$ be the vector field on $\mathbb{R}^3$ given by

$$\mathbf{F}(x, y, z) = (y \cos z - yze^x, x \cos z - ze^x, -xy \sin z - ye^x).$$

(a) (4 points) Show that $\mathbf{F}$ is conservative.

(b) (4 points) Find a potential function for $\mathbf{F}$.

4. In this question we will calculate the surface area of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{a^2} + z^2 = 1$.
   (a) (4 points) Find a parameterisation of the ellipsoid given above.
   (b) (5 points) Find the normal vector to this parameterisation and its length.

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