This wee on the problem set we are practice finding maxima and minima of functions. The first two questions you should be able to master quickly. Questions 3-8 are more difficult and require thought. Don't be demoralised if you don't solve these quickly, I don't expect you to get them all this week. Lastly question 9 provides some practice calculating antiderivatives.

Homework: The first homework will be due on Friday 7 October, at 2pm, the *start* of the lecture. It will consist of questions:

$$2(a),\ 3 \text{ and } 8$$

*Numbers in parentheses indicate the question has been taken from the textbook:

S. J. Schreiber, *Calculus for the Life Sciences*, Wiley,

and refer to the section and question number in the textbook.

1. (4.2) Find the critical points and classify them (as global/local maxima/minima or neither) using the *first derivative test.*
   (a) (4.2-6) $f(x) = 10 + 6x - x^2$
   (b) (4.2-7) $f(t) = t^2 e^{-t}$
   (c) (4.2-9) $f(x) = \frac{x}{1-x}$
   (d) (4.2-12) $y = e^{t^2-2t+1}$
2. (4.2) Find the critical points and classify them (as global/local maxima/minima or neither) using the *second derivative test.*
   (a) (4.2-14) $y = 1 - \exp\left(-x^2\right)$
   (b) (4.2-15) $y = x + \frac{1}{2+x}$
   (c) (4.2-16) $y = \frac{2x^2-x^4}{4}$
3. (4.2-35) In Problem 30 of 4.1 (see problem 9 on Problem Set 1), we saw that the weekly mortality rate during the outbreak of the Black Plague in Bombay (1905-1906) can be reasonably well described by the function
   $$f(t) = 890\,\mathrm{sech}^2(0.2t - 3.4) \quad \text{deaths/week}$$
   where t is measured in weeks. Find the global maximum of this function. Recall that
   $$\mathrm{sech}(x) = \frac{2}{e^x + e^{-x}}.$$
4. (4.2-36) A particular species of plant (for example, bamboo) flowers once and then dies. A well-known formula for the average growth rate r of a semelparous species (a species that breeds only once) that breeds at age x is
   $$r(x) = \frac{\ln\left[s(x)n(x)p\right]}{x}$$
   where $s(x)$ represents the proportion of plants that survive from germination to age x, $n(x)$ is the number of seeds produced at age x, and p is the proportion of seeds that germinate.
   (a) Find the age of reproduction that maximizes r in terms of the parameters a, b, c and p where
   $$s(x) = e^{-ax} \quad a > 0$$
   and
   $$n(x) = bx^c \quad b > 0$$
   $0 < c < 1$.
   (b) Sketch the graph of $y = r(x)$ for the case where $a = 0.2, b = 3, c = 0.8$, and $p = 0.5$.

5. (4.2-40) An epidemic spreads through a community in a such a say that t weeks after its outbreak, the number of residents who have been infected is given by a function of the form

$$f(t) = \frac{A}{1 + Ce^{kt}}$$

where A is the total number of susceptible residents. Show that the epidemic is spreading most rapidly when half the susceptible residents have been infected.

(Hint: Read the question carefully, which function should you be maximising/minimising? Is it f or a function related to f? The key word is "spreading").

6. (4.3-24) In a species of fish, the growth rate function is given by $G(x) = 1.5x(1 - x/K)$ where $K = 6000$ metric tons (i.e. the population of fish, x, is measured in metric tons rather than number of individuals). The price a fisherman can get is $p = \$600$ per metric ton. If the amount the fisherman can harvest is determined by the function $H = hx$, where teach unit of h costs the fisherman $c = \$100$, what is the maximum amount of money the fisherman can expect to make on a sustainable basis. (Hint: The fishermans sustainable income is given by $pH - ch$ where H is a sustainable harvesting rate).

(Further hint: read Example 2 in 4.2 and the preceding two paragraphs.)

7. (4.3-32) During the winter, a species of bird migrates from the coast of a mainland to an island 500 miles southeast. If the energy the bird requires to fly one mile over the water is twice more than the amount of energy it requires to fly over the land, determine what path the species should fly to minimize the amount of energy used.

8. You are a Biologist and you are studying the population of a species of fish that has just been reintroduced to a local river system. You believe the number of fish is increasing linearly over time. That is, you think the number of fish $P(t)$ at time t is of the form

$$P(t) = mt + I$$

for some number m and where I is the initial number of fish introduced. To test this theory you make n measurements.

t	$P(t)$
t_1	p_1
t_2	p_2
$\vdots$	
t_n	p_n

Your statistician friend tells you that the error between your hypothesised function $P(t) = mt$ and the data you actually described is given by

$$E(m) = \sum_{i=1}^{n} (p_i - mt_i - I)^2 .$$

(a) What m should you pick to best model the population of fish? Your answer should be given in terms of the t_i's, p_i's and I.

(b) Suppose the actual data you observe is

t	$P(t)$
1	104
2	110
3	120
4	126

Sketch the graph of $P(t) = mt$ with your chosen m from above and also plot the above data points.

9. (5.1) Find the general antiderivative of the functions
   (a) (5.1-2) $f(x) = 4$
   (b) (5.1-8) $f(t) = 4t + 4t^2$
   (c) (5.2-10) $f(x) = \frac{1}{2x}$
   (d) (5.1-14) $f(x) = 4\sin(5x)$
   (e) (5.1-16) $f(x) = 14e^x$
   (f) (5.1-22) $f(u) = 6u + 3\cos u$