# Midterm 1 practice

## UCLA: Math 31B, Spring 2017

*Instructor:* Noah White
*Date:*
*Version: practice*

- This exam has 4 questions, for a total of 40 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- All final answers should be exact values. Decimal approximations will not be given credit.
- Indicate your final answer clearly.
- Full points will only be awarded for solutions with correct working.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section: ______________________________

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| Total: | 40 | |

1. Calculate the following integrals.

   (a) (3 points)
   $$\int e^{x^2+x} + 2xe^{x^2+x}\ dx.$$

   (b) (3 points)
   $$\int_0^1 \frac{x+1}{(x+1)^2+1}\ dx.$$

   (c) (4 points)
   $$\int \sec x\ dx.$$

   *Hint: this one is quite tricky, multiply by* $\frac{\sec x+\tan x}{\sec x+\tan x}$.

**Solution:**

(a) $\int (2x+1)e^{x^2+x}\ dx = e^{x^2+x} + c.$

(b) $\int_0^1 \frac{x+1}{(x+1)^2+1}\ dx = \int_1^2 \frac{u}{u^2+1}\ dx = \left[\frac{1}{2}\ln(u^2+1)\right]_1^2 = \frac{1}{2}[\ln 5 - \ln 2].$

(c) $\int \sec x\ dx = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\ dx = \int u^{-1}\ du = \ln u + C$ where $u = \sec x + \tan x$.

2. (a) (1 point) Let $f(x) = x^3 - 3x^2 + 3x - 1$.
   What are the Taylor polynomials $T_2(x)$ and $T_4(x)$ for $f(x)$ centered at 0?

   (b) (3 points) Let $f(x) = x^3 - 3x^2 + 3x - 1$.
   What are the Taylor polynomials $T_2(x)$ and $T_4(x)$ for $f(x)$ centered at 1?

   (c) (6 points) Let $T_n(x)$ be the $n$-th Taylor polynomial for

$$f(x) = 2\cosh x + (x-3)^{2017}$$

   centered at 0.
   Find an $n$ such that $|f(1) - T_n(1)| \leq 1$.

**Solution:**

(a) $T_2(x) = -3x^2 + 3x - 1$, $T_4(x) = x^3 - 3x^2 + 3x - 1$.

(b) $T_2(x) = 0$, $T_4(x) = (x-1)^3$.

(c) Let $n = 2017$. Then $|f^{(n+1)}(u)| = 2|\cosh u| \leq e + e^{-1}$ for all $|u| \leq 1$.
So Taylor's Error Bound says that

$$|f(1) - T_n(1)| \leq \frac{(e+e^{-1})|1-0|^{n+1}}{(n+1)!} \leq \frac{e+e^{-1}}{4} \leq 1.$$

3. (a) (6 points) Suppose $a > 0$.
Calculate the following definite integral using a $u$-substitution and the knowledge of fundamental integrals which relate to inverse trigonometric functions.

$$\int_0^{\frac{a}{2}} \frac{1}{\sqrt{a^2 - x^2}}\, dx$$

(b) (3 points) Give a formula for $\sin(\arctan x)$ which does not involve trignometric functions.

(c) (1 point) Calculate $\lim_{x \to -\infty} \sin(\arctan x)$.

**Solution:**

(a) Let $u = \frac{x}{a}$. Then $dx = a\ du$. So

$$\int_0^{\frac{a}{2}} \frac{1}{\sqrt{a^2 - x^2}}\, dx = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 - u^2}}\, dx = \Big[ \arcsin(u) \Big]_0^{\frac{1}{2}} = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

(b) $\frac{x}{\sqrt{1+x^2}}$.

(c) $-1$.

4. (10 points) Calculate the following indefinite integral

$$\int \frac{3x^2 - 6x + 2}{(x-1)^2(x-2)}\, dx.$$

**Solution:** $\int \frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{2}{x-2}\, dx = \ln|x-1| - \frac{1}{x-1} + 2\ln|x-2| + c.$

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