This week on the problem set you will get practice with continuous random variables, cumulative density functions and joint PDFs. Especially challenging questions, or questions that are not appropriate for an exam, are indicated with one or more asterisks.

Homework: The third homework will be due on Friday 8 December, at 12pm, the *start* of the lecture. It will consist of questions:

8 (from supp. problems) and 5.

1. From the textbook, chapter 2, problems 7, 8, 11, 13, 15, 16.

2. From the supplementary problems, chapter 2, problem 6, 8, 9, 14. Problem 8 has been typed out below since there was a typo in the supplementary problems as posted on the textbook website.

3. If X is an exponential random variable with parameter λ find both PDF and CDF of the random variable $Y = e^X$.

4. A random variable X is said to have arcsine law if it's CDF is

$$F_X(t) = \frac{2}{\pi} \arcsin(\sqrt{t}).$$

   (a) What are the possible values that X can have?

   (b) Find the median of X.

   (c) Find the PDF of X (you might want to recall some derivatives of typical functions).

   (d) Find the mean $\mathbb{E}(X)$ (to compute the integral you might want to use the trigonometric substitution and trigonometric double angle identities).

5. Let A be the set of all pairs (x, y) which satisfy each of the conditions

$$x \geq 0,\ y \geq 0,\ 1 \leq x + y \leq 2.$$

   Let the random variables X and Y be jointly continuous with the joint PDF

$$f_{X,Y}(x, y) = \begin{cases} C(x + y), & \text{if } (x, y) \in A, \\ 0, & \text{otherwise.} \end{cases}$$

   Find the value of the constant C. Find the marginal PDFs and CDFs.

6. A food store just got a delivery of 10000 potatoes. It is known that each potato is rotten with probability 0.1. What is the expected number of healthy potatoes? Use the normal approximation to estimate the probability that there are at least 8970 healthy potatoes.

7. (Problem 8 from supplementary problems, with typo fixed.) A signal of amplitude $s = 2$ is transmitted from a satellite but is corrupted by noise, and the received signal is $Z = s + W$, where W is noise. When the weather is good, W is normal with zero mean and variance 1. When the weather is bad, W is normal with zero mean and variance 4. Good and bad weather are equally likely. In the absence of any weather information:

   (a) Calculate the PDF of Z.

   (b) Calculate the probability that Z is between 1 and 3.