This week on the problem set you will get practice manipulating sets and applying them to probability models. Especially challenging questions, or questions that are not appropriate for an exam, are indicated with one or more asterisks.

Homework: The first homework will be due on Friday 13 October, at 12pm, the *start* of the lecture. It will consist of questions:

4 and 6.

1. From the textbook, chapter 1, problems 14, 17, 50, 55.
2. From the supplementary problems, chapter 1, problems 13, 31.
3. (*) From a well shuffled, standard deck of 52 cards (`https://en.wikipedia.org/wiki/Standard_52-card_deck`), calculate the probabilities of the standard poker hands (`https://en.wikipedia.org/wiki/List_of_poker_hands`) being drawn from the top,
   - one pair,
   - two pair,
   - three of a kind,
   - straight,
   - flush,
   - full house,
   - four of a kind, and
   - straight flush.

   Conclude that this list is ordered from most to least likely.
4. If a day is sunny the probability that the next day will be rainy is $\frac{1}{2}$. If exactly k consecutive days have been rainy probability that the following day will be sunny is $\frac{1}{k+1}$. If today is sunny what is the probability that the next n days will all be rainy?
5. A burglar just stole your keychain which has n keys, exactly two of which open your apartment door. He will try to open your door using the keys one by one in a random order. Whenever a key does not open your door, he will not attempt to use this key again. What is the probability that the burglar opens your door on the kth attempt.
6. (*) You are running a day care center and today you are looking after n babies. At the end of the day, n parents arrive to collect their respective children. You are lazy and terrible at running a day care center so you give every parent a random baby.
   (a) What is the probability that no parent ends up with the correct baby?
   (b) What is the probability that exactly k parents end up with the correct baby?
   (c) What do these probabilities tend to as n becomes very large?
7. A friend of yours plays a game with his opponent. The game goes as follows: your friend draws two cards from a well shuffled deck of twenty cards marked with numbers 1 to 20 and his opponent then draws two cards from the rest of the deck. The winner is the person who has the highest card. After dealing the cards, both players have the option to withdraw from the game. Your friend asks you to tell him if it is more likely to loose (in which case he will want to withdraw) or to win (in which case he will not withdraw). After receiving his cards he tells you that the higher of his two cards is k. For which values of k would you advise him to withdraw and for which values of k would you advise him to stay in the game.
8. You have d indistinguishable balls that need to be placed into n labelled buckets. How many ways are there of doing this?

9. Suppose Ω is a finite set. How many elements does the power set $\mathcal{P}(\Omega)$ have?

10. You are walking on the plane $\mathbb{R}^2$ starting at the origin $(0,0)$. You can only take steps of size 1 and only in the positive x or y directions. So your first step will either be to $(0,1)$ or $(1,0)$.
    (a) How many paths are there to (n,m)?
    (b) How many paths are there to (n,m) that pass through (i,j) where $i \leq n$ and $j \leq m$?
    (c) If (i,j) is north-east of $(p.q)$, who many paths are there that pass though (i,j) *and* (p,q)?
    (d) What about paths that pass through (i,j) *or* (p,q)?
    (e) Repeat the previous two parts with the assumption that (i,j) is to the north-west of (p,q).

    It is fine to leave your answers in terms of binomial coefficients.