~~Ans~~ Mathematical Finance

Assignment 1

Rajat Singh
190678

Ans 1. Probability distribution :

Assuming $P(H) = p$, $P(T) = 1-p$

X	$P(x)$
1	p
2	$p(1-p)$
3	$p(1-p)^2$
⋮	
k	$p(1-p)^{k-1}$

$$P(X > m+n) = p(1-p)^{m+n}(1 + (1-p) + (1-p)^2 \cdots)$$

$$P(X > n) = p(1-p)^n(1 + (1-p) + (1-p)^2 \cdots)$$

$$P(X > m) = p(1-p)^m(1 + (1-p) + (1-p)^2 \cdots)$$

$$P((X > m+n) | (X > n)) = \frac{\cancel{p}(1-p)^{m+n}\cancel{(1 + (1-p) + (1-p)^2 \cdots)}}{\cancel{p}(1-p)^n\cancel{(1 + (1-p) + (1-p)^2 \cdots)}}$$

$$\boxed{= (1-p)^m}$$

$$P(X > m) = \cancel{p}(1-p)^m \times \frac{1}{\cancel{1-(1-p)}} \boxed{= (1-p)^m}$$

$$\therefore P((X > m+n) | (X > n)) = P(X > m)$$ Hence proved.

<u>Mathematical Finance:</u>

Ans2. Poisson distribution with parameter λ is:

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\text{mean}(\mu) = E(X) = \sum_x x p(x)$$

$$= \sum_{x=0}^{\infty} x \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda}.\lambda.\sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!}$$

$$= \lambda . e^{-\lambda} . e^{\lambda} = \lambda$$

$$\therefore \boxed{\text{Mean} = \lambda}$$

$$\text{Variance}(\sigma^2) = E(X^2) - [E(X)]^2$$

$$E(X^2) = ?\ , E(X) = \lambda$$

$$E(X^2) = \sum_{x=0}^{\infty} \frac{x^2 \lambda^x e^{-\lambda}}{x!}$$

let's find $E(X(X-1))$ to make things easier:

$$E(X(X-1)) = \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \lambda^2 e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$$= \lambda^2 e^{-\lambda} . e^{\lambda} = \lambda^2$$

$$E(X^2) - E(X) = \lambda^2$$

$$\therefore E(X^2) = \lambda^2 + \lambda$$

$$\therefore \text{Variance} = E(X^2) - (E(X))^2$$

$$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2} = \lambda$$

$$\boxed{\therefore \text{Variance} = \lambda}$$

Ans3: Let $X \sim N(u, \sigma^2)$

Let $Z = \frac{X-u}{\sigma}$

Since $u(X) = u$. (mean of X), $u(X-u) = 0$

$$\therefore u\left(\frac{X-u}{\sigma}\right) = 0$$

Since $\sigma^2(X) = \sigma^2$, $\therefore \sigma^2\left(\frac{X-u}{\sigma}\right) = 1$

$\therefore Z = N(0,1)$, hence it is standard normal variable