Attix Ch.7 pt 2

Book - don't waste your time with Attix. Just read through the Kissick Lectures.

Review:

Maxwell's Eqns + empirical E & M

Plane wave solutions - EM radiation

↓

Poynting Vector - energy transferred by EM radiation

Larmor Formula - energy radiated by oscillating dipole

↓

Thomson Scattering - free charge in low v, $\sigma = \frac{P_{out}}{S}$

Rayleigh Scattering - neutral atom ''

comment: $\sigma_{tr} = 0$

Bremsstrahlung - relativistic Larmor formula to describe radiation emitted by linear "de"acceleration

Today - Photoelectric effect, Compton effect, Pair production, Total CS & Mixtures

Photoelectric Effect

Lenard 1902: experimentally observed PE effect

"election energy does not show the slightest dependence on light intensity." ← it should based on $\bar{S} = \frac{1}{\mu}\bar{E}\times\bar{B}$

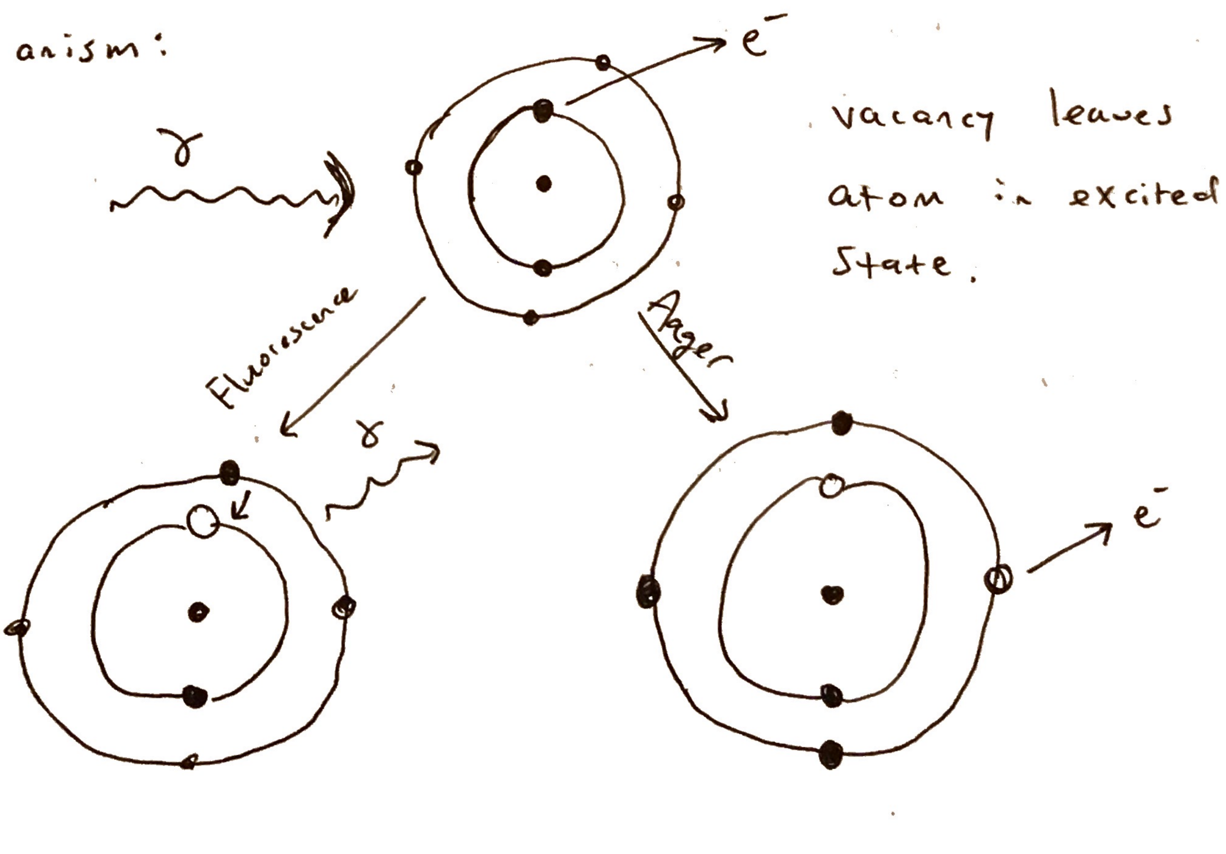
Einstein 1905: Suppose light is quantized with $E = h\nu$

Then elections emitted with

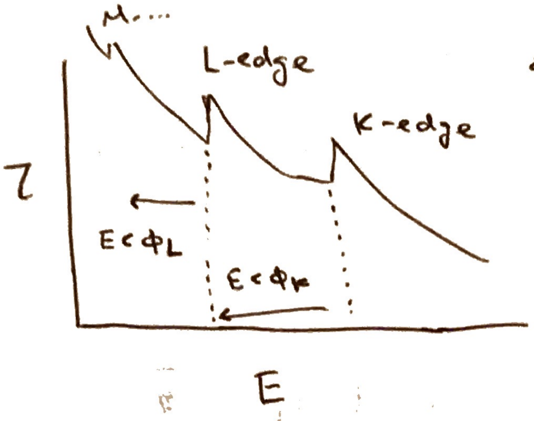
$$E_{e^-} = \begin{cases} 0 \,, & h\nu < \phi \\ h\nu - \phi \,, & h\nu > \phi \end{cases}$$

- experimentally verified → Nobel Prize 1921
- This, with Planck (1900), Compton (1923) inspires quantum revolution in physics.

Mechanism:

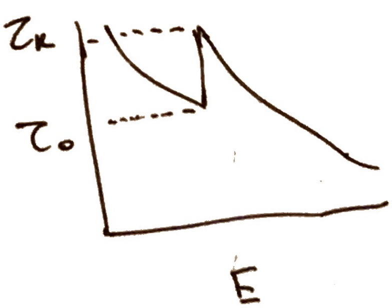


Energy Dependence: $\tau \approx \frac{Z^3}{E^3}$ - experimental



& Total CS by Energy. "edges" are where input photons have sufficient energy to free bound electrons. (work function ϕ)

The majority of PE interactions happen in outer ~~shells~~. orbitals. We care especially about inner-orbital PE because fluorescence contributes significantly. We estimate the abundance of PE interactions in an ~~shell~~ orbital with the participation fraction.



$$P_K = \frac{\tau_K - \tau_0}{\tau_K}$$

Energy Transfer:

Assume we have some measured Cross section $\frac{\tau}{\rho}$.

For every PE interaction in orbital n,

Say a fraction Y_n of the atoms

de-excite through fluorescence. Therefore $1 - Y_n$ de-excite through Auger. Only Auger de-excitations will contribute significantly to the energy transferred.

$$\frac{\tau_{tr}}{\rho} = \frac{\tau}{\rho}\left[\frac{h\nu - P_K Y_K \Phi_K - (1-P_K) P_L Y_L \Phi_L \cdots}{h\nu}\right]$$

take total energy — subtract fluorescence — from every shell...

Compton Effect

Compton 1923: Takes quantized photons having energy $h\nu$ as suggested by Einstein. Do Kinematics.

Conservation of energy:

$$mc^2 + h\nu = h\nu' + \sqrt{p_e^2 c^2 + m^2 c^4}$$

$$\rightarrow p_e^2 c^2 = (mc^2 + h\nu - h\nu')^2 - m^2 c^4$$

$$= h^2\nu^2 + h^2\nu'^2 + 2h\nu mc^2 - 2h\nu' mc^2 - 2h^2\nu\nu' \quad (1)$$

Conservation of momentum:

$$p_e^2 = (\bar{P}_\gamma - \bar{P}_{\gamma'}) \cdot (\bar{P}_\gamma - \bar{P}_{\gamma'})$$

$$= P_\gamma^2 + P_{\gamma'}^2 - 2 p_\gamma p_{\gamma'} \cos\theta$$

$$P_e^2 c^2 = h^2 \nu^2 + h^2 \nu'^2 - 2h^2 \nu\nu' \cos\theta \qquad (2)$$

$$(1) = (2)$$

$$h^2\nu^2 + h^2\nu'^2 + 2h\nu mc^2 - 2h\nu' mc^2 - 2h^2\nu\nu'$$

$$= h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta$$

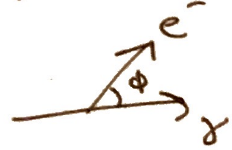
$$\rightarrow \quad mc^2(\nu - \nu') = h\nu\nu'(1-\cos\theta)$$

divide by $mc\nu\nu'$

$$\rightarrow \quad \frac{c}{\nu'} - \frac{c}{\nu} = \boxed{\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)}$$

can do same thing for $P_{\gamma'}^2 = (\bar{P}_\gamma - \bar{P}_e)\cdot(\bar{P}_\gamma - \bar{P}_e)$

$$\rightarrow \quad \boxed{\cot\phi = \left(1 + \frac{h\nu}{mc^2}\right)\tan\left(\frac{\theta}{2}\right)}$$



Compton showed that the energy of the scattered radiation should depend on angle. This was experimentally verified (Hoo ~1925), and Compton was awarded the Nobel Prize (1927).

At this point, there still hadn't been a successful attempt at modeling Compton/Thomson Scattering in QM.

Klein-Nishina Equation:

There is an alternative formulation to classical mechanics (apart from Newton's Laws, Lagrangian mech., Hamiltonian mechanics, etc.) which states that the time-evolution of a system with generalized coordinates $\bar{q}$ is governed by their Hamiltonian according to the Hamilton-Jacobi equation:

$$-\frac{\partial}{\partial t} S(\bar{q},t) = H\left(\bar{q}, \frac{\partial S}{\partial \bar{q}}, t\right)$$

where $S(q,t) = \int L\left(q, \frac{\partial S}{\partial q}, t\right) dt$.

Notice that his looks like the Schrödinger equation for

$$-\frac{\partial}{\partial t} \rightarrow i\hbar \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial x} \rightarrow -i\hbar \frac{\partial}{\partial x}, \quad S(q,t) \rightarrow \psi(\bar{r},t)$$

Klein and Gordon (1927) noticed the correspondence between Hamilton Jacobi Mechanics and the Schrodinger equation. They took the Hamilton-Jacobi Equation for an electron in a potential,

$$-\frac{1}{c^2}\left(\frac{\partial S}{\partial t} - eV\right)^2 + \sum_{k=1}^{3}\left(\frac{\partial S}{\partial t} + \frac{e}{c}A_k\right)^2 + m^2c^2 = 0$$

Where V is the electric potential and $\bar{A}$ is the magnetic potential, and used the same transformations as Schrodinger.

$$\left[\frac{-1}{c^2}\left(i\hbar\frac{\partial}{\partial t}+eV\right)^2+\sum_{k=1}^{3}\left(\frac{\hbar}{i}\frac{\partial}{\partial x_k}+\frac{e}{c}A_k\right)^2+m^2c^2\right]\psi=0$$

This is the Klein-Gordon Equation for an electron in potential $(V, \bar{A})$. They use first-order perturbation theory to approximate the momentum of the particle, from which they calculate the induced classical $\bar{E}$ & $\bar{B}$, and Poynting vector magnitude. They found

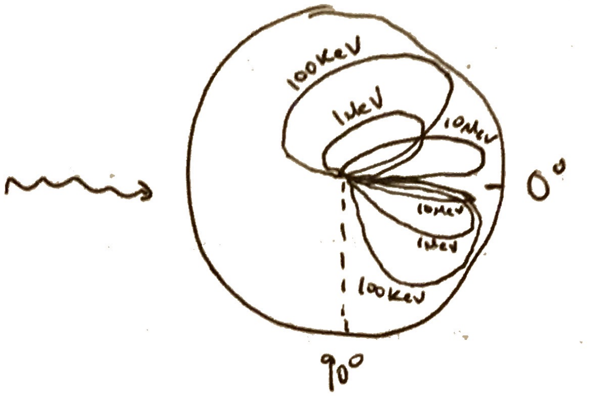
via K-G:
$$\frac{d\sigma}{d\Omega}=\frac{1}{2}\frac{e^4}{m^2c^4}\left(\frac{\nu'}{\nu}\right)^2$$

~~[illegible]~~ In 1929, Klein and Nishina ~~[illegible]~~ performed the same series of calculations, using the Dirac equation ~~[illegible]~~ instead of K-G, and found

$$\boxed{\text{K-N:}\quad \frac{d\sigma}{d\Omega}=\frac{1}{2}\frac{e^4}{m^2c^4}\left(\frac{\nu'}{\nu}\right)^2\left[\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}\right)-\sin^2\theta\right]}$$

This is very similar to the calculation we did for Rayleigh / Thomson Scattering. Integrate over solid angle to get total

$$\sigma=\int\frac{d\sigma}{d\Omega}\,d\Omega=\left\{\ \text{monster}\ \right\}$$



Preferred scattering angle depends on incident energy.

Energy Transfer Coefficient:

For calculating dose, we only care about the fraction of energy transferred to the electron. $\left(\frac{T}{h\nu}\right)$

For every σ, there is a distribution of energy transfers $\frac{d\sigma}{dT}$. To get a complete picture of energy transfer for a given σ, we want the cross section averaged over the distribution of Ts.

$$\sigma_{tr} = \int_0^{T_{max}} \frac{d\sigma}{dT}\left(\frac{T}{\nu h}\right) dT$$

where

$$\frac{d\sigma}{dT} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{d\theta}\frac{d\theta}{dT} = \frac{\pi e^4}{\alpha_0 m^2 c^4 h\nu}\left[2 - \frac{2T}{\alpha(h\nu - T)} + \frac{T^2}{\alpha^2(h\nu - T)^2} + \frac{T^2}{h\nu(h\nu - T)}\right]$$

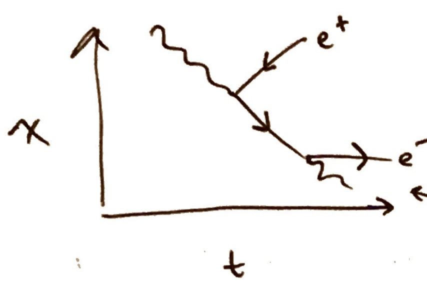
Values for $\frac{\sigma_{tr}}{\rho}$ are tabulated.

Pair production

Blackett ~~1948~~ observe d in cloud chamber. ~~1950~~
1948 Nobel prize in physics.

Mechanism:

Neutral boson creates a particle and an antiparticle.



Feynman Diagram : why does pp have to happen near a nucleus / electron? This guy.

It's not possible to conserve energy and momentum otherwise. Energy and momentum are interchangable for photons. When energy is "spent" making mass, the available momentum of the photon decreases and is not transferred to the electron / positron. This is accounted for by the recoil of a nearby nucleus or electron.

If the recoiling particle is an electron it is often ejected and this is called <u>triplet production</u>.

The electron and positron do not need to split their energy evenly or travel in opposite directions but on average...

$$\bar{T}_{e^-} = \frac{T_{available}}{2} \qquad \bar{\theta} = \frac{m_e c^2}{\bar{T}_{e^-}} \text{ (radians)}$$

The total PP cross section is

$$\kappa = \left(4.8\times10^{-28}\ \frac{cm^2}{at}\right) Z^2 \bar{P}$$

where

$$\bar{P} \approx \frac{28}{9} \ln\left(\frac{2h\nu}{mc^2}\right) - \frac{218}{27}$$

for $h\nu \ll 137mc^2 Z^{-1/3}$, and

$$\bar{P} \approx \frac{28}{9} \ln\left(183 Z^{-1/3}\right) - \frac{2}{27}$$

In this case, all of the energy is being transferred to charged particles (minus $2mc^2$), so

$$\frac{\kappa_{tr}}{\rho} = \frac{\kappa}{\rho}\left[\frac{h\nu - 2m_ec^2}{h\nu}\right]$$

Photonuclear Interactions

Similar to atomic orbitals, nuclei exist in shells, which can absorb photons and emit nucleons, similar to Auger electron emission.

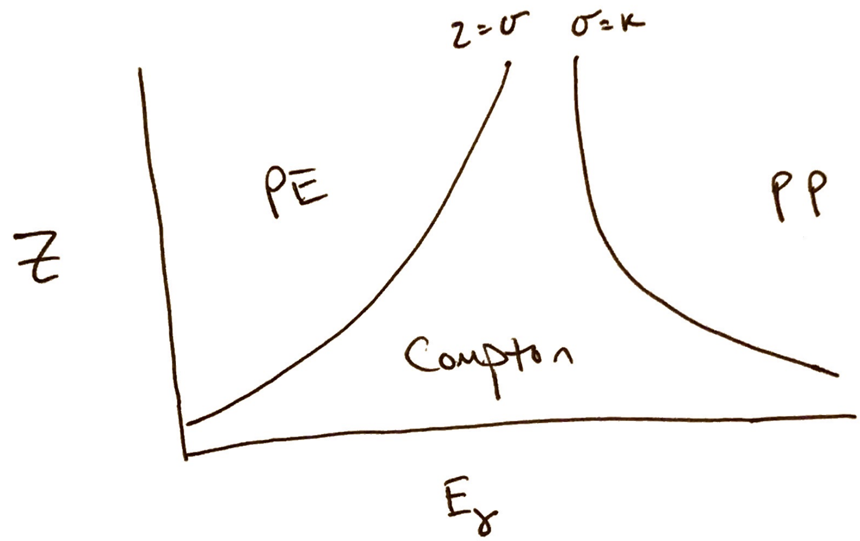
$(\gamma, n), (\gamma, p), (\gamma, \alpha), (\gamma, 2n), \ldots$

These are relatively infrequent, so we dont consider them during dose calculations. At high energies we must consider (γ, n) for shielding.

All together now!

The total mass attenuation coefficient is the sum of its components:

$$\frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_{Ray}}{\rho} + \frac{\sigma_{(\gamma,n)}}{\rho} + \frac{\sigma_{(\gamma,p)}}{\rho} + \ldots$$



We deal with transferred energy radiating away with a fudge factor

$$\frac{\mu_{en}}{\rho} = (1-g)\frac{\mu_{tr}}{\rho}$$

Absorption in mixtures is calculated wrt mass fraction.

$$\left(\frac{\mu}{\rho}\right) = \left(\frac{\mu}{\rho}\right)_A f_A + \left(\frac{\mu}{\rho}\right)_B f_B + \ldots$$