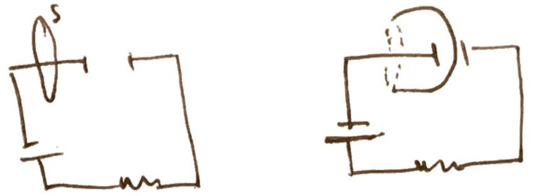
Displacement Current" Inconsistency



$$\int_{\partial S} \bar{B}\cdot d\bar{l} = \mu_0 \iint_S \bar{J}\cdot d\bar{S} + \cancel{\mu_0\epsilon_0 \iint_S \bar{E}\cdot d\bar{S}}$$

$$\mu_0\epsilon_0 \frac{d}{dt}\iint_S \bar{E}\cdot d\bar{S}$$

So what? Field equations become coupled.

Boring $\longrightarrow$ Not Boring

$$\nabla\cdot\bar{B} = \nabla\cdot\bar{E} = 0 = \bar{\nabla}\times\bar{B} \qquad\qquad \nabla\cdot\bar{B} = \nabla\cdot\bar{E} = 0$$

$$\nabla\times\bar{E} = \frac{-\partial\bar{B}}{\partial t} \qquad\qquad \nabla\times\bar{E} = \frac{-\partial\bar{B}}{\partial t}$$

$$\cancel{\nabla\times\bar{B}=0} \qquad\qquad \nabla\times\bar{B} = \mu_0\epsilon_0\frac{\partial E}{\partial t}$$

We ~~cannot~~ solve the equations on the right for $\bar{E}$

$$\frac{\partial}{\partial t}\left[\mu_0\epsilon_0\frac{\partial\bar{E}}{\partial t} = \nabla\times\bar{B}\right]$$

$$\mu_0\epsilon_0\frac{\partial^2\bar{E}}{\partial t^2} = \nabla\times\frac{\partial\bar{B}}{\partial t} \longrightarrow \frac{\partial B}{\partial t} = -\nabla\times\bar{E}$$

$$= -\nabla\times(\nabla\times\bar{E}) = -\nabla(\cancelto{0}{\nabla\cdot\bar{E}}) + \nabla^2\bar{E}$$

$$\rightarrow \frac{1}{c^2}\frac{\partial^2\bar{E}}{\partial t^2} = \nabla^2\bar{E} \qquad \text{Where } c = \sqrt{\frac{1}{\mu_0\epsilon_0}} \approx 3\times10^8 \text{ m/s}$$

Same for B $\rightarrow \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = \nabla^2 B$

["The velocity of the transverse" ...]

= "Oh shit." J. C. Maxwell, paraphrased by me.

[Greece - fur on Amber 600 BC ηλεκτρον | Optics 400 BC Lenses & Euclid

One of the first great unifications in physics.]

Plane wave solutions to Wave Eqns / Maxwells Eqns:

$$\bar{E} = \bar{E}_0 e^{i(\bar{k}\cdot\bar{x} - \omega t)} \qquad \bar{B} = \bar{B}_0 e^{i(\bar{k}\cdot\bar{x} - \omega t)}$$

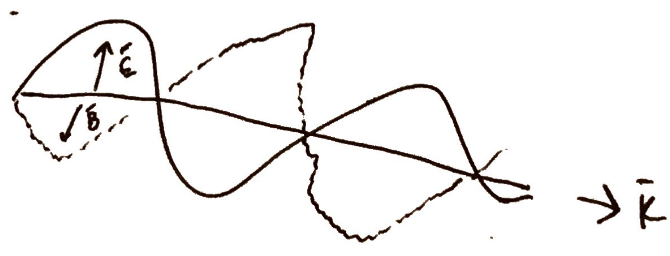
are solutions when $\bar{k} = k\hat{k}$, $\omega = ck$, and

$$\bar{k}\cdot\bar{E}_0 = \bar{k}\cdot\bar{B}_0 = 0, \quad \bar{k}\times\bar{E}_0 = kE_0\hat{B} = \omega\bar{B}_0$$

[Writing $\bar{E}$ and $\bar{B}$ as complex seems strange, but they satisfy Maxwells' Eqns and are easier to manipulate than sines & cosines. Just take the real part before doing any $\bar{E}^2$ or shenanigans.]

[I'm sure you all have seen this before, but we talk about EM waves as propagating in $\hat{k}$ with perp $\bar{E}$ and $\bar{B}$. When we talk about <u>polarization</u>, we are just referring to the direction $\hat{E}$ is pointing]

Here's the picture:



Any old plant will tell you that EM waves carry energy.

Starting with

$$\mathcal{U} = \int_V d^3x \left(\frac{\epsilon_0}{2} \bar{E}\cdot\bar{E} + \frac{1}{2\mu_0} \bar{B}\cdot\bar{B} \right)$$

Energy of Fields in V

Calculate

$$\frac{d\mathcal{U}}{dt} = \int_V d^3x \left(\epsilon_0 \bar{E}\cdot\frac{\partial \bar{E}}{\partial t} + \frac{1}{\mu_0} \bar{B}\cdot\frac{\partial \bar{B}}{\partial t} \right)$$

$$= \int_V d^3x \left(\frac{1}{\mu_0} \bar{E}\cdot(\nabla\times\bar{B}) - \bar{E}\cdot\bar{J} - \frac{1}{\mu_0} \bar{B}\cdot(\nabla\times\bar{E}) \right)$$

use $\bar{E}\cdot(\nabla\times\bar{B}) - B\cdot(\nabla\times E) = -\nabla\cdot(E\times B)$ + div thm.

$$\rightarrow \frac{d\mathcal{U}}{dt} = -\int_V d^3x\, (\bar{J}\cdot\bar{E}) - \frac{1}{\mu_0} \int_{\partial V} (\bar{E}\times\bar{B})\cdot d\bar{S}$$

work done on $\bar{J}$ by $\bar{E}$
(don't care, not radiation)

energy radiated away

Poynting Vector $\bar{S} = \frac{1}{\mu_0} \bar{E}\times\bar{B}$ $\approx$ Energy fluence

- energy loss is $\oint_\Sigma \bar{S}\cdot d\bar{\Sigma}$

Thomson Scattering: ~30 years after Maxwell, pre QM

People are begining to ask how matter is effected by EM waves.

Larmor Formula:

Given: for $r \gg \lambda$ the magnetic field is well approximated by $\bar{B}(t,\bar{x}) = \frac{-\mu_0}{4\pi r c}\,\hat{x}\times\ddot{\bar{p}}(t - r/c)$

("electric dipole approximation"

where $\quad \bar{p} = \int_V d^3x\ \rho(\bar{x})\,\bar{x}\quad$ — "center of charge"

[So we can have some arbitrary charge distribution far away doing crazy things and still describe $\bar{B}$ from]

~~for~~

In this case we can write the Poynting vector

$$\bar{S} = \frac{1}{\mu_0}\bar{E}\times\bar{B} = \frac{c}{\mu_0}|\bar{B}|^2\hat{x} = \frac{\mu_0}{16\pi^2 r^2 c}|\hat{x}\times\ddot{\bar{p}}|^2\,\hat{x}$$

So for oscillations in the $\hat{z}$ direction

$$\bar{S} = \frac{\mu_0}{16\pi^2 r^2 c}|\ddot{\bar{p}}|^2\sin^2\theta\,\hat{z}$$

take $\ddot{\bar{p}} = |\ddot{\bar{p}}|\hat{z}$

$\rightarrow |\hat{x}\times\ddot{p}| = |\ddot{p}|\sin\theta$

Integrate over sphere $\qquad P = \frac{\mu_0}{6\pi c}|\ddot{\bar{p}}|^2 \; - \; \bar{P} = \frac{\mu_0}{\cancel{8}\pi c}|\ddot{\bar{p}}|^2_{max}$

12

For $\bar{p} = \cancel{p_0}\, Qd\hat{z}e^{i\omega t}$, $\ddot{\bar{p}} = -Qd\omega^2\hat{z}e^{i\omega t}$

$$\boxed{\bar{P}_{rad} = \frac{\mu_0 Q^2 d^2 \omega^4}{12\pi c} \quad \text{- Laimor Formula}}$$

<u>Thomson Scattering</u> - First application of radiation formulas to Scattering

~ 30 yrs after Maxwell, but ~~b~~ pre QM

Suppose we have a free charged particle sitting in an ~~electr~~ oscillating electric field, and it doesn't ever move far from the origin. ~~~~

i.e. long wavelength.

Strated?-

Newton: $m\ddot{\bar{x}} = q\bar{E}$

$$\bar{x}(t) = \frac{-qE_0}{m\omega^2}\sin(\omega t)$$

$$\frac{qE_0}{m\omega^2} << \frac{c}{\omega} \Rightarrow \underbrace{\frac{qE_0}{m\omega}}_{V_{max}} << c$$

According to Laimor:

$$\bar{P}_{radiated} = \frac{\mu_0 q^2 (q^2 E_0^2)\cancel{\omega^4}}{12\pi m^2 \cancel{\omega^4} c}$$

$$\bar{S}_{incident} = \frac{1}{\mu_0}\bar{E}\times\bar{B} = \frac{cE_0^2}{\mu_0}\hat{k}\sin^2(\bar{k}\cdot\bar{x} - \omega t)$$

~~All energy in~~ Avg. energy in

$$S_{inc}^{war} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{cE_0^2}{\mu_0}\sin^2(kx-\omega t)\,dt$$

$$= \frac{cE_0^2\,\omega}{2\pi\mu_0}\cdot\frac{1}{2}\int_0^{2\pi/\omega}\left[1-\cos(\cancel{2kx - 2\omega t})\right]dt$$

$$= \frac{cE_0^2}{2\mu_0}$$

$$\frac{P_{radiated}}{S_{incident}} = \frac{\mu_0 q^4 E_0^2}{12\pi m^2 c}\cdot\frac{2\mu_0}{cE_0^2} = \boxed{\frac{\mu_0^2 q^4}{6\pi m^2 c} = \sigma_{Th} \approx 10^{-30}\ m^2}$$

Rayleigh Scattering:

We can imagine doing the same thing for a neutral atom, like hydrogen. When you put hydrogen in an oscillating field, the com does not change, but the constituents will want to move.

Given: $\cancel{\alpha \approx q^2/\omega_0^2 m}$, $\bar{p} = \alpha\bar{E}$ – good assumption $\cancel{\omega_0 = \frac{q^2}{4\pi a^3 \epsilon_0 m}}$

~~ω_0 is the resonant frequency of the atom~~

$$\alpha = \frac{q^2}{\omega_0^2 m} \qquad \omega_0 = \frac{q^2}{4\pi a^3 \epsilon_0 m}$$

$$\bar{P}_{rad} = \frac{\mu_0 p^2 \omega^4}{12\pi c} = \frac{\mu_0 \alpha^2 E_0^2 \omega^4}{12\pi c} = \frac{\mu_0 q^2 E_0^2 \omega^4}{12\pi c \omega_0^4 m^2}$$

$$\bar{S}_{inc} = \frac{c E_0^2}{2\mu_0} \rightarrow \boxed{\sigma_{Ray} = \frac{\mu_0^2 q^4}{6\pi m^2 c^2} \left(\frac{\omega}{\omega_0}\right)^4}$$

Depends on atomic structure ω_0.

Goes as ~~~~ $\sigma \sim \omega^4$ – short wavelengths scatter more. Blue light appears to come from all regions of the sky.

Not really γ scattering

Relativistic Larmor Formula & Bremsstrahlung

The derivation of the Relativistic Larmor formula is significantly more complicated, but follows the same steps as the previous derivation. Same thing, but

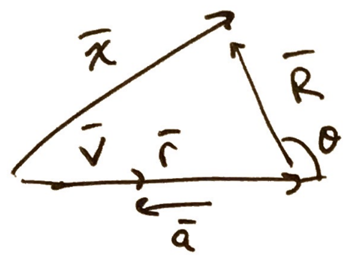
1. not far away
2. account for light travel time. relativistic

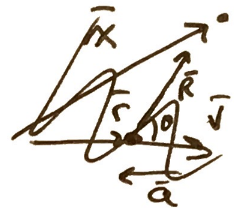
$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2 \epsilon_0 c^3} \frac{\left[\hat{R} \times \left[\left(\hat{R} - \bar{v}/c\right) \times \bar{a}\right]\right]^2}{k^5}$$

where $k = 1 - \frac{\hat{R} \cdot \bar{v}}{c}$, and $\bar{R}(t') = \bar{x} - \bar{r}(t')$

In the case of Bremsstrahlung, we assume linear acceleration in the opposite direction as $\bar{v}$

$$\hat{a} = -\hat{v} \qquad \hat{R}\cdot\bar{v} = v\cos\theta$$





$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0 c}\,\frac{[\hat{R}\times(\hat{R}\times\bar{a})]^2}{k^5} = \cancel{\frac{q^2}{16\pi^2\epsilon_0 c}\,\frac{\hat{R}\times[-a\sin\theta\,\widehat{R\times a}]^2}{k^5}}$$

$$= \frac{q^2}{16\pi^2\epsilon_0 c}\,\frac{[\hat{R}\times(-a\sin\theta(\hat{R}\times\hat{a}))]^2}{k^5}$$

Perp so we get the whole thing dotted into itself

$$\boxed{= \frac{q^2 a^2}{16\pi^2\epsilon_0 c}\,\frac{\sin^2\theta}{\left(1-\frac{v}{c}\cos\theta\right)^5}}$$

For $v \ll c$ $\rightarrow \dfrac{q^2a^2}{16\pi^2\epsilon_0 c}\sin^2\theta$, max @ $\theta/2$.

@ relativistic speeds - two lobes in the forward direction.

$$\int d\Omega\left(\frac{dP}{d\Omega}\right) = P = \frac{q^2\gamma^6 a^2}{6\pi\epsilon_0 c^3}$$

Attix Ch. 7. Gamma and X-ray Interactions 1

Introductions - Name, Undergrad, favorite movie

How's the class going? Any questions abt. Hmw?

Outline: - You will not be tested on today's - feel free to leave

For me...

Friday - Scattering in Classical Electrodynamics:

Review, Thompson ~~Scattering~~, Rayleigh ~~scattering~~,

Bremsstrahlung

Tuesday - "~~Modern~~" ~~Physics Interactions~~:

Compton, Photoelectric, Pair prod, Photonuclear,

Summary ~~as it relates to us~~.

Maxwell's Equations:

"Gauss' Law" $\nabla \cdot \bar{E} = \frac{\rho}{\epsilon_0}$

No Magnetic Monopoles $\nabla \cdot \bar{B} = 0$

Faraday's Law $\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$

Ampere's Law $\nabla \times \bar{B} = \mu_0 \left(\bar{J} + \epsilon_0 \frac{\partial \bar{E}}{\partial t} \right)$

What did Maxwell actually contribute? Why are these "Maxwell's Equations"?