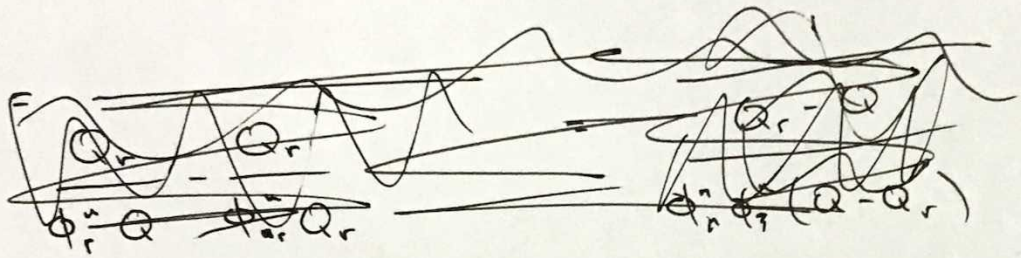
$$\left(1+\frac{k}{\phi^n}\right)Q = \left(1+\frac{k}{\phi_r^n}\right)Q_r \qquad Q - Q_r = \left(\frac{Q_r}{\phi_r^n} - \frac{Q}{\phi^n}\right)k$$

$$Q + Q\frac{k}{\phi^n} = Q_r + \frac{kQ_r}{\phi_r^n} \qquad k = \frac{Q-Q_r}{\frac{Q_r}{\phi_r^n} - \frac{Q}{\phi^n}}$$

$$k = \frac{Q-Q_r}{\frac{Q_r}{\phi_r^n} - \frac{Q}{\phi^n}} \cdot \frac{1/(Q-Q_r)}{1/(Q-Q_r)} = \frac{1}{\frac{Q_r}{\phi_r^n(Q-Q_r)} - \frac{Q}{\phi^n(Q-Q_r)}}$$



$$= \frac{\phi_r^n \phi^n (Q-Q_r)}{Q_r\phi^n - Q\phi_r^n}$$

$$1 + \frac{k}{\phi^n} = 1 + \frac{\phi_r^n(Q-Q_r)}{Q_r\phi^n - Q\phi_r^n} = \frac{Q_r\phi^n - Q\phi_r^n + \phi_r^n Q - \phi_r^n Q_r}{Q_r\phi^n - Q\phi_r^n}$$

$$= \frac{Q_r\phi^n - \phi_r^n Q_r}{Q_r\phi^n - Q\phi_r^n} = \frac{Q_r(\phi^n - \phi_r^n)}{Q_r(\phi^n - \frac{Q}{Q_r}\phi_r^n)} \cdot \frac{1/\phi_r^n}{1/\phi_r^n}$$

$$= \frac{\left(\frac{\phi}{\phi_r}\right)^n - 1}{\left(\frac{\phi}{\phi_r}\right)^n - \left(\frac{Q}{Q_r}\right)} \cdot \frac{-1}{-1} = \boxed{\frac{1-\left(\frac{\phi}{\phi_r}\right)^n}{\left(\frac{Q}{Q_r}\right) - \left(\frac{\phi}{\phi_r}\right)^n}}$$