Attix Ch 10 - Bragg Gray

Check in.

Today:

- Bragg Gray Thy + Corollaries
- "Spencer's Derivation" of Bragg Gray

Tuesday:

- Spencer-Attix Thy
- Burlin Cavity Thy

Calorimetry

Want to measure energy deposition (J/kg) in tissue. Instinct should be to measure change in temperature.

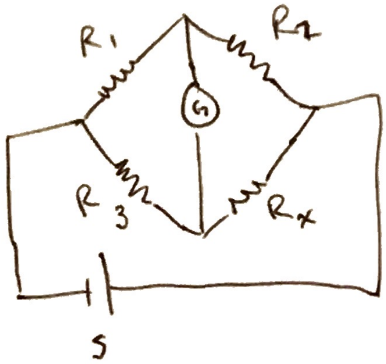
Problem: $c_{water} = 4180 \frac{J}{kg\,°C}$

$$\Delta T = \frac{2\ Gy}{4180\ \frac{J}{kg\,°C}} = 0.00048\ °C$$

$$= 480\ \mu K$$

This is a very small temperature change to measure.

Can be done in a controlled environment using a wheatstone bridge and thermistors.



$$R_x = \frac{R_2 V_s - (R_1 + R_2) V_G}{R_1 V_s + (R_1 + R_2) V_G} R_3$$

1) Apply V_s 2) Measure V_G

3) Calculate R_x 4) Calculate $T(R_x)$

This is done for detector calibration at accredited dosimetry labs, but is not feasible for the clinic.

Ion Chambers

Roentgen (1896) - air "conducts electricity" when traversed by x-rays.

Ion chambers are an ~~quick~~ easy and reliable way to measure dose, usually in air.

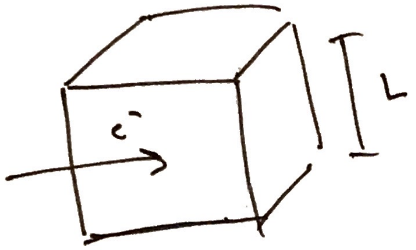
$$D_g = \frac{Q}{m}\left(\frac{W}{e}\right)_g \quad - \quad 33.97 \text{ J/C for air}$$

Problem: we dont care about the dose to the gas in the chamber; we care about the dose to the surrounding medium.

Solution: Use ~~em~~ insights from physics to estimate D_M (D_g). BG, SA, Burlin, ...

Bragg-Gray Theory

How to calculate ~~dose~~ local energy deposition in a medium? Only care about electrons.



$$\text{energy lost} = L\left(\frac{dT}{dx}\right)$$

$$\text{mass} = \rho L^3$$

$$\text{dose} = \frac{L\left(\frac{dT}{dx}\right)}{\rho L^3} = \frac{1}{L^2}\left(\frac{dT}{\rho dx}\right)_c$$

only care about collisional portion of stopping power

For a beam of N electrons

$$D = \frac{N}{L^2}\left(\frac{dT}{\rho dx}\right)_c = \phi\left(\frac{dT}{\rho dx}\right)_c$$

{Note that this is for monoenergetic ϕ.}

For a polyenergetic beam, we should use

$$\mathbb{E}_{e^- \sim \phi}\left[\frac{dT}{\rho dx}\right] = \int_{\mathbb{R}} p(T)\left(\frac{dT}{\rho dx}\right) dT$$

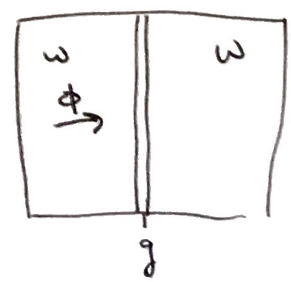
$$p(T) = \frac{\phi_T(T)}{\int_{\mathbb{R}} \phi_T(T)\, dT} = \frac{N(T)}{N}$$

$$\rightarrow \mathbb{E}_{e^- \sim \phi}\left[\frac{dT}{\rho dx}\right] = \frac{\int_{\mathbb{R}} \phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT}{\int_{\mathbb{R}} \phi_T\, dT} \equiv \bar{S}_w$$

So if we want to know the local dose in material w, we could use $D_w = \phi \bar{S}_w$.

When people talk about Bragg Gray Theory, they usually draw this picture:

If we take some medium w, and insert a slab of material g which is so thin that ϕ is not changed...



$$\frac{D_w}{D_g} = \frac{\phi_w \bar{S}_w}{\phi_g \bar{S}_g} \xrightarrow[\phi_g = \phi_w]{\text{"small cavities"}} \frac{D_w}{D_g} = \frac{\bar{S}_w}{\bar{S}_g} = \bar{S}^w_g$$

$$\boxed{D_w = \bar{S}^w_g D_g}$$ ← BG main result

Corollaries:

1) Different gasses, same medium.

$$D_w = \bar{S}^w_{g_1} \frac{Q_1}{m_1} \left(\frac{W}{e}\right)_1 = \bar{S}^w_{g_2} \frac{Q_2}{m_2} \left(\frac{W}{e}\right)_2$$

$$\rightarrow \frac{Q_1}{Q_2} = \frac{\rho_1 V_1}{\rho_2 V_2} \frac{(W/e)_2}{(W/e)_1} \bar{S}^{g_2}_{g_1}$$

$$\Rightarrow \frac{Q_1}{Q_2} \propto \frac{\rho_1}{\rho_2} \propto \frac{P_2 T_1}{P_1 T_2}$$ → Tells us how to make P_{TP} corrections

2) Same gas, different media.

Assume CPE, so dose in medium can be calculated w/ photon attenuation

$$D_{w1} = \bar{S}^{w1}_{g} \frac{Q_1}{m_1} = \Psi \left(\frac{\mu_{en}}{\rho}\right)_1$$

can write D_{w2} and set both equal to $\bar{S}_g$

$$\rightarrow \frac{Q_1}{Q_2} = \frac{V_1}{V_2} \frac{(\mu_{en}/\rho)_{w_1}}{(\mu_{en}/\rho)_{w_2}} \bar{S}^{w2}_{w_1}$$

$$\frac{Q_1}{Q_2} \propto \frac{(\mu_{en}/\rho)_{w_1}}{(\mu_{en}/\rho)_{w_2}}$$

– expect measured charge to depend on total photon attenuation. Sanity check.

Spencer's "Derivation": – not really a ~~cal~~ derivation. A calculation ~~one~~ trick where the calculation is wrong

Problem w/BG:

$$\frac{D_w}{D_g} = \frac{\int_{\mathbb{R}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT}{\int_{\mathbb{R}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT}$$

$\uparrow$

How to calculate electron spectrum for clinical beams? Monte Carlo

Spencer says assume CPE, so the electrons freed ~~by~~ effectively deposit their energy locally, i.e.

$$N T_0 \overset{CPE}{=} \int_0^{T_0} \Phi_T \left(\frac{dT}{\rho dx}\right) dT \overset{?}{\Longrightarrow} \Phi_T = \frac{N}{\left(dT/\rho dx\right)_w}$$

Spencer's spectrum satisfies the integral, but it is not the only one. For example,

$$\Phi_T = \frac{N T_0}{\left(dT/\rho dx\right)} \delta(T-\alpha), \quad 0<\alpha<T_0$$

also satisfies the integral but is clearly nonsensical.

Anyway, assuming $\Phi_T = \dfrac{N}{\left(dT/\rho dx\right)_w}$

$$D_g = N \int_0^{T_0} \frac{\left(dT/\rho dx\right)_g}{\left(dT/\rho dx\right)_w} dT$$

$$\boxed{\frac{D_g}{D_w} = \frac{1}{T_0} \int_0^{T_0} \frac{\left(\frac{dT}{\rho dx}\right)_g}{\left(\frac{dT}{\rho dx}\right)_w} dT}$$

Actually not bad for approximations.

For average compton electron energy $\bar{T}_0$, you can look up $\bar{S}^g_w\left(\bar{T}_0/2\right)$ for decent results.