Attix Ch 10 pt 2

Today:
- Spencer Attix Thy
- Burlin Cavity Thy
- Fano ~~Theory~~ Thm

Review

- Ion chamber dosimetry is more clinically viable than calorimetry.
- Photon fluences induce electron fluences, which describe local energy deposition.
- Bragg-Gray Thy relates chamber dose to medium dose for <u>small cavities</u>

$$\frac{D_w}{D_g} = \frac{\Phi_w \bar{S}_w}{\Phi_g \bar{S}_g} \xrightarrow[\text{cavities}]{\text{small}} = \frac{\bar{S}_w}{\bar{S}_g} = \bar{S}^w_g$$

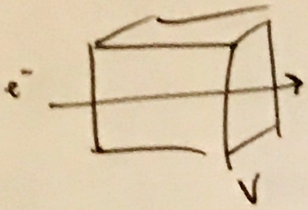
$$D_w = \bar{S}^w_g \left(\frac{W}{e}\right)_g \frac{Q}{m}$$

- Corollaries tell us about clinical use.
- Spencer's Derivation assumes electron spectrum to make $\bar{S}^w_g$ tractable.

$$\frac{D_g}{D_w} \approx \frac{1}{T_0} \int_0^{T_0} \left(\frac{dT}{\rho dx}\right)_{c,g} \Big/ \left(\frac{dT}{\rho dx}\right)_{c,w} dT$$

Spencer-Attix Thy

We made an approximation during our derivation of Bragg-Gray which isn't entirely fair.



"The energy deposited in V is the energy lost by e^-."

In this approximation, we had let the dose be equal to the charged particle analog of the KERMA. (Converted Energy per Mass (CEMA))

$$D_{e^-} \approx CEMA = L \left(\frac{dT}{\rho dx}\right)_c \frac{1}{\rho L^3}$$

In reality, electron interactions can happen in two modes

$$\left(\frac{dT}{\rho dx}\right)_c$$

"Hard" collisions

- Fast secondary electrons
- $T' \geq \Delta$
- non-local energy deposition

"Soft" collisions

- Slow secondary electrons
- $T' < \Delta$
- deposit energy on the spot

Spencer-Attix Thy handles hard collisions by partitioning the spectrum into two groups

- fast: $T \geq \Delta$, cross cavity if they strike it
- slow: $T < \Delta$, deposit on the spot

Let's rework our expression for dose...

We have currently

$$D_w = \phi \bar{S}_w = \frac{\cancel{\phi} \int_{IR} \phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT}{\cancel{\int_{IR} \phi_T \, dT}} = \int_{IR} \phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT$$

We want our electron spectrum to include δ-rays (secondary electrons from hard collisions).

Let's assume we know how to do this ($R(T)$)

$$R(T) \equiv \frac{\phi_T^\delta}{\phi_T} \rightarrow \phi_T^\delta = R(T)\,\phi_T$$

We are only interested in energy transferred to secondary electrons having energy $T' < \Delta$.

So we only want this part of the CEMA.

$$L_w(T,\Delta) \equiv \left(\frac{dT}{\rho dx}\right)_{c,w,\,T'<\Delta}$$ "Restricted Stopping Power"

Take the expression for dose and let

$$\left(\frac{dT}{\rho dx}\right)_{c,w} \rightarrow L_w(T,\Delta)$$

$$\Phi_T \rightarrow \Phi_T^{\delta} = R(T)\,\Phi_T$$

$$D_w = \int_0^{T_{max}} \Phi_T\, R(T)\, L_w(T,\Delta)\, dT$$

We should be careful. Remember that secondary electrons with $T' < \Delta$ should be delivered on the spot. Here, we've allowed slow elections to continue to count. We can fix this by changing our lower integration bound:

$$\boxed{D_w = \int_\Delta^{T_{max}} \Phi_T\, R(T)\, L_w(T,\Delta)\, dT}$$

We can write the ratio of doses as in B-G:

$$\frac{D_w}{D_g} = \frac{\int_\Delta^{T_{max}} \Phi_T\, R(T)\, L_w(T,\Delta)\, dT}{\int_\Delta^{T_{max}} \Phi_T\, R(T)\, L_g(T,\Delta)\, dT} = \frac{\bar{L}_w}{\bar{L}_g} = \bar{L}_g^w$$

$$\boxed{D_w = \bar{L}_g^w\, D_g}$$ ← Spencer-Attix Main result

Again, we have a completely legit expression for the ratio of doses, but it is completely intractable without Monte Carlo.

Spencer provides an identical calculation as he did with Bragg-Gray. Assuming

- CPE
- Source of NT_0 electrons

$$NT_0 = \int_{\Delta}^{T_0} \Phi_T \, R(T,T_0) \, L_w(T,\Delta) \, dT$$

$$\xrightarrow[\text{Math}]{\text{Spencer}} \quad \Phi_T^S = \frac{N R(T,T_0)}{\left(dT/\rho dx\right)}$$

$$\rightarrow \boxed{\frac{D_g}{D_w} \approx \frac{1}{T_0} \int_{\Delta}^{T_{max}} \frac{R(T,T_0)}{\left(dT/\rho dx\right)_{c,w}} L_g(T,\Delta) \, dT}$$

We can actually calculate this. R, L tabulated in Attix for several media.

Experimentally, slightly better than Bragg-Gray.

We use $\bar{L}^w_g$ for TG-51 etc.

Large Cavity Theory

If the cavity is large relative to the mean free path of electrons generated in the medium, CPE will be established in the cavity, i.e.

- Dose in the cavity will be due to electrons generated in the cavity.
- Dose in the medium will be due to electrons generated in the medium.

$$D_w = K_c = \Psi \left(\frac{\mu_{en}}{\rho} \right)_w \qquad D_g = \Psi \left(\frac{\mu_{en}}{\rho} \right)$$

Assuming that the cavity is still small enough for Ψ to not be perturbed.

$$\boxed{\frac{D_w}{D_g} = \frac{\int_{\mathbb{R}} \Psi_T \left(\frac{\mu_{en}}{\rho} \right)_w dT}{\int_{\mathbb{R}} \Psi_T \left(\frac{\mu_{en}}{\rho} \right)_g dT} = \left(\frac{\overline{\mu_{en}}}{\rho} \right)_g^w}$$

Burlin Cavity Theory

Small Cavities

$$\frac{D_w}{D_g} = \bar{S}^w_g ,$$

$$\frac{D_w}{D_g} = \bar{L}^w_g$$

Large Cavities

$$\frac{D_w}{D_g} = \left(\frac{\overline{\mu_{en}}}{\rho}\right)^w_g$$

Medium (?) Cavities

$$\boxed{\frac{D_w}{D_g} = d\,\bar{S}^w_g + (1-d)\left(\frac{\overline{\mu_{en}}}{\rho}\right)^w_g}$$

← Burlin CT main result

What is d? Some measure of "who" makes the dose.

$\bar{S}^w_g$ claims wall electrons make cavity dose

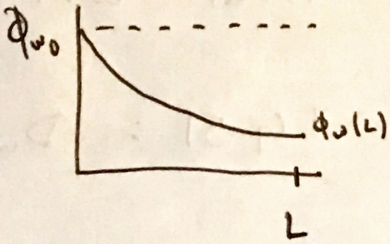
$\left(\frac{\overline{\mu_{en}}}{\rho}\right)^w_g$ claims cavity electrons make cavity dose

Model:

Chamber of length L with

(a) exponential build-down of wall electrons

(b) exponential build-up of chamber electrons



In B-G, we assumed that every electron which strikes the cavity wall crosses it. We'll let d be the extent to which this assumption fails.

$$d \equiv \frac{\text{integral fluence w/ attenuation}}{\underset{\text{(B-G)}}{\text{integral fluence w/o attenuation}}} = \frac{\int_0^L \Phi_{wo} e^{-\beta l}\, dl}{\int_0^L \Phi_{wo}\, dl}$$

$$d = \frac{\Phi_{wo}}{\Phi_{wo} L} \int_0^L e^{-\beta l}\, dl = \frac{1}{L}\left[\frac{-1}{\beta} e^{-\beta l}\right]_{l=0}^{l=L}$$

$$\boxed{d = \frac{1-e^{-\beta L}}{\beta L}}$$

where β describes attenuation of wall electrons in cavity

Empirically: $\beta = \frac{16\rho}{(T_{max} - 0.036)^{1.4}}$ or $e^{-\beta t_{max}} = 0.01$ or 0.04

Clinical Considerations

- Typical threshold $\Delta = 10$ keV

- Ion chambers usually have thin walls, which we treat with perturbation factors

$$\text{TG-51}: \quad D_w = \frac{Q}{m}\left(\frac{w}{e}\right)_g \bar{L}_g^w \cdot \left(P_{fl} \cdot P_{dis} \cdot P_{wall} \cdot P_{elec}\right)$$

Fano's Thm (slightly off topic)

Concerning irradiating a material with density ρ.

let $\rho \to \rho' = x\rho$

- electron range changes by $\dfrac{\rho}{\rho'} = \dfrac{1}{x}$

- electron production changes by $\dfrac{\rho'}{\rho} = x$

→ electron fluence is unchanged, dose is unchanged

(to first order, neglects screening effects)