

Attix Ch. 13 - Ion Chamber measurements & Calibration

OUTLINE :

- 1) Motivation for N_{gas}
- 2) TG 21 and Correction factors

Motivation

Recall that we can estimate D_w , given D_g , according to our cavity theories:

$$\text{- Bragg-Gray: } D_w = \bar{S}_g^w D_g$$

$$\text{- Spencer-Attix: } D_w = \bar{L}_g^w D_g$$

$$\text{- Large: } D_w = \left(\frac{\mu}{\rho}\right)_g^w D_g$$

$$\text{- Berlin: } D_w = \left[d \bar{S}_g^w + (1-d) \left(\frac{\bar{\mu}}{\rho}\right)_g^w \right] D_g$$

All of these are contingent on being able to accurately measure D_g .

$$D_g = \frac{1}{m} \left(\frac{\bar{w}}{c} \right) Q$$

This is all fine and dandy if we are able to accurately determine

- Q ✓
- $(\frac{\bar{w}}{c})$ ✓
- m ✗

For a function of many variables $f(x_1, x_2, \dots, x_n)$,

$$\delta f = \sqrt{\sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \delta x_i}$$

Assuming $\delta(\frac{w}{e})$, δQ are negligible — $\delta(\frac{w}{e}) \approx \delta Q \approx 0$,

$$\delta D_g = \frac{\delta m}{m^2} \left(\frac{w}{e} \right) Q = D_g \frac{\delta m}{m}$$

$$\rightarrow \boxed{\frac{\delta D_g}{D_g} = \frac{\delta m}{m} = \frac{\delta V}{V}}$$

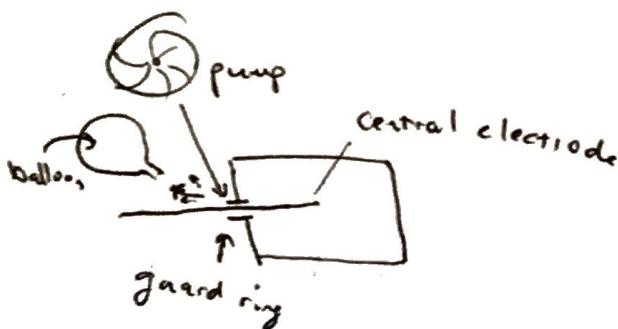
That is to say, if we want 1% uncertainty in our ion chamber dose, we need 1% uncertainty in our measurement of the chamber "volume".

This is ~~probably~~ actually hard to achieve, particularly in the small chambers used for IMRT validation.

ASIDE:

We can imagine a system to very accurately measure V (or m). We might equip our open chamber with a pump, evacuate it, and measure the volume of ~~displaced~~ air that

We removed (under the appropriate temp & pressure conditions).



The problem is that the volume we measure does not represent the active volume of the chamber. Ion chambers have a guard ring which is kept at the potential of the central electrode, to reduce leakage and restrict the active volume. This causes the active volume to be sensitive to the potential ΔV , guard ring geometry, etc.

So we have $D_g = \frac{1}{m} \left(\frac{\bar{w}}{e} \right) Q$, but we

cannot accurately know m , so

- 1) Call dose / charge the calibration coefficient N_{gas}

$$D_g = N_{\text{gas}} Q \left(\frac{1}{A_{\text{ion}}} \right)$$

Here, I have introduced the factor $A_{\text{ion}} < 1$ to account for unmeasured charge due to ion recombination. $Q_{\text{freed}} = \left(\frac{1}{A_{\text{ion}}} \right) Q_{\text{measured}}$.

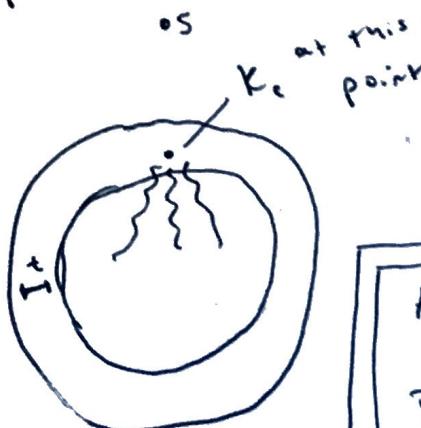
2) Design an experiment where we know D_g , then we find $N_{\text{gas}} = \frac{D_g^* A_{\text{ion}}}{Q^*}$.

Put ^(Co-60) \downarrow a source having known exposure rate Γ at some distance from our ion chamber.

For exposure $X = \frac{\Gamma AT}{r^2}$, we can find the dose

in air $D = X \left(\frac{w}{e} \right)_g$. We know, however, that

the ion chamber wall will attenuate some photons.



$$D_g \approx X \left(\frac{w}{e} \right) e^{-\mu t} (1 + \bar{x}\mu) \\ \approx X \left(\frac{w}{e} \right) A_w \beta_w$$

ASIDE: TCRE says dose = KERMA upstream at distance \bar{x}

$$D(x) = K_c(x - \bar{x}), K_c \propto e^{-\mu x}$$

$$\rightarrow K_c(x - \bar{x}) = e^{\mu \bar{x}} K_c(x) \approx (1 + \bar{x}\mu) K_c(x) \\ = D(x)$$

We've corrected for the presence of the chamber wall. Additionally, we can correct for the fact that chamber dose will be mostly due to chamber electrons.

We can model electron production with a ratio of attenuation coefficients $(\frac{\mu_{tr}}{p})_{gas}^{wall}$ and electron "absorption" with a ratio of stopping powers $(\frac{L}{p})_{gas}^{wall}$.

Our finished model for dose given exposure X is

$$D_g = X \left(\frac{\bar{w}}{e} \right)_{gas} A_w \beta_w \left(\frac{\mu_{tr}}{p} \right)_{gas}^{wall} \left(\frac{L}{p} \right)_{wall}^{gas}$$

and $D_g = N_{gas} Q \left(\frac{1}{A_{ion}} \right)$, so for our experiment,

$$N_{gas} = \frac{A_{ion}}{Q} X \left(\frac{\bar{w}}{e} \right) A_w \beta_w \left(\frac{\mu_{tr}}{p} \right)_{gas}^{wall} \left(\frac{L}{p} \right)_{wall}^{gas}$$

ASIDE:

Our formalism is slightly different when using an ion chamber with a buildup cap. Berlin-like correction give

$$N_{gas} = \frac{\frac{1}{Q} X \left(\frac{\bar{w}}{e} \right) A_{ion} A_w \beta_w}{\alpha L_g^w \left(\frac{\mu_{tr}}{p} \right)_w^2 + (1-\alpha) L_g^{cap} \left(\frac{\mu_{tr}}{p} \right)_{cap}^2}$$

3) Use the measured calibration coefficient for future measurements:

$$D_{\text{med}} = Q N_{\text{gas}} L_{\text{gas}}^{\text{med}} \prod_i P_i$$

where P_i are perturbation factors for measurement conditions.

Correction Factors

There are two sources of error in our current mechanism

$D_{\text{med}} = Q N_{\text{gas}} L_{\text{gas}}^{\text{med}}$: charge measurement error and cavity theory error. These are both addressed with factors P_i .

Charge Measurement Corrections ($P_{\text{ion}}, P_{\text{TP}}, P_{\text{elec}}, P_{\text{pol}}$)

P_{elec} - addresses disagreement between true charge and measured charge. If ion chamber and electrometer are each calibrated together, $P_{\text{elec}} = 1$.

P_{TP} - freed charge is proportional to chamber gas density.

$$\rightarrow P_{\text{TP}} = \frac{T}{T_{\text{STP}}} \frac{P_{\text{STP}}}{P}$$

P_{pol} - ion chambers will sometimes exhibit a sensitivity to polarity. Ideally, this dependence wouldn't exist, since the charge fixed is independent of polarity. We account for this by slightly increasing or decreasing our measured value based on the chamber setting.

$$P_{\text{pol}} = \frac{|Q_+(v)| + |Q_-(v)|}{2Q}$$

where $Q = Q_+$ or $Q = Q_-$ depending on the setting we are using. For example, if $Q_+ = 2 \text{ nC}$ and $Q_- = -1 \text{ nC}$, then

$$\text{In the + setting} - P_{\text{pol}} = \frac{2+1}{2+2} = \frac{3}{4} = 75\%.$$

$$\text{In the - setting} - P_{\text{pol}} = \frac{2+1}{2+1} = \frac{3}{2} = 150\%.$$

So we are effectively correcting to the average if our measurements were taken in each polarity. We measure 1.5 nC either way.

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P_{ion} - P_{ion} attempts to correct for ion recombination
in the chamber.

The number density of electrons in the chamber
has the following time derivative:

$$\frac{dN_e}{dt} = \Gamma_{\text{ion}} - \Gamma_{\text{collection}} - \Gamma_{\text{recombination}}$$

where Γ_{ion} is the ionization rate, Γ_{col} is the
collection rate, and Γ_{rec} is the recombination rate.
We can say some ~~more~~ things about these rates.

$$\Gamma_{\text{rec}} = \alpha N_e N_{\text{ion}} = \alpha N_e^2,$$

where α is a property of the gas.

$$\Gamma_{\text{col}} = \beta N_e \phi,$$

where ϕ is the applied potential and β is again a
gas property.

$$\Gamma_{\text{ion}} \propto \frac{dD}{dt}$$

$$|\Gamma_{\text{rec}}| \ll |\Gamma_{\text{ion}}|, |\Gamma_{\text{col}}|$$

CONTINUOUS IRRADIATION:

Under continuous irradiation we will reach equilibrium.

$$\frac{\partial n_e}{\partial t} = \Gamma_{ion} - \Gamma_{rec} - \Gamma_{col} = 0$$

$$\rightarrow \frac{\Gamma_{ion}}{\Gamma_{col}} - \frac{\Gamma_{rec}}{\Gamma_{col}} - 1 = \rho_{ion} - \frac{\Gamma_{rec}}{\Gamma_{col}} - 1 = 0$$

$$\rightarrow \rho_{ion} = \frac{\Gamma_{rec}}{\Gamma_{col}} + 1 = \frac{\alpha n_e^2}{\beta n_e \phi} + 1 = \frac{\alpha n_e}{\beta \phi} + 1$$

For Γ_{rec} small, $\Gamma_{ion} \approx \Gamma_{col} \rightarrow \rho_{ion} \approx \beta n_e \phi$.

$$n_e \approx \frac{\rho_{ion}}{\beta \phi}, \text{ so}$$

$$\boxed{\rho_{ion} = \frac{\alpha \rho_{ion}}{\beta^2 \phi^2} + 1 = \frac{k}{\phi^2} + 1}$$

$$\text{where } k = \frac{\cancel{\alpha} \rho_{ion}}{\beta^2}$$

PULSED IRRADIATION:

We will assume ion pairs are created in an instant, then track recombination following that pulse.

$$\frac{\partial n_e}{\partial t} = -\Gamma_{col} - \Gamma_{rec}, \quad (\Gamma_{rec} \ll \Gamma_{col})$$

$$\dot{n}_e = -\rho n_e \phi \rightarrow n_e(t) = n_e(0) e^{-\beta \phi t}$$

The total number recombined following a single pulse is

$$R = \alpha \int_0^\infty n_e^2(t) dt = \frac{\alpha n_e^2(0)}{-2\beta\phi} \left[e^{-2\beta\phi t} \right]_{t=0}^{t=\infty}$$

$$\rightarrow R = \frac{\alpha n_e^2(0)}{2\beta\phi}$$

We can write $P_{ion}(n_e(0))$

$$P_{ion} = \frac{n_e(0)}{n_e(0) - R} \stackrel{\text{small } R}{\approx} 1 + \frac{R}{n_e(0)}$$

$$\rightarrow P_{ion} = \frac{\alpha n_e(0)}{2\beta\phi} + 1 = \frac{k}{\phi} + 1$$

$$\text{where } k = \frac{\alpha n_e(0)}{2\beta}$$

MEASURING P_{ion}

If we want to operate an ion chamber at potential ϕ , the corrected reading should be in agreement with the corrected measurement at a reference potential ϕ_{ref} .

$$\left(1 + \frac{k}{\phi^n}\right) Q = \left(1 + \frac{k}{\phi_{ref}^n}\right) Q_{ref}$$

Some algebra gives

$$k = \frac{Q - Q_{ref}}{\frac{Q}{\phi^n} - \frac{Q_{ref}}{\phi_{ref}^n}}$$

We can then write

$$P_{ion}(\phi) = 1 + \frac{k}{\phi^n} = \frac{1 - \left(\frac{\phi}{\phi_{ref}}\right)^n}{\frac{Q}{Q_{ref}} - \left(\frac{\phi}{\phi_{ref}}\right)^n}$$

$n=1$ - pulsed

$n=2$ - continuous

Cavity Theory Corrections

Real ion chambers are not the ideal cavities that we assumed in chapter ~~number~~ 10.

Effects

P_{wall} - Adjustment for the presence of the chamber wall.
We use a Bethe-like deposit equation to account for production of electrons in the chamber wall.

$$[_{air}^{med}] \rightarrow \alpha \left(\frac{M_{tr}}{p} \right)_{wall} [_{air}^{wall}] + (1-\alpha) [_{air}^{med}]$$

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with $\alpha = \frac{1 - e^{-\beta t}}{\beta t}$, t the thickness of the wall,

and β modeled empirically. This gives

$$\rho_{\text{wall}} = \frac{\alpha \bar{E}_{\text{air}}^{\text{med}} \left(\frac{M_H}{P} \right)_{\text{air}} + (1-\alpha) \bar{E}_{\text{air}}^{\text{med}}}{\bar{E}_{\text{air}}^{\text{med}}}$$

P_{cpl} - This factor is meant to account for the replacement of medium with ion chamber material. It is often written as the product of two other factors; P_{gr} , P_{f1} .

P_{gr} - When taking measurements at points other than d_{max} , the dose distribution will vary over the volume of the chamber, and our charge measurement will succumb to volume-averaging effects.

Empirical data suggests that we can fix this simply by shifting our chamber $0.4 r_{\text{car}}$ "up stream", where r_{car} is the cavity radius. Or we can use the factor

$$P_{\text{gr}} = \frac{Q(d_{\text{ref}} + 0.4 r_{\text{car}})}{Q(d_{\text{ref}})}$$

P_{fi} - The fluence correction factor is generally not a big deal for photon beams (outside of the build-up region). It describes any significant changes to the electron fluence due to the absence of scattering material in the chamber volume. Determined empirically.

P_{cel} - We must account for the presence of the central electrode just as we do for P_{well} .

AAPM Task Groups / Review

- $D_g = \frac{1}{m} \left(\frac{w}{e} \right) Q$ is insufficient for small ion chambers because we cannot accurately determine m .
- We determine

$$N_{\text{gas}} = \frac{A_{\text{ion}}}{Q} \times \left(\frac{w}{e} \right) A_w \beta_w \left(\frac{M_w}{P} \right)_g^w \left(\frac{E}{P} \right)_{\text{env}}^{\infty}$$

in a controlled environment, then we find dose using N_{gas} .

According to TG-21,

$$D_w = \rho_{ion} P_{pp} P_{elec} Q N_{gas} \bar{L}_g^w P_{wall} P_{prep}$$

P_{pol} and P_{cel} are not used until TG-51.