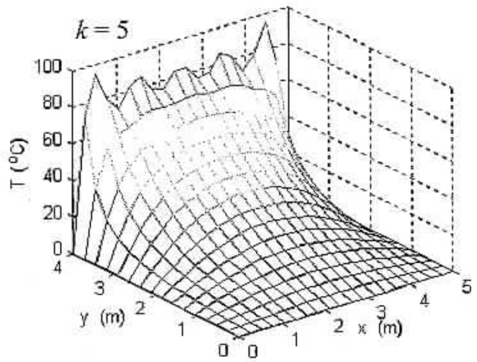
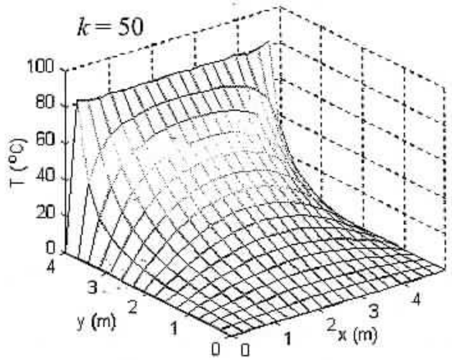
The program was executed twice, first using 5 terms ($k = 5$) in the Fourier series to calculate the temperature at each point, and then with $k = 50$. The mesh plots created in each execution are shown in the figures below. The temperature should be uniformly 80°C at $y = 4$ m. Note the effect of the number of terms (k) on the accuracy at $y = 4$ m.





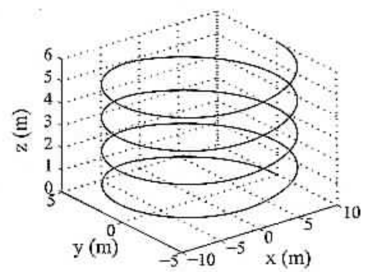
9.6 *PROBLEMS*

1. The position of a moving particle as a function of time is given by:

$$x = (1 + 0.1t)\cos(t)$$
$$y = x = (1 + 0.1t)\sin(t)$$
$$z = 0.2\sqrt{t}$$

Plot the position of the particle for $0 \le t \le 30$.

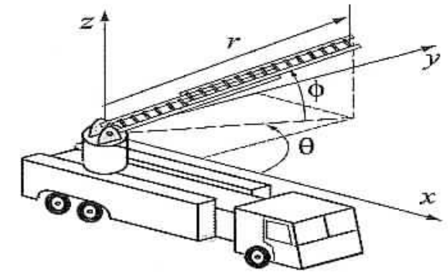
2. An elliptical staircase of height h can be modeled by the parametric equations:



$$x = r\cos(t)$$
$$y = r\sin(t)$$
$$z = \frac{ht}{2\pi n}$$

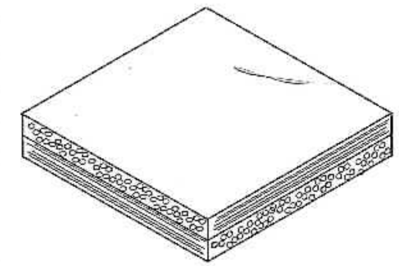
where $r = \dfrac{b}{\sqrt{1 - \varepsilon^2\cos^2(t)}}$. ε is the eccentricity of the ellipse defined by $\varepsilon = \sqrt{1 - \dfrac{b^2}{a^2}}$. a and b are the semimajor and semiminor axes of the ellipse, and n is the number of revolutions that the staircase makes. Make a 3-D plot of an elliptical staircase with $a = 10$ m, $b = 5$ m, $h = 6$ m, and $n = 4$. (Create a vector t for the domain 0 to $2\pi n$, and use the `plot3` command.)

3. The ladder of a fire truck can be elevated (increase of angle ϕ), rotated about the z axis (increase of angle θ), and extended (increase of r). Initially the ladder rests on the truck, $\phi = 0$, $\theta = 0$, and $r = 8$ m. Then the ladder is moved to a new position by raising the ladder at a rate of 5 deg/s, rotating at a rate of 8 deg/s, and extending the ladder at a rate of 0.6 m/s. Determine and plot the position of the tip of the ladder for 10 seconds.



4. Make a 3-D surface plot and a 3-D contour plot of the function $z = 4\sqrt{\dfrac{x^2}{2} + \dfrac{y^2}{2} + 1}$ in the domain $-2 \le x \le 2$ and $-2 \le y \le 2$.

5. An anti-symmetric cross-ply composite laminate has two layers where the fibers are aligned perpendicular to one another. A laminate of this type will deform into a saddle shape due to residual thermal stresses as described by the equation:



$$w = k(x^2 - y^2)$$

where x and y are the in-plane coordinates, w is the out-of-plane deflection and k is the curvature (a complicated function of material properties and geometry). Make a surface plot showing the deflection of a six-inch square plate ($-3 \le x \le 3$ in., $-3 \le y \le 3$ in.) assuming $k = 0.01$ in^{-1}.

6. Make a 3-D surface plot of the function $z = \sin\left(\dfrac{2\pi x}{60}\right)\sin\left(\dfrac{3\pi y}{60}\right)$ in the domain $0 \le x \le 60$ and $0 \le y \le 60$.

7. The van der Waals equation gives a relationship between the pressure p (in atm.), volume V, (in L), and temperature T (in K) for a real gas:

$$P = \frac{nRT}{V - b} - \frac{n^2a}{V^2}$$

where n is the number of moles, $R = 0.08206$ (L atm)/(mole K) is the gas constant, and a (in L^2 atm/mole^2), and b (in L/mole) are material constants.

Consider 1.5 moles of nitrogen ($a = 1.39$ L^{2}atm/mole^2, $b = 0.03913$ L/mole). Make a 3-D plot that shows the variation of pressure (dependent variable, z axis) with volume (independent variable, x axis) and temperature (independent variable, y axis). The domains for the volume and temperature are: $0.3 \le V \le 1.2$ L, and $273 \le T \le 473$ K.

8. Molecules of a gas in a container are moving around at different speeds. Maxwell's speed distribution law gives the probability distribution $P(v)$ as a function of temperature and speed:

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{(-Mv^2)/(2RT)}$$

where M is the molar mass of the gas in kg/mol, $R = 8.31$ J/mol-K is the gas constant, T is the temperature in degrees K, and v is the molecules speed in m/s.

Make a 3-D plot of $P(v)$ as a function of v and T for $0 \le v \le 1000$ m/s and $70 \le T \le 320$ K for oxygen (molar mass 0.032 kg/mol).

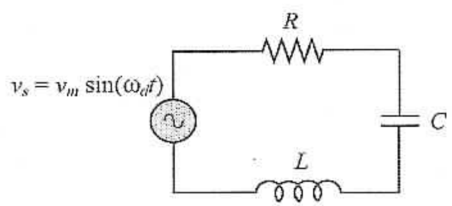
9. The heat index (sometimes called apparent temperature) is a measure of how the temperature actually feels when the effect of relative humidity is added. One formula that is used for calculating the heat index is (www.noaa.gov):

$$HI = -42.379 + 2.04901523T + 10.14333127R_H - 0.22475541\,T R_H -$$
$$-6.83783 \times 10^{-3}T^2 - 5.481717 \times 10^{-2} R_H^2 + 1.22874 \times 10^{-3} T^2 R_H +$$
$$+ 8.5282 \times 10^{-4} T R_H^2 - 1.99 \times 10^{-6} T^2 R_H^2$$

where HI is the heat index, T is the temperature in degrees F, and R_H in %.

Make a 3-D plot of HI as a function of T and R_H for $80° \le T \le 105°$F and $30 \le R_H \le 90$ %.

10. An RLC circuit with an alternating voltage source is shown. The source voltage v_s is given by $v_s = v_m \sin(\omega_d t)$ where $\omega_d = 2\pi f_d$ in which f_d is the driving frequency. The amplitude of the current, I, in this circuit is given by:



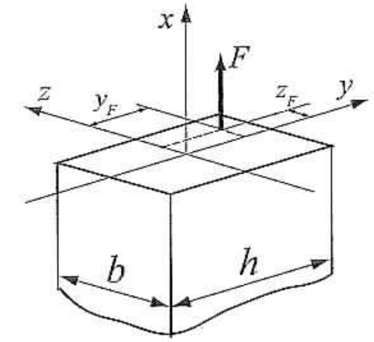
$$I = \frac{v_m}{\sqrt{R^2 + (\omega_d L - 1/(\omega_d C))^2}}$$

where R and C are the resistance of the resistor and capacitance of the capacitor, respectively. For the circuit in the figure $C = 15 \times 10^{-6}$ F, $L = 240 \times 10^{-3}$ H, and $v_m = 24$ V.

a) Make a 3-D plot of I (z axis) as a function of ω_d (x axis) for $60 \le f \le 110$ Hz, and a function of R (y axis) for $10 \le R \le 40\ \Omega$.

b) Make a plot that is a projection on the x-z plane. Estimate from this plot the natural frequency of the circuit (the frequency at which I is maximum). Compare the estimate with the calculated value of $1/(2\pi\sqrt{LC})$.

11. The normal stress σ_{xx} at point (y, z) in the cross section of a rectangular beam, due to the applied force F at a point (y_F, z_F), is given by:



$$\sigma_{xx} = \frac{F}{A} + \frac{F z_F z}{I_{yy}} + \frac{F y_F y}{I_{zz}}$$

where I_{zz} and I_{yy} are the area moments of inertia defined by:

$$I_{zz} = \frac{1}{12}bh^3 \text{ and } I_{yy} = \frac{1}{12}hb^3.$$

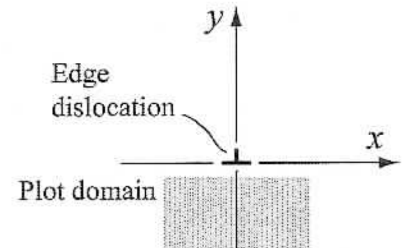
Determine and plot the normal stress in the cross-sectional area shown in the figure, given that: $h = 40$ mm, $b = 30$ mm, $y_F = -15$mm, $z_F = -10$mm, and $F = -250000$N. Plot the coordinates y and z in the horizontal plane, and the normal stress in the vertical direction.

12. A defect in a crystal lattice where a row of atoms is missing is called an edge dislocation. The stress field around an edge dislocation is given by:

$$\sigma_{xx} = \frac{-Gb}{2\pi(1-\nu)} \frac{y(3x^2 + y^2)}{(x^2 + y^2)^2}$$

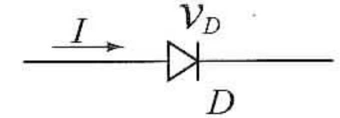
$$\sigma_{yy} = \frac{Gb}{2\pi(1-\nu)} \frac{y(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\tau_{xy} = \frac{Gb}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}$$



where G is the shear modulus, b is the Burgers vector, and ν is Poisson's ratio. Plot the stress components (each in a separate figure) due to an edge dislocation in aluminum for which $G = 27.7 \times 10^9$ Pa, $b = 0.286 \times 10^{-9}$m, and $\nu = 0.334$. Plot the stresses in the domain $-5 \times 10^{-9} \le x \le 5 \times 10^{-9}$ m and $-5 \times 10^{-9} \le y \le -1 \times 10^{-9}$ m. Plot the coordinates x and y in the horizontal plane, and the stresses in the vertical direction.

13. The current I flowing through a semiconductor diode is given by:



$$I = I_S\left(e^{\frac{qv_D}{kT}} - 1\right)$$

where $I_S = 10^{-12}$ A is the saturation current,

$q = 1.6 \times 10^{-19}$ coulombs is the elementary charge value, $k = 1.38 \times 10^{-23}$ joule/K is Boltzmann's constant, v_D is the voltage drop across the diode, and T is the temperature in Kelvins. Make a 3-D plot of I (z axis) versus v_D (x axis) for $0 \le v_D \le 0.4$, and versus T (y axis) for $290 \le T \le 320\text{K}$.

14. The equation for the streamlines for uniform flow over a cylinder is:

$$\psi(x, y) = y - \frac{y}{x^2 + y^2}$$

where ψ is the stream function. For example, if $\psi = 0$ then $y = 0$. Since the equation is satisfied for all x, the x-axis is the zero ($\psi = 0$) streamline. Observe also that the collection of points where $x^2 + y^2 = 1$ is also a streamline. Thus, the stream function above is for a cylinder of radius 1. Make a 2-D contour plot of the streamlines around a one inch radius cylinder. Set up the domain for x and y to range between -3 and 3. Use 100 for the number of contour levels. Add to the figure a plot of a circle with radius of 1. Note that MATLAB also plots streamlines inside the cylinder. This is a mathematical artifact.

15. The Verhulst model, given in the following equation, describes the growth of a population that is limited by various factors such as overcrowding, lack of resources etc.:

$$N(t) = \frac{N_\infty}{1 + \left(\frac{N_\infty}{N_0} - 1\right)e^{-rt}}$$

where $N(t)$ is the number of individuals in the population, N_0 is the initial population size, N_∞ is the maximum population size possible due to the various limiting factors, and r is a rate constant. Make a surface plot of $N(t)$ versus t and N_∞ assuming $r = 0.1\ \text{s}^{-1}$, and $N_0 = 10$. Let t vary between 0 and 100 and N_∞ between 100 and 1000.

Chapter 10
Applications in Numerical Analysis

Numerical methods are commonly used for solving mathematical problems that are formulated in science and engineering where it is difficult or even impossible to obtain exact solutions. MATLAB has a large library of functions for numerically solving a wide variety of mathematical problems. This chapter explains how to use a number of the most frequently used of these functions. It should be pointed out here that the purpose of this book is to show users how to use MATLAB. Some general information on the numerical methods is given, but the details, which can be found in books on numerical analysis, are not included.

The following topics are presented in this chapter: solving an equation with one unknown, finding a minimum or a maximum of a function, numerical integration, and solving a first-order ordinary differential equation.

10.1 Solving an Equation with One Variable

An equation with one variable can be written in the form $f(x) = 0$. The solution is the value of x where the function crosses the x axis (the value of the function is zero), which means that the function changes sign at x. An exact solution is a value of x for which the value of the function is exactly zero. If such a value does not exist or is difficult to determine, a numerical solution can be determined by finding an x that is very close to the point where the function changes its sign (crosses the x-axis). This is done by the iterative process where in each iteration the computer determines a value of x that is closer to the solution. The iterations stop when the difference in x between two iterations is smaller than some measure. In general, a function can have none, one, several, or infinite number of solutions.