11.10 PROBLEMS

1. Define x as a symbolic variable and create the two symbolic expressions:

$$S_1 = (x-4)^2-(x+3)^2+16x-4 \text{ and } S_2 = x^3-6x^2-x+30$$

Use symbolic operations to determine the simplest form of following expressions:

a) $S_1 \cdot S_2$.

b) $\frac{S_1}{S_2}$.

c) $S_1 + S_2$.

d) Use the `subs` command to evaluate the numerical value of the result from part *c* for $x = 2$.

2. Define y as a symbolic variable and create the two symbolic expressions:

$$S_1 = (\sqrt{3}+x)^2-2\left(\sqrt{3}x+\frac{x}{2}+\frac{x^2}{2}\right) \text{ and } S_2 = x^2+3x+9$$

Use symbolic operations to determine the simplest form of following expressions:

a) $S_1 \cdot S_2$.

b) $\frac{S_1}{S_2}$.

c) $S_1 + S_2$.

d) Use the `subs` command to evaluate the numerical value of the result from part *c* for $x = 4$.

3. Define u as a symbolic variable and create the two symbolic expressions:

$$Q = u^3+2u^2-25u-50 \text{ and } R = 3u^3+4u^2-75u-100$$

Use symbolic operations to determine the simplest form of the product Q/R.

4. Define x as a symbolic variable.

a) Show that the roots of the polynomial:

$$f(x) = x^5-x^4-27x^3+13x^2+134x-120$$

are 1, 2, 5, –3, and –4 by using the `factor` command.

b) Derive the equation of the polynomial that has the roots: $x = 5$, $x = -3$, $x = -2$, and $x = 4$.

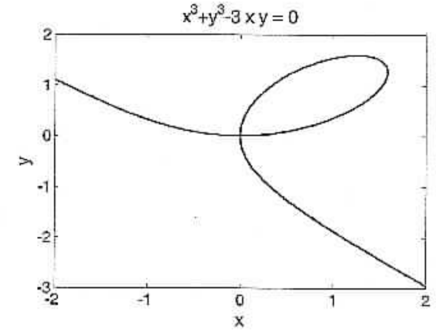
5. Use the commands from Section 11.2 to show that:

a) $\cos(3x) = 4\cos^3x-3\cos x$.

b) $\sin x\cos y = \frac{1}{2}[\sin(x-y)+\sin(x+y)]$

c)
$$\cos(x+y+z) = \cos x\cos y\cos z - \sin x\sin y\cos z - \sin x\cos y\sin z - \cos x\sin y\sin z$$

6. The folium of Descartes is the graph shown in the figure. In parametric form its equation is given by:



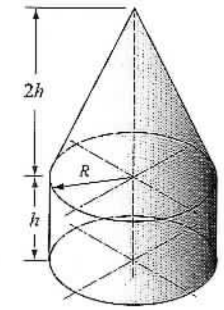
$$x = \frac{3t}{1+t^3} \text{ and } y = \frac{3t^2}{1+t^3} \text{ for } t \neq -1$$

a) Use MATLAB to show that the equation of the folium of Descartes can also be written as:

$$x^3+y^3 = 3xy$$

b) Make a plot of the folium, for the domain shown in the figure by using the `ezplot` command.

7. A cylindrical silo with a height of $h = 8$ m and a roof shaped as a cone with a height of $2h$ has a surface area of 370 m^2. Determine the radius R of the base. (Write an equation for the surface area in terms of the radius and the height. Solve the equation for the radius, and use the `double` command to obtain a numerical value.)



8. The relation between the tension T and the steady shortening velocity v in a muscle is given by the Hill equation:

$$(T+a)(v+b) = (T_0+a)b$$

where a and b are positive constants and T_0 is the isometric tension, i.e. the tension in the muscle when $v = 0$. The maximum shortening velocity occurs when $T = 0$.

a) Using symbolic operations, create the Hill equation as a symbolic expression. Then use `subs` to substitute $T = 0$, and finally solve for v to show that $v_{max} = (bT_0)/a$.

b) Use v_{max} from part *a* to eliminate the constant b from the Hill equation, and show that $v = \frac{a(T_0-T)}{T_0(T+a)}v_{max}$.

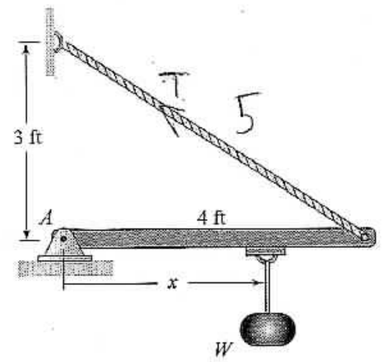
9. Consider the two circles in the x-y plane given by the equations:

$$(x-2)^2+(y-3)^2 = 16 \text{ and } x^2+y^2 = 25$$

a) Use the `ezplot` command to plot the two circles in the same plot.

b) Determine the coordinates of the points where the circles intersect.

10. A 4 ft long ~~boom~~ beam supports a weight W positioned a distance x from support A as shown. The tension T in the cable and the x and y components of the force at A (F_{Ax} and F_{Ay}) can be calculated from the equations:



$$\frac{12}{5}T - Wx = 0$$

$$F_{Ax} - \frac{4}{5}T = 0$$

$$F_{Ay} + \frac{3}{5}T - W = 0$$

a) Use MATLAB to derive expressions for the forces T, F_{Ax}, and F_{Ay} in terms of x, and W.

b) Use the `subs` command to substitute $W = 200\,\text{lb}$ into the expressions that were derived in part *a*. This will give the forces as a function of the distance x.

c) Use the `ezplot` command to plot the forces (all three in the same plot) as a function of x, for x starting at 0 and ending at 4 ft.

11. The mechanical power output P in a contracting muscle is given by:

$$P = Tv = \frac{kvT_0\left(1 - \frac{v}{v_{max}}\right)}{k + \frac{v}{v_{max}}}$$

where T is the muscle tension, v is the shortening velocity (max of v_{max}), T_0 is the isometric tension (i.e. tension at zero velocity) and k is a non-dimensional constant that ranges between 0.15 and 0.25 for most muscles. The equation can be written in non-dimensional form:

$$p = \frac{ku(1-u)}{k+u}$$

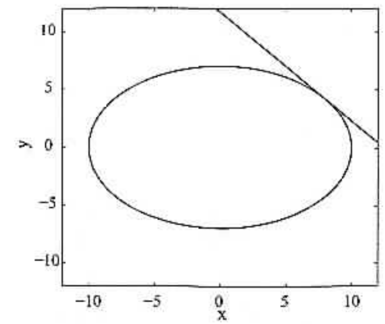
where $p = (Tv)/(T_0 v_{max})$, and $u = v/v_{max}$. Consider the case $k = 0.25$.

a) Plot p versus u for $0 \le u \le 1$.

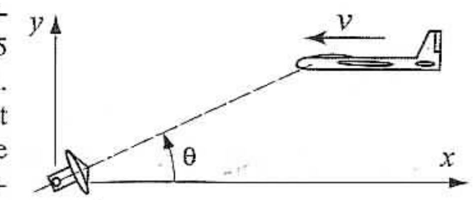
b) Use differentiation to find the value of u where p is maximum.

c) Find the maximum value of p.

12. The equation of an ellipse is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $2a$ and $2b$ are the lengths of the major and minor axis, respectively. Write a program in a a script file that first derives the equation (symbolically) of the tangent line to the ellipse at the point (x_0, y_0) on the upper part of the ellipse (i.e. for $-a < x_0 < a$ and $0 < y_0$). Then, for specific values of a, b, x_0, and y_0 the program makes a plot, like the one shown on the right, of the ellipse and the tangent line. Execute the program with $a = 10$, $b = 7$, and $x_0 = 8$.



13. A tracking radar is locked on an airplane flying at a constant altitude of 5 km, and a constant speed of 540 km/h. The airplane travels along a path that passes exactly above the radar. The radar starts the tracking when the airplane is 100 km away.



a) Derive an expression for the angle θ of the radar antenna as a function of time.

b) Derive an expression for the angular velocity of the antenna, $\frac{d\theta}{dt}$, as a function of time.

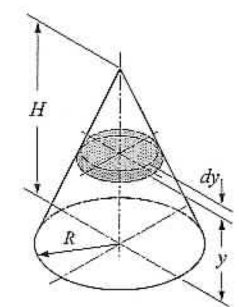
c) Make two plots on the same page, one of θ vs. time and the other of $\frac{d\theta}{dt}$ vs. time, where the angle is in degrees and the time is in minutes for $0 \le t \le 20$ min.

14. Evaluate the indefinite integral $I = \int \frac{\sin^2 x \cos x}{(2 + 3\sin x)^2} dx$.

15. Show that the differential volume element of the cone shown is given by:

$$dV = \pi R^2\left(1 - \frac{y}{H}\right)^2 dy$$

Use MATLAB to evaluate the integral of dV from 0 to H symbolically and show that the volume of the cone is $V = \frac{1}{3}\pi R^2 H$.



16. The equation of an ellipse is:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

Show that the area A enclosed by the ellipse is given by $A = \pi ab$.

believe this might be able to shown w/ symbolic math box

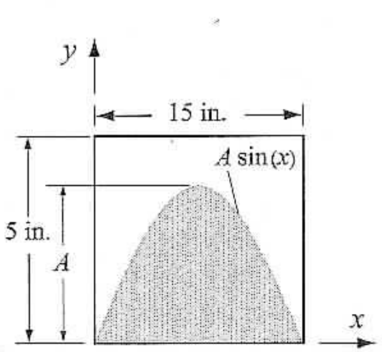
17. A ceramic tile has the design shown in the figure. The shaded area is painted red and the rest of the tile is white. The border line between the red and the white areas follows the equation:

this should be

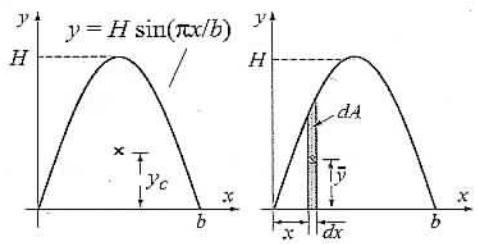
$$y = A\sin(x) \quad A\sin(kx)$$

Determine k such that the area of the white and red colors will be the same.

do you mean A?



18. Show that the location of the centroid y_c of the cross-sectional area shown is given by $y_c = \frac{H\pi}{8}$. The coordinate y_c can be calculated by:



$$y_c = \frac{\int \bar{y}\,dA}{\int dA}$$

I thought $y_c = \bar{y}$ ...?

I don't believe that this question can be done with the symbolic math toolbox

19. The *rms* value of an AC voltage is defined by:

$$v_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t')dt'}$$

where T is the period of the waveform.

a) A voltage is given by $v(t) = V\cos(\omega t)$. Show that $v_{rms} = \frac{V}{\sqrt{2}}$ and is independent of ω. (The relationship between the period T and the radian frequency ω is: $T = \frac{2\pi}{\omega}$.)

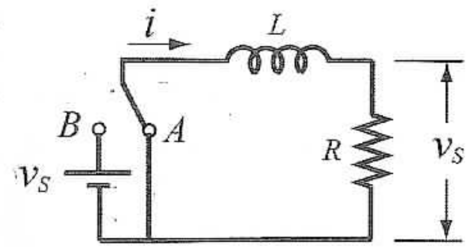
b) A voltage is given by $v(t) = 2.5\cos(350t) + 3$ V. Determine v_{rms}.

20. The spread of an infection from a single individual to a population of N uninfected persons can be described by the equation:

$$\frac{dx}{dt} = -Rx(N+1-x) \text{ with initial condition } x(0) = N$$

where x is the number of uninfected individuals and R is a positive rate constant. Solve this differential equation symbolically for $x(t)$. Also, determine symbolically the time t at which the infection rate dx/dt is maximum.

21. A resistor R ($R = 0.4\Omega$) and an inductor L ($L = 0.08$ H) are connected as shown. Initially, the switch is connected to point A and there is no current in the circuit. At $t = 0$ the switch is moved from A to B, such that the resistor and the inductor are connected to v_S ($v_S = 6$ V), and current starts flowing in the circuit. The switch remains connected to B until the voltage on the resistor reaches 5V. At that time (t_{BA}) the switch is moved back to A.



The current i in the circuit can be calculated from solving the differential equations:

$iR + L\frac{di}{dt} = v_S$ During the time from $t = 0$ and until the time when the switch is moved back to A.

$iR + L\frac{di}{dt} = 0$ From the time when the switch is moved back to A and on.

The voltage across the resistor v_R at any time is given by $v_R = iR$.

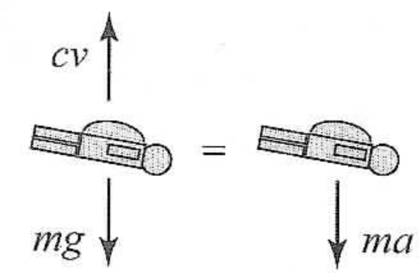
a) Derive an expression for the current i in terms of R, L, v_S, and t for $0 \le t \le t_{BA}$ by solving the first differential equation.

b) Substitute the values of R, L, and v_S in the solution of i, and determine the time t_{BA} when the voltage across the resistor reaches 5V.

c) Derive an expression for the current i in terms of R, L, and t, for $t_{BA} \le t$ by solving the second differential equation.

d) Make two plots (on the same page), one for v_R vs. t for $0 \le t \le t_{BA}$, and the other for v_R vs. t for $t_{BA} \le t \le 2t_{BA}$.

22. The velocity of a sky-diver when his parachute is still closed can be modeled by assuming that the air resistance is proportional to the velocity. From Newton's second law of motion the relationship between the mass m of the sky-diver and his velocity v is given by (down is positive):



$$mg - cv = m\frac{dv}{dt}$$

where c is a drag constant and g is the gravitational constant $g = 9.81\ \text{m/s}^2$.

a) Solve the equation for v in terms of m, g, c, and t, assuming that the initial velocity of the sky-diver is zero.

b) It is observed that 4 s after a 90 kg sky-diver jumps out of an airplane, his velocity is 28 m/s. Determine the constant c.

c) Make a plot of the sky-diver velocity as a function of time for $0 \le t \le 30$ s.

23. Determine the general solution of the differential equation:

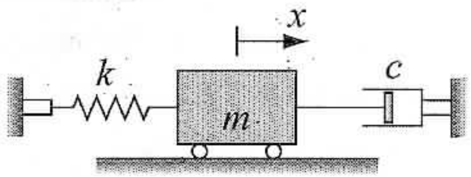
$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 1 = 0$$

Show that the solution is correct. (Derive the first and second derivatives of the solution, and then substitute back in the equation.)

24. Determine the solution of the following differential equation that satisfies the given initial conditions. Plot the solution for $0 \le x \le 5$.

$$\frac{d^3y}{dx^3} + 5\frac{dy}{dx} + 0.5x = 0,\quad y(0) = 0,\quad \left.\frac{dy}{dx}\right|_{x=0} = 0,\quad \left.\frac{d^2y}{dx^2}\right|_{x=0} = 1$$

25. Damped free vibrations can be modeled by considering a block of mass m that is attached to a spring and a dashpot as shown. From Newton's second law of motion, the displacement x of the mass as a function of time can be determined by solving the differential equation:



$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where k is the spring constant, and c is the damping coefficient of the dashpot. If the mass is displaced from its equilibrium position and then released, it will start oscillating back and forth. The nature of the oscillations depends on the size of the mass and the values of k and c.

For the system shown in the figure $m = 10\,\text{kg}$, and $k = 28\,\text{N/m}$. At time $t = 0$ the mass is displaced to $x = 0.18\,\text{m}$, and then released from rest. Derive expressions for the displacement x and the velocity v of the mass, as a function of time. Consider the following two cases:

a) $c = 3$ N-s/m.

b) $c = 50$ N-s/m.

For each case, plot the position x and the velocity v vs. time (two plots on one page). For case (*a*) take $0 \le t \le 20$ s, and for case (*b*) take $0 \le t \le 10$ s.

Appendix: Summary of Characters, Commands, and Functions

The following tables list MATLAB's characters, commands, and functions that are covered in the book. The items are grouped by subjects.

Characters and arithmetic operators

Character	Description	Page
+	Addition.	11, 58
–	Subtraction.	11, 58
*	Scalar and array multiplication.	11, 59
.*	Element-by-element multiplication of arrays.	66
/	Right division.	11, 65
\	Left division.	10, 64
./	Element-by-element right division.	66
.\	Element-by-element left division.	66
^	Exponentiation.	11
.^	Element-by-element exponentiation.	66
:	Colon; creates vectors with equally spaced elements, represents range of elements in arrays.	35, 41
=	Assignment operator.	16
( )	Parentheses; sets precedence, encloses input arguments in functions and subscripts of arrays.	11, 40, 41, 158
[ ]	Brackets; forms arrays. encloses output arguments in functions.	34, 35, 37, 158
,	Comma; separates array subscripts and function arguments, separates commands in the same line.	9, 17, 40-43, 158
;	Semicolon; suppresses display, ends row in array.	10, 37
'	Single quote; matrix transpose, creates string.	39, 50-52
...	Ellipsis; continuation of line.	9
%	Percent; denotes a comment, specifies output format.	10

Relational and logical operators

Character	Description	Page
<	Less than.	192
>	Greater than.	192
<=	Less than or equal.	192
>=	Greater than or equal.	192
==	Equal.	192
~=	Not equal.	192
&	Logical AND.	195
|	Logical OR.	195
~	Logical NOT.	195