```
global k m
k=30; m=1500; v0=90;
xspan=[0:0.2:3];                     A vector that specifies the interval of the solution.
v0mps=v0*1000/3600;                  Changing the units of v0 to m/s.
[x v]=ode45(@bumper,xspan,v0mps)     Solving the ODE.
plot(x,v)
xlabel('x (m)'); ylabel('velocity (m/s)')
```

Note that a function handle `@bumper` is used for passing the user-defined function `bumper` into `ode45`. The listing of the user-defined function with the differential equation, named `bumper`, is:

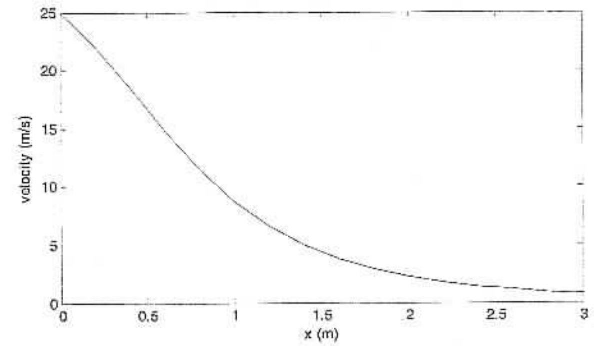
```
function dvdx=bumper(x,v)
global k m
dvdx=-(k*v^2*(x+1)^3)/m;
```

unlike integration, it appears that element-by-element operators are not required

When the script file executes (was saved as Chap10SamPro6) the vectors `x` and `v` are displayed in the Command Window (actually they are displayed on the screen one after the other, but to save room they are displayed below next to each other).

```
>> Chap10SamPro6
x =              v =
         0           25.0000
    0.2000           22.0420
    0.4000           18.4478
    0.6000           14.7561
    0.8000           11.4302
    1.0000            8.6954
    1.2000            6.5733
    1.4000            4.9793
    1.6000            3.7960
    1.8000            2.9220
    2.0000            2.2737
    2.2000            1.7886
    2.4000            1.4226
    2.6000            1.1435
    2.8000            0.9283
    3.0000            0.7607
```


The plot of the velocity as a function of distance generated by the program is:



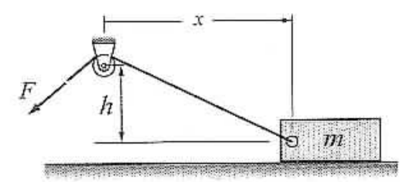
10.6 PROBLEMS

1. Determine the solution of the equation: $10(e^{-0.2x} - e^{-1.5x}) = 4$.

2. Determine the three positive roots of the equation: $e^{-0.2x}\cos(2x) = 0.15x^2 - 1$.

3. Determine the positive roots of the equation: $x^3 - 7x^2\cos(3x) + 3 = 0$.

4. A box of mass $m = 18$ kg is being pulled by a rope as shown. The force that is required to move the box is given by:

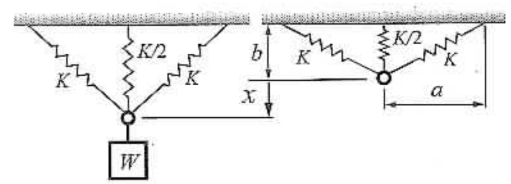


$$F = \frac{\mu m g\sqrt{x^2 + h^2}}{x + \mu h}$$

where $h = 10$ m, $\mu = 0.55$ is the friction coefficient, and $g = 9.81$ m/s². Determine the distance x when the pulling force is equal to 90 N.

5. A scale is made of three springs, as shown in the figure. Initially, the springs are not stretched. When an object is attached to the ring, the springs stretch and the ring is displaced down a distance x. The weight of the object can be expressed in terms of the distance x by:

$$W = \frac{2K}{L}(L - L_0)(b + x) + \frac{K}{2}x$$

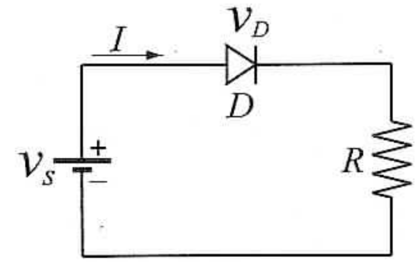


where $L_0 = \sqrt{a^2 + b^2}$ is the initial length of a diagonal spring, and

$L = \sqrt{a^2 + (b + x)^2}$ is the stretched length of a diagonal spring.

For the given scale $a = 0.18$ m, $b = 0.06$ m, and the springs' constant is $K = 2600$ N/m. Determine the distance x when a 200 N object is attached to the scale.

6. The diode in the circuit shown is forward biased. The current I flowing through the diode is given by:



$$I = I_S\left(e^{\frac{qv_D}{kT}} - 1\right)$$

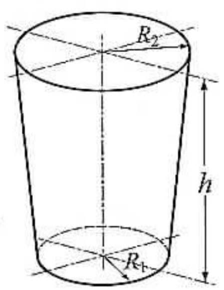
where v_D is the voltage drop across the diode, T is the temperature in Kelvins, $I_S = 10^{-12}$ A is the saturation current, $q = 1.6 \times 10^{-19}$ coulombs is the elementary charge value, and $k = 1.38 \times 10^{-23}$ joule/K is Boltzmann's constant. The current I flowing through the circuit (the same as the current in the diode) is given also by:

$$I = \frac{v_S - v_D}{R}$$

Determine v_D if $v_S = 2$ V, $T = 297$ K, and $R = 1000\ \Omega$. (Substitute I from one equation into the other equation and solve the resulting nonlinear equation.)

7. Determine the maximum of the function: $f(x) = 10(e^{-0.2x} - e^{-1.5x}) - 4$.

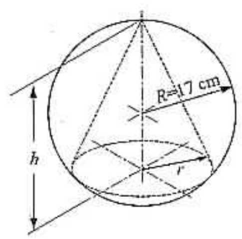
8. A paper cup shaped as a frustum of a cone with $R_2 = 1.3R_1$ is designed to have a volume of 240 cm^3. Determine R_1 and height of the cup such that the least amount of paper will be used for making the cup.



answer corresponds to $V = 266$ cm^3

9. Consider again the box that is being pulled in Problem 4. Determine the distance x at which the force that is required to pull the box is the smallest. What is the magnitude of this force?

10. Determine the dimensions (radius r and height h) and the volume of the cone with the largest volume that can be made inside of a sphere with a radius R of 17 cm.



11. Planck's radiation law gives the spectral radiancy R as a function of the wave length λ and temperature T (in degrees K):

$$R = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{(hc)/(\lambda kT)} - 1}$$

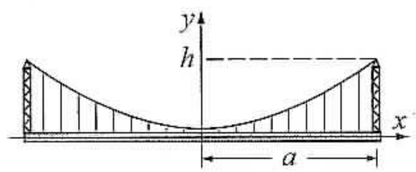
where $c = 3.0 \times 10^8$ m/s is the speed of light, $h = 6.63 \times 10^{-34}$ J-s is the Planck constant, and $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant.

Plot R as a function of λ for $0.2 \times 10^{-6} \le \lambda \le 6.0 \times 10^{-6}$ m at $T = 1500$ K, and determine the wavelength that gives the maximum R at this temperature.

12. Use MATLAB to calculate the following integral:

$$\int_1^6 \frac{3 + e^{0.5x}}{0.3x^2 + 2.5x + 1.6}dx$$

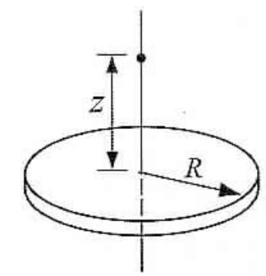
13. The length L of the main supporting cable of a suspension bridge can be calculated by:



$$L = 2\int_0^a\left(1 + \frac{4h^2}{a^4}x^2\right)^{1/2}dx$$

where a is half the length of the bridge, and h is the distance from the deck to the top of the tower where the cable is attached. Determine the length of a bridge with $a = 60$ m, and $h = 15$ m.

14. The electric field E due to a charged circular disk at a point at a distance z along the axis of the disk is given by:



$$E = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)dr$$

where σ is the charge density, ε_0 is the permittivity constant, $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2, and R is the radius of the disk. Determine the electric field at a point located 5 cm from a disk with a radius of 6 cm, charged with $\sigma = 300\ \mu$C/m^2.

15. The variation of gravitational acceleration g with altitude y is given by:

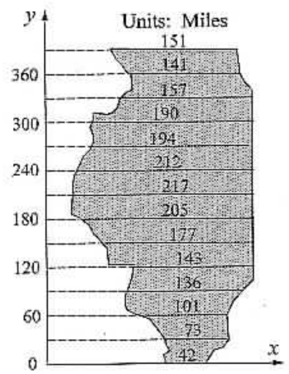
$$g = \frac{R^2}{(R + y)^2}g_0$$

where $R = 6371$ km is the radius of the earth, and $g_0 = 9.81$ m/s^2 is the gravitational acceleration at sea level. The change in the gravitational potential energy, ΔU, of an object that is raised up from the Earth is given by:

$$\Delta U = \int_0^h mgdy$$

Determine the change in the potential energy of a satellite with a mass of 500 kg that is raised from the surface of the earth to a height of 800 km.

16. An approximate map of the state of Illinois is shown in the figure. Measurements of the width of the state are marked at intervals of 30 miles. Use numerical integration to estimate the area of the state. Compare the result with the actual area of Illinois, which is 57,918 square miles.

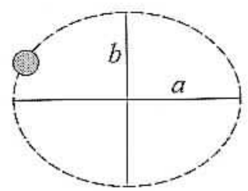


17. The time-dependent relaxation modulus $G(t)$ of many biological materials can be described by Fung's reduced relaxation function:

$$G(t) = G_\infty\left(1 + c\int_{\tau_1}^{\tau_2}\frac{e^{(-t)/x}}{x}dx\right)$$

Use numerical integration to find the relaxation modulus at 10 s, 100 s, and 1000 s. Assume $G_\infty = 5\,\text{ksi}$, $c = 0.05$, $\tau_1 = 0.05\,\text{s}$, and $\tau_2 = 500\,\text{s}$.

18. The orbit of Pluto is elliptical in shape, with $a = 5.9065 \times 10^9\,\text{km}$ and $b = 5.7208 \times 10^9\,\text{km}$. The perimeter of an ellipse can be calculated by:



$$P = 4a\int_0^{\pi/2}\sqrt{1 - k^2\sin^2\theta}\,d\theta$$

where $k = \frac{\sqrt{a^2 - b^2}}{a}$. Determine the distance Pluto travels in one orbit. Calculate the average speed that Pluto travels (in km/h) if one orbit takes about 248 years.

19. Solve:

$$\frac{dy}{dx} = x + \frac{xy}{3} \quad \text{for } 1 \le x \le 4 \text{ with } y(1) = 2.$$

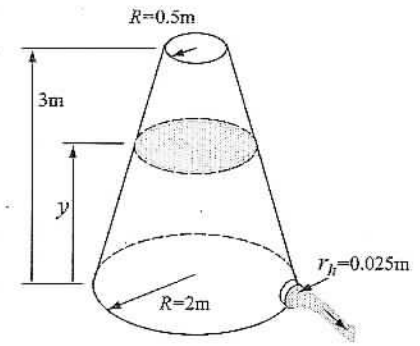
Plot the solution.

20. Solve:

$$\frac{dy}{dx} = 0.6x\sqrt{y} + 0.5y\sqrt{x} \quad \text{for } 0 \le x \le 4 \text{ with } y(0) = 2.5.$$

Plot the solution.

21. A water tank shaped as an inverted frustum cone has a circular hole at the bottom on the side, as shown. According to Torricelli's law, the speed v of the water that is discharging from the hole is given by:



$$v = \sqrt{2gh}$$

where h is the height of the water and $g = 9.81\,\text{m/s}^2$. The rate at which the height, y, of the water in the tank changes as the water flows out through the hole is given by:

$$\frac{dy}{dt} = \frac{\sqrt{2gy}r_h^2}{(2 - 0.5y)^2}$$

where r_h is the radius of the hole.

Solve the differential equation for y. The initial height of the water is $y = 2\,\text{m}$. Solve the problem for different times and find the time where $y = 0.1\,\text{m}$. Make a plot of y as a function of time.

22. The sudden outbreak of an insect population can be modeled by the equation:

$$\frac{dN}{dt} = RN\left(1 - \frac{N}{C}\right) - \frac{rN^2}{N_c^2 + N^2}$$

The first term relates to the well known logistic population growth model where N is the number of insects, R is an intrinsic growth rate, and C is the carrying capacity of the local environment. The second term represents the effects of bird predation. Its effect becomes significant when the population reaches a critical size N_c. r is the maximum value that the second term can reach at large values of N.

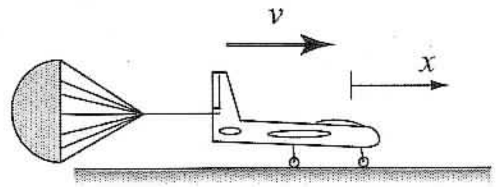
Solve the differential equation for $0 \le t \le 50$ days and two growth rates, $R = 0.55$ and $R = 0.58\,\text{days}^{-1}$, and with $N(0) = 10,000$. The other parameters are $C = 10^4$, $N_c = 10^4$, $r = 10^4$ 1/day. Make one plot comparing the two solutions and discuss why this model is called an "outbreak" model.

23. The growth of a fish is often modeled by the von Bertalanffy growth model:

$$\frac{dw}{dt} = aw^{2/3} - bw$$

where w is the weight and a and b are constants. Solve the equation for w for the case $a = 5\,\text{lb}^{1/3}$, $b = 2\,\text{day}^{-1}$, and $w(0) = 0.5\,\text{lb}$. Make sure that the selected time span is just long enough so that the maximum weight is approached. What is the maximum weight for this case? Make a plot of w as a function of time.

24. An airplane uses a parachute and other means of braking as it slows down on the runway after landing. Its acceleration is given by $a = -0.0035v^2 - 3$ m/s². Since $a = \frac{dv}{dt}$, the rate of change of the velocity is given by:



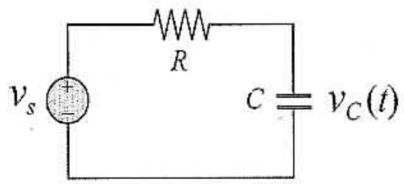
$$\frac{dv}{dt} = -0.0035v^2 - 3$$

Consider an airplane with a velocity of 300 km/h that opens its parachute and starts decelerating at $t = 0$ s.

a) By solving the differential equation, determine and plot the velocity as a function of time from $t = 0$ s until the airplane stops.

b) Use numerical integration to determine the distance x the airplane travels as a function of time. Make a plot of x vs. time.

25. An RC circuit includes a voltage source v_s, a resistor $R = 50\ \Omega$, and a capacitor $C = 0.001$ F, as shown in the figure. The differential equation that describes the response of the circuit is:



$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{1}{RC}v_s$$

where v_c is the voltage of the capacitor. Initially, $v_s = 0$, and then at $t = 0$ the voltage source is changed. Determine the response of the circuit for the following three cases:

do you mean v_c?

a) $v_s = 12$ V for $t \geq 0$.

b) $v_s = 12\sin(2 \cdot 60\pi t)$ V for $t \geq 0$.

c) $v_s = 12$ V for $0 \leq t \leq 0.01$ s, and then $v_s = 0$ for $t \geq 0.01$ s (rectangular pulse).

Each case corresponds to a different differential equation. The solution is the voltage of the capacitor as a function of time. Solve each case for $0 \leq t \leq 0.2$ s. For each case plot v_s and v_c vs. time (make two separate plots on the same page).

Chapter 11
Symbolic Math

All the mathematical operations done with MATLAB in the first ten chapters were numerical. The operations were carried out by writing numerical expressions that could contain numbers and variables with preassigned numerical values. When a numerical expression is executed by MATLAB, the outcome is also numerical (a single number or an array with numbers). The number, or numbers, are either exact or a floating point-approximated value. For example, typing 1/4 gives 0.2500 – an exact value, and typing 1/3 gives 0.3333 – an approximated value.

Many applications in math, science, and engineering require symbolic operations, which are mathematical operations with expressions that contain symbolic variables (variables that don't have specific numerical values when the operation is executed). The result of such operations is also a mathematical expression in terms of the symbolic variables. One simple example is solving an algebraic equation which contains several variables for one variable in terms of the others. If a, b, and x are symbolic variables, and $ax - b = 0$, x can be solved in terms of a and b to give $x = b/a$. Other examples of symbolic operations are analytical differentiation or integration of mathematical expressions. For instance, the derivative of $2t^3 + 5t - 8$ with respect to t is $6t^2 + 5$.

MATLAB has the capability of carrying out many types of symbolic operations. The numerical part of the symbolic operation is carried out by MATLAB exactly without approximating numerical values. For example, the result of adding $\frac{x}{4}$ and $\frac{x}{3}$ is $\frac{7}{12}x$ and not $0.5833x$.

Symbolic operations can be performed by MATLAB when the Symbolic Math Toolbox is installed. The Symbolic Math Toolbox is a collection of MATLAB functions that are used for execution of symbolic operations. The commands and functions for the symbolic operations have the same style and syntax as those for the numerical operations. The symbolic operations themselves are executed primarily by Maple®, which is mathematical software designed for this purpose. The Maple software is embedded within MATLAB and is automatically activated when a symbolic MATLAB function is executed. Maple also exists as separate independent software. That software, however, has a completely different struc-