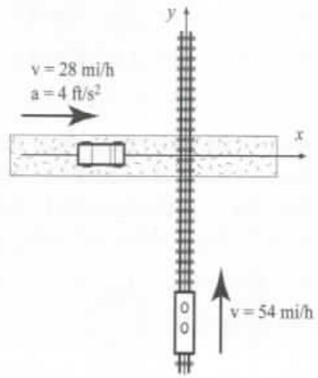
Sample Problem 3-5: Motion of two particles



A train and a car are approaching a road crossing. At $t = 0$ the train is 400 ft. south of the crossing traveling north at a constant speed of 54 mi/h. At the same time the car is 200 ft. west of the crossing traveling east at a speed of 28 mi/h and accelerating at 4 ft/s². Determine the positions of the train and the car, the distance between them, and the speed of the train relative to the car every second for the next 10 seconds.

To show the results, create an 11×6 matrix in which each row has the time in the first column and the train position, car position, distance between the train and the car, car speed, and the speed of the train relative to the car, in the next five columns, respectively.

Solution

The position of an object that moves along a straight line at a constant acceleration is given by $s = s_o + v_o t + \frac{1}{2}at^2$ where s_o and v_o are the position and velocity at $t = 0$, and a is the acceleration. Applying this equation to the train and the car gives:

$$y = -400 + v_{otrain}t \quad \text{(train)}$$

$$x = -200 + v_{ocar}t + \frac{1}{2}a_{car}t^2 \quad \text{(car)}$$

The distance between the car and the train is: $d = \sqrt{x^2 + y^2}$.

The velocity of the train is constant and in vector notation is: $\mathbf{v}_{train} = v_{otrain}\mathbf{j}$. The car is accelerating and its velocity at time t is given by: $\mathbf{v}_{car} = (v_{ocar} + a_{car}t)\mathbf{i}$. The velocity of the train relative to the car, $\mathbf{v}_{t/c}$ is given by: $\mathbf{v}_{t/c} = \mathbf{v}_{train} - \mathbf{v}_{car} = -(v_{ocar} + a_{car}t)\mathbf{i} + v_{otrain}\mathbf{j}$. The magnitude (speed) of this velocity is the length of the vector.

The problem is solved in the following program written in a script file. First a vector `t` with 11 elements for the time from 0 to 10 s is created, then the positions of the train and the car, the distance between them, and the speed of the train relative to the car at each time element are calculated.

```
v0train=54*5280/3600; v0car=28*5280/3600; acar=4;
```
Create variables for the initial velocities (in ft/s) and the acceleration.

```
t=0:10;
```
Create the vector `t`.

```
y=-400+v0train*t;
x=-200+v0car*t+0.5*acar*t.^2;
```
Calculate the train and car positions.

```
d=sqrt(x.^2+y.^2);
```
Calculate the distance between the train and car.

```
vcar=v0car+acar*t;
```
Calculate the car's velocity.


```
speed_trainRcar=sqrt(vcar.^2+v0train^2);
```
Calculate the speed of the train relative to the car.

```
table=[t' y' x' d' vcar' speed_trainRcar']
```
Create a table (see note below).

Note: In the commands above, `table` is the name of the variable that is a matrix containing the data to be displayed.

When the script file is executed, the following is displayed in the Command Window:

```
table =
```

Time (s)	Train position (ft)	Car position (ft)	Car-train distance (ft)	Car speed (ft/s)	Train speed relative to the car; (ft/s)
0	-400.0000	-200.0000	447.2136	41.0667	89.2139
1.0000	-320.8000	-156.9333	357.1284	45.0667	91.1243
2.0000	-241.6000	-109.8667	265.4077	49.0667	93.1675
3.0000	-162.4000	-58.8000	172.7171	53.0667	95.3347
4.0000	-83.2000	-3.7333	83.2837	57.0667	97.6178
5.0000	-4.0000	55.3333	55.4777	61.0667	100.0089
6.0000	75.2000	118.4000	140.2626	65.0667	102.5003
7.0000	154.4000	185.4667	241.3239	69.0667	105.0849
8.0000	233.6000	256.5333	346.9558	73.0667	107.7561
9.0000	312.8000	331.6000	455.8535	77.0667	110.5075
10.0000	392.0000	410.6667	567.7245	81.0667	113.3333

In this problem the results (numbers) are displayed by MATLAB without any text. Instructions on how to add text to output generated by MATLAB are presented in Chapter 4.

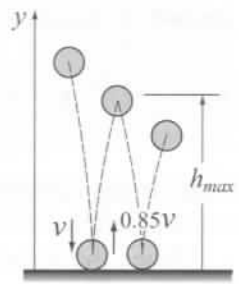
3.9 PROBLEMS

Note: Additional problems for practicing mathematical operations with arrays are provided at the end of Chapter 4.

1. For the function $y = \frac{(2x^2 - 5x + 4)^3}{x^2}$, calculate the value of y for the following values of x: –2, –1, 0, 1, 2, 3, 4, 5 using element-by-element operations.

2. For the function $y = 5\sqrt{t} - \frac{(t+2)^2}{0.5(t+1)} + 8$, calculate the value of y for the following values of t: 0, 1, 2, 3, 4, 5, 6, 7, 8 using element-by-element operations.

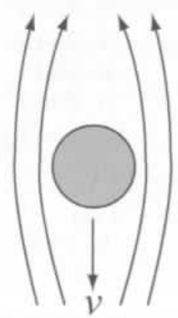
3. A ball that is dropped on the floor bounces back up many times, reaching a lower height after each bounce. When the ball impacts the floor its rebound velocity is 0.85 times the impact velocity. The velocity v that a ball hits the floor after being dropped from a height h is given by $v = \sqrt{2gh}$, where $g = 9.81\ \text{m/s}^2$ is the acceleration of the Earth. The maximum height h_{max} that a ball reaches is given by $h_{max} = \frac{v^2}{2g}$, where v is the upward velocity after impact. Consider a ball that is dropped from a height of 2 m. Determine the height the ball reaches after the first 8 bounces. (Calculate the velocity of the ball when it hit the floor for the first time. Derive a formula for h_{max} as a function of the bounce number. Then create a vector $n = 1, 2, \ldots, 8$ and use the formula (use element-by element operations) to calculate a vector with the values of h_{max} for each n.)



4. If a basketball is dropped down from a helicopter, its velocity as a function of time $v(t)$ can be modeled by the equation:

$$v(t) = \sqrt{\frac{2mg}{\rho A C_d}}\left(1 - e^{-\sqrt{\frac{\rho g C_d A}{2m}}\,t}\right)$$

where $g = 9.81\ \text{m/s}^2$ is the gravitation of the Earth, $C_d = 0.5$ is the drag coefficient, $\rho = 1.2\ \text{kg/m}^3$ is the density of air, $m = 0.624$ kg is the mass of the basketball, and $A = \pi r^2$ is the projected area of the ball ($r = 0.117$ m is the radius). Determine the velocity of the basketball for $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,$ and 10 s. Note that initially the velocity increases rapidly, but then due to the resistance of the air, the velocity increases more gradually. Eventually the velocity approaches a limit that is called the terminal velocity.

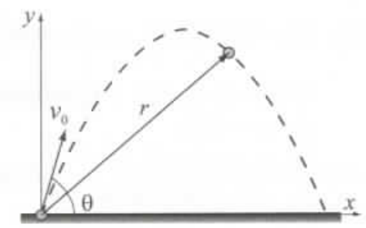


5. The length $|\mathbf{u}|$ (magnitude) of a vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = 14\mathbf{i} + 25\mathbf{j} - 10\mathbf{k}$, determine its length two ways:
   a) Define the vector in MATLAB, and then write a mathematical expression that uses the components of the vector.
   b) Define the vector in MATLAB, then use element-by element operation to create a new vector with elements that are the square of the original vector. Then use MATLAB built-in functions `sum` and `sqrt` to calculate the length. All of these can be written in one command.

6. The position as a function of time $(x(t), y(t))$ of a projectile fired with a speed of v_0 at an angle θ is given by:

$$x(t) = v_0 \cos\theta \cdot t \qquad y(t) = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$$

where $g = 9.81\ \text{m/s}^2$ is the gravitation of the Earth. The distance r to the projectile at time t can be calculated by $r(t) = \sqrt{x(t)^2 + y(t)^2}$. Consider the case where $v_0 = 100\ \text{m/s}$ and $\theta = 79°$. Determine the distance r to the projectile for $t = 0, 2, 4, \ldots, 20$ s.



7. Two vectors are given:

   $\mathbf{u} = 4\mathbf{i} + 9\mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$

   Use MATLAB to calculate the dot product $\mathbf{u} \cdot \mathbf{v}$ of the vectors in two ways:
   a) Define **u** as a row vector and **v** as a column vector, and then use matrix multiplication.
   b) Use MATLAB built-in function `dot`.

8. Define x and y as the vectors $x = 2, 4, 6, 8, 10$ and $y = 3, 6, 9, 12, 15$. Then use them in the following expression to calculate z using element-by-element calculations.

$$z = \left(\frac{y}{x}\right)^2 + (x+y)^{\left(\frac{y-x}{x}\right)}$$

9. Define h and k as scalars, $h = 0.7$, and $k = 8.85$, and x, y and z as the vectors $x = [1, 2, 3, 4, 5]$, $y = [2.1, 2.0, 1.9, 1.8, 1.7]$, and $z = [2.0, 2.5, 3.0, 3.5, 4.0]$. Then use these variables to calculate G using element-by-element calculations for the vectors.

$$G = \frac{hx + ky}{(x+y)^h} + \frac{e^{\left(\frac{hy}{z}\right)}}{z^{(y/x)}}$$

10. Show that $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

    Do this by first creating a vector x that has the elements: 1 0.5 0.1 0.01 0.001 0.00001 and 0.0000001. Then, create a new vector y in which each element is determined from the elements of x by $\frac{e^x - 1}{x}$. Compare the elements of y with the value 1 (use format long to display the numbers).

11. Use MATLAB to show that the sum of the infinite series $4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$ converges to π. Do it by computing the sum for:
    a) $n = 100$
    b) $n = 10{,}000$
    c) $n = 1{,}000{,}000$

    For each part create a vector n in which the first element is 0, the increment is 1 and the last term is 100, 10,000, or 1,000,000. Then, use element-by-element calculation to create a vector in which the elements are $\frac{(-1)^n}{2n+1}$. Finally, use MATLAB built-in function sum to add the terms of the series (and multiply the sum by 4). Compare the values obtained in parts *a*, *b*, and *c* with the value of π. (Don't forget to type semicolons at the end of commands that otherwise will display large vectors.)

12. Use MATLAB to show that the sum of the infinite series $\sum_{n=0}^{\infty}\frac{1}{(2n+1)(2n+2)}$ converges to $ln\ 2$. Do this by computing the sum for:
    a) $n = 50$
    b) $n = 500$
    c) $n = 5{,}000$

    For each part create a vector n in which the first element is 0, the increment is 1 and the last term is 50, 500, or 5,000. Then, use element-by-element calculation to create a vector in which the elements are $\frac{1}{(2n+1)(2n+2)}$. Finally, use the function sum to add the terms of the series. Compare the values obtained in parts *a*, *b*, and *c* to $ln\ 2$.

13. Fisheries commonly estimate the growth of a fish population using the von Bertalanffy growth law:

$$L = L_{max}(1 - e^{-K(t+\tau)})$$

    where L_{max} is the maximum length, K is a rate constant, and τ is a time constant. These constants vary with species of fish. Assume $L_{max} = 50$ cm, and $\tau = 0.5$ years, calculate the length of a fish at 2 years of age for $K = 0.25$, 0.5, and 0.75 years^{-1}.

14. Create the following three matrices:

$$A = \begin{bmatrix} 5 & 2 & 4 \\ 1 & 7 & -3 \\ 6 & -10 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 11 & 5 & -3 \\ 0 & -12 & 4 \\ 2 & 6 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 14 & 1 \\ 10 & 3 & -2 \\ 8 & -5 & 9 \end{bmatrix}$$

    a) Calculate $A+B$ and $B+A$ to show that addition of matrices is commutative.
    b) Calculate $A+(B+C)$ and $(A+B)+C$ to show that addition of matrices is associative.
    c) Calculate $5(A+C)$ and $5A+5C$ to show that, when matrices are multiplied by a scalar, the multiplication is distributive.
    d) Calculate $A*(B+C)$ and $A*B+A*C$ to show that matrix multiplication is distributive.

15. Use the matrices *A*, *B*, and *C* from the previous problem to answer the following:
    a) Does $A*B = B*A$?
    b) Does $A*(B*C) = (A*B)*C$?
    c) Does $(A*B)^t = B^t*A^t$? (^{t} means transpose)
    d) Does $(A+B)^t = A^t + B^t$?

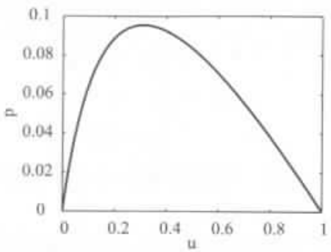
16. Two projectiles, *A* and *B*, are shot at the same instant from the same spot. Projectile *A* is shot at a speed of 680 m/s at an angle of 65° and projectile *B* is shot at a speed of 780 m/s at an angle of 42°. Determine which projectile will hit the ground first. Then, take the flying time t_f of that projectile and divide it into ten increments by creating a vector *t* with 11 equally spaced elements (the first element is 0, the last is t_f). Calculate the distance between the two projectiles at the eleven times in vector *t*.

17. The mechanical power output *P* in a contracting muscle is given by:

$$P = Tv = \frac{kvT_0\left(1 - \frac{v}{v_{max}}\right)}{k + \frac{v}{v_{max}}}$$

    where *T* is the muscle tension, *v* is the shortening velocity (max of v_{max}), T_0 is the isometric tension (i.e. tension at zero velocity) and *k* is a nondimensional constant that ranges between 0.15 and 0.25 for most muscles. The equation can be written in nondimensional form:

$$p = \frac{ku(1-u)}{k+u}$$



where $p = (Tv)/(T_0 v_{max})$, and $u = v/v_{max}$. A figure with $k = 0.25$ is shown on the right.

a) Create a vector u ranging from 0 to 1 with increments of 0.05.

b) Using $k = 0.25$, calculate the value of p for each value of u.

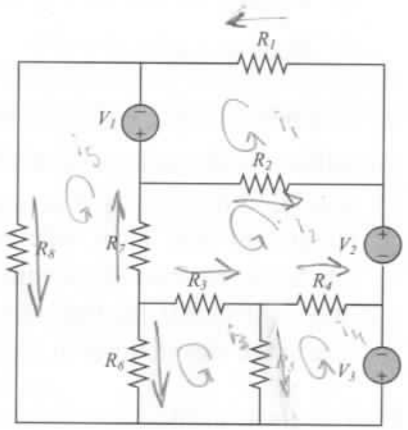
c) Using MATLAB built-in function `max`, find the maximum value of p.

d) Repeat the first three steps with increments of 0.01 and calculate the percent relative error defined by: $E = \left|\frac{p_{max_{0.01}} - p_{max_{0.05}}}{p_{max_{0.05}}}\right| \times 100$.

18. Solve the following system of five linear equations:

$$1.5x - 2y + z + 3u + 0.5w = 7.5$$
$$3x + y - z + 4u - 3w = 16$$
$$2x + 6y - 3z - u + 3w = 78$$
$$5x + 2y + 4z - 2u + 6w = 71$$
$$-3x + 3y + 2z + 5u + 4w = 54$$

19. The electrical circuit shown consists of resistors and voltage sources. Determine the current in each resistor, using the mesh current method that is based on Kirchhoff's second voltage law (see Sample Problem 3-4).



$V_1 = 38$ V, $V_2 = 20$ V, $V_3 = 24$ V
$R_1 = 15\,\Omega$, $R_2 = 18\,\Omega$, $R_3 = 10\,\Omega$
$R_4 = 9\,\Omega$, $R_5 = 5\,\Omega$, $R_6 = 14\,\Omega$
$R_7 = 8\,\Omega$, $R_8 = 13\,\Omega$

Chapter 4

Using Script Files and Managing Data

A script file (see Section 1.8) is a list of MATLAB commands, called a program, that is saved in a file. When the script file is executed (run), MATLAB executes the commands. Section 1.8 describes how to create, save, and run a simple script file in which the commands are executed in the order that they are listed, and in which all the variables are defined within the script file. The present chapter gives more details of how to input data to a script file, how data is stored in MATLAB, various ways to display and save data that is created in script files, and how to exchange data between MATLAB and other applications. (How to write more advanced programs where commands are not necessarily executed in a simple order is covered in Chapter 7.)

In general, variables can be defined (created) in several ways. As shown in Chapter 2, variables can be defined implicitly by assigning values to a variable name. Variables can also be assigned values by the output of a function. In addition, variables can be defined with data that is imported from files outside MATLAB. Once defined (either in the Command Window or when a script file is executed) the variables are stored in MATLAB's Workspace.

Variables that reside in the Workspace can be displayed in different ways, saved, or exported to applications outside MATLAB. Similarly, data from files outside MATLAB can be imported to the Workspace and then used in MATLAB.

Section 4.1 explains how MATLAB stores data in the Workspace and how the user can see the data that is stored. Section 4.2 shows how variables that are used in script files can be defined in the Command Window and/or in Script files. Section 4.3 shows how to output data that is generated when script files are executed. Section 4.4 explains how the variables in the Workspace can be saved and then retrieved, and Section 4.5 shows how to import and export data from and to applications outside MATLAB.