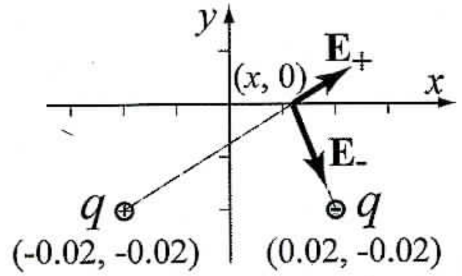
Solution



The electric field **E** at any point $(x, 0)$ along the x-axis is obtained by adding the electric field vectors due to each of the charges.

$$\mathbf{E} = \mathbf{E}_- + \mathbf{E}_+$$

The magnitude of the electric field is the length of the vector **E**.

The problem is solved by following these steps:

Step 1: Create a vector x for points along the x-axis.

Step 2: Calculate the distance (and distance^2) from each charge to the points on the x-axis.

$$r_{minus} = \sqrt{(0.02-x)^2 + 0.02^2} \qquad r_{plus} = \sqrt{(x+0.02)^2 + 0.02^2}$$

Step 3: Write unit vectors in the direction from each charge to the points on the x-axis.

$$\mathbf{E}_{minusUV} = \frac{1}{r_{minus}}((0.02-x)\mathbf{i} - 0.02\mathbf{j})$$

$$\mathbf{E}_{plusUV} = \frac{1}{r_{plus}}((x+0.02)\mathbf{i} + 0.02\mathbf{j})$$

Step 4: Calculate the magnitude of the vector $\mathbf{E}_-$ and $\mathbf{E}_+$ at each point by using Coulomb's law.

$$E_{minusMAG} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{minus}^2} \qquad E_{plusMAG} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{plus}^2}$$

Step 5: Create the vectors $\mathbf{E}_-$ and $\mathbf{E}_+$ by multiplying the unit vectors by the magnitudes.

Step 6: Create the vector **E** by adding the vectors $\mathbf{E}_-$ and $\mathbf{E}_+$.

Step 7: Calculate E, the magnitude (length) of **E**.

Step 8: Plot E as a function of x.

A program in a script file that solves the problem is:

```
q=12e-9;
epsilon0=8.8541878e-12;
x=[-0.05:0.001:0.05]';
rminusS=(0.02-x).^2+0.02^2; rminus=sqrt(rminusS);
rplusS=(x+0.02).^2+0.02^2; rplus=sqrt(rplusS);
EminusUV=[((0.02-x)./rminus), (-0.02./rminus)];
EplusUV=[((x+0.02)./rplus), (0.02./rplus)];
```

Create a column vector x.

Step 2, each variable is a column vector.

Steps 3 & 4, each variable is a two column matrix. Each row is the vector for the corresponding x.

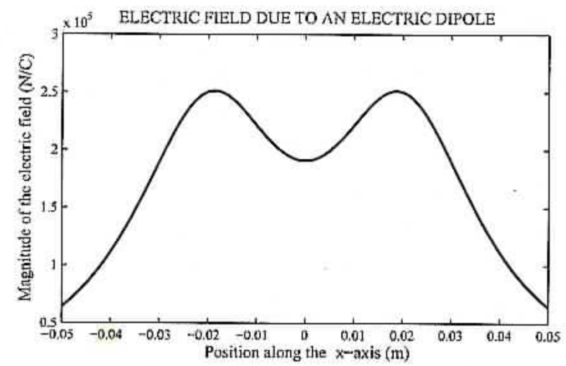

```
EminusMAG=(q/(4*pi*epsilon0))./rminusS;
EplusMAG=(q/(4*pi*epsilon0))./rplusS;
Eminus=[EminusMAG.*EminusUV(:,1), EminusMAG.*EminusUV(:,2)];
Eplus=[EplusMAG.*EplusUV(:,1), EplusMAG.*EplusUV(:,2)];
E=Eminus+Eplus;
EMAG=sqrt(E(:,1).^2+E(:,2).^2);
plot(x,EMAG,'k','linewidth',1)
xlabel('Position along the x-axis (m)','FontSize',12)
ylabel('Magnitude of the electric field (N/C)','FontSize',12)
title('ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE','FontSize',12)
```

Step 5.

Step 6.

Step 7.

When this script file is executed in the Command Window the following figure is created in the Figure Window:



5.13 PROBLEMS

1. Make two separate plots of the function $f(x) = 0.01x^4 - 0.45x^2 + 0.5x - 2$; one plot for $-4 \le x \le 4$, and one for $-8 \le x \le 8$.

2. Plot the function $f(t) = \frac{5}{1+e^{5.5-1.5x}} - \frac{x^2}{20}$ for $-10 \le x \le 10$.

3. Use the `fplot` command to plot the function:
$f(x) = \frac{40}{1+(x-4)^2} + 5\sin\left(\frac{20x}{\pi}\right)$ in the domain $0 \le x \le 10$.

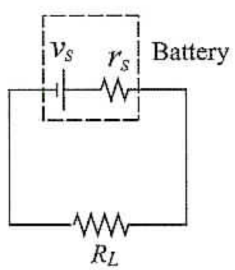
4. Plot the function $f(x) = \frac{x^2-4x-5}{x-2}$ for $-4 \le x \le 8$. Notice that the function has a vertical asymptote at $x = 2$. Plot the function by creating two vectors for the domain of x, the first vector (call it $x1$) with elements from -4 to 1.7, and

the second vector (call it $x2$) with elements from 2.3 to 8. For each x vector create a y vector (call them $y1$ and $y2$) with the corresponding values of y according to the function. To plot the function make two curves in the same plot ($y1$ vs. $x1$, and $y2$ vs. $x2$).

5. Plot the function $f(x) = \dfrac{4x-30}{x^2-3x-10}$ for $-10 \le x \le 10$. Notice that the function has two vertical asymptotes. Plot the function by dividing the domain of x into three parts: one from -10 to near the left asymptote, one between the two asymptotes, and one from near the right asymptote to 10. Set the range of the y-axis from -20 to 20.

6. Plot the function $f(x) = 3x\cos^2 x - 2x$ and its derivative, both on the same plot, for $-2\pi \le x \le 2\pi$. Plot the function with a solid line, and the derivative with a dashed line. Add a legend and label the axes.

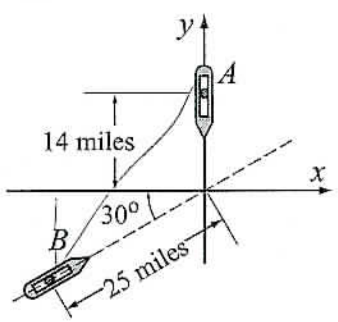
7. An electrical circuit that includes a voltage source v_S with an internal resistance r_S and a load resistance R_L is shown in the figure. The power P dissipated in the load is given by:



$$P = \frac{v_S^2 R_L}{(R_L + r_S)^2}$$

Plot the power P as a function of R_L for $1 \le R_L \le 10\,\Omega$, given that $v_S = 12$ V, and $r_S = 2.5\,\Omega$.

8. Ship A travels south at a speed of 6 miles/hour, and a ship B travels 30° north to the east at a speed of 14 miles/hour. At 7 AM the ships are positioned as shown in the figure. Plot the distance between the ships as a function of time for the next 4 hours. The horizontal axis should show the actual time of day starting at 7 AM, while the vertical axis shows the distance. Label the axes. If visibility is 8 miles, estimate from the graph the time when people from the two ships can see each other.

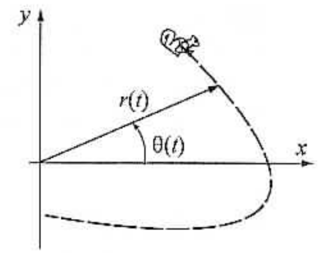


9. The Gateway Arch in St. Louis is shaped according to the equation:

$$y = 693.8 - 68.8\cosh\left(\frac{x}{99.7}\right) \text{ ft.}$$

Make a plot of the arch.

10. The position as a function of time $[x(t), y(t)]$ of a squirrel running on a grass field is given by:



$$x(t) = -0.28t^2 + 6.5t + 61 \text{ m and}$$
$$y(t) = 0.18t^2 - 8.5t + 65 \text{ m}$$

a) Plot the trajectory (position) of the squirrel for $0 \le t \le 30$ s.

b) Plot (a second plot) the length of the position vector $r(t) = x(t)i + y(t)j$ of the squirrel for $0 \le t \le 30$ s.

c) Plot (a third plot) the angle $\theta(t)$ of the position vector for $0 \le t \le 30$ s.

11. In astronomy, the relationship between the relative luminosity L/L_{Sun} (brightness relative to the sun), the relative radius R/R_{Sun}, and the relative temperature T/T_{Sun} of a star is modeled by:

$$\frac{L}{L_{Sun}} = \left(\frac{R}{R_{Sun}}\right)^2 \left(\frac{T}{T_{Sun}}\right)^4$$

The HR (Hertzsprung-Russell) diagram is a plot of L/L_{Sun} versus the temperature. The following data is given:

	Sun	Spica	Regulus	Alioth	Barnard's Star	Epsilon Indi	Beta Crucis
Temp (K)	5840	22400	13260	9400	3130	4280	28200
L/L_{sun}	1	13400	150	108	0.0004	0.15	34000
R/R_{sun}	1	7.8	3.5	3.7	0.18	0.76	8

To compare the data with the model, use MATLAB to plot a HR diagram. The diagram should have two sets of points. One uses the values of L/L_{Sun} from the Table (use asterisk markers), and the other uses values of L/L_{Sun} that are calculated by the equation by using R/R_{Sun} from the Table (use circle markers). In the HR diagram both axes are logarithmic. In addition, the values of the temperature on the horizontal axis are decreasing from left to right. This is done with the command: `set(gca,'XDir','reverse')`. Label the axes and use a legend.

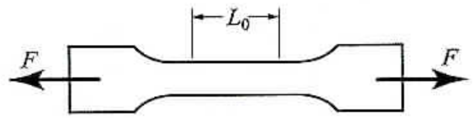
12. The position x as a function of time of a particle that moves along a straight line is given by:

$$x(t) = -0.1t^4 + 0.8t^3 + 10t - 70 \text{ m}$$

The velocity $v(t)$ of the particle is determined by the derivative of $x(t)$ with respect to t, and the acceleration $a(t)$ is determined by the derivative of $v(t)$ with respect to t.

Derive the expressions for the velocity and acceleration of the particle, and make plots of the position, velocity, and acceleration as a function of time for $0 \le t \le 8$ s. Use the `subplot` command to make the three plots on the same page with the plot of the position on the top, the velocity in the middle, and the acceleration at the bottom. Label the axes appropriately with the correct units.

13. In a typical tension test a dog-bone-shaped specimen is pulled in a machine. During the test, the force F needed to pull the specimen and the length L of a gauge section are measured. This data is used for plotting a stress-strain diagram of the material. Two definitions, engineering and true, exist for stress and strain. The engineering stress σ_e and strain ε_e are defined by:



$\sigma_e = \dfrac{F}{A_0}$ and $\varepsilon_e = \dfrac{L - L_0}{L_0}$, where L_0 and A_0 are the initial gauge length and the initial cross-sectional area of the specimen, respectively. The true stress σ_t and strain ε_t are defined by: $\sigma_t = \dfrac{F}{A_0}\dfrac{L}{L_0}$ and $\varepsilon_t = \ln\dfrac{L}{L_0}$.

The following are measurements of force and gauge length from a tension test with an aluminum specimen. The specimen has a round cross section with radius of 6.4 mm (before the test). The initial gauge length is $L_0 = 25$mm. Use the data to calculate and plot the engineering and true stress-strain curves, both on the same plot of the material. Label the axes and label the curves.
Units: When the force is measured in Newtons (N), and the area is calculated in m², the units of the stress are Pascals (Pa).

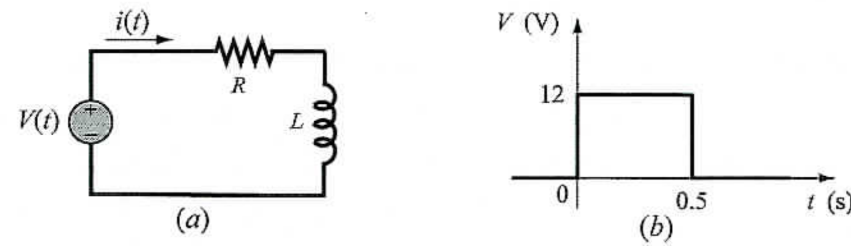
F (N)	0	13345	26689	40479	42703	43592	44482	44927
L (mm)	25	25.037	25.073	25.113	25.122	25.125	25.132	25.144
F (N)	45372	46276	47908	49035	50265	53213	56161	
L (mm)	25.164	25.208	25.409	25.646	26.084	27.398	29.150	

14. The area of the aortic valve, A_V in cm^2, can be estimated by the equation (Hakki Formula):

$$A_V = \frac{Q}{\sqrt{PG}}$$

where Q is the cardiac output in L/min, and PG is the difference between the left ventricular systolic pressure and the aortic systolic pressure (in mmHg). Make one plot with two curves of A_V versus PG, for $2 \le PG \le 60$ mmHg. One curve for $Q = 4$ L/min and the other for $Q = 5$ L/min. Label the axes and use a legend.

15. A resistor, $R = 4\ \Omega$, and an inductor, $L = 1.3$ H, are connected in a circuit to a voltage source as shown in Figure (a) (RL circuit). When the voltage source



applies a rectangular voltage pulse with an amplitude of $V = 12$ V and a duration of 0.5 s, as shown in Figure (b), the current $i(t)$ in the circuit as a function of time is given by:

$$i(t) = \frac{V}{R}(1 - e^{(-Rt)/L}) \quad \text{for } 0 \le t \le 0.5 \text{ s}$$

$$i(t) = e^{-(Rt)/L}\frac{V}{R}(e^{(0.5R)/L} - 1) \quad \text{for } 0.5 \le t \text{ s}$$

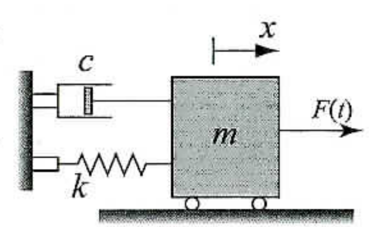
Make a plot of the current as a function of time for $0 \le t \le 2$ s.

16. The dynamic storage modulus G' and loss modulus G'' are measures of a material mechanical response to harmonic loading. For many biological materials these moduli can be described by Fung's model:

$$G'(\omega) = G_\infty\left\{1 + \frac{c}{2}\ln\left[\frac{1 + (\omega\tau_2)^2}{1 + (\omega\tau_1)^2}\right]\right\} \quad \text{and} \quad G''(\omega) = cG_\infty[\tan^{-1}(\omega\tau_2) - \tan^{-1}(\omega\tau_1)]$$

where ω is the frequency of the harmonic loading, and G_∞, c, τ_1, and τ_2 are material constants. Plot G' and G'' versus ω (two separate plots on the same page) for $G_\infty = 5$ ksi, $c = 0.05$, $\tau_1 = 0.05$s, and $\tau_2 = 500$s. Let ω vary between 0.0001 and 1000 s^{-1}. Use log scale for the ω axis.

17. The vibrations of the body of a helicopter due to the periodic force applied by the rotation of the rotor can be modeled by a frictionless spring-mass-damper system subjected to an external periodic force. The position $x(t)$ of the mass is given by the equation:



$$x(t) = \frac{2f_0}{\omega_n^2 - \omega^2}\sin\left(\frac{\omega_n - \omega}{2}t\right)\sin\left(\frac{\omega_n - \omega}{2}t\right)$$

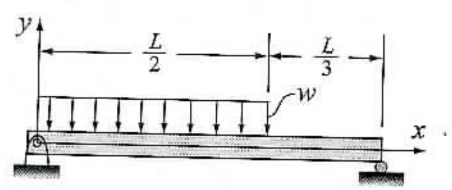
where $F(t) = F_0\sin\omega t$, and $f_0 = F_0/m$, ω is frequency of the applied force, and ω_n is the natural frequency of the helicopter. When the value of ω is close to the value of ω_n the vibration consists of fast oscillation with slowly changing amplitude called beat. Use $F_0/m = 12\,\text{N/kg}$, $\omega_n = 10\,\text{rad/s}$, and $\omega = 12\,\text{rad/s}$ to plot $x(t)$ as a function of t for $0 \le t \le 10\,\text{s}$.

18. The ideal gas equation states that $\frac{PV}{RT} = n$, where P is the pressure, V is the volume, T is the temperature, $R = 0.08206$ (L atm)/(mole K) is the gas constant, and n is the number of moles. For one mole ($n = 1$) the quantity $\frac{PV}{RT}$ is a constant equal to 1 at all pressures. Real gases, especially at high pressures, deviate from this behavior. Their response can be modeled with the van der Waals equation:

$$P = \frac{nRT}{V-nb} - \frac{n^2 a}{V^2}$$

where a and b are material constants. Consider 1 mole ($n = 1$) of nitrogen gas at $T = 300\text{K}$. (For nitrogen gas $a = 1.39\,\text{L}^2\text{atm/mole}^2$, and $b = 0.0391$ L/mole.) Use the van der Waals equation to calculate P as a function of V for $0.08 \le V \le 6\,\text{L}$, using increments of 0.02L. At each value of V calculate the value of $\frac{PV}{RT}$ and make a plot of $\frac{PV}{RT}$ versus P. Does the response of nitrogen agrees with the ideal gas equation?

19. A simply supported beam that is subjected to a constant distributed load w over half of its length is shown in the figure. The deflection y, as a function of x, is given by the equations:



$$y = \frac{-wx}{24LEI}\left(Lx^3 - \frac{16}{9}L^2x^2 + \frac{64}{81}L^4\right) \text{ for } 0 \le x \le \frac{2}{3}L$$

$$y = \frac{-wL}{54EI}\left(2x^3 - 6Lx^2 + \frac{40}{9}L^2x - \frac{4}{9}L^3\right) \text{ for } \frac{2}{3}L \le x \le L$$

where E is the elastic modulus, I is the moment of inertia, and L is the length of the beam. For the beam shown in the figure $L = 20\,\text{m}$, $E = 200 \times 10^9\,\text{Pa}$ (steel), $I = 348 \times 10^{-6}\,\text{m}^4$, and $w = 5 \times 10^3\,\text{N/m}$. Make a plot of the deflection of the beam y as a function of x.

Chapter 6
User-Defined Functions and Function Files

A simple function in mathematics, $f(x)$, associates a unique number to each value of x. The function can be expressed in the form $y = f(x)$, where $f(x)$ is usually a mathematical expression in terms of x. A value of y (output) is obtained when a value of x (input) is substituted in the expression. Many functions are programed inside MATLAB as built-in functions, and can be used in mathematical expressions simply by typing their name with an argument (see Section 1.5); examples are `sin(x)`, `cos(x)`, `sqrt(x)`, and `exp(x)`. Frequently, in computer programs, there is a need to calculate the value of functions that are not built-in. When a function expression is simple and needs to be calculated only once, it can be typed as part of the program. However, when a function needs to be evaluated many times for different values of arguments it is convenient to create a "user-defined" function. Once a user-defined function is created (saved) it can be used just like the built-in functions.

A user-defined function is a MATLAB program that is created by the user, saved as a function file, and then can be used like a built-in function. The function can be a simple single mathematical expression or a complicated and involved series of calculations. In many cases it is actually a subprogram within a computer program. The main feature of a function file is that it has an input and an output. This means that the calculations inside the function file are carried out using the input data, and the results of the calculations are transferred out of the function file by the output. The input and the output can be one or several variables, and each can be a scalar, vector, or an array of any size. Schematically, a function file can be illustrated by: