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MATH 4720

Homework 5

1

Let G be a finite group and G a normal subgroup of G. Let H' be a subgroup of G/N and let $H1 = \pi^{-1}(H)$ be the corresponding subgroup of G via the Correspondence Theorem. Show that $|H1| = |H| \cdot |N|$.

2

In the group $\mathbb{Z}_{40}$, let $H = \langle 4 \rangle$ and $N = \langle 10 \rangle$.

a Find the elements of $H + N$ and $H \cap N$.

$$H \cap N = \langle (4, 10) \rangle$$

s= {0,20}

$H + N = \langle 4 \rangle + \langle 10 \rangle s \quad = \langle 14 \rangle s = \{14, 28, 2, 16, 30, 4, 18, 32, 6, 20, 34, 8, 22, 36, 10, 24, 38, 12, 26, 0\}s \quad = \{0, 2, 4$

H $\cap N = \langle (4, 10) \rangle$

$s = \{0, 20\}$

b List the cosets of $\frac{H+N}{N}$.

Note that this quotient group is $a + N$ for all $a \in H + N$.

$$\frac{H+N}{N} = \{\{0, 10, 20, 30\}, \{2, 12, 22, 32\}, \{4, 14, 24, 34\}, \{6, 16, 26, 36\}, \{8, 18, 28, 38\}\}.$$

c List the cosets of $\frac{H}{H \cap N}$. Note that this is $a + N$ for all $a \in H \cap N$.

$$\frac{H+N}{N} = \{\{0, 20\}, \{4, 24\}, \{8, 28\}, \{12, 32\}, \{16, 36\}\}$$