1. $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ **Base:** $1^2 = \frac{1(1+1)(2+1)}{6} = \frac{6}{6} = 1$

Assume: $1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Show: $1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$

$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} =$

$\frac{k(k+1)(2k+1)6(k+1)^2}{6} = \frac{(k+1)[k(2k+1)6(k+1)]}{6} = \frac{(k+1)(2k^2+k+6k+6)}{6} =$

$\frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$

2. $1^3 + 2^3 + \cdots + n^3 = (\frac{n(n+1)}{2})^2$ **Base:** $1^3 = (\frac{1(1+1)}{2})^2 = (\frac{2}{2})^2 = 1$

Assume: $1^3 + 2^3 + \cdots + k^3 = (\frac{k(k+1)}{2})^2$

Show: $1^3 + 2^3 + \cdots k^3 + (k+1)^3 = (\frac{(k+1)((k+1)+1)}{2})^2$

$(\frac{k(k+1)}{2})^2 + (k+1)^3 = \frac{k^4+k^2+2k}{4} + (k^3 + 3k^2 + 3k + 1) = \frac{k^4+k^2+2k}{4} + \frac{4(k^3+3k^2+3k+1)}{4}$

$= (\frac{(k+1)((k+1)+1)}{2})^2$

$\frac{k^4+6k^3+13k^2+12k+4}{4} = \frac{[(k+1)(k+2)]^2}{4} = (\frac{(k+1)((k+1)+1)}{2})^2$

3. $1 * 1! + 2 * 2! + \cdots + n * n! = (n+1)! - 1$ **Base:** $1 * 1! = (1+1)! - 1 = 1$

Assume: $1 * 1! + 2 * 2! + \cdots + k * k! = (k+1)! - 1$

Show: $1 * 1! + 2 * 2! + \cdots + k * k! + (k+1)(k+1)! = ((k+1)+1)! - 1$

$(k+1)! - 1 + (k+1)(k+1)! = (k+2)! - 1$

$(k+2)(k+1)! - 1 = (k+2)! - 1$

$((k+1)+1)! - 1 = (k+2)! - 1$

4. $2^n > n^2$ *where* $n > 4$ **Base:** $2^5 > 5^2 \rightarrow 32 > 25$

Assume: $2^k > k^2$

Show: $2^{k+1} > (k+1)^2$ *where* $k > 4$

$2^{k+1} = 2^k * 2 > 2k^2 > k^2 > 2k + 1 + k^2 = (k+1)^2$

5. $1^3 + 3^3 + \cdots + (2n-1)^3 = n^2(2n^2 - 1)$ **Base:** $1^5 = 1^2(2(1)^2 - 1) = 1$

Assume: $1^3 + 3^3 + \cdots + (2k-1)^3 = k^2(2k^2 - 1)$

Show: $1^3 + 3^3 + \cdots + (2k-1)^3 + (2(k+1)-1)^3 = (k+1)^2(2(k+1)^2 - 1)$

$k^2(2k^2 - 1) + (2(k+1) - 1)^3 = (2k^4 - k^2) + (2k+1)^3 =$

$(2k^4 - k^2) + (8k^3 + 12k^2 + 6k + 1) = 2k^4 + 8k^3 + 11k^2 + 6k + 1 =$

$(k+1)^2(2(k+1)^2 - 1) = (k+1)^2(2k^2 + 4k + 1) =$

$2k^2(k+1)^2 + 4k(k+1)^2 + (k+1)^2 =$

$(2k^4 + 4k^3 + 2k^2) + (4k^3 + 8k^2 + 4k) + (k^2 + 2k + 1) =$

$2k^4 + 8k^3 + 11k^2 + 6k + 1 = (k+1)^2(2(k+1)^2 - 1)$

6. $\frac{1}{1*2}+\frac{1}{2*3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

Base: $\frac{1}{1*2}=\frac{1}{1+1}=\frac{1}{2}$

Assume: $\frac{1}{1*2}+\frac{1}{2*3}+\cdots+\frac{1}{k(k+1)}=\frac{k}{k+1}$

Show: $\frac{1}{1*2}+\frac{1}{2*3}+\cdots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$

$$\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)}=\frac{k+1}{k+2}$$

7. $S=\sum_{j=0}^{n} ar^j=\frac{ar^{n+1}-a}{r-1}\ ,r\neq 1$

Base: $n=1;\ ar^0=\frac{ar^{0+1}-a}{r-1}=\frac{a\cancel{(r-1)}}{\cancel{r-1}}$

Assume: $\sum_{j=0}^{k} ar^j=\frac{ar^{(k+1)}-a}{r-1}$

Show: $\sum_{j=0}^{k+1} ar^j=\frac{ar^{k+2}-a}{r-1}$

That is to say: $ar^0+ar^1+\cdots+ar^k+ar^{k+1}=\frac{ar^{k+2}-a}{r-1}$

$\frac{ar^{(k+1)}-a}{r-1}+ar^{k+1}=\frac{ar^{k+2}-a}{r-1}$ by assumption substitution

$$\frac{ar^{(k+1)}-a}{r-1}+ar^{k+1}=\frac{ar^{(k+1)}-a}{r-1}+\frac{(r-1)ar^{k+1}}{r-1}=\frac{ar^{(k+1)}-a}{r-1}+\frac{ar^{k+2}-ar^{k+1}}{r-1}=\frac{ar^{(k+1)}-a+ar^{k+2}-ar^{k+1}}{r-1}=$$
$$\frac{-a+ar^{k+2}}{r-1}=\frac{ar^{k+2}-a}{r-1}$$

8. $S=\sum_{i=1}^{n+1} i*2^i=n*2^{n+2}+2\ , for\ all\ integers\ n\geq 0$

Base: $n=0\ \sum_{i=1}^{n+1} i*2^i=n*2^{n+2}+2$

$\sum_{i=1}^{1} i*2^i=0*2^2+2=2$

Assume: $\sum_{i=1}^{k+1} i*2^i=k*2^{k+2}+2$

Show: $\sum_{i=1}^{(k+1)+1} i*2^i=(k+1)*2^{(k+1)+2}+2$

$(0*2^0)+\cdots+((k+1)*2^{k+1})+((k+2)*2^{(k+2)})=(k+1)*2^{(k+1)+2}+2$

$(2^{k+2}k+2)+\left(2^{(k+2)}k+2^{(k+3)}\right)=2^{k+2}k+2^{(k+2)}k+2^{(k+3)}+2=$

$2k(2^{k+2})+2^{(k+3)}+2=2^{k+3}k+2^{(k+3)}+2=2^{(k+3)}(k+1)+2$