

1)

$$2) \quad T(n) = \begin{cases} a & n = 1 \\ T\left(\frac{n}{2}\right) + b & n \geq 2 \end{cases}$$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + b$$

$$T(n) = \left(T\left(\frac{n}{4}\right) + b\right) + b$$

$$T(n) = T\left(\frac{n}{4}\right) + 2b$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + b$$

$$T(n) = \left(T\left(\frac{n}{8}\right) + b\right) + 2b$$

$$T(n) = T\left(\frac{n}{8}\right) + 3b$$

$$T\left(\frac{n}{8}\right) = T\left(\frac{n}{16}\right) + b$$

$$T(n) = \left(T\left(\frac{n}{16}\right) + b\right) + 3b$$

$$T(n) = T\left(\frac{n}{16}\right) + 4b$$

$$T(n) = T\left(\frac{n}{2^k}\right) + kb$$

$$k \text{ such that } T\left(\frac{n}{2^k}\right) = T(a)$$

That is $\left(\frac{n}{2^k}\right) = 1$ or more simply $2^k = n$, thus $n = \log n$

$$T(n) = T\left(\frac{n}{2^{\log n}}\right) + \log n b$$

$$T(n) = T\left(\frac{n}{n}\right) + (\log n)b \rightarrow (a) + \log n b$$

thus $T(n)$ is $O(\log n)$

$$3) \quad T(n) = \begin{cases} a & n = 1 \\ 2T\left(\frac{n}{2}\right) + b & n \geq 2 \end{cases}$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + b$$

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + b\right) + b$$

$$T(n) = 4T\left(\frac{n}{4}\right) + 2b + b$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + b$$

$$T(n) = 4\left(2T\left(\frac{n}{8}\right) + b\right) + 2b + b$$

$$T(n) = 8T\left(\frac{n}{8}\right) + 4b + 2b + b$$

$$T\left(\frac{n}{8}\right) = 2T\left(\frac{n}{16}\right) + b$$

$$T(n) = 8\left(2T\left(\frac{n}{16}\right) + b\right) + 4b + 2b + b$$

$$T(n) = 16T\left(\frac{n}{16}\right) + 8b + 4b + 2b + b$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + (2^k - 1)b$$

$$k \text{ such that } T\left(\frac{n}{2^k}\right) = T(a)$$

That is $\left(\frac{n}{2^k}\right) = 1$ or more simply $2^k = n$, thus $n = \log n$

$$T(n) = 2^{\log n} T\left(\frac{n}{2^{\log n}}\right) + (2^{\log n} - 1)b$$

$$T(n) = nT\left(\frac{n}{n}\right) + (n - 1)b \rightarrow n(a) + (n - 1)b$$

thus $T(n)$ is $O(n)$

$$4) \quad T(n) = \begin{cases} a & n = 1 \\ T(n-1) + n^k & n \geq 2 \end{cases}$$

$$T(n-1) = T(n-2) + (n-1)^k$$

$$T(n) = T(n-2) + (n-1)^k + n^k$$

$$T(n-2) = T(n-3) + (n-2)^k$$

$$T(n) = T(n-3) + (n-2)^k + (n-1)^k + n^k$$

$$T(n-3) = T(n-4) + (n-3)^k$$

$$T(n) = T(n-4) + (n-3)^k + (n-2)^k + (n-1)^k + n^k$$

$$T(n) = T(n-j) + \left(\frac{n(n+1)}{2}\right)^k$$

$$T(n) = T(n-j) + \left(\frac{n^2+n}{2}\right)^k$$