CSE/IT 122: Homework 4B

For problems involving running time, make a table of costs and the number of times each line runs. Have a column for line numbers. You do not need to show the work for the count, but your answer needs to be exact for each line. If a problem involves best case and a worst case scenario, make a column for each case.

Type your answers using Latex, or Word, or similar, and convert the file to a pdf file named `cse122_firstname_lastname_hw4b.pdf`.

Problems

1. Using the iteration (aka substitution) method find the explicit formula for the recurrence relation:
$$b_k = \frac{b_{k-1}}{1 + b_{k-1}}$$

   for all integers with $k \geq 1$ and $b_0 = 1$

   Prove that the formula is correct by induction. Type your answer. Show your work.

2. Solve (find an explicit formula) the following recurrence relation for the running time $T(n)$ using the substitution method:
$$T(n) = \begin{cases} a & \text{if } n = 1 \\ T(n/2) + b & \text{if } n \geq 2 \end{cases}$$

   a and b are constants. What is the $bigO$ of the running time? Type your answer. Show your work. Clearly show what $T(n)$ is after k unrollings.

3. Solve the following recurrence relation for the running time $T(n)$ using the substitution method:
$$T(n) = \begin{cases} a & \text{if } n = 1 \\ 2T(n/2) + b & \text{if } n \geq 2 \end{cases}$$

   a and b are constants. What is the $bigO$ of the running time? Type your answer. Show your work. Clearly show what $T(n)$ is after k unrollings.

4. Show for the following recurrence relation for the running time $T(n)$ is $O(n^{k+1})$. Use the substitution method:
$$T(n) = \begin{cases} a & \text{if } n = 1 \\ T(n-1) + n^k & \text{for } n > 1 \end{cases}$$

a and k are constants. Type your answer. Show your work. Clearly show what $T(n)$ is after j unrollings.

Submission

Upload your pdf file to Canvas before the due date.