
CSE589, CSE/IT441 Cryptography & Applications – Fall 2017

Department of Computer Science, New Mexico Tech

Lecturer: Dongwan Shin

HOMEWORK 3 - Assigned: 10/25, **Due: 11/1** (*midnight*)¹

Please **type your answers** and **submit** them to Canvas.

Problem 1. [5 points] Does the set of residue classes modulo 21 form a group

- a. with respect to addition? (*Show why or why not*)
- b. with respect to multiplication? (*Show why or why not*)

Problem 2. [5 points] Consider the set $S = \{a, b\}$ with addition and multiplication defined by the following tables:

| | | |
|---|---|---|
| + | a | b |
| a | b | a |
| b | a | b |

| | | |
|---|---|---|
| x | a | b |
| a | b | b |
| b | b | a |

Is S a ring? Justify your answer.

Problem 3. [5 points] Determine which of the following are reducible over $\text{GF}(2)$:

- a. $x^4 + 1$
- b. $x^6 + x^2 + 1$
- c. $x^5 + 1$

Problem 4. [5 points] Prove the following:

- a. $[(a \bmod n) - (b \bmod n)] \bmod n = (a - b) \bmod n$
- b. $[(a \bmod n) \times (b \bmod n)] \bmod n = (a \times b) \bmod n$

Problem 5. [10 points] This problem provides a numerical example of a portion of AES encryption. Please refer to AES S-boxes and MixColumns matrix in your textbook in order to solve this problem. Given a plaintext and a key as follows,

The Plaintext:

00 01 02 03 04 05 06 07 08 09 0A 0B 0C 0D 0E 0F

The Key:

10 11 10 11 10 11 10 11 10 11 10 11 10 11 10 11

- a. Show the original contents of **State**, displayed as a 4×4 matrix.
- b. Show the value of **State** after initial AddRoundKey.
- c. Show the value of **State** after SubBytes.
- d. Show the value of **State** after ShiftRows.
- e. Show the value of **State** after MixColumns.

¹Except for Programming Lab, which is due on 11/10 (Friday). The solution for the programming lab should be submitted by email (dongwan.shin@nmt.edu).

Programming Lab 1. [100 points] This lab is to conduct a linear cryptanalysis attack to find the key used to encrypt messages.

The cipher used for encryption is the basic Substitution-Permutation Network (SPN) you implemented for the 2nd homework, with the following S-Box used instead.

Table 1: S-Box Representation

| | | | | | | | | | | | | | | | | |
|--------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| input | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| output | 2 | C | 4 | 1 | 7 | A | B | 6 | 8 | 5 | 3 | F | D | 0 | E | 9 |

A number of pairs of plaintext and ciphertext are provided in the attached **knownpairs.txt** file. Referring to *A Tutorial on Linear and Differential Cryptanalysis* by Howard M. Heys, answer the following:

- a. **[20 points]** Provide a linear approximation table similar to Table 4.
- b. **[30 points]** Discuss your linear approximation approach for the complete cipher, as in Section 3.4.
- c. **[40 points]** Provide a table for your experimental result for linear attack similar to Table 5.
- c. **[10 points]** What is the key used for encryption?