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Homework 4

MATH5131

03/15/2023

1)

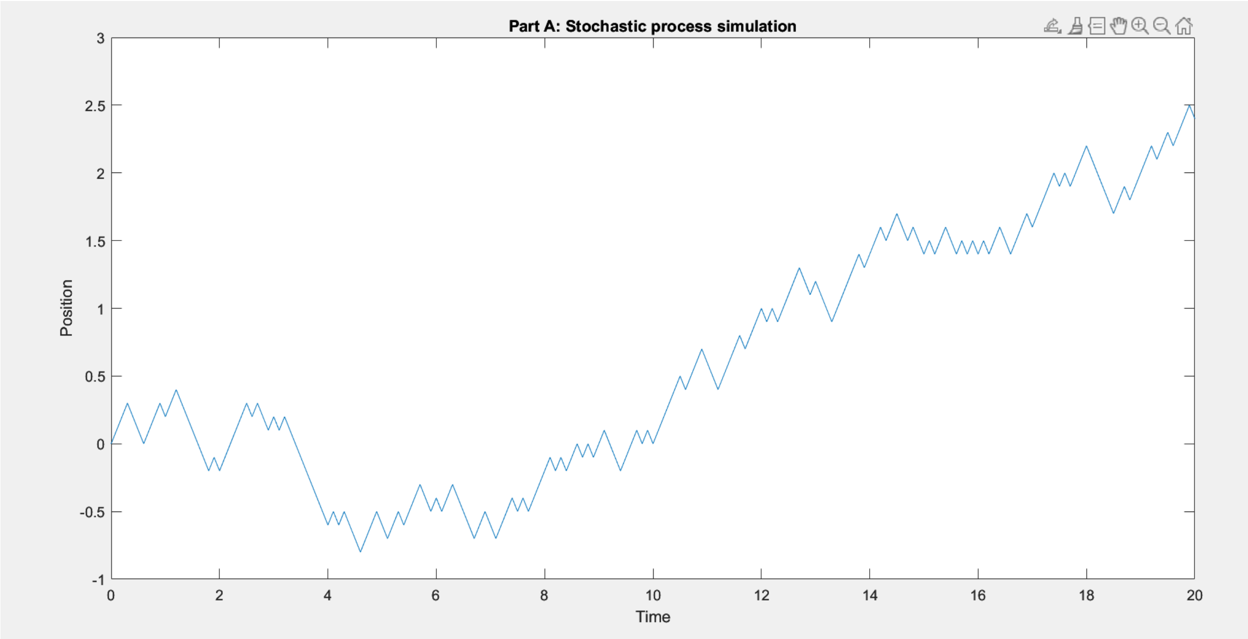
a)

```
% parameters
% step size
delta_x = 0.1;
% time
T = 20;
% time steps
n_steps = T/delta_x;
% store particle position vector
x = zeros(n_steps+1,1);
% initial position is zero
x(1) = 0;

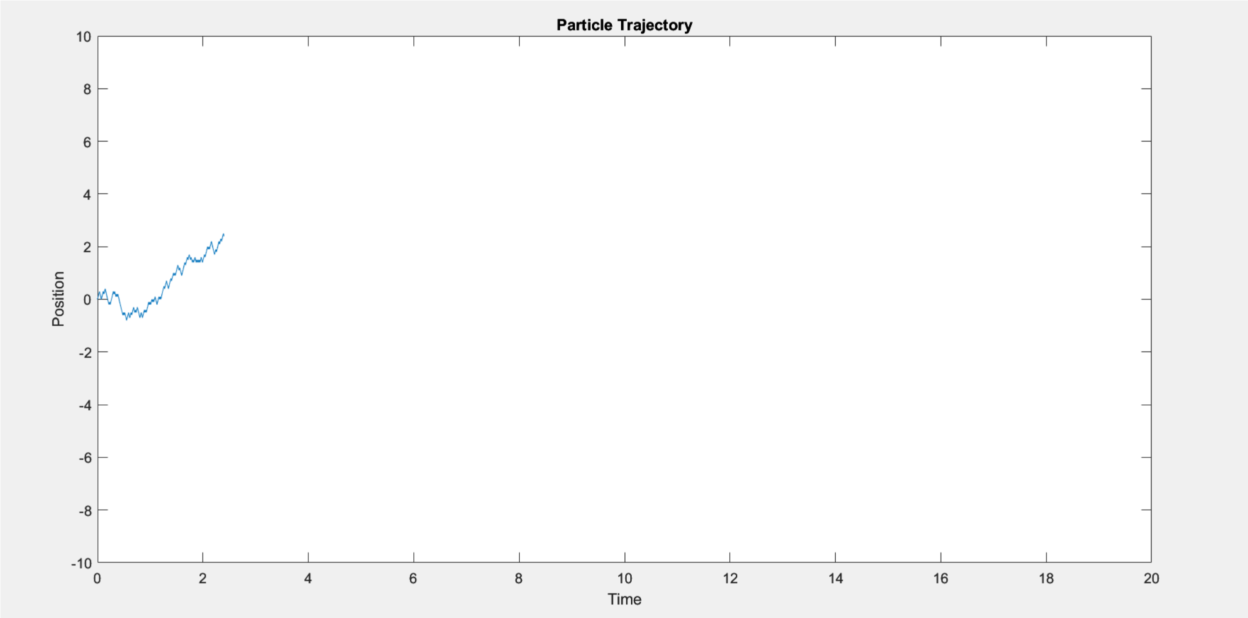
% movement
for i = 2:n_steps+1
    % if else statement
    % Move to the right with probability 1/2
    if rand < 0.5
        x(i) = x(i-1) + delta_x;
    % Move to the left with probability 1/2
    else
        x(i) = x(i-1) - delta_x;
    end
end

% Trajectory plot
plot(0:delta_x:T,x)
xlabel('Time')
ylabel('Position')
title('Part A: Stochastic process simulation')

% Animation
figure
for i = 1:length(x)
    t = linspace(0,x(i),length(x(1:i)));
    plot(t,x(1:i))
    xlabel('Time')
    ylabel('Position')
    title('Particle Trajectory')
    xlim([0 T])
    ylim([-10 10])
    drawnow
end
```



Attached below is a screenshot of the final moment the movie produced:



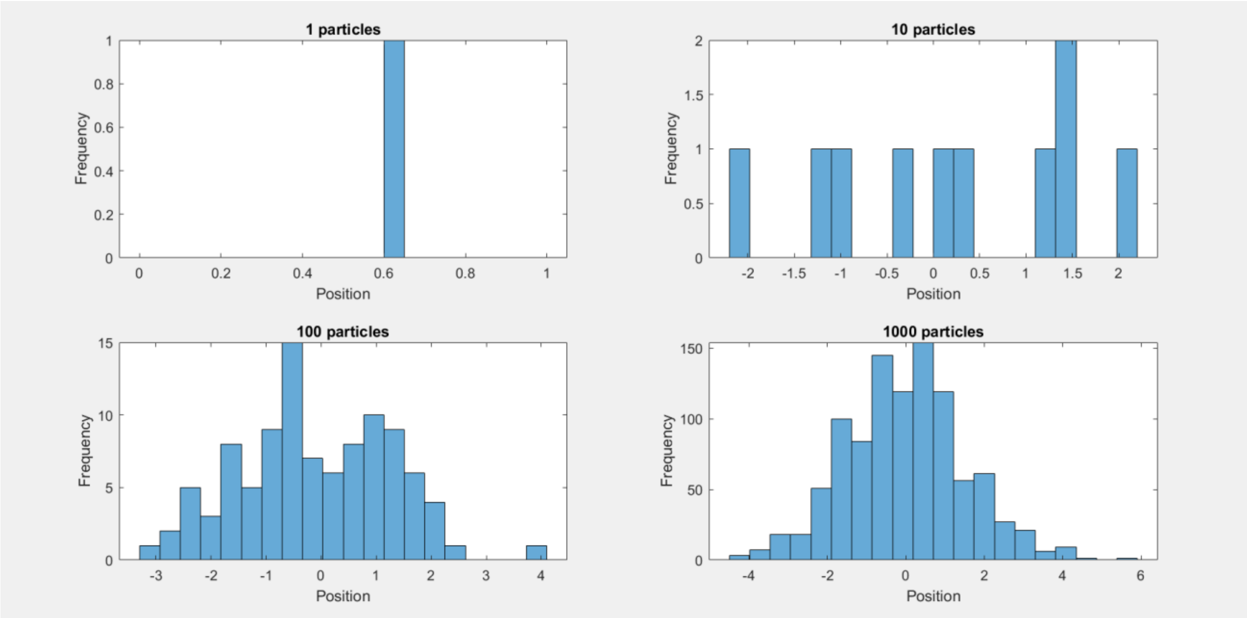
b)

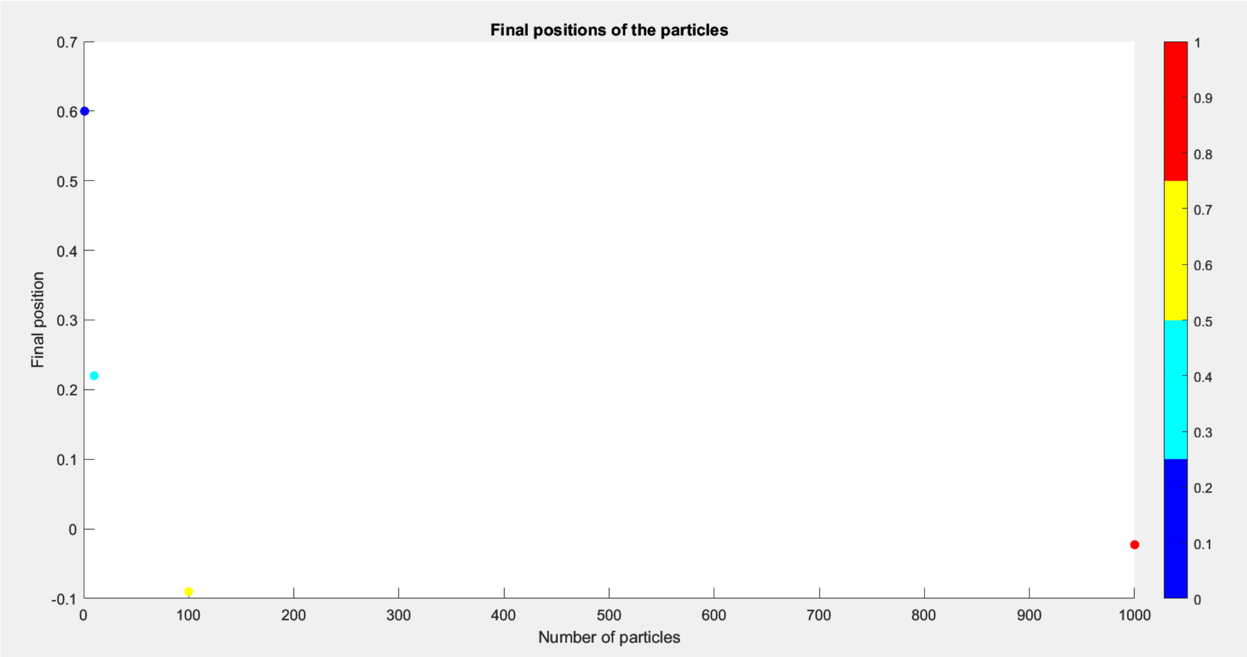
```
% parameters
% step size
delta_x = 0.1;
% time
T = 20;
% time steps
n_steps = T/delta_x;
% particles amount mentioned
n_particles = [1, 10, 100, 1000];
% store zeros for final position vector
final_positions = zeros(length(n_particles),1);

figure(1)
% movement of multiple particles
for i = 1:length(n_particles)
    % Store particle position
    x = zeros(n_steps+1,n_particles(i));
    % Set the initial position to zero: Initial condition
    x(1,:) = 0;
    for j = 2:n_steps+1
        % Simulate the particle movement
        x(j,:) = x(j-1,:) + delta_x*(rand(1,n_particles(i)) < 0.5)*2 - delta_x;
    end
    % final position
    final_positions(i) = mean(x(end,:));

    % histogram: final positions
    subplot(2,2,i)
    histogram(x(end,:),20)
    xlabel('Position')
    ylabel('Frequency')
    title(sprintf('%d particles',n_particles(i)))
end

% Plot: final positions of particles
figure(2)
cmap = colormap(jet(length(n_particles)));
for i = 1:length(n_particles)
    scatter(n_particles(i),final_positions(i),[],cmap(i,:),'filled')
    hold on
end
xlabel('Number of particles')
ylabel('Final position')
title('Final positions of the particles')
colorbar
```





Looking at the histogram, as the number of particles increases, it looks like the distribution of final positions becomes more uniform.

2)

2) $x' = x(2-x-y)$ $\qquad$ $y' = y(3-2x-2y)$

$x' = 2x - x^2 - xy$ $\qquad$ $y' = 3y - 2xy - 2y^2$

$x' = 0$ $\qquad$ $y' = 0$

$0 = x(2-x-y)$ $\qquad$ $0 = y(3-2x-2y)$

$x = 0$ $\qquad$ $y = 0$

$2-x-y = 0$ $\qquad$ $3-2x-2y=0$

$y = 2-x$ $\qquad$ $2y = 3-2x$ $\qquad$ $(0,0)$

$x = 2-y$ $\qquad$ $y = \frac{3}{2} - x$

$x = 2-0$

$x = 2$ $\qquad$ $y = \frac{3}{2} - 0$

$(2,0)$

$(0, 3/2)$

$$\begin{bmatrix} \frac{dx'}{dx} & \frac{dx'}{dy} \\ \frac{dy'}{dx} & \frac{dy'}{dy} \end{bmatrix} = \begin{bmatrix} 2-2x-y & -x \\ -2y & 3-2x-4y \end{bmatrix}$$

(0,0) Source

$$\begin{bmatrix} 2-2(0)-0 & -0 \\ -2(0) & 3-2(0)-4(0) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

trace $= 2+3 = 5$, det $= ad-bc = 2(3)-(0)(0) = 6$

(0, 3/2) Saddle

$$\begin{bmatrix} 2-2(0)-3/2 & -0 \\ -2(3/2) & 3-2(0)-4(3/2) \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -3 & -3 \end{bmatrix}$$

trace $= \frac{1}{2} - 3 = \frac{1}{2} - \frac{6}{2} = \frac{-5}{2}$, det $= ad-bc = \frac{-3}{2} - (-3)(0) = \frac{-3}{2}$

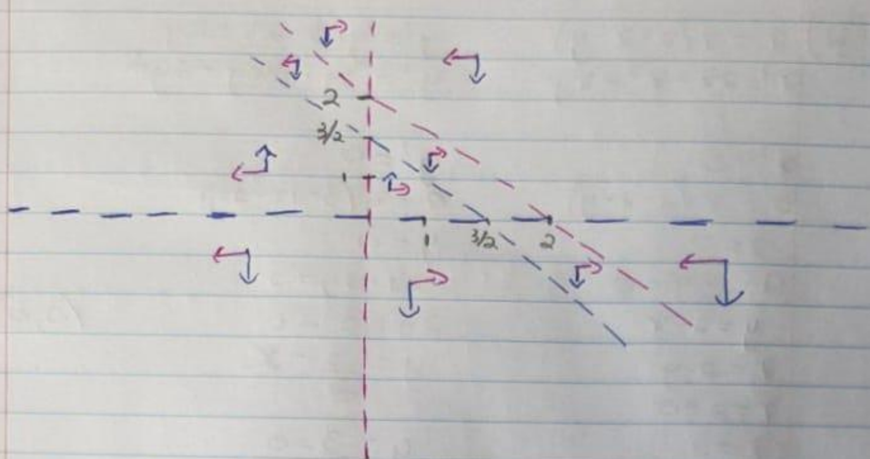
(2,0) Sink

$$\begin{bmatrix} 2-2(2)-0 & -2 \\ -2(0) & 3-2(2)-4(0) \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$$

trace $= -2-1 = -3$ det $= ad-bc = -2(-1)-(0)(-2) = 2$

$y = 2 - x$

$y = 3/2 - x$



(x,y)	dx/dt
(3,3)	−
(1,−1)	+
(−1,−1)	−
(−1,4)	+

(x,y)	dy/dt
(3,3)	−
(−1,4)	−
(1,−1)	−
(−2,2)	+

$x' = 2(3) - (3)^2 - 3(3) = -12$

$x' = 2(1) - (1)^2 - (1)(-1) = 2$

$x' = 2(-1) - (-1)^2 - (-1)(-1) = -4$

$x' = 2(-1) - (-1)^2 - (-1)(4) = 1$

$y' = 3(3) - 2(3)(3) - 2(3)^2 = -27$

$y' = 3(4) - 2(-1)(4) - 2(4)^2 = -12$

$y' = 3(-1) - 2(1)(-1) - 2(-1)^2 = -3$

$y' = 3(2) - 2(-2)(2) - 2(2)^2 = 6$

3)

3) $\dot{h} = -khs$ $\qquad \dot{s} = khs - ls$

$(h) \xrightarrow[-khs]{} (S) \xrightarrow[-ls]{} (R)$

$\dot{h} + \dot{s} + \dot{R} = 0$

$-khs + khs - ls + ls = 0$

$0 = 0$

$kh - l = 0$

$kh = l$

$h = \frac{l}{k}, \; R_0 = \frac{k}{l}$

$h = \frac{1}{R_0}$

a) $\dot{h} = -khs$

$\dot{h} = 0$

$-khs = 0$

$\dot{h} = 0 \Rightarrow h = 0, s = 0$

$\dot{s} = khs - ls$

$\dot{s} = 0$

$0 = khs - ls$

$0 = s(kh - l)$

$s = 0$

$kh - l = 0$

$kh = l$

$h = \frac{l}{k}$

(h, s)

$(0, 0)$

$(l/k, 0)$

$$J = \begin{bmatrix} \frac{d\dot{h}}{dh} & \frac{d\dot{h}}{ds} \\ \frac{d\dot{s}}{dh} & \frac{d\dot{s}}{ds} \end{bmatrix}$$

$$J = \begin{bmatrix} -ks & -kh \\ ks & kh - l \end{bmatrix}$$

$$(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & -l \end{bmatrix} \qquad (l/k, 0) = \begin{bmatrix} 0 & -l \\ 0 & 0 \end{bmatrix}$$

trace $= -l$

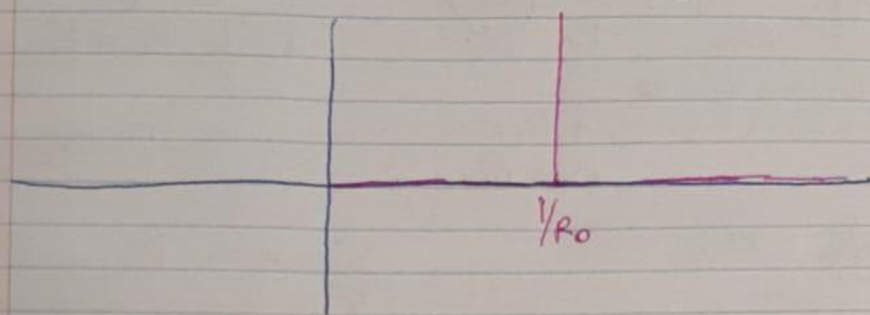
determinant $= ad - bc = 0$

trace $= 0$

determinant $= ad - bc = 0$

infinite fixed points [mentioned in class]

b) $\dot{h} = -khs$ $\quad$ $\dot{s} = khs - ls$ $\quad$ look at this quadrant



c) $\frac{ds}{dh} = \frac{khs - ls}{-khs} = \frac{s(kh-l)}{-khs}$

$\frac{ds}{dh} = \frac{kh-l}{-kh}$

$u = kh$ $\qquad$ $u = kh$, $du = k\,dh$ $\qquad$ $dh = \frac{1}{k}du$

$ds = \frac{kh-l}{-kh}\,dh$

$\int ds = \int \frac{kh-l}{-kh}\,dh$ $\quad$ Substitute in

$\int ds = \int \frac{u-l}{-u} \times \frac{1}{k}\,du$

$\int ds = \frac{1}{k}\int \frac{u-l}{-u}\,du$

$$\int ds = \frac{-1}{k}\int \frac{u - c}{u}\, du$$

$$S = -\frac{1}{k}\int \left(\frac{u}{u} - \frac{c}{u}\right) du$$

$$S = -\frac{1}{k}\int \left(1 - \frac{c}{u}\right) du$$

$$S = -\frac{1}{k}\left(u - c\ln|u|\right) + C$$

$$S = -\frac{1}{k}\left(u - c\ln|kh|\right) + C$$

$$C = S + \frac{1}{k}\left(u - c\ln|kh|\right)$$

d)

(h, s)	h'
$(1, 1)$	$-$
$(2, 2)$	$-$

$1(0.2) - 1 = -0.8$ $\quad 1(0.1) - 1$

$1(1) - 1 = 0$ $\quad 0.1 - 1$

$1(2) - 1 = 2 - 1 = 1$ $\quad -0.9$

$R_0 = \frac{k}{c}$

$S' = khs - 1s$

$S' = k(1)(1) - c(1)$ $\quad R_0 = \frac{0.5}{1} = 0.5$

$S' = k - c$

$S' = S(kh - c)$ $\quad 2(2) - 2$

$S = 0$ $\quad 4 - 2$

$kh - c = 0$ $\quad 2$

$kh = c$ $\quad 0.5(3) - 1$

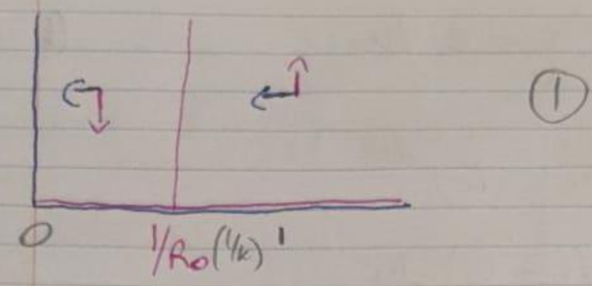
$h = \frac{c}{k}$ $\quad 1.5 - 1$

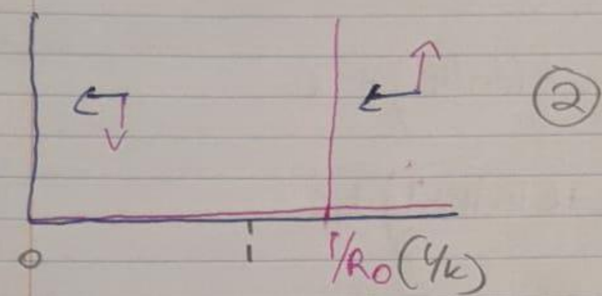
0.5

$h' = -khs$

$h' = -k(1)(1) = -k \quad (1, 1)$

$h' = -k(2)(2) = -4k \quad (2, 2)$





e) Epidemic occurs when $R_0 > 1$, when $\frac{k}{l} > 1$

As seen in graph 1

$\frac{ds}{dt} = khs - ls = s(kh-l)$

$h = \frac{l}{k}$

$R_0 = \frac{k}{l}$

$h = \frac{1}{R_0}$

$s(kh-l) > 0$ $\qquad$ $kh - l < 0$

$kh > l$ $\qquad$ $kh < l$

$h > \frac{l}{k}$ $\qquad$ $h < \frac{l}{k}$

$\frac{ds}{dt} > 0$ $\qquad$ $\frac{ds}{dt} < 0$

$s > 0$

Occurs when $h > \frac{l}{k}$

Code

1)

a)

```
% parameters
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    end
end

% Trajectory plot
plot(0:delta_x:T,x)
xlabel('Time')
ylabel('Position')
title('Part A: Stochastic process simulation')

% Animation
figure
for i = 1:length(x)
    t = linspace(0,x(i),length(x(1:i)));
    plot(t,x(1:i))
    xlabel('Time')
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    title('Particle Trajectory')
    xlim([0 T])
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    drawnow
end
```

b)

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    % final position
    final_positions(i) = mean(x(end,:));

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    histogram(x(end,:),20)
    xlabel('Position')
    ylabel('Frequency')
    title(sprintf('%d particles',n_particles(i)))
end

% Plot: final positions of particles
figure(2)
cmap = colormap(jet(length(n_particles)));
for i = 1:length(n_particles)
    scatter(n_particles(i),final_positions(i),[],cmap(i,:),'filled')
    hold on
end
xlabel('Number of particles')
ylabel('Final position')
title('Final positions of the particles')
colorbar
```