# Lecture Notes: Hot Big Bang
# Feb 2024

## 1 CMB

We will now give a somewhat qualitative description of the early history of the universe, the generation of the CMB, BBN.

The cosmic microwave background (CMB) was discovered in 1965 by Penzias and Wilson. While the discovery was serendipitous, such a background had been theorized as early as 1948 on the basis of the effects it would have on nucleosynthesis. Big bang theory makes two fundamental predictions when Gamow first proposed it. First, actually the reason that big bang was proposed, was to explain, and to make predictions of the abundance of elements in stars, why stars do not consist of H only. This was the days before nuclear synthesis in stellar interior was worked out by Hans Bethe, and later by Burbidges, Hoyle, and Flower. So there was no explanation yet on why there are 25% of Helium in stellar atmosphere etc. It turned out that BBN can not explain the synthesis of elements beyond the lightest ones, and was not a successful theory. But it did make predictions on the primordial abundance pattern, and of course, on the relic photon field that we now know as CMB.

The CMB has three truly remarkable properties:

1. It is nearly perfectly isotropic.
2. It is a nearly perfect blackbody.
3. It is a lot of energy!

Moreover, it has not been resolved into discrete sources, even at arcsecond resolution.

These facts strongly favor an origin of the CMB that lies beyond the usual astrophysical processes that involve stars and galaxies. Energetically, the CMB dwarves the other known contributions to the intergalactic radiation field. The CMB is now known to have a temperature of 2.725 Kelvin which corresponds to an energy density of $4.2 \times 10^{-13}$ ergs $\mathrm{cm}^{-3}$. All other sources of extragalactic background light (e.g. the optical, X-ray, and submillimeter backgrounds) add up to $60 \pm 20\,\mathrm{nW\,m^2\,sr^{-1}}$, about 200 times less than the CMB ($12,600\,\mathrm{nW\,m^2\,sr^{-1}}$). So it is the dominant sources of photons in the universe.

**this figure** summarizes the amount of radiation background as a function of wavelength. $\nu I_\nu$ notation; origin of each components.

The nearly perfect blackbody spectrum of the CMB means that it must have been emitted from a very optically thick source. Since we can see radio and optical sources out to $z > 6$, the optical depth to that redshift must be small. Hence, the CMB was emitted prior to $z > 6$.

For a value of $\Omega_b h^2 = 0.02$ (explain this), the CMB today amounts to 1 MeV per baryon in the universe, and this increases as $1 + z$ at earlier times. Converting hydrogen to helium produces about 4.5 MeV per baryon, so even burning all of the hydrogen in the universe would only barely produce the CMB at modest redshift.

All these features point to the natural explanation that CMB came from the hot big bang, at redshift >> 1. If that were the case, then as we have derived from previous lectures, and this is something that you should know: for a relativistic specie, $p = \rho/3c^2$ as the equation of state, and the energy density goes as $\rho \sim R^{-4} \sim (1+z)^4$. Remember that the easy way to remember this is the number density goes as $R^{-3}$ and the frequency of the photons go as $R^{-1}$. Since we have, for photons, $\rho \sim T^4$, that means the temperature goes as $T \sim 1/R \sim (1+z)$. So the tempertaure of the CMB would increase as $(1+z)$ to high redshift. Now if this can be observed, then it would be a very strong evidence that CMB really comes from high-redshift sources.

It is indeed observed. Example: work by Jian Ge, who was a graduate student here at Steward at the time, and now a professor at Florida building exoplanet instruments, and prof Jill Bechtold, using the MMT at Mt. Hopkins, measured the CI absorption line in bright high redshift quasars, which is caused by very cold gas clouds along the line of sight. You can see the different absorption lines caused by fine structure splitting of energy levels; and the relative strength of the lines are determined by the number of atoms in these fine structure states (with different J number, as L and S are coupled differently). The relative numbers are determined by the excitation temperature. In these very cool clouds, with no stars etc., the excitation temperature is thought to be the same as CMB temperature, which is the only heat source. So, by measuring these lines in high-z quasar, you can measure CMB temperature at high-z as well. To make the long story short, what these groups found was that the CMB T goes exactly as $T_0(1+z)$ in these quasars.

## 2 Radiation dominated era

So we know that the matter density goes as $(1+z)^3$, while the radiation density goes as $(1+z)^4$. Then however small the CMB density, in term of mass density, is, there must be a time when the radiation were dominating, as it increases faster with redshift. When does it happen? Here we are looking for the time when the matter energy density equals the radiation density:

$$\frac{\rho_r}{\rho_m} = \frac{\sigma T^4}{\Omega_m (1+z)^3 c^2} = \frac{2.48 \times 10^{-5}(1+z)}{\Omega_m h^2}.$$

Here $H = 100h$km s$^{-1}$Mpc$^{-1}$. So at redshift $z > 4 \times 10^4 \Omega_m h^2 \sim 3000$, the universe was radiation-dominated, even though now it is matter-dominated. This is actually not quite complete yet. Here we are assuming that the radiation, or relativistic particle in the universe is only photons. It is not, there are also three kinds of neutrinos the same as photons. For reasons that we will discuss later this week, it turns out that there is also a neutrino background with a temperature of 1.95K. And adding that into the balance, the radiation dominated error lasted somewhat longer, to $z \sim 2.4 \times 10^4 \Omega_m h^2 \sim 1800$. We have derived before the dynamics in the radiation dominated era. Note that in this case, $\Omega$ is always one and cosmological constant is always negligible. So we have: $R \sim t^{1/2}$, or $T \sim t^{-1/2}$ as the universe cools down from the big bang.

The present-day photon-to-baryon radio is another key cosmological parameter. Assuming $T_0 = 2.725K$,

$$\frac{N_\gamma}{N_B} = \frac{3.6 \times 10^7}{\Omega_b h^2}.$$

Note that as far as we are not creating photons or baryons, this relation will hold to high redshift. Let's see how long it will hold. This is the same as measuring $\Omega_b h^2$, which we will talk about towards the end of this week.

## 3 Recombination Epoch

The hot Big Bang model explains the CMB as the relic radiation from the hot, early universe. At earlier times, the CMB photons were more energetic, with the temperature of the blackbody scaling as $1 + z$. At $z < 1000$, the photons are too cold to keep the hydrogen in the universe ionized, and so the optical depth is low. At $z > 1000$, the photons are hot enough to ionize hydrogen. This creates a high-optical depth plasma that can thermalize the spectrum. The transition from an ionized plasma to a neutral gas at $z \approx 1000$ is called "recombination".

At first, the fact that recombination occurs at $z \approx 1000$ may be surprising, since the CMB temperature at that epoch is only around 0.25 eV, far less than the 13.6 eV needed to ionize hydrogen. The temperature corresponds to 13.6eV is about 150,000 K. The discrepancy occurs because the number density of photons is so much larger than that of baryons—there are roughly 2 billion CMB photons per baryon—that even the very energetic tail of the Planck spectrum supplies enough photons to ionize the universe.

Now let's see how this works out: since we are only considering the highest energy photons, we are only worrying about the Wien, or exponential tail, of the black-body curve. So the total number of photons with $h\nu > E$ in the limit of $h\nu >> kT$ is:

$$n(>E) = \int_{E/h}^{\infty} \frac{8\pi\nu^2}{c^3} \frac{d\nu}{e^{h\nu/kT}} = \frac{1}{\pi^2}\left(\frac{2\pi kT}{hc}\right)^3 e^{-x}(x^2 + 2x + 2),$$

where $x = h\nu/kT$. Now the total number density of photons in a BB spectrum at temperature T is:

$$N = 0.244\left(\frac{2\pi kT}{hc}\right)^3.$$

Therefore, the fraction of photons of the BB spectrum with energy greater than E is then:

$$\frac{n(>E)}{n} = \frac{e^{-x}(x^2 + 2x + 2)}{0.244\pi^2}.$$

Roughly speaking, the gas will be ionized if there are as many photons with $h\nu > 13.6eV$ as there are hydrogen atoms, that is, we need only one photon in $3.6 \times 10^7/\Omega_b h^2$ of the photons in the CMB to have energy greater than 13.6eV to ionize the gas. And since the fraction above is determined by T, and $T \sim (1+z)$, that will determine the time when recombination happened. Now assume $\Omega_b h^2 = 0.04$, then this fraction is $10^{-9}$, so then we will find: $x = E/kT = 26.5$, and the temperature needed at that time is: $T = 150,000/26.5 = 5600K$. Since the CMB temperature now is 2.7K, then the recombination happens at $z = 5600/2.7 \sim 2000$. Detailed calculations show that the pregalactic gas was 50% ionized at a redshift of $z_r \sim 1500$. This is the epoch of recombination. The universe was fully ionized before then, it was hot plasma, which recombined at $z \sim 1500$. Before the recombination epoch, the universe was full of electrons. It has very important consequences.

We know that electrons and photons interact, through the simple process of Thompson scattering. Without going to detail, which is described in Longair's book, we have the optical depth of Thompson scattering for a completely ionized plasma as:

$$\tau = 0.035\Omega_b/\Omega_m^{1/2} h z^{3/2}.$$

So for any reasonable value, $\tau >> 1$. Before the recombination era, the universe was opaque. Therefore, the universe beyond redshift of about 1100 is unobservable due to this huge optical depth. Any photons originating from larger redshift were scattered many times before they propagated to the Earth and all information about their origin is lost. It is called the photon barrier. Also, if there is no further scattering of photons of CMB, the redshift of about 1000

| Before | $z \sim 1100$ | After |
|---|---|---|
| $p + e^-$ plasma | recombination/phase transition | HI |
| $\tau >> 1$ opaque | last scattering/photon barrier | $\tau << 1$, transparent |
| $T_e = T_{CMB}$ | decoupling | $T_e < T_{CMB}$ adiabatic expansion |

becomes the last scattering surface. So what we are observing the CMB is really this last scattering surface, which can be regarded as the photosphere of the big bang. After the last scattering, the universe becomes optically thin to CMB photons, and they do not interact with matter anymore, in other words, electrons and photons are decoupled.

During the radiation era and before CMB era, we can no longer ignore the small energy change during the scattering process, i.e., the Compton effect. Because of the Compton effect, and the very large scattering optical depth, photon, or energy field, and matter are strongly coupled and they have the exact same temperature. The radiation dominated era leads us to the idea that the universe was much hotter in the past and that it was in fact gravitationally dominated by radiation.

Recombination is a landmark epoch in the history of the universe. It is the photon barrier, the last scattering surface for Thompson scattering, and when radiation and matter decoupled.

Recombination is an epoch, not an event, in the sense that it takes a period of time. We will show that later, but it is about $\Delta z = 200$.

One good way to look at the CMB is the analogy of stellar photosphere. In this sense, recombination is the photosphere of the universe, when optical depth is one. The other good reason for this analogy is that CMB spectrum is similar to stellar photosphere spectrum as well. It is a black body continuum plus recombination lines. More on that later. Since there is no other opacity source, because no matter, the spectrum is far simpler. But one does need to model recombination.

One more note before we look at what is going on in the matter dominated era. If the CMB photons last scattered at some high redshift, e.g. 1000, then photons that arrive to us from very different directions were emitted by regions in the universe that were apparently out of causal contact. The age of the universe at CMB era is about 1 million years. So size of the universe that could have causal contact, the signal could travel is $c \times t$ which is about about 0.3 Mpc, which has an angular diameter of about 4 deg on the sky. All the patches on the sky with separation larger than this are out of causal contact at the recombination time. If these patches were actually out of causal contact, then there would be no way for a causal process to arrange that the CMB in those patches would have the same temperature.

Hence, the near isotropy of the CMB is an amazing fact about the universe, implying that the causal structure that we extrapolate from the matter that we know about *must be wrong.* Inflation provides a striking solution to this problem.

## 4 Early Epochs

Before recombination and during radiation dominated era, matter and radiation are strongly coupled by Compton scattering. In this case, the electrons and photons are transferring energy with each other during the collisions, and matter will have the same temperature as photons, or CMBs.

The thermal history of earlier universe is well described by simple thermal dynamics and particle physics. Let me outline briefly here (Figure):

(1) we can extrapolate back to redshift of about $3 \times 10^8$ when the radiation temperature was about $10^9 K$. This temperature is sufficiently high for the background photons to have $\gamma$-ray energies of about 100 keV. In this case, the high energy photons in the tail of the Planck distribution will have enough energy to dissociate light nuclei such as helium and D. At earlier epochs, all nuclei are dissociated in protons and neutrons. Now we are travelng back in time. To travel forward in time, this is the epoch when the process of primordial nucleosynthesis took place, we will discuss that in just a minute.

(2) At $z \sim 10^9$, the energy of the photon field is about half Mev. This is the energy of electron-positron annihilation. So the e-p pari production from the thermal background can take place and the universe was then full of e-p pairs. So now if we are travelng forward, the e and p are annihilating at this epoch, annihilate, their energy is transferred to the radiation field. THis is the little discontinuity in the figure.

(3) At slightly earlier epoch, the opacity of the universe for weak interactions became unity. We know that weak interactions involve neutrinos, in the same way that E&M interaction involves photons. So this epoch is the epoch of neutrino barrier, since to the photon barrier at $z \sim 1000$. At this epoch, neutrinos become uncoupled.

(4) Extrapolating to about $z \sim 10^{12}$ when the temperature of the radiation field was high enough for baryon-antibaryon pair production. Just as in the case of the epoch of e-p pair-production, the universe before then was flooded with baryons and anti-baryons. Therefore, there is any a slight discontinuity in temperature. Here there is one of the cosmological problems, namely the baryon asymmetry problem. In order to produce the matter-dominated universe we live in today, there must have been a tiny asymmetry between matter and antimatter in the very

early universe, about one in 1 billion excess of matter. This asymmetry is the reason for the photon/baryon ratio we have today. It must originate from the very early universe.

(5) Finally, we can this to as far as we trust our particle physics, which is probably at least believable to about 100 GeV. How far back more one can push is then a matter of taste. Some have no hesitation to go all the way to the Planck era, when particle physics and GR merge and the physics as we know become irrelevant. Note that Planck scale is defined as when Schwardschild scale, $\lambda = Gm/c^2$, or the size of horizon, is the same as the Compton wavelength, or the size determined by uncertainty principles: $\lambda = h/(mc)$. Combining these two, we have:

$$\lambda_p = (Gh/c^3)^{1/2},$$

and Planck time:

$$t_p = (Gh/c^5)^{1/2} = 1.35 \times 10^{-43} sec.$$

Beyond this, our view of space-time has to be changed. We are entering the regime of string theory, which is beyond this course.

Most part of these early history of the universe will be beyond our class, which, as we noted in our first lecture, is a cosmology class from an extragalactic viewpoint. So we will not discuss in detail things that are really beyond our current means of astronomical observations. But how far can we push? We talked about photon barrier, that is, at the CMB era, $z \sim 1200$, the optical depth due to Thompson scattering is large. CMB acts as a photosphere of the universe, so we can't see through it. It is a perfect BB, with no more information other than a temperature. How do we know, or test our model beyond that? There are a few ways that early universe left imprints on later era observations.

- CMB fluctuations. The T fluctuation of CMB is a result of early physical processes, in particular, inflation. We will discuss this in a few weeks.
- g-wave. We don't know how to do that. But in principle. Also, it has consequences on CMB structure.
- Neutrino. As we discussed, the neutrino barrier is at much higher redshift, $z \sim 10^{10}$ or so. It left us with a neutrino background at $T = 1.95K$, a temperature we will justify later. So if we can observe $\nu$ background as we do with photons, maybe. But we don't know how to do that either.
- BBN. What the last point says is that the signatures of weak interaction (and strong interaction) in the universe are preserved at higher redshift. We can observe phenomena related to weak and strong interaction, i.e., nuclear interaction, at higher redshift to

test our physics. The key result of such interactions is the synthesis of heavy element in the universe a process similar to the nuclear reaction that we are familiar with in the interior of stars, with one major difference. In stars, we think about nucleosynthesis as a process with an increasing T. In BBN, temperature are decreasing rapidly as the universe expands. So your nuclear reaction rate is always competing with the Hubble expansion. When it is longer than Hubble time, it is not relevant anymore. So during BBN, only a small number of light elements formed. The abundance of these element provides a sensitive test to our cosmology, especially to the number density of baryons.

## 5 Equilibrium Abundances in the Early Universe

Before we discuss BBN, we should consider a bit further the role of neutrinos and weak interaction in the early universe. I hope you still remember some of your particle physics and statistical physics stuff.

The CMB and big bang leads us to the idea that the universe was much hotter in the past and that it was in fact gravitationally dominated by radiation. It has important consequence to the species of particles in early universe.

Imagine what would happen if the universe were very hot. For example, let's consider that the temperature of the CMB were 10 MeV/k. Two photons colliding could produce an electron-positron pair.

$$e^+ + e^- \leftrightarrow \gamma + \gamma$$

How frequent is such a reaction? A given photon will have a reaction rate $nc\sigma$, where $\sigma$ is the cross-section of the reaction. The cross-section is approximately the Thompson cross-section. In detail, it's about $(3/16)\sigma_T(m_ec^2/E)^2$, which is about $3 \times 10^{-28}$ cm$^2$ at 10 MeV.

The density of photons today is 411 cm$^{-3}$ at a temperature of 0.00023 eV. The redshift to make the CMB be at 10 MeV is $4 \times 10^{10}$. Hence, the density of photons is then $2.5 \times 10^{34}$ cm$^{-3}$. So the reaction rate is $2 \times 10^{17} s^{-1}$.

What is the age of the universe at this redshift? The Hubble constant will be

$$H^2 = \frac{8\pi G\rho}{3}$$

At these redshifts, the density of the CMB is far higher than that of the protons, neutrons, and other matter. Using just the density of the CMB, we would have $\rho c^2 = aT^4$ where $a = 7.56 \times 10^{-15}$ ergs cm$^{-3}$ K$^{-4}$. We had $4.2 \times 10^{-13}$ ergs cm$^{-3}$ at $z = 0$; here we have

$(1+z)^4$ more, $1.1 \times 10^{30}$ ergs cm$^{-3}$! Dividing by $c^2$ gives a mass density of $1.2 \times 10^9$ g cm$^{-3}$. That makes $H = 25s^{-1}$. The age of the universe is of order $H(z)^{-1}$, which works out to 0.04 seconds.

Hence, a photon can interact of order $10^{16}$ times in the age of the universe. This means that the number of photons and electrons will come to a statistical equilibrium. How many of each should be around?

For weakly interacting gas of particles in thermodynamic equilibrium, the number density of particles is

$$n = g \int \frac{d^3p}{h^3} f(E)$$

where $g$ is the number of spin states and $f(E)$ is the occupation number of the non-interacting states. Statistical mechanics says that the occupation of a given state is

$$f(E) = \frac{1}{\exp[(E-\mu)/kT] \pm 1}$$

for fermions and bosons. Here, $E^2 = m^2c^4 + p^2c^2$ and $\mu$ is the chemical potential of the species.

The energy density is

$$u = \rho c^2 = g \int \frac{d^3p}{h^3} f(E)E$$

We can switch variables to $E$ by using $d^3p = 4\pi\sqrt{E^2 - m^2c^4}E\,dE/c^3$. Then

$$\rho c^2 = \frac{4\pi g}{c^3h^3} \int dE \frac{E^2\sqrt{E^2 - m^2c^4}}{\exp[(E-\mu)/kT] \pm 1}$$

For a relativistic species ($m = 0$) with $\mu = 0$, the energy density integral is

$$\rho c^2 = \frac{4\pi^5 k^4}{15c^3h^3} gT^4 = \frac{g}{2} aT^4$$

for bosons. You know this result from the Planck spectrum. For fermions, there is a reduction by a factor of 7/8.

The number density of particles is

$$n = \frac{8\pi k^3}{c^3h^3} \zeta(3) gT^3 \propto T^3$$

for bosons. For fermions, there is a reduction by a factor of 3/4.

We are dealing with three different kind of particles in the early universe:

- photons. They are massless bosons with g=2.

$$N = 0.244(2\pi kT/hc)^3 m^{-3}, E = aT^4;$$

- nucleons, electrons and their antiparticles. They are fermions with g=2.

$$N = N^{-} = 0.183(2\pi kT/hc)^3 m^{-3}, E = 7/8aT^4;$$

- neutrinos and antineutrinos. We are aware of three kinds of neutrinos. They are fermions with g=1.

$$N = N^{-} = 0.091(2\pi kT/hc)^3 m^{-3}, E = 7/16aT^4.$$

Hence, we would predict that the number of electrons should be 7/8 of the number of photons. This is vastly more electrons than we observe in the universe today. What happened to them?

As the universe cools, the temperature drops below the rest mass energy of the electrons. The annihilation rate of electrons and positrons became faster than the production rate. The electrons and positrons disappear in favor of photons.

The energy in the electrons and positrons (which was 7/4 of that in the CMB) becomes extra energy in photons. But this means that the photons have too much energy for a blackbody of their temperature. As the photons scatter off the residual charge in the universe, they can share their energy and achieve a new blackbody temperature. If the annihilations were faster than the redistribution, then the change in temperature would be at constant energy, which would mean that $T$ would increase by $(11/4)^{1/4}$. However, in fact the annihilations are slower than the redistribution, which means that the change is accomplished adiabatically, i.e. at constant entropy. The entropy of a relativistic distribution goes $(\rho + p)/T = 4\rho/3T$, which scales as $T^3$. Hence, the temperature actually increases by $(11/4)^{1/3}$.

Note that this breaks the relation between $R$ and $T$. As the universe expands, normally $T \sim R^{-1}$, but as the electrons and positrons are annihilating, the temperature drops less quickly.

Of course, as the temperature drops, the equilibrium density of electrons would go to zero. We apparently would have no electrons in the universe! Two effects intervene.

First, the annihilation rate depends on the density of electrons and positrons and eventually this becomes so small that the electrons can't annihilate in the age of the universe. The resulting abundance is known as the "freeze-out" abundance. Clearly, the result depends on the annihilation cross-section. Larger cross-sections make the final abundance smaller.

However, for electrons the freeze-out abundance would be far too small (moreover, there'd be a lot of positrons around!). Instead, the residual electron density is set by a tiny asymmetry ($10^{-9}$) in the number of electrons relative to the number of positrons. All the positrons annihilate on electrons, but the asymmetry is left over.

This is known as the lepton asymmetry. We don't know where it comes from!

In the early universe, the relativistic species determine the density. Any species with $kT > mc^2$ will have a number density similar to the CMB. We can count them!

$$\rho c^2 = (g_{boson} + \frac{7}{8} g_{fermi}) \frac{a}{2} T^4$$

In detail, we should allow for each species to have a separate temperature. We define

$$g_* = \Sigma_{boson} g_i \left(T_i/T_{cmb}\right)^4 + \frac{7}{8} \Sigma_{boson} g_i \left(T_i/T_{cmb}\right)^4$$

and then write $\rho c^2 = g_*(a/2)T_{cmb}^4$.

Then we have the Hubble constant, which is $H^2 = 8\pi G\rho/3$. This gives $H \propto g_*^{1/2} T^2$, which means that the age of the universe is roughly $t \sim T_{MeV}^{-2}$ seconds.

As the universe cools, species for which the rest mass is important annihilate and become trace constituents.

## 6 neutrinos in early universe

Next, consider neutrinos. These have too slow a reaction; they stop scattering or annihilating at 1 MeV, so their number density is frozen, regardless of whether they are relativistic today.

The cross-section of the neutrinos are weak, typically $G_F^2 T^2 = 5 \times 10^{-44} T_{MeV}^2$ cm$^2$.

The redshift to achieve 1 MeV temperatures is roughly $4 \times 10^9$ (but this is actually a bit too high by $(4/11)^{1/3}$), which makes the density $2.5 \times 10^{31}$ cm$^{-3}$. The interaction rate is $\Gamma = n\sigma c = 0.04\ s^{-1}$. This is comparable to the Hubble constant, so we have roughly 1 encounter per Hubble time (this came out to 25, but we dropped lots of factors).

Above 1 MeV, there are plenty of interactions to make neutrinos and antineutrinos. We get full blackbody populations.

However, the neutrinos stop interacting at $T < 1$ MeV. This is known as decoupling. These backgrounds of 3 species of neutrino and antineutrino should still be around in the universe! There should be roughly as many low-energy neutrinos as there are photons!

This cosmic neutrino background has never been detected directly! The neutrinos have a tiny energy and no one has thought of a way to detect them directly. However, we can infer their presence cosmologically, by their effect on nucleosynthesis in the early universe.

However, as a minor detail, the early decoupling of the neutrinos means that they do not share in the energy released by the annihilation of the electrons and positrons. Hence, the photon temperature rises by $(11/4)^{1/3}$ relative to the neutrinos. In other words, the neutrinos have a temperature of $(4/11)^{1/3} \times 2.725$ Kelvin, which is 1.95 K. This means that there are fewer neutrinos than photons (each spin state is down by a factor of $(3/4) \times (4/11)$ relative to that of photons, so we have $6 \times (3/4) \times (4/11) \times 411/2 = 336$ neutrinos per cubic centimeter.

When the neutrino's decouple, they are very relativistic and hence have a momentum distribution given by a (fermionic) blackbody. However, we think the neutrino does have a small mass. Today, this mass is considerably higher than the kinetic energy. This can make a significant mass contribution to the universe! If a particular species of neutrino had a mass of about 30 eV, the 112 neutrinos per cubic centimeter would suffice to make $\Omega = 1$!

Hence, we can infer that the neutrino (if stable) cannot have a mass between about 30 eV and 1 MeV! Otherwise, the universe would have $\Omega \gg 1$, in conflict with observations. In fact, we think the limit can be placed lower, but we'll get to that later.

One of the remarkable properties of the Hot Big Bang is that standard particle and nuclear physics predicts the formation of certain elements in the early universe. Indeed, the large amount of helium and simple existence of deuterium would be very hard to explain were it not for the Big Bang. It is a remarkable piece of corroborating evidence for the Hot Big Bang that the predicted abundances work out to be close to those observed.

At very high energies, $T \gg 1$ GeV, there are lots of quarks and antiquarks around. As the temperature drops to a few MeV, the quarks and antiquarks annihilate, leaving only a small (and unexplained) residual population of quarks.

These quarks are bound into protons and neutrons. The neutron is slightly heavier than the proton, by about 1.293 MeV. Neutrons and protons can transmute by weak interactions

$$n + \nu_e \leftrightarrow p + e^-$$

If these reactions were fast (compared, as usual, to the Hubble constant), then the relative density of the two would be

$$\left[\frac{n}{p}\right] = \exp\left(-\frac{1.3\ \mathrm{MeV}}{kT}\right)$$

As the temperature dropped well below an MeV, all the neutrons would disappear and the universe would be left with only hydrogen.

Or would it? As you know from stellar physics, there are heavier elements that are energetically favored. If we really had equilibrium all the way to low temperatures, these nuclei would be favored and the universe would be made of iron!

Both of these predictions are wrong because the interaction rates are not arbitrarily fast. The process of nucleosynthesis is guided through a few particular channels that yield the standard predictions.

## 7 The decoupling of neutrinos and neutrino barrier

First, let's consider the weak interaction that governs the conversion of protons and neutrons. We saw before that two neutrino interactions froze out at about 1 MeV. We get a similar answer here.

Roughly, the time-scale for weak interaction is

$$t_{weak} = (\sigma N c)^{-1}$$

where $\sigma$ is the cross section of weak interactions, $\sigma \propto E^2 \propto T^2$, and N is the number density $N \sim R^{-3} \sim T^3$, so $t_{weak} \sim T^{-5}$. And the age of the universe $t_H \sim R^2 \sim T^{-2}$. It is important to remember that the cross-sections fall sufficiently quickly with energy that the ratio of the interaction rates to the Hubble constant is a steep power of temperature, $T^3$. So the reactions really do shut off.

Detailed calculations find that the ratio of neutrons to protons drops to about 1:6 as the temperature drops below 1 MeV, corresponds to $T \sim 10^{10} K$. Remember that the age of the universe is about 1 second.

A by-product of this is that we have also derived the time when the universe was transparent to neutrinos. Analogy to the photon barrier at $z \sim 1200$, we have a neutrino barrier, and thus a neutrino background. We can in principle use neutrino background to probe the universe when it was 1 sec old!

## 8 The Synthesis of the light elements

We now have 1 neutron and 6 protons in a cooling universe. If nothing else happened, then the neutrons would decay back into protons with a half life of 887 seconds. Remember that free neutrons are unstable!

However, the neutrons are saved. After a few minutes, the temperature of the universe is down to about 100 keV. At these temperatures, $^4He$ is favored. Most of the neutrons become bound into the Helium.

There are two questions that arise here:

1) In fact, $^4He$ is favored even at slightly higher temperatures (say 300 keV). Why doesn't form a little earlier?

2) Why, at some lower temperature, doesn't the helium fuse into $^{12}C$ and so on into heavier elements?

The answers are that the reaction pathways have bottlenecks. To form the helium, we have to fuse 4 protons. 4-body interactions are very rare; instead, nature prefers to link together 2-body interactions. The relevant one in this case involves deuterium, $n + p \to D + \gamma$. The deuterium then combines with another $p$ or $n$ to form $^3He$ or $^3H$, and then one last nucleon to reach $^4He$. Alas, deuterium is rather fragile and there are a billion times more photons than nucleons. So at hotter temperatures, the abundance of deuterium rises slowly. The production of $^4He$ is starved waiting for it!

Why stop with helium? The same reason that helium on earth doesn't fuse: the Coulomb barrier. As the universe cools, heavier elements are energetically favored, but the reaction rates are very slow because of the Coulomb repulsion of the nuclei. The delay in the formation of Helium helps here, as does the lack of a tightly bound nucleon with mass 5 or 8 (there's no good intermediate point between $^4He$ and $^{12}C$).

So the process ends with most of the neutrons bound into $^4He$. How much helium? In the delay of a few minutes to cool to 100 keV, the neutron fraction decays from 1:6 to 1:7. This means that there are 2 neutrons for every 14 protons. That makes 1 helium and 12 hydrogen. Helium is about 25% by mass ($Y = 0.25$).

This is very close to what is observed!

To a part in $10^4$, all of the neutrons end up in $^4He$, but the calculation of the exact neutron-proton ratio is actually rather detailed.

The neutron-proton ratio depends sharply on temperature as the weak interactions are freezing out, so these reactions must be modeled carefully.

Moreover, at about this time, the neutrino-neutrino interactions are slowing, so the neutrinos aren't necessarily held into a thermal distribution.

Moreover, the electron-positron pairs are annihilating, so the universe is getting reheated. It spends longer at certain temperatures than one would expect.

So, the actual freeze-out abundance present at $t \sim 100$ seconds requires a lot of detailed calculation. However, the principles are generally straight-forward: we know how to calculate electroweak cross-sections. Theoretical predictions are thought to be good to better than

1%.

## 9 Dependence on cosmological parameters

What dependences does the result have?

For a long time, the uncertainty in the overall amplitude of the weak interaction rates dominated the uncertainties. Weaker interactions mean earlier freeze-out's and higher $n/p$. However, this parameter is now well-measured.

If one increases $g_*$ by adding extra relativistic species to the universe, then one gets faster expansion at a given temperature. This means that reactions freeze-out a bit earlier ($\Gamma = H$). In detail, $T_{fo} \propto g_*^{1/6}$. The result is higher $n/p$ and more helium.

If the baryon-to-photon ratio is larger, then deuterium forms more easily. This makes $^4He$ production a bit earlier. However, this is a weak effect on $^4He$. Factors of 10 in the baryon density make 3% effects on helium.

So, one reaches the remarkable conclusion that the abundance of helium depends primarily on the number of relativistic species in the universe! The argument has generally been that BBNS rules out a 4th neutrino species (even a sterile species)! The best fit value so far has $n_\nu = 2.3$ with $n_\nu = 3$ consistent at about 1-$\sigma$ level. This constraint is quite bit earlier than the experimental physics constraint and it is truly remarkable.

**Figure, Longair 10.2**

$\eta = 10^{10} n_B/n_\gamma = 274\Omega_b h^2$

## 10 Observations

### 10.1 Helium-4

Observationally, the fraction of $^4He$ is close to what's predicted, but different groups do disagree at the 2% level. In any case, stars also make $^4He$. Plotting $^4He$ against metallicity in stars or HII regions reveals a small slope (solar stars are $\sim 0.29$). The game is to assume that helium enrichment and metal enrichment are related and to try to extrapolate to zero metals. This isn't so bad, but the measurement of helium abundances in stellar atmospheres or the ISM is never perfect. Many estimates of the primordial helium are a bit lower than the BBNS prediction ($Y_p = 0.238 \pm 0.007$), but the uncertainties seem large enough to be consistent. Nevertheless, it is a triumph that the helium predicted from the early universe is a close match to that observed in metal-poor stars!

**figure 10.3 of Longair**

### 10.2 Deuterium and He-3

Trace amounts of other nuclei are also produced. Deuterium and $^3He$ are made on the way to $^4He$, but as the temperature drops the Coulomb barriers cause the final reactions to freeze-out. This leaves a small amount of $D$ and $^3He$ around (and $^3H$, but that decays). The levels are a few parts in $10^5$.

This production is *very* sensitive to the baryon density. The reaction rates are proportional to the density, so higher density means faster reactions, which means more complete burning to $^4He$ and lower D,$^3He$ abundances.

If we could measure the "primordial" (i.e. pre-stellar) fraction of $D$ and/or $^3He$ we would have a measurement of the baryon density!

$D$ is relatively fragile. All known and common astrophysical processes destroy it. In particular, it is burned easily in stars as part of the pre-main-sequence evolution. Conservatively, one takes the observed $D$ abundance as a lower limit. This means that it is an upper limit on the baryon density! In terms of the critical density, this limit is $\Omega_b h^2 < 0.025$. With $h > 0.5$, that means $\Omega_b < 0.1$ (and probably a fair bit less).

We observe the density of the local universe to be higher than this, typically $\Omega_m$ at least 0.2. This is strong evidence for non-baryonic dark matter!

It has always been dicey to argue about whether the $D/H$ ratio in the Milky Way is primordial. Recently, we have gotten measurements at high redshift from absorption in QSO spectra. This argues for $D/H \approx 3 \times 10^{-5}$. If this is primordial, and it may well be, then $\Omega_b h^2 = 0.022 \pm 0.002$! We have a 10% measure of the baryon density! Measurement of deuterium in quasar absorption line is generally regarded as primordial. But because of the low D abundance, the observation is highly difficult. It is a weak transition next to H, because of the slight different in the reduced mass when having atomic number of 2 instead of one. So on the blue wing of Ly$\alpha$ line. It requires: (1) accurate measurement of H, which is always saturated in order to see D; (2) removal of contaminant in Ly alpha forest.

Local $D/H$ measures are generally lower than this value, around $1.5 \times 10^{-5}$, and display significant scatter. It is thought that much of the deuterium in the ISM has been destroyed by cycling through stars.

This baryon density does prefer a relatively high value for $^4He$, around $Y = 0.245$ or 0.25. This is a bit of a stretch.

**Figure 10.4 of Longair**

For a few years, there were claims of very high $D/H$ ratios, leading to low baryon densities. I think these are all defunct now.

$^3$He is observed in oldest meteorites which is regarded as the value for solar nebula. The interpretation is complicated. D burns into He-3 which burns in He-4.

## 10.3 Lithium-7

$^7Li$ is also produced in BBNS. It is produced by combining $^4$He with $^3$H at low baryon density and with $^3$He at high baryon density. This produces a minimum abundance near where the predicted baryon density is.

**Figure 10.5 of Longair**

The predicted abundance is about $3 \times 10^{-10}$. This is a factor of 2 higher than what is observed in stars $(1 - 2 \times 10^{-10})$. The destruction of Lithium in stars is a fairly active and contested field. Lithium burns fairly easily in stars. Convective stars will destroy most of their surface lithium. But it can also be enhanced by collisions of cosmic ray protons and cold ISM gas. Figure shows a lithium platoon at low metallicity.

What makes the heavier elements? Everything above Carbon is made in stars. Most of the $B$ and $Be$ are made when cosmic rays hit heavy nuclei and split them in two. This is called spallation. Note that the spallation abundances are far below that of $D$; it is implausible that deuterium is produced by spallation.

Review by Olive astro-ph/0202486. Note that much of the BBNS literature uses the baryon-to-photon ratio $\eta = 5.36 \times 10^{-10}(\Omega_b h^2/0.02)$.

Most of the leverage in BBNS currently comes from $D/H$, as this puts an upper bound on $\Omega_b h^2$ that is very similar to the results from the CMB. Indeed, the latest CMB results push the baryon slightly higher, which would require a slightly lower $D/H$. Also, $^4$He and $^7$Li don't fit too well with that value.

BBN is one of the sometimes overlooked triumph of big bang. It is a highly specialized field. But this overlook might be just due to the simplicity and success of the whole prediction. I regard the BBN constraints on things like the number of relativistic species some of the most beautiful physics I encountered. It is the highest redshift direct probe of the universe up to now, with the exception of CMB constraints on inflation which is indirect and highly model dependent. With the BBN, we are safely trust our basic understanding of the expansion and thermal physics in early universe up to $z = 10^{10}$. Actually, things are most predictable at between photon barrier and neutrino barrier, at redshift between thousand and billion. Before that, we are more and more dependent on particle physics at the highest energy. At lower redshift, the inhomogeneity of the universe becomes important which results in the growth of structures, which we occupy our discussion for the rest of the semester.