AST541 Lecture Notes: CMB March 2024

The power spectrum of angular fluctuations in the CMB and its polarization properties provide a wealth of information of cosmological significance, both for the determination of cosmological parameters and for understanding the structure formation in the Universe. Starting with ground-based and balloon experiments, the experimental success was highlighted by COBE, WMAP and Planck missions, Nobel prize worthy experiments.

Our goal here us to related the models of structure formation to the imprint they leave on the CMB. COBE, WMAP and Planck, combined with low-z redshift survey, bring us to the era of precision cosmology, in which we are now able to determine cosmological parameters to better than 10%.

We will not discuss in great details about the experimental aspects of things. It is a highly specialized field. The experimental achievements now is at the precision that requires very detailed modeling. To include all the relevant physics, the only feasible approach is to find numerical solutions of the coupled Einstein - gravity, Boltzmann - statistical, and fluid dynamical equations. There are now great off-the-shelf codes to do this for you, for example CMBFast. If you want to interpret CMB results, you will need to rely on those. Our goal here is to outline the general physical processes, and to explain the general principles of why we can use CMB anisotropy to determine cosmological parameters.

We will first establish the basic scales of our problem. Then introduce the tools CMB people uses to describe the angular anisotropy. Next class, we will talk about features on the CMB power spectrum and results on cosmological parameters.

1 The Recombination Era

As shown before,

$$\tau = 0.035\Omega_B/\Omega_m^{1/2}hz^{3/2}$$

where τ is the optical depth due to Thompson scattering. So at $z \sim 1000$, optical depth is very large. Any radiation originating from higher redshift than the epoch of recombination was scattered many times until the last scattering surface. This is very similar to the problem of stellar atmosphere. The redshift at which $\tau \sim 1$ corresponds to the stellar photosphere. One can define a visibility function:

$$v(z) = e^{-\tau}d\tau/dz,$$

which tells you at which redshift most of the photons reach us.

Figure.

We discussed a little bit about calculation of recombination before. One needs to include carefully all processes about Lyman line and continuum photons, in particular two-photon process in which two photons are liberated from the 2s state to the ground state in a rare quadrupole transition, which turns out to be the dominant way to get hydrogen to the ground state, because one photon process will generate a high energy photons that can be used immediately to ionize another hydrogen at large optical depth.

The important thing about this figure is that recombination, or last scattering, didn't happen at a single redshift. Rather, for a maximum visibility at z=1090, the half maxima are at 1178 and 983, with an interval of $\delta z = 195$. It takes about 120,000 years. This finite depth of large scattering layer has important consequence.

2 Scales of Fluctuations

Before we look at the CMB power spectrum, let's work out a few physical and angular scales that are important to the CMB first. Here we assume a LCDM cosmology with concordance numbers.

2.1 The last scattering layer

Taking the thickness of the last scattering layer to correspond to a redshift interval of 195 at z=1090, the comoving radial distance is then

$$\Delta r = 16.2(\Omega_m h^2)^{-1/2} = 42\text{Mpc}.$$

THe mass within this cale is roughly $2 \times 10^{15} M_\odot$, which is the mass of a cluster. The angular size is $\theta \sim 3'$.

On comoving scale less than 42Mpc at current epoch, we expect a number of independent fluctuations to be present along the line of sight through the last scattering layer. So the random superposition of these perturbations leads to a statistical reduction in the amplitude of the observed intensity fluctuation by a factor of $N^{-1/2}$.

2.2 The sound horizon and acoustic peaks

We briefly mentioned acoustic peaks in the last lecture. The existence, and the properties of acoustic peaks are the most fundamental predictions of CMB anisotropy. We want to calculate the sound horizon at the last scattering, which is defined to be $\lambda_s = c_s t$, where c_s is the sound speed and t is the age of the universe at that time. This is the maximum distance which sound waves could travel and undergo coherent oscillations, therefore, it is the longest wavelength which acoustic waves could have at the epoch of recombination. The sound speed of a photon fluid is $c/\sqrt{3}$. When there are baryons, it slows down a bit. But roughly, $c_s \sim 0.5c$. The comoving size of sound horizon is given by

$$s = \int_0^{t_*} dt\, c_s(z)\, (1+z) = \frac{c}{\sqrt{3}} \int_0^{t_*} dt\, (1+z)$$

For our standard cosmology, this is about 100 h^{-1} Mpc, and it corresponds to $\sim$ 0.6 deg on the sky. This wavelength corresponds to the first maximum in the power spectrum of temperature fluctuations. It is when a single wavelength of oscillation between the entry of horizon and the epoch of recombination. Because after recombination, the acoustic wave could not grow anymore and will freeze. So we are predicting that there will be a very pronounced peak in the CMB power spectrum at this scale, which is the same thing we saw in the galaxy LSS. There will be higher order resonance at smaller scales. The acoustic scale is a standard ruler, or really a sounding rod. Note that we are only talking about baryons. There will be a background of perturbation from dark matter which is a result of our Harrison-Zeldovich spectrum. The baryon features discussed here are all on top of it.

2.3 Silk damping scale

Another important physical process related to perturbations in baryons is the effect of Silk damping, which is basically the photon diffusion, or dissipation process of the baryon-photon fluid in the pre-recombination era.

Although at that time, matter and radiation were closely coupled, the coupling was not perfect and photons could diffuse out of the density perturbations. Perturbations in baryon density travels as acoustic waves, or sound waves, in the universe. Radiation pressure provides the restoring force which maintained these oscillations. If the photons diffused out of the perturbations, the wave will be damped out. These was first recognized by Joe Silk in 1968. It was only observed in the CMB small scale power spectrum very recently. We want to find out the scale to which photons can diffuse. At smaller scales, baryon perturbation will be damped. The radial distance over which photons can diffuse is

$$r_S \sim (Dt)^{1/2} = (1/3\lambda ct)^{1/2},$$

where D is the diffusion coefficient which goes roughly as the product of mean free path of photons due to Thomspon scattering, the reason that photons will diffuse, and the sound speed of the relativistic fluid. Plug in the age of the universe at recombination, we find that Silk damping scale is 9 Mpc, and the angular scale is about $\sim 5'$ on the sky. What Silk damping is going to do is to damp out high order acoustic peaks in the CMB. There is a homework problem asking you to work out some more details.

2.4 Particle Horizon Scale

Finally, just to remind you that one can calculate the scale of particle horizon at recombination. This is the maximum distance in the universe at which causal connection can be established. We worked out these scale before, it is just a few times larger than the sound horizon. $r_H \sim 500Mpc$ in comoving units, and angular scale of about 2 deg on the sky.

So to ramp up:

- recombination time
- particle horizon: scale large than 2 deg not conected.
- scale between 0.01 and 2 deg, lots of baryonic features, affected by silk damping and finite thickness of last scattering layer. Important cosmological probe,

3 Power Spectra of the CMB

Now we introduce the technique and nomenclature to express the fluctuation in the temperature of the CMB. We are observing a temperature distribution over the surface of a sphere, and we are interested in very large angles. So we need the two-D polar equivalent of the relation between density distribution and power spectrum. The standard technique is to use the spherical harmonic functions as our orthonormal base functions. We first make a spherical harmonic expansion of the temperature distribution of the whole sky:

$$\frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - T_0}{T_0} = \sum_{l=0}^{\infty} \sum_{m=-1}^{m=l} a_{lm} Y_{lm}(\theta, \phi),$$

where the normalized function Y_{lm} is given by:

$$Y_{lm}(\theta\phi) = \left[\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(sl+|m|)!}\right]^{1/2} P_{lm}(\cos\theta) e^{im\phi} \times \frac{(-1)^m, m \geq 0}{1, m < 0},$$

here Y_{lm} is called a spherical harmonic function of degree l and order m, and $P_{lm}(\cos\theta)$ is an associated Legendre function. Note that for each l, there are 2l+1 values of m. The power of each mode (l,m) is a_{lm}, which can be found as:

$$a_{lm} = \int \frac{\Delta T}{T}(\theta, \phi) Y^*_{lm} d\Omega$$

where Y^*_{lm} is the complex conjugate of the spherical harmonic function. Now we have the two-D power spectrum. Roughly speaking, the degree l is related to the angular separation as $\theta \sim \pi/l$. Therefore l is oftern referred to as the multipole moment, $l = 1$ is the dipole, $l = 2$ is the quadrupole etc.

a_{lm} is a complete description of the density fluctuation. But how is that related to our 3-D power spectrum? In general, the temperature distribution over the sky need not to be Gaussian. If it were Gaussian, then the distribution of a_{lm} for the same l will have random phases, Inflation strongly predict a random phase distribution, since there the origin of fluctuation is quantum fluctuation in the early universe. Other theories that invoking seeding of fluctuations by topological defects, cosmic strings or cosmic textures, suggest non-Gaussian distribution, as they cause abrupt temperature discontinuities. In this case, a_{lm} will be correlated.

If it were Gaussian then each a_{lm} represents an independent estimate of the power at multipole l, so we just average them, and introduce the power spectrum C_l:

$$C_l =< |a_{lm}|^2 >$$

C_l will be a complete description of the temperature fluctuation. It can be shown that for a Harrison-Zeldovich power spectrum,

$$C_l \propto 1/l(l+1)$$

so you usually see people plot $l(l+1)C_l$, because on large scale, it will be relatively flat.

one important issue this discussion brings up is the issue of **cosmic variance,**, which ultimately limits the precision of our estimates of temperature fluctuations. The spherical harmonic analysis shows the we obtain (2l+1) independent estimates of the value C_l for a given multipole. The precision with which value of C_l is known is:

$$\sigma(C_l)/C_l \sim [2/2l+1]^{1/2}.$$

This is the result of we do not have enough sample of the universe, and can not be overcome by better precision in our measurements. So there is a limit that we can know about the property of the largest scale structure in the universe. For COBE, at $l < 20$, the measurements are cosmic variance rather than noise limited, For WMAP, $l < 400$ are cosmic variance limited. In other words, for $l < 20$, COBE is basically as good as it will ever get and future experiment will not improve upon it. This does cause problems. For example, our CMB features a small quadrupole, by 1.5σ or so, comparing to best-fit models. Is there anything wrong with our cosmology, that there is less quadrupole? Or is it just because when you look at many multipoles, some of them by definition will fall out of 1-σ?

Another note: you hear people working on galaxies talking about cosmic variance a lot. Most of the time they use the term wrong. They sometimes say that HDF is too small and the measurement is subject to cosmic variance, since there is strong clustering at that scale, and one is not getting a fair sample of the universe by looking only at a small field. But this is not cosmic variance. The correct term is **sample variance**. Cosmic variance basically says that we have only one universe, and at certain level, our homogeneous cosmology breaks down because we don't have enough independent measurements to figure out what the average value of certain quantity of the universe actually is.

4 CMB power spectrum

Figure 1 Dipole,

Moving through a bath of blackbody photons creates a temperature dipole: hotter in one direction, but does remain a blackbody. The universe does have a preferred frame. Intrinsic dipole would be 10^{-5}, not 10^{-3}. heliocentric dipole has an amplitude of 3.353mK (350km/s), towards $l = 264$, $b = 48$. Part of this is earth rotation in the Galaxy. Correcting for that, LG motion is 600 km/s toward (l,b) = 270, 30; RA=10.5h, dec -26. This is peculiar velocity of the LG.

Figure 2.

Dipole was discovered in early 70's, but other anisotropies weren't discovered until 1992, COBE. At 10 deg scale, CMB fluctuation is about 30μK, or about 10^{-5}.

To get to the picture of WMAP, one has to get rid of: (1) the free-free from galaxy; (2) point sources. One does that by observing in multi-frequency, and then filter out foreground.

Figure.

Finally, CMB power spectrum. Features:

- large scale: relatively flat: a result of Harrison-Zeldovich power spectrum. One can fit the initial power spectrum very precisely, and find out if it is scale invariant, or with a little tile, as inflation predicts. It also somewhat depends on expansion history.
- middle scale: strong acoustic peaks. The first location of the first peak is basically the

angular diameter distance to the large scattering sound horizon. This is determined by a combination of Ωh^2 and expansion history, with small dependence on baryon.

- small scale: small scale effects are complicated, including the silk damping, contribution from clusters of galaxies etc.

So by fitting C_l carefully, one can infer a lot of cosmological parameters. Even more powerful when combining with low-z constraints.

5 Large Scale: Sachs-Wolf effect

On the very largest scales, the dominant source of temperature fluctuation results from the fact that the photons we observe originated from the perturbations at the last scattering layer. Here we consider things bigger than the baryonic acoustic peak and larger than the thickness of the last scattering layer. The only effect one needs to consider, except reionization, is the fact that the photons will pass through gravitational potential fluctuations, and therefore they will be gravitationally redshifted. In general, one can show that as far as the perturbation grow linearly with redshift, what they gain by falling into them is exactly compensated by the redshift coming out. So it is the escape from density perturbations at the epoch of recombination which provides the gravitational redshift. This is known as the Sachs-Wolfe (1967) effect.

A potential well has two effect:

(1) the photon is redshifted as it climbs out of the well

$$\delta T/T = \delta\Phi/c^2$$

(2) the region is time dilated, $\delta t/t = \delta\Phi/c^2$. Clocks in that region run slow relative to the outside world, so that region is younger. This means that it is hotter. Since during matter dominated era, $R \propto t^{2/3}$, $\delta R/R = 2/3\delta t/t$, and $\delta R \sim \delta T$, so the region has $\delta T/T = -(2/3)\delta\Phi/c^2$. So the total is $(1/3)\delta\Phi/c^2$.

So what's the potential perturbation $\delta\Phi$?

$$\Delta\Phi \sim \frac{G\Delta M}{d},$$

and mass fluctuation $\Delta M \sim \Delta\rho d^3$, where $\Delta\rho$ is the amplitude of the density fluctuations, $\Delta\rho = \Delta \times \rho_0$, where Δ is the relative density fluctuation that in $\Omega = 1$ universe, goes like

$\Delta \sim (1+z)$, and $\rho_0 \sim (1+z)^{-3}$, so:

$$\Delta\rho = \Delta\rho_0(1+z)^2,$$

And the size of the fluctuation $d = d_0/(1+z)$, therefore,

$$\Delta\phi \sim G\rho_0 d_0^2,$$

the perturbation of gravitational potential is independent of the cosmic epoch as long as density perturbation grows as 1/(1+z). This justified our earlier statement that we only need to consider the potential when the photon climbs out of recombination epoch.

We can also use the results from the power spectrum analysis we had before, for $P(k) \sim k^n$: $\Delta \sim M^{-(n+3)/6}$, therefore, $\Delta\rho_0 \sim \rho_0 M^{-(n+3)/6}$, and $M \sim \rho_0 d_0^3$, to derive:

$$\Delta\phi \sim \delta\rho_0 d_0^2 \sim d_0^{(1-n)/2} \sim \theta^{(1-n)/2},$$

so the large scale temperature fluctuation is then:

$$\Delta T/T \sim 1/3\Delta\phi/c^2 \sim \theta^{(1-n)/2},$$

and for Harrison-Zeldovich, one has a flat temperature power spectrum.

This was detected by the COBE satellite. If one ascribes the power observed to gravitational fluctuations, then one gets a normalization of the power spectrum of fluctuation at Gpc scales at z = 1000. This is already a powerful constraint on models of structure formation. It is a striking aspect of cold dark matter cosmologies that they can reproduce the level of the CMB and the level of large-scale structure today.

It should be noted that the independence of the power spectrum on muliple moments depends on the assumption that the development of the perturbation is linear from the recombination to current epoch. For models with $\Omega \neq 1$, in particular for ΛCDM model, this is no longer true at small redshift. In this case, we need to integrate Sachs-Wolfe effect over all redshift back to recombination. The integrated SachsWolfe effect is also caused by gravitational redshift, however it occurs between the surface of last scattering and the Earth, so it is not part of the primordial CMB. It occurs when the Universe is dominated in its energy density by something other than matter. If the Universe is dominated by matter, then large-scale gravitational potential energy wells and hills do not evolve significantly. If the Universe is dominated by radiation, or by dark energy, though, those potentials do evolve, subtly changing the energy of photons passing through them. A signature of the late-time ISW is a non-zero cross-correlation function between the galaxy density and the temperature of the CMB. It was observed in the SDSS, lending additional evidence to dark energy.

6 Intermediate Scale; the Acoustic Peaks

Plasma before recombination has lots of pressure from the photons. This allows it to oscillate rather than collapse.

$$\delta\phi = \phi_* - \phi_0 = \int_0^{t_*} c_s k(1+z)dt = ks$$

Hence, all modes of a given k have the same phase at recombination. We have fluctuations in the early universe that have a characteristic scale. In the limit that $\Omega_b h^2$ is small and $\Omega_m h^2$ is large, the sound horizon can be computed to be

$$\frac{2c}{H(z_*)}\frac{1}{\sqrt{3}}(1+z_*)$$

We see this scale projected on the sky at the angular diameter distance to $z = 1000$.

It turns out that the angular scale depends very strongly on the curvature of the universe and only weakly on Ω_m and Λ. Open universes would make the scale smaller. Hence, the location of the acoustic peaks in the CMB are a very precise test of the curvature of the universe!

We just discussed the monopole mode of the acoustic peak. Multipoles also present. One can show that, for the temperature fluctuation $\Theta = \Delta T/T$,

$$\frac{d^2\Theta}{dt^2} = -1/3k^2\Phi - k^2 c_s^2 \Theta,$$

so this equation of a forced harmonic oscillator, and one can show the equation and find out how the acoustic peaks of different multipoles will behalf, which is summarized in the figure. Note that:

(1) the amplitude is sensitively dependent on baryon density, not surprisingly. This is mostly due to the slowing down of sound wave with the presence of baryon.

(2) the location of the peak depends mostly on curvature, with little dependence on mass density and very little on cosmological constant.

(3) The first peak, which actually corresponds to half wave to recombination, is the highest, second is lower. Then, high order peaks are suppressed due to various small scale effect that we are going to discuss now.

7 Small Scales

Small scale effects include:

- statistical and silk damping. Both statistical damping due to the finite thickness of last scattering layer, which will result in statistical reduction for small scale fluctuations. SIlk damping is the photon diffusion effect before recombination and decoupling, which will also suppress small scale power. Both has scale of 10Mpc, or about a few arcmin, resulting in damping at $l > 500$, with severe damping at $l > 2000$.
- The Sunyaev-Zeldovich effect. We will discuss S-Z later. Angular scale of high-redshift clusters is about 1 arcmin. Hot plasma in the cluster will scatter photons. Since electrons are hotter, in this process photons will gain energy. In the microwave sky, which is on the R-J tail of black body, this will result in a reduction in temperature. There is also a kinematic effect caused by the peculiar velocity of clusters. They need to be modeled and taken into account for high-l.
- Confusion due to discrete sources. In cm wavelength, discrete sources are mostly quasars, and also radio galaxies, BL Lacs. For high frequency observations, such as those with Planck, small scale power from clustering of sub-mm galaxies can affect things as well.

8 Reionization

At some epoch well after the epoch of recombination, the IGM gas must have been heated and reionized. We will discuss this in great detail in a few weeks. This is the process of reionization, when the universe changed from HI to HII. Figure.

- When: z=6-20; latest Planck, $z \sim 8 - 9$.
- By what: most likely high-energy photons from early galaxies.
- how: not clear at all

From CMB point of view, because reionization released hot photons, there will be a finite τ for Thompson scattering. The next effect of this to attenuate temperature fluctuation and therefore the power spectrum on scales less than reionization horizon size.

Figure. Note that $\tau = 0.1$ and 0.2 correspond to $z = 12$ and 20. This is probably a bit too high.