Notes on Cosmological Parameters
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In matter dominated universe, we have:

$$\left[\frac{H(z)}{H_0}\right]^2 = \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda.$$

This is the form of Friedmann equation I find most useful, because it connects Hubble constant with other cosmological parameters: density, curvature and cosmological constant. Obviously, at $z = 0$:

$$1 = \Omega_m + \Omega_k + \Omega_\Lambda,$$

What Friedmann equation gives us is the expansion history of the universe. Through it, we also introduced a number of cosmological parameters.

- $H_0 = \dot{R}/R|_0$, is the current expansion rate of the universe.
- t_0 is the current age of the universe since the BB.
- Ω_m is the current density parameter of the Universe.
- k is the curvature of the universe, deciding the geometry.
- Λ is the cosmological constant.
- q_0 is the deceleration parameter.

Clearly, they are not all independent. Indeed, they are tied by Friedmann equations. In matter dominated era, as the equation we showed above, the entire expansion history can be described by three parameters, including a scale (Hubble constant, or age of the universe), and two parameters that specify the relative contribution of matter (including dark matter), curvature and cosmological constant to the total energy-density budget of the universe. This is our **Robertson-Walker-Friedmann world model.** The most important task of a cosmologist is to understand what our world model is. And our biggest task is to test whether this world model, and which version of it, is supported by our observations. This is called the **cosmological tests.**

In fact, our cosmological test, or our cosmological model, includes even more parameters, because we are interested in not only the expansion history of the material dominated era, but (1) the state of the universe in radiation dominated era, i.e., CMB, (2) the growth of fluctuation in the universe. We will visit those later in the semester.

Cosmological tests:

- **Expansion history**
- **Radiation-dominated era – anisotropy of CMB**
- **growth of fluctuation – galaxy clustering, IGM**

This figure shows the number of parameters presented by the Planck summary. The main conclusion of this is that our cosmology can be described by essentially 6/7 parameters that fit all existing observations satisfactorily. These observations are (almost all):

(1) the expansion history of the universe, including measurements such as supernova, which we will discuss next lecture;

(2) the anisotropy of the CMB;

(3) the large scale structure in low-z universe, including measurements of galaxy clustering and structure of the IGM, as well as clusters of galaxies

And all of them can be fit by seven parameters (paper said six, because it is assuming a flat geometry: $H_0 = 70$, $\Omega_m = 0.28$, $\Omega_b = 0.05$, $k = 0$, and then two parameters, the zero point and slope that specify the density fluctuation power spectrum, and one parameter describes the cross section of free electrons to CMB that will affect CMB photons through Compton scattering. We will discuss Ω_b, and the other three later. For the moment, we will concentrate on the three parameters that appear in Friedmann equations, i.e., the expansion history of the universe, which we generally call the **classical cosmological tests**. But the cosmological tests will really dominate our discussions in this class in the next 1.5 months of the class. We will discuss how to measure Ω_b in the BBN lecture, and discuss CMB in details, as well as large scale structure tests in details later.

1 Hubble Constant

We already heard from Matthew about history of Hubble Constant.

The distance scale path has been a long and tortuous one, but with the imminent launch of HST there seems good reason to believe that the end is finally in sight. – Marc Aaronson

The determination of the distance scale of the universe, the Hubble constant, has been one of the most fascinating chapter of modern astronomy. ALMOST all of last century's great astronomers had a role, both positive and negative, on it. It involves a large amount of astrophysics, and a large amount of resources, it is controversial and had many ups and downs. It had a lot of characters, a lot of stories that we can tell. Some of these are older stories now, but you will get a sense of how our field developed. For the stories, I will refer you to an old, excellent book, 'The Lonely Hearts of the Cosmos", by Dennis Overbye. The central scheme of the book is about the determination of Hubble constant.

Hubble constant has sometimes been called the most variable constant in science. As an example of that, look at (Huchra Figure 1). A bit of history, the first published determination of Hubble constant is not from Hubble himself, but from Lemaitre in 1927 based on Hubble's observations, and it is still the largest value to date, about 630 km/s/Mpc. Hubble himself finally weighted in in 1929 at 500. Also, very early on, the Dutch astronomer, Jan Oort, thought something was wrong with Hubble's scale and published a value of 290 km/s/Mpc, but this was largely forgotten.

The first major revision to Hubble's value was made in the 1950's due to the discovery of Population II stars by W. Baade. To distinguish pulsating variables RR Lyr (PopII) with Cepheids (PopI). That was followed by other corrections for confusion, etc. that pretty much dropped the accepted value down to around 100 km/s/Mpc by the early 1960's.

But after that, it entered an era of a very and intense debate. (Huchra Figure 2). So what is shown here is the Hubble constant determination from 1970 on. Note two symbols here, (1) open stars are values determined by two Gerard de Vaucouleurs and Sidney Van Ben Bergh. At least until 1990s, they always got H=100. (2) star symbols are values determined by another prominent group, by Allan Sandage, Hubble's student and hand-picked heir, who might be the most important astronomer after Hubble, and his friend Gustav Tammam, a Swiss astronomer. For about 40 years, their values were always 50 - 55.

Both can't be right.

Then there are the third group of people, the people who initially didn't not belong to these two camps. They are mostly "young" astronomers, at least when this group of astronomers entered the game in the 1970s. They invented a whole new arrays of distant indicators, and were much more adapted to the new technology at the time, such as CCDs. Their values more or less lied in the middle, like 75.

Then came HST. The single most important mission of HST is to measure Hubble constant, there is no doubt about that. The group that in charge of that is called the "Key Project" team. The measurement of Hubble constant is one of the two Key Projects of HST. Key project team used a combination of distance indicators, and did exclusively detailed work on the Cepheids in galaxies in Virgo clusters to calibrate their distance indicator. The final result of the HST Key Project, which was published about two years ago, was 72 ± 8. One of the most important goal of HST is to determine the Hubble constant within 10%, and they were able to do that.

I will argue that it is also an extraordinarily difficult constant to determine. Let's see why:

we have Hubble's law:

$$cz = v = Hr$$

so the recession velocity, or the redshift, is proportional to the distance of a distant object. In order to determine H_0, obviously, we need to determine two things: measure the redshift of a distant galaxy, and measure the distance to it with an independent method. So how far a galaxy can you use? We mentioned that the random motion of a galaxy due to its local gravitational field, i.e., the motion that has nothing to do with the Hubble flow, the uniform expansion of the universe, is of the order 500 - 1000 km/s. So in order to measure the Hubble constant that is not affected by this deviation from Hubble flow, we then have to go to sufficiently large distance when the velocity of the galaxy is dominated by the the expansion, not by the local velocity field. So in order to measure Hubble constant to an accuracy of 10%, we need to go to a expansion velocity of, say 7,000 km/s. If we assume that the Hubble constant is 70 km/s/Mpc, plug this back to Hubble's law, we find that it required us to measure the distance of a galaxy at 100 Mpc away. This is very far away.

The other difficulty we have is that although using relations such as Tully-Fisher relation, Faber-Jackson relation and likes, we can measure the distance of faraway galaxies, these distance measurements are all **Relative.** It means that these relations more or less tell us how to determine the proportionality of galaxy properties. For example, T-F tells us that $L \propto v^4$, so if you measure the rotation curve of two galaxies, you can then know the relative intrinsic brightness of these two galaxies. Therefore, if you know the distance of one galaxy, and the relative observed brightnesses, then you can get the distance of the other galaxy. But it still requires you to know at least one galaxy, and therefore the relation needs to be calibrated. There are two flavors of distance indicators. Most of the distance indicators that we know of that apply to distant galaxies are relative distance indicators, they need to be tied to an absolute distance indicator in order to be applied to get Hubble constant. This process is very non-trivial. The simplest absolute distance indicator we know is the familiar trigonometric parallax. But it can only be applied to objects within few tens of parsecs with sufficient accuracy. On the other hand, in order to calibrate such as T-F relation, we need to measure the absolute distance to at least a few nearby spiral galaxies. We only know two spiral galaxies, M31 and M33, hardly enough. So we need to go to the nearest clusters of galaxies, such as Virgo, and Fornax clusters, which are a few Mpc away. To tie the distance of galaxies a few Mpc away with stars at few tens of parsecs away is very difficult.

The classical procedure for estimating distances in the Universe typically uses a series of relative distance indicators; the distance of a nearby galaxy might be determined by comparing the apparent brightness of its individual stars to those of similar stars in the Milky Way; the distance of those Milky Way stars are compared to some more nearby stars that we can use absolute distance indicators, and so on. This bookstrapping approach to measuring distances is called the cosmic distance ladder, where each rung up the ladder takes us to greater distance, until we are out into the uniform Hubble flow to measure the distance indicated by the redshift.

You obviously DON'T want to have too many steps on the ladder. None of the distance indicator is perfect, they all have their errors, that is, the relation such as Cepheids, T-F, can not uniquely predict the distance, but only with a degree of uncertainty. When you begin to build the distance ladder, the error will build up, the relative error term more or less increases by the errors adding in quadrature,

$$(\sigma(D)/D)^2 = \Sigma(\sigma(D_i)/D_i)^2.$$

So if you require the final Hubble constant to be 10% accurate, and you have 10 steps to reach it from trigonometric parallax in your distance ladder, than you need each step to be about 3% accurate, which is very hard to achieve.

Because the error budget is so tight, there are two things that people should worry about, (1) to find absolute distance indicator (other than parallax) to minimize the number of steps; (2) to try to make sure that the result is not biased, no systematic errors.

The HST key project, whose goal is to measure H_0 with 10% accuracy, was formed in 1986, by Marc Aaronson, and started their observations in 1991. The final result of the key project is published in 2001. The key project way to determine H_0 is (1) to find and calibrate Cepheid distance in a large number of nearby ($<$ 20 Mpc) galaxies, and use it as the primary distance indicator. and (2) then to use T-F, F-P, type Ia, and SBF as the secondary indicator to greater distance to determine H_0. They were able to achieve both goals. With the Cepheids calibration, (Table 1), they were able to much better calibrate the secondary indicators, the error on these indicates decreased from 10-20%, to 4-10%, because of that.

This plot (Figure 4) shows Hubble diagram, the diagram that shows the relation between distance and velocity, of key project results. Hubble diagram is a very important concept, which you should all remember. It plots D vs. v, and the slope of Hubble diagram is H_0. Also, when you go really far, it is going to curve, deviate from a straight line due to GR effect.

So this figure shows the Hubble diagram, with different indicators of Hubble constant, T-F, F-P, SBF, Ia and II (expanding photosphere). They are all consistent, no systematic differences among them. Table 12 shows the values and uncertainties. The final answer from the Key Project team is.

$$H_0 = 72 \pm 8\text{km s}^{-1}\text{Mpc}^{-1}.$$

However, this is not the end of the story.

2 Tension in H_0

However, this is not the end of the story. You might ask in this era of precision cosmology from WMAP and Planck, why do we still worry about Hubble constant from traditional method. The result is local distance measurements presented a completed independent way from CMB, which relies of growth of structure in the early universe.

It started with Adam Riess and colleagues (arXiv: 1604.01424) published the results from their most recent HST observations, in which they lowered the uncertainly of H_0 to 2.4%. The bulk of the improvement comes from new, near-IR observations of Cepheids in typeIa SN host galaxies. In 2016, the best estimate is $H_0 = 73.24 \pm 1.74$. This is 3.5-σ than the 2016 Planck of 66.93 ± 0.62.

The main improvement comes from (1) a much fewer step in the "distance ladder". There are now three steps involved:

- Local geometric measurements (parallex!!) to Cepheids
- Cepheids to SN1a
- SN1a in Hubble flow to determine H_0.

(2) a much better determination of Cepheids distance in local galaxies with good Cepheids and SN1a, usually using IR relation.

(3) a larger number of paralllex with Cepheids, with GAIA and HST parallex. This is in the newest 2018 paper.

Figures show:

- History of H_0
- New distance ladder
- Error budget (Riess 2016 Figure 1)

- HST observations of SN1a hosts (Figure 2)
- Planck H(z) determination
- Riess+16 Figure 13

The very latest paper using GAIA DR3: $H_0 = 73.2 \pm 1.3$.

This is to be compared with Planck 2018 result: $H_0 = 67.27 \pm 0.60$ km s^{-1} Mpc^{-1}. This is a 4.2-σ result.

Riess et al. (2016) discussed possible solutions, other than observational issues. Changing most assumptions in our cosmological models do not help very much. However, An increase in the number of relativistic species in the early Universe increases the radiation density and expansion rate during the radiation- dominated era, shifting the epoch of matter-radiation equality to earlier times. The resulting reduction in size of the sound horizon (which is used as a standard ruler for the CMB and BAO) by a few percent for one additional species (N_{eff} = 4) increases H_0 by about about 10% for a flat Universe, more than enough to bridge the divide between the local and high-redshift scales. This N_{eff} is basically the number of relativistic species in the early universe (other than photon). We only now three species of neutrinos. If there is another species, which can be called "dark radiation", then we can reconcile those two.

We are still early in the days of this puzzle. But don't be surprised if this becomes serious discussion. Another example of cosmological as a tool for new physics.

3 Age of the Universe

Deceleration makes the universe younger than a Hubble time. Looking back in time, the expansion was faster and therefore we reach the Big Bang more quickly. *Draw pictures.*

For $\Omega = 1$ universe, the age of the universe is $2/3H_0$. Alternatively, $H_0 t_0 = 2/3$.

For open universes without lambda, $H_0 t_0$ can be larger. An empty universe $\Omega_m = 0$ has $H_0 t_0 = 1$; this is sometimes called a coasting universe. For reasonable $\Omega_m = 0.3$ or so, the result is about 0.85.

Positive Λ allows $H_0 t_0 > 1$. In fact, as $\Omega_m \to 0$, the Hubble parameter becomes time-independent, $\dot{R} = H_0 R$ means that $R \propto \exp(H_0 t)$, and we have an infinitely old universe. More on that later!

Lesson: Lower matter densities yield less deceleration and hence older universes for a fixed Hubble constant.

Observationally, the prejudice toward $\Omega = 1$ caused higher Hubble constants to be in conflict with age estimates of old stellar populations, in particular globular clusters. $H_0 = 50$ km/s/Mpc and $\Omega = 1$ gave an age of 13.3 Gyr. But $H_0 = 70$ would be only 10 Gyr. This was called the 'age crisis'.

Globular cluster ages: typically people say that the ages are 15 Gyr. Like many branches of astrophysics, errors bars have been quoted that in time turn out to have neglected important systematic effects. Opacities, abundance ratios, and the local distance scale have all caused these estimates to move around in the last 10 years.

One can also try to infer ages from white dwarf cooling.

With the currently favored Λ universes, the age of the universe is about 13–14 Gyr. My outsider's opinion is that this is within the errors on the stellar ages. I would also stress that it's remarkable that two completely disjoint programs agree to about 10%.

4 Ω_m

The debate on the value of Ω_m, the mass density, is a quite old one. The first attempt to measure Ω_m is to measure the deceleration of the universe to high redshift, using something called the brightest cluster galaxies, the brightest member of a cluster, which seems to be a quite good (10-15%) relative distance indicator. So what they were doing is really to measure $q_0 = 1/2\Omega_m$ if $\Lambda = 0$. The game started from as early as late 1950s, but it was never successful. The resulted q_0 tends to be all over the place, and it turned out that at that time, people didn't know how to model the evolution of galaxy luminosity itself, and at high redshift, the properties of BCGs changed, they are younger and brighter, so they are not good standard candles. In the end, this test didn't go anywhere. But such attempts unexpectedly grew into a whole new field, the field of galaxy evolution, when a group of young astronomers in mid 1970s began to question this method.

The second setback for measuring Ω_m is the realization that the universe is dark-matter dominated, and the visible, baryonic matter is but only a small fraction of it. So by counting stars and galaxies, it is very difficult then, to add up the total mass density of the universe, and get Ω_m in this way.

- $q_0 = \Omega_0/2$; brightest cluster galaxies;

- $\Omega_0 = 8\pi G\rho_0/3H_0^2$: dark matter??

Because of the difficulty of measuring Ω_m with deceleration, and with counting galaxies, for a long time, it is a rather confusing field. On one hand, people have known for a long time that baryonic matter is only a few percent, $\Omega_b < 0.05$. On the other hand, the nature of dark matter prevented people how to correctly estimate total Ω for a while. By 1980s, there are two obvious camps, one favors $\Omega = 1$, on the basis that it is the most elegant universe, inflation predicted it, flatness problem solved and so forth, the other favors $\Omega < 0.5$, as they just can't come up with enough dark matter in their measurement to close the universe. Actually, the majority astronomers who thought they knew the answer believed $\Omega_m = 1$. But in this case the minority won.

In the 1990s, the possibility of a low-mass universe gained substantial support through a number of very convincing observations, many of them independent. Although each observation has its strengths, weaknesses and assumptions, they all indicate that $\Omega_m < 1$.

Many of these methods that determined a low density universe involved using clusters of galaxies as the main tool. The idea is that since cluster of galaxies are the most massive and largest bound objects in the universe, they should have a very good mix of baryonic and dark matter, since the ratio of baryons to dark matter, or the mass to light ratio, would be close to the cosmic mean. And because they are bound, you can measure the cluster mass reasonably well, and use this to somehow derive the mean cosmic density. We will defer those to much latte because they require us to understand how cluster of galaxies are formed.

So by late 1990s, the overwhelming evidence is that Ω_m is small, we live in a light weighted, low density universe; but as I explained last time, theorists really want the universe to be flat, and with our cosmic triangle,

$$1 = \Omega_m + \Omega_\kappa + \Omega_\Lambda,$$

a low density universe can not be flat unless there is a cosmological constant. Is there? Now let's turn our attention to the second side of the cosmic triangle, the cosmological constant term, which determines the acceleration and the fate of the universe.

5 Dark Energy

You have heard from Jasmin about accelerating universe. What's the meaning of Λ? It is a replusive force in the universe that accelerates the universe. Looking at the energy equation of the Friedmann equation:

$$\ddot{R} = -\frac{4\pi G}{3} R \left(\rho + \frac{3p}{c^2} \right) + \frac{1}{3} \Lambda R$$

First, it causes the universe to accelerate. Second, it looks like some sort of energy term just as matter or photons... but there is nothing there. In elementary particle physics, it represents some sort of vacuum energy.

The story might stop right here with a happy ending– a complete physics model of the cosmic expansion– were it not for a chorus of complaints from the particle theorists. The standard model of particle physics has no natural place for a vacuum energy density of the modest magnitude required by the astrophysical data. The simplest estimates would predict a vacuum energy 10120 times greater.

(In supersymmetric models, it's "only" 1055 times greater.) So enormous a Λ would have engendered an acceleration so rapid that stars and galaxies could never have formed. Therefore it has long been assumed that there must be some underlying symmetry that precisely cancels the vacuum energy. Now, however, the supernova data appear to require that such a cancellation would have to leave a remainder of about one part in 10120. That degree of fine tuning is most unappealing.

The cosmological constant model requires yet another fine tuning. In the cosmic expansion, mass density becomes ever more dilute. Since the end of inflation, it has fallen by very many orders of magnitude. But the vacuum energy density ρ_Λ, a property of empty space itself, stays constant. It seems a remarkable and implausible coincidence that the mass density, just in the present epoch, is within a factor of 2 of the vacuum energy density.

Given these two fine-tuning coincidences, it seems likely that the standard model is missing some fundamental physics. Perhaps we need some new kind of accelerating energy– a "dark energy" that, unlike Λ, is not constant. with the goal of solving the coincidence problems.

The dark energy evinced by the accelerating cosmic expansion grants us almost no clues to its identity. Its tiny density and its feeble interactions presumably preclude identification in the laboratory. By construction, of course, it does affect the expansion rate of the universe, and different dark-energy models imply different expansion rates in different epochs. So we must hunt for the fingerprints of dark energy in the fine details of the history of cosmic expansion.

The wide-ranging theories of dark energy are often characterized by their equation-of-state parameter

$$w = p/\rho c^2,$$

the ratio of the dark energy's pressure to its energy density. The deceleration (or acceleration) of an expanding universe, given by the general relativistic energy equation:

$$\ddot{R} = -\frac{4\pi G}{3}R(\rho + 3p/c^2) + 1/3\Lambda R.$$

In case we don't have cosmological constant:

$$\ddot{R} = -\frac{4\pi G}{3}R(\rho + 3p/c^2) = -\frac{4\pi G}{3}\rho(1 + 3w).$$

So the expansion of the universe would depend on this ratio w. Thus the expansion accelerates whenever w is more negative than ?1/3, after one includes all matter, radiation, and dark-energy components of the cosmic energy budget.

Each of the components has its own w: negligible for nonrelativistic matter, obviously, for them, $p = 0$, pressureless. $w = +1/3$ for radiation and relativistic matter, for them $p = \rho/3c^2$. and $w = ?1$ for Λ. Let's say how this is the case:

$$\Omega_\Lambda = \Lambda/3H^2,$$

We can write:

$$\Omega_\Lambda = 8\pi G\rho_\Lambda/3H^2,$$

similar to the way we write $\Omega_m = 8\pi G\rho/3H^2$. So $\Lambda = 8\pi G\rho_\Lambda$. Now for the energy equation, if we have only Λ term:

$$\ddot{R} = 1/3\Lambda R = 8/3\pi R\rho_\Lambda,$$

so comparing this with the equation above, we then have:

$$-2 = 1 + 3w \to w = -1.$$

That is, Λ exerts a peculiar negative pressure!

In fact, the energy equation tells us (see notes before):

$$\ddot{R} = -4\pi G/3\rho(1 + 3w),$$

$$dR/d\rho = -3\rho(1 + w),$$

$$\rho \propto R^{-3(1+w)}.$$

General relativity tells us that each component's energy density falls like $R^{-3(1+w)}$ as the cosmos expands. Therefore, radiation's contribution falls away first, so that nonrelativistic matter and dark energy now predominate. Given that the dark-energy density is now about twice the mass density, the only constraint on dark-energy models is that w must, at present, be more negative than -1/3 to make the cosmic expansion accelerate. However, most dark-energy alternatives to a cosmological constant have a w that changes over time. If we can learn more about the history of cosmic expansion, we can hope to discriminate among theories of dark energy by better determining w and its time dependence. Unfortunately, the differences between the expansion histories predicted by the current crop of dark-energy models are extremely small. Distinguishing among them will require measurements an order of magnitude more accurate than those shown in figure 3, and extending twice as far back in time. There are many telescope, both ground-based, and from space, that plan to observe many more distance SNe, to constrain the expansion of the universe much better, and to constrain the equation of state.

To summarize:

- $w = 0$: matter;
- $w = 1/3$: relativistic particles, photons, neutrinos;
- $w < -1/3$: accelerating universe;
- $w = -1$: cosmological constant

6 Cosmic Dynamics Experiment

We will discuss other dark energy experiments later on in this class. But I do want to mention one of the more ambitious ideas that I encountered. Issue with many DE experiments is that they suffer from systematics. SN tests you always worry about dust and evolution; other tests based on large scale structure have subtle effect with the growth of structures and the bias of tracers. People always wonder about what if one can directly measure the dynamical expansion of the universe, i.e., . first derivative of redshift, or, basically wait long enough to see the redshift of the object change with time. You can show (which is a homework problem) that:

$$dz/dt = (1 + z)H_0 - H(z),$$

so for an object at rest, if you long enough, then because of the expansion of universe, the line will shift. If you observe this over a range of redshift, then you are mapping H(z), or Friedmann equation directly. The only problem is that this shift happens at Hubble time scale, i.e., dz/dt is extremely small. $\sim 10^{-10}$ per year, requiring velocity accuracy of cm/s, or about two orders of magnitude better than exoplanet surveys have achieved.

But that didn't stop people from planning. In fact, this is one of the science drivers for E-ELT....

Use Lyα forest observations. Very stable. 4000 hours on E-ELT over 20 years for one set of measurements. 10% of time available. Accuracy and model independent measurements of expansion history. Worth it?