Franz_ASTR540_HW4_NB

November 19, 2024

1 ASTR540 Homework 4

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[1]:
```
import warnings
from functools import partial
from copy import deepcopy

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

from astropy.coordinates import SkyCoord
from astropy import units as u
from astropy import constants as c
from astropy.table import vstack

from astroquery.gaia import Gaia
from astroquery.sdss import SDSS
```

1.1 Problem 1

Proper Motion of Bo¨otes III. Let's revisit the proper motion of the disrupting satellite Bo¨otes III, as discussed in Carlin & Sand 2018. Since that study, there has been the 3rd data release of Gaia (Gaia DR3), so it is possible that there are more proper motion members in the catalog, and that the overall uncertainties have improved.

a. First, go to Table 1 of the Carlin paper, which displays all of the 'radial velocity confirmed' members of Bo¨otes III, and make your own table with updated proper motion values.

b. By how much have the uncertainties going from Gaia DR2 to Gaia DR3 improved? Have there been any large changes in proper motion for any of these stars (>5)? |

c. Calculate your own error-weighted proper motion to Bo¨otes III, compare it with the DR2 value, and make a plot similar to Figure 1 of the paper.

d. Try to identify further proper motion members in Bo¨otes III, as described in Section 2.3 of the paper. Do the improved measurements of Gaia DR3 lead to more plausible proper motion members of Bo¨otes III?

1.1.1 Problem 1a

[2]:
```
# read in the Table 1 data from Carin & Sand (2018)
data_dr2 = pd.read_csv("Carlin_and_Sand_2018_Table1.txt", sep="\t")

# and then clean it up
data_dr2 = data_dr2.dropna(how="all", axis=1)

data_dr2 = data_dr2.replace({
    "cdots": np.nan,
    "N" : False,
    "Y" : True
})

data_dr2["v_helio_err"] = [v.split(" +or- ")[1] if not pd.isna(v) else v for v in data_dr2.v_helio]
data_dr2["v_helio"] = [v.split(" +or- ")[0] if not pd.isna(v) else v for v in data_dr2.v_helio]

data_dr2["mu_alpha_err"] = [v.split(" +or- ")[1]  if not pd.isna(v) else v for v in data_dr2.mu_alpha]
data_dr2["mu_alpha"] = [v.split(" +or- ")[0]  if not pd.isna(v) else v for v in data_dr2.mu_alpha]

data_dr2["mu_delta_err"] = [v.split(" +or- ")[1]  if not pd.isna(v) else v for v in data_dr2.mu_delta]
data_dr2["mu_delta"] = [v.split(" +or- ")[0]  if not pd.isna(v) else v for v in data_dr2.mu_delta]
```

[3]:
```
data_dr3 = []
idxs = []
radius = 1*u.arcsec

for i, row in data_dr2.iterrows():
    coord = SkyCoord(row.alpha, row.delta, unit=u.deg)
    job = Gaia.cone_search(coord, radius=radius)
    res = job.get_results()

    if len(res) != 1:
        warnings.warn(f"Length of result not equal to 1 for {row.source_id}!")
        continue

    data_dr3.append(res)
    idxs.append(i) # keep track of index for joining

data_dr3 = vstack(data_dr3).to_pandas()
data_dr3 = data_dr3.set_index(np.array(idxs))
```

```
/tmp/ipykernel_466903/39562995.py:11: UserWarning: Length of result not equal to
1 for -9223372036854775808!
  warnings.warn(f"Length of result not equal to 1 for {row.source_id}!")
```

[4]:
```
data_dr3.columns = [c+"_dr3"  if "_dr3" not in c else c for c in data_dr3.
 ↪columns]
data_dr2.columns = [c+"_dr2" if "_dr2" not in c else c for c in data_dr2.
 ↪columns]

data = pd.merge(data_dr2, data_dr3, left_index=True, right_index=True,␣
 ↪how="outer")
```

[22]:
```
def get_sdss_colors(df, ra_key="ra", dec_key="dec", filters=["u", "g", "r",␣
 ↪"i", "z"]):

    all_data = []

    for idx, row in df.iterrows():

        coord = SkyCoord(row[ra_key], row[dec_key], unit="deg")

        try:
            res = SDSS.query_region(
                coord,
                radius=1*u.arcsec,
                spectro=False,
                fields=["ra", "dec", "objid"] + filters,
            )
        except Exception as e:
            all_data.append(row)
            continue

        if res is None or not len(res):
            all_data.append(row)
            warnings.warn(f"Skipping {row.DESIGNATION} because no SDSS data!")
            continue

        res = res.to_pandas()

        res_filtered = res[res.g > -100]

        if not len(res):
            all_data.append(row)
            warnings.warn(f"Skipping {row.DESIGNATION} because no high quality␣
 ↪SDSS data!")
            continue

```

```
        # take the u, g, r, i, z from the closest row
        min_sep = np.argmin(
            coord.separation(
                SkyCoord(
                    res_filtered.ra.values,
                    res_filtered.dec.values,
                    unit="deg"
                )
            )
        )
        for filt in filters:
            row[f"{filt}_sdss"] = res_filtered[filt].values[min_sep]

        all_data.append(row)

    return pd.DataFrame(all_data)

data = get_sdss_colors(data, ra_key="ra_dr3", dec_key="dec_dr3")

data[["source_id_dr2", "alpha_dr2", "delta_dr2", "mu_alpha_dr2",␣
 ↪"mu_delta_dr2", "SOURCE_ID_dr3", "ra_dr3", "dec_dr3", "pmra_dr3",␣
 ↪"pmdec_dr3"]]
```

[22]:

```
           source_id_dr2   alpha_dr2  delta_dr2 mu_alpha_dr2 mu_delta_dr2  \
0    1450811159128205568  208.955653  26.502208        1.078        5.259
1    1450817000283805824  209.013621  26.661679       -0.466       -0.583
2    1450826552290908800  209.128453  26.840015       -1.211       -1.139
3    1450826724089602816  209.159600  26.872888        -1.84       -1.784
4    1451038792395234560  209.174766  26.979976       -1.932        0.579
5    1450823051893369984  209.261163  26.829991       -1.709       -0.332
6    1450803222028673280  209.331958  26.573608       -1.492       -0.928
7    1450833767836315392  209.467272  26.804392       -0.207       -0.927
8    1450828682594980736  209.479318  26.664838       -0.462       -0.479
9    1450835314024556544  209.500251  26.835870       -1.006       -1.389
10   1450748486965605760  209.520145  26.287895       -0.566       -1.793
11   1450842185972291840  209.573497  26.944967       -1.435        0.541
12   1450781163077088000  209.610213  26.651188        0.058       -1.044
13   1450782949783229824  209.668856  26.711657       -2.084       -2.503
14   1450751617997283840  209.681364  26.467320       -5.797       -2.153
15   1450739381635432192  209.700291  26.407019         0.24       -1.969
16   1450785045727265792  209.810712  26.693513       -1.595       -1.315
17   1450788584780752256  209.845448  26.753301       -7.457       -2.497
18   1450783774417361280  209.916855  26.655120      -10.529      -11.886
19  -9223372036854775808  209.489916  26.858467          NaN          NaN
20   1258556500130302080  210.143865  25.931296        -1.34       -0.978

    SOURCE_ID_dr3      ra_dr3    dec_dr3   pmra_dr3  pmdec_dr3
```

0	1.450811e+18	208.955653	26.502208	-1.957361	-2.901003
1	1.450817e+18	209.013621	26.661679	-1.147453	-0.904130
2	1.450827e+18	209.128453	26.840015	-0.837238	-0.971798
3	1.450827e+18	209.159600	26.872888	-0.857254	-1.206666
4	1.451039e+18	209.174766	26.979976	-1.652480	-0.706780
5	1.450823e+18	209.261163	26.829991	-1.400544	-1.098103
6	1.450803e+18	209.331957	26.573608	-1.378667	-0.721432
7	1.450834e+18	209.467272	26.804392	-1.324451	-0.947451
8	1.450829e+18	209.479318	26.664838	-0.470838	-0.927059
9	1.450835e+18	209.500250	26.835871	-1.611917	-0.534405
10	1.450748e+18	209.520145	26.287895	-0.832462	-1.081806
11	1.450842e+18	209.573497	26.944967	-1.064027	-0.821582
12	1.450781e+18	209.610213	26.651188	-0.600902	-0.834171
13	1.450783e+18	209.668855	26.711657	-3.194013	-1.795397
14	1.450752e+18	209.681363	26.467319	-4.928035	-3.255653
15	1.450739e+18	209.700291	26.407018	-0.215862	-3.008471
16	1.450785e+18	209.810711	26.693513	-1.239291	-1.117548
17	1.450789e+18	209.845446	26.753300	-7.274916	-2.051706
18	1.450784e+18	209.916853	26.655119	-11.452348	-7.981481
19	NaN	NaN	NaN	NaN	NaN
20	1.258557e+18	210.143865	25.931296	-1.235442	-0.889244

1.1.2 Problem 1a Discussion

A summary of the results of merging the DR2 catalog from Carlin & Sand (2018) and the Gaia DR3 measurements of the same objects is shown in the above table!

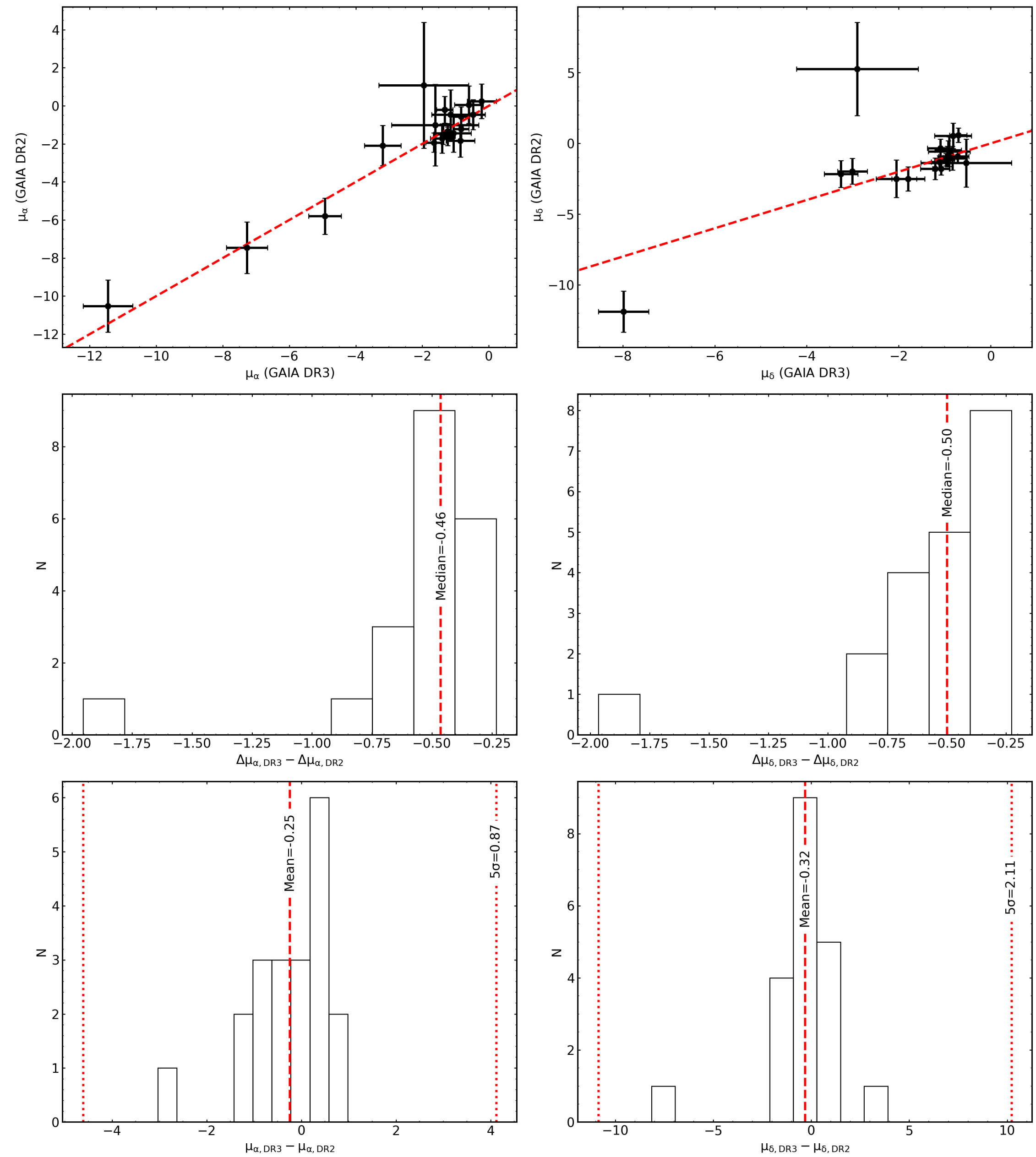
1.1.3 Problem 1b

[6]:
```
fig, axs = plt.subplots(3,2,figsize=(16,18))
ax1, ax2, ax3, ax4, ax5, ax6 = axs.flatten()

# first plot the proper motion from DR2 to DR3
ax1.errorbar(
    data.pmra_dr3,
    data.mu_alpha_dr2.astype(float),
    xerr = data.pmra_error_dr3,
    yerr = data.mu_alpha_err_dr2.astype(float),
    linestyle="none",
    marker="o",
    color="k"
)

ax1.set_ylabel(r"$\mu_\alpha$ (GAIA DR2)")
ax1.set_xlabel(r"$\mu_\alpha$ (GAIA DR3)")

ylim = ax1.get_ylim()
xlim = ax1.get_xlim()
```

```
x=y=np.linspace(*ylim)
ax1.plot(x,y,linestyle='--',color='r')
ax1.set_ylim(ylim)
ax1.set_xlim(xlim)

ax2.errorbar(
    data.pmdec_dr3,
    data.mu_delta_dr2.astype(float),
    xerr = data.pmdec_error_dr3,
    yerr = data.mu_delta_err_dr2.astype(float),
    linestyle="none",
    marker="o",
    color="k"
)

ax2.set_ylabel(r"$\mu_\delta$ (GAIA DR2)")
ax2.set_xlabel(r"$\mu_\delta$ (GAIA DR3)")

ylim = ax2.get_ylim()
xlim = ax2.get_xlim()
x=y=np.linspace(*ylim)
ax2.plot(x,y,linestyle='--',color='r')
ax2.set_ylim(ylim)
ax2.set_xlim(xlim)

# Then a histogram of the difference in the uncertainties
err_diff = data.pmra_error_dr3 - data.mu_alpha_err_dr2.astype(float)
med = np.nanmedian(err_diff)
ax3.hist(err_diff, color='k', fill=False)
ax3.set_xlabel(r"$\Delta \mu_{\alpha,DR3} - \Delta \mu_{\alpha,DR2}$")
ax3.set_ylabel("N")
ax3.axvline(med, linestyle='--', color='r')
ax3.text(med, 5, f"Median={med:.2f}", rotation=90, verticalalignment="center",␣
 ↪horizontalalignment="center", backgroundcolor="white")

err_diff = data.pmdec_error_dr3 - data.mu_delta_err_dr2.astype(float)
med = np.nanmedian(err_diff)
ax4.hist(err_diff, color='k', fill=False)
ax4.set_xlabel(r"$\Delta \mu_{\delta,DR3} - \Delta \mu_{\delta,DR2}$")
ax4.set_ylabel("N")
ax4.axvline(med, linestyle='--', color='r')
ax4.text(med, 6.5, f"Median={med:.2f}", rotation=90,␣
 ↪verticalalignment="center", horizontalalignment="center",␣
 ↪backgroundcolor="white")

# then a histogram of the difference in the proper motion values
diff = data.pmra_dr3 - data.mu_alpha_dr2.astype(float)
```

```
mean = np.nanmean(diff)
std = np.nanstd(diff)

ax5.hist(diff, color='k', fill=False)
ax5.set_xlabel(r"$\mu_{\alpha,DR3} - \mu_{\alpha,DR2}$")
ax5.set_ylabel("N")
ax5.axvline(mean, linestyle='--', color='r')
ax5.axvline(mean+5*std, linestyle=':', color='r')
ax5.axvline(mean-5*std, linestyle=':', color='r')
ax5.text(mean, 5, f"Mean={mean:.2f}", rotation=90, verticalalignment="center",␣
 ↪horizontalalignment="center", backgroundcolor="white")
ax5.text(mean+5*std, 5, rf"$5\sigma$={std:.2f}", rotation=90,␣
 ↪verticalalignment="center", horizontalalignment="center",␣
 ↪backgroundcolor="white")

diff = data.pmdec_dr3 - data.mu_delta_dr2.astype(float)
mean = np.nanmean(diff)
std = np.nanstd(diff)

ax6.hist(diff, color='k', fill=False)
ax6.set_xlabel(r"$\mu_{\delta,DR3} - \mu_{\delta,DR2}$")
ax6.set_ylabel("N")
ax6.axvline(mean, linestyle='--', color='r')
ax6.axvline(mean+5*std, linestyle=':', color='r')
ax6.axvline(mean-5*std, linestyle=':', color='r')
ax6.text(mean, 6.5, f"Mean={mean:.2f}", rotation=90,␣
 ↪verticalalignment="center", horizontalalignment="center",␣
 ↪backgroundcolor="white")
ax6.text(mean+5*std, 6.5, rf"$5\sigma$={std:.2f}", rotation=90,␣
 ↪verticalalignment="center", horizontalalignment="center",␣
 ↪backgroundcolor="white")
```

```
[6]: Text(10.219034500144446, 6.5, '$5\\sigma$=2.11')
```



1.1.4 Problem 1b Discussion

The median improvement in the uncertainties in proper motions is ~0.50 mas/yr. See the above histograms (row 2 in the above plot) for the distribution of improvements. As shown in the third row of the above plot, there have not been any > 5 changes in the proper motion between gaia DR3 and DR2 for this small sample.

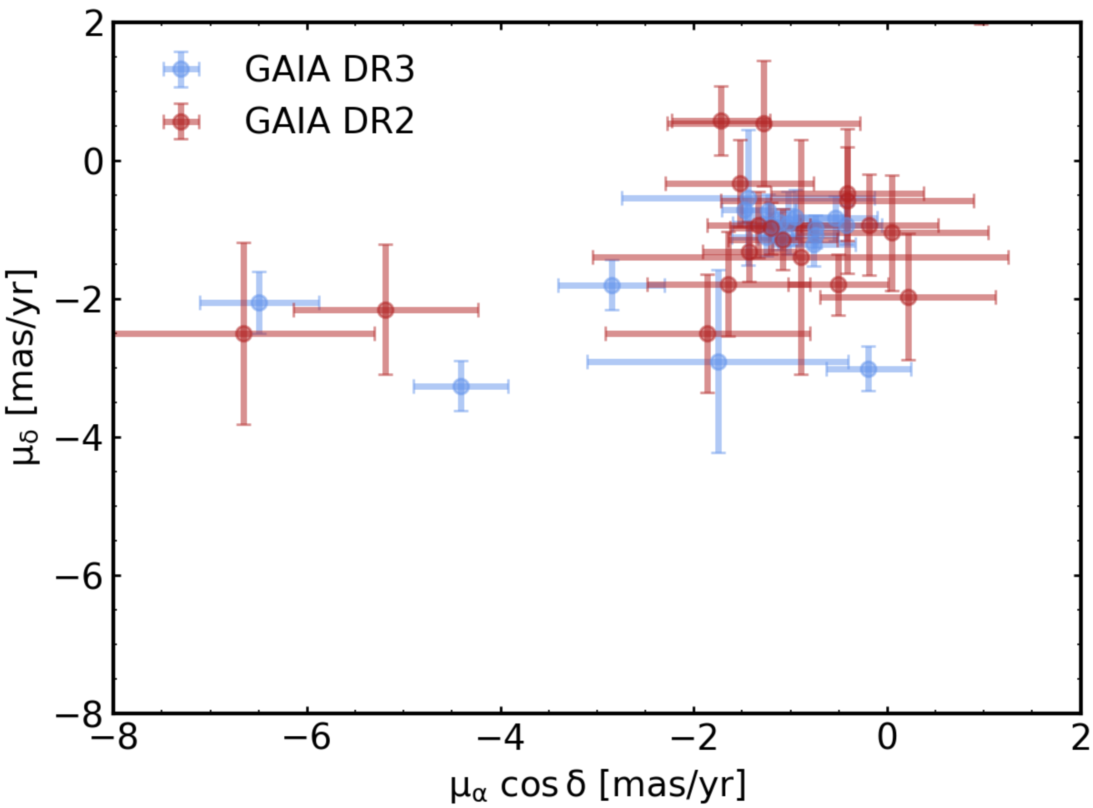
1.1.5 Problem 1c

[7]:
```
# compute the error weighted mean
pmra, pmra_wt = data.pmra_dr3[~pd.isna(data.pmra_dr3)], data.pmra_error_dr3[~pd.
↪isna(data.pmra_dr3)]
pmdec, pmdec_wt = data.pmdec_dr3[~pd.isna(data.pmdec_dr3)], data.
↪pmdec_error_dr3[~pd.isna(data.pmdec_dr3)]

where_not_outlier = pmra*np.cos(data.dec_dr3 * np.pi/180) > -4
pmra = pmra[where_not_outlier]
pmra_wt = pmra_wt[where_not_outlier]
pmdec = pmdec[where_not_outlier]
pmdec_wt = pmdec_wt[where_not_outlier]

mean_pmra = np.average(pmra, weights=pmra_wt)
std_pmra = np.sqrt(np.average((pmra-mean_pmra)**2, weights=pmra_wt))

mean_pmdec = np.average(pmdec, weights=pmdec_wt)
std_pmdec = np.sqrt(np.average((pmdec-mean_pmdec)**2, weights=pmdec_wt))

print(rf"($\mu_a~\cos\delta$, $\mu_\delta$) = ({mean_pmra:.2f}, {mean_pmdec:.
↪2f}) $\pm$ ({std_pmra:.2f}, {std_pmdec:.2f})")

# make plot similar to Fig. 1 from the paper

fig, ax = plt.subplots()

ax.errorbar(
    data.pmra_dr3*np.cos(data.dec_dr3 * np.pi/180),
    data.pmdec_dr3,
    xerr=data.pmra_error_dr3,
    yerr=data.pmdec_error_dr3,
    linestyle="none",
    marker='o',
    label="GAIA DR3",
    color='cornflowerblue',
    alpha=0.5
)

ax.errorbar(
    data.mu_alpha_dr2.astype(float)*np.cos(data.delta_dr2.astype(float) * np.pi/
↪180),
    data.mu_delta_dr2.astype(float),
    xerr=data.mu_alpha_err_dr2.astype(float),
    yerr=data.mu_delta_err_dr2.astype(float),
    linestyle="none",
    marker='o',
```

```
    label="GAIA DR2",
    color='firebrick',
    alpha=0.5
)

ax.set_ylabel(r"$\mu_\delta$ [mas/yr]")
ax.set_xlabel(r"$\mu_\alpha~\cos\delta$ [mas/yr]")

ax.set_ylim(-8, 2)
ax.set_xlim(-8, 2)

ax.legend()
```

```
($\mu_a~\cos\delta$, $\mu_\delta$) = (-1.39, -1.44) $\pm$ (0.69, 0.92)
```

[7]: <matplotlib.legend.Legend at 0x7d2d117a41d0>



1.1.6 Problem 1c Discussion

I get a mean proper motion, weighted by the uncertainties, of (μ_a $\cos\delta$, μ_δ) = (-1.39, -1.44) $\pm$ (0.69, 0.92) from the Gaia DR3 dataset. This is consistent with the value from Gaia DR2 within 1σ. Although, interestingly, the uncertainties on the mean are much larger for DR3 than DR2.

I'm not sure how it was computed in the paper, but I did did a weighted standard deviation to compute the statistical uncertainty.

1.1.7 Problem 1d

[8]:
```
isofile = "fehm20afem2.SDSSugriz"
boo3_distance = 46.5 # kpc, from the paper

def M_to_m(M, d=boo3_distance):
    return M + 5*np.log10(d/10)

with open(isofile, 'r') as f:
    iso_data = f.readlines()

hdr = iso_data[0:6]
iso_data = iso_data[6:]

hdr_vals = [val.strip() for val in hdr[3].split(' ') if val not in {'#', ''}]
mix_len, Y, Z, Zeff, Fe_H, a_Fe = hdr_vals

isochrones_in_this_file = []
base_iso = dict(
    idx = [],
    M = [],
    LogTeff = [],
    LogG = [],
    LogL = [],
    u = [],
    g = [],
    r = [],
    i = [],
    z = [],
    age = [],
    mix_len = [],
    Y = [],
    Z = [],
    Zeff = [],
    Fe_H = [],
    a_Fe = [],
)
iso = deepcopy(base_iso)
age = None

for j in range(len(iso_data)):
    if iso_data[j] == '\n':
        continue
```

```
    if '#AGE' in iso_data[j]:
        isochrones_in_this_file.append(pd.DataFrame(iso))

        age = float(iso_data[j].split(" EEPS=")[0].split("AGE=")[1].strip())
        iso = deepcopy(base_iso)
        continue

    if '#' == iso_data[j][0]:
        # this line is a header
        continue


    line = iso_data[j]
    goodline = [val for val in line.strip().split(' ') if len(val) > 0]

    if len(goodline) != 10:
        print(goodline)
        continue

    for val, key in zip(goodline, iso.keys()):
        # if key == 'age': continue
        iso[key].append(val)

    iso['age'].append(age)
    iso['mix_len'].append(mix_len)
    iso['Y'].append(Y)
    iso['Z'].append(Z)
    iso['Zeff'].append(Zeff)
    iso['Fe_H'].append(Fe_H)
    iso['a_Fe'].append(a_Fe)

iso_data = pd.concat(isochrones_in_this_file).reset_index(drop=True)

# select just the 10 Gyr one
iso_data = iso_data[iso_data.age == 10]

# scale all of the magnitudes to boo3_distance
for filt in ["u", "g", "r", "i", "z"]:
    iso_data[filt] = M_to_m(iso_data[filt].astype(float), d=boo3_distance*1e3)

iso_data
```

[8]:

	idx	M	LogTeff	LogG	LogL	u	g	r \
7068	12	0.103511	3.5376	5.3748	-2.8181	36.404765	32.468665	30.781265
7069	13	0.113801	3.5502	5.3310	-2.6829	35.490765	31.976365	30.362465
7070	14	0.125317	3.5628	5.2860	-2.5455	34.679065	31.476965	29.948765
7071	15	0.138532	3.5743	5.2421	-2.4122	33.980765	30.999365	29.558465

```
7072   16  0.152713  3.5827  5.2041  -2.2982  33.447165  30.609265  29.235965
…    …       …       …       …       …         …          …          …
7331  275  0.856466  3.6502  0.7935   3.1312  18.666965  16.462765  15.466865
7332  276  0.856473  3.6486  0.7629   3.1556  18.667265  16.425365  15.414365
7333  277  0.856479  3.6471  0.7331   3.1795  18.669365  16.389665  15.363365
7334  278  0.856486  3.6457  0.7042   3.2026  18.672265  16.355565  15.314365
7335  279  0.856491  3.6444  0.6775   3.2240  18.675365  16.324365  15.269265

              i          z   age mix_len       Y          Z        Zeff  \
7068  30.051865  29.662965  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7069  29.705465  29.342965  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7070  29.348765  29.010965  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7071  29.001365  28.684265  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7072  28.706265  28.403265  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
…          …          …    …      …       …          …          …
7331  15.087665  14.876465  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7332  15.029165  14.814465  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7333  14.972265  14.754065  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7334  14.917465  14.695765  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04
7335  14.866865  14.641965  10.0  1.9380  0.2452  1.2990E-04  1.6990E-04

      Fe_H   a_Fe
7068  -2.01  -0.20
7069  -2.01  -0.20
7070  -2.01  -0.20
7071  -2.01  -0.20
7072  -2.01  -0.20
…      …    …
7331  -2.01  -0.20
7332  -2.01  -0.20
7333  -2.01  -0.20
7334  -2.01  -0.20
7335  -2.01  -0.20

[268 rows x 17 columns]
```

[9]:

```python
boo3_distance = 46.5 # kpc, from the paper
boo3_delta_distance = 2 # kpc, the value used in the paper
boo3_radius = 30 # arcmin, value used for the cone search in the paper
pmra_min, pmra_max = (-3, 1) # mu_alpha * cos(delta) in mas/yr
pmdec_min, pmdec_max = (-3, 1) # mu_delta in mas/yr

# Color-magnitude filter: this requires stars to be within
# 0.1 mag at g0 = 16, increasing linearly to 0.2 mag width at g0 = 22.5
def cmd_filter(g_mag):
    p = np.polyfit(np.linspace(16, 22.5), np.linspace(0.1,0.2), 1)
    cmd_filter = np.polyval(p=p, x=g_mag)
```

```python
    return cmd_filter

def is_within_cmd_filter(g_mag, r_mag, isochrone):
    filt = cmd_filter(g_mag)

    # find closest isochrone point
    # good enough for now
    min_idx = np.argmin(np.abs(g_mag-isochrone.g.values))
    g, r = isochrone.g.values[min_idx], isochrone.r.values[min_idx]

    color = g_mag - r_mag

    return color <= (g-r+filt) and color >= (g-r-filt)
```

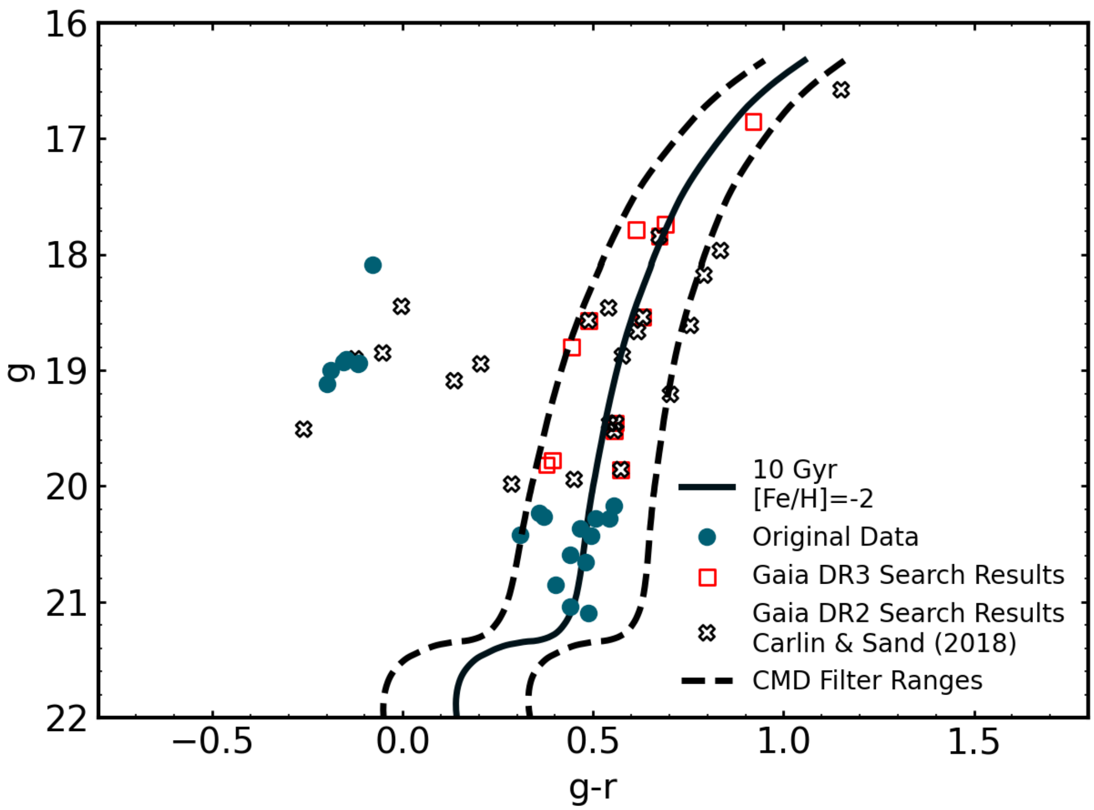
[10]:
```python
# first do a cone search in Gaia DR3
Gaia.ROW_LIMIT = 100_000
boo3_coord = SkyCoord(209.3, 26.8, unit=u.deg)
res = Gaia.query_object(
    coordinate=boo3_coord,
    radius=boo3_radius*u.arcmin,
    verbose=False
)

# filter on proper motion
ra_res = res[(res["pmra"]*np.cos(res["ra"].to(u.radian)) > pmra_min) *␣
 ↪(res["pmra"]*np.cos(res["ra"].to(u.radian)) < pmra_max)]
dec_res = ra_res[(ra_res["pmdec"] > pmdec_min) * (ra_res["pmdec"] < pmdec_max)]

# filter on the CMD cuts
res = get_sdss_colors(dec_res.to_pandas())

res["is_within_cmd_filter"] = res.apply(
    lambda row : is_within_cmd_filter(row.g_sdss, row.r_sdss, iso_data),
    axis=1
)

res = res[res.is_within_cmd_filter]

# filter on g_0
res = res[res.g_sdss < 20]
```

```
/tmp/ipykernel_466903/2148086655.py:22: UserWarning: Skipping Gaia DR3
1450837238169921280 because no SDSS data!
  warnings.warn(f"Skipping {row.DESIGNATION} because no SDSS data!")
```

[11]:
```python
# read in Table 2 from Carlin & Sand (2018)
dr2_res = pd.read_csv("Carlin_and_Sand_2018_Table2.txt", sep="\t")
```

```
dr2_res = get_sdss_colors(dr2_res, ra_key="alpha", dec_key="delta")

# then plot it up!
fig, ax = plt.subplots()

ax.plot(iso_data.g.astype(float)-iso_data.r.astype(float), iso_data.g.
 ↪astype(float), label="10 Gyr\n[Fe/H]=-2")

ax.plot(data.g_sdss-data.r_sdss, data.g_sdss, marker='o', linestyle='none',␣
 ↪label="Original Data")
ax.scatter(res.g_sdss-res.r_sdss, res.g_sdss, marker='s', edgecolor='r',␣
 ↪facecolor="none", label="Gaia DR3 Search Results")
ax.scatter(dr2_res.g_sdss-dr2_res.r_sdss, dr2_res.g_sdss, marker='X',␣
 ↪edgecolor='k', facecolor="none", label="Gaia DR2 Search Results\nCarlin &␣
 ↪Sand (2018)")

ax.plot(
    iso_data.g.astype(float)-iso_data.r.astype(float) + cmd_filter(iso_data.g.
 ↪astype(float)),
    iso_data.g.astype(float),
    linestyle='--',
    color='k',
    label = "CMD Filter Ranges"
)
ax.plot(
    iso_data.g.astype(float)-iso_data.r.astype(float) - cmd_filter(iso_data.g.
 ↪astype(float)),
    iso_data.g.astype(float),
    linestyle='--',
    color='k'
)
ax.invert_yaxis()
ax.set_xlabel("g-r")
ax.set_ylabel("g")

ax.legend(fontsize=10)

ax.set_ylim(22, 16)
ax.set_xlim(-0.8, 1.8)
```

[11]: (-0.8, 1.8)



1.1.8 Problem 1d Discussion

We used the same proper motion filter, same color-magnitude diagram filter, and maximum $g < 20$ from Carlin & Sand (2018) to search for plausible main sequence and red giant branch proper motion members of Bootes III. The results are shown in the figure above.

From the SDSS colors and Isochrones (from HW 1), we find 7 new proper motion members of Bootes III. The Isochrone shown is for a 10 Gyr cluster with a metallicity $[Fe/H] = -2$. Note that we do not find some of the original members from Carlin & Sand (2018) for two primary reasons:

1. We use SDSS magnitudes instead of Panstarrs magnitudes. This is because the Isochrone we have access to is for SDSS magnitudes. This changes the exact width of the Color Magnitude Diagram filter and eliminates some of them.
2. The Isochrone we are using does not include the population of bluer stars on the right side of the color-magnitude diagram. This is a limitation of this analysis.

Additionally, it is important to note that these are simply **plausible** members of Bootes III that need true RV measurements to confirm. As described in Carlin & Sand (2018), there are numerous sources of contaminants for these proper motion calculations.

1.2 Problem 2

Congratulations! You have discovered a dwarf galaxy around the Milky Way. Wishing to refine it's distance, you took time series imaging data of this new system and identified 3 RR Lyrae stars, whose proper- ties are in the data file "rrl.dat". Infer the distance to the new dwarf galaxy using the basic relations in Caceres & Catelan 2008, ApJSS, 179, 242. You can assume a metallicity of Z=0.0001, but assume a 10% uncertainty on this value. For the average i and z band magnitudes, you can assume 0.03 mag uncertainties, and a 0.01 day uncertainty on the RRL periods. What is the distance (with uncertainty) to your newly discovered dwarf galaxy?

The relevant equations in Caceres & CAtelan (2008) are equations (6) and (7):

$M_z = 0.839 - 1.295\ \log P + 0.211\ \log Z$

$M_i = 0.908 - 1.035\ \log P + 0.220\ \log Z$

Which has the general form $M_f = \gamma - \alpha\ \log P + \beta\ \log Z$

From these we can use the distance modulus equation to find the distance to each RR Lyrae Star

$m - M = 5 \log d - 5 \rightarrow \log d = \frac{m-M+5}{5}$

Substituting in the general formulation of M

$\log d_f = \frac{1}{5}\left[m_f - \gamma + \alpha\ \log P - \beta\ \log Z + 5\right]$

Which then, propagating uncertainty, has an uncertainty of

$\sigma_d = d_f \left[\left(\alpha \frac{\sigma_P}{\ln(10)\ P}\right)^2 + \left(\beta \frac{\sigma_Z}{\ln(10)\ Z}\right)^2 + \left(\frac{\sigma_m}{m_f}\right)^2 \right]^{1/2}$

[12]:
```
def log_distance(P, sigma_P, Z, sigma_Z, m, sigma_m, alpha, beta, gamma):
    log_d = 1/5 * (m - gamma + alpha*np.log10(P) - beta*np.log10(Z) + 5)
    sigma_d = log_d * ( (alpha*sigma_P/(np.log(10)*P))**2 + (beta*sigma_Z/(np.
 ↪log(10)*Z))**2 + (sigma_m/m)**2 )**0.5

    return log_d, sigma_d
```

[13]:
```
rr_lyrae = pd.read_csv("rrl.dat", sep=' ')

rr_lyrae["Z"] = 0.0001
rr_lyrae["Z_err"] = 0.1*rr_lyrae.Z

rr_lyrae["i_mag_err"] = 0.03
rr_lyrae["z_mag_err"] = 0.03

rr_lyrae["Period_err"] = 0.01

i_band_kwargs = dict(
    alpha = 1.035,
    beta = -0.220,
    gamma = 0.908
)
```

```
z_band_kwargs = dict(
    alpha = 1.295,
    beta = -0.211,
    gamma = 0.839
)

rr_lyrae['log_d_i'], rr_lyrae['log_d_i_err'] = log_distance(
    rr_lyrae["Period(d)"],
    rr_lyrae.Period_err,
    rr_lyrae.Z,
    rr_lyrae.Z_err,
    rr_lyrae.i_mag,
    rr_lyrae.i_mag_err,
    **i_band_kwargs
)

rr_lyrae['log_d_z'], rr_lyrae['log_d_z_err'] = log_distance(
    rr_lyrae["Period(d)"],
    rr_lyrae.Period_err,
    rr_lyrae.Z,
    rr_lyrae.Z_err,
    rr_lyrae.z_mag,
    rr_lyrae.z_mag_err,
    **z_band_kwargs
)

rr_lyrae
```

[13]:

```
   Name     RA(deg)   DEC(deg)  Period(d)  i_mag  z_mag    A_i    A_z       Z  \
0  RRL1  189.570323 -40.939879   0.389918  20.92  20.86  0.192  0.146  0.0001
1  RRL2  189.633635 -40.878072   0.422481  20.88  20.83  0.203  0.155  0.0001
2  RRL3  189.584351 -41.101214   0.735898  20.67  20.59  0.177  0.135  0.0001

     Z_err  i_mag_err  z_mag_err  Period_err   log_d_i  log_d_i_err   log_d_z  \
0  0.00001       0.03       0.03        0.01  4.741731     0.071321  4.729462
1  0.00001       0.03       0.03        0.01  4.740942     0.068136  4.732484
2  0.00001       0.03       0.03        0.01  4.748831     0.054291  4.746906

   log_d_z_err
0     0.081105
1     0.076786
2     0.057062
```

[14]:

```
def print_results(log_dist, log_dist_err, varname):

    dist = 10**log_dist
```

```
    dist_err = log_dist_err * dist/log_dist

    print(fr"${varname}$ = {dist/1e3:.2f} $\pm$ {dist_err/1e3:.2f} kpc")

print_results(rr_lyrae.log_d_z.mean(), rr_lyrae.log_d_z_err.mean(), "d_z")
print_results(rr_lyrae.log_d_i.mean(), rr_lyrae.log_d_i_err.mean(), "d_i")
```

```
$d_z$ = 54.49 $\pm$ 0.82 kpc
$d_i$ = 55.44 $\pm$ 0.75 kpc
```

1.2.1 Problem 2 Discussion

The full results of the distance calculations for these three RR Lyrae stars are shown in the table above. Since I compute the distance in both z-band and i-band independently, we get two measurements of the distance. These measurements, which I get by averaging the log distance results, are consistent within uncertainties and give a distance of $d \sim (55 \pm 1)kpc$.

1.3 Problem 3

Black Hole Accretion Time Scales As a mass m of gas falls into a black hole, at most 0.1mc2 is likely to emerge as radiation; the rest is swallowed by the black hole. Show that the Eddington luminosity for a black hole of mass M is equivalent to 2×10−9 M c2 yr−1. Explain why we expect the black hole's mass to grow by at least a factor of e every 5 × 107 years.

We start by deriving the eddington luminosity in a typical way

$$F_g = F_{rad} \tag{1}$$

$$\frac{GMm_p}{r^2} = \frac{\sigma_T L_{edd}}{4\pi c r^2} \tag{2}$$

Solving for L_{edd}

$$L_{edd} = \frac{4\pi G m_p}{c\sigma_T} M c^2$$

(3)

Or in terms of years

$$L_{edd} = \frac{1.261 \times 10^8 \pi G m_p}{c\sigma_T} \frac{Mc^2}{yr} \tag{4}$$

[17]:

```
sec_to_year = 3600 * 24 * 365 * u.s / u.yr
# print(f"{sec_to_year*4:.2e}")

res_factor = 4*sec_to_year * np.pi * c.G * c.m_p / (c.c * c.sigma_T)
```

```
res_factor
```

[17]: $2.2182837 \times 10^{-9}\ \frac{1}{\mathrm{yr}}$

Using the above calculations

$$L_{edd} = 2.22 \times 10^{-9} M c^2 yr^{-1}$$

(5)

We can also write a mass accretion rate

$$\dot{M}_{edd} = \frac{L_{edd}}{\epsilon c^2} = \frac{2.22 \times 10^{-9} M}{\epsilon} yr^{-1} \tag{6}$$

We are given $\epsilon = 0.1$ in the problem giving an equation

$$\dot{M}_{edd} = 2.22 \times 10^{-8} M yr^{-1} \tag{7}$$

Integrating the mass from $M_0 \to eM_0$ and from $t = 0 \to t = \tau$ for an e-folding time τ gives

$$\int_{M_0}^{eM_0} \dot{M}_{edd} dM = 2.22 \times 10^{-8} yr^{-1} \int_0^{\tau} M dt \tag{8}$$

Assuming an exponential growth from $M_0 \to eM_0$ then we have a functional form $M = M_0 e^{t/\tau}$

$$M_0(e-1) = 2.22 \times 10^{-8} M_0 yr^{-1} \int_0^{\tau} e^{t/\tau} dt \tag{9}$$

$$M_0(e-1) = 2.22 \times 10^{-8} M_0 yr^{-1} \tau (e-1) \tag{10}$$

Simplifying gives a timescale

$$\tau = \frac{yr}{2.22 \times 10^{-8}} = 4.5 \times 10^{7} yr \tag{11}$$

[16]:
```
f"{(1/2.22e-8):.2e} yr"
```

[16]: '4.50e+07 yr'

1.4 Problem 4

Closed Box Enrichment In a scenario where stars are made from gas that is initially free of metals, so Z(t=0)=0, what is the mean metal abundance of stars once all of the gas is gone?

My Solution

The average stellar metalicity is defined as

$$< z_s >= \frac{M_z}{M_s} \tag{12}$$

Where M_z is the mass of the metals and M_s is the stellar mass. At some time t, when $M_g = 0$ (ie all the gas was used up to make stars), we know that $M_{tot} = M_s$ giving

$$< z_s >= \frac{M_z}{M_s} \tag{13}$$

For an outer shell on the star δM_s,

$$M_z = \int_0^t z(t)\delta M_s \tag{14}$$

Changing variables to M_g

$$M_z = -\int_{M_{tot}}^0 z(M_g)\delta M_g \tag{15}$$

Using the equation for $z(M_g)$ derived in class gives an integral

$$M_z = P\int_{M_{tot}}^0 \ln\left(\frac{M_g}{M_{tot}}\right)\delta M_g \tag{16}$$

$$M_z = P\left[M_g\left(\ln\frac{M_g}{M_{tot}} - 1\right)\right]_{M_{tot}}^0 \tag{17}$$

$$M_z = P\left[0 + M_g\right] \tag{18}$$

$$M_z = P\ M_g \tag{19}$$

Substituting this back into the equation for $< z_s >$ gives

$$\boxed{< z_s >= P} \tag{20}$$

```
[ ]:
```