

# MONETARY MECHANISMS\*

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## Abstract

We provide a series of results for a standard model where exchange is facilitated by liquid assets. Compared to past work, minimal structure is imposed on the mechanism determining the terms of trade. Four simple axioms lead to a class of mechanisms encompassing common bargaining theories, competitive price taking and other solution concepts. Using only the axioms, we establish existence and (perhaps more surprisingly) uniqueness of stationary monetary equilibrium. We also show how to support desirable outcomes using creatively designed mechanisms. Special cases include pure currency economies, but we also consider extensions to incorporate real assets and credit.

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# 1 Introduction

This paper provides a series of useful results for the model in Lagos and Wright (2005) where exchange is facilitated by liquid assets. A novelty is that we impose minimal structure on the mechanism determining the terms of trade. Instead of, e.g., Nash bargaining or Walrasian pricing, we show how four simple axioms lead to a class of mechanisms that encompass common bargaining theories, competitive price taking, and other solution concepts. Using only properties implied by the axioms, we establish existence under certain conditions (as is standard) and generic uniqueness (as is perhaps more surprising) of nondegenerate steady state, i.e., of stationary monetary equilibrium. The argument is related to one used in Wright (2010) for Nash bargaining, but some problems there are corrected, and, moreover, we establish results for *any* mechanism in the class under consideration. We also show how to support desirable outcomes using a variant of Hu et al. (2009). While pure-currency economies are a special case, we also consider real assets and credit.

These results extend much previous work. Lagos and Wright (2005) has bilateral random matching and Nash bargaining. Extensions by Lagos and Rocheteau (2005), Rocheteau and Wright (2005) and others use price taking and price posting with directed search. Alternative axiomatic and strategic bargaining solutions are studied by Aruoba et al. (2007), Zhu (2015) and others. Hu et al. (2009) use pure mechanism design, Galenianos and Kircher (2008) and Dutu et al. (2012) use auctions, and Silva (2015) considers monopolistic competition. See Nosal and Rocheteau (2011) or Lagos et al. (2015) for surveys of the literature – suffice it to say here that the framework has become a workhorse in monetary economics. The relevance of this is that the model is being used extensively in theory and applied work. Hence, while our analysis is motivated by technical rather than substantive issues, the results should be of interest to many people working on applications.<sup>1</sup>

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<sup>1</sup>This is not to say the approach has been universally adopted in monetary economics, since for some issues, heterogeneity based on history is crucial (e.g., Molico 2006, Menzio et al. 2013, Wallace 2014, Lippi et al. 2015 and Rocheteau et al. 2015). In the model studied here, by design,

## 2 Environment

Each period in discrete time has two markets: first, a decentralized market, or DM, with frictions detailed below; second, a frictionless centralized market, or CM. There are two types of agents, buyers  $b$ , and sellers  $s$ . Types are permanent, but the results also apply when types are determined randomly each period. The DM involves bilateral trade: a buyer meets a seller with probability  $\alpha$ , while a seller meets a buyer with probability  $\alpha/n$ , where  $n$  is buyer/seller ratio. In the DM sellers can produce but do not consume, while buyers want to consume but cannot produce, to preclude barter. In the baseline model there is no record keeping of DM activity, to preclude credit. This generates an essential role for a payment instrument (Kocherlakota 1998), a role played here by cash. In the CM all agents work, consume, adjust portfolios and settle debts, depending on the version of the model.

Preferences between the CM and DM are separable, and linear in CM labor. Thus, the period utility functions of buyers and sellers are

$$\mathcal{U}^b(q, x, \ell) = u(q) + U(x) - \ell \text{ and } \mathcal{U}^s(q, x, \ell) = -c(q) + U(x) - \ell,$$

where  $q$  is the DM good,  $x$  is the CM good, and  $\ell$  is labor. One unit of  $\ell$  produces one unit of  $x$  in the CM, so the real wage is one (this is easily relaxed). The constraints  $x \geq 0$ ,  $q \geq 0$  and  $\ell \in [0, 1]$  are assumed not to bind, as can be guaranteed in the usual way. Also,  $U$ ,  $u$  and  $c$  are twice continuously differentiable, where  $U', u', c' > 0$ ,  $U'' < 0$  and  $u'' \leq 0 \leq c''$  with one equality strict. Also,  $u(0) = c(0) = 0$ . Agents discount between the CM and DM according to  $\beta = 1/(1 + r)$ , with  $r > 0$ .<sup>2</sup>

Goods  $q$  and  $x$  are nonstorable. There is a storable asset called fiat money, with supply per buyer  $M$ . Assume  $M_{+1} = (1 + \pi) M$ , where subscript  $+1$  indicates next

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money demand is history independent, allowing us to focus on other issues, and to derive analytic rather than numerical results.

<sup>2</sup>Quasi-linear CM utility is common in this literature because it implies history independence. However, as in Wong (2015), similar results hold for any CM utility function  $\tilde{U}(x, 1 - \ell)$  with  $\tilde{U}_{11}\tilde{U}_{22} = \tilde{U}_{12}^2$ . See Wong (2016) and Gu et al. (2016) for applications using the more general specification. Alternatively, one can use any monotone and concave  $\tilde{U}$  and get the same results by assuming indivisible labor (Rocheteau et al. 2008). Separability between the CM and DM can also be relaxed (Rocheteau et al. 2007).

period. Changes in  $M$  are accomplished by lump sum transfers if  $\pi > 0$  or taxes if  $\pi < 0$ , but the results also apply if instead government uses new money to buy CM goods. For convenience, only buyers pay taxes or get transfers. Let  $\phi$  be the price of money in terms of CM numeraire  $x$ . The focus here is on stationary outcomes, where all real variables are constant, including real balances  $z = \phi M$ . Hence, inflation is  $\phi_{+1}/\phi = 1 + \pi$ . We also use the Fisher equation,  $1 + i = (1 + \pi)(1 + r)$ . As usual,  $i$  is the nominal return an agent requires in the next CM to give up a dollar in this CM, and we can price such trades even if they do not occur in equilibrium. Attention is restricted to  $\pi > \beta - 1$ , or the limit  $\pi \rightarrow \beta - 1$ , which means  $i \rightarrow 0$  and is called the Friedman rule; there are no monetary equilibria with  $\pi < \beta - 1$ .

### 3 Baseline Model

The state of an agent in the baseline model is real balances in terms of CM numeraire,  $z = \phi m$ . The value functions in the CM and DM are  $W(z)$  and  $V(z)$ .

#### 3.1 The CM Problem

The CM problem for an agent of type  $j = b, s$  is

$$W_j(z) = \max_{x, \ell, \hat{z}} \{U(x) - \ell + \beta V_j(\hat{z})\} \text{ st } z + \ell = x + (1 + \pi)\hat{z} + T,$$

where  $\hat{z}$  is real money carried to the next DM and  $T$  is the tax. FOC's are

$$U'(x) - 1 = 0 \tag{1}$$

$$z + \ell - (1 + \pi)\hat{z} - x - T = 0 \tag{2}$$

$$-(1 + \pi)U'(x) + \beta V'_j(\hat{z}) \leq 0, = \text{ if } \hat{z} > 0. \tag{3}$$

From (1),  $x = x^*$  where  $x^*$  solves  $U'(x^*) = 1$ . The envelope condition is  $W'_j(z) = 1$ . From (3),  $\hat{z}$  does not depend on  $z$ . As shown below,  $\hat{z} > 0$  for buyers in monetary equilibrium, while  $\hat{z} = 0$  for sellers, since given the class of mechanisms under consideration they have no use for liquidity in the DM.

### 3.2 The DM Problem

When a buyer and seller meet in the DM they choose a quantity  $q$  and payment  $p$  according to some *general trading mechanism*  $\Gamma$  mapping the buyer's  $\hat{z}_b$  into a pair  $(p, q)$ .<sup>3</sup> Note  $(p, q)$  depends on  $\hat{z}_b$  because of the feasibility constraint  $p \leq \hat{z}_b$ , but not on  $\hat{z}_s$  because we restrict attention to mechanisms that only depend on variables that are relevant for the trading surplus of the buyer and seller. These are

$$\begin{aligned} S_b &= u(q) + W_b(z_b - p) - W_b(z_b) = u(q) - p \\ S_s &= -c(q) + W_s(z_s + p) - W_s(z_s) = p - c(q), \end{aligned}$$

by the envelope condition  $W'_j = 1$ . Hence they do not depend on  $\hat{z}_b$  or  $\hat{z}_s$  directly, and depend only on  $\hat{z}_b$  indirectly through the constraint  $p \leq \hat{z}_b$ .<sup>4</sup>

To guarantee gains from trade are positive but finite, assume  $u'(0) > c'(0)$  and  $\exists \bar{q} > 0$  such that  $u(\bar{q}) = c(\bar{q})$ . Then define the unconstrained efficient quantity  $q^*$  by  $u'(q^*) = c'(q^*)$ , and let  $p^* = \inf \{\hat{z}_b : \Gamma_p(\hat{z}_b) = q^*\}$  be the minimum payment required for a buyer to get  $q^*$ .

We focus on mechanisms of the form

$$\Gamma_p(z) = \begin{cases} z & \text{if } z < p^* \\ p^* & \text{otherwise} \end{cases} \quad \text{and} \quad \Gamma_q(z) = \begin{cases} v^{-1}(z) & \text{if } z < p^* \\ q^* & \text{otherwise} \end{cases} \quad (4)$$

where  $v$  is some strictly increasing function with  $v(0) = 0$  and  $v(q^*) = p^*$ . Section 4 presents axioms that imply  $\Gamma$  must take the form in (4) and discusses examples. In terms of economics, it says a buyer gets the efficient quantity  $q^*$  and pays some amount  $p^* = v(q^*)$ , determined by the mechanism, if he can afford it in the sense that  $p^* \leq z$ ; and if he cannot afford it, he pays  $p = z$  and gets some  $q = v^{-1}(z) < q^*$

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<sup>3</sup>This definition assumes bilateral matching, and hence does not encompass the setting in Galenianos and Kircher (2008) and Dutu et al. (2012), where a seller can meet multiple buyers, or the one in Head et al. (2012) and Liu et al. (2015), where a buyer can meet multiple sellers. There may be a way to extend some of our results to those environments but we do not attempt to do so here. However, we do consider Walrasian price taking below, which is sometimes motivated by having a large number of buyers and sellers in the market, but can still be applied formally to bilateral trade; for more discussion, see Corbae et al. (2003) and Rocheteau and Wright (2005).

<sup>4</sup>This is not to say one cannot consider a mechanism that depends on both,  $\tilde{\Gamma}(\hat{z}_b, \hat{z}_s)$  – e.g., one could assume bargaining power depends on  $\hat{z}_s$  – we simply do not allow that here.

determined by the mechanism. Thus,  $v^{-1}(z)$  is the quantity a constrained buyer gets, while  $v(q)$  is how much it costs. For convenience, let us also assume  $v$  is twice continuously differentiable almost everywhere.

Sellers carry no money to the DM in equilibrium. Still, we write

$$V_s(z) = W_s(z) + \frac{\alpha}{n} [p - c(q)],$$

where  $p = \Gamma_p(\tilde{z})$  and  $q = \Gamma_q(\tilde{z})$  depend on the  $\tilde{z}$  of the buyer a seller meets, which he takes as given. For a buyer,

$$V_b(z) = W_b(z) + \alpha [u(q) - p],$$

where  $p = \Gamma_p(z)$  and  $q = \Gamma_q(z)$  depend on his own real balances. One can check that in equilibrium  $z \leq p^*$  and  $q \leq q^*$ .<sup>5</sup>

After simplifying we can write

$$W_b(z) = \bar{W} + \beta \max_{\hat{z}, q_{+1}} \{-i\hat{z} + \alpha [u(q_{+1}) - v(q_{+1})]\},$$

where  $i$  comes from the Fisher equation, and  $q_{+1}$  depends on  $\hat{z}$  through the mechanism  $\Gamma$ . Here  $\bar{W}$  is a constant that is irrelevant for the choice of  $\hat{z}$ . Thus,  $W_b(z) = \bar{W} + \alpha\beta \max_{q_{+1}} J(q_{+1}; i)$  where

$$J(q_{+1}; i) = u(q_{+1}) - (1 + i/\alpha) v(q_{+1}).$$

This change of variable replaces the choice of  $\hat{z}$  with the direct choice of  $q_{+1}$ . Without loss of generality, impose  $q \in [0, q^*]$  and represent the problem by

$$\max_q J(q; i) \text{ st } q \in [0, q^*]. \quad (5)$$

Any solution  $q_i > 0$  satisfies the FOC  $u'(q) - (1 + i/\alpha) v'(q) = 0$ .

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<sup>5</sup>To see this, substitute  $V_b$  into  $W_b$  and use the linearity of  $W_b$  to reduce the choice of  $\hat{z}$  to  $\max_{\hat{z}, q_{+1}} \{-(1 + \pi)\hat{z} + \beta\hat{z} + \beta\alpha [u(q_{+1}) - p_{+1}]\}$ , where  $q_{+1} = q^*$  and  $p_{+1} = p^*$  if  $\hat{z} \geq p^*$ , and  $q_{+1} = v^{-1}(\hat{z})$  and  $p_{+1} = \hat{z}$  if  $\hat{z} < p^*$ . If  $\hat{z} > p^*$ , the derivative wrt  $\hat{z}$  is  $-(1 + \pi) + \beta < 0$ , given  $\pi > \beta - 1$ .

### 3.3 Equilibrium

Formally, a stationary monetary equilibrium is a CM allocation  $(x, \ell)$ , a DM outcome  $(p, q)$  and real balances  $z$  such that  $x$  and  $\ell$  solve (1)-(2),  $q$  solves (5) and  $p = z = v(q) > 0$ . A nonmonetary equilibrium is similar except  $p = q = z = 0$ . Given  $\mathcal{U}^b(q, x, 1 - \ell)$  is separable between the CM and DM,  $(p, q)$  is independent of  $(x, \ell)$ , so we can discuss some properties of the former without reference to the latter. To proceed, assume  $\exists q > 0$  such that  $v(q) < u(q)$ . This means buyers would get gains from trade absent liquidity considerations, and holds automatically for most reasonable mechanisms (although not, e.g., take-it-or-leave-it offers by sellers).

**Proposition 1** *Stationary monetary equilibrium exists iff  $i < \bar{i}$ , where  $\max_q J(q; \bar{i}) = 0$ . For generic parameters it is unique, and  $q_i \in (0, q_0) \Rightarrow \partial q_i / \partial i < 0$ .*

**Proof:** We have reduced equilibrium to a solution  $q_i$  to (5), where  $J(q; i)$  is twice continuously differentiable and  $J(0; i) = 0$ . Clearly a solution exists. Since  $v(q) < u(q)$  for some  $q > 0$ ,  $q_i = 0$  is not a solution when  $i \rightarrow 0$ . Since  $J_i < 0$ , by the envelope condition, there is a  $\bar{i}$  below which equilibrium exists and above which it does not, although note that we could have  $\bar{i} < \infty$  or  $\bar{i} = \infty$ .

Since we do not know  $J_{qq} < 0$ , in general, there may be multiple local maximizers. This is shown in Figure 1a, where the higher curve is for  $i = i_1$  and the lower curve is for  $i_2 > i_1$ . At any local maximum  $J_q(q; i) = u' - (1 + i/\alpha)v' = 0$  and  $J_{qq} = u'' - (1 + i/\alpha)v'' < 0$ . We claim the global maximizer is unique for generic  $i$ . To see this, suppose  $J(q_1^*; i) = J(q_2^*; i) = \max_q J(q; i)$  with  $q_2^* > q_1^*$ . Since  $q_1^*$  is the lowest global maximizer,  $J(q_1^*; i) > J(q; i) \forall q < q_1^*$ . Increase  $i$  to  $i + \varepsilon$ . For any  $q < q_1^*$ , by continuity,  $J(q_1^*; i + \varepsilon) > J(q; i + \varepsilon)$  if  $\varepsilon$  is small. The same argument applies to  $q > q_1^*$  except  $q = q_2^*$ . Since  $J_i(q; i) = -v(q) < 0$  and  $v(q)$  is increasing,  $J_i(q_2^*; i) < J_i(q_1^*; i)$ . Thus  $J(q_1^*; i + \varepsilon) > J(q_2^*; i + \varepsilon)$ , and now the global maximizer is unique at  $q$  near  $q_1^*$ .

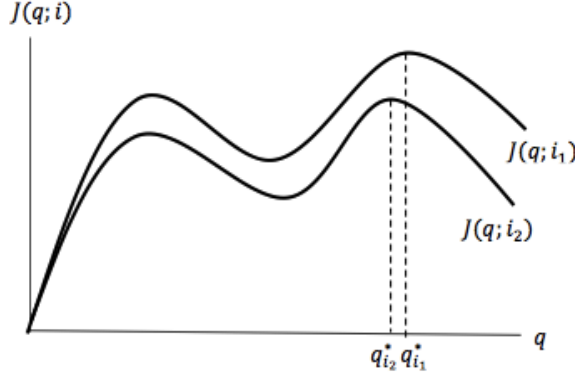


Figure 1a: The Objective Function

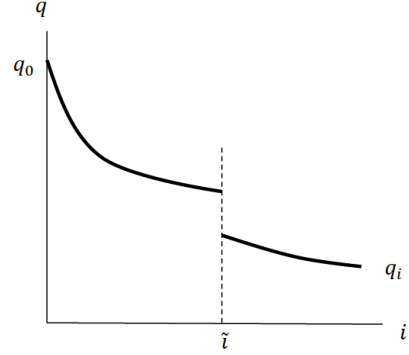


Figure 1b: The Solution  $q_i$  vs  $i$

Thus, increasing  $i_1$  to  $i_2$  shifts  $J$  down and shifts each local maximizer to the left. In particular, the global maximizer shifts to the left, and hence  $\partial q_i / \partial i < 0$  when  $q_i$  is single valued. If we increase  $i$  from  $i \approx 0$ , we may reach a nongeneric point  $\tilde{i}$  where there are multiple global maximizers, say  $q_1^*$  and  $q_2^* > q_1^*$ . By the argument used above, for  $\tilde{i} + \varepsilon$  the unique global maximizer is close to  $q_1^*$  and for  $\tilde{i} - \varepsilon$  the unique global maximizer is close to  $q_2^*$ . So  $q_i$  is continuously decreasing in  $i$  and single-valued function except possibly for  $i$  in a set of measure 0 where it has multiples values, and jumps to the left as  $i$  increases. Since for generic  $i$  there is a unique  $q_i$ , there cannot be multiple equilibria. ■

The key step in the demonstration is turning the equilibrium problem into a decision problem, although this is clearly not the same as the usual characterization of competitive equilibrium by a planner's problem. Still, the idea is that buyers take  $q = v^{-1}(z)$  as given, similar to taking price as given in Walrasian markets. Figure 1b shows  $q_i$ , which is strictly decreasing, and happens to have a jump in this example. One can interpret  $z_i = v(q_i)$  as the demand for liquidity, the cost of which is  $i$ . At  $i = \tilde{i}$ , buyers are indifferent between two values on the demand correspondence, and we can assign a fraction to each; changing  $i$  to  $\tilde{i} + \varepsilon$  implies a unique  $q_i$ . Note the intercept is  $q_0 \leq q^*$  – e.g.,  $q_0 = q^*$  with Kalai bargaining, while  $q_0 < q^*$  with



Nash bargaining unless  $\theta = 1$  where  $\theta$  is buyers' bargaining power. Also, in the requirement for existence  $i < \bar{i}$ , we can have  $\bar{i} < \infty$  or  $\bar{i} = \infty$  – e.g., under standard Inada conditions, with Kalai bargaining one can show  $\bar{i} = \alpha\theta/(1 - \theta)$ , while with Nash one can show  $\bar{i} = \infty$ .<sup>6</sup>

## 4 Mechanisms

We do not want the results to depend on a particular way of determining the terms of trade. How general is our class of mechanisms? Consider the following:

**Axiom 1** (*Feasibility*):  $\forall z, 0 \leq \Gamma_p(z) \leq z, 0 \leq \Gamma_q(z)$ .

**Axiom 2** (*Individual Rationality*):  $\forall z, u \circ \Gamma_q(z) \geq \Gamma_p(z)$  and  $\Gamma_p(z) \geq c \circ \Gamma_q(z)$ .

**Axiom 3** (*Monotonicity*):  $\Gamma_p(z_2) > \Gamma_p(z_1) \Leftrightarrow \Gamma_q(z_2) > \Gamma_q(z_1)$ .

**Axiom 4** (*Bilateral Efficiency*):  $\forall L, \nexists(p', q')$  with  $p' \leq z$  such that  $u(q') - p' \geq u \circ \Gamma_q(z) - \Gamma_p(z)$  and  $p' - c(q') \geq \Gamma_p(z) - c \circ \Gamma_q(z)$  with one inequality strict.

Note Axiom 3 does not say the surpluses increase with  $z$ , only that  $p$  increases with  $q$ , so that Nash bargaining satisfies this even though  $S_b$  may not be increasing in  $z$ . Also, Axiom 4 it is not critical for most results – e.g., they hold for the monopsony mechanism discussed below. Also, Axiom 4 is an ex post condition in the DM saying that we cannot make the parties better off conditional on  $z$ ; it does not say the ex ante choice of  $z$  in the CM is efficient.

**Proposition 2** *Any  $\Gamma$  satisfying Axioms 1-4 takes the form given in (4).*

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<sup>6</sup>While stationary monetary equilibrium is generically unique, with fiat currency there is always a nonmonetary equilibrium, plus nonstationary monetary equilibria. However, since many applications use stationary equilibrium, the results are useful. Moreover, some applications integrate the specification with other models – e.g., Berentsen et al. (2011) combine it with a Pissarides (2000) labor market. That setup can have multiple monetary steady states, because there is feedback from the goods market to the labor market, and vice versa, but our result is still useful because it at least guarantees the goods market outcome is unique given any labor market outcome. Finally, as mentioned, the proof strategy is similar to the one used in Wright (2010) for Nash bargaining, although there is problem with that argument (not the result) because it is assumed the buyer maximizes  $u(q)/v(q) - (1 + i/\alpha)$  instead of  $u(q) - (1 + i/\alpha)v(q)$ .

**Proof:** First consider  $z < p^*$ . By Axiom 3 and the definition of  $p^*$ , we have  $q = \Gamma_q(z) < q^*$ . We prove  $p = z$  by contradiction. Suppose  $p \neq z$ . We cannot have  $p > z$ , by Axiom 1, so  $p < z$ . Consider  $p' = p + \varepsilon_p < z$  and  $q' = q + \varepsilon_q < q^*$ , which is feasible for small  $(\varepsilon_p, \varepsilon_q)$ . If  $\varepsilon_p = u(q') - u(q)$ , one can easily check that the buyer's surplus  $S_b$  does not change, while for the seller

$$dS_s = p' - c(q') - p + c(q) = u(q') - c(q') - u(q) + c(q). \quad (6)$$

Since  $q^* > q' > q$ ,  $u(q) - c(q)$  is increasing in  $q$ . Therefore  $S_s$  increases, contradicting Axiom 4. Hence,  $z < p^* \Rightarrow p = z$ .

Next, consider  $z \geq p^*$ . We prove  $q = q^*$  by contradiction. Suppose  $q = \Gamma_q(z) < q^*$ . We know  $p = \Gamma_p(z) < p^*$  by Axiom 3. Let  $p' = p + \varepsilon_p$  and  $q' = q + \varepsilon_q$ . As in the previous step, one can check  $(p', q')$  dominates  $(p, q)$ , contradicting Axiom 4. Suppose instead  $q > q^*$ . Let  $p' = p - \varepsilon_p$  and  $q' = q - \varepsilon_q > q^*$ , where  $\varepsilon_p = u(q) - u(q')$ , which satisfies Axiom 1 and Axiom 3 for small  $(\varepsilon_p, \varepsilon_q)$ . One can check that  $S_b$  does not change while the change in  $S_s$  is the same as (6). Since  $q > q' > q^*$ ,  $u(q) - c(q)$  is decreasing in  $q$ . Therefore  $S_s$  increases, contradicting Axiom 4. So  $z \geq p^*$  implies  $q = q^*$ , which implies  $p = p^*$  by the definition of  $p^*$  and Axiom 3. Hence,  $v^{-1}(p^*) = q^*$ . By Axiom 3,  $v^{-1}$  and  $v$  are strictly increasing. By Axiom 2,  $v^{-1}(0) = 0$ . By definition,  $v^{-1}(p^*) = q^*$ . This shows  $\Gamma$  is as described by (4). ■

A simple example is provided by Kalai's proportional bargaining solution, with  $\theta$  the buyers' bargaining power. In this context, Kalai bargaining maximizes  $S_b$  wrt  $(p, q)$  subject to  $S_b = \theta(S_b + S_s)$  and  $p \leq z$ . Let

$$v(q) = \theta c(q) + (1 - \theta) u(q) \quad (7)$$

and let  $p^* = v(q^*)$ . Then Kalai's solution is: if  $p^* \leq z$  then  $(p, q) = (p^*, q^*)$ ; and if  $p^* > z$  then  $p = z$  and  $q = v^{-1}(z)$  with  $v$  given by (7). Another example is generalized Nash bargaining, which maximizes  $S_b^\theta S_s^{1-\theta}$  wrt  $(p, q)$  subject to  $p \leq z$ . This gives a similar qualitative outcome, except

$$v(q) = \frac{\theta u'(q) c(q) + (1 - \theta) c'(q) u(q)}{\theta u'(q) + (1 - \theta) c'(q)}.$$

The Nash and Kalai solutions are the same if  $\theta = 1$ ; if  $\theta \in (0, 1)$ , given  $u'' < 0$  or  $c'' > 0$ , they are different when  $p \leq z$  binds. Yet both take the form in (4).

Zhu (2015) shows that a simple strategic bargaining game also satisfies our axioms. This game has the seller make an initial offer  $(p, q)$ . If the buyer agrees, they trade; otherwise, a coin flip determines which one of them makes a final offer. If the final offer is accepted, they trade; otherwise, they part with no trade. This game is nice, for our applications, for several reasons discussed in Zhu (2015).<sup>7</sup> He also shows a different game, where the buyer instead of the seller makes the initial offer, does not satisfy our axioms. In particular, it violates strict monotonicity, because the buyer might pay a higher  $p$  for the same  $q$ .

We can also use Walrasian pricing, motivated by saying that agents trade in large groups, not bilaterally, but otherwise keeping the environment the same (Rocheteau and Wright 2005). This gives a similar qualitative outcome, except now  $v(q) = \bar{p}q$ , where agents take  $\bar{p}$  as given, even though in equilibrium  $\bar{p} = c'(q)$ . Silva (2015) provides another example using monopolistic competition. An example that violates Axiom 4 is a monopsonist that takes as given  $c'(q)$  rather than the price,

$$\max_q [u(q) - qc'(q)] \text{ st } qc'(q) \leq z.$$

Let  $\tilde{q}$  be a solution without the liquidity constraint (the standard monopsonist outcome) and let  $\tilde{p} = \tilde{q}c'(\tilde{q})$ . This mechanism looks like (4), except the critical value is  $\tilde{p} = v(\tilde{q})$  rather than  $p^* = v(q^*)$ .

Next, consider trying to construct  $v(q)$  to support a desirable  $q^o$ , which could be  $q^o = q^*$  or something else, as in Hu et al. (2009) and Wong (2016).

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<sup>7</sup>One is that axiomatic bargaining is problematic for nonstationary equilibria (Coles and Wright 1998; Coles and Muthoo 2003). Another is that his game ends for sure, on or off the equilibrium path, in finite time; standard bargaining (Rubinstein 1982; Binmore et al. 1986) can go on forever at least off the equilibrium path, which is awkward when the DM closes and CM opens. His game is also better than simply flipping a coin to see who makes a take-it-or-leave-it offer as in some search models with linear utility (Gale 1987; Mortensen and Wright 2001): when agents are risk averse, it is Pareto superior to use the initial offer to avoid the coin flip.

**Proposition 3** *Let  $\hat{q}$  solve  $u(\hat{q}) = (1 + i/\alpha)c(\hat{q})$ . Then there exists a mechanism to support any  $q^o \leq \min\{q^*, \hat{q}\}$ . In particular, if  $q^* \leq \hat{q}$  we can achieve  $q^o = q^*$ , even if  $i > 0$ . There is no way to support  $q^o > q^*$ .*

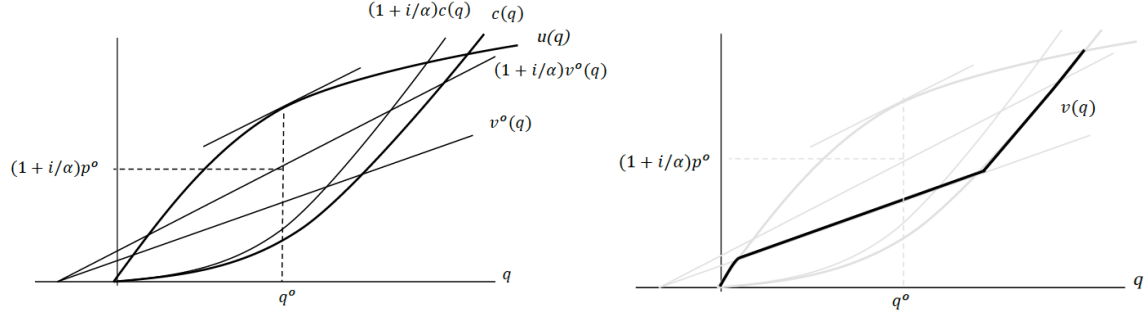


Figure 2: A HKW Style Mechanism

**Proof:** Consider  $q^o \leq \min\{q^*, \hat{q}\}$ . Figure 2 shows  $u(q)$  and  $(1 + i/\alpha)c(q)$ . Pick  $(1 + i/\alpha)p^o$  such that  $(1 + i/\alpha)c(q^o) < (1 + i/\alpha)p^o < u(q^o)$ , which is possible since  $q^o \in [0, \hat{q}]$ . Draw a line through  $(q^o, (1 + i/\alpha)p^o)$  with slope  $u'(q^o)$ , labelled  $(1 + i/\alpha)v^o(q)$  in the graph. Now define  $v(q)$  on  $[0, \bar{q}]$  as follows: first rotate  $(1 + i/\alpha)v^o(q)$  to get  $v^o(q)$ ; then truncate it above by  $u(q)$  or  $c(q)$ , whichever  $v^o$  meets first from the left; and truncate it below by  $u(q)$  or  $c(q)$ , whichever  $v^o$  meets first from the right. If  $v^o$  is truncated by  $u$  then  $J$  is negative on that truncation. If it is truncated by  $c$  then  $J$  is concave and the maximizer is not on the truncation. Since  $v(\cdot)$  is strictly increasing,  $v^{-1}(\cdot)$  is well defined, and a mechanism  $\Gamma$  is given by (4). This mechanism is consistent with trading  $q^o$  and  $p^o = v(q^o)$  ex post, in the DM, because  $c(q^o) \leq p^o \leq u(q^o)$ . And it is consistent with the ex ante decision to bring enough  $z$  from the CM, because  $q^o$  is the global maximizer of  $J(q; i) = u(q) - (1 + i/\alpha)v(q)$ . Hence we support  $q^o$ .

We now claim  $q^o > q^*$  cannot be supported. Even if a buyer has an ex ante incentive to bring enough  $\hat{z}$  from the CM to pay  $p^o$  and get  $q^o$ , in the DM there is an alternative  $(p, q)$  that Pareto dominates  $(p^o, q^o)$ , involving a reduction in  $q$  from  $q^o$  toward  $q^*$  and a reduction in  $p^o$ . So Axiom 4 implies we cannot support  $q^o > q^*$ .

Note this is not an issue for  $q^o < q^*$ , as when a buyer only brings enough to get  $q^o$ , renegotiation toward  $q^* > q^o$  violates Axiom 1. So we can construct mechanisms that deliver any  $q^o \leq q^*$  when  $\hat{q} > q^*$ , while  $\hat{q}$  is the highest  $q$  we can support when  $\hat{q} < q^*$ . ■

Intuitively, the idea in the proof is that  $v(q)$  should give buyers the incentive in the CM to choose the right  $\hat{z}$ . This implies  $u'(q^o) = (1 + i/\alpha) v'(q^o)$ , which tells us something about  $v(q)$ , but we also have to ensure agents want to trade  $q^o$  after they meet in the DM. Another observation is that  $q > \hat{q}$  cannot be supported, and  $i$  big makes  $\hat{q}$  small. So when  $i$  is high, we cannot achieve  $q^*$  even with this type of mechanism. We can support  $q^*$  for  $i > 0$  as long as  $i$  is not too big. Also, there is more than one way to construct such mechanisms, and while our approach is similar in spirit to Hu et al. (2009), the details are rather different. In particular, our mechanism is linear over the range where the DM incentive conditions are slack. Also, our mechanism satisfies the monotonicity axiom, whereas the one in Hu et al. (2009) only satisfies the other three.

## 5 Extensions

Consider replacing fiat money by an asset  $a$ , say a Lucas tree, with dividend  $\rho > 0$  in terms of numeraire each CM. The asset supply is fixed at  $A$ , and its price in terms of  $x$  is  $\psi$ . Then the CM constraint becomes  $x = \ell + (\psi + \rho)a - \psi\hat{a}$ , and the DM constraint  $v(q) \leq (\psi + \rho)\hat{a}$ . Also, to avoid a minor technicality, assume  $q_0 = q^*$ .<sup>8</sup> Then, if the DM constraint does not bind, we get  $\psi = \psi^* \equiv \rho/r$  and  $q = q^*$ . But if it does bind, which happens iff  $\rho A$  is low, we get  $\psi > \psi^*$  and  $q < q^*$ .

Suppose  $\rho A$  is low – the interesting case – and consider

$$J(q; \psi) = u(q) - \left[ 1 + \frac{r\psi - \rho}{\alpha(\psi + \rho)} \right] v(q).$$

---

<sup>8</sup>This is true for Walrasian pricing or Kalai bargaining, e.g., but not necessarily Nash bargaining. The technicality is that buyers may hold some assets they do not bring to the DM, something that never happens with fiat money. See Geromichalos et al. (2007) or Lagos and Rocheteau (2008).

The FOC is

$$u'(q) = \left[ 1 + \frac{r\psi - \rho}{\alpha(\psi + \rho)} \right] v'(q).$$

It is not hard to show, similar to Proposition 1, that the solution  $q_\psi$  exists, is generically unique and strictly decreases with  $\psi$ , with  $q_\psi \rightarrow q^*$  as  $\psi \rightarrow \psi^*$  and  $q_\psi \rightarrow 0$  as  $\psi \rightarrow \infty$ . This can be again interpreted as the demand for liquidity. The supply is given by  $(\psi + \rho)A$ , which is endogenous and strictly increasing with  $\psi$ . Hence, there is a unique equilibrium at  $\psi > \psi^*$ .<sup>9</sup>

Next assume that a buyer can make a promise of payment in numeraire at the next CM, subject to  $p \leq D$ . Here the debt limit  $D$  is exogenous, but Gu et al. (2016) show how to endogenize it along the lines of Kehoe and Levine (1993). The CM state variable is now wealth  $z - d$ , where  $d$  is debt from the previous DM. Then

$$W_j(z - d) = \max_{x, \ell, \hat{z}} \{U(x) - \ell + \beta V_j(\hat{z})\} \text{ st } z - d + \ell = x + (1 + \pi)\hat{z} + T$$

again implies  $W'_j = 1$ . A mechanism now maps a buyer's *liquidity position*,  $L \equiv z + D$ , into  $(p, q)$ . All of the above results go through with  $L$  replacing  $z$ .

One can easily show that  $D \geq v(q^*)$  implies there is no monetary equilibrium, as buyers can get  $q = q^*$  on credit. If  $D < v(q^*)$ , their choice reduces to

$$\tilde{q}_i = \arg \max J(q; i) \text{ st } q \in [q_D, q^*],$$

where  $q_D = v^{-1}(D)$ . If  $\tilde{q}_i > q_D$ , there is a monetary equilibrium where buyers use cash plus credit, while if  $\tilde{q}_i = q_D$  there is no monetary equilibrium. One can show  $\tilde{q}_i$  is unique and  $\partial \tilde{q}_i / \partial i < 0$  in monetary equilibrium by an argument similar to the one for Proposition 1. Further, suppose for the sake of illustration that  $J$  is single peaked at, say,  $\bar{q}_i$ . Then  $q_D < \bar{q}_i$  implies  $\tilde{q}_i = \bar{q}_i$ , while  $q_D \geq \bar{q}_i$  implies  $\tilde{q}_i = q_D$ .

Thus, monetary equilibrium exists iff  $D$  is not too big.

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<sup>9</sup>This is stronger than the result for fiat money, which establishes only generic uniqueness, because with  $\rho = 0$  the analog of supply is perfectly elastic and could be like  $\tilde{z}$  in Figure 1b. Another difference from fiat money is that  $\rho > 0$  rules out a stationary equilibria with  $\psi = 0$ , as well as nonstationary equilibria where  $\psi \rightarrow 0$ , but not necessarily other nonstationary equilibria, including cyclic, chaotic and stochastic equilibria (e.g., see Rocheteau and Wright 2013). Also, one can study the model with both real assets and fiat money, which exists when  $\rho A$  and  $i$  are both low; we leave that as an exercise.

As above, we can construct a mechanism to support some  $q^o$  when credit is available up to limit  $D$ . Define  $q_L \equiv u^{-1}(D)$  and  $q_H = c^{-1}(D)$ . Let  $\hat{q}_c$  solve  $u(q) - (1 + i/\alpha)c(q) + iD/\alpha = 0$ . This is the maximum  $q$  that a buyer will accept ex ante given the best terms of trade. The next result is proved the Appendix.

**Proposition 4** *If  $D \geq u(q^*)$  then only  $q^*$  can be supported. If  $D < u(q^*)$  then we have the following: any  $q^o \in [q_L, \min\{q_H, q^*\}]$  can be supported by a nonmonetary equilibrium; any  $q^o \in [q_L, \min\{\hat{q}_c, q^*\}]$  can be supported by a monetary equilibrium. No other  $q^o$  can be supported.*

Now assume that a buyer can make a promise of any payment  $p > z$  at a cost described by  $\gamma(p - z)$ , where  $\gamma', \gamma'' > 0$  and  $\gamma(0) = \gamma'(0) = 0$ .<sup>10</sup> A trading mechanism  $\Gamma$  now delivers an outcome in the constrained core, constructed as follows. First, solve

$$\max_{p,q} S_b = [u(q) - p - \gamma(p - z) \mathbb{I}_{\{p > z\}}] \text{ st } p - c(q) = S_s$$

The FOC implies

$$u'(q) = \{1 + \gamma'[S_s + c(q) - z]\} c'(q), \quad (8)$$

if  $z < S_s + c(q^*)$ , and  $q = q^*$  otherwise. Then impose  $S_b, S_s \geq 0$  to get

$$\mathcal{C} \equiv \{(p, q) \mid q \text{ solves (8), } p = c(q) + S_s, S_s \geq 0, \text{ and } S_b \geq 0\}.$$

We now revise Axiom 1 by dropping the constraint  $\Gamma_p \leq z$  and Axiom 4 by dropping  $p' \leq z$ . Given this, the next result is proved in the Appendix:

**Proposition 5** *Any  $\Gamma$  satisfying the revised Axioms takes the form*

$$\Gamma(z) = \begin{cases} (p, v^{-1}(p)) \in \mathcal{C}, & p > z \text{ if } z < p^* \\ (p^*, q^*) & \text{otherwise} \end{cases} \quad (9)$$

where  $v$  is some strictly increasing function with  $v(0) = 0$  and  $v(q^*) = p^*$ .

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<sup>10</sup>See Lotz and Zhang (2016) and Bethune et al. (2015), e.g., for recent papers taking this approach. Note the cost can be imposed on sellers, rather than buyers, with similar results.

To describe the outcome in more detail, first rewrite (8) as

$$u'(q) = \{1 + \gamma'[v(q) - z]\} c'(q). \quad (10)$$

Then differentiate to get,

$$\frac{\partial q}{\partial z} = \frac{-\gamma'' c'}{u'' - c''(1 + \gamma') - c' \gamma'' v'} > 0$$

Hence,  $q$  is monotone, and we write (10) as  $z = f(q)$ , where  $f' > 0$ . As  $p$  increases with  $q$ , it also increases with  $z$ . Then (10) pins down the use of credit as  $v(q) - z = g(q)$ , where  $g(q) \equiv \gamma'^{-1}[u'(q)/c'(q) - 1]$  with  $g'(q) < 0$ . Given this,

$$V(z) = \begin{cases} W(z) + \alpha[u(q) - v(q) - \gamma \circ g(q)] & \text{if } z < p^* \\ W(z) + \alpha[u(q^*) - v(q^*)] & \text{otherwise} \end{cases}$$

Now write the CM problem as

$$\tilde{J}(q; i) = u(q) - \gamma \circ g(q) - g(q) - (1 + i/\alpha) f(q).$$

It is easy to see that  $z \geq p^*$  is not a solution. Let  $q^c$  solve  $g(q) = v(q)$ , which gives the amount of trade without money, in which case a buyer's ex ante surplus is equal to his ex post surplus. Also, if  $i$  changes,  $\tilde{J}$  rotates around  $q^c$ . Then

$$\tilde{q}_i = \arg \max_{q \in [0, q^*]} \tilde{J}(q; i).$$

As  $u - \gamma - g$  and  $f$  are strictly increasing, the argument used in the proof of Proposition 1 implies  $\tilde{q}_i$  is unique. There are two cases: (i)  $\tilde{q}_i = q^c$  and buyers use credit only; (ii)  $\tilde{q}_i > q^c$  and they use cash plus credit. In the second case, as in Proposition 1,  $\partial \tilde{q}_i / \partial i < 0$ .

Again we can design a mechanism to support  $q^o$ , although the construction is more complicated. Let  $q_L^c$  solve  $g(q_L^c) = u(q_L^c) - \gamma \circ g(q_L^c)$  and let  $q_H^c$  solve  $g(q_H^c) = c(q_H^c)$ . With credit only,  $q_L^c$  is the lowest incentive compatible  $q$  for a buyer, and  $q_H^c$  is the highest for a seller. Define  $h(q) = u(q) - \gamma \circ g(q) + i g(q) / \alpha$ . By monotonicity of  $u - \gamma$  and  $g$ , there is a threshold  $i$ , say  $\hat{i}$ , below which  $h$  is strictly increasing. The following is proved in the Appendix.



**Proposition 6** *Suppose  $i < \hat{i}$ . If  $q_L^c < q_H^c$ , any  $q^o \in [q_L^c, q_H^c]$  can be supported as a nonmonetary equilibrium. Any  $q^o \in Q \cap [q_L^c, q^*]$  can be supported as a monetary equilibrium, where  $Q = \{q | h(q) - (1 + i/\alpha) c(q) \geq 0\}$ . No other  $q^o$  can be supported.*

## 6 Conclusion

This paper provides a series of new results for a standard monetary model while imposing minimal structure on the terms of trade. Simple axioms generate a class of mechanisms that encompass various solution concepts and are sufficient to establish uniqueness, monotone comparative statics and other results. We also showed how to support outcomes using creatively designed mechanisms in the spirit of Hu et al. (2009), for pure-currency economies as well as those with real assets or credit. These results should be of interest to people using versions of the framework in applications. While it is far from true that all monetary economists use this model, as mentioned in fn.1, there are enough that do to make the findings potentially quite useful.

## Appendix

**Proof of Proposition 4:** First consider  $D < u(q^*)$ . As  $q^o > q^*$  violates Axiom 4, it cannot be supported. Figure 3 shows the construction of a  $v$  that supports  $q^o \in [q_L, \min\{q_H, q^*\}]$  as a nonmonetary equilibrium. Draw a line with slope  $u'(q^o)$  through  $D$ , labeled  $v^o(q)$  in the graph. Truncate it above by  $u(q)$  or  $c(q)$ , whichever  $v^o$  meets first from the left, and below by  $u(q)$  or  $c(q)$ , whichever  $v^o$  meets first from the right. In Figure 3,  $v^o$  is truncated by  $c$  on both sides. Given  $D$  and  $v(q)$ , buyers pay  $v(q^o)$  using  $D$  if  $z = 0$ . Given  $v(q)$ , suppose they pick  $z > 0$  to get  $q > q^o$ . As  $u$  is concave,  $u(q) - v(q) < u(q^o) - v(q^o)$ . Ignoring common terms, ex ante expected utility is  $u(q) - v(q) - i/\alpha [v(q) - D]$  in monetary equilibrium and  $u(q^o) - v(q^o)$  in nonmonetary equilibrium. Since  $v(q) > D$ , carrying money yields lower expected utility. Therefore, buyers set  $z = 0$  and get  $q^o$ .

To deliver  $q^o \in (q_L, \min\{\hat{q}_c, q^*\}]$ , we construct  $v(q)$  as in the benchmark case. Pick  $p^o$  such that  $(1 + i/\alpha) \min\{c(q^o), D\} < (1 + i/\alpha) p^o < u(q^o) + iD/\alpha$ . Draw a line through  $(q^o, (1 + i/\alpha) p^o)$  with slope  $u'(q^o)$ , labeled  $(1 + i/\alpha) v^o$ . Rotate the line to get  $v^o(q)$  then truncate it above by  $c(q)$ , and below by the  $y$  axis and replace  $v(0) = 0$ . The ex ante utility is represented by  $J + iD/\alpha$ . To show  $J(q^o; i) > J(q_D; i)$ , note that if  $q_D$  is on the linear part of  $v$  or the part truncated by  $c(q)$ , by convexity,  $J(q^o; i) > J(q_D; i)$ . This mechanism is consistent the ex ante choice of  $z$ , as illustrated in Figure 4.

To get  $q^o < q_L$ , buyers are willing to pay  $v(q^o) < D \leq L$ , as otherwise they get negative surplus. However, this violates bilateral efficiency since  $q^o < q^*$  but  $v(q^o) < L$ . Therefore,  $q^o < q_L$  cannot be supported. A nonmonetary equilibrium cannot support  $q^o > q_H$ , as the seller would get a negative surplus. It cannot support  $q^o > q^*$ , as it violates Axiom 4. By the argument in the proof of Proposition 3,  $q^o > \min\{\hat{q}_c, q^*\}$  cannot be supported in monetary equilibrium. Next, consider  $D \geq u(q^*)$ . If  $q^o < q^*$  then  $v(q^o) = D$ . However, this yields a negative surplus for the buyer. If  $q^o = q^*$ , we can pick any  $v(q^*) \in [c(q^*), u(q^*)]$  and construct a

nonmonetary equilibrium as in the case of  $D < u(q^*)$ . ■

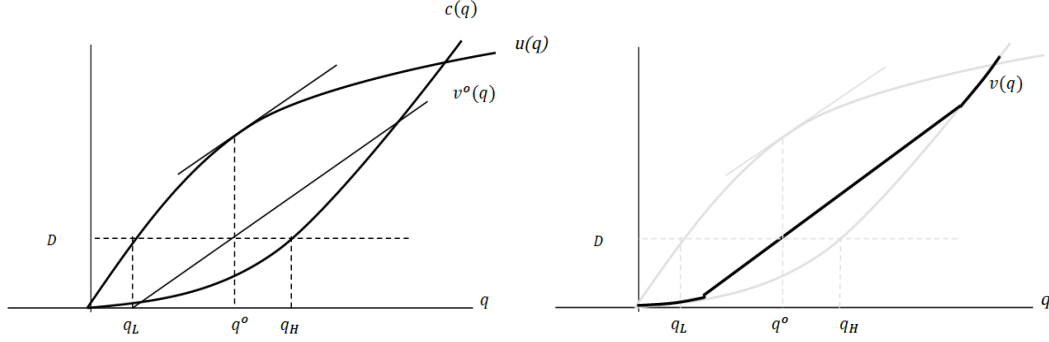


Figure 3: HKW Style Mechanism with Free Credit – Nonmonetary Equilibrium

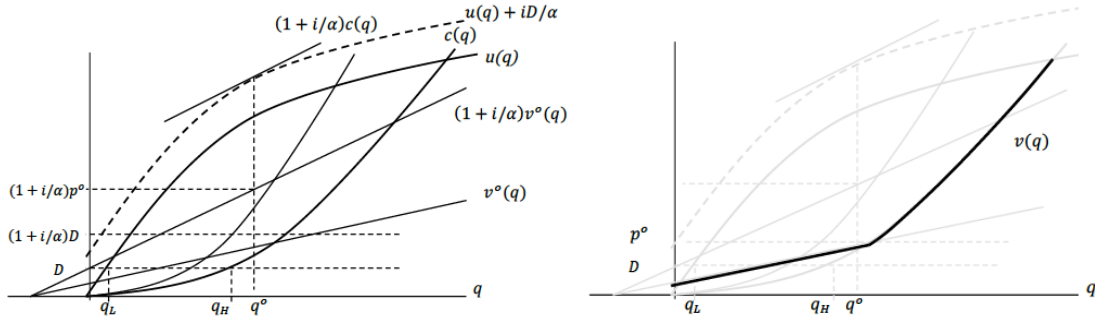


Figure 4: HKW Style Mechanism with Free Credit – Monetary Equilibrium

**Proof of Proposition 5:** First consider  $z < p^*$ . We show  $(p, q) \in \mathcal{C}$  by contradiction. Suppose  $(p, q) \notin \mathcal{C}$ . Now, suppose  $u'(q) > \{1 + \gamma'[S_s + c(q) - z]\} c'(q)$ . Let  $q' = q + \varepsilon_q$  and  $p' = p + \varepsilon_p$ , where  $\varepsilon_p = c(q') - c(q)$ . Then  $S_s$  does not change, while  $S_b$  increases, contradicting Axiom 4. Similarly, if  $u'(q) < \{1 + \gamma'[S_s + c(q) - z]\} c'(q)$  then  $p' = p - \varepsilon_p$  and  $q' = q - \varepsilon_q$ , with  $\varepsilon_p = c(q) - c(q')$ , improves  $S_b$  and leaves  $S_s$  unchanged, contradicting Axiom 4. We now show  $p > z$  if  $z < p^*$ . Suppose  $p \leq z$ . Then  $\gamma = 0$ , and  $(p, q) \in \mathcal{C}$  iff  $u'(q) = c'(q)$ , which implies  $q = q^*$ . By Axiom 3 and the definition of  $p^*$ , we must have  $p = p^*$ , which is a contradiction.

Next consider  $z \geq p^*$ . As in the proof of Proposition 2, one can show  $q = \Gamma_q(z) < q^*$  violates Axiom 4. Suppose  $q > q^*$ . Let  $p' = p - \varepsilon_p$  and  $q' = q - \varepsilon_q > q^*$ ,

where  $\varepsilon_p = u(q) - u(q')$ . One can check  $S_b$  weakly increases, as buyers save on costly credit, while  $S_s$  increases. This contradicts Axiom 4. Hence,  $z \geq p^*$  implies  $q = q^*$ , and  $p = p^*$  by the definition of  $p^*$  and Axiom 3. This implies  $v^{-1}(p^*) = q^*$ . By Axiom 3,  $v$  is strictly increasing. By Axiom 2,  $v(0) = 0$ . ■

**Proof of Proposition 6:** Define the set of incentive feasible trades  $\mathcal{I}$ . For  $S_b$ ,  $S_s \geq 0$  this requires

$$u(q) - \gamma \circ g(q) - p \geq 0 \quad (11)$$

$$p - c(q) \geq 0 \quad (12)$$

Any outcome in the core must satisfy  $p \geq g(q)$ . As usual,  $q \leq q^*$ , and therefore

$$\mathcal{I} \equiv \{(p, q) \mid (p, q) \text{ satisfies (11)-(12), } p \geq g(q) \text{ and } q \leq q^*\}.$$

which implies an ex post feasible  $q$  is in  $[q_L^c, q^*]$ .

The buyer's CM objective function is  $\tilde{J}(q; i) = h(q) - (1 + i/\alpha)v(q)$ . Note that as we do not require  $h$  to be concave,  $Q$  is not necessarily convex. We look for  $v(q)$  such that the distance between  $h$  and  $(1 + i/\alpha)v(q)$  is maximized at some  $q^o$ .

In Figure 5, the area surrounded by dashed curves  $(1 + i/\alpha)(u - \gamma)$ ,  $(1 + i/\alpha)c$ ,  $(1 + i/\alpha)g$  and  $q = q^*$  is the set of  $(1 + i/\alpha)v(q)$  that is ex post feasible. Pick  $(1 + i/\alpha)v(q^o)$  in the area surrounded by the dashed curves. Consider first non-monetary equilibrium, with payment  $p^o = g(q^o)$ . Draw a line with slope  $\varepsilon > 0$  that goes through  $(1 + i/\alpha)g(q^o)$ , labeled  $(1 + i/\alpha)v^o$ . Denote the intersection of  $h$  and  $(1 + i/\alpha)v^o$  by  $q^2$ . The slope  $\varepsilon$  is chosen to be small so that the distance between the line and  $h$  is maximized on  $[q^2, q^o]$ . This can be done as  $h$  is strictly increasing for small  $i$ . Denote the intersection of  $h$  and  $(1 + i/\alpha)c$  by  $q^1$ . The constructed  $(1 + i/\alpha)v$  is as follows.

$$(1 + i/\alpha)v = \begin{cases} (1 + i/\alpha)c & \text{if } q \leq q^1 \\ (1 + i/\alpha)v^o & \text{if } q^2 < q \leq q^o \\ h & \text{if } q > q^o \text{ and } \max\{q^1, 0\} < q \leq q^2 \end{cases} \quad (13)$$

Now  $v$  is strictly increasing and  $h - (1 + i/\alpha)v$  is maximized at  $q^o$ . Ex ante, buyers will not bring  $z > 0$  as  $p^o = g(q^o)$  does not require cash.

Next, consider monetary equilibrium. For  $q^o \in Q \cap [q_L^c, q^*]$ , pick  $p^o > g(q^o)$  in  $\mathcal{I}$ . Draw a line with slope  $\varepsilon > 0$  through  $(1 + i/\alpha)p^o$ , labeled  $(1 + i/\alpha)v^o$ . Denote the intersection of  $h$  and  $(1 + i/\alpha)v^o$  by  $q^2$ . Let  $(1 + i/\alpha)v$  take the form in (13). Now,  $v$  is strictly increasing and  $h - (1 + i/\alpha)v$  is maximized at  $q^o$ . See Figure 6. For  $q^o \notin Q$ , any incentive compatible  $p^o$  results in  $z = 0$ . If  $q^o \in [q_L^c, q_H^c]$ , a nonmonetary equilibrium can support  $q^o$ , but monetary equilibrium is not feasible. For  $q^o \notin [q_L^c, q^*]$ , the allocation is also not feasible. ■

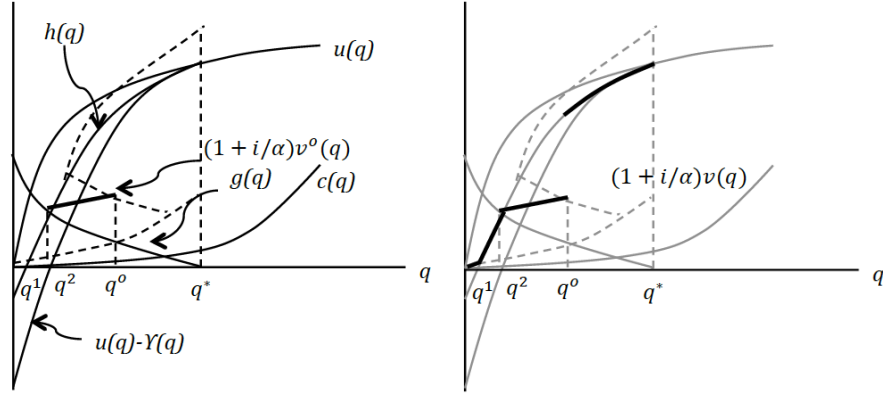


Figure 5: HKW Style Mechanism with Costly Credit – Nonmonetary Equilibrium

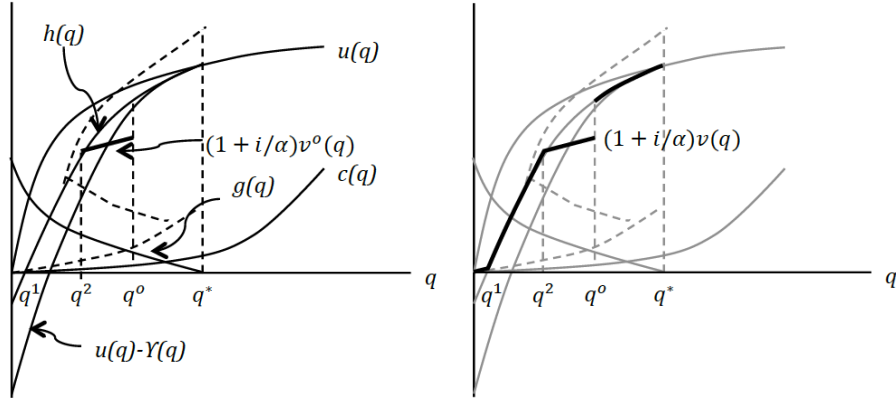


Figure 6: HKW Style Mechanism with Costly Credit – Monetary Equilibrium

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