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# Designing radio-mobile access networks based on synchronous digital hierarchy rings

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**Abstract**

In this paper, we address the SDH network design problem (SDHNDP) which arises while designing the fixed part of global system for mobile communications access networks using synchronous digital hierarchy (SDH) rings.

An SDH ring is a simple cycle that physically links a subset of antennae to a single concentrator. Inside a ring, a concentrator handles the total traffic induced by antennae. Technological considerations limit the number of antennae and the total length of a ring.

The SDHNDP is a new problem. It belongs to a class of location-routing problems that introduce location into the multi-depot vehicle routing problem. In this paper, we precisely describe the SDHNDP and propose a mixed integer programming-based model for it. Furthermore, we devise a heuristic algorithm that computes a feasible solution.

We report the results of our computational experiments using the CPLEX software, on instances comprising up to 70 antennae or six concentrator sites. An analysis provides insight into the behavior of the lower bound obtained by the LP relaxation of the model, in response to the network density. This lower bound can be improved by adding some valid inequalities. We show that an interesting cut can be obtained by approximating the minimum number of rings in any feasible solution. This can be achieved by solving a "minimum capacitated partition problem". Finally, we compare the lower bound to the heuristic solution value for a set of instances.


*Keywords:* SDH/SONET network design; Location routing; Minimum capacitated partition problem; Mixed integer programming

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## 1. Introduction

Progress in telecommunication technology results in improved services, but can also lead to difficult and challenging design problems. For example, networks in which nodes are connected by synchronous digital hierarchy (SDH) rings offer high capacities and can be used to provide a rapid service restoration in the case of a failure. As a result, network designers are faced with a new problem of designing networks based on ring structures.

In this paper, we address the SDH network design problem (SDHNDP) which arises while designing the fixed part of the GSM access network into SDH rings. For a detailed presentation of the SDH technology, see [1], and of the GSM access networks, see [2]. The SDHNDP is a large and complex combinatorial problem with several optimization levels. The interests are not only theoretical, but also economical and technological.

In Section 2, we describe the SDHNDP, its data and its constraints as well as the objective function to be optimized. We also give an overview of some studies dealing with a class of similar location-routing problems. In Section 3, we give the notations used to define the SDHNDP.

In Section 4, we propose a mixed integer programming-based model. The main source of difficulty comes from the number and the type of constraints in the SDHNDP. Another difficulty arises with the elimination of sub-tours from feasible solutions. Different formulations can be used to overcome this difficulty. The formulation discussed in this paper uses a polynomial number of variables and constraints. The continuous relaxation of the proposed model can be improved significantly for some instances by adding a few valid inequalities. An interesting cut is based on the calculation of a trivial lower bound of the number of rings in a feasible solution of the SDHNDP. In Section 5, we improve this lower bound by solving a "minimum capacitated partition problem".

In Section 6, we present our heuristic solution method. Section 7 is devoted to computational experiments. We present our results for a set of real instances having up to 70 antennae or six concentrator sites. An analysis provides insight to the behavior of the model, specially the LP relaxation in response to the network density. We show that the continuous relaxation of the proposed model with the additional cuts often gives a good lower bound for the SDHNDP, except for some instances corresponding to sparse networks.

## 2. General context and description of the SDHNDP

Today, the GSM access network has a tree topology and is composed of two parts: radio and switching subnetworks (see Fig. 1). The radio subnetwork is composed of antennae called base transceiver stations (BTS) and concentrators known as base station controllers (BSC). The switching part contains the mobile switching centers (MSC).

Each mobile station communicates with a BTS and generates traffic that flows through the GSM access network from the BTS to the related BSC and, as the case may be, to the related MSC. The total mobile traffic induced by the BTSs must satisfy capacity constraints and it flows through the GSM access network.

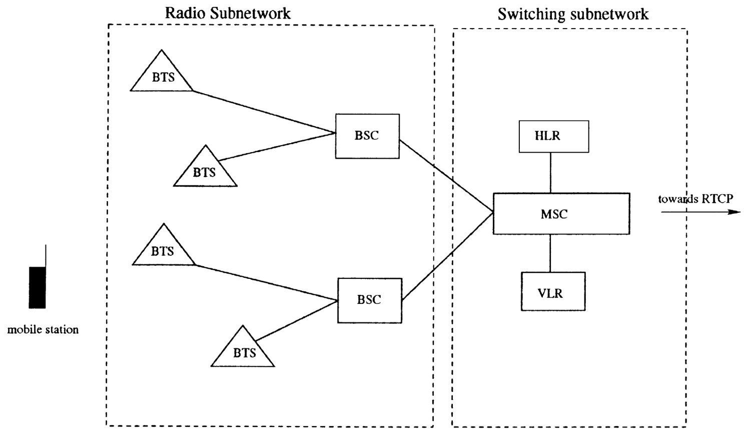


Fig. 1. GSM access network with a tree topology.

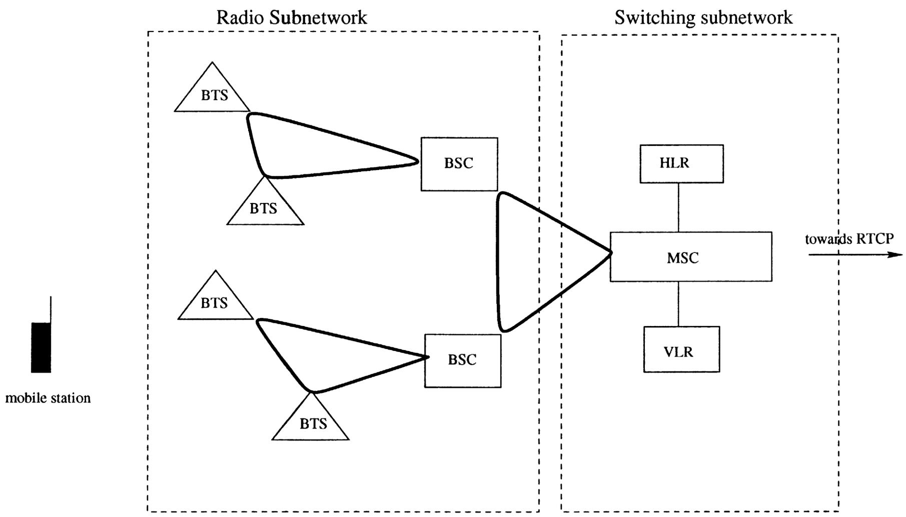


Fig. 2. GSM access network with ring structures.

For the future GSM access network, the use of SDH rings offers better network reliability and greater mobile traffic capacities. As shown in Fig. 2, there are two types of rings:

- BTS-BSC ring, a simple cycle which links a set of BTSs and a BSC, within the radio subnetwork.
- BSC-MSC ring, a simple cycle which connects a subset of BSCs to an MSC. These rings link the radio subnetwork to the switching subnetwork.

The mobile traffic from a BTS (respectively, a BSC) to a BSC (respectively, an MSC) flows through the ring connecting these two nodes. In this paper, we will focus on the design problem inside the radio subnetwork. This problem includes the location of BSCs and the *topological* design

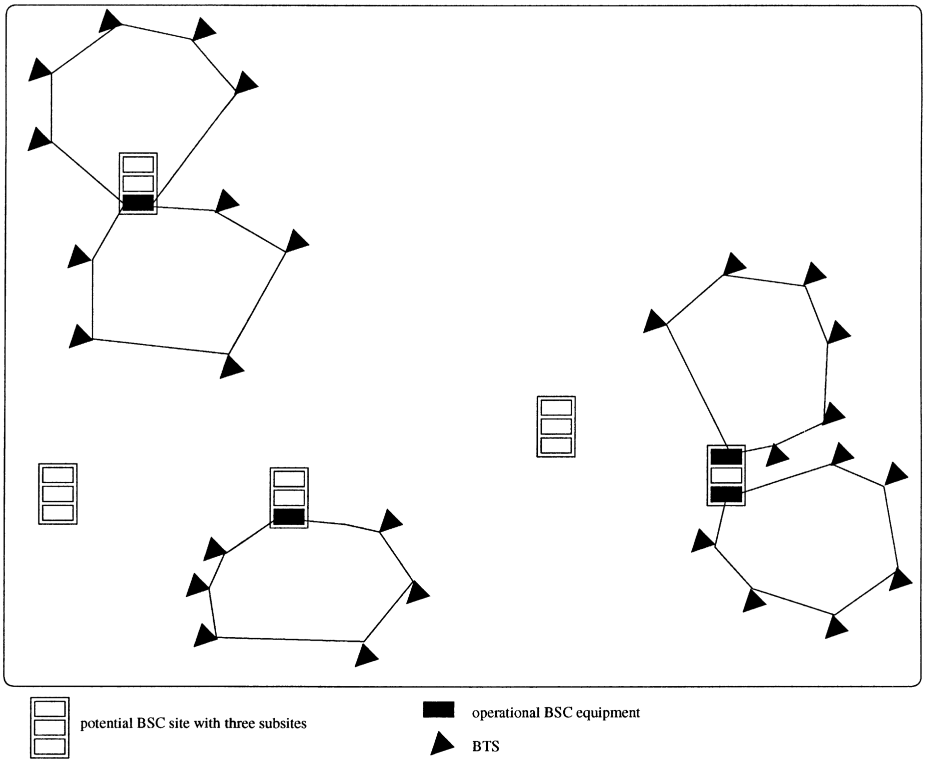


Fig. 3. Description of the SDHNDP.

of the BTS-BSC rings. As defined in [3], the topological design indicates where links are present, i.e. which node pairs are directly connected with fiber.

In the SDHNDP, BTS locations and traffic demands are known but, for the BSCs, only the sites where the BSCs can be placed and the list of available BSC types are given. The capacity of a BSC, i.e. the amount of traffic it can handle, depends on its type. A site can receive up to three BSCs and is opened if at least one BSC is placed on it (see Fig. 3).

The SDHNDP consists in finding the number and the type of the BSCs to locate at each site on the one hand and, on the other hand, in defining SDH rings such that each BTS is in exactly one ring (see Fig. 3). Technological considerations limit the number of BTSs and the total length of a ring. The capacity of a ring in only limited by the characteristics of the BSC handling that ring. The objective is to minimize the total costs including the site opening costs, the BSC costs and the SDH ring costs. Each SDH ring generates fixed establishment costs and variable link costs per unit of link length.

Several studies are interested in the ring network design problem, with different considerations. Many authors (see [3,4]) consider the additional problem of the *physical* design of the rings, once their topology has been decided. As defined in [3], the physical design problem is to determine the number of stacked rings in each topological ring, as well as the nodes and demands served by each of the stacked ring. The physical rings, called self-healing rings, are rings with redundant capacity and automatic protection, which will handle a reliable traffic inside the topological ring.

Another aspect of those problems is to optimize the hierarchical ring problem, i.e. in our context, the problem obtained by including the BSC-MSC rings in the design problem. In [3,4], a single ring connects the nodes of a backbone network, and a collection of rings connects the access nodes to the backbone. Another problem where the backbone is a ring is described in [5,6], where the access nodes are linked by a tree rather than a ring. Moreover, like in the SDHNDP, the location of equipments of different types have to be decided in their problem.

The SDHNDP belongs to a class of location-routing problems that introduce location into the multi-depot vehicle routing problem (MDVRP). For a review, see [7,8]. In [7], the authors propose integer programming-based modeling for some location-routing problems and a branch and cut algorithm similar to the one they propose for the vehicle routing problem. In [8], Laporte et al. examine a class of asymmetrical MDVRPs and location-routing problems, under capacity or maximum cost restrictions. They propose a graph representation and extension in order to transform these problems into equivalent constrained assignment problems. Instances involving up to 80 nodes are solved using a branch and bound algorithm. The location-routing problems considered in [7,8] are less complex than the SDHNDP. In fact, the costs optimized in these location-routing problems include only depot opening costs, vehicle costs and variable transportation costs. In addition, the capacity or cost constraints are related to each vehicle.

The SDHNDP is similar to the "modified warehouse location-routing problem" (MWLRP), in which the objective is to locate the warehouses and to design a set of vehicle tours so that all customers are served and the total costs are minimized (see [9,10]). Each vehicle corresponds, in the SDHNDP context, to an SDH ring and a warehouse corresponds to a BSC. The MWLRP tries to minimize the depot variable costs in addition to the costs considered in the classical location-routing problems. The depot variable costs represent depot consumption costs and are different from BSC costs which, instead, are modular costs. The main difference between the two problems is that the number and the capacity of the BSCs within a site have to be determined in the SDHNDP, whereas in the MWLRP, exactly one warehouse of fixed capacity is located on an open site. In addition, the MWLRP considers depot capacity, but the SDHNDP considers BSC capacity. The MWLRP was introduced for the first time by Perl in 1983 (see [11]). In [9], Perl and Daskin propose a mixed integer programming based-model and develop a three-phase decomposition method to solve the MWLRP. This heuristic was improved by Hansen et al. [10] and, more recently, by Wu et al. [12].

## 3. Notations and definitions

In order to define, the SDHNDP we use the following notation:

| | |
|---|---|
| $N$ | number of BTSs |
| $M$ | number of potential BSC sites |
| $N_s$ | maximal number of BSCs on a BSC site |
| $N_\omega$ | number of available BSC types |

$I = \{1, \dots, N\}$ is the subset of indexes of BTSs, where $BTS_i$ represents a BTS site indexed $i \in I$.
$J = \{N+1, \dots, N+M\}$ is the subset of indexes of BSC sites, where $BSC_j$ represents a BSC site indexed $j \in J$.

$d_{gh}$ is the Euclidean distance between the BTS or BSC sites $g$ and $h$.

Since up to $N_s$ BSCs can be placed in a BSC site, we split each BSC site into $N_s$ BSC subsites, each one being able to receive at most one BSC equipment. Hence the $s$th subsite of a site $BSC_j$ is represented by two indices $j$ and $s$. So, let $S = \{1, \ldots, N_s\}$ be the subset of indexes of the BSC subsites located at each BSC site.

Let $q_i = (IT_i, Erlang_i, NBTS_i, TRX_i)$ be the vector characterizing a $BTS_i$. This vector is composed of the mobile traffic demand and other technical parameters described below:

| | |
|---|---|
| $IT_i$ | traffic volume ($IT$) |
| $Erlang_i$ | traffic intensity ($Erlang$) |
| $NBTS_i$ | number of antennae |
| $TRX_i$ | number of TRX terminals |

Let $\Omega = \{1, \ldots, N_\omega\}$ be the subset of the available BSC types, where $TBSC_\omega$ denotes a BSC equipment-type indexed $\omega \in \Omega$; and let $Q_\omega = (IT_\omega, Erlang_\omega, NBTS_\omega, TRX_\omega)$ be the technical capacities vector of a BSC equipment of type $TBSC_\omega$, where

| | |
|---|---|
| $IT_\omega$ | maximal traffic volume ($IT$) |
| $Erlang_\omega$ | maximal traffic intensity ($Erlang$) |
| $NBTS_\omega$ | maximal number of antennae handled by this BSC |
| $TRX_\omega$ | maximal number of TRX terminals handled by this BSC |

Each BSC can handle a limited mobile traffic volume (measured in $IT$) and intensity (measured in $Erlang$). Also, the number of antennae and TRX terminals handled by a BSC are limited.

Each SDH ring must respect the following restrictions:

| | |
|---|---|
| $N_{\max}$ | maximal number of BTSs on a ring ($N_{\max} = 14$) |
| $N_{\min}$ | minimal number of BTSs on a ring ($N_{\min} = 2$) |
| $D_{\max}$ | maximal length of a ring ($D_{\max} = 100$ km) |

The objective is to minimize the network costs, where

| | |
|---|---|
| $F_j$ | the opening cost of site $BSC_j$ |
| $M_\omega$ | the acquisition cost of a BSC equipment of type $TBSC_\omega$ |
| $CU$ | the cost per unit of ring length |
| $CF$ | the fixed cost per ring |

## 4. A mixed integer programming model for the SDHNDP

Let $G = (V, A)$ be a simple symmetric and directed graph. The set of nodes $V$ is composed of the BTS sites and the BSC sites. The set of arcs $A$ is deduced from the complete digraph by dropping:

- arcs between two BSC sites (these arcs are never included into a ring),
- arcs $(g, h)$ satisfying $d_{gh} + \min_{j \in J}(d_{jg} + d_{hj}) > D_{\max}$. Indeed, these arcs are too long to be included into a ring with a total length of at most $D_{\max}$.

The problem consists in finding some elementary cycles of $G$ such that each BTS belongs to one and only one cycle, and each cycle contains one and only one BSC site. Our formulation considers the following 0–1 variables. For each arc $(g,h) \in A$, we define

$$v_{gh} = \begin{cases} 1 & \text{if arc } (g,h) \text{ is selected (i.e. included into a ring),} \\ 0 & \text{otherwise.} \end{cases}$$

Note that, as $G$ is symmetric, a distinct variable $v_{hg}$, associated to arc $(h,g)$, is also considered. For each $j \in J$, $s \in S$ and $\omega \in \Omega$

$$u_{js\omega} = \begin{cases} 1 & \text{if the } s\text{th subsite of } BSC_j \text{ contains a BSC equipment of type } TBSC_\omega, \\ 0 & \text{otherwise.} \end{cases}$$

Decision variables $v_{gh}$ and $u_{js\omega}$ completely define a feasible solution of the SDHNDP. But, they are not sufficient to formulate this problem by an integer linear program. For this, we use the following additional variables:

$$z_j = \begin{cases} 1 & \text{if the potential site } BSC_j \text{ is open (i.e. at least one of its subsites contains a} \\ & \text{BSC equipment),} \\ 0 & \text{otherwise,} \end{cases}$$

$$t_{ijs} = \begin{cases} 1 & \text{if } BTS_i \text{ is assigned to the subsite } s \text{ of } BSC_j, \\ 0 & \text{otherwise,} \end{cases}$$

$$l_{gh} = \begin{cases} \text{length of the ring portion from one BSC site to node } g \text{ if arc } (g,h) \text{ is selected,} \\ 0 \quad \text{otherwise,} \end{cases}$$

$$n_i = \left\{ \text{number of BTSs visited before } BTS_i \text{ by the ring containing } BTS_i. \right.$$

Length variables $l_{gh}$ are introduced to express the ring length limitation. To each selected arc $(g,h)$ corresponds only one ring which passes through this arc and only one BSC subsite that belongs to this ring. Variable $l_{gh}$ represents the length of the portion of this ring from the BSC subsite to node $g$. For each $BTS_i$, we define the variable $n_i$ in order to express the ring cardinality restriction.

The SDHNDP can be formulated as follows:

$$\text{minimize} \quad \sum_{j\in J} F_j z_j + \sum_{j\in J}\sum_{s\in S}\sum_{\omega\in\Omega} M_\omega u_{js\omega} + CU \sum_{(g,h)\in A} d_{gh} v_{gh} + CF \sum_{j\in J}\sum_{i\in I} v_{ji} \tag{1}$$

subject to

$$\sum_{h:(h,i)\in A} v_{hi} = 1, \quad i \in I, \tag{2}$$

$$\sum_{h:(i,h)\in A} v_{ih} = 1, \quad i \in I, \tag{3}$$

$$\sum_{s\in S} t_{ijs} \geqslant (v_{ji} + v_{ij}), \quad i\in I,\ j\in J, \tag{4}$$

$$1 + t_{i_2js} \geqslant t_{i_1js} + (v_{i_1i_2} + v_{i_2i_1}), \quad i_1\in I,\ i_2\in I\ :\ (i_1 \neq i_2),\ j\in J,\ s\in S, \tag{5}$$

$$\sum_{j\in J}\sum_{s\in S} t_{ijs} = 1, \quad i\in I, \tag{6}$$

$$\sum_{i\in I} q_i t_{ijs} \leqslant \sum_{\omega\in\Omega} Q_\omega u_{js\omega}, \quad j\in J,\ s\in S, \tag{7}$$

$$n_i \leqslant (N_{\max} - 1)\left(1 - \sum_{j\in J} v_{ji}\right), \quad i\in I, \tag{8}$$

$$n_{i_1} - n_{i_2} + 1 \leqslant N_{\max}(1 - v_{i_1i_2}), \quad i_1\in I,\ i_2\in I(i_1 \neq i_2), \tag{9}$$

$$l_{ji} = 0, \quad j\in J,\ i\in I, \tag{10}$$

$$l_{ih} \leqslant \left(D_{\max} - d_{ih} - \min_{j\in J} d_{hj}\right) v_{ih}, \quad i\in I,\ h\in V(i\neq h), \tag{11}$$

$$\sum_{h\in V} (l_{hi} + d_{hi}v_{hi}) = \sum_{h\in V} l_{ih}, \quad i\in I, \tag{12}$$

$$\sum_{\omega\in\Omega} u_{js\omega} \leqslant z_j, \quad j\in J,\ s\in S, \tag{13}$$

$$\sum_{\omega\in\Omega} u_{js\omega} \geqslant t_{ijs}, \quad i\in I,\ j\in J,\ s\in S, \tag{14}$$

$$\sum_{j\in J, i\in I} v_{ji} \geqslant \left\lceil \frac{N}{N_{\max}} \right\rceil, \tag{15}$$

$$v_{gh}\in\{0,1\}, \quad l_{gh} \geqslant 0, \quad (g,h)\in A, \tag{16}$$

$$u_{js\omega}\in\{0,1\}, \quad j\in J,\ s\in S,\ \omega\in\Omega, \tag{17}$$

$$z_j\in\{0,1\}, \quad j\in J, \tag{18}$$

$$t_{ijs}\in\{0,1\}, \quad i\in I,\ j\in J,\ s\in S, \tag{19}$$

$$n_i \geqslant 0, \quad i\in I. \tag{20}$$

Objective (1) is to minimize the network costs. The site opening costs are equal to $\sum_{j\in J} F_j z_j$. The BSC acquisition costs are equal to $\sum_{j\in J}\sum_{s\in S}\sum_{\omega\in\Omega} M_\omega u_{js\omega}$. The variable ring costs are linear in function of the total ring length $\sum_{(g,h)\in A} d_{gh}v_{gh}$. Finally, the fixed costs are equal to $CF \sum_{j\in J}\sum_{i\in I} v_{ji}$ since $\sum_{j\in J}\sum_{i\in I} v_{ji}$ gives the number of rings.

Constraints (2) (resp. (3)) impose that exactly one selected arc arrives (resp. starts) at each BTS, i.e. each BTS is visited by exactly one ring. Constraints (4) mean that if $BTS_i$ is linked to $BSC_j$ by a selected arc, then $BTS_i$ is assigned to at least one of the subsites of $BSC_j$. Constraints (5) oblige $BTS_{i_1}$ and $BTS_{i_2}$ to be assigned to the same subsite if arc $(i_1, i_2)$ is selected. Constraints (6) mean that $BTS_i$ is assigned to exactly one subsite. Constraints (4)–(6) together oblige a ring to start and terminate at the same subsite and all the BTSs of a ring to be assigned to the same subsite. They also discard rings having only one BTS site since constraints (4) and (6) imply $v_{ij} + v_{ji} \leqslant 1$. Constraints (7) satisfy BSC capacities at each subsite.

Values of variables $n_i$ define a ranking of the BTS sites within the same ring. The first BTS site visited by a ring is ranked 0. This is ensured by constraints (8). The following BTSs in the same ring are ranked $1, 2, \ldots,$ etc. For this, constraints (9) guarantee that if arc $(i_1, i_2)$ is selected then $n_{i_2} \geqslant n_{i_1} + 1$. Constraints (8) also prevent the rank values from being larger than $(N_{\max} - 1)$. Hence, in a feasible solution, the cardinality of a ring is at most $N_{\max}$. Variables $n_i$ have an additional utility. They discard solutions having subtours of BTS sites, in the same way as in the formulation by Miller et al. [13] for the traveling salesman problem.

Constraints (10)–(12) concern variables $l_{gh}$ and aim to satisfy the length restrictions on the rings. Recall that, if arc $(g,h)$ belongs to a ring (i.e. is selected), $l_{gh}$ is the length of the ring portion from the BSC site of the ring to vertex $g$. Hence, $l_{gh} = 0$ if $g$ corresponds to a BSC site or if arc $(g,h)$ is not selected. This is imposed by constraints (10) and (11). Moreover, constraints (12) give the relation between two consecutive selected arcs. Indeed, by constraints (2) and (3), exactly one selected arc $(h_1, i)$ arrives at node $i$ and one selected arc $(i, h_2)$ starts at node $i$. Then constraints (12) set $l_{ih_2} = l_{h_1 i} + d_{h_1 i}$. Finally, the ring passing through arc $(i,h)$ has a length of at least $l_{ih} + d_{ih} + \min_{j\in J} d_{hj}$. Thus, constraints (11), together with the others, represent necessary and sufficient conditions to restrict the total length of any feasible ring to $D_{\max}$.

Constraints (13) impose the opening of a BSC site if one of the corresponding subsites is opened.

Constraints (14) and (15) are redundant constraints whose objective is to get a better LP relaxation of SDHNDP. Constraints (14) state that if $BTS_i$ is assigned to subsite $s$ of $BSC_j$ then that subsite contains an equipment. Constraints (15) say that any feasible solution contains at least $\lceil N/N_{\max}\rceil$ rings. This is trivial due to the restriction on the number of BTSs on a ring.

Constraints (16)–(20) represent domain restrictions.

Note that the MIP (1)–(20) admits multiple equivalent solutions. This is for example due the fact that rings are not actually directed, and that $N_s$ identical subsites are associated with each site. These equivalent solutions could partially explain the difficulties in solving the MIP problem. We tried to add some kind of constraints to discard some of these equivalent solutions. However, our model does not include these constraints because a short experiment led us to conclude it was not efficient. This does not exclude the possibility of finding a better technique to break the symmetry of the problem.

We show in the following section how to compute a lower bound $\beta$ on the number of rings in any feasible solution, which is at least as good as $\lceil N/N_{\max}\rceil$. The computational experiments show that, when $\lceil N/N_{\max}\rceil$ is replaced by $\beta$ in constraints (15), this leads to a better solution for problem SDHNDP.

## 5. Improvement of the model by computing a better lower bound on the number of rings

In this section, we propose a new lower bound $\beta$ that improves the lower bound $\lceil N/N_{\max}\rceil$, on the number of rings in a feasible solution of the SDHNDP. This lower bound will be used to tighten constraints (15). Recall that $G=(V,A)$ is the graph obtained after a few preprocessing rules are applied as mentioned in the beginning of Section 4. Let $\Delta=(V,E)$ be the nondirected graph obtained from $G$ by replacing every arc by an edge and discarding multiple edges. Given an integer $K$, we call $P_K$ the problem of partitioning graph $\Delta$ into $K$ subsets subject to constraints that we will make precise below. We define $\beta$ as the smallest $K$ such that problem $P_K$ is feasible.

Problem $P_K$ is the problem of partitioning the nodes set $V$ of graph $\Delta$ into $K$ subsets, and selecting some edges from $E$ in such a way that

- each BTS belongs to exactly one subset,
- every subset induces a clique in graph $\Delta$,
- the number of BTSs within a subset $\in[N_{\min},N_{\max}]$,
- each subset contains exactly one BSC site,
- each node within a subset has two adjacent selected edges in this subset,
- the total length of the selected edges within a subset is lower than $D_{\max}$,
- the total length of the selected edges is minimized. With this objective function, one also obtain a lower bound on the optimal value of SDHNDP (see formulations (21)–(32) and related comments).

Note that, in $P_K$, subtours within a subset are allowed, and the BSC capacities are supposed to be unlimited. From a feasible solution to SDHNDP containing $v$ rings, one can easily deduce a solution to $P_v$ by a one-to-one correspondence between rings and the subsets of the partition. Hence, if $P_K$ is not feasible for a given $K$ then $(K+1)$ is a lower bound on the number of rings in a feasible solution to SDHNDP. From our definition of $\beta$, it is clear that $\beta\geqslant\lceil N/N_{\max}\rceil$.

Moreover, let $\Delta'$ be the subgraph of $\Delta$ induced by the subset $I$ of BTS sites. Recall that, in a graph, a stable set is a set of nodes such that no pair of nodes is linked by an edge, and the stability number is the maximal cardinality of a stable set. If $\mathcal{S}\subset I$ is a stable set for graph $\Delta'$, then the nodes of $\mathcal{S}$ must belong to different subsets in any partition of graph $\Delta$. Thus $\beta\geqslant\alpha(\Delta')$, where $\alpha(\Delta')$ is the stability number of graph $\Delta'$.

We perform the following algorithm to compute $\beta$:

*Step* 1: **Init**. $K=\max(\lceil N/N_{\max}\rceil,\alpha(\Delta'))$.
*Step* 2: Solve $P_K$.
*Step* 3: *If* ($P_K$ is feasible) *then* $\{\beta=K$; **STOP**.$\}$ *else* $\{K=K+1$; *goto* Step 2$\}$.

In Step 1, one needs to compute $\alpha(\Delta')$. For this, we use the classical formulation for the maximal cardinality stable set problem by a 0–1 linear problem and use an MIP solver to find a stable set $\mathcal{S}$ such that $|\mathcal{S}|=\alpha(\Delta')$.

In Step 2, problem $P_K$ is solved. We formulate it as a 0–1 linear problem. Our formulation uses the stable set $\mathcal{S}$ found in Step 1. Let $\mathcal{S}=\{b_1,b_2,\ldots,b_{|\mathcal{S}|}\}$. By Step 1, $K\geqslant|\mathcal{S}|$.

We consider the decision variables $x_{gk}=1$ iff node $g$ is in subset $k$, and $y_{ghk}=1$ iff edge $(g,h)$ is selected in subset $k$. Then we formulate problem $P_K$ as follows:

$$(\mathrm{P}_K) \quad \text{minimize} \quad \sum_{k\in\{1,\dots,K\}} \sum_{(g,h)\in E \;\; (g<h)} d_{gh}\, y_{ghk} \tag{21}$$

$$\text{subject to} \quad \sum_{k\in\{1,\dots,K\}} x_{ik} = 1, \quad i\in I, \tag{22}$$

$$\sum_{j\in J} x_{jk} = 1, \quad k\in\{1,\dots,K\}, \tag{23}$$

$$\sum_{i\in I} x_{ik} \leqslant N_{\max}, \quad k\in\{1,\dots,K\}, \tag{24}$$

$$\sum_{i\in I} x_{ik} \geqslant N_{\min}, \quad k\in\{1,\dots,K\}, \tag{25}$$

$$x_{gk} + x_{hk} \leqslant 1, \quad (g,h)\notin E,\ \ k\in\{1,\dots,K\}, \tag{26}$$

$$\sum_{h\in V(g<h)} y_{ghk} + \sum_{h\in V(h<g)} y_{hgk} = 2x_{gk}, \quad g\in V,\ \ k\in\{1,\dots,K\}, \tag{27}$$

$$\sum_{(g,h)\in E \;\; (g<h)} d_{gh}\, y_{ghk} \leqslant D_{\max}, \quad k\in\{1,\dots,K\}, \tag{28}$$

$$\sum_{k\in\{1,\dots,K\}} y_{ghk} \leqslant 1, \quad (g,h)\in E\ \ (g<h), \tag{29}$$

$$x_{b_k k} = 1, \quad k\in\{1,\dots,|\mathscr{S}|\}, \tag{30}$$

$$y_{ghk}\in\{0,1\}, \quad (g,h)\in E,\ \ k\in\{1,\dots,K\}, \tag{31}$$

$$x_{gk}\in\{0,1\}, \quad g\in V,\ \ k\in\{1,\dots,K\}. \tag{32}$$

Constraints (22) assign each BTS to exactly one subset. Constraints (23) impose the assignment of one BSC site to each subset. Constraints (24) and (25) limit the cardinality of each subset. Constraints (26) avoid the assignment of a pair of nodes to a given subset if there is no edge between the two nodes in $E$. Constraints (27) represent degree constraints. If node $g$ is in subset $k$ then exactly two different arcs, incident to $g$, must be selected in subset $k$. Otherwise no arc incident to $g$ can be selected in subset $k$.

The total length of the edges selected within each subset is limited to $D_{\max}$ by constraints (28). Constraints (29) are additional constraints introduced to tighten the LP relaxation of $P_K$. Constraints (30) assign $BTS_{b_k}$ of the stable set $\mathscr{S}$ to subset $k$. This guarantees the assignment of all the nodes in $\mathscr{S}$ to different subsets, and since any numbering of the subsets is valid, this variable fixing does not discard feasible solutions of $P_K$. Once these variables are definitely fixed, the subsets become less interchangeable, and the MIP resolution of problem $P_K$ should be faster. Finally, objective function

(21) minimizes the total length of the selected edges. Observe that, if $K$ is the smallest value such that problem $P_K$ is feasible, and $v(P_K)$ is the optimal value of $P_K$, then $v(P_K) + K \times CF$ is a lower bound of the optimal value of the SDHNDP. We do not use this lower bound below, but we think it could be interesting.

## 6. A heuristic solution method for the SDHNDP

The heuristic method that we developed is composed of three phases. Again, we use graph $G = (V, A)$ described in the beginning of Section 4. The edges are assigned values equal to the distance between the vertices they join.

*Phase* 1: The goal of this phase is to connect each BTS to a BSC site by means of a tree. The trees thus obtained must be partial subgraphs of $G$ and each of these trees comprises a set of BTS and only one BSC site. The partial subgraph of $G$ composed by these trees is called a forest spanning the BTSs. One seeks a spanning forest of minimal weight, i.e. that minimizes the sum of the lengths of the edges that it uses.

To determine a forest of minimal weight, we use the following algorithm:

*Step* 1: Add to the graph $G$ edges of null weight, linking all pairs of BSC sites. Let $G'$ be the obtained graph.

*Step* 2: Use Prim's algorithm to find a minimum spanning tree for graph $G'$. Let $A'$ be the obtained tree.

*Step* 3: Remove from $A'$ the edges linking two BSC sites. One then obtains a spanning forest. One can easily show that this forest is of minimal weight and that the weight of this forest is a lower bound to the total length of the rings in any feasible solution of SDHNDP.

In fact, we replace Step 2 of the above algorithm by a heuristic. In order to take into account the constraints of SDHNDP, we adapt Prim's algorithm of Step 2 in such a way that:

- the total demand of the set of BTSs connected to a BSC site by a tree does not exceed the maximum capacity of a BSC site, i.e. the capacity which one obtains by equipping its $N_s$ subsites by an equipment of maximum capacity;
- a BTS is never connected to a tree whose BSC (each tree comprises one and only one BSC) is at a distance greater than $\alpha D_{\max}/2$ of this BTS ($0 < \alpha \leqslant 1$ is a parameter of the heuristic). In this way, one expects to avoid obtaining a forest whose number of vertices in each tree is too much unbalanced.

Note that once Step 2 is modified, the obtained forest no longer has optimality properties.

*Phase* 2: The spanning forest obtained at the end of Phase 1 defines an assignment of each BTS to a BSC site. Indeed, each BTS belongs to one and only one tree and each tree contains one and only one BSC site. One thus decides to open these BSC sites and to close those which do not belong to the tree. Then one considers the trees one by one, and forms SDH rings which connect the BTS to the BSC sites. These rings are formed by solving a vehicle routing problem with only one deposit, under the following constraints: a ring contains at most $N_{\max}$ BTSs and its length is lower than or equal to $D_{\max}$. In this problem, the objective function to be minimized is equal to the sum of the costs associated, on the one hand, with the number of rings and, on the other hand, with the total length of these rings. For each tree, we solve the problem by a heuristic method which is an adaptation of that of Clarke and Wright [14]. Observe that, at the end of Phase 1, the total demand of BTSs assigned to a BSC does not exceed $N_s$ times the maximal capacity of a BSC equipment. This enables us to forget the capacity restrictions in Phase 2.

*Phase* 3: At the end of Phase 2, one has the SDH rings connecting the BTSs and, for each ring, the corresponding BSC site. It remains to decide how to equip the subsites of the open BSC sites. Hence it is necessary to decide, for each subsite, whether it is equipped or not and, if yes, which type of equipment is to be used for this subsite. In fact, for each open BSC site, one has to solve a simple capacitated facility location problem where the clients are the rings assigned to that BSC site, and the potential depots are the $N_s$ subsites of that BSC site. We solve these problems using a mixed integer programming solver.

## 7. Computational experiments

We consider real instances containing up to 70 BTS, six BSC sites, and two types of BSC equipment. For all these instances, the fixed cost per ring is $CF = 100\,000$ (monetary units). This is also the maximal cost due to the length of a given ring since $D_{\max} = 100\,000$ (length units) and $CU = 1$ (monetary unit per length unit). Recall that the number of BTSs per ring varies between $N_{\max} = 14$ and $N_{\min} = 2$. Table 1 gives some additional characteristics of the considered instances. $N$ is the number of BTS, $M$ is the number of BSC sites, $N_s$ is the number of subsites per site and $N_\omega$ is the number of available equipment types. The last column gives the density of graph $G$. Recall that $G = (V, A)$ is the symmetric digraph where $V$ is the set of all the BTS and all the BSC sites and $A$ is obtained from the complete graph by removing "impossible" arcs (see Section 4). Density is therefore equal to $|A|/((N + M)(N + M - 1))$. Instances pb1–pb5 correspond to networks of high density (occurring in urban area for example) and instances pb6–pb10 correspond to networks with a lower density. In the later case, distances between BTSs and from BTSs to BSC sites are larger, hence we discard a larger number of impossible arcs.

Table 1
The main characteristics of our test instances

| pb | $N$ | $M$ | $N_s$ | $N_\omega$ | Density (%) |
|---|---|---|---|---|---|
| pb1 | 20 | 3 | 3 | 2 | 99 |
| pb2 | 30 | 3 | 3 | 2 | 99 |
| pb3 | 40 | 6 | 3 | 2 | 99 |
| pb4 | 50 | 6 | 3 | 2 | 99 |
| pb5 | 60 | 6 | 3 | 2 | 99 |
| pb6 | 30 | 4 | 3 | 2 | 50 |
| pb7 | 40 | 4 | 3 | 2 | 49 |
| pb8 | 50 | 5 | 3 | 2 | 48 |
| pb9 | 60 | 5 | 3 | 2 | 48 |
| pb10 | 70 | 5 | 3 | 2 | 50 |

Table 2
Results for the instances of Table 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| pb | $s^*$ | $\bar{s}$ | $\frac{\bar{s}-s^*}{s^*}$ (%) | $b_0$ | $\frac{s^*-b_0}{s^*}$ (%) | $b_1$ | $\frac{s^*-b_1}{s^*}$ (%) | $b_2$ | $\frac{s^*-b_2}{s^*}$ (%) | $\frac{\bar{s}-b_1}{\bar{s}}$ (%) |
| pb1 | 217723 | 217723 | 0 | 213884 | 2 | 213884 | 2 | 215475 | 1 | 2 |
| pb2 | 331858 | 331858 | 0 | 320671 | 3 | 320671 | 3 | 321502 | 3 | 3 |
| pb3 | 340947 | 441898 | 30 | 323646 | 5 | 323646 | 5 | 323646 | 5 | 27 |
| pb4 | 457892 | 544827 | 19 | 430293 | 6 | 430293 | 6 | 430293 | 6 | 21 |
| pb5 | 548303 | 548303 | 0 | 534667 | 2 | 534667 | 2 | 534667 | 2 | 2 |
| pb6 | 934635 | 934703 | 0 | 737818 | 21 | 896013 | 4 | 908821 | 3 | 4 |
| pb7 | 997237 | 1108545 | 11 | 840098 | 16 | 932949 | 6 | 944339 | 5 | 16 |
| pb8 | 1344134 | 1447388 | 8 | 1078821 | 20 | 1155491 | 14 | 1157487 | 14 | 20 |
| pb9 | 1491262 | 1505517 | 1 | 1185502 | 21 | 1208283 | 19 | 1209294 | 19 | 20 |
| pb10 | 1661551 | 1679399 | 1 | 1224937 | 26 | 1228846 | 26 | 1230096 | 26 | 27 |
| Ave. | | | 7 | | 12 | | 9 | | 8 | 14 |

We use a PC with 384 MB of RAM and a 400 MHz PentiumII processor. Further, we use C++ and CPLEX 7.0 [15]. Table 2 gives the results of our heuristic solution method and of different ways of computing lower bounds:

- $s^*$ is the cost of the best-known solution, i.e. the best of the two following solutions: the one computed by our heuristic method described in Section 6, and the one obtained by running the MIP solver of CPLEX for 4 h,
- $\bar{s}$ is the cost of the solution computed by our heuristic method, described in Section 6, with
$$\alpha = \frac{1}{2} + \frac{1}{D_{\max}} \max_{i \in I} \min_{j \in J} d_{ij},$$
- $b_0$ is the lower bound computed by LP-relaxation of the MIP formulation of the SDHNDP with $\beta = \lceil N/N_{\max} \rceil$,
- $b_1$ is the lower bound computed by LP-relaxation of the MIP formulation of the SDHNDP, where $\beta$ is the lower bound on the number of rings in any feasible solution, computed by our algorithm of Section 5,
- $b_2$ is the improvement of $b_1$ obtained by CPLEX (MIP) after 2 h of CPU time. Hence $b_0 \leqslant b_1 \leqslant b_2$.

Column 4 (resp. 6, 8, and 10) gives the relative error of $\bar{s}$ (resp. $b_0$, $b_1$, and $b_2$) compared to the value of the best-known solution $s^*$. Column 11 gives the relative error between $b_1$ and $\bar{s}$ that is the approximation we get if we limit the computation effort to a relatively small time. Indeed, the computation of $\bar{s}$ by the heuristic algorithm takes less than 10 min of CPU time for each of the instances and the computation of $b_1$, by solving a linear program takes from 1 s to 22 min for instances corresponding to high meshed networks and from 2 s to 2 min for the other instances.

If we compare Columns 6 and 8, we can evaluate the contribution of the model improvement method which is based on the calculation of the tightest lower bound $\beta$ of the number of rings (cf. Section 5):

- For instances corresponding to high meshed networks (pb1–pb5), the improvement method has no contribution. This is due to the fact that the lower bound $\beta$ is not greater than $\lceil N/N_{\max} \rceil$ for these instances. Moreover, $\beta$ is reached for all these instances because it equals the number of rings found in the best-known solutions.
- But for instances corresponding to lower density networks (pb6–pb10) the contribution of the improvement method is considerable on the average error. In fact, we pass from an average error before improvement ($(s^* - b_0)/s^*$) equal to 21% to an average error after improvement ($(s^* - b_1)/s^*$) equal to 12%. Besides, the lower bound $\beta$ of the number of rings obtained by the improvement method is strictly greater than $\lceil N/N_{\max} \rceil$ for each of these instances. On the other side, this lower bound equals the number of rings found in the best known solutions for only instances pb6 and pb7. To carry on solving the model using CPLEX during 2 h does not allow the lower bound to be significantly improved.

## 8. Conclusion

In this paper, we address a new industrial optimization problem which arises while designing the fixed part of the radio-mobile access networks with the SDH technology. This optimization problem is complex and includes well-known combinatorial optimization subproblems: vehicles routing problems, discrete location, and sizing problems. Our approach consists in modeling the overall problem as a mixed integer linear programming model. This model is concise, but contains a large number of variables and constraints and cannot be used to exactly solve real instances. However, it allows us to compute a lower bound of each solution value. We improve this lower bound by refining some of the model constraints. Moreover, we develop a greedy heuristic method to obtain a feasible solution. Our results show that the average gap between the lower bound and the feasible solution obtained by our heuristic is equal to 15% for all the considered instances.

This study constitutes the first step to resolve this new problem. In order to try to better solve it, a future research could consist in testing a cutting plane method based on known cuts used for close problems, as subtours elimination constraints.

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