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Dynamic Location-routeing Problems

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Dynamic location-routeing problems involve the determination of the least-cost sequence of depot, vehicle fleet and route configurations in a distribution system, over a given planning horizon. This paper presents two solution approaches to such problems. The first is an exact method which is appropriate for small-scale problems. It consists of representing the problem by a suitable network and of solving to optimality an integer linear programme associated with the network. In the second approach, some of the system costs are approximated, and a global solution is then obtained by determining a shortest path on a directed graph. Under some hypotheses, this approach is suitable for large-scale problems. It is illustrated on a simple example.

Key words: distribution systems, dynamic location-routeing problems, facility location, vehicle routeing

INTRODUCTION

The design of distribution systems frequently involves locating facilities among candidate sites and determining delivery or collection routes from these facilities to customers or users. When customer demand or spatial distribution changes significantly over time, location and routeing decisions may have to be updated periodically, resulting in facility opening and closing costs. Dynamic location-routeing problems (DLRPs) involve the determination of a least-cost sequence of depot, vehicle fleet and route configurations over a planning horizon of τ time periods. This paper presents two solution approaches to such problems. It is restricted to cases in which there is a single product and a single transportation mode.

It is convenient to represent a distribution system by a multi-layer diagram such as the one depicted in Figure 1. Here, there are three layers, representing primary facilities, secondary facilities and users. Primary facilities often correspond to production sites; secondary facilities may be depots, distribution centres, warehouses or transfer points; and users usually are customers. In this paper, the terms 'secondary facility' and 'depot' are used interchangeably. Goods usually flow from primary facilities to secondary facilities to users in the case of deliveries, or in the opposite direction in the case of collections. Here we assume vehicles make deliveries. Some authors (see, for example, Mercer *et al.*[1] and Dejax[2]) distinguish between warehouses and depots, and consider four layers instead of three, while others (for example, Watson-Gandy and Dohrn[3]) allow the possibility of making trips directly from layer 1 to layer 3. We shall exclude this case, and restrict our attention to the classical case of three-layer configurations in which

(i) primary facility locations are fixed for the duration of the planning horizon, and the number of vehicles based at each of these facilities is also fixed;
(ii) secondary facility locations and the size of vehicle fleets have to be determined periodically and may change over time; and
(iii) the number and location of users gradually change over time, although such movements are often hard to predict with any great degree of precision.

Our results extend easily to cases where the number of layers is different from three.

Several distribution rules can be defined for a λ-layer system according to whether vehicles make return trips (R) (or direct shipments[4]) between layer l and layer $l + 1$ ($l = 1, \ldots, \lambda - 1$) or whether they make tours (T) (or peddling[4]); i.e. each of their trips may include several points as opposed to only one in the first case. Thus, in a three-layer system, there are four possibilities, described by (R, R), (R, T), (T, R) and (T, T). Here the lth component of each vector represents the distribution rule for trips between layer l and layer $l + 1$. Real-life examples and references for each of these cases are reported in Laporte.[5] These cover situations as diverse as distribution in the food and drink industry,[1,3] collection of raw latex in Malaysia,[6] post-box location,[7] newspaper delivery,[8] location of NATO aircraft bases,[9] etc.

Consider, for the time being, the static case, i.e. problems from which the time dimension is absent ($\tau = 1$). If all locations are predetermined, problems (R, T), (T, R) and (T, T) are vehicle-

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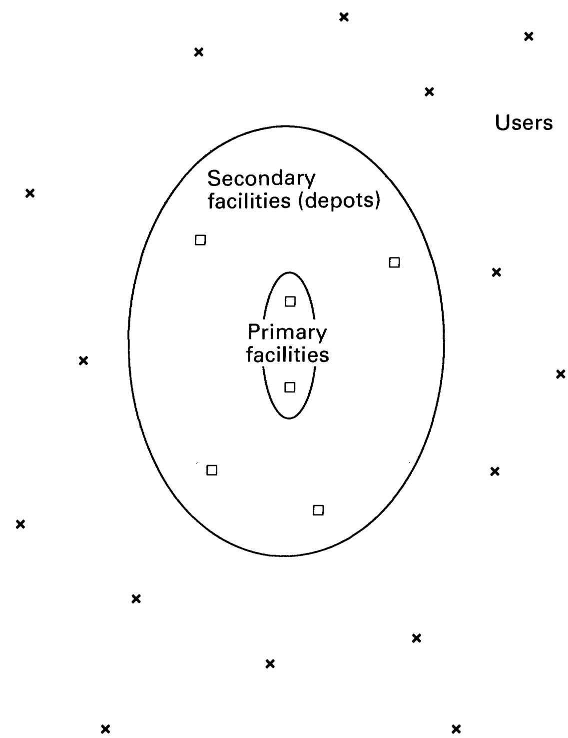

FIG. 1. *A three-layer diagram for a distribution system.*

routeing problems (VRPs), about which an extensive literature exists (see Laporte and Nobert[10] for single-depot VRPs and Laporte *et al.*[11] for multi-depot VRPs). The case (*R*, *R*) is simpler in that it corresponds to a trans-shipment problem and can therefore easily be handled by means of a standard network-flow algorithm. When locational decisions have to be made, the three cases (*R*, *T*), (*T*, *R*) and (*T*, *T*) define location-routeing problems (LRPs) (see Laporte[5]), whereas the case (*R*, *R*) belongs to the families of simple plant-location problems (see Krarup and Pruzan[12] and Aikens[13]) and of location distribution problems (see Bookbinder and Reece,[14] Crainic *et al.*[15] and Dejax[2]).

The dynamic case ($\tau > 1$) has been less widely studied. Psaraftis[16] examines dynamic routeing problems in which vehicle routes must be constructed when user locations are gradually defined over time (e.g. as in taxi or ambulance operations). The hierarchical vehicle-routeing problem described by Marchetti Spaccamela *et al.*[17] concerns vehicle acquisition at time $t = 0$, given that the spatial distribution of users may vary in subsequent periods. Erlenkotter[18] provides a survey of dynamic location-distribution problems belonging to the (*R*, *R*) category. These problems have also been studied by Van Roy and Erlenkotter,[19] Khumawala *et al.*[20] and Zinelabdine and Dejax.[21] However, we are not aware of any study of dynamic problems combining locational decisions and tours. This paper attempts to fill this gap.

In the next section, we develop a general network representation for such problems, and suggest a mathematical programming formulation as well as an optimal branch-and-bound algorithm. In the following section, we introduce an approximation scheme for problems which cannot be tackled by an exact algorithm. A numerical example for the latter scheme is then presented, and the conclusion follows.

AN EXACT NETWORK-BASED ALGORITHM

The model developed in this section is based upon the following idea: represent the τ-period DLRP by means of a suitable directed network constructed in such a way that the optimal solution to the problem defined on the network corresponds to a number of Hamiltonian circuits. These Hamiltonian circuits can then be interpreted as a solution to the DLRP. In order for the network representation to be valid, its arcs must adequately describe the various physical and

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logical connections associated with the problem and must bear the appropriate costs. The advantage of these representations is that least-cost Hamiltonian circuits on directed networks can be determined for problems of relatively large sizes (see, for example, Carpaneto and Toth[22] or Balas and Christofides[23]).

The idea of using such transformations can be traced back to the work of Eilon *et al.*:[24] through an appropriate graph extension, these authors showed how the m-travelling salesman problem (m-TSP) over n cities could be solved as a single TSP over $n + m - 1$ cities. This work was later improved by Lenstra and Rinnooy Kan[25] and extended to the case of the capacity constrained VRP[26] and of the multi-depot VRP.[5,11] We shall now describe the network representation with a minimum of formal notation.

The extended network (see Figure 4) contains, *for every time-period t*, a node for every user, a number of nodes for every depot (two plus the maximum number of vehicles based at the depot) and a number of nodes for each primary facility (one plus the maximum number of vehicles based at that facility). In addition, nodes are required to represent the initial depot locations ($t = 0$) and, if applicable, the target depot configuration ($t = \tau + 1$). The arcs of the network belong to three categories:

(i) logical arcs illustrating the sequences of depot openings and closures;
(ii) physical arcs corresponding to vehicle movements; and
(iii) dummy arcs introduced to complete Hamiltonian circuits.

We shall concentrate on logical arcs first. Figure 2 depicts a logical network for a three-period, three-depot problem, with a maximum of two vehicles based at each depot. The last two nodes of

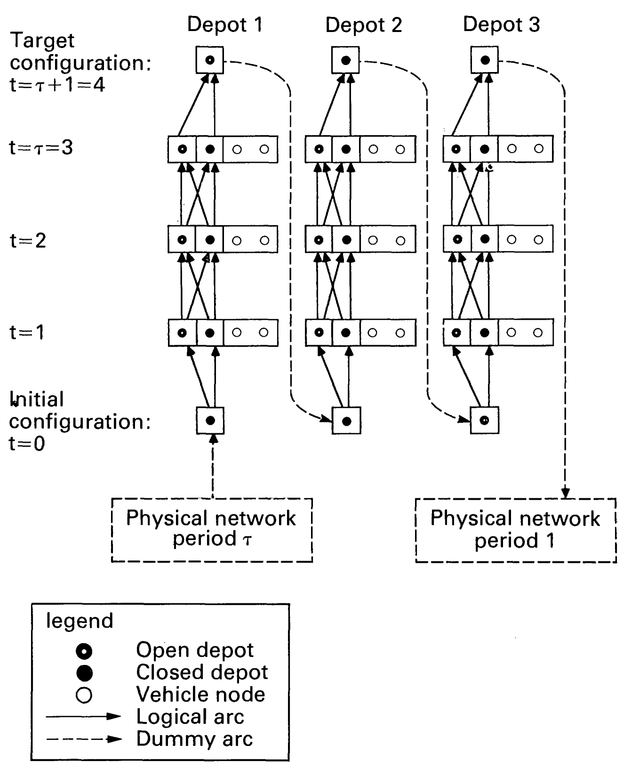

FIG. 2. *Logical network.*

each depot at each time-period are 'vehicle nodes': each corresponds to the end of a vehicle trip, and they are not required at this stage. The remaining nodes correspond to an 'open depot' (black and white nodes) or to a 'closed depot' (black nodes). The cost associated with a given combination of 'open' and 'closed' depots can be written on the appropriate arc. [*Note*: All costs mentioned in this paper are discounted to the beginning of the planning horizon. Variable and

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fixed costs are scaled so that they refer to the same time-period.] Define:

r_{it}: the cost of opening depot i at the beginning of period t ($t = 1, \ldots, \tau + 1$); $r_{i,\tau+1} = 0$ if there is no target configuration;

s_{it}: the cost of closing depot i at the beginning of period t (or at the end of period $t - 1$) ($t = 1, \ldots, \tau + 1$); $s_{i,\tau+1} = 0$ if there is no target configuration;

g_{it}: the fixed cost of operating depot i during period t ($t = 0, \ldots, \tau$ and $g_{i0} = 0$).

Let i be a depot node (open or closed) at the beginning of period t, and j a node (open or closed) corresponding to the same depot at the beginning of period $t + 1$. Then a transition cost c_{ij} is defined according to one of the four possibilities shown in Table 1.

TABLE 1. *Transition costs for four combinations of depot states*

State of depot at the beginning of period t	State of depot at the beginning of period $t + 1$	Transition cost c_{ij}
open	open	g_{it}
open	closed	$g_{it} + s_{i,t+1}$
closed	open	$r_{i,t+1}$
closed	closed	0

The dummy arcs to and from the physical network and those joining the nodes of the final and initial configurations have no particular meaning and have zero costs. The optimal Hamiltonian path on the logical network provides the optimal depot opening and closing sequence.

We now describe the physical network. In Figure 3, we have represented a feasible solution on the physical network for one period and two primary facilities with two vehicles each. This network is similar to that used in static LRPs,[5] except that now, there is an additional node per depot, indicating whether it is open or not. Note that the logical and physical networks share the

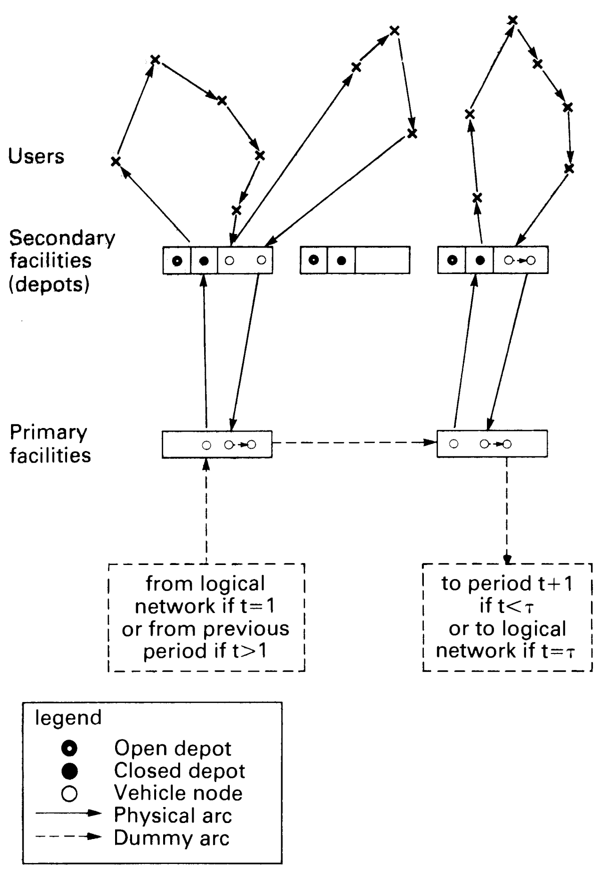

FIG. 3. *Physical network for period* t.

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depot nodes. User nodes are usually not the same at different time-periods. Here an arc leaving a primary or a secondary facility means that a vehicle is used. The appropriate vehicle fixed cost can be added to the travel cost of the arc. An arc linking two consecutive nodes within a primary facility or a depot means that the corresponding vehicle is not used. Such arcs have zero costs. At a given time-period t, arcs connect

(i) all primary facility nodes (except the last one) to the first *black* node of every depot;
(ii) the last node of every depot to the primary facility nodes (except the first of these nodes) and to the first black node of every other depot;
(iii) the last node of a primary facility to the first node of the next primary facility;
(iv) all depots and users whenever real direct connections exist.

An interesting fact about this representation is that primary facility vehicles enter open depots through a black node. This is so because a facility is open if and only if its black node is *not* used in the logical network, and, since we are seeking Hamiltonian circuits in the complete network (i.e. the union of the logical and physical networks), no node can be entered more than once.

The DLRP can now be formulated as an integer linear programme. Let (i, j) be an arc in the complete network, c_{ij} its cost, and x_{ij} a binary variable indicating whether it is used ($x_{ij} = 1$) or not ($x_{ij} = 0$) in the optimal solution. Moreover, since some nodes i may not be incident to any arc in the optimal solution, it is necessary to define for such nodes a variable x_{ii} equal to 0 *if the node is used* and to 1 if it is *not used*. In what follows, x_{ii} is assumed to take the value 0 whenever it has not been defined. Let N be the set of all nodes. The dynamic location-routeing problem can then be formulated as follows:

Maximize

$$\sum_{i \neq j} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{i \in N} x_{ij} = 1 \qquad (j \in N) \tag{2}$$

$$\sum_{j \in N} x_{ij} = 1 \qquad (i \in N) \tag{3}$$

$$\sum_{\substack{i, j \in S \\ i \neq j}} x_{ij} \leqslant |S| - 1 \qquad \text{(for all } S \subseteq N \text{ such that } |S| \geqslant 2 \text{ and subtours with node set } S \text{ are illegal)} \tag{4}$$

$$\sum_{k=1}^{p-1} x_{i_k i_{k+1}} \leqslant p - 2 \qquad [\text{for all illegal vehicle routes } (i_1, \ldots, i_p)] \tag{5}$$

$$x_{kk} = x_{uu} \qquad \text{(for all } k \text{ and } u \text{ such that } u \text{ corresponds to a white depot node, and } k \text{ to a vehicle node of the same depot)} \tag{6}$$

$$x_{ij} = 0, 1 \qquad (i, j \in N). \tag{7}$$

In this formulation, constraints (2) and (3) indicate that all nodes i which are visited ($x_{ii} = 0$) are entered and left exactly once; nodes i which are not visited ($x_{ii} = 1$) will have no incident arc. All dummy arcs belong to the solution, and their corresponding x_{ij} is fixed at 1. Any integer solution satisfying (2) and (3) will consist of a number of Hamiltonian circuits. Only the following circuits are allowed:

(i) those starting at a depot, going through some users and coming back to the same depot,
(ii) those including all remaining nodes i which must be visited.

All other subtours (having a node set S) are illegal. They are eliminated by imposing the classical TSP subtour-elimination constraints (4).[27] Constraints (5) prohibit illegal vehicle routes, i.e. routes which do not start and end at the same (primary or secondary) facility and routes which violate some side constraints (e.g. total customer demand exceeding vehicle capacity, excessive length or duration, etc.—for an overview of such constraints, see Laporte and Nobert[10]). In order to prevent illegal routes, at least one of their arcs must be eliminated; thus at most $p - 2$ arcs of an illegal chain linking p nodes may be preserved. Finally, constraints (6) are imposed to ensure that

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vehicle nodes of closed depots are not used in the solution. Note that the number of subtour-elimination constraints (4) and of illegal vehicle-route constraints (5) can be astronomical; however, as will be seen in the algorithm, these are initially relaxed and generated as they are found to be violated. In practice, only a very small proportion of these constraints are generated.

We have represented in Figure 4 a feasible solution to a two-period, two-depot problem on the complete network. The sequences of open and closed depots, vehicle movements and connections between the logical and physical networks are clearly identified. This solution consists of two Hamiltonian circuits. One of these circuits corresponds to the route of a vehicle based at depot 1, period 2. The second circuit contains all remaining nodes used in the solution.

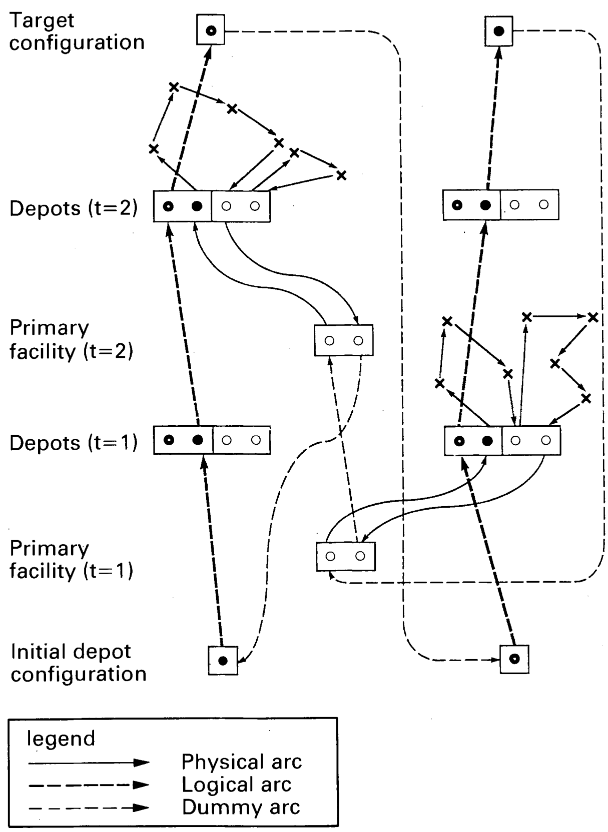

FIG. 4. *Feasible solution to a problem on the complete network. There are two periods, two depots and two vehicles per depot.*

When the number of arcs in the complete network is not too large and all problem parameters are known, it is possible to solve the problem to optimality by using the following relaxation approach.

Step 1

Solve the relaxed problem defined by (1), (2), (3) and (7). This is an easy-to-solve assignment problem. If none of the relaxed constraints is violated, the solution is optimal; stop. Otherwise, an enumerative process is initiated.

Step 2

Introduce the violated constraints (4), (5) or (6) by successively fixing at 0 or at 1 the appropriate variables in a branch-and-bound fashion (see Carpaneto and Toth[22]). All subproblems are

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restricted assignment problems, and thus integrality is preserved throughout the process. Stop when all non-dominated branches of the search tree have been explored.

Such an approach was used with success by Laporte *et al.*[11] to solve to optimality comparable problems (static LRPs) involving a total of up to 80 facilities and users. This would be the size, for example, of a three-period DLRP having one primary facility with one vehicle, two depots with two vehicles each and 15 users in each period.

AN APPROXIMATION SCHEME

The above exact approach is inadequate for problems involving too many nodes or when the location or other characteristics of users over the different time-periods cannot be known with any degree of precision. In such cases, it may be more appropriate to use an approximation scheme. We now describe such a method based for the most part on the work of Beardwood *et al.*[28] and of Haimovich and Rinnooy Kan.[29] Since this method requires, for each time-period, the complete enumeration of all non-empty subsets of the set J of secondary facilities, it will only be feasible if $\tau(2^{|J|} - 1)$ is not too large. It can be applied if the following hypotheses are satisfied:

(i) the user-space is bounded, convex and compact;
(ii) users are uniformly distributed in that space;
(iii) a Euclidean metric is used;
(iv) the number of users is sufficiently large.

The method consists of first approximating, for every non-empty subset S of J and for every period t, the overall distribution cost $z(S, t)$. Let $(S, 0)$ and $(S', \tau + 1)$ denote the initial and final open-depot configurations respectively [with $z(S, 0) = z(S', \tau + 1) = 0$], and let $c[(S, t), (S', t + 1)]$ be the value of $z(S, t)$ plus the transition cost from (S, t) to $(S', t + 1)$, $(S, S' \subseteq J;\ S, S' \neq \emptyset;\ t = 0, \ldots, \tau)$. The solution is then obtained by computing a least-cost path from $(S, 0)$ to $(S', \tau + 1)$. The procedure can be broken down into the following steps.

Repeat steps 1 to 3 for every pair (S, t), $(S \subseteq J;\ S \neq \emptyset;\ t = 1, \ldots, \tau)$

Step 1

Let S be the set of open depots at period t. Compute the distribution cost $z_1(S, t)$ between all primary facilities and the secondary facilities of S. Since $|S|$ is assumed to be small, this cost can be determined by means of an exact algorithm.[10]

Step 2

Define the catchment area of every facility of S as its *Voronoi area*,[30] i.e. the set of users closer to that facility than to any other facility.

Step 3

Now consider in turn the Voronoi areas defined in step 2. For given t and S, there are $|S|$ such areas, each containing an open depot near its centre. Partition each area according to the circular region partitioning scheme (CRP) described by Haimovich and Rinnooy Kan.[29] Variations of this scheme can also be used—see, for example, Daganzo and Newell[31] or Langevin and Soumis.[32] First draw h equally spaced radii around the depot, thus defining h 'sectors' intersecting with the catchment area. For situations where vehicles make at most q visits each, Haimovich and Rinnooy Kan define h as

$$h = \lceil (4\pi n \bar{r} / 3 q r_{\max})^{1/2} \rceil,$$

where

n is the number of users in the catchment area;
$\bar{r}$ is the average distance between users and the depot;
$r_{\max}$ is the maximum distance between a user and the depot;
$\lceil x \rceil$ is the smallest integer greater than or equal to x.

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Subdivide every sector of the catchment area into zones by drawing parallel arcs. If a given sector contains v zones, then the $v-1$ zones furthest away from the depot each correspond to the tightest vehicle route that can feasibly be covered (or covered with some probability $1-\alpha$) by a single vehicle, taking into account the various side constraints. The zones which are closest to the depot are then merged and repartitioned by radial cuts, so as to use the smallest possible number of vehicles (see Figure 5).

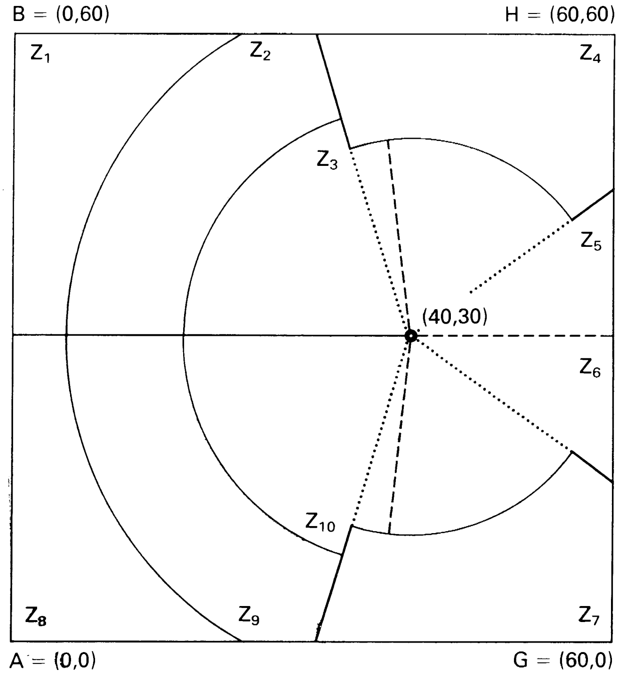

FIG. 5. *Circular region partitioning scheme in a square catchment area. The dotted lines correspond to the initial partitioning into 11 zones. The broken lines show the 10 zones obtained after repartitioning the central area.*

Define:

n_k: the number of users in zone Z_k;
A_k: the area of zone Z_k;
d_k: the length of the shortest return trip between the depot and the first user of zone Z_k.

The expected optimal route length in zone Z_k is then $T_k = \beta\sqrt{A_k n_k}$ (see Beardwood *et al.*[28]), where β is a constant. Stein[33] shows that β can be taken as 0.765 provided the zone satisfies hypotheses (i)–(iii) and $n_k \geqslant 15$. Other values for β can be derived under different hypotheses (see, for example, Daganzo[34]).

Using these results, the total distribution cost $z_2(S, t)$ between the depots and the users can now be approximated for every pair (S, t). To illustrate, assume $z_2(S, t)$ is a linear function of the number of vehicles and of distance, with fixed vehicle costs f_t and variable routeing costs c_t. If V_{it} is the number of zones (vehicles) associated with depot i, then

$$z_2(S, t) = \sum_{i \in S} \left[f_t V_{it} + \sum_{k=1}^{V_{it}} c_t(d_k + \beta\sqrt{A_k n_k}) \right]. \tag{8}$$

Alternative, more sophisticated distribution-cost formulas can of course be used.

Step 4

Construct a graph G in which the nodes correspond to the pairs (S, t) ($S \subseteq J$; $S \neq \emptyset$; $t = 0, \ldots, \tau+1$). For $t = 0, \ldots, \tau$, define an arc between every node (S, t) and every node $(S', t+1)$

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having a cost

$$c[(S, t), (S', t+1)] = z_1(S, t) + z_2(S, t) + \sum_{i \in S \cap S'} g_{it} + \sum_{i \in S \cap (J - S')} (g_{it} + s_{i,t+1}) + \sum_{i \in (J-S) \cap S'} r_{i,t+1}, \quad (9)$$

where $z_1(S, 0) = z_2(S, 0) = 0$ and $z_1(S, t) + z_2(S, t) = z(S, t)$.

Step 5

The solution is obtained by computing on G a least-cost path between the source $(S, 0)$ and the sink $(S', \tau + 1)$.

When the number of modes in G is not too large, this procedure constitutes a practical approach to the DLRP. The only term of (9) which represents an approximate cost is $z_2(S, t)$. The quality of the approximation depends on two key factors: (i) the partitioning rule and (ii) the formula used for the expected length of a single, optimal vehicle route. As far as the first factor is concerned, CRP schemes are shown by Haimovich and Rinnooy Kan[29] to be superior to a variety of other simple schemes, and to be asymptotically optimal when vehicles visit at most q users each and there is no upper limit on route lengths. The approximation formula, on the other hand, seems quite robust and reliable provided the zones each contain a sufficient number of uniformly distributed points; in practice, zones need only be 'fairly compact and fairly convex' (Larson and Odoni,[35] p. 408).

For problems in which, at each period, a small number of facilities have to be selected from a relatively large set, the number of candidate locations could be reduced as follows. Determine for each period t a set J_t of candidate facilities by applying a heuristic algorithm for the corresponding static location-routeing problem. Here, the Jacobsen and Madsen[8] ALA-SAV algorithm can be used (for a survey of alternative algorithms, see Laporte[5]). Then define the set J of potential depot sites at each period t as $J = \bigcup_{t=1}^{\tau} J_t$. Our approximate scheme could then be applied provided $\tau(2^{|J|} - 1)$ is not too large.

NUMERICAL EXAMPLE FOR THE APPROXIMATION SCHEME

We now illustrate the approximation scheme on a simple two-period problem with no primary facilities. In the first period, there are 400 users uniformly distributed in the rectangle $ABCD$, where $A = (0, 0)$, $B = (0, 60)$, $C = (100, 60)$ and $D = (100, 0)$. There are two potential depot locations at $E = (40, 30)$ and $F = (80, 30)$ (see Figure 6). In the second period, the user space is shifted 20 units to the right, corresponding to the rectangle $A'B'C'D'$, but the number of users and the potential depot locations remain the same. Vehicles can deliver to a maximum of 25 users each. Initially, only depot E is open, and the target configuration consists of depot F. The Voronoi areas of the two depots are determined by the user-space boundary and by the line segment GH.

In each period, travel distances must be estimated for each of the three subsets $S = \{E\}$, $S = \{F\}$ and $S = \{E, F\}$. In the first two cases, the catchment area of the selected depot is the whole user space; in the third case, it corresponds to the Voronoi area just defined. The CRP scheme is then applied to each of these areas separately, and the distances are approximated. For example, let $t = 1$, $S = \{E, F\}$, and consider the Voronoi area of E bounded by $ABHG$. It contains 240 users. Using the formula developed by Eilon *et al.*[24] (p. 154) for the expected Euclidean distance between a fixed point and a random point in a rectangle, we find $\bar{r} = 24.42$. We then compute $r_{max} = 50$ and $h = 5$. Eleven zones are first defined, and the five closest to the depot are then repartitioned into four (see Figure 5). The expected distances in each zone are reported in Table 2. The total expected distance travelled is equal to 1006.56, i.e. twice the sum of the d_k and T_k columns.

Repeating the same procedure for all pairs (S, t), we obtain the results reported in Table 3. The first line is computed from Table 2, and the last column is obtained by using formula (8), with $\beta = 0.765, f_t = 200$ and $c_t = 0.5$.

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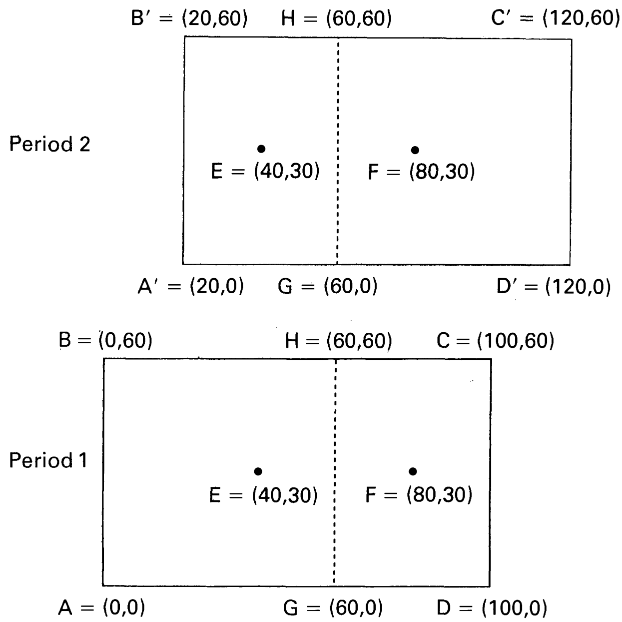

FIG. 6. *User areas for a two-period problem.*

TABLE 2. *Expected travel distances in a square catchment area partitioned into 10 zones*

Zones Z_k	d_k	n_k	A_k	T_k
Z_1, Z_8	65.86	25.0	375.0	74.07
Z_2, Z_9	43.96	25.0	375.0	74.07
Z_3, Z_{10}	0.00	22.5	337.5	66.66
Z_4, Z_7	37.92	25.0	375.0	74.07
Z_5, Z_6	0.00	22.5	337.5	66.66

TABLE 3. *Distribution costs for a two-depot, two-period problem*

Node (S, t) of G	Depot	Voronoi area	Number of vehicles V_{it}	Total distance	Total distribution cost $z_2(S, t)$
$(\{E, F\}, 1)$	E	$ABHG$	10	1006.56	4208.01
	F	$GHCD$	7	609.45	
$(\{E\}, 1)$	E	$ABCD$	16	1966.60	4183.30
$(\{F\}, 1)$	F	$ABCD$	16	2173.10	4286.55
$(\{E, F\}, 2)$	E	$A'B'HG$	7	609.45	4208.01
	F	$GHC'D'$	10	1006.56	
$(\{E\}, 2)$	E	$A'B'C'D'$	16	2173.10	4286.55
$(\{F\}, 2)$	F	$A'B'C'D'$	16	1966.60	4183.30

Assuming $g_{it} = r_{it} = s_{it} = 20$, for all admissible indices i and t, the shortest path on G is given by the sequence $(\{E\}, 0)$, $(\{E\}, 1)$, $(\{F\}, 2)$, $(\{F\}, 3)$ with a total cost of $0 + (4183.30 + 20 + 20 + 20) + (4183.30 + 20) = 8446.60$. In other words, the solution consists of selecting depot E in the first period and depot F in the second period.

CONCLUSION

We have defined in this paper an important class of distribution problems with location and routeing components and a time dimension. Such problems occur in a variety of contexts where customer spatial distribution and demand change significantly over time. The first solution

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approach presented is an exact algorithm based on a network representation of the problem. It can only be applied if the number of delivery points is relatively small and all problem data are known. The second approach is an approximation scheme which works well under some hypotheses and is relatively easy to implement. It combines early results on the asymptotic expected length of a travelling salesman tour and more recent ones on the partitioning of a catchment area into zones which can be served by one vehicle each. Both methods are general in the sense that they can easily take into account a wide variety of costs and side constraints.

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