

1. (20 points) Use this vocabulary to translate the following sentences into SL

- A. He will go fishing.
B. He will buy soda.
C. It will rain.

a) He won't go fishing, whether he buys soda, or not.

$\sim A$

B

$$(\sim A \ \& \ (B \vee \sim B))$$

b) He will go fishing and buy soda, unless it rains.

A

B

C

$$((A \ \& \ B) \ \& \ \sim C)$$

c) He will go fishing even though he will buy soda.

A

B

$$(A \ \& \ B)$$

2. (20 points) Fill in this truth table to tell whether these sentences are logically equivalent in SL:

A	B	$A \vee B$	$\sim B$	$\sim A$	$\sim B \supset \sim A$
T	T	T	F	F	T
T	F	T	T	F	T
F	T	T	F	T	F
F	F	F	T	T	T

$(A \vee B)$ is not logically equivalent to $(\sim B \supset \sim A)$

3. Are the sentences $A \vee B$ and $\sim B \supset \sim A$ in Question 2 logically equivalent in SL?

✓ no, these two sentences are not logically equivalent.

4. (40 points) Correctly complete these sentences:

Two sentences are *logically equivalent* in sentential logic when:

✓ Every row in the truth table evaluates to the same logical value for both sentences

A *connective* is:

✓ a word or words which modify the meaning of
-4 a sentence

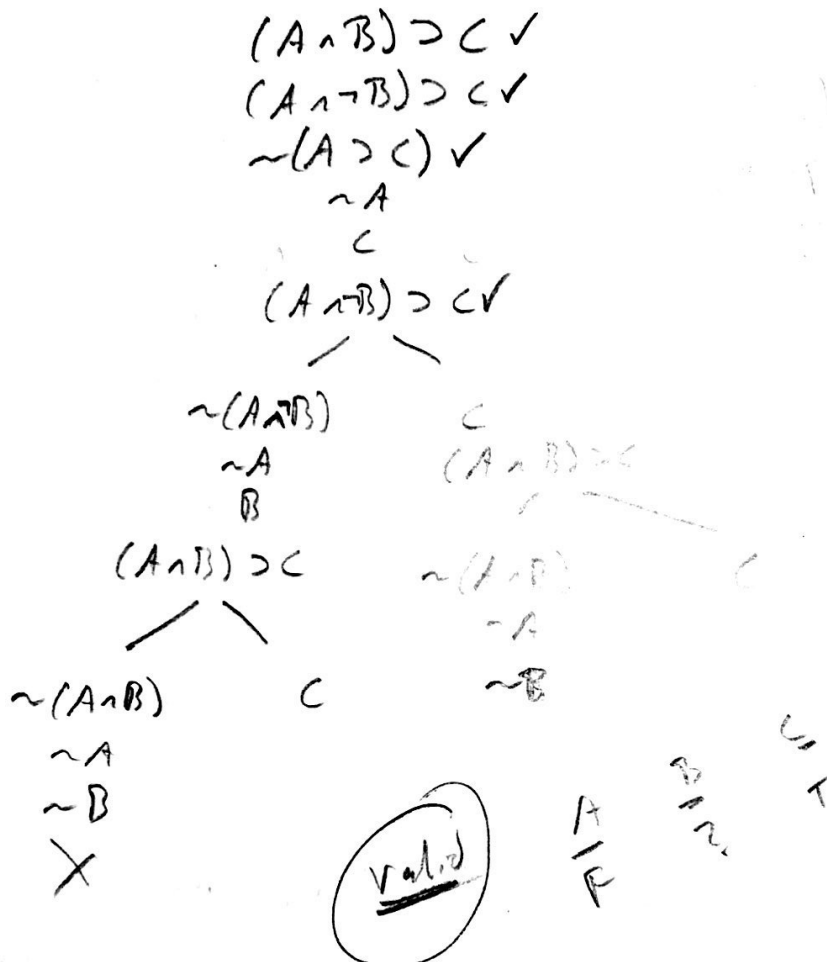
A connective is *truth functional* when:

✓ it modifies the truth value of the
-4 sentence it is connected to.

Use truth trees to tell whether these arguments are valid in SL. For each one that is not valid give a counterexample (you might want to do one on the back):

$$\begin{aligned} (A \wedge B) &\supset C \\ (A \wedge \neg B) &\supset C \\ \therefore A &\supset C \end{aligned}$$

$$\begin{aligned} A &\supset \neg C \\ C &\supset \neg(A \wedge D) \\ \therefore A &\equiv \neg C \end{aligned}$$



valid

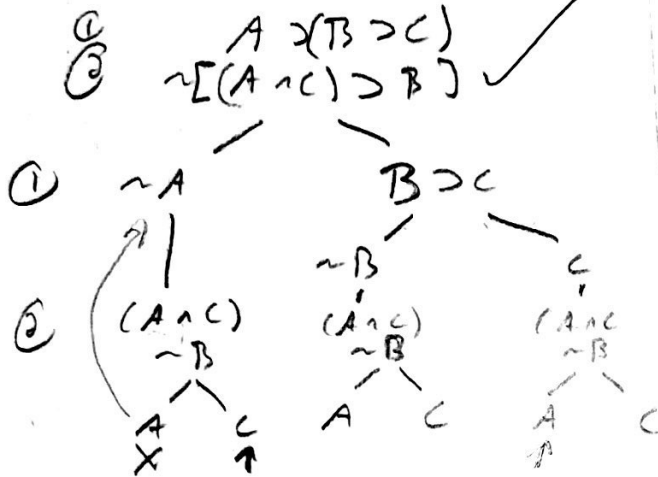
Many
Open
branches

on
tree,
the tree is wrong, though.

1. (30 points) Use truth trees to tell whether the lefthand pair of sentences are logically equivalent in SL, and do the same for the righthand pair. For each pair that is not logically equivalent give a counterexample:

$$A \supset (B \supset C)$$

$$(A \wedge C) \supset B$$



Counter Example

A	B	C
T	F	T

$$A \supset (B \supset C) = T \supset (F \supset T)$$

$$T \supset T$$

$$T$$

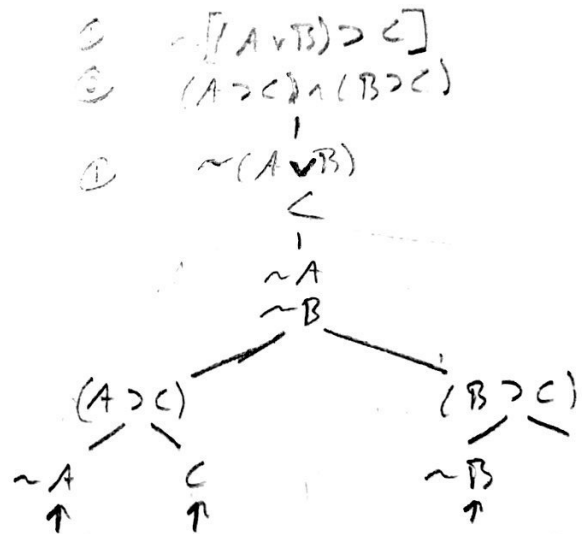
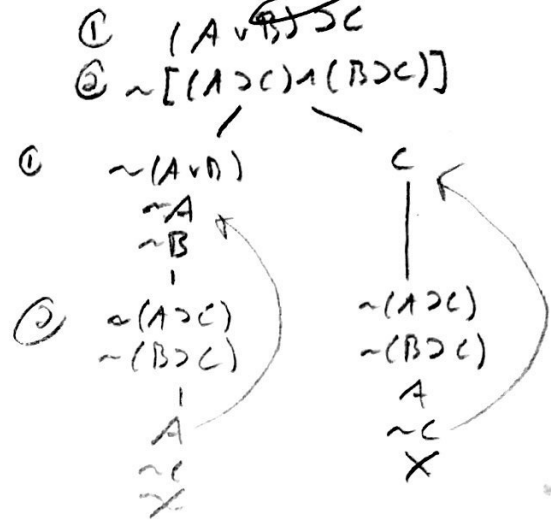
$$(A \wedge C) \supset B = (T \wedge T) \supset F$$

$$T \supset F$$

$$F$$

$$(A \vee B) \supset C$$

$$(A \supset C) \wedge (B \supset C)$$

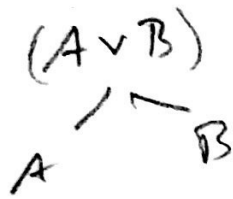


These are logically equivalent.

2. (40 points, in two parts) To prove soundness of the tree test for validity in SL we have to verify a certain property of each rule, which the handout calls Lemma One. In class we have called it the Soundness property, and said truth "goes down trees." State this property precisely. And describe what it has to do with soundness of the test.

this
gets
truth
goes
down

For each rule, fulfillment of the post means fulfillment of the whole. For example



fulfillment of either A or B makes $(A \vee B)$ true as well



fulfillment of the single branch B makes the statement $(A \wedge B)$ true as well.

If this is true for all rules, then "truth" can propagate down the tree to help us solve the tree.

3. (30 points) Let us add a new rule to our tree test for validity of arguments in SL: If every branch gets 100 lines long just stop and give no answer. Is this modified test sound? is it complete? is it both? Explain.

Soundness is the property that a test gives no false positives. This new test is sound because it gives no false positives, though do note that this is because it gives no answers at all.

Completeness is the property that if the item is true, the test evaluates to true. This new test is not complete because the test doesn't evaluate at all!

Use this vocabulary for translating PL in questions 1 and 2:

UD: Students at Case.

Px: x is a poet.

Ex: x is an engineer.

Kxy: x knows y.

1. (60 points) Translate these sentences into PL.

A. No one knows everyone.

$$\sim \exists x \forall y (Kxy)$$

B. Some engineer knows everyone.

$$\exists x (Ex \wedge \forall y (Kxy))$$

C. Every poet knows every engineer.

$$\forall x (Px \rightarrow \forall y (Ey \rightarrow Kxy))$$

D. Some poet engineer knows a poet taller than all the engineers.

$$\exists x (Px \wedge Ex) \wedge \exists y (Py \wedge \forall z (Ez \rightarrow Kxz))$$

$T_{\alpha\beta} \triangleq \alpha$ is taller than β

$$\exists x \exists y \forall z [(Px \wedge Ex) \wedge (Py) \wedge (Ez) \wedge (Kxz) \wedge (Tyz)]$$

Poet Engineer

Poet

all engineers

x knows y

y taller than all engineers

2. (40 points) Tell whether these arguments are valid in PL. For each that is, give a completed tree, and for each that is not give a counter example:

$$\exists x \forall y (Px \supset Kxy) \\ \therefore \exists x Px$$

$$\exists x \forall y (Px \supset Kxy) \quad (1) \\ \sim \exists x Px \quad (2)$$

$$\forall x \sim Px \quad (3)$$

$$\exists x \forall y (Px \supset Kxy) \quad \text{frank} \quad (4)$$

$$\forall y (P_{\text{frank}} \supset K_{\text{frank}} y) \quad (5)$$

$$\forall x \sim Px \quad \text{frank} \quad (6)$$

$$\sim P_{\text{frank}} \quad (7)$$

$$\forall y (P_{\text{frank}} \supset K_{\text{frank}} y) \quad \text{bob} \quad (8)$$

$$P_{\text{frank}} \supset K_{\text{frank}} \text{bob} \quad (9)$$

$$\sim P_{\text{frank}}$$

$$K_{\text{frank}} \text{bob}$$

Not Valid

Counter Example:

$$U = \{0, 1\}$$

$$P = \{ \}$$

$$K = \{ \}$$

$$\text{frank} \quad (2)$$

$$\text{bob} \quad (2)$$

Both Frank & bob are not poets, and neither knows the other.

$$\exists x \exists y (Px \& Kxy) \\ \forall x Px \\ \therefore \forall x \exists y Kxy$$

$$\exists x \exists y (Px \& Kxy) \quad (1)$$

$$\forall x Px \quad (2)$$

$$\sim \forall x \exists y Kxy \quad (3)$$

$$\exists x \sim \exists y Kxy \quad (4)$$

$$\exists x \forall y \sim Kxy \quad (5)$$

$$\exists x \exists y (Px \& Kxy) \quad \text{frank} \quad \text{bob} \quad (6)$$

$$P_{\text{frank}} \& K_{\text{frank}} \text{bob}$$

$$P_{\text{frank}}$$

$$K_{\text{frank}} \text{bob}$$

$$\exists x \forall y \sim Kxy \quad \text{frank} \quad \text{bob} \quad (7)$$

$$\sim K_{\text{frank}} \text{bob}$$

$$X$$

Valid

NY.

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