A Multiple-Detection Joint Probabilistic Data Association Filter

B. Habtemariam, R. Tharmarasa, T. Thayaparan, M. Mallick, and T. Kirubarajan

***Abstract*—Most conventional target tracking algorithms assume that a target can generate at most one measurement per scan. However, there are tracking problems where this assumption is not valid. For example, multiple detections from a target in a scan can arise due to multipath propagation effects as in the over-the-horizon radar (OTHR). A conventional multitarget tracking algorithm will fail in these scenarios, since it cannot handle multiple target-originated measurements per scan. The Joint Probabilistic Data Association Filter (JPDAF) uses multiple measurements from a single target per scan through a weighted measurement-to-track association. However, its fundamental assumption is still one-to-one. In order to rectify this shortcoming, this paper proposes a new algorithm, called the Multiple-Detection Joint Probabilistic Data Association Filter (MD-JPDAF) for multitarget tracking, which is capable of handling multiple detections from targets per scan in the presence of clutter and missed detection. The multiple-detection pattern, which can account for many-to-one measurement set-to-track association rather than one-to-one measurement-to-track association, is used to generate multiple detection association events. The proposed algorithm exploits all the available information from measurements by combinatorial association of events that are formed to handle the possibility of multiple measurements per scan originating from a target. The MD-JPDAF is applied to a multitarget tracking scenario with an OTHR, where multiple detections occur due to different propagation paths as a result of scattering from different ionospheric layers. Experimental results show that multiple-detection pattern based probabilistic data association improves the state estimation accuracy. Furthermore, the tracking performance of the proposed filter is compared against the Posterior Cramér-Rao Lower Bound (PCRLB), which is explicitly derived for the multiple-detection scenario with a single target.**

***Index Terms*—Multitarget tracking in clutter, data association, probabilistic data association, multiple-detection per target per scan, over-the-horizon radar (OTHR), multiple-detection JPDAF (MD-JPDAF).**

I. Introduction

MOST detection-based target tracking algorithms assume that a target generates at most one detection per scan with probability of detection less than unity. In this case, the data association uncertainty is only the measurement origin uncertainty [2]. Thus, given a set of measurements in a scan, at most one of them can originate from the target and the rest have to be false alarms. This basic assumption results in the formulation of one-to-one measurement-to-track association as an optimization or enumeration problem. For example, in the Probabilistic Data Association Filter (PDAF) [2], [5], [15] and its multitarget version, the Joint Probabilistic Data Association Filter (JPDAF) [7], [20], weights are assigned to measurements based on the Bayesian assumption that at most one of the measurements is from the target and the rest are false alarms. In the Multiple Hypothesis Tracker (MHT) [18], [6], [16] using the multiframe assignment (MFA) algorithm [4], the measurement-to-track association is evaluated as one-to-one combinatorial optimization in the best global hypothesis. In all these cases, the one-to-one assumption is fundamental for measurement-to-track association and target state estimation.

However, a target can generate multiple detections in a scan due to, for example, multipath propagation or extended nature of the target with a high resolution radar. In this paper, we address multiple detections due to multipath propagation only. When multiple detections from the same target fall within the association gate, the PDAF and its multitarget version, the JPDAF, tend to apportion the association probabilities, but still with the fundamental assumption that only one of them is correct. When the measurements are not close to one other, as in the case of multipath detections, the PDAF and JPDAF initialize multiple tracks for the same target. The MHT algorithm tends to generate multiple tracks to handle the additional measurements from the same target due to the basic assumption that at most one measurement originated from each target. Such ad hoc handling of multiple detections has undesirable side effects.

Thus, an algorithm that explicitly considers multiple detections from the same target in a scan needs to be developed so that all useful information in the received measurements about the target is processed with the correct assumption. Presence of multiple detections per target per scan increases the complexity of a tracking algorithm due to uncertainty in the number of target-originated measurements, which can vary from time to time, in addition to the measurement origin uncertainty. However, estimation accuracy can be improved and the number of false tracks can be reduced using the correct assumption with multiple-detection.

Multiple-detection is a common phenomenon in over-the-horizon radars (OTHRs) [8], [11], which provides the motivation for this work. This is due to the OTHR's reliance on the ionospheric layers for signal transmission and reception. The signal transmitted from an OTHR will be scattered by one of the ionospheric layers, then scattered from the target, and finally scattered by another or the same ionospheric layer be-

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fore it is collected by a receiver. Hence, there will be multiple propagation paths due to multiple ionospheric layers that grow with the number of ionospheric layers. Among the various possible signal propagation paths, a target might be either detected or missed in one or more propagation paths. A tracker has to determine from which propagation path(s) a target is detected while processing multiple target-originated measurements. In the literature, different techniques have been proposed to solve the multipath problem with OTHRs. With respect to target localization, the maximum likelihood coordinate registration for the OTHR is developed in [17]. Furthermore, using the OTHR, target localization based on a Markov model is presented in [1] and maneuvering target detection using adaptive clutter rejection is presented in [27]. A multiple hypothesis tracking-based multipath fusion algorithm is presented in [23] and a multipath track association with Langragian relaxation is presented in [22]. A multipath PDAF (MPDAF) is proposed in [21]. However, in this MPDAF approach, identical probabilities of target detection for all propagation modes are assumed, which might not be always the case. Furthermore, the MPDAF is limited to a single target tracking problem and its formulation assumes that the target exists or is observable in all propagation modes although it may not be detected by one or more modes in practice. A related work is also done to jointly associate measurement from multiple sensors [13], [14] and multiple scans [12] using probabilistic data association. A random finite set (RFS) based approach to solve the problem of multiple detections per target per scan is proposed in [26] using 2D position-only measurements. In our previous work [9], we developed a Multiple-Detection Probabilistic Data Association Filter (MD-PDAF) to solve multiple-detection problem for a single-target tracking problem. We developed the PCRLB for this problem in [10]. A 2D radar with range and bearing measurements was used in [9], [10] to demonstrate the algorithms.

In this paper, we present a new Multiple-Detection Joint Probabilistic Data Association Filter (MD-JPDAF) for multitarget tracking. While the algorithm is motivated by the OTHR problem, it is applicable to any multitarget tracking problem with multiple detections per target. The filtering algorithm is derived with the explicit assumption of multiple detections so that a multiple-detection pattern would account for all possible many-to-one measurement set-to-track association rather than a one-to-one measurement-to-track association. In the proposed MD-JPDAF, combinatorial association events are formed to handle the possibility of multiple measurements from the same target in a scan. Multiple association events are formed by creating φ out of m combinations of multiple measurements to track assignment, where φ is the number of target-originated measurements and m is the total number of measurements in the validation gate. Priori information on the number of target-originated measurements can also be used, if available, to determine the probability of detection conditioned on the number of target-originated measurements and to reduce the total number of association events. For each association event, the event probabilities are calculated based on probabilistic inference made on no measurement, one measurement or a set of measurements originating from the target. With this explicit multiple-detection, many-to-one measurement set-to-track formulation, the proposed algorithm can handle the uncertainty in the number of target-originated measurements in addition to measurement origin uncertainty.

If the target is detected only once per scan, the MD-JPDAF will reduce to the conventional JPDAF. In simulations, the performance of different tracking algorithms are analyzed by generating multiple-detection measurements from a target observed in clutter. In addition, the proposed algorithm is applied to tracking in multipath with OTHRs. Performance of the proposed MD-JPDAF is compared with that of the conventional JPDAF. Results of performance evaluation show the effectiveness of the new algorithm with respect to estimation accuracy. However, the computational complexity of the proposed algorithm is higher due to increased number of association events.

The paper is organized as follows. Section II discusses the multiple-detection pattern, the basic input to the MD-JPDAF. Models for combinatorial events in the presence of multiple-detection are presented in this section. The MD-PDAF and its mulitarget extension, MD-JPDAF, are presented in Section III, where theoretical development of the algorithms is discussed. The PCRLB, which is explicitly derived for multiple detections per target in a scan, is presented in Section IV for a single target tracking scenario. Simulation results are presented in Section V, based on a single target and multitarget tracking in clutter where observations are made with a sensor that returns multiple detections and with the OTHR. For the OTHR, different target detection probabilities are considered for each propagation mode. Finally, we present concluding remarks and future directions of research in Section VI.

II. Multiple-Detection Pattern

If multiple detections from the same target fall within the association gate, a measurement or a set of measurements will be associated with a track. Data association uncertainty corresponding to a number of target-originated measurements as well as measurement source can be resolved by generating a multiple-detection pattern. The multiple-detection pattern will consider all possible events for many-to-one measurement set-to-track association.

We assume that the target state x evolves according to the nearly constant velocity model (NCVM) in 2D [3]

$$x(k) = Fx(k-1) + w(k-1), \tag{1}$$

where F is the state transition matrix and $w(k-1)$ is a zero-mean white Gaussian process noise with covariance Q given as

$$F = I \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{2}$$

and

$$Q = I \otimes q \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix}, \tag{3}$$

where q is the power spectral density [3] of the process noise, T is the scan time, $\otimes$ is the Kronecker product and I is the identity matrix.

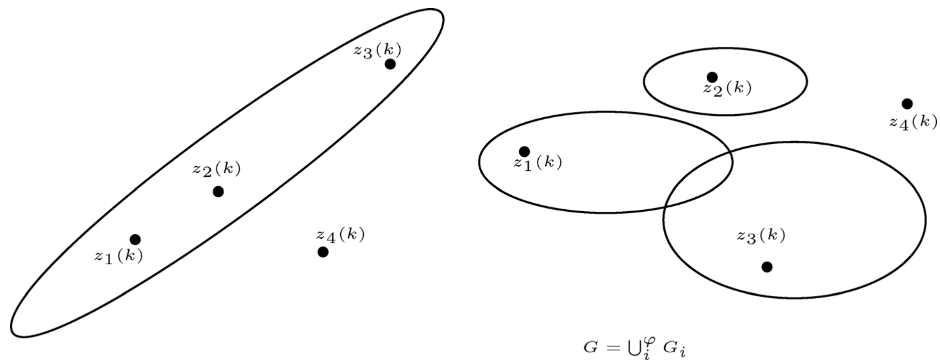

Fig. 1. Validation gate: Single source (left) and multiple sources (right).

The nonlinear measurement model for the measurement z of dimension n_z is described by

$$z(k) = h_\varphi(x(k)) + v_\varphi(k), \tag{4}$$

where h_φ and $v_\varphi(k)$ are the nonlinear measurement function and measurement noise, respectively, corresponding to the φ-th measurement mode. The measurement noise is assumed to be zero-mean white Gaussian with covariance $R_\varphi(k)$. The process and measurement noises are assumed to be independent.

After signal processing, thresholding and detection, gating has to be applied to reduce the number of measurements as shown in Fig. 1. As a result, the number of feasible events that are generated based on the measurements will be reduced. The validation gate is an n_z-dimensional ellipse [2] given by

$$\begin{aligned} V(k,\gamma) &= \bigcup_{\varphi=1}^{\varphi_{\max}} V_\varphi(k,\gamma) \\ V_\varphi(k,\gamma) &= \{z(k) : [z(k) - \hat{z}_\varphi(k \mid k-1)]' S_\varphi(k)^{-1} \\ &\qquad [z(k) - \hat{z}_\varphi(k \mid k-1)] \leqslant \gamma\}. \end{aligned} \tag{5}$$

Here, γ is the gate threshold, $\hat{z}_\varphi(k \mid k-1)$ is the predicted measurement, and $S_\varphi(k)$ is the innovation covariance that corresponds to the φ-th measurement, which is given by

$$S_\varphi(k) = H_\varphi(k) P(k \mid k-1) H_\varphi(k)' + R_\varphi(k), \tag{6}$$

where $P(k \mid k-1)$ is the prediction covariance and $H_\varphi(k)$ is the Jacobian matrix of the nonlinear measurement function h_φ.

For $m(k)$ measurements inside the validation gate, φ out of $m(k)$ association events are evaluated. Here, φ runs from 1 to the maximum number of target-originated measurements. Then,

$$N_a = \sum_{i=0}^{\varphi_{\max}} C_i^{m(k)}, \tag{7}$$

where N_a is the total number of measurement set-to-track association events and C_x^y is the number of combinations of x out of y objects defined by

$$C_x^y = \begin{cases} \frac{y!}{x!(y-x)!}, & 1 \le x \le y, \\ 1, & x = 0. \end{cases} \tag{8}$$

The total association event count, N_a, represents all possible events from zero target-originated measurement to all of the measurements being target-originated. For example, as depicted in Fig. 1, there are four measurements, $(z_1(k), z_2(k), z_3(k), z_4(k))$, in the scan. Out of the four measurements, three of them, $(z_1(k), z_2(k), z_3(k))$, are inside the validation gate. Combinatorial association events are created only for those measurements that fall inside the validation gate. The maximum number of target-originated measurement is assumed to be $\varphi_{\max} = 3$. Thus the possible events are:

- none of the measurements is target-originated
  — $\varphi = 0$, $n_\varphi = 1$.
- one of the measurements is target-originated
  — $\varphi = 1$, $n_\varphi = C_1^3 = 3$,
  — 3 measurement set-to-track association events,
  — $z_1(k)$ or $z_2(k)$ or $z_3(k)$ originated from a target,

$$z_{1,1}(k) = z_1(k), \tag{9}$$
$$z_{1,2}(k) = z_2(k), \tag{10}$$
$$z_{1,3}(k) = z_3(k). \tag{11}$$

- two of the measurements are target-originated
  — $\varphi = 2$, $n_\varphi = C_2^3 = 3$,
  — 3 measurement set-to-track association events,
  — $z_1(k), z_2(k)$ or $z_1(k), z_3(k)$ or $z_2(k), z_3(k)$ originated form a target

$$z_{2,1}(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix}, \tag{12}$$
$$z_{2,2}(k) = \begin{bmatrix} z_1(k) \\ z_3(k) \end{bmatrix}, \tag{13}$$
$$z_{2,3}(k) = \begin{bmatrix} z_2(k) \\ z_3(k) \end{bmatrix}. \tag{14}$$

- all of the measurements are target-originated
  — $\varphi = 3$, $n_\varphi = C_3^3 = 1$,
  — 1 measurement set-to-track association event,
  — $z_1(k), z_2(k), z_3(k)$ originated from a target,

$$z_{3,1}(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \\ z_3(k) \end{bmatrix}. \tag{15}$$

Accordingly, the measurement equation for the (φ, n_φ) event becomes

$$z_{\varphi,n_\varphi}(k) = \begin{bmatrix} h_1(x(k)) \\ \vdots \\ h_\varphi(x(k)) \end{bmatrix} + \begin{bmatrix} v_1(k) \\ \vdots \\ v_\varphi(k) \end{bmatrix}. \tag{16}$$

III. MD-PDAF AND MD-JPDAF

The approach of the standard PDAF is to calculate the association probabilities for each measurement that falls in the validation region around the predicted measurement [2]. If two of the measurements are target-originated, the algorithm apportions the total weight among the validated measurements with more weight to target-originated measurements, with the assumption that only one of them is target-originated. It is not the correct approach especially when there are false alarms in the validation gate. This is because the weight assigned to false alarms becomes significant compared to the divided weight assigned to target-originated measurements.

The proposed multiple-detection based algorithms evaluate the association probabilities of the events generated by multiple-detection pattern. These event probabilities are calculated based on probabilistic inference made on the number of validated measurements, the number of target-originated measurements and the measurements locations.

A. MD-PDAF for Single Target Tracking

The MD-PDAF is formulated under the following assumptions:

- Among the validated measurements, a measurement or a set of measurements can originate from a target.
- The target detections occur independently with known probabilities.
- Clutter is uniform/Poisson distributed within the measurement validation gate [2].
- There is only one target of interest whose state evolves according to a dynamic equation driven by process noise as stated in (1). Multiple targets are considered in Section III-B.
- Track has been initiated. Note that this is not really a limitation since one- or two-point initialization technique can be used for track initialization [3].

The MD-PDAF calculates the probability that each set of measurements, rather than a single measurement, is attributable to the target of interest. The sets of measurement candidates for association are generated from the multiple-detection pattern discussed in Section II. This probabilistic (Bayesian) information based on the candidate set of measurements is used in a tracking filter that updates the target states. The flow of MD-PDAF is shown Fig. 2.

Accordingly, based on the multiple-detection pattern presented in Section II, the measurement set-to-target association events are given as

$$\theta_{\varphi,n_\varphi}(k) = \begin{cases} \text{chosen } \varphi \text{ measurements are target-originated,} \\ \qquad\qquad n_\varphi = 1,\ldots,c_{\varphi m}(k), \\ \text{none of the other measurements are} \\ \text{target-originated,} \\ \qquad\qquad n_\varphi = 0, \end{cases} \tag{17}$$

where n_φ is a variable that enumerates the events under the chosen φ target-originated measurements, and $c_{\varphi m}(k)$ is φ combinations out of $m(k)$ measurements given by

$$c_{\varphi m}(k) = \frac{m(k)!}{\varphi!(m(k)-\varphi)!}. \tag{18}$$

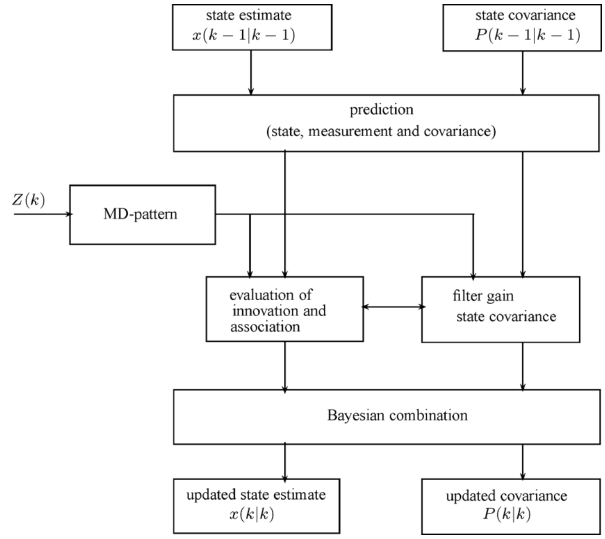

Fig. 2. Flow of MD-PDAF.

TABLE I
NUMBER OF MULTIPLE-DETECTION ASSOCIATION EVENTS

Number of Measurements $m(k)$	Number of Association Events, N_a		
	$\varphi_{max}=2$	$\varphi_{max}=3$	$\varphi_{max}=m(k)$
2	4	4	4
3	7	8	8
4	11	15	16
5	16	26	32
6	22	42	64
7	29	64	128
8	37	93	256

The number of association events grows rapidly for $\varphi > 2$ as shown in Table I. However, for a practical system, the expected number of target-originated measurements can be used as a priori to reduce the number of association events. For example for over-the-horizon radars with four possible propagation paths (e.g., only E and F layers [11]) the maximum number of target-originated measurements will be four and the association variable φ runs from zero to four.

Thus the conditional mean is given by

$$\begin{aligned} \hat{x}(k\,|\,k) &= E(x(k)\,|\,Z^k) \\ &= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} E(x(k)\,|\,\theta_{\varphi,n_\varphi}(k), Z^k) \\ &\quad \times p(\theta_{\varphi,n_\varphi}(k)\,|\,Z^k) \qquad (19) \\ &= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \hat{x}_{\varphi,n_\varphi}(k\,|\,k)\beta_{\varphi,n_\varphi}(k). \qquad (20) \end{aligned}$$

The estimate conditioned on n_φth combination of φ measurements being correct is

$$\hat{x}_{\varphi,n_\varphi}(k\,|\,k) = \hat{x}(k\,|\,k-1) + W_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k), \tag{21}$$

where the corresponding innovation is

$$\nu_{\varphi,n_\varphi}(k) = \begin{bmatrix} (z(k) - \hat{z}_1(k\,|\,k-1))' \\ \vdots \\ (z(k) - \hat{z}_\varphi(k\,|\,k-1))' \end{bmatrix}. \tag{22}$$

The Kalman gain $W_{\varphi,n_\varphi}(k)$ is given as

$$W_{\varphi,n_\varphi}(k) = P(k \mid k-1) H_{\varphi,n_\varphi}(k)' S_{\varphi,n_\varphi}(k)^{-1}, \tag{23}$$

where

$$S_{\varphi,n_\varphi}(k) = H_{\varphi,n}(k) P(k \mid k-1) H_{\varphi,n_\varphi}(k)' + R_{\varphi,n_\varphi}(k), \tag{24}$$

$$H_{\varphi,n_\varphi}(k) = [H_1(k), \ldots, H_\varphi(k)]', \tag{25}$$

$$R_{\varphi,n_\varphi}(k) = \begin{bmatrix} R_1(k) & 0 & \ldots & 0 \\ 0 & R_2(k) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R_\varphi(k) \end{bmatrix}. \tag{26}$$

Here $\beta_{\varphi,n_\varphi}(k) \propto p(\theta_{\varphi,n_\varphi}(k) \mid Z^k)$ is the conditional probability of the event where the probabilistic inference is made on the number of validated measurements $(m(k))$, number of target-originated measurements (φ) and measurements locations (see Appendix VII-A). Then,

$$\beta_{\varphi,n_\varphi}(k) = \frac{1}{c} p(Z^k \mid \theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}) \times p(\theta_{\varphi,n_\varphi}(k) \mid m(k), \varphi). \tag{27}$$

The first term in (27) refers to the joint density of the pdf of the correct measurement is given in (28) where P_G is the factor that accounts for restricting the normal density to the validation gate. Thus,

$$p(Z^k \mid \theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}) = \begin{cases} P_G^{-1} \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{-m(k)+\varphi} \mathcal{N}(\nu_{\varphi,n_\varphi}(k); 0, S_{\varphi,n_\varphi}(k)), & n_\varphi = 1, \ldots, c_{\varphi m}(k), \\ \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{-m(k)}, & n_\varphi = 0. \end{cases} \tag{28}$$

The second term in (27) is the probability of the association events conditioned only on $m(k)$ and φ. Here,

$$p(\theta_{\varphi,n_\varphi}(k) \mid m(k), \varphi) = \begin{cases} \frac{1}{m(k)} \frac{P_{D\varphi} P_G \mu(m(k)-\varphi)}{\sum_{\varphi=1}^{m(k)} P_{D\varphi} P_G \mu(m(k)-\varphi) + (1-P_D P_G)\mu(m(k))}, & n_\varphi = 1, \ldots, c_{\varphi m}(k), \\ \frac{(1-P_D P_G)\mu(m(k))}{\sum_{\varphi=1}^{m(k)} P_{D\varphi} P_G \mu(m(k)-\varphi) + (1-P_D P_G)\mu(m(k))}, & n_\varphi = 0. \end{cases} \tag{29}$$

where $\mu(\cdot)$ is the probability mass function of the number of false alarms, $P_{D\varphi}$ is the probability of detecting a target φ times per scan and $P_D = \sum_{\varphi=1}^{\varphi_{\max}} P_{D\varphi}$ is the total probability of target detection.

The state update equation is given by

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} W_{\varphi,n_\varphi}(k) \beta_{\varphi,n_\varphi}(k) \nu_{\varphi,n_\varphi}(k), \tag{30}$$

and the covariance associated with the updated state is

$$P(k \mid k) = E\{[x(k) - \hat{x}(k \mid k)][x(k) - \hat{x}(k \mid k)]' \mid Z^k\} \tag{31}$$

$$= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \beta_{\varphi,n_\varphi}(k) \times E\{[x(k) - \hat{x}(k \mid k)] \times [x(k) - \hat{x}(k \mid k)]' \mid \theta_{\varphi,n_\varphi}(k), Z^k\}. \tag{32}$$

Parametric or nor-parametric [2] MD-PDA can be developed based on the assumed Poisson or diffuse a prior model used for the probability mass function of the number of false measurements.

Hence,

- Poisson model (parametric MD-PDA):

$$\mu(m(k)) = e^{-\lambda\left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}} \frac{\left(\lambda \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}\right)^{m(k)}}{m(k)!}, \tag{33}$$

  where λ is the spacial density.
- Diffuse a prior model (non-parametric MD-PDA):

$$\mu(m(k)) = \mu(m(k) - \varphi) = K, \tag{34}$$

  where K is a constant.

B. MD-JPDAF for Multitarget Tracking

If the targets are close to one another with overlapping validation regions, a measurement or set of measurements could be generated from more than one target. This will result in further measurement source uncertainty in addition to measurement uncertainties associated with single target or widely separated targets.

This additional uncertainty can be resolved by considering joint measurement set-to-track association events [2]. With multiple detection the conditional probabilities based on joint association events are given as:

$$\Theta_J(k) = \bigcap_{\varphi_t=0}^{m(k)} \bigcap_{n_{\varphi_t}=1}^{c_{\varphi_t m}(k)} \theta_{\varphi_t,n_{\varphi_t}}(k) \tag{35}$$

where $\theta_{\varphi,n_{\varphi_t}}(k)$ is the event that measurement set $z_{\varphi_t,n_{\varphi_t}}$ originated from target t, $t = 0, 1, \ldots, \tilde{T}$, for $\tilde{T}$ number of targets. The event matrix for the joint measurement set-to-track association event $\Theta_J(k)$ is given by

$$\hat{\Omega}_J(\Theta_J(k)) = [\hat{\omega}_{\varphi_t,n_{\varphi_t}}(\Theta_J(k))], \tag{36}$$

where $\hat{\omega}_{\varphi_t,n_{\varphi_t}}(\Theta_J(k)) = 1$ if $\theta_{\varphi_t,n_{\varphi_t}}(k) \in \Theta_J(k)$, and is zero otherwise. A given measurement set can also originate from a target so that $\delta_t(\Theta_J(k)) := \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \hat{\omega}_{\varphi_t,n_{\varphi_t}} \leq 1$. Here, define a binary measurement set association indicator $\tau_{\varphi,n_\varphi}(\Theta_J(k)) = \sum_{t=1}^{\tilde{T}} \hat{\omega}_{\varphi_t,n_{\varphi_t}}(\Theta_J(k))$ in order to indicate if the measurement set $z_{\varphi_t,n_{\varphi_t}}$ is associated to a target in the event $\Theta_J(k)$. In addition, the number of unassociated measurement sets in the event $\Theta_J(k)$ is given by

$$\phi(\Theta_J(k)) = \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} [1 - \tau_{\varphi,n_\varphi}(\Theta_J(k))]. \tag{37}$$

In an approach similar to (27), the joint measurement set-to-track association probability $p(\Theta_J(k) \mid Z^k)$ is equivalent to the product of the measurement likelihood function, $p(Z(k) \mid \Theta_J(k), m(k), \varphi_t, Z^{k-1})$, and prior probability, $p(\Theta_J(k) \mid m(k), \varphi_t, Z^{k-1})$. Therefore, the measurement set likelihood function is evaluated as

$$\begin{aligned} &p(Z(k) \mid \Theta_{Jt}(k), m(k), \varphi_t, Z^{k-1}) \\ &= \prod_{\varphi_t=0}^{m(k)} \prod_{n_{\varphi_t=1}}^{c_{\varphi_t m}(k)} p\left(z_{\varphi_t, n_{\varphi_t}}(k) \middle| \theta_{\varphi_t, n_{\varphi_t}}(k), m(k), \varphi_t, Z^{k-1}\right), \end{aligned} \tag{38}$$

where the conditional pdf of a measurement given its origin is

$$\begin{aligned} &p\left(z_{\varphi_t, n_{\varphi_t}}(k) \middle| \theta_{\varphi_t, n_{\varphi_t}}(k), m(k), \varphi_t, Z^{k-1}\right) \\ &= \begin{cases} P_G^{-1} N\left(z_{\varphi_t, n_{\varphi_t}}(k); \hat{z}_{\varphi_t, n_{\varphi_t}}(k \mid k-1), S_{\varphi_t, n_{\varphi_t}}(k)\right) \\ \qquad\qquad \text{if } \tau_{\varphi, n_\varphi}(\Theta_J(k)) = 1, \\ \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{-1} \\ \qquad\qquad \text{if } \tau_{\varphi, n_\varphi}(\Theta_J(k)) = 0, \end{cases} \end{aligned} \tag{39}$$

and the prior joint association probability is given as

$$\begin{aligned} p\left(\Theta_{Jt}(k) \mid m(k), \varphi_t, Z^{k-1}\right) &= \left(\frac{\phi(\Theta_J(k))!}{c_{\varphi_t m}(k)!} \mu(\phi)\right) \\ &\times \prod_{t=1}^{\tilde{T}} \left(P_{D_{\varphi_t}} P_G\right)^{\delta_t(\Theta_J(k))} \left(1 - P_{D_t} P_G\right)^{1-\delta_t(\Theta_J(k))}, \end{aligned} \tag{40}$$

where $P_{D_{\varphi_t}}$ is the probability of detecting the t-th, φ times per scan and $P_{D_t} = \sum_{\varphi=1}^{\varphi_{\max}} P_{D_{\varphi_t}}$ is the total probability of target detection.

Furthermore, target birth and target existence modes can be incorporated into the MD-JPDAF to handle varying number of targets [19], [20]. The main focus of this paper is integrating the multiple detection pattern into the probabilistic data association filter for a single and multiple targets problem. However, with similar approach a Multiple-Detection Joint Integrated Probabilistic Data Association filter (MD-JIPDAF) can be developed.

IV. Posterior Cramér-Rao Lower Bound

In this section, the PCRLB [25], which is explicitly derived for the multiple-detection purpose, is presented. The PCRLB provides a theoretical lower bound that can be used as a benchmark for estimation performance evaluation.

The standard PCRLB also makes the one-to-one assumption, which necessitates the new derivation of the modified PCRLB that accounts for multiple detections. First, a review on PCRLB is provided and the derivation considering the effect of multiple-detection is presented next.

A. Background

Let $\hat{x}(k \mid k)$ be an unbiased estimate of the state vector $x(k)$ based on measurements $Z_k = z(1), \ldots, z(k)$ and prior initial density $p(x_0)$. Let $P(k \mid k)$ be the covariance of $\hat{x}(k \mid k)$. Then $P(k \mid k)$ has a lower bound known as the PCRLB, i.e.,

$$\begin{aligned} P(k \mid k) &= E[(\hat{x}(k) - x(k))(\hat{x}(k) - x(k))' \mid Z_k] \\ &\geq J(k)^{-1}, \end{aligned} \tag{41}$$

where $J(k)$ is the Fisher information matrix (FIM). A recursive formula for the evaluation of the posterior FIM [24], [25] is given by

$$J(k) = J_x(k) + J_z(k). \tag{42}$$

The first term in (42), i.e., the prior information regarding the target states, is given by

$$J_x(k+1) = D_{22}(k) - D_{21}(k)(J(k) + D_{11}(k))^{-1} D_{12}(k), \tag{43}$$

where

$$D_{11}(k) = E\left[-\frac{\partial^2}{\partial x(k) \partial x(k)} \ln p(x(k+1) \mid x(k))\right], \tag{44}$$

$$\begin{aligned} D_{12}(k) &= (D_{21}(k))', \\ &= E\left[-\frac{\partial^2}{\partial x(k+1) \partial x(k)} \ln p(x(k+1) \mid x(k))\right], \end{aligned} \tag{45}$$

$$\begin{aligned} D_{22}(k) = E\Bigg[&-\frac{\partial^2}{\partial x(k+1) \partial x(k+1)} \\ &\times \ln p(x(k+1) \mid x(k))\Bigg]. \end{aligned} \tag{46}$$

Here, for a linear and Gaussian system transition model, it can be shown that [25]

$$J_x(k+1) = [Q + F(J(k)^{-1})F']^{-1} \tag{47}$$

The second term in (42), i.e., the measurement contribution factor, is given by

$$\begin{aligned} J_z(k+1) = E\Bigg[&-\frac{\partial^2}{\partial x(k+1) \partial x(k+1)} \\ &\times \ln p(z(k+1) \mid x(k))\Bigg]. \end{aligned} \tag{48}$$

B. Effect of Multiple Detections

Let $m(k)$ be the total number of measurements from sensor at time k. Thus,

$$z(k) = \{z_i(k)\}_{i=1}^{m(k)}. \tag{49}$$

Under the assumption that false alarms are uniformly distributed in the measurement space and the number of false alarms is Poisson distributed, the probability of getting $m(k)$ number of measurements out of which φ are target-originated is

$$\begin{aligned} p(m(k), \varphi) &= (1 - P_{D\varphi}) \\ &\times \frac{\left(\lambda \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}\right)^{m(k)} e^{-\lambda\left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}}}{m(k)!} \\ &+ P_{D\varphi} \frac{\left(\lambda \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}\right)^{m(k)-\varphi} e^{-\lambda\left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}}}{(m(k)-\varphi)!}. \end{aligned} \tag{50}$$

In the above, $P_{D\varphi}$ is the probability of detecting a target φ times per scan and V is the gated volume of the measurement space.

The probability that φ measurements are target generated is then given by

$$\epsilon(m(k),\varphi) = \frac{P_{D\varphi}}{p(m(k),\varphi)} \times \frac{\left(\lambda\left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}\right)^{m(k)-\varphi} e^{-\lambda\{\bigcup_{i=1}^{\varphi} V_i(k)\}}}{(m(k)-\varphi)!}. \quad (51)$$

With the assumption of more than one target-originated measurements, the measurement information matrix is given by

$$J_z(k) = \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} p(m(k),\varphi) J_{z_{\varphi,n_\varphi}}(k), \quad (52)$$

where

$$J_{z_{\varphi,n_\varphi}}(k) = E\left[-\frac{\partial^2}{\partial x(k+1)\partial x(k+1)} \times \ln p(z_{\varphi,n_\varphi} \mid x(k), m(k), \varphi)\right]. \quad (53)$$

Here, $p(z_{\varphi,n_\varphi} \mid x(k), m(k), \varphi)$ is given by

$$\begin{aligned} p(z_{\varphi,n_\varphi} \mid x(k), m(k), \varphi) &= \frac{1-\epsilon(m(k),\varphi)}{\left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{m(k)}} \\ &+ \frac{\epsilon(m(k),\varphi)}{(m(k)-\varphi)\left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{m(k)-\varphi}} \\ &\times \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} p(z_{\varphi,n_\varphi} \mid x(k)), \end{aligned} \quad (54)$$

where $p(z_{\varphi,n_\varphi} \mid x(k))$ is the pdf of the measurement set originated from a target.

V. SIMULATIONS

In the first part of this section, the comparison of the MD-PDAF with PDAF in terms of estimation accuracy is studied. The simulation is performed using a 2D sensor that returns multiple target-originated detections per scan. Furthermore, a multitarget scenario is considered in this simulation that demonstrates the performance of the proposed MD-JPDA algorithm.

In the second part of simulation, the MD-JPDAF is applied to multipath tracking with OTHR and performance analysis is made with respect to estimation accuracy.

A. 2D Sensor Scenario

A surveillance region covering a 1500 m × 1500 m is considered. In this region, there is a target that starts from [500 m, 800 m] with initial velocity [8 m/s, 5 m/s]. Track initialization is done using the two-point target initialization method. The scan interval (sampling period) is 1 s and the dataset consists of 30 scans.

A multiple-detection sensor that returns more than one target-originated measurement per scan is used in this experiment. A 2D radar with low probability of detecting a target once per scan of the measurement data ($P_{D1} = 0.01$) and with high probability of detecting a target twice per scan of the measurement data ($P_{D2} = 0.95$) is considered. Hence, the total probability of detecting a target in a scan of the measurement data (i.e., P_D used for PDAF) will become $P_D = P_{D1} + P_{D2} = 0.96$. The false alarm rate is 5 false alarms per scan.

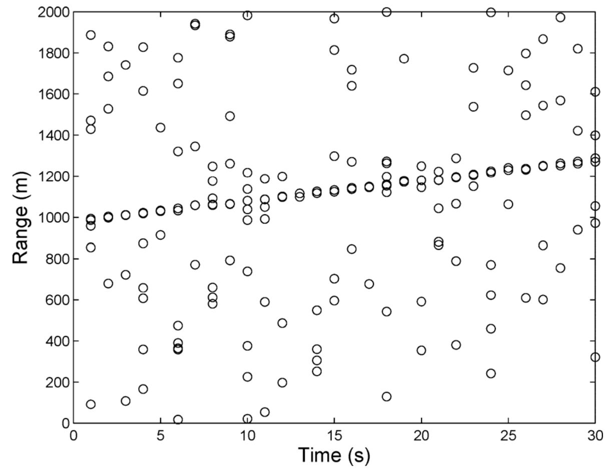

Fig. 3. Range measurements in a single run.

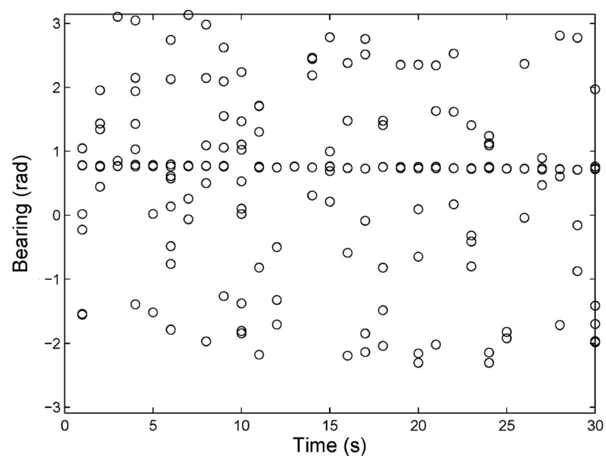

Fig. 4. Bearing measurements in a single run.

Figs. 3 and 4 show the range and bearing measurements from a multiple-detection sensor. In the multiple target-originated measurements case, the probabilities of detection used with the MD-PDAF are P_{D1} and P_{D2} while $P_D = P_{D1} + P_{D2}$ is the total probability of detecting a target in PDAF.

Fig. 5 shows the Root Mean Square Error (RMSE) for position estimation based on 500 MonteCarlo runs. The figure demonstrates the improved performance of the multiple-detection approach over the conventional probabilistic data association approach. While PDAF tends to apportion the weights among the target originated measurement, MD-PDAF assigns weights to measurement sets, rather than to a single measurement, that are originated from a target. From the same figure it can be also seen that the state estimation accuracy of the MD-PDAF is close to the PCRLB that was derived for multiple detections.

Furthermore, a multitarget scenario with two closely spaced targets is considered to evaluate the performance of the proposed MD-JPDA algorithm. The first target (labeled as T-1) starts from [700 m, 700 m] with initial velocity [10 m/s, 3 m/s] and the second target (labeled as T-2) starts from [700 m, 750 m] with initial velocity [10 m/s, −3 m/s]. The measurement valida-

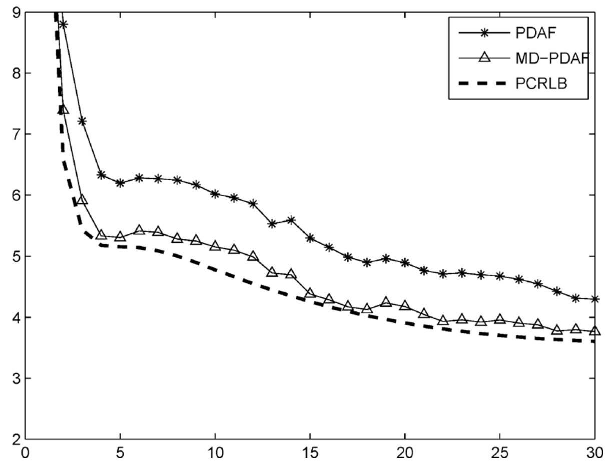

Fig. 5. Position RMSE evaluation for MD-PDAF vs. PDAF and PCRLB.

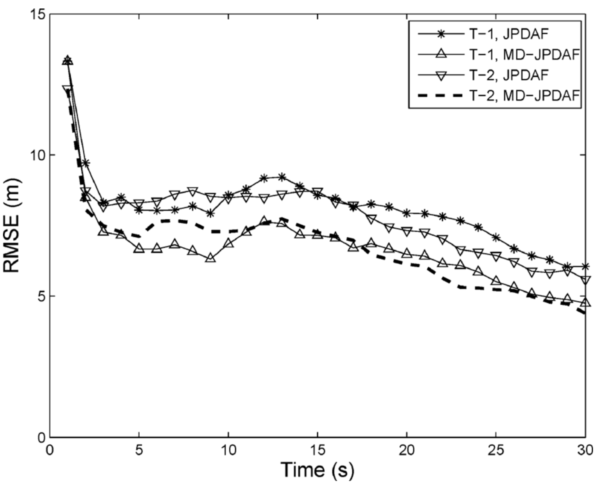

Fig. 6. Position RMSE evaluation for MD-JPDAF vs. JPDAF (T-1 denotes the first target and T-2 denotes the second target).

tion regions of the targets overlap during the duration of 9–18 that necessitates the evaluation of joint measurement set association events.

The targets are observed with a multiple detection sensor with similar multiple detection probabilities as the single target case above. The estimation error results, which are based on 100 MonteCarlo runs, are shown in Fig. 6 and, as expected, it reveals the performance gain of the MD-JPDAF over the JPDAF in a multitarget scenario with multiple detections.

B. Over-the-Horizon Radar Scenario

In the OTHR scenario, transmitted signals scattered by the target arrive at the receiver via different propagation paths [9]. Multipath propagation arises from the presence of regions with relatively high electron density in the ionosphere. As shown in Fig. 7, a signal transmitted from the senor can be scattered by one of the two ionospheric layers, scattered from the target and scattered again by one of the two ionospheric layers before it reaches the receiver. This phenomenon leads to more than one detection of the same target in a scan.

The OTHR measurement model used in this paper is based on [21]. According to the model, the signal paths from the transmitter to the target and from the target to the receiver are scatterings from idealized ionospheric layers with virtual heights h_E

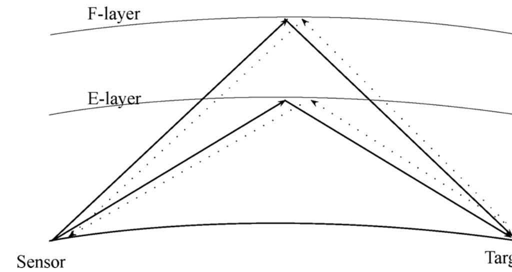

Fig. 7. Representative OTHR propagation modes [21].

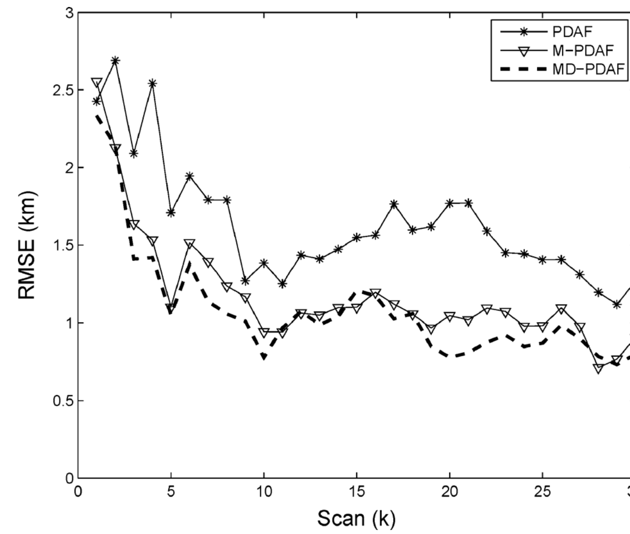

Fig. 8. Position RMSE evaluation for PDAF, MPDAF and MD-PDAF with OTHR data.

and h_F. With two ionospheric layers, denoted by E and F, there are four propagation modes:

- EE—transmit on E and receive on E,
- EF—transmit on E and receive on F,
- FE—transmit on F and receive on E,
- FF—transmit on F and receive on F.

The ionospheric layer heights are assumed to be constant for the duration of the tracking. Actual models that take multiple-hop modes, ionospheric tilt, the curvature of the earth and uncertainty in the heights of ionospheric layers are not included for simplicity [11]. But they can be incorporated as well.

The measurements from the OTHR are mode-dependent azimuth, slant range and Doppler given by

$$z(k) = \begin{cases} h_{\mathrm{EE}}(x(k)) + w_{\mathrm{EE}}(k) & \text{mode EE, with } P_{D_{\mathrm{EE}}}, \\ h_{\mathrm{EF}}(x(k)) + w_{\mathrm{EF}}(k) & \text{mode EF, with } P_{D_{\mathrm{EF}}}, \\ h_{\mathrm{FE}}(x(k)) + w_{\mathrm{FE}}(k) & \text{mode FE, with } P_{D_{\mathrm{FE}}}, \\ h_{\mathrm{FF}}(x(k)) + w_{\mathrm{FF}}(k) & \text{mode FF, with } P_{D_{\mathrm{FF}}}, \\ \text{clutter} & \text{otherwise,} \end{cases} \tag{55}$$

where the target state variables are represented in ground coordinates as, $x(k) = [r(k), \dot{r}(k), \vartheta(k), \dot{\vartheta}(k)]'$. Initially, a single target that starts at [850 km, 670 km] with initial velocity [0.25 km/s, 0.2 km/s] and that follows a system transition matrix (2) with process noise model (3) is considered to compare the performance of the single target tracking filters (PDAF, MPDAF and MD-PDAF). Afterwards, a second target that starts [855 km, 695 km] with initial velocity [0.15 km/s, -0.3

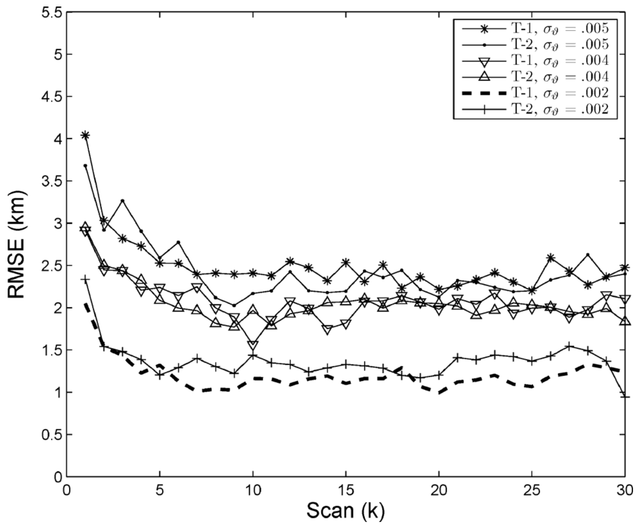

Fig. 9. Position RMSE evaluation MD-JPDAF with OTHR data.

km/s] is added in the simulation experiment in order to evaluate the performance of the MD-JPDAF.

The standard deviation of the bearing, slant range and Doppler measurement errors are $\sigma_\vartheta = 0.002$ rad, $\sigma_r = 0.1$ km, and $\sigma_{\dot{r}} = 0.005$ km/s, respectively. In contrast to the assumption made in the formulation of MPDAF [21], a signal propagation mode dependent probabilities of target detection are considered ($P_{D_{\mathrm{EE}}} = P_{D_{\mathrm{FF}}} = 0.7$ and $P_{D_{\mathrm{EF}}} = P_{D_{\mathrm{FE}}} = 0.8$). For the MPDAF the average probability of detection $P_D = 0.75$ is used. However, in the MD-PDAF, the probability of detection to a given measurement set-to-track association event is calculated as $P_{D_\varphi} = \prod_{i=1}^{\varphi} P_{D_i}, i \in (\mathrm{EE}, \mathrm{EF}, \mathrm{FE}, \mathrm{FF})$. The PDAF initializes and maintains on average three false tracks due to measurements being received via different modes from the same target while the MD-PDAF combines these measurements into a single track, thereby reducing the number of false tracks significantly.

Fig. 8 shows the RMSE comparison between the PDAF and MPDAF (assuming $P_D = 0.75$ in both cases) and MD-PDAF. As shown in the figure, in a single target scenario the performances of MPDAF and MD-PDAF are significantly better than of the PDAF with a small performance gain of MD-PDAF over MPDAF due to the correct handling of mode dependent probabilities of detection.

Finally, the performance of the multitarget tracking algorithm MD-JPDAF is evaluated in a multiple targets scenario with OTHR. The estimation errors for various bearing measurement standard deviation are shown in Fig. 9. Note that the MPDAF does not handle multiple targets.

VI. Conclusion

In this paper, we presented a new Multiple-Detection Joint Probabilistic Data Association filter (MD-JPDAF) for tracking multiple targets when multiple detections from a target per scan arises. When multiple detections from a target fall within the association gate, the standard JPDAF returns degraded state estimates due to violation of the one-measurement-per-scan assumption. In the proposed MD-JPDAF, combinatorial association events are formed to handle the possibility of multiple measurements from a target in a scan. Modified association probabilities are calculated with the explicit assumption of multiple detections. In order to provide a benchmark for performance comparison, we also derived the PCRLB for this multiple-detection scenario for a single target tracking problem. We performed Monte Carlo simulations with a 2D sensor and an OTHR to test the performance of the proposed filter. Experimental results demonstrate the effectiveness of the proposed algorithm.

Our future work will consider applying MD-JPDAF to a multisensor-multitarget tracking problem and integrating multiple-detection pattern into other multitarget tracking algorithms like the MFA based MHT. Further work will also be performed to initialize targets with multiple detections including target birth and continuity as well as death probabilities. Future work will also consider the realistic 3D tracking problem where a spherical model of the Earth will be used.

Appendix

A. Probabilistic Inference

In the MD-PDAF, multiple-detection association probabilities are evaluated by probabilistic inference, which is made on

- the number of measurements in the validation region, $m(k)$,
- the number of target-originated measurements, φ,
- the location of measurements.

This is expressed as

$$\beta_{\varphi,n_\varphi}(k) = p(\theta_{\varphi,n_\varphi}(k) \mid Z^k, m(k), \varphi, Z^{k-1}) \tag{56}$$

$$= \frac{1}{c} p\left(Z^k \mid \theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}\right) \times p\left(\theta_{\varphi,n_\varphi}(k) \middle| m(k), \varphi, Z^{k-1}\right). \tag{57}$$

The first and second terms in (57) refer to measurement location and source inference, respectively.

1) Measurement Location Inference: This is the joint pdf of the correct measurement

$$p\left(Z^k \mid \theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}\right) = \begin{cases} \frac{1}{P_G} \times \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{-m(k)+\varphi} \mathcal{N}\left(\nu_{\varphi,n_\varphi}(k); 0, S_{\varphi,n_\varphi}(k)\right), & n_\varphi = 1, \ldots, c_{\varphi m}(k), \\ \left\{\bigcup_{i=1}^{\varphi} V_i(k)\right\}^{-m(k)}, & n_\varphi = 0. \end{cases} \tag{58}$$

2) Measurement Source Inference: This evaluates the event $\theta_{\varphi,n_\varphi}$ conditioned on the total number of validated measurements $\mathcal{M} = m(k)$ and number of target-originated measurements, φ. Here $\mathcal{M}$ denotes the random variable and $m(k)$ its realization [2]. Then,

$$p\left(\theta_{\varphi,n_\varphi}(k) \middle| m(k), \varphi, Z^{k-1}\right) = p\left(\theta_{\varphi,n_\varphi} \middle| \mathcal{M} = m(k), \varphi\right) \tag{59}$$

$$= p\left(\theta_{\varphi,n_\varphi} \middle| \Psi = m(k) - \varphi, \mathcal{M} = m(k)\right) \times p(\Psi = m(k) - \varphi \mid \mathcal{M} = m(k)) + p\left(\theta_{\varphi,n_\varphi} \middle| \Psi = m(k), \mathcal{M} = m(k)\right) \times p(\Psi = m(k) \mid \mathcal{M} = m(k)), \tag{60}$$

where Ψ is the number of false measurements. For φ target-originated measurements, Ψ must be either $m(k)-\varphi$ or $m(k)$. Thus,

$$p\left(\theta_{\varphi,n_\varphi}(k)\middle|\, m(k),\varphi,Z^{k-1}\right) = \begin{cases} \frac{1}{m(k)} \times p(\Psi = m(k)-\varphi \,|\, \mathcal{M}=m(k)), \\ \qquad\qquad n_\varphi = 1,\ldots,c_{\varphi m}(k), \\ p(\Psi = m(k) \,|\, \mathcal{M}=m(k)), \\ \qquad\qquad n_\varphi = 0, \end{cases} \tag{61}$$

where

$$\begin{aligned} p(\Psi = m(k)-\varphi \,|\, \mathcal{M}=m(k)) &= \frac{p(\mathcal{M}=m(k) \,|\, \Psi=m(k)-\varphi)p(\Psi=m(k)-\varphi)}{p(\mathcal{M}=m(k))} \\ &= \frac{P_{D\varphi}P_G\mu(m(k)-\varphi)}{p(\mathcal{M}=m(k))}, \end{aligned} \tag{62}$$

and

$$\begin{aligned} p(\Psi = m(k) \,|\, \mathcal{M}=m(k)) &= \frac{p(\mathcal{M}=m(k) \,|\, \Psi=m(k))p(\Psi=m(k))}{p(\mathcal{M}=m(k))} \\ &= \frac{(1-P_DP_G)\mu(m(k))}{p(\mathcal{M}=m(k))}, \end{aligned} \tag{63}$$

$P_{D\varphi}$ is the probability of detecting a target φ times per scan. The total probability of detection P_D will become the superposition of detection probabilities of $P_{D\varphi}$. Also, $P_{D\varphi}P_G$ is the probability that the target has been detected and φ measurements originated from it are inside the gate and $(1-P_DP_G)$ is the probability that the measurements in the gate are false alarms. Thus,

$$p(\mathcal{M}=m(k)) = \sum_{\varphi=1}^{m(k)} P_{D\varphi}P_G\mu(m(k)-\varphi) + (1-P_DP_G)\mu(m(k)). \tag{64}$$

Substituting (64) in (62) and (63), the result in (61), we get

$$p\left(\theta_{\varphi,n_\varphi}(k)\middle|\, m(k),\varphi,Z^{k-1}\right) = \begin{cases} \frac{1}{m(k)} \frac{P_{D\varphi}P_G\mu(m(k)-\varphi)}{\sum_{\varphi=1}^{m(k)} P_{D\varphi}P_G\mu(m(k)-\varphi)+(1-P_DP_G)\mu(m(k))}, \\ \qquad\qquad n_\varphi = 1,\ldots,c_{\varphi m}(k), \\ \frac{(1-P_DP_G)\mu(m(k))}{\sum_{\varphi=1}^{m(k)} P_{D\varphi}P_G\mu(m(k)-\varphi)+(1-P_DP_G)\mu(m(k))}, \\ \qquad\qquad n_\varphi = 0. \end{cases} \tag{65}$$

3) Special Case: With only one target-originated measurement $(P_{D1},\ldots,P_{D\varphi}=P_D)$

$$p(\theta_{\varphi,n_\varphi}(k) \,|\, m(k),\varphi,Z^{k-1}) = \begin{cases} \frac{1}{m(k)} \frac{P_DP_G\mu(m(k)-1)}{P_DP_G\mu(m(k)-1)+(1-P_DP_G)\mu(m(k))}, \\ \qquad\qquad n_\varphi = 1,\ldots,c_{\varphi m}(k), \\ \frac{(1-P_DP_G)\mu(m(k))}{P_DP_G\mu(m(k)-1)+(1-P_DP_G)\mu(m(k))}, \\ \qquad\qquad n_\varphi = 0. \end{cases} \tag{66}$$

Then the MD-PDAF reduces to the standard PDAF.

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