Seasonal SIR model

Undoubtedly the SIR model with extensions is very important to describe and predict the dynamics of the pandemic. In this exam you will investigate a Seasonal SIR model.

(a, 10) The reproduction number is now a periodic function of time. Therefore, in this model the unit of time is **year**. Assume that **during the first half of each year** the reproduction number is (hint: use Piecewise)

```
In[ ]:= minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2

Out[ ]= minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2
```

and in the remainder of the year it has a value of ***minR01***. Write a function for R01t.

```
In[ ]:= minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2

Out[ ]= minR01 + 1/2 (-minR01 + R01) (1 - Cos[12.5664 Mod[t, 1]])
```

If you cannot write R01t you can ask for help, but you will receive no points for part a.

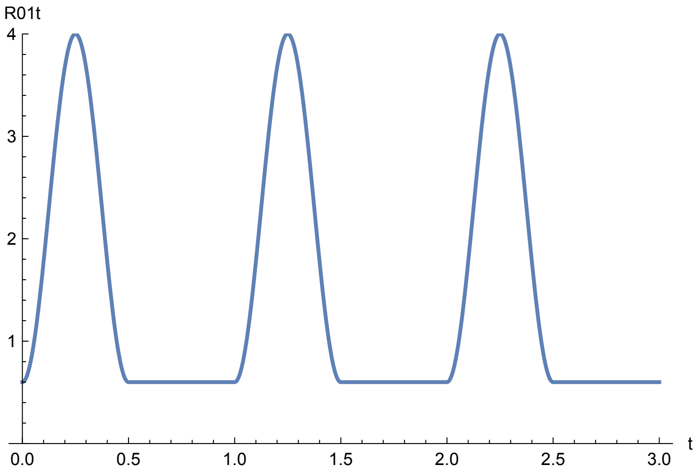
(b, 2) Now plot this function for **R01 = 4** and **minR01=0.6**.

```
In[ ]:= values = {R01 → 4, minR01 → 0.6}

Out[ ]= {R01 → 4, minR01 → 0.6}
```

```
In[ ]:= R01t = Piecewise[
          {{minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2, Mod[t, 1] < 0.5}},
          minR01]

Out[ ]= { minR01 + 1/2 (-minR01 + R01) (1 - Cos[12.5664 Mod[t, 1]])   Mod[t, 1] < 0.5
        { minR01                                                     True
```

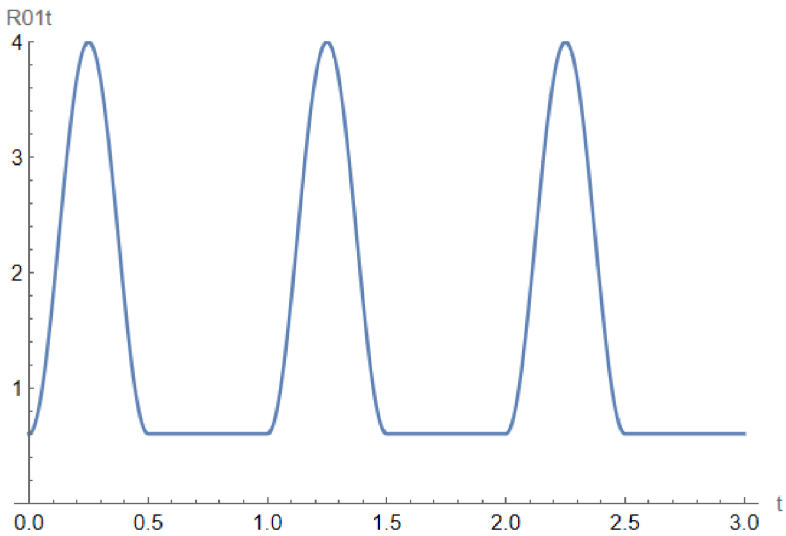
In[ ]:=
```
Plot[R01t /. values, {t, 0, 3},
  PlotRange → {Automatic, {0, 4}}, AxesLabel → {"t", "R01t"}]
```

Out[ ]=

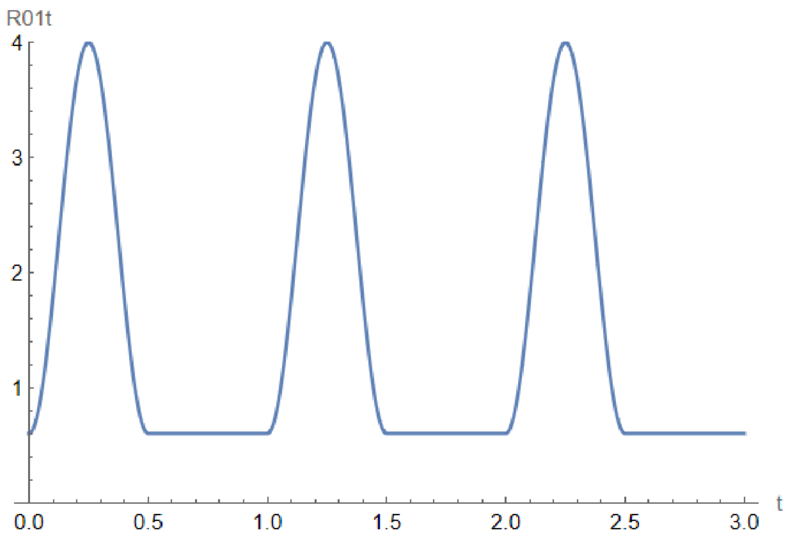


Your plot should look like this:

In[ ]:=



Out[ ]=



(c, 25) The first model contains three compartments susceptible, s(t); infected1, i1(t); recovered1, r1(t). It is described by three coupled odes:

```
In[ ]:=
s'[t] == -R01t[t] * γ * s[t] * i1[t] + δ * (r1[t]),
i1'[t] == R01t[t] * γ * s[t] * i1[t] - γ * i1[t],
r1'[t] == γ * i1[t] - δ * r1[t]
```

```
Out[ ]=
s'[t] == -R01t[t] * γ * s[t] * i1[t] + δ * (r1[t]),
i1'[t] == R01t[t] * γ * s[t] * i1[t] - γ * i1[t],
r1'[t] == γ * i1[t] - δ * r1[t]
```

The reproduction number is now a periodic function of time as defined under part (a).

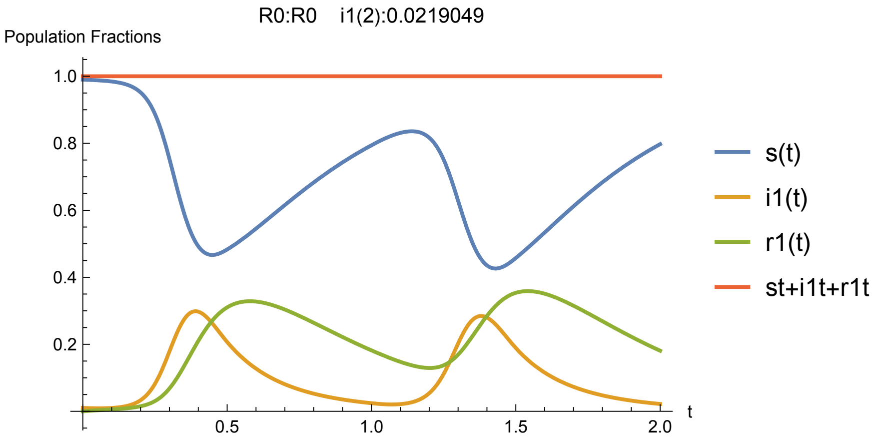
The initial conditions are s(0)=1-i01, r1(0)=0, i1(0)=i01. The model contains four parameters: the maximum reproduction number ***R01***, the minimum reproduction number ***minR01***, initial condition ***i01***, and the recovery rates ***γ*** and ***δ***.
Write a function **mySIR** with arguments **γ,δ,i01,R01,minR01** and ***tend*** to compute the model outcome. Compute the values of r1(t) at ***tend*** and call it ***r1tend***. Make a plot (with legend) of the concentrations s(t), i1(t), r1(t), and of their sum. Label the horizontal axis. Use PlotLabel to write the values of R01, i1(tend) (hint: using the functions StringExpression and ToString).

```
In[ ]:= mySIR[γ_, δ_, i01_, R01_, minR01_, tend_] := Module[{R01t, eqs, sol},
  R01t = Piecewise[
    {{minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2, Mod[t, 1] < 0.5}},
    minR01];
  eqs = {s'[t] == -R01t * γ * s[t] * i1[t] + δ * (r1[t]),
    i1'[t] == R01t * γ * s[t] * i1[t] - γ * i1[t],
    r1'[t] == γ * i1[t] - δ * r1[t], s[0] == 1 - i01, r1[0] == 0, i1[0] == i01};
  sol = NDSolve[eqs, {s[t], i1[t], r1[t]}, {t, 0, tend}] // Flatten;
  {st, i1t, r1t} = {s[t], i1[t], r1[t]} /. sol;
  i1tend = i1t /. {t → tend};
  Plot[{st, i1t, r1t, st + i1t + r1t}, {t, 0, tend},
   PlotLegends → {"s(t)", "i1(t)", "r1(t)", "st+i1t+r1t"},
   AxesLabel → {"t", "Population Fractions"}, PlotLabel → "R0:" ~~
     ToString[R0] ~~ "    i1(" ~~ ToString[tend] ~~ "):" ~~ ToString[i1tend]]
 ]
```

```
In[ ]:=
```

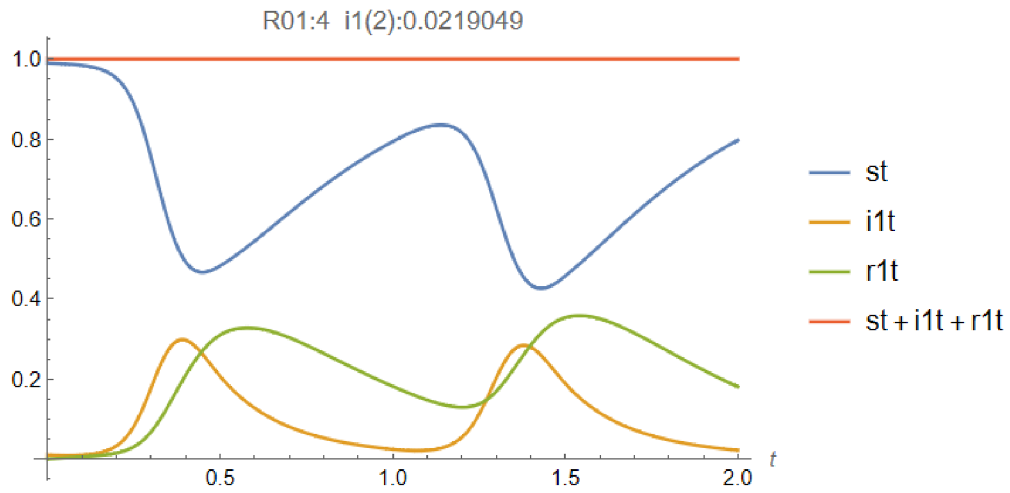
Plot the Seasonal SIR model with the values γ = 7,δ=3,i01 = 0.01, R01 = 4, minR01=0.6, and tend = 2

In[ ]:=
```
mySIR[7, 3, 0.01, 4, 0.6, 2]
```
Out[ ]=



In[ ]:=

Your plot should look like this:



If you cannot write **mySIR** you can ask for help, and mail the notebook you have written so far.

(d,5) Write a caption (about 40 words) in which you describe all four curves in the plot from (a).

Initially this looks like the original SIR model in which S decreases, I increases and R increases. However this normal epidemic is interrupted when seasonal effects kick in and the system tries to restore itself back to default values, however this process also gets interrupted by a seasonal flair up (1.0 < t < 1.5), and a new epidemic wave takes place. The st + it +rt curve remains constant at 100%.

(e, 5) Demonstrate that with these parameters the pandemic is periodic, and explain why this is the case (20 words)

Decrease in S[t] and increase in i1[t] (.i.e. an epidemic wave) depend on seasonal/periodic R01[t], which is never 0 for these values.

In[ ]:=

(f,6) Write a function mySIRitend (based upon mySIR) that returns i1(tend).

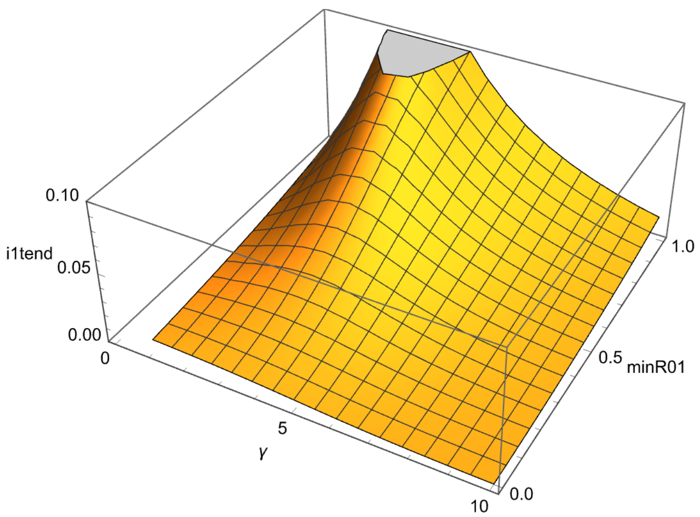
In[ ]:=
```
mySIRitend[γ_, δ_, i01_, R01_, minR01_, tend_] := Module[{},
  R01t = Piecewise[
    {{minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2, Mod[t, 1] < 0.5}},
    minR01];
  eqs = {s'[t] == -R01t * γ * s[t] * i1[t] + δ * (r1[t]),
    i1'[t] == R01t * γ * s[t] * i1[t] - γ * i1[t],
    r1'[t] == γ * i1[t] - δ * r1[t], s[0] == 1 - i01, r1[0] == 0, i1[0] == i01};
  sol = NDSolve[eqs, {s[t], i1[t], r1[t]}, {t, 0, tend}] // Flatten;
  {st, i1t, r1t} = {s[t], i1[t], r1[t]} /. sol;
  i1tend = i1t /. {t → tend};
  i1tend]
```

In[ ]:=

(g,5) Make a 3D plot of i1(tend) with γ from 1 to 10 and minR01 from 0 to 1, with δ=3, R01=4, i01 = 0.01, and tend = 2. Use options to label the axes and make a plot with a range for i1tend from 0 to 0.1.

In[ ]:=
```
Plot3D[mySIRitend[γ, 3, 0.01, 4, minR01, 2], {γ, 1, 10},
 {minR01, 0, 1}, AxesLabel → {Automatic, Automatic, "i1tend"},
 PlotRange → {Automatic, Automatic, {0, 0.1}}]
```

Out[ ]=



In[ ]:=

(h,25) A more realistic model contains more than one virus type, and unfortunately, also **disabled**. The SIRD model contains seven compartment: susceptible, s(t); infected1, i1(t); infected2, i2(t); recovered1, r1(t), recovered2, r2(t), disabled1, d1(t), disabled2, d2(t). It is described by seven coupled non-linear ODEs:

In[◦]:=
```
s'[t] == -R01t[t] * γ * s[t] * i1[t] - R02t[t] * γ * s[t] * i2[t] + δ * (r1[t] + r2[t]),
i1'[t] == R01t[t] * γ * s[t] * i1[t] - (γ + μ1) * i1[t],
i2'[t] == R02t[t] * γ * s[t] * i2[t] - (γ + μ2) * i2[t],
r1'[t] == γ * i1[t] - δ * r1[t],
r2'[t] == γ * i2[t] - δ * r2[t],
d1'[t] == μ1 * i1[t],
d2'[t] == μ2 * i2[t]
```

Out[◦]=

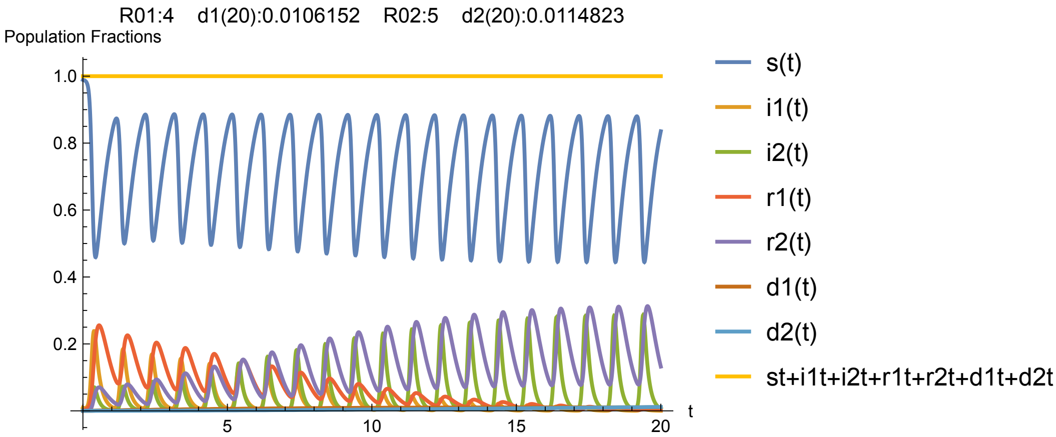
```
s'[t] == -R01t[t] * γ * s[t] * i1[t] - R02t[t] * γ * s[t] * i2[t] + δ * (r1[t] + r2[t]),
i1'[t] == R01t[t] * γ * s[t] * i1[t] - (γ + μ1) * i1[t],
i2'[t] == R02t[t] * γ * s[t] * i2[t] - (γ + μ2) * i2[t],
r1'[t] == γ * i1[t] - δ * r1[t],
r2'[t] == γ * i2[t] - δ * r2[t],
d1'[t] == μ1 * i1[t],
d2'[t] == μ2 * i2[t]
```


R02t is a function analogous to R01t from part (a). The model contains four additional parameters: the maximum reproduction number R02, initial condition i02, and the disable rates μ1 and μ2. The additional initial conditions are i2(0)=i02, r2(0)=0, d1(0)=0, d2(0)=0, and s(0)=1-i01-i02. Write a function **mySIRD** with arguments ***γ*,*δ*,*μ*1,i01,R01,minR01,*μ*2,i02,R02,minR02** and ***tend*** to compute the model outcome. Compute the values of d1(t) and d2(t) at ***tend*** and call them ***d1tend, d2tend***. Make a plot (with legend) of the concentrations s(t), i1(t), i2(t), r1(t), r2(t), d1(t), d2(t) and the sum of all concentrations. Use PlotLabel to write the values of R01, R02, d1(tend), d2(tend).

```
In[ ]:= mySIRD[γ_, δ_, μ1_, i01_, R01_, minR01_, μ2_, i02_, R02_, minR02_, tend_] :=
 Module[{R01t, R02t, eqs, sol2},
  R01t = Piecewise[
    {{minR01 + (R01 - minR01) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2, Mod[t, 1] < 0.5}},
    minR01];
  R02t = Piecewise[
    {{minR02 + (R02 - minR02) (1 - Cos[2 * Pi * Mod[t, 1] / 0.5]) / 2, Mod[t, 1] < 0.5}},
    minR02];
  eqs = {s'[t] == -R01t * γ * s[t] * i1[t] - R02t * γ * s[t] * i2[t] + δ * (r1[t] + r2[t]),
    i1'[t] == R01t * γ * s[t] * i1[t] - (γ + μ1) * i1[t],
    i2'[t] == R02t * γ * s[t] * i2[t] - (γ + μ2) * i2[t],
    r1'[t] == γ * i1[t] - δ * r1[t],
    r2'[t] == γ * i2[t] - δ * r2[t],
    d1'[t] == μ1 * i1[t],
    d2'[t] == μ2 * i2[t],
    r1[0] == 0, i1[0] == i01, i2[0] == i02,
    r2[0] == 0, d1[0] == 0, d2[0] == 0, s[0] == 1 - i01 - i02};
  sol2 = NDSolve[eqs,
     {s[t], i1[t], i2[t], r1[t], r2[t], d1[t], d2[t]}, {t, 0, tend}] // Flatten;
  {st, i1t, i2t, r1t, r2t, d1t, d2t} =
   {s[t], i1[t], i2[t], r1[t], r2[t], d1[t], d2[t]} /. sol2;
  d1tend = d1t /. {t → tend};
  d2tend = d2t /. {t → tend};
  Plot[{st, i1t, i2t, r1t, r2t, d1t, d2t, st + i1t + i2t + r1t + r2t + d1t + d2t},
   {t, 0, tend}, PlotLegends → {"s(t)", "i1(t)", "i2(t)", "r1(t)",
     "r2(t)", "d1(t)", "d2(t)", "st+i1t+i2t+r1t+r2t+d1t+d2t"},
   AxesLabel → {"t", "Population Fractions"},
   PlotLabel → "R01:" ~~ ToString[R01] ~~ "     d1(" ~~ ToString[tend] ~~
     "):" ~~ ToString[d1tend] ~~ "     R02:" ~~ ToString[R02] ~~
     "     d2(" ~~ ToString[tend] ~~ "):" ~~ ToString[d2tend]]
 ]

In[ ]:=
```

(i,2) Plot the SIRD model with $\gamma = 7$, δ=3, μ1=0.02, i01 = 10^-2, R01 = 4,minR01=0.4, μ2=0.01, i02 = 10^-3, R02 = 5,minR02=0.1,   and tend = 20

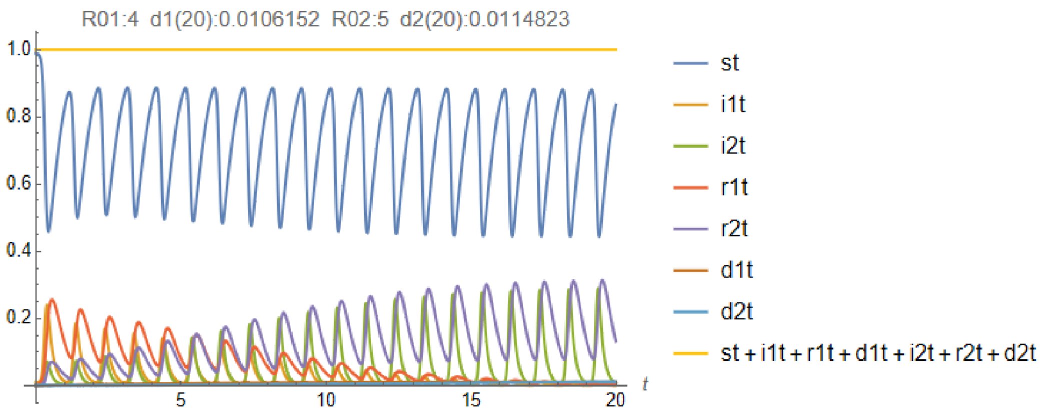
In[ ]:=
```
mySIRD[7, 3, 0.02, 10^-2, 4, 0.4, 0.01, 10^-3, 5, 0.1, 20]
```
Out[ ]=



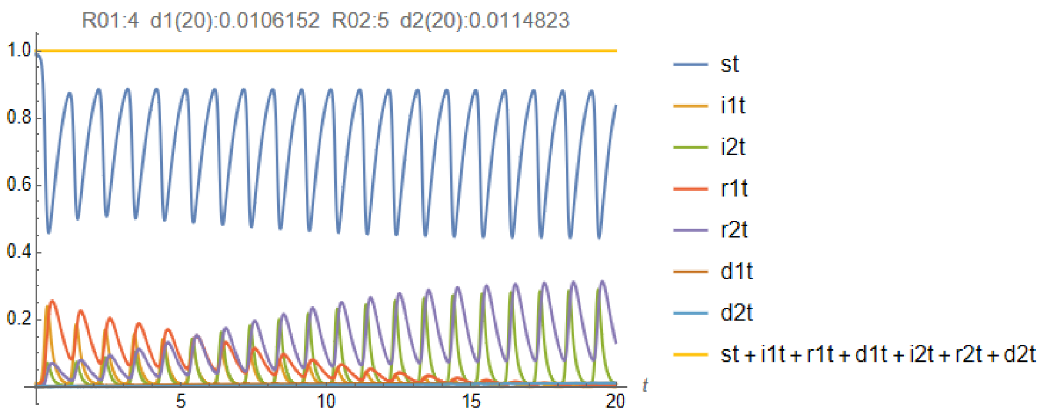
In[ ]:=

Your plot should look like this:

In[ ]:=



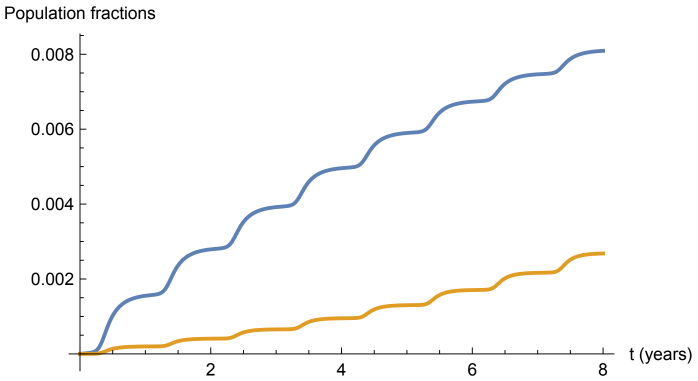
Out[ ]=



(j,2) Make a separate plot of d1t and d2t until 8 years.

In[ ]:=
```
Plot[{d1t, d2t}, {t, 0, 8}, PlotLegends → {"d1(t)", "d2(t)"},
 AxesLabel → {"t (years)", "Population fractions"}]
```

Out[ ]=



In[ ]:=

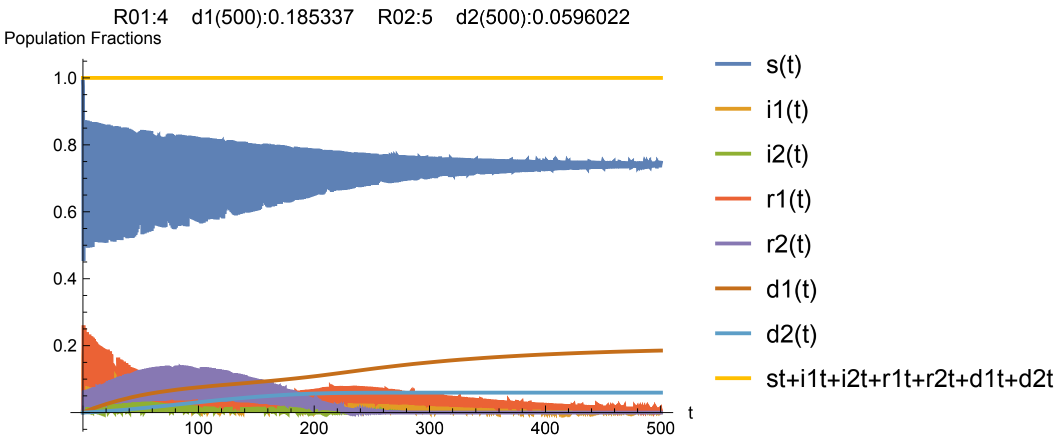
(k,4) Write a caption (about 40 words) in which you describe the main differences between the curves of the two virus types in the plots from (i) and (j).

Virus 1 impacts the system more than virus 2 at the beginning, i.e. higher infections, but after about 6 years this turns around, virus 2 impacts the system more than virus 1 which looks to die out. Virus 1 causes more disabled than virus 2, so it may be easier to recover from virus 2.

(k, 2) Plot the SIRD model with $\gamma = 7$, $\delta = 3$, $\mu1 = 0.02$, i01 = 10^-2, R01 = 4, minR01 = 0.462, $\mu2 = 0.01$, i02 = 10^-3, R02 = 5, minR02 = 0.1, and tend = 500

In[ ]:=
```
mySIRD[7, 3, 0.02, 10^-2, 4, 0.462, 0.01, 10^-3, 5, 0.1, 500]
```

Out[ ]=



In[ ]:=

(l, 7) Write a caption (about 40 words) in which you describe the main outcomes of the plot from (k). What will be the final state?

It becomes clear that the impact of both viruses diminishes with each season. Seasonal flair ups no longer lead to outbreaks like they did, and the whole system eventually remains more or less constant over time, leaving a population of mainly susceptible and disabled individuals.