Statistics Problems

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Statistics: Part I

1. Two fair dice are rolled and their results are recorded.
   (a) Write out the sample space Ω for this experiment (it should have 36 options).
   (b) Consider the random variables $X : \Omega \to \mathbb{R}$ that sums the two numbers on each die. What is $P(X \geq 7)$? (Hint: think about what the underlying probability measure on Ω looks like.)
2. A group of 300 students at a high school were surveyed and asked if 1) they ride a bike to school and 2) they take any AP classes. Results are recorded below.

Bike/AP	Yes	No	Total
Yes	60	30	90
No	140	70	210
Total	200	100	300

   (a) What is $P(\text{Bike} \cap \text{AP})$?
   (b) What is $P(\text{AP}|\text{doesn't Bike})$?
   (c) Are the events Bike and AP independent?
3. Urn A has three red balls and two white balls, and urn B has two red balls and five white balls. A fair coin is tossed. If it lands heads up, a ball is drawn from urn A; otherwise, it's drawn from urn B.
   (a) Use the law of total probability to find $P(\text{red ball is drawn})$.
   (b) Use Bayes' rule to find $P(\text{heads was flipped} \mid \text{red ball was drawn})$.
4. A player throws darts at a target. On each trial, independently of the other trials, he hits the bulls-eye with probability .05. How many times should he throw so that his probability of hitting the bulls-eye at least once is .5?

Statistics: Part II

You will need to refer to a density cheat sheet or the slides to solve these problems. You can also look up different distributions/densities via google as you need.

1. Which is more likely: 9 heads in 10 tosses of a fair coin or 18 heads in 20 tosses?

2. A multiple-choice test consists of 20 items, each with four choices. A student is able to eliminate one of the choices on each question as incorrect and chooses randomly from the remaining three choices. A passing grade is 12 items or more correct. What is the probability the student passes?

3. Suppose that the random variable X has density

   $$p(x) = \begin{cases} cx^2, 0 \leq x \leq 5 \\ 0, \text{otherwise} \end{cases}.$$

   What value must c be?

4. Suppose X is a exponential random variable with parameter $\lambda = 1$. Find the number c such that

   (a) $P(X \leq c) = 0.8$

   (b) $P(X \leq c) = 0.5$

   This second number is the *median* of the distribution P.

5. The *cumulative distribution function* or *cdf* of a continuous random variable X is defined as

   $$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} p(u)du.$$

   Write the quantity $P(a \leq X \leq b)$ as an expression that only involves terms of the cdf F_X.

6. Compute

   (a) $\lim_{x\to\infty} F_X(x)$

   (b) $\lim_{x\to-\infty} F_X(x)$

   Hint: no calculus needed.

Statistics: Part III

1. Let X be Poisson random variable with parameter λ. Use the fact that $\sum_{j=0}^{\infty} \frac{\lambda^j}{j!} = e^{\lambda}$ to compute $\mathbb{E}(X)$.

2. Evaluate
$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}. \tag{1}$$

3. For a Gaussian random variable $X \sim \mathcal{N}(\mu, \sigma^2)$, it turns out that
$$\mathbb{E}(X) = \mu$$
$$\mathrm{Var}(X) = \sigma^2.$$

   Use this information to compute $\mathbb{E}(X^2)$.

4. Let X be a Bernoulli random variable with parameter p, i.e.
$$P(X = 1) = p$$
$$P(X = 0) - 1 - p.$$

   Calculate $\mathbb{E}(X)$ and $\mathrm{Var}(X)$.

5. Suppose that $\mathbb{E}(X) = \mu$ and $\mathrm{Var}(X) = \tau^2$. Let $Z = \frac{X-\mu}{\tau}$. Compute
   (a) $\mathbb{E}(Z)$
   (b) $\mathrm{Var}(Z)$

6. Use the definition of variance and covariance to prove that
$$\mathrm{Var}(\alpha X + \beta Y) = \alpha^2 \mathrm{Var}(X) + \beta^2 \mathrm{Var}(Y) + 2\alpha\beta \mathrm{Cov}(X,Y)$$

   for random variables X,Y and scalars $\alpha, \beta \in \mathbb{R}$.