The Effect of Airplane Design, Paper Size, and Paper Type on the Flight Distance of Paper Airplanes

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Introduction

This experiment aims to determine which factors most impact the distance that a paper airplane flies. Paper airplanes have long been an easy and fun arts and crafts activity that can create a model capable of flying over 200 feet in the air using a piece of paper and some basic folding techniques.

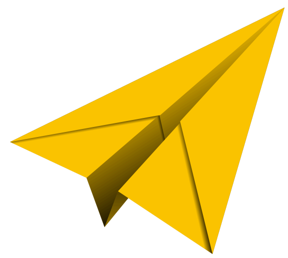
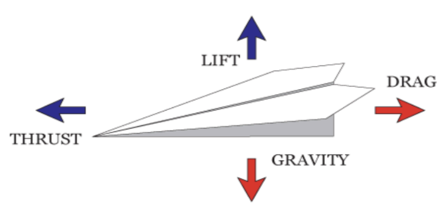


Figure 1: A Standard Paper Airplane

The basics of a paper airplane design are relatively simple. Four main forces act on paper airplanes while in flight. 1) The person or mechanism that throws the airplane provides thrust to push the plane forward. 2) The air that pushes against the design is called drag, which eventually stops the airplane from moving forward continuously. 3) Earth's gravitational force, which acts on any object with mass, pulls the airplane toward the ground. 4) The air pushing up under an airplane's wings provides lift, which counteracts the gravitational force and keeps the plane airborne. The distance, height, and duration for which a paper airplane flies are determined by the relationship between thrust vs. drag and lift vs. gravity.

Figure 2: The 4 Forces that Act on a Paper Airplane



In this experiment, the goal is to hold the effect of thrust and gravity constant and change the various aspects of the paper airplane design (paper size, paper type, and airplane design) to alter the effects of lift and drag to see which factors most significantly affect the distance (measured in inches, rounded to the nearest half-inch, and calculated using the part of the paper airplane that traveled the furthest) that a paper airplane flies. The central question of this experiment is: which levels of the following factors (paper size, paper type, and airplane design) produce a paper airplane that flies the furthest?

Table 1: Factors and Factor Levels

Factor	**Level**	
	(-)	**(+)**
Airplane Design (A)	The Racer	The Classic Dart
Paper Type (B)	Lined Notebook Paper	Blank Printer Paper
Paper Size (C)	11in x 17in	8.5in x 11in

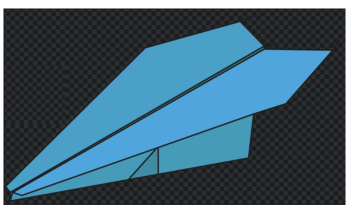


Figure 3: The Classic Dart

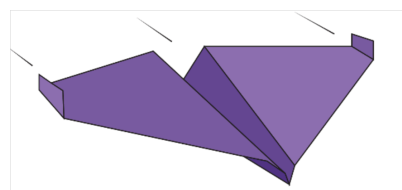


Figure 4: The Racer

Experimental Design

This experiment will analyze the flight distance of paper airplanes using a full factorial design at two levels, with the three factors and their levels as given in Table 1. Each factor-level combination has three replicates, and analysis is run on the mean and variance of the three replicates. There are 2^3 factor-level combinations and eight runs at each combination, resulting in 24 trials overall. This experiment design allows for a comprehensive analysis of each factor's location and dispersion effects and insight into all factor combinations' main effects and interaction effects. The planning matrix given in Table 2 provides an outline of the run order for each of the factor-level combinations.

Table 2: Planning Matrix

Run	**Airplane Design**	**Paper Type**	**Paper Size**
1	Classic Dart	Lined Notebook	8.5in x 11in
2	Classic Dart	Lined Notebook	11in x 17in
3	Classic Dart	Blank Printer	8.5in x 11in
4	Classic Dart	Blank Printer	11in x 17in
5	The Racer	Lined Notebook	8.5in x 11in
6	The Racer	Lined Notebook	11in x 17in
7	The Racer	Blank Printer	8.5in x 11in
8	The Racer	Blank Printer	11in x 17in

Statistical Equations and Methods Overview

To evaluate the effects of each factor and interaction, we will use the following equations to calculate the mean and variance of each factor level combination *i,* each replicate *j*, and the number of replicates for each combination *n*:

$$\bar{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij} \quad (1) \qquad\qquad s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_i)^2 \quad (2)$$

The main effects, 2-factor interaction effects, and 3-factor interaction effects will be calculated using the following equations: where ȳ is the average response at a specific factor level or conditional combination of factor levels. The dispersion effects will be calculated using the same equations with the variance estimates.

1. ME(A) = ȳ(A+) - ȳ(A-) (3)
2. INT(A,B) = ½{(ȳ(A+|B+) + ȳ(A-|B-)}- ½{(ȳ(A+|B-) + ȳ(A-|B+)} (4)
3. INT(A,B,C) = ½{INT(A,B|C+) - ½{INT(A,B,C-) (5)

Finally, the significance of these effects will be calculated in 2 different ways to provide both qualitative and quantitative analysis.

1) Initial insight into the effects of the interactions between 2 different factors will be evaluated using two-factor interaction plots to qualitatively assess how the combination of 2-factor levels affects the response of distance traveled.
2) To provide further analysis of the significance of the different main effects and interaction effects, I will use Lenth's Method for Effect Significance to assign quantitative values to the estimates using the following steps:
    1. Compute the pseudo-standard error (PSE), where $|\hat{\theta}_i|$ is the absolute value of the effect/interaction estimate and

       $PSE = 1.5 \cdot \text{median}_{\{|\hat{\theta}_i| < 2.5 s_0\}} |\hat{\theta}_i|$, (6)
    2. Compute the $t_{PSE,\,i}$, for each main and interaction effect estimate i

       $t_{PSE,i} = \frac{\hat{\theta}_i}{PSE}$ (7)
        a) If $|t_{PSE,\,i}|$ exceeds the IER critical value at the specified significance level α for I = the number of main and interaction effects, then the estimate is considered significant at the specified level α

Analysis and Results

For the analysis of the results, I used *R* to efficiently compute the mean and variance values (using equations (1) and (2)), create the interaction plots, and calculate the effect significances using Lenth's method. Table 3 shows the complete design matrix, including the design setup, the response values for each of the three replicates, and the calculated mean and variance values for each factor level combination, rounded to 2 decimal points

Table 3: Model Matrix

Run	Airplane Design	Paper Type	Paper Size	Response: Airplane Distance Traveled (in)			$\bar{y}$	s^2	$\ln(s^2)$
1	+	-	+	123.0	90.5	185.0	132.83	2305.08	7.74
2	+	-	-	252.0	293.0	213.0	252.67	1600.33	7.37
3	+	+	+	198.0	161.0	255.0	204.67	2242.33	7.72
4	+	+	-	288.0	397.0	267.5	317.67	4885.08	8.49
5	-	-	+	109.0	56.5	54.0	73.17	964.58	6.87
6	-	-	-	76.0	90.5	60.0	75.50	232.75	5.44
7	-	+	+	142.0	216.0	190.5	182.83	1413.08	7.25
8	-	+	-	177.5	157.0	124.0	152.83	728.58	6.59

As shown in Table 3, run 2 and run 4 have the highest mean flight distances (252.67in and 317.67 in, respectively). The factor-level combinations utilize the Classic Dart airplane design and the smaller paper size. It is worth noting that the variances for these two runs are high: run 4, specifically, has the highest variance out of all eight factor-level combinations. These high variances indicate inconsistencies in the results and that the longer flight distances could be due to unmeasured factors. Runs 5 and 6 have the two smallest mean flight distances (73.17in and 75.5in, respectively). The Racer airplane design and the lined notebook paper type are used in both designs, suggesting that the negative factor level for these factors contributes to a shorter flight distance. These two runs have two of the smallest, with run 6 corresponding to the lowest variance of all eight factor-level combinations. These small variance values suggest that these factor-level combinations have a more robust effect on the flight distances, especially when compared to runs 2 and 4 and their corresponding factor-level combinations.

In Table 4, we can see all of the location and dispersion estimates, using equations (3), (4), and (5), for the main effects, two-factor interactions, and three-factor interactions.

Table 4: Factorial Effects

Effect	**Location ($\bar{y}$)**	**Dispersion ($\ln(s^2)$)**
A	105.875	6.869
B	80.958	6.255
C	-51.292	4.178
AB	-12.542	5.651
AC	-65.125	6.039
BC	9.792	6.051
ABC	-6.375	6.022

Given the values in the table, airplane design (A) has the largest effect on the distance flown. Paper type (B) and paper size (C) also significantly affect the distance flown. Regarding interactions, the only significant effect that impacts the response is the interaction between A and C. However, the dispersion estimates for ME(A) and ME(B) are unusually high, indicating that these results may be inconclusive. The optimal factor levels for the airplane design, paper type, and paper size are the Classic Dart (A+), blank printer paper (B+), and 11in by 17in (C-). Still, it is worth noting that the high variance diminishes the confidence of these results.

The results in Figure 5, showing the half-normal plots for each main and interaction effect, echo the previous sentiment regarding significant factors. The plot indicates that main effect A is most important as it varies most from the other points. Additionally, main effects B and C and interaction effect AC appear significant due to their placement in the plot.

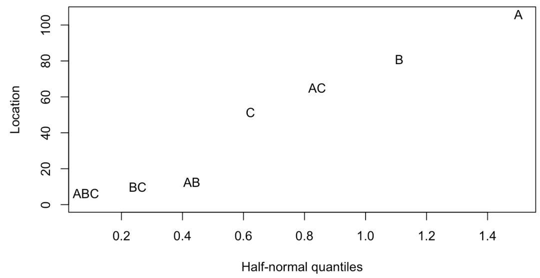


Figure 5: Half-normal Plot of Location Effects

The following six plots show the interaction between all of the two-factor combinations.

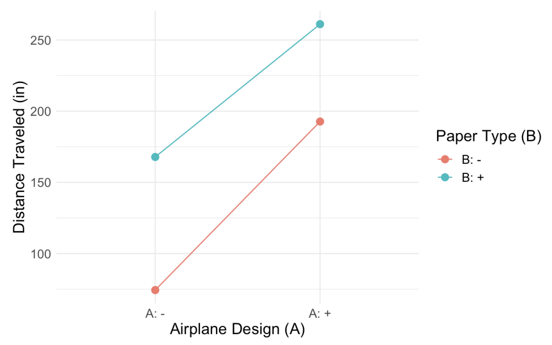


Figure 6: Airplane Design vs. Paper Type

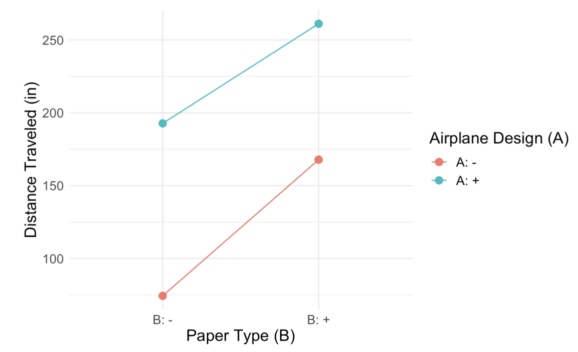


Figure 7: Paper Type vs. Airplane Design

Figures 6 and 7 show the relationship between Airplane Design and Paper Type. These plots indicate that factors A and B should be set to the positive factor level to increase the flight distance. The A vs. B plot and the B vs. A plot show a weak antagonistic relationship, indicating that the relationship may be more complex than the data reveals.

Figure 8: Airplane Design vs Paper Size

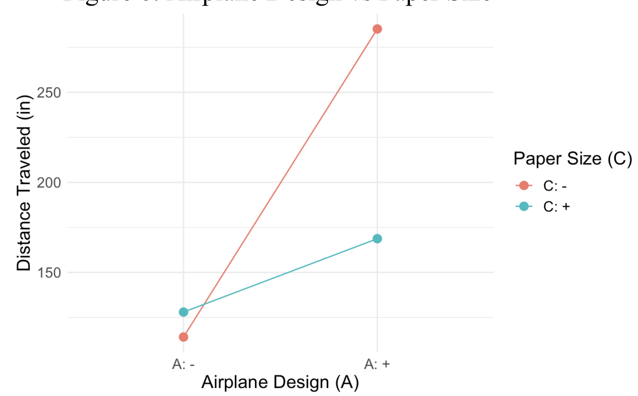
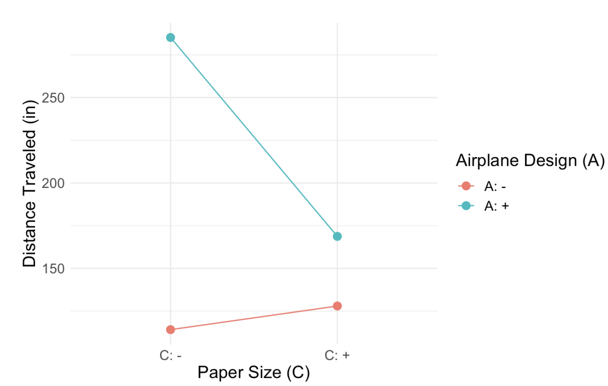


Figure 9: Paper Size vs Airplane Design



Figures 8 and 9, which show the plots of interaction between airplane design and paper type, contradict the previous notion that the interaction effect of AC is significant. In both plots, the 2 points with the highest distance traveled measurement corresponds to a positive level of A, indicating that the significance of INT(A, C) may rely on the previously established significance of ME(A).

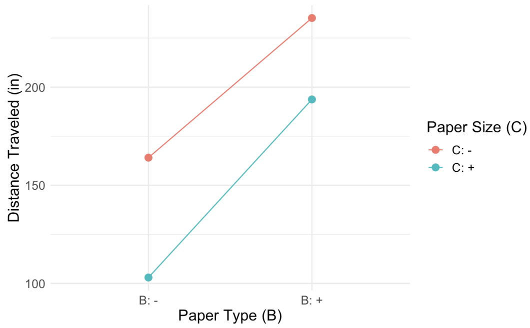


Figure 10: Paper Type vs Paper Size

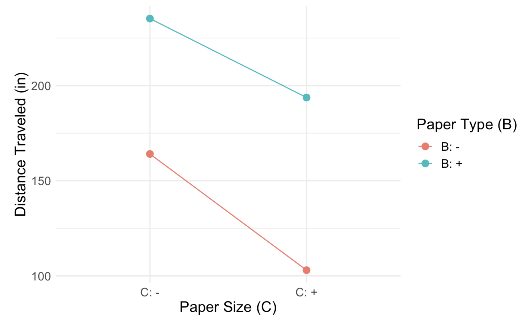


Figure 11: Paper Size vs Paper Type

Figures 10 and 11 depict the interaction plots between factors B and C. While Figure 10 shows a slightly antagonistic relationship, Figure 11 shows a synergistic effect. However, similarly to Figures 8 and 9, the two points corresponding to the highest distance traveled statistic are when B is at the + factor level, reaffirming the takeaway that ME(B) is significant while ME(C) and INT(B, C) are less significant, if significant at all.

Finally, effect significance will be evaluated quantitatively using Lenth's method, as outlined in equations (6) and (7) above. Table 5 displays the results using Lenth's method at both $\alpha = 0.05$ and $\alpha = 0.2$.

Table 5: Lenth's Method Results

Effect	**t_{PSE}**	**Significance at α = 0.05**	**Significance at α = 0.2**
A	1.376	False	True
B	1.052	False	False
C	0.667	False	False
AB	0.163	False	False
AC	0.846	False	False
BC	0.127	False	False
ABC	0.083	False	False

The IER value associated with $I = 7$ and $\alpha = 0.05$ is 2.3, and the IER value associated with $I = 7$ and $\alpha = 0.2$ is 1.2. Table 5 indicates that at the standard α of 0.05, none of the effects are significant, and the results are dictated by extraneous variables not measured in this experiment. Additionally, Lenth's method is evaluated at $\alpha = 0.2$ because this is the lowest α at which any effect estimate is significant. These results indicate that none of the factors, Airplane Design, Paper Type, and Paper Size, are largely significant when evaluating the distance traveled response variable. However, if one effect did affect the response, it is likely airplane design due to ME(A)'s significance at $\alpha = 0.2$.

Discussion

The main effect and interaction effect estimates, the half-normal plot for location, and the interaction plots clearly show that main effect A is the most significant effect with main effects B and C, and interaction effect BC are somewhat significant. However, when evaluating these effects using Lenth's method, none of the effect estimates are significant at α of 0.05. In fact, the lowest α at which any effect is significant is $\alpha = 0.2$, where only main effect A is valid. Given the values in the model matrix (Table 3), the optimal design for factors A, B, and C is the Classic

Dart folded on 11in x 17in blank printer paper. However, given the high variance, a more robust design would be the Classic Dart folded on 11in x 17in lined notebook paper with a high average distance traveled and a lower variance. While this second factor-level combination produces an average distance of 65 inches shorter, the variance is over 3200 units smaller. We can also conclude that the worst two factors-level combinations consist of the Racer folded on 8.5in x 11in lined notebook paper and the Racer folded on 11in x 17in lined notebook paper, corresponding to the two lowest average distances traveled and two of the lowest variances.

Multiple sources of error may have affected the results. First, the environment and process of throwing a paper airplane are very error-prone. While the experiment was conducted inside, any air pressure and movement shift would have significantly altered the results. Paper airplanes are incredibly light, and wind movements were unaccounted for, which affected any of the aerial forces on the design, which likely skewed the results. Additionally, there are multiple sources of user error. When throwing each paper airplane, I attempted to standardize the amount of thrust by extending my arm from 90° to 180°. However, no measurement was taken to determine the amount of thrust given for each individual trial. Another unaccounted-for factor is the quality of the airplane's design. While constructing the Classic Dart design is standard and straightforward, the Racer was much more complex and took multiple attempts to build a viable design. Therefore, the higher quality of the Classic Dart paper airplanes may have resulted in long response distances compared to the lower-quality Racer designs. While ease of creation may be valuable when considering which type of paper airplane to construct, the design quality was not considered. Thus, the significance of the effect of airplane design may be confounded by the differences in quality of the two designs. These factors likely contributed to the vast discrepancies in each run’s three replicates. For example, while run 3 yielded the highest mean response, the range of replicate responses is over 130 inches. This trend is also visible in the other runs and can be seen numerically in the variance measurements, which are exceptionally high even given the high response values.

In the future, the extraneous factors listed above should be considered when determining the most critical effects on paper airplane flight distance. One way to improve this experimental design would be to use a machine to throw the paper airplanes instead of myself. This procedural change would remove the human error caused by unidentical amounts of thrust for each run and minimize one of the most significant outside factors. Additionally, while most experiments of this level do not have the means or budget to completely stabilize the airflow in the room when running trials, using a more confined room with fewer airflow entry points could minimize the effect of inconsistent wind flow during the trials.

Another experiment using more factor levels should be used to reaffirm further or discount the results. Factor levels using cardstock and parchment papers could further investigate the effect of paper type. Notecards and poster-sized paper could be used to gain more insight into how paper size affects flight distance. Other paper airplane designs could be used to determine which design or type of design maximizes flight distance. Of course, new planning matrices would be needed to conduct these follow-up experiments.

Finally, this experiment provided an additional understanding of the intricacies of full factorial designs. While the equations and analyses can be complex, ensuring that the experimental environment is as constant as possible can be more demanding. Extraneous variables greatly impacted the results, reemphasizing the discrepancy between working through theoretical statistical problems and applying these concepts in real-world scenarios.

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