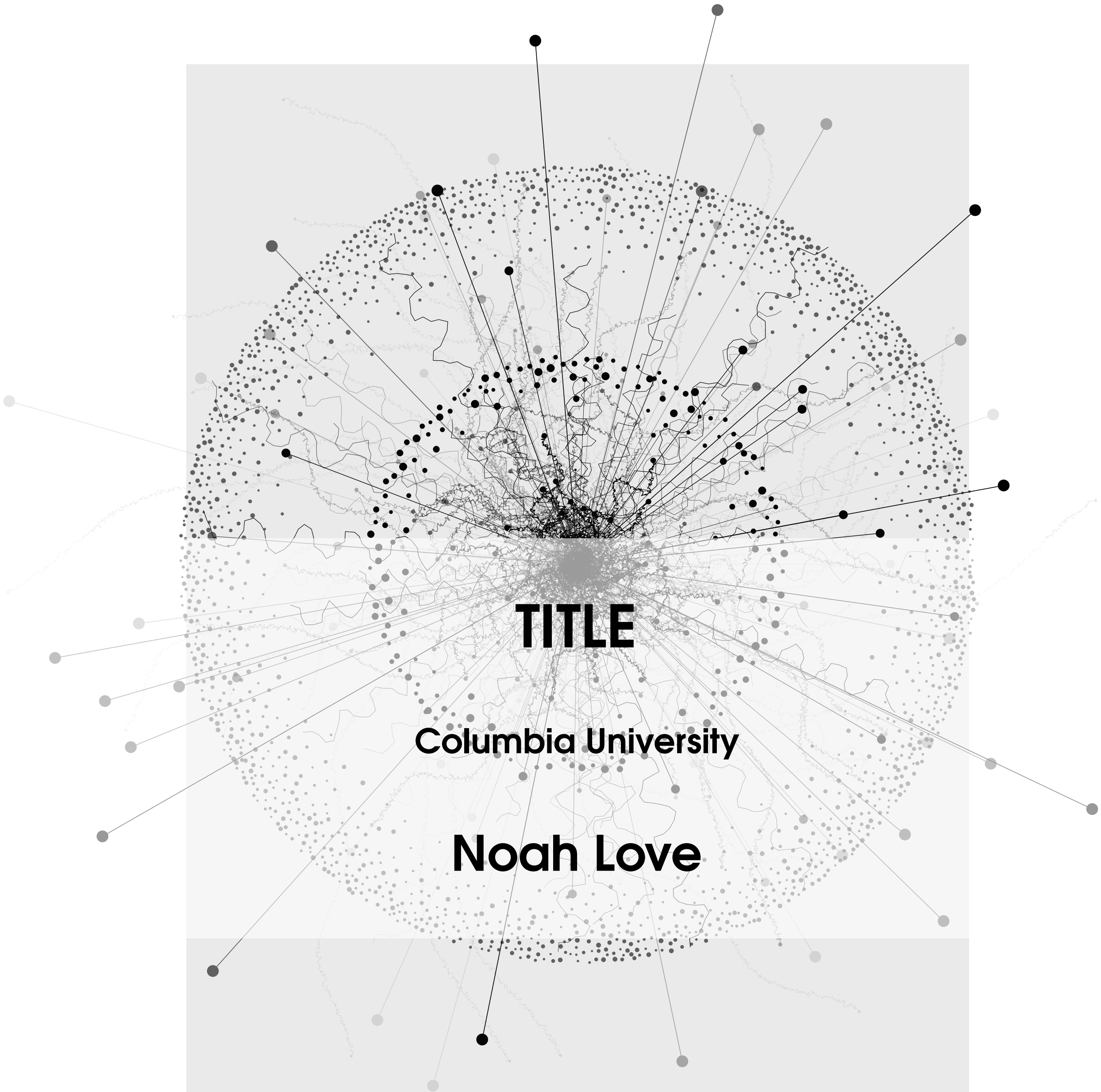


TITLE

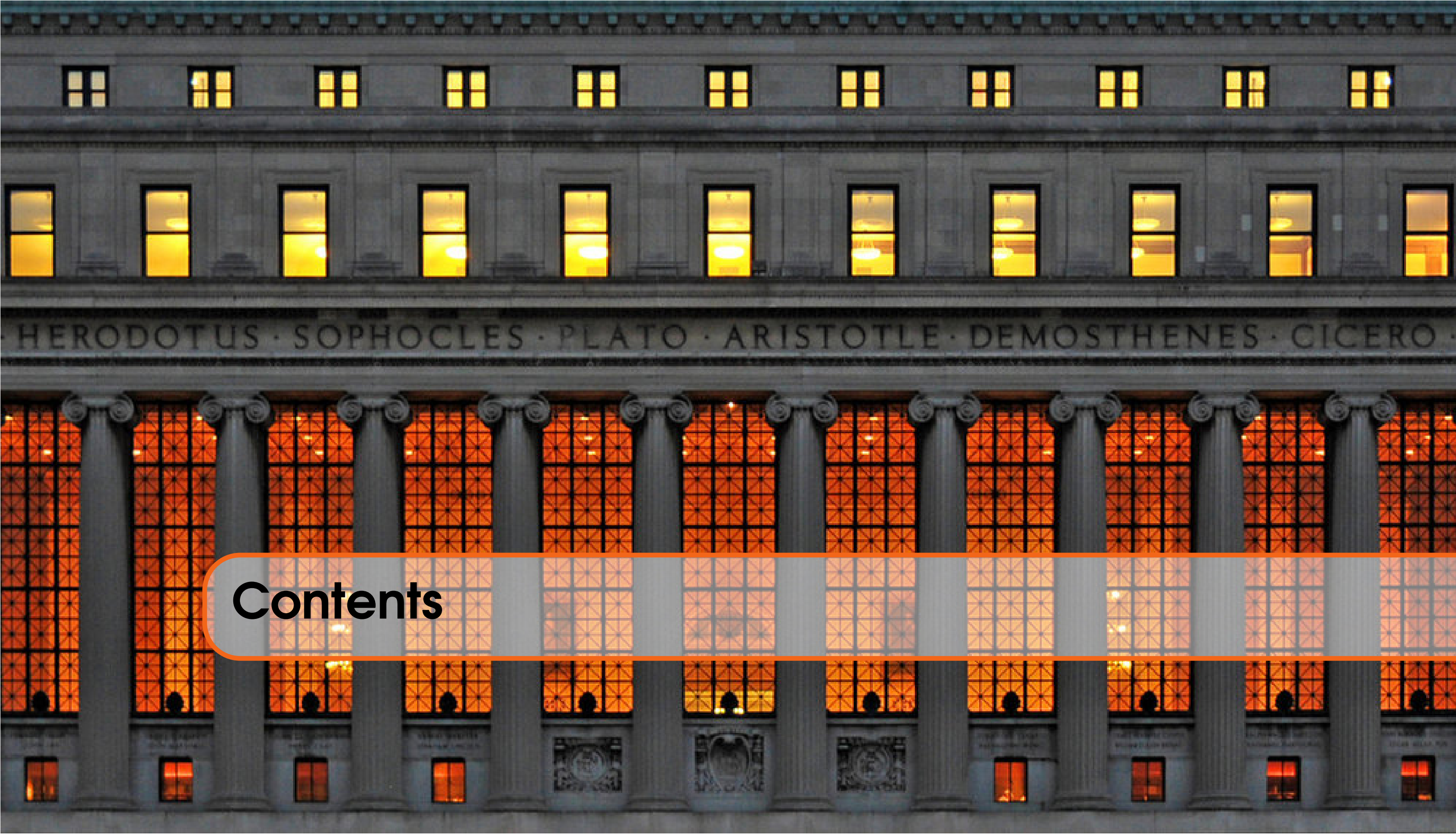
Columbia University

Noah Love

CLASS TAUGHT BY PROFESSOR NAME

NOAH.LOVE@COLUMBIA.EDU

Columbia University YEARS
Written in LaTeX



Contents

1. Math Review

1.1 Notation

This is an important section so that you can understand all of the following information. Although typically written in English, mathematicians have their own language and that is a language of signs.

The most important symbol to know is:

$$\mathbb{Z}$$

$\leq$	`\leq`	$\geq$	`\geq`	$\neq$	`\neq`	$\approx$	`\approx`
$\times$	`\times`	$\div$	`\div`	$\pm$	`\pm`	$\cdot$	`\cdot`
$^{\circ}$	`^{\circ}`	$\circ$	`\circ`	$\prime$	`\prime`	$\cdots$	`\cdots`
∞	`\infty`	$\neg$	`\neg`	$\wedge$	`\wedge`	$\vee$	`\vee`
$\supset$	`\supset`	$\forall$	`\forall`	$\in$	`\in`	$\rightarrow$	`\rightarrow`
$\subset$	`\subset`	$\exists$	`\exists`	$\notin$	`\notin`	$\Rightarrow$	`\Rightarrow`
$\cup$	`\cup`	$\cap$	`\cap`	$\mid$	`\mid`	$\Leftrightarrow$	`\Leftrightarrow`
$\dot{a}$	`\dot a`	$\hat{a}$	`\hat a`	$\bar{a}$	`\bar a`	$\tilde{a}$	`\tilde a`
α	`\alpha`	β	`\beta`	γ	`\gamma`	δ	`\delta`
ϵ	`\epsilon`	ζ	`\zeta`	η	`\eta`	ε	`\varepsilon`
θ	`\theta`	ι	`\iota`	κ	`\kappa`	ϑ	`\vartheta`
λ	`\lambda`	μ	`\mu`	ν	`\nu`	ξ	`\xi`
π	`\pi`	ρ	`\rho`	σ	`\sigma`	τ	`\tau`
υ	`\upsilon`	ϕ	`\phi`	χ	`\chi`	ψ	`\psi`
ω	`\omega`	Γ	`\Gamma`	Δ	`\Delta`	Θ	`\Theta`
Λ	`\Lambda`	Ξ	`\Xi`	Π	`\Pi`	Σ	`\Sigma`
Υ	`\Upsilon`	Φ	`\Phi`	Ψ	`\Psi`	Ω	`\Omega`

2. Review of Linear Algebra

2.1 Matrices

Definition 2.1.1 A $m \times n$ matrix is a rectuangular array with m rows and n columns:

$$\mathbf{A} = (a_{ij})_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \tag{2.1}$$

Above we have $m \times n$ individual entries, each of which can be identified by a_{ij} where i is the row and j is the column it is located in.

THis is a test to see if it ever redoes it. Does this work?

2.1.1 Matrix Operations

Let's assume that we have two equal dimension matrices:

$$\mathbf{A} = (a_{ij})_{m \times n}, \quad \mathbf{B} = (b_{ij})_{m \times n} \tag{2.2}$$

2.2 Partitioned Matrices

2.3 Inverses

2.4 Linear Independence

2.5 Rank of a matrix

2.6 Results from Linear Systems

2.7 Eigenvalues and Eigenvectors

Definition 2.7.1 An eignevector of $A_{n \times n}$ is a nonzero vector x satisfying

$$Ax = \lambda x \tag{2.3}$$

for some $\lambda \in \mathbb{C}$. The associated value λ is called the eigenvalue.

Example 2.1

$$A = \begin{pmatrix} a_{11} & a_{12} \\ 1_{21} & a_{22} \end{pmatrix} \Rightarrow (A - \lambda I)x = 0 \tag{2.4}$$

For this to be true, $(A - \lambda I)$ must have a non-zero determinent. In the case above:

$$\begin{pmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{pmatrix} = \lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0$$

Then for the roots, we have:

$$\lambda_1 + \lambda_2 = a_{11} + a_{22} = \operatorname{tr}(A) \tag{2.5}$$

$$\lambda_1 \lambda_2 = a_{11}a_{22} - a_{12}a_{21} = \det(A) \tag{2.6}$$

■

Example 2.2

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} \Rightarrow \lambda^2 - \lambda - 6 = 0 = (\lambda - 3)(\lambda + 2)$$

From this, we know the λ values are 3 and -2. From here, we have the equations:

$$x_1 + 2x_2 = 2x_1$$
$$3x_1 + 0 = 3x_2$$

From this: we see that $x_1 = x_2$ meaning there is an infinite amount of eigenvectors. They can be written generally as:

$$x = \begin{pmatrix} t \\ t \end{pmatrix}$$

For the case where $\lambda = -2$ we have the equations:

$$x_1 + 2x_2 = 2x_1$$
$$3x_1 + 0 = -2x_2$$

Then we get a similar equation:

$$x = \begin{pmatrix} -2/3t \\ t \end{pmatrix}$$

Again there is an infinite amount because you can multiple by a scalar. ■

In general:

$$Ax = \lambda x \iff (A - \lambda I)x = 0 \iff det(A - \lambda I) = 0 \tag{2.7}$$

Definition 2.7.2 — characteristic polynomial.

$$p(\lambda) = det(A - \lambda I) = (-\lambda)^n + b_{n-1}(-\lambda)^{n-1} + \cdots + b_1(-\lambda) + b_0$$

The zeros of the characteristic polynomial are precisely the eigenvalues of A.

2.8 Quadratic Forms

2.9 Quadratic Forms with Linear Constraints