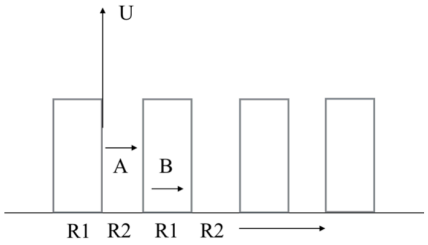
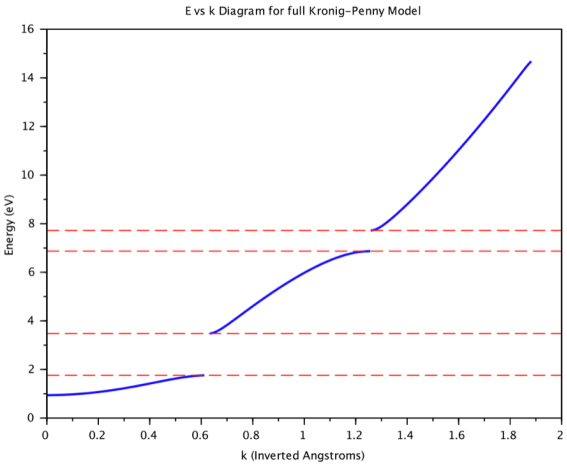
Problem 1.

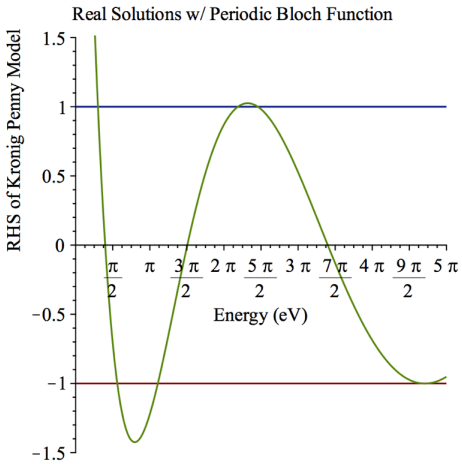
The Kronig Penny model is an application of the time independent Schrodinger equation for an electron in a potential well but with a periodicity accounted for by the Bloch function. That is, the Kronig Penny model is a solution to the Schrodinger equation for a series of equally spaced potential wells (1D in our case)- see the pictorial below. Deriving the KP relation is [theoretically] straightforward- one solves the potential well problem and arrives at a product exponentials and a periodic Bloch function. By substituting this result back into the SE and accounting for boundary conditions, the KP relation is found. However, it cannot be solved algebraically and therefore must be solved numerically. This is done by realizing that in order for solutions to exist, the domains and ranges of both the right and left hand side of the relation must be equal. By plotting the RHS and LHS as a function of energy one arrives at the series of curves given below. By solving for the points where the range of the RHS exceeds that of the LHS we can determine the limits for k and therefore the boundaries of the BZ. By taking these ranges we can solve for the corresponding wavenumber and arrive at the familiar E.K diagram as given below. One important result of this analysis is the realization that the restrictions on k (because of the incommensurate ranges of the RHS and LHS) also restrict the values of E that are allowed for the system. Because we solved the SE for a series of potential wells rather than just a single one this gives rise to a quasi-continuous "band" of allowed energy levels. The values of k at which a jump discontinuity in E occurs are referred to Brillouin Zones. Another observation is that as k increases the magnitude of the forbidden energy regions decreases and the system approaches a fully continuous range of allowed energies as predicted by classical models.



Periodic potential wells



Resulting EK diagram when numerically solving for k.



Incommensurate ranges of RHS and LHS of KP model.

Problem 2.

a.

A purity of 1 in 10^11 (10 ppt) is pretty freaking pure. As such, I think for this problem it's safe to approximate the material as an intrinsic semiconductor. Plus, the problem doesn't state anything about the nature of the dopant (impurities). That is, if the problem really wanted us to assume anything besides intrinsic semi conductivity, it would have told us whether or not we're dealing with electron acceptors or donors- something we need to know because the mobility of holes and electrons are not equal. From table 5.1 the concentration of free electrons in Ge is 2.3*10^13/cc. We also know this to be the concentration of holes in the valence band. The mobility of electrons and holes respectively are 3900 and 1900 cm^2/Vsec (@ 300K).

$$\sigma = en_i(\mu_e + \mu_h)$$

$$\sigma = 1.6*10^{-19}*2.3*10^{13}*(3900+1900) = 21.34\_m\Omega^{-1}cm^{-1}$$

b.

From table 5.1 Nc=1.04*10^19/cc and Nv=6*10^18/cc. The band gap is 0.66 eV. k=8.62*10^-5 eV/K. Using equation 5.20 from the book and taking T=300K we see we get n=n[i] which is what we would expect. Plugging in T=450K:

$$n = \sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}}$$

$$n = \sqrt{1.04*10^{19}*6*10^{18}}\, e^{\frac{-0.66}{2*8.62*10^{-5}*450}}$$

$$n \approx 1.60*10^{15}\ /\ cc$$

$$\sigma = en_i(\mu_e + \mu_h)$$

$$\sigma = 1.6*10^{-19}*1.60*10^{15}*(3900+1900) = 1.48\_\Omega^{-1}cm^{-1}$$

c.

The important assumption here is that we are assuming full ionization of dopant atoms. Phosphorous is an electron donor to the system, so it's concentration in the Ge will be equal to the concentration of free electrons (e0). Then we can solve for the concentration of holes (p0). We will, as usual, let the conductivity equal the sum of the products of electron charge, concentration and mobility:

$$e_0 = 10^6\ /\ cc$$

$$p_0 = \frac{n_i^2}{e_0} = \frac{(2.3*10^{13})^2}{10^6} = 5.3*10^{20}\ /\ cc$$

$$\sigma = e(n_e\mu_e + n_h\mu_h)$$

$$\sigma = 1.6*10^{-19}\left(10^6*3900 + 5.3*10^{20}*1900\right) = 161.1\_k\Omega^{-1}cm^{-1}$$

Problem 4.

a.

In this region the Si is doped with B, an electron acceptor, making this region a p-type semiconductor. The Fermi energy is calculated as follows. First we need to know the concentration of dopants (10^18/cc) and then use the law of mass action to find electron concentration:

$$e_0 = \frac{n_i^2}{p_0} = \frac{(1*10^{10})^2}{1*10^{18}} = 10^2$$

$$\therefore$$

$$e_0 << p_0, n_0 \approx p_0$$

Fermi level and charge carrier concentration are linked by the following equation:

$$n_0 = n_i e^{\frac{E_f - E_i}{kT}} \Rightarrow E_f - E_i = kT \ln\left(\frac{n_0}{n_i}\right)$$

And taking Ei as zero:

$$-E_i = kT \ln\left(\frac{n_0}{n_i}\right)$$

Plugging in T=300K, 10^18/cc Boron dopants and the intrinsic concentration of Si, 10^10:

$$E_f = 8.62*10^{-5}*300 \ln\left(\frac{10^{18}}{10^{10}}\right) \approx -0.48\_eV$$

Thus, in the p-type region the Fermi energy is ≈0.48 eV lower than in pure Si.

b.

Since the semiconductor has been compensation doped with an excess amount of electron donors compared to acceptors there is a net increase in the concentration of e-. This increase is equal to the differences of the concentrations of the dopants assuming full ionization. Therefore, this region is an n-type extrinsic semiconductor with an electron concentration as follows:

$$e_0 = [P] - [B] = 4*10^{18} - 10^{18} = 3*10^{18} / cc$$

Using the same logic as in part a:

$$p_0 = \frac{n_i^2}{e_0} = \frac{(1*10^{10})^2}{3*10^{18}} = \frac{100}{3} / cc$$

$$p_0 << e_0, n_0 \approx e_0$$

$$n_0 = n_i e^{\frac{E_f - E_i}{kT}} \Rightarrow E_f - E_i = kT \ln\left(\frac{n_0}{n_i}\right)$$

$$E_f = 8.62 * 10^{-5} * 300 \ln\left(\frac{3*10^{18}}{10^{10}}\right) \approx 0.51\_eV$$

c.

The contact potential at the junction is given as the difference in work functions between the two materials. We will define the magnitude of the work function for pure Si as x such that the work function for the p type material is x+0.51 eV and the work function for the n type is x-0.48 eV. Taking the difference of these work functions:

$$(x + 0.51) - (x - 0.48) = 0.51 + 0.48 = 0.99\_eV$$

d.