RADIATION PATTERN OF A LIGHT-EMITTING DIODE

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1 INTRODUCTION

The intention of this experiment is to characterize the radiation pattern of an LED empirically and verify it analytically. To do this, we make a few critical assumptions.

1. We treat the LED as uniform planar light source.
2. The radiation pattern of the LED is independent of angle ϕ.
3. The experiment is performed at a scale such that the we can treat LED as a point source and the aperture area as differential.

We mounted an LED on a rotating platform and recorded power perceived by a power detector at chosen distance R from the LED for angles between -90° and 90°. Using this data, we are able to determine the total power emitted by the LED and compare it with our analytically derived expected results. We refined our model in an attempt to account for numerical discrepancies between the expected and observed results. Unfortunately, as we were in a hurry, we forgot to record our station number. However, examination of the rigorous procedure used to setup the apparatus leads us to conclude that any variation in between our results and the correct answer would be due to variation in LEDs. Support for this conclusion is offered in the Analysis section.

2 DATA

2.1 Collection Methodology

The following precautions were taken to minimize any error introduced in collecting data.

- We set the aperture radius precisely by closing it flush around a nominally 0.7 mm stick of lead. We later measured the section of the lead stick that we inserted in the aperture with a caliper and found it was actually 0.69 mm in diameter.
- We re-purposed multimeter boxes - which we used as four black walls surrounding our apparatus, minimizing variations in light noise which we noticed could be attributed to movements of the observer. The resulting change in millivolts which corresponds to power detected after installation of the black walls was notable, having decreased by about 20 mV. Inevitable background radiation was still present, but having audited previous measurements at certain angles at random, and finding no difference, we safely conclude the background radiation was effectively constant, which we account for in our subsequent computations.
- We once again used an ordinary pencil to precisely determine the radius of the hemisphere, that is, the distance from the the approximate center of the LED where light is emitted, to the sensor plate within the power detector. We place the pencil's eraser's tip on the curved tip of the LED and the other end of the pencil towards the aperture of the power detector. We then clicked the pencil so that the pencil's lead extended past the opening of the aperture just 5 mm in front of the sensor. We then used a caliper to measure the length of the extended pencil and added 4 mm back to account for the radius of the LED, and another 2 mm for the distance from the lead tip to the sensor plate.

2.2 Collected Data

The raw collected voltage measurements can be found in Appendix A The most important parameters that we measured are listed below in Table 1.

Table 1: Measured Parameters

Parameter	Value
Aperture Diameter	0.69 mm
Aperture Area	$0.374\,\text{mm}^2$
Distance from LED to Sensor ("R")	153.63 mm
Maximum Recorded Voltage	527.7 mV
Maximum Recorded Power	1.0352 µW
Resistor Voltage	6.54 V
Battery Voltage	9.00 V
Bias Resistance	1.182 kΩ
Ambient Noise	5.1 mV

3 RESULTS

3.1 Normalized Voltages

A table containing Normalized Voltages can be found in Appendix B. To normalize our data, we subtracted the ambient voltage (5.1 mV) from all of our measured values and then divided by the maximum voltage observed (517.6 mV at $\theta = -14°$).

3.2 Raised Cosine Parameter

The value of the Raised Cosine Parameter n was found using the MATLAB Code in Appendix C. We read in our data, and normalized it, and found the best least squares fit for our data using the equation:

$$\text{error} = \frac{1}{N}\sum_{i=1}^{N}[V_{norm}(\theta_{si}) - \cos^n(\theta_{si})]^2$$

We used a range of exponents ranging from 0 to 100 with a 0.1 step size and found the best fit exponent to be 33.1 with a normalized error of 0.0077.

3.3 Peak Radiation Intensity

To find the approximate peak radiation intensity, we used the equation:

$$K_0 \approx \frac{P_0}{A_r}R^2 \;\; [W/sr]$$

which uses the maximum power P_0, determined from our measurements to be 1.0352 µW, the aperture area measured as $0.374\,\text{mm}^2$, and the distance R from the LED to the aperture of 153.63 mm.

3.4 Power and Efficiency

To calculate the power supplied to the LED, we measured the voltage across a resistor of know value. The resulting power consumed by the LED is $(9V - 6.54V) * (.0005A) = 13\,\text{mW}$. We also numerically computed the power emitted by the LED in the hemisphere facing the apparatus using the equation provided below.

$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}} K(\theta)\cdot\sin(\theta)d\theta d\phi$$

We obtained an observed total power of 4.7 mW. Thus, the output efficiency of our LED is $\frac{4.7}{13} = 36$. Various resources in academia confirm that this is not unreasonable.

$$P = \frac{2\pi K_0}{n+1}\ [W]$$

$$P_s = IV$$

3.5 Resultant Data

Table 2: Calculated Results

Parameter	Value
Raised Cosine Exponent, n	33.1
Peak Radiation Intensity, K_0	66.2 mW/sr
Total Emitted Power, P_e	16.7 mW
Supplied Electrical Power, P_s	12.2 mW
Wall-Plug Efficiency	63.235%

4 DISCUSSION

Our results reveal two modes within our radiation pattern. Cursory research revealed a point source is a poor model for a domed LED. The semiconductor die that sits within the post-anvil structure inside the LED must be cut with saw cuts into a cuboid. The typical manufacturing processes for LED's yield dies with radiation patterns that are best characterized by 6 light cones emerging from a center in an octahedral fashion. The emitted power plot(Figure 1) reveals two modes - what actually appears to be the sum of two raised cosines centered left and right of zero degrees. An easy explanation for this might be two emission cones that are mirrored across the on-axis. This would result in the observed dip at 0 degrees. We propose a new model, the sum of two cosines shifted left and right of center to peak at the two maximums, to account for the two light cones. Using a least squares fit yielded

$$\cos^{127}(\theta - .1396) + \cos^{127}(\theta + .1396)$$

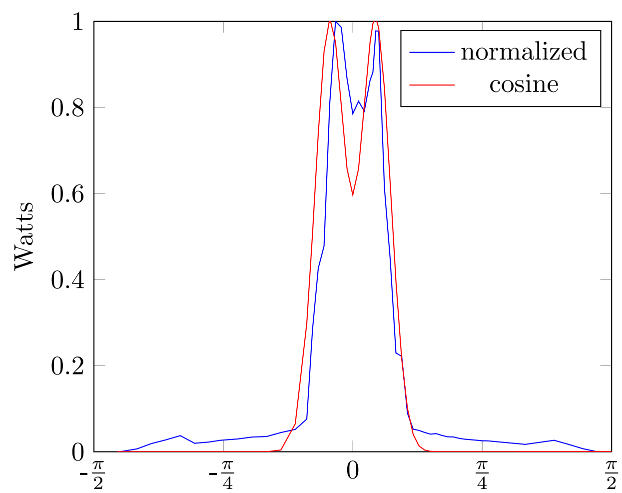

Figure 3: Observed Power (Normalized) vs.Predicted Power (Two Cone Model)

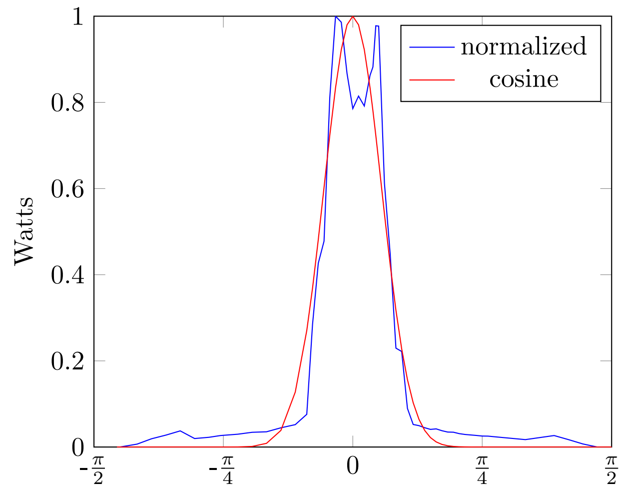

Figure 1: Observed Power (Normalized) vs. Predicted Power (Lambertian Cosine)

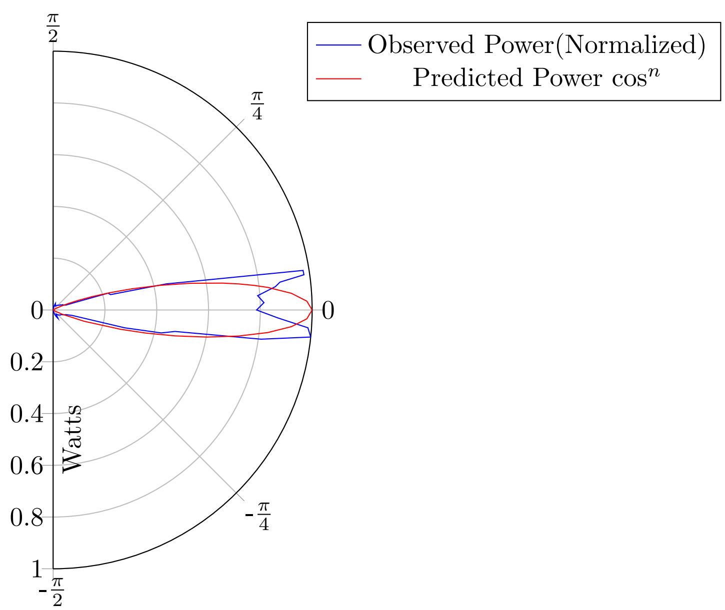

Figure 2: Observed Power (Normalized) vs. Predicted Power (Lambertian Cosine)

Appendices

Appendix A Collected Raw Data

Angle		Voltage (mV)	Angle		Voltage (mV)
Measured	Adjusted	Measured	Measured	Adjusted	Measured
-42	-90	4.5	49	1	510.8
-35	-83	8.6	51	3	322.1
-30	-78	15	53	5	237.1
-25	-73	19.4	55	7	123.9
-20	-68	24.5	57	9	119.9
-15	-63	15.3	59	11	51.5
-10	-58	16.8	61	13	32.2
-6	-54	18.9	63	15	30.8
0	-48	20.5	65	17	28.3
5	-43	22.8	67	19	26.3
10	-38	23.4	69	21	26.9
15	-33	28.2	71	23	24.7
20	-28	32	73	25	23.1
22	-26	35.6	75	27	22.9
24	-24	44.4	77	29	21.2
26	-22	152.3	79	31	20.2
28	-20	225.9	81	33	19.6
30	-18	252.4	83	35	19
32	-16	424.8	85	37	18.3
34	-14	522.7	87	39	18.2
36	-12	515.5	90	42	17.2
38	-10	453.8	95	47	15.6
40	-8	411.7	100	52	14
42	-6	426.7	110	62	18.9
44	-4	414.8	115	67	14
46	-2	451.7	120	72	8.8
47	-1	461.7	125	77	5.1
48	0	511.2	130	82	4.1

Appendix B Normalized Voltages

Angle		Voltage (mV)	Angle		Voltage (mV)
Measured	Adjusted	Normalized	Measured	Adjusted	Normalized
-42	-90	-0.0012	49	1	0.9770
-35	-83	0.0068	51	3	0.6124
-30	-78	0.0191	53	5	0.4482
-25	-73	0.0276	55	7	0.2295
-20	-68	0.0375	57	9	0.2218
-15	-63	0.0197	59	11	0.0896
-10	-58	0.0226	61	13	0.0524
-6	-54	0.0267	63	15	0.0497
0	-48	0.0298	65	17	0.0448
5	-43	0.0342	67	19	0.0410
10	-38	0.0354	69	21	0.0421
15	-33	0.0446	71	23	0.0379
20	-28	0.0520	73	25	0.0348
22	-26	0.0589	75	27	0.0344
24	-24	0.0759	77	29	0.0311
26	-22	0.2844	79	31	0.0292
28	-20	0.4266	81	33	0.0280
30	-18	0.4778	83	35	0.0269
32	-16	0.8109	85	37	0.0255
34	-14	1.0000	87	39	0.0253
36	-12	0.9861	90	42	0.0234
38	-10	0.8669	95	47	0.0203
40	-8	0.7855	100	52	0.0172
42	-6	0.8145	110	62	0.0267
44	-4	0.7915	115	67	0.0172
46	-2	0.8628	120	72	0.0071
47	-1	0.8821	125	77	0.0000
48	0	0.9778	130	82	-0.0019

Appendix C Matlab Code

```
close all
%% Conversions
radians = (angle/360)*2*pi;
nW = (mV - 5.1)*2;
W = nW*10^(-9);
WNorm = W/max(W); %Ask buck about this

%% Measured and Computed Parameters
apetureRadius = .69/2*10^-3;
sphericalRadius = (149.63+5.00)*10^-3;
Ar = pi*(apetureRadius^2);
P0 = max(W);
I0 = P0/Ar;
K0 = I0*(sphericalRadius^2);

%% Least Squares fit
N = length(radians);
x = 0:.1:100;
tempFit = zeros(N,1);
F_Least_Squares = zeros(length(x),1);

for index = 1:length(x)
%index = 2;
for i = 1:N
 tempFit(i) = (WNorm(i) - (cos(radians(i)) .^x(index))).^2;
end
F_Least_Squares(index) = (1/N)*sum(tempFit);
end
[minVal minIndex] = min(F_Least_Squares);
powerBestFit = x(minIndex)
smallestError = F_Least_Squares(minIndex)
%% Integral
angleIntegrand = radians(27:end);
WIntegrand = W(27:end);
integral_angle = ((WIntegrand*sphericalRadius^2)/Ar).*sin(angleIntegrand).*[diff(angleIntegrand); 0];
Total_power = 2*pi*sum(integral_angle)
Total_power_predicted = (2*pi*K0)/(powerBestFit +1)

 %% Plots

% Radians vs. Power Emitted
%Rad_vs_Power = figure;
%axes1 = axes('Parent',Rad_vs_Power);
%hold(axes1,'on');
%plot(radians,WNorm);
%hold on
%plot(radians,cos(radians).^powerBestFit);
%ylabel({'Power(Watts)'});
%xlabel({'Radians'});
%title({'Radians vs. Power Emitted'});
%hold off
```

```
% Least Squares Plot
figure2 = figure;
plot(x,F_Least_Squares)

% Radians vs. Power Emitted polar plot
figure3 = figure;
%axes1 = axes('Parent',Rad_vs_Power);
radiansplot = radians;
polarplot(radiansplot,WNorm);
hold on
polarplot(radians,cos(radians).^powerBestFit);
hold off
title({'Radians vs. Power Emitted Polar'});
```