CS50: Introduction to Artificial Inteligence Notes

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May 13, 2023

Abstract

CS50: Introduction to Artificial Inteligence notes. Note template by Pingbang Hu.

Contents

Chapter 1

Knowledge

1.1 Propositional Logic

We use standard logic notation:

- $\neg p$
- $p \vee q$
- $p \wedge q$
- $p \Rightarrow q$:

p	q	$p \Rightarrow q$
false	false	true
false	true	true
true	false	false
true	true	true

- $p \Leftrightarrow q$:

p	q	$p \Leftrightarrow q$
false	false	true
false	true	true
true	false	false
true	true	true

Now we must establish *what* is considered to be "true" in our world by defining a **model**. We need to represent that knowledge. We do so by defining it via a **knowledge base**.

Definition 1.1.1 (Model). Assignnment of a truth value to every propositional symbol.

Definition 1.1.2 (Knowledge Base). A set of sentences known by a knowledge-based agent.

Definition 1.1.3 (Enatilment).

$$\alpha \models \beta \text{ "}\alpha \textit{ entails } \beta\text{"}$$

In every model in which sentence α is true, sentence β is also true.

1.2 Inference

Our aim is to see if our knowledge base, KB, entails some query about the world, α:

$$KB \models \alpha?$$

We first define a **model checking algorithm** to determine if $KB \models \alpha$. We can determine this by doing the following:

- enumerate all possible models
- if in every model where KB is true, α is also true, then $KB \models \alpha$

1.3 Inference By Resolution

To determine if $KB \models \alpha$ via knowledge resolution:

- Check if $KB \wedge \neg\alpha$ is a contradiction
  - Convert $KB \wedge \neg\alpha$ to Conjunctive Normal Form
  - Keep checking to see if we can use resolution to produce new clause
  - If we ever produce the empty clause (equivalent to False), we have a contradiction and so $KB \models \alpha$
- If so, then $KB \models \alpha$
- Otherwise, no entailment

Problem 1.3.1. Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A ?

Answer. First, we convert to CNF:

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

We can resolve $(\neg B \vee C)$ and $(\neg C)$ by concluding that $\neg B$. With the knowledge of $\neg B$ we now see that, considering $A \vee B$, we can conclude A. We see that

$$A \wedge \neg A \Rightarrow False$$

and so we can conclude that the clause entails A. ⊛

Chapter 2

Probability

2.1 Introduction to Probability

We can represent a **possible world** using ω where all possible worlds is the set Ω . Thus, we can define total probability as:

$$\sum_{\omega \in \Omega} p(\omega).$$

We build machine models to predict outcomes based on data using **conditional probability**. In other words, we have probability based on some evidence. We can calculate conditional probability using the following formula:

$$P(a|b) = \frac{P(a \land b)}{P(b)}.$$

2.2 Random Variables

In probability we define a variable with a set of possible values as a **random variable**:

> **Definition 2.2.1** (Random Variable)**.** A variable possessing a distribution of probabilities for various "states".

2.3 Baye's Rule

From the above equation we have:

$$P(a \land b) = P(b)P(a|b) = P(a)P(b|a)$$

> **Definition 2.3.1** (Independence)**.** The knowledge that one event occurs does not affect the probability of the other event. So we have
> $$P(a \land b) = P(a)P(b)$$
> since $P(b) = P(b|a)$ if a and b are independent.

Thus, we can derive Baye's rule, which relates the probability of one event on the condition of another event, to the reverse relationship.

> **Definition 2.3.2** (Baye's Rule)**.**
> $$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

What Baye's rule allows us to do is that given:

$$P(\text{visible effect} \mid \text{unknown cause})$$

we can calculate

$$P(\text{unknown cause} \mid \text{visible effect}).$$

2.4 Bayesian Networks

There are a number of different probabilistic models. The first we discuss are **Bayesian Networks**.

Definition 2.4.1 (Bayesian Network). A data structure that represents the dependencies among random variables. They have the following characteristics:

- directed graph
- each node represents a random variable
- arrow from X to Y means X is a parent of Y
- each node X has probability distribution $P(X|\text{Parents}(X))$

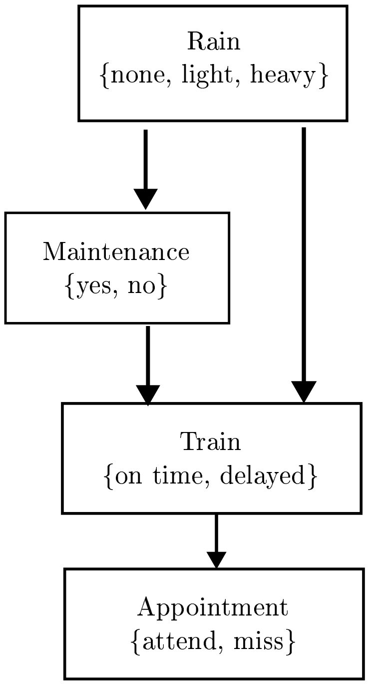

Figure 2.1: A basic example of a Bayesian Network.

We now aim to make an *inference* using the Bayesian Network. Given the following:

- Query X: variable for which to compute distribution
- Evidence variables E: observed variables for event e
- Hidden variables Y: non-evidence, non-query variable

our goal is to calculate $P(X|e)$.

Problem 2.4.1. Calculate $P(\text{Appointment}|\text{light, no})$

Answer. Here, the *evidence* is that there is light rain and no maintenance. The *query* is the status of Appointment. The *hidden layer* is the status of the train, since you are not given the train's status and you are not querying it; it's just a confounding variable.

We note that

$$P(\text{Appointment}|\text{light, no}) = \alpha P(\text{Appointment, light, no})$$

and by the marginalization technique:

$$= \alpha[P(\text{Appointment, light, no, on time}) + P(\text{Appointment, light, no, delayed})].$$

⊛

Remark. The **marginalization technique** essentially states that

$$\alpha P(Q, E) = \alpha \left[P(Q, E, Y) + P(Q, E, \neg Y) \right]$$

2.5 Inference by Enumeration

The above is an example of **inference by enumeration**. More formally, for the following:

- X: the query variable
- e: the evidence
- y: ranges over values of hidden variables
- α: normalizes the result

we have:

$$P(X|e) = \alpha P(X, e) = \alpha \sum_y P(X, e, y)$$

2.6 Sampling

Rather than attempting to calculate an exact probability, we can approximate a probability instead via **sampling**. By randomly generating samples for $n = 1000$ or $n = 10000$, we can get fairly useful results.

Definition 2.6.1 (Rejection Sampling). **Rejection sampling** is the process of simulating numerous examples from a distribution while considering only the samples that possess the attributes of the desired query.

Remark. Rejection sampling is not particularly effective when the evidence you are looking for is fairly unlikely, since you are rejecting a lot of samples. This is inefficient since you are throwing away a large portion of your samples.

Definition 2.6.2 (Likelihood Weighting). Rather than sampling everything, in **likelihood weighting**, we start by *fixing* the values for evidence variables. We then sample the non-evidence variables using conditional probabilities in the Bayesian Network. Finally we weight each sample by its *likelihood*.

2.7 Markov Models

As opposed to assigning one random variable to a value, but rather an array of random variables to a value over a timescale. This leads to a lot more data, and so we must make some assumptions.

Definition 2.7.1 (Markov Assumption). The assumption that the current state depends only on a finite fixed number of previous states.

Definition 2.7.2 (Markov Chain). A sequence of random variables where the distribution of each variable follows the Markov assumption.

For example, consider the basic Markov Chain

$$X_t \to X_{t+1}$$

where X_t represents the weather today, and X_{t+1} represents the weather tomorrow. We can construct a **transition model** to describe the relationship as follows:

	X_{t+1}: Sunny	X_{t+1}: Rainy
X_t: Sunny	0.8	0.2
X_t: Rainy	0.3	0.7

The first line, for example, translates to "given today was sunny, tomorrow will be sunny with a probability of 0.8 and rainy with probability 0.2". We can refine our definition of the Markov Model further, by introducing *hidden* states.

Definition 2.7.3 (Hidden Markov Model). A Markov model for a system with hidden states that generate some observed event.

Definition 2.7.4 (Filtering). Given observations from start until now, calculate distribution for current state.

Definition 2.7.5 (Prediction). Given observations from start until now, calculate the distribution for a future state.

Definition 2.7.6 (Smoothing). Given observations from start until now, calculate the distribution for a past state.

Definition 2.7.7 (Most Likely Explanation). Given observations from start until now, calculate most likely sequence of states.

Chapter 3

Optimization

3.1 Hill Climbing

The most basic form of optimization we explore is that of the local search.

> **Definition 3.1.1** (Local Search). Search algorithms that maintain a single node and searches by moving to a neighboring node.

We can implement a local searching using **hill climbing** in which we do the following:

- Check values of neighbors of current best value
- If a neighbor has a value closer to the desired value, set that value as current best

Algorithm 3.1: HillClimb

```
Data: problem
cur = initial state of problem
repeat
    neighbor = best val neighbor of cur
    if neighbor not better than current then
        return cur
    cur = neighbor
until better neighbor does not exist
```

The problem with this naive approach (**steepest ascent**) is that we may get stuck at a local extremum rather than the global extremum. Further, we may encounter a "shoulder", where several neighboring values are the same, and get stuck.

3.2 Simulated Annealing

Although there are a variety of hill climbing algorithms (stochastic, first-choice, beam, etc.). They have their flaws but the theme is that we never go from a good value to a worse value. Thus, we must overcome local extrema. We can tackle this with **simulated annealing**.

> **Definition 3.2.1** (Simulated Annealing). Simulated annealing is akin to a "cooling" physical process.
>
> - Early on, higher "temperature": more likely to accept neighbors that are worse than current state
> - Later on, lower "temperature": less likely to accept neighbors that are worse than current state

Algorithm 3.2: SimulatedAnnealing

```
Data: problem, max
cur = initial state of problem
for t = 1 to max do
    T = Temperature(t)
    neighbor = random neighbor of cur
    ΔE = how much better neighbor is than cur
    if ΔE > 0 then
        cur = neighbor
    with probability e^(ΔE/T), set cur = neighbor
```

3.3 Linear Programming

In the context where we are trying to optimize for some function or when we have real values we are often trying to minimize or maximize a cost function given a variety of constraints. This is where **linear programming** becomes useful.

Definition 3.3.1 (Linear Programming). Linear programming problems often entail the following:

- Minimize a cost function $c_1x_1 + c_2x_2 + \ldots + c_nx_n$
- With constraints of form $a_1x_1 + a_2x_2 + \ldots + a_nx_n \leq b$ or of form $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- With bounds for each variable $l_i \leq x_i \leq u_i$

Problem 3.3.1. Two machines X_1 and X_2 which cost \$50/hr and \$80/hr to run, respectively. We have the following constraints:

- X_1 requires 5 units of labor, X_2 requires 2 units of labor per hour; we have total of 20 units of labor to spend; goal is to minimize cost
- X_1 produces 10 units of output per hour, X_2 produces 12 units of output per hour; company needs 90 units of output

Answer. We can create a cost function:

$$50x_1 + 80x_2$$

and our constraints:

$$5x_1 + 2x_2 \leq 20$$
$$10x_1 + 12x_2 \geq 90$$

or equivalently:

$$-10x_1 + -12x_2 \leq -90$$

and then solve using a standard linear programming technique such as Simplex, Interior-Point, etc.

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3.4 Constraint Satisfaction

Constraint satisfaction problems often have some number of variables that must be optimized, but they are subject to some constraints.

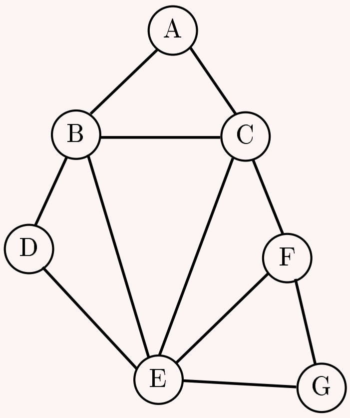
Definition 3.4.1 (Constraint Satisfaction Problem). A general constraint satisfaction problem consists of the following:

- Set of variables $\{x_1, \ldots, x_n\}$
- Set of domains for each variable $\{D_1, \ldots, D_n\}$
- Set of constraints C

Problem 3.4.1. We must schedule exams for classes $A, B, \ldots, G$ such that no conflicts arise for the students taking the courses. The four students' schedules are:

- *Student 1*: A, B, C
- *Student 2*: B, D, E
- *Student 3*: C, E, F
- *Student 4*: E, F, G

We can represent this graphically using an undirected graph, where the nodes represent exams and the edges indicate that the two exams *cannot* be scheduled for the same time.



Here we define our variables, domains, and constraints:

- Variables: $\{A, B, C, D, E, F, G\}$
- Domains: $\{Monday, Tuesday, Wednesday\}$
- Constraints: $\{A \neq B, A \neq C, B \neq C, \ldots, E \neq G, F \neq G\}$

Our goal is to find an assignment of a day to each of the classes such that we don't have any conflicts between the classes. In other words, we are aiming for **node consistency**.

Definition 3.4.2 (Node Consistency). When all the values in a variable's domain satisfy the variable's *unary* constraints.

Furthermore, we are seeking **arc consistency**.

Definition 3.4.3. When all the values in a variable's domain satisfy the variable's binary constraints. More formally: to make X arc-consistent with respect to Y, remove elements from X's domain until every choice for X has a possible choice for Y.

Definition 3.4.4 (Unary Contraint). A constraint involving a *single* variable. (e.g.$\{A \neq Wednesday\}$)

Definition 3.4.5 (Binary Constraint). A constraint involving *two* variables. (e.g.$\{A \neq B\}$)

Note that constraints can come of different forms. **Hard** constraints are absolute requirements, whereas **soft** constraints are preferences.

3.5 Arc Consistency

We first aim to define a function that, given some consistency problem *csp*, can make some variable X **arc consistent** with respect to another variable, Y.

Algorithm 3.3: Revise

```
Data: csp, X, Y
revised = false
for x in X.domain do
    if no y in Y.domain satisfies constraint for (X, Y) then
        delete x from X.domain
        revised = true
return revised
```

We can enforce arc consistency across an entire consistency problem:

Algorithm 3.4: AC-3

```
Data: csp
queue = all arcs in csp
while queue not empty do
    (X, Y) = Dequeue(queue)
    if Revise(csp, X, Y) then
        if size of X.domain == 0 then
            return false
        for each Z in X.neighbors - {Y} do
            Enqueue(queue, (Z, X))
return true
```

3.5.1 CSPs as Search Problems

We can reframe CSPs as *search problems*. We do so by defining a CSP as follows:

- initial state: empty assignment (no variables)
- actions: add a $\{variable = value\}$ to assignment
- transition model: shows how adding an assignment changes the assignment
- goal test: check if all variables assigned and constraints all satisfied
- path cost function: all paths have same cost

3.6 Backtracking

We can find a solution to a CSP by simply applying arbitrary assignments to variables one by one until a constraint is broken. Then, we can just backtrack and try another assignment. We eventually find a solution or check every possible assignment.

However, we can be more clever in our approach and apply the idea of **inference** to our approach. We can operate just as before, but when we find ourselves about to backtrack we can instead observe the graph and look for *arc inconsistencies*. Using the information from neighboring nodes, we can deduce what a particular node's possible assignments may be. Thus, we can backtrack less than we originally did.

Algorithm 3.5: Backtrack

```
Data: assignment, csp
if assignmennt complete then
    return assignment
var = SelectUnassignedVar(assignment, csp)
for value in DomainValues(var, assignment, csp) do
    if value consistent with assignment then
        add {var = value} to assignment
        inferences = Inference(assignment, csp) // get inferences
        if inferences ≠ failure then
            add inferences to assignment // add inferenced nodes
        result = Backtrack(assignment, csp)
        if result ≠ failure then
            return result
    remove {var = value} and inferences from assignment
return failure
```

We note that we can make the search process more efficient by being smarter about which variable we select in our `SelectUnassignedVar` function. We use various heuristics:

- Minimum Remaining Values (MRV) heuristic: select the variable that has the smallest domain
- Degree heuristic: select the variable that has the highest degree; works because the highest degree variable has the most constraints, removing it is most helpful

Additionally, we can refine our `DomainValues` function further. We utilize another set of heuristics:

- Least-constraining values heuristic: return variables in order by number of choices that are ruled out for neighboring variables; try least-constraining values first

Appendix