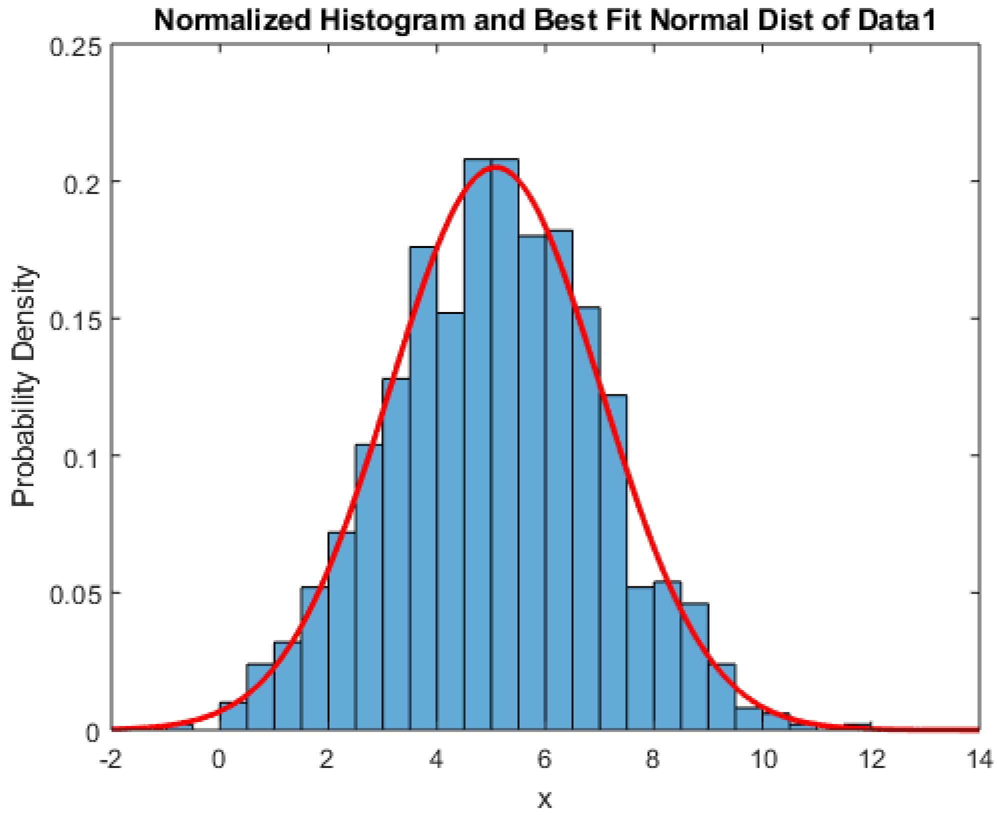
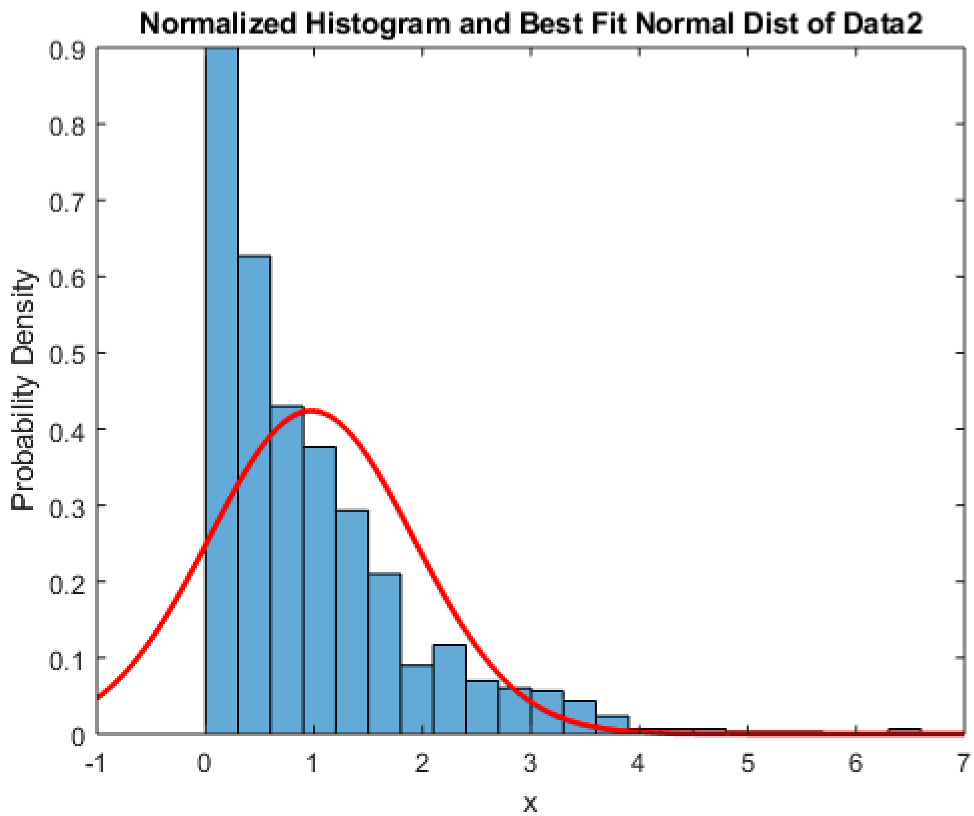
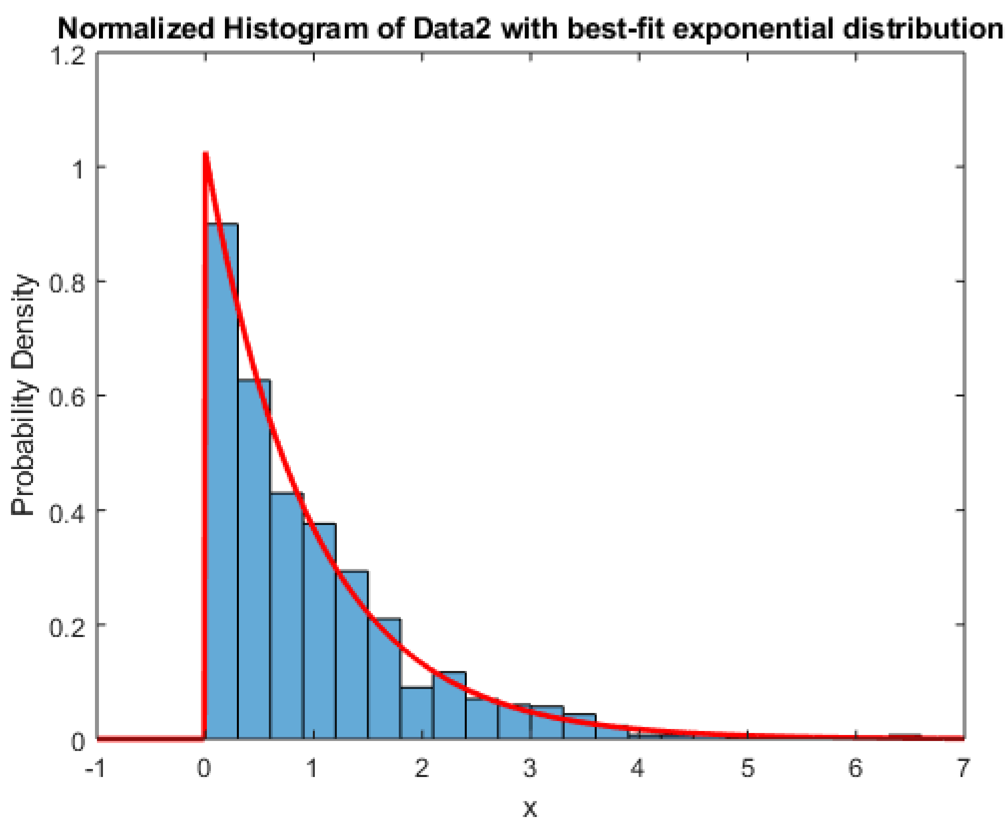
```
% Author: Noah Rondeau
% ECE 485: Data Analysis and Pattern Recognition
% Assignment 2
% Question 1
clc;
clear all;

fprintf('ECE 485: Data Analysis and Pattern Recognition\n');
fprintf('Author: Noah Rondeau\n');
fprintf('Assignment 2, Question 1\n\n');

% Load data from files into column vectors
load Data/Data1;
load Data/Data2;

Data1 = Data1';
Data2 = Data2';

% ========= PART A and B ==========

% calculate best fit gaussians
x1 = -2:0.01:14;
x2 = -1:0.01:7;
Data1_dist = fitdist(Data1, 'Normal');
Data2_dist = fitdist(Data2, 'Normal');
pdf1 = pdf(Data1_dist, x1);
pdf2 = pdf(Data2_dist, x2);

% print message
fprintf('PART A and B: plot histograms with best fit gaussian\n\n');

% Plot Histograms for each data set
figure(1);
histogram(Data1,'Normalization', 'pdf');
line(x1, pdf1, 'Color', 'red', 'Linewidth', 2);
title('Normalized Histogram and Best Fit Normal Dist of Data1');
xlabel('x');
ylabel('Probability Density');
figure(2);
histogram(Data2, 'Normalization', 'pdf');
line(x2, pdf2, 'Color', 'red', 'Linewidth', 2);
title('Normalized Histogram and Best Fit Normal Dist of Data2');
xlabel('x');
ylabel('Probability Density');

% PART C: Chi-square testing
fprintf('PART C: Chi-square testing\n\n');
% default is 5% confidence level
[h1, p1] = chi2gof(Data1);
[h2, p2] = chi2gof(Data2);

fprintf('Chi-square results for Data1:\n');
```

```matlab
fprintf('h = %d, p = %f\n', h1, p1);
fprintf('Chi-square results for Data2:\n');
fprintf('h = %d, p = %f\n', h2, p2);

fprintf('\nChi-square test does not reject null hypothesis for Data1
 at alpha = 0.05.\n');
fprintf('Chi-square test rejects null hypothesis for Data2 at alpha =
 0.05.\n');

% PART D

fprintf('\nPART D: Re-test Data 2 with exponential distribution');
fprintf('Based on the shape of the distribution, try again with an
 exponential distribution\n');

Data2_dist2 = fitdist(Data2, 'Exponential');
pdf3 = pdf(Data2_dist2, x2);
[h3, p3] = chi2gof(Data2, 'CDF', Data2_dist2);

figure(3);
histogram(Data2, 'Normalization', 'pdf');
line(x2, pdf3, 'Color', 'red', 'Linewidth', 2);
title('Normalized Histogram of Data2 with best-fit exponential
 distribution');
xlabel('x');
ylabel('Probability Density');


fprintf('Chi-square results for Data2 with an Exponential Dist:\n');
fprintf('h = %d, p = %f\n', h3, p3);
fprintf('Since h = 0, the null hypothesis is not rejected\n\n');
```

```
ECE 485: Data Analysis and Pattern Recognition
Author: Noah Rondeau
Assignment 2, Question 1

PART A and B: plot histograms with best fit gaussian

PART C: Chi-square testing

Chi-square results for Data1:
h = 0, p = 0.330722
Chi-square results for Data2:
h = 1, p = 0.000000

Chi-square test does not reject null hypothesis for Data1 at alpha =
 0.05.
Chi-square test rejects null hypothesis for Data2 at alpha = 0.05.

PART D: Re-test Data 2 with exponential distributionBased on the shape
 of the distribution, try again with an exponential distribution
Chi-square results for Data2 with an Exponential Dist:
h = 0, p = 0.130439
Since h = 0, the null hypothesis is not rejected
```


Normalized Histogram and Best Fit Normal Dist of Data1
Probability Density
0.25
0.2
0.15
0.1
0.05
0
-2
0
2
4
6
8
10
12
14
x


Normalized Histogram and Best Fit Normal Dist of Data2
Probability Density
x
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
0
1
2
3
4
5
6
7


Normalized Histogram of Data2 with best-fit exponential distribution
Probability Density
x
1.2
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
7

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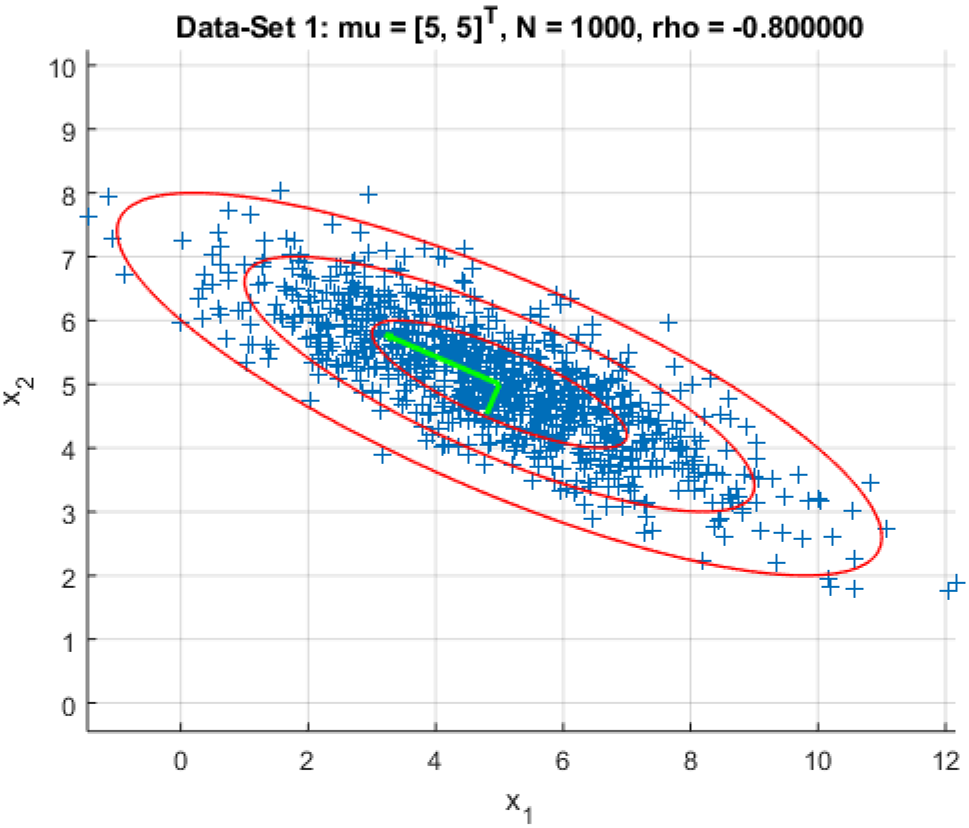
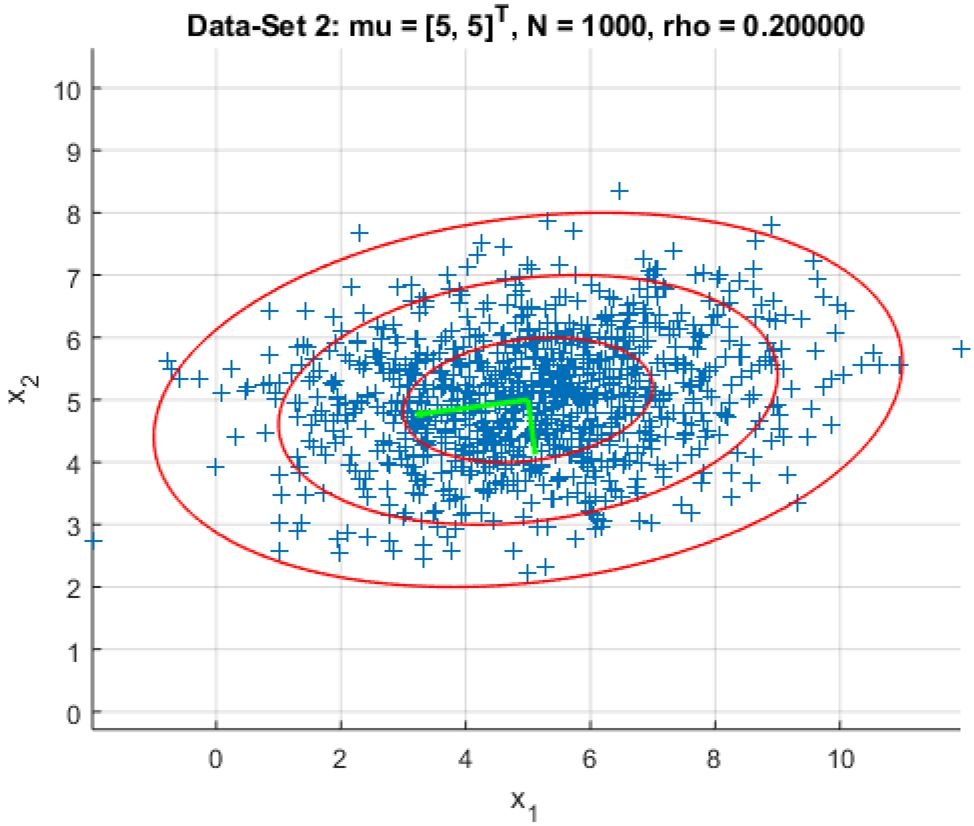
```
% Author: Noah Rondeau
% ECE 485: Data Analysis and Pattern Recognition
% Assignment 2
% Question 2
clc;
clear all;
close all;

fprintf('ECE 485: Data Analysis and Pattern Recognition\n');
fprintf('Author: Noah Rondeau\n');
fprintf('Assignment 2, Question 2\n\n');

fprintf('PART A: Generate 3 data sets\n');
fprintf('PART B: Plot the data sets as scatter plots\n');
fprintf('PART C: Overlay eigenvectors of each data set and 1-, 2-, and
 3-sigma contours\n');
fprintf('For this question, each of A, B, and C will be done on one
 set of data at a time\n\n');

mu = [5 5]';
sigma1 = 2;
sigma2 = 1;
N = 1000;
rho = [-0.8 0.2 0.9];

for i=1:3

    fprintf('Data set %i: mean = [5, 5]^T, N = 1000, rho = %d\n\n',
 i,rho(i));
    %calculate covariance matrix
    Sigma = Q2_CovFrom(sigma1, sigma2, rho(i));
    %generate random data from distribution
    Data1 = Q2_RandomGenerate(N, mu, Sigma);

    figure(i);
    hold on;
    scatter(Data1(:,1), Data1(:,2), '+');
    Q2_PlotEllipse(mu, Sigma, 1);
    Q2_PlotEllipse(mu, Sigma, 2);
    Q2_PlotEllipse(mu, Sigma, 3);
    Q2_PlotEigen(mu, Sigma, 1);
    t = sprintf('Data-Set %d: mu = [5, 5]^T, N = 1000, rho = %f', i,
 rho(i));
    title(t);
    xlabel('x_1');
    ylabel('x_2');
    axis equal;
    grid on;
    hold off;
end
```

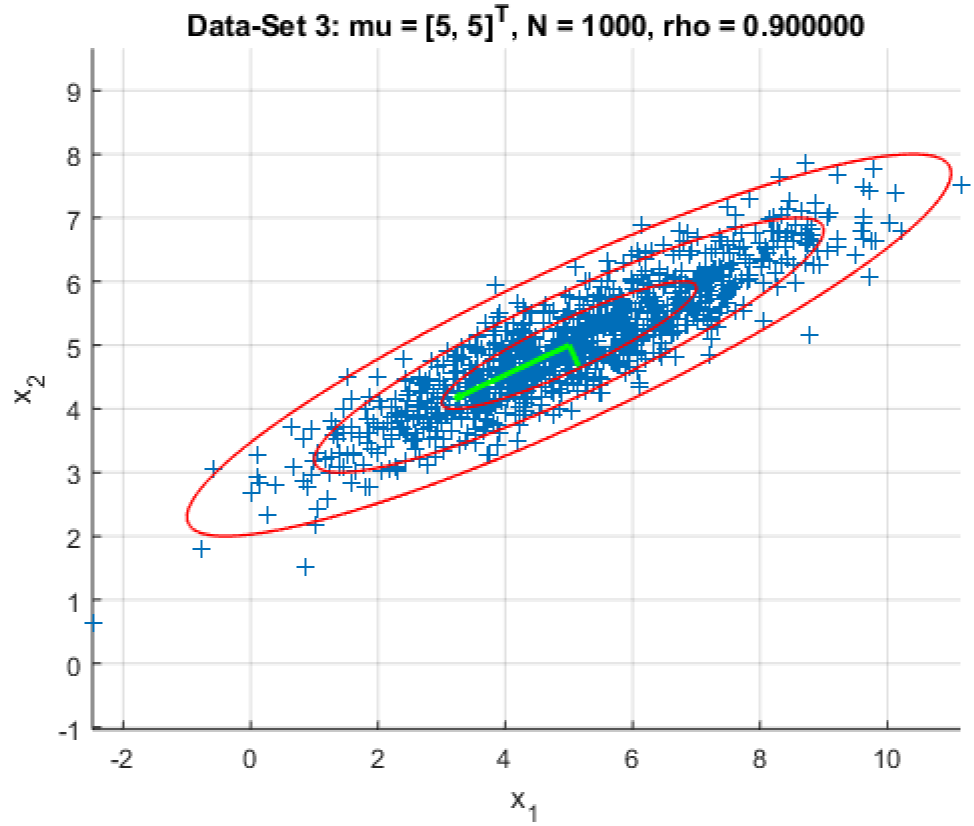
ECE 485: Data Analysis and Pattern Recognition

```
Author: Noah Rondeau
Assignment 2, Question 2

PART A: Generate 3 data sets
PART B: Plot the data sets as scatter plots
PART C: Overlay eigenvectors of each data set and 1-, 2-, and 3-sigma
 contours
For this question, each of A, B, and C will be done on one set of data
 at a time

Data set 1: mean = [5, 5]^T, N = 1000, rho = -8.000000e-01

Data set 2: mean = [5, 5]^T, N = 1000, rho = 2.000000e-01

Data set 3: mean = [5, 5]^T, N = 1000, rho = 9.000000e-01
```





Data-Set 2: mu = [5, 5]^T, N = 1000, rho = 0.200000
x$_2$
x$_1$



Data-Set 3: mu = [5, 5]^T, N = 1000, rho = 0.900000
x$_2$
x$_1$

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```matlab
% Author: Noah Rondeau
% ECE 485: Data Analysis and Pattern Recognition
% Assignment 2
% Question 2
clc;
clear all;
close all;

fprintf('ECE 485: Data Analysis and Pattern Recognition\n');
fprintf('Author: Noah Rondeau\n');
fprintf('Assignment 2, Question 3\n\n');

%load data and extract into classes
load Data/Data3;
class1 = Data3( Data3(:,3)==1, 1:2);
class2 = Data3( Data3(:,3)==2, 1:2);
class3 = Data3( Data3(:,3)==3, 1:2);

length_c1 = length(class1);
length_c2 = length(class2);
length_c3 = length(class3);
length_d3 = length(Data3);

prior_c1 = length_c1 / length_d3;
prior_c2 = length_c2 / length_d3;
prior_c3 = length_c3 / length_d3;


mu1 = mean(class1)';
Sigma1 = cov(class1);
mu2 = mean(class2)';
Sigma2 = cov(class2);
mu3 = mean(class3)';
Sigma3 = cov(class3);

fprintf('PART A: estimated class means and covariances\n');
fprintf('Class 1:\n');
disp('Mean:');
disp(mu1);
disp('Covariance:');
disp(Sigma1);
fprintf('Class 2:\n');
disp('Mean:');
disp(mu2);
disp('Covariance:');
disp(Sigma2);
fprintf('Class 3:\n');
disp('Mean:');
disp(mu3);
disp('Covariance:');
disp(Sigma3);
```

```
if prior_c1 == prior_c2 && prior_c2 == prior_c3
    fprintf('Note: all three classes have the same apriori
 probabilities\n');
    fprintf('All three classes have the same number of records in the
 data\n');
end

% PART B
fprintf('\nPART B: Test point classification by Mahalanobis distance
\n');

x1 = [10,2];
x2 = [-3,4];
x3 = [2,2];
x4 = [5, -7];
X = [x1;x2;x3;x4];

% determine mahalanobis distance for each x relative to each class
dist1 = mahal(X, class1);
dist2 = mahal(X, class2);
dist3 = mahal(X, class3);

% determine class for each x
for i=1:4
    fprintf('x_%d Mahalanobis Distances:\n', i);
    fprintf('From:\n\tClass1: %f\n\tClass2: %f\n\tClass3: %f\n',
 dist1(i), dist2(i), dist3(i));
    min_dist = min([dist1(i), dist2(i), dist3(i)]);
    if min_dist == dist1(i)
        x_class(i) = 1;
    elseif min_dist == dist2(i)
        x_class(i) = 2;
    elseif min_dist == dist3(i)
        x_class(i) = 3;
    end
    fprintf('Point belongs to: Class%d\n\n', x_class(i));
end

% Tag each x with its class value and label
X = [X x_class'];
for i=1:4
    data_labels(i,1) = cellstr(sprintf('x%d, (C%d)', i, X(i,3)));
end

% PART D
fprintf('PART D: Supervised vs Unsupersised Learning\n');
fprintf('If we did not know which points belonged to which class, the
 approach above does not work.\n');
fprintf('It becomes a clustering problem, not a classification
 problem.\n');
fprintf('We have not yet gone over these issues in class.\n');
fprintf('But, I could imagine a sort of finite-element analysis which
 would\n');
```

```
fprintf('assign a density to a region based on the number of data
 points it contains.\n');
fprintf('Assuming either:\n\ta) that I know the number of classes the
 data represents, or\n');
fprintf('\tb) that I assume they are distributed according to a
 particular distribution:\n');
fprintf('I would then attempt to fit distributions centered at the
 density peaks (assuming they exist).\n');
fprintf('This would not generally work if two classes overlapped
 significantly\n\n');

%PART E: Get decision boundaries
coeff1 = log(prior_c1 / (2*pi*sqrt(det(Sigma1))));
coeff2 = log(prior_c2 / (2*pi*sqrt(det(Sigma2))));
coeff3 = log(prior_c3 / (2*pi*sqrt(det(Sigma3))));
syms p1(xs1,xs2) p2(xs1, xs2) p3(x21, xs2);
syms b12(xs1) b13(xs1) b23(xs1);
assume(xs1, 'real');
assume(xs2, 'real');
p1(xs1,xs2) = coeff1 - 0.5 * ([xs1 xs2]' - mu1)' * inv(Sigma1) *([xs1
 xs2]' - mu1);
p2(xs1,xs2) = coeff2 - 0.5 * ([xs1 xs2]' - mu2)' * inv(Sigma2) *([xs1
 xs2]' - mu2);
p3(xs1,xs2) = coeff3 - 0.5 * ([xs1 xs2]' - mu3)' * inv(Sigma3) *([xs1
 xs2]' - mu3);
b12 = simplify(p1==p3);
b13 = simplify(p1==p3);
b23 = simplify(p2==p3);

fprintf('PART E: 2-class decision boundaries\n\n');
fprintf('Boundary between classes 1 and 2:\n');
disp(vpa(b12/(1e48), 4));

fprintf('Boundary between classes 1 and 3:\n');
disp(vpa(b13/(1e48), 4));

fprintf('Boundary between classes 2 and 3:\n');
disp(vpa(b23/(1e20), 4));


figure(1);
hold on;
scatter(class1(:,1), class1(:,2), '+b');
scatter(class2(:,1), class2(:,2), 'om');
scatter(class3(:,1), class3(:,2), 'xy');
scatter(X(:,1), X(:,2), '^k', 'HandleVisibility', 'off');
text(X(:,1) + 0.5, X(:,2) + 0.2, data_labels);
Q2_PlotEllipse(mu1, Sigma1, 1, 'on');
Q2_PlotEllipse(mu1, Sigma1, 2, 'off');
Q2_PlotEllipse(mu1, Sigma1, 3, 'off');
Q2_PlotEllipse(mu2, Sigma2, 1, 'off');
Q2_PlotEllipse(mu2, Sigma2, 2, 'off');
Q2_PlotEllipse(mu2, Sigma2, 3, 'off');
Q2_PlotEllipse(mu3, Sigma3, 1, 'off');
```

```
Q2_PlotEllipse(mu3, Sigma3, 2, 'off');
Q2_PlotEllipse(mu3, Sigma3, 3, 'off');
Q2_PlotEigen(mu1, Sigma1, 1, 'on', 'off');
Q2_PlotEigen(mu2, Sigma2, 1, 'off', 'off');
Q2_PlotEigen(mu3, Sigma3, 1, 'off', 'off');

axis equal;
grid on;

% plot where the boundaries are. Do this by creating a mesh grid and
% calculating all three mvnpdf() against it. Then subtract the
 resulting
% matrices to get the subtraction of the two functions of interest.
 Then
% plot the contour where that new matrix is 0.

%use the existing plot bounds
x1_lims = xlim;
x2_lims = ylim;
x1_range = x1_lims:0.1:x1_lims(2);
x2_range = x2_lims(1):0.1:x2_lims(2);
[X1, X2] = meshgrid(x1_range, x2_range);

pdf1 = mvnpdf([X1(:) X2(:)], mu1', Sigma1);
pdf2 = mvnpdf([X1(:) X2(:)], mu2', Sigma2);
pdf3 = mvnpdf([X1(:) X2(:)], mu3', Sigma3);

bound12 = pdf1 - pdf2;
bound12 = reshape(bound12, length(x2_range), length(x1_range));
bound13 = pdf1 - pdf3;
bound13 = reshape(bound13, length(x2_range), length(x1_range));
bound23 = pdf2 - pdf3;
bound23 = reshape(bound23, length(x2_range), length(x1_range));

contour(X1, X2, bound12, [0 0], '-k', 'LineWidth', 1);
contour(X1, X2, bound13, [0 0], ':k', 'LineWidth', 1);
contour(X1, X2, bound23, [0 0], '--k', 'LineWidth', 1);

title({'Data with 1,2,3-\sigma contours,','eigenvectors, and decision
 boundaries'});
xlabel('x_1');
ylabel('x_2');

legend('Class1', 'Class2', 'Class3', ...
       '1,2,3-\sigma ellipses',...
       'Eigenvectors', ...
       'Decision bound 1-2', ...
       'Decision bound 1-3', ...
       'Decision bound 2-3', ...
       'Location', 'bestoutside' );

hold off;
```

```
Author: Noah Rondeau
Assignment 2, Question 3

PART A: estimated class means and covariances
Class 1:
Mean:
    0.0355
    7.4948

Covariance:
    2.2196   -1.9762
   -1.9762    2.2166

Class 2:
Mean:
   -0.9853
   -1.0147

Covariance:
    1.5050    3.0342
    3.0342    7.6511

Class 3:
Mean:
    2.0016
   -2.9851

Covariance:
    1.0089    0.1126
    0.1126    0.7451

Note: all three classes have the same apriori probabilities
All three classes have the same number of records in the data

PART B: Test point classification by Mahalanobis distance
x_1 Mahalanobis Distances:
From:
 Class1: 69.688371
 Class2: 318.835858
 Class3: 86.275973
Point belongs to: Class1

x_2 Mahalanobis Distances:
From:
 Class1: 88.183168
 Class2: 56.407509
 Class3: 102.463145
Point belongs to: Class2

x_3 Mahalanobis Distances:
From:
 Class1: 32.434386
 Class2: 11.804292
 Class3: 33.924289
```

Point belongs to: Class2

x_4 Mahalanobis Distances:
From:
Class1: 233.164291
Class2: 236.259443
Class3: 34.734344
Point belongs to: Class3

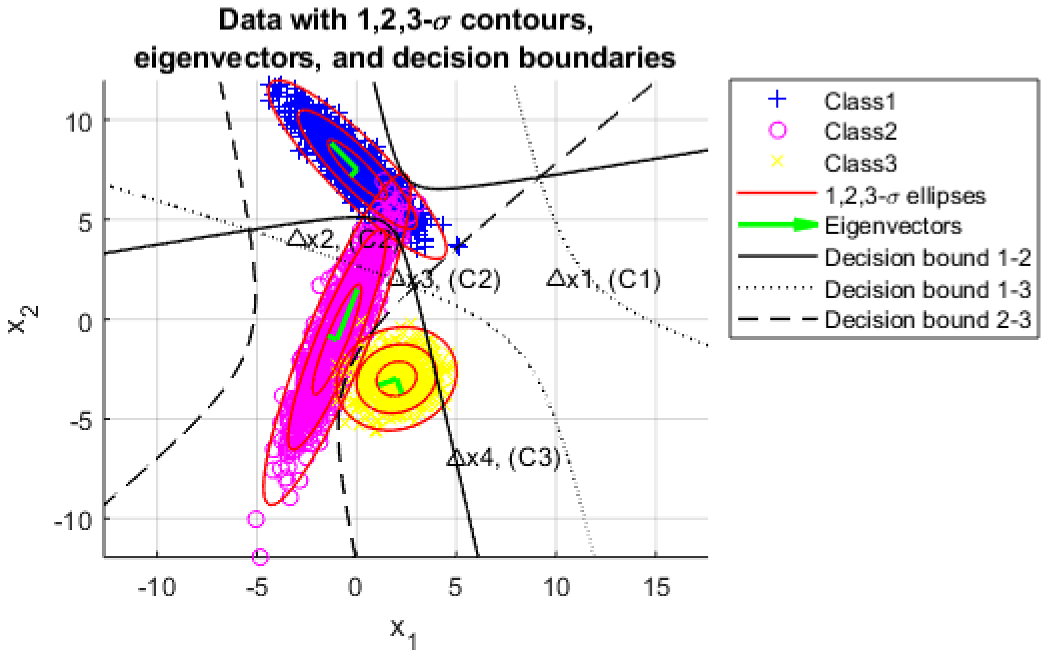
PART D: Supervised vs Unsupersised Learning
If we did not know which points belonged to which class, the approach above does not work.
It becomes a clustering problem, not a classification problem.
We have not yet gone over these issues in class.
But, I could imagine a sort of finite-element analysis which would assign a density to a region based on the number of data points it contains.
Assuming either:
a) that I know the number of classes the data represents, or
b) that I assume they are distributed according to a particular distribution:
I would then attempt to fit distributions centered at the density peaks (assuming they exist).
This would not generally work if two classes overlapped significantly

PART E: 2-class decision boundaries

Boundary between classes 1 and 2:
*0.4299*xs1^2 + 1.535*xs1*xs2 + 0.3006*xs2^2 + 38.81 == 8.918*xs1 + 15.23*xs2*
symbolic function inputs: xs1, xs2

Boundary between classes 1 and 3:
*0.4299*xs1^2 + 1.535*xs1*xs2 + 0.3006*xs2^2 + 38.81 == 8.918*xs1 + 15.23*xs2*
symbolic function inputs: xs1, xs2

Boundary between classes 2 and 3:
*4.504e-5*xs1*(9.355e31*xs1 + 3.573e32) == 1.832e28*xs2 + 4.246e27*xs1*xs2 + 1.303e27*xs2^2 + 2.854e28*
symbolic function inputs: xs1, xs2



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```
function SIGMA = Q2_CovFrom(sigma1, sigma2, rho)
%Q2_CovFrom()
%   Creates a 2x2 covariance matrix for bivariate normal distribution
 based
%   on the x1 and x2 standard deviations and the correlation
 coefficient

sigma12 = sigma1*sigma2*rho;
SIGMA = [(sigma1^2) sigma12; sigma12 (sigma2^2)];

end
```

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```
function Q2_PlotEigen(mu, Sigma, scale, vis_1, vis_2)
%Q2_PlotEigen plots eigenvectors of covariance matrix centered on mean
%   mu : mean, Sigma: covariance, scale: how many sigmas
%   vis_1, vis_2: display in plot legend or not: either 'on' or 'off'
[V, D] = eig(Sigma);
V = V*sqrt(D)*scale;
quiver(mu(1), mu(2), V(1,1), V(2,1), 'LineWidth',
 2, 'Color', 'g', 'HandleVisibility', vis_1);
quiver(mu(1), mu(2), V(1,2), V(2,2), 'LineWidth',
 2, 'Color', 'g', 'HandleVisibility', vis_2);
end
```

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```
function Q2_PlotEllipse(mu, Sigma, scale, visibility)
%Q2_PlotEllipses Plot a contour ellipse at scale*stdev centered on
 mean
%   mu: mean
%   Sigma: covariance
%   scale: how many standard deviations
%   visibility: whether in plot legend or not, 'on' or 'off'

t = linspace(0, 2*pi, 1000);
[Eigvec, Eigval] = eig(Sigma);
V = Eigvec*sqrt(Eigval)*scale;
e = V*[cos(t); sin(t)] + mu;
plot(e(1,:), e(2,:), '-r', 'LineWidth', 1, 'HandleVisibility',
 visibility);
end
```

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```
function dataout = Q2_RandomGenerate(N, mu, sigma)
%UNTITLED Generates N random numbers from multivariate dist
%   This is really just a wrapper function around mvnrand...
dataout = mvnrnd(mu, sigma, N);

end
```

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